



**The Abdus Salam  
International Centre for Theoretical Physics**



**2139-3**

**School on Synchrotron and Free-Electron-Laser Sources and their  
Multidisciplinary Applications**

*26 April - 7 May, 2010*

**Production of synchrotron and  
Production of synchrotron and FEL radiation  
Part 1**

Emanuel Karantzoulis  
*Sincrotrone Trieste, S.C.p.A.*



# Production of synchrotron and FEL radiation

*Part 1*

*April 26, 2010*

*ICTP*

**Emanuel Karantzoulis**

*Sincrotrone Trieste S.C.p.A*

- ❑ Accelerators
- ❑ Electromagnetic radiation
- ❑ What users want
- ❑ Synchrotron radiation (SR) history
- ❑ Characteristics of SR

## Devices that accelerate and/or accumulate beams of particles

It all started with nuclear physics. The pioneers of nuclear and particle physics used naturally-occurring sources for their particle beams: Rutherford discovered the atomic nucleus and then disintegrated the nucleus of nitrogen using alpha particles from naturally occurring radioactive isotopes while many of the early discoveries in particle physics were made using cosmic rays. However, there are problems with both methods. There is an upper energy limit of  $\sim 10 \text{ MeV}$  for alpha particles which is insufficient for them to penetrate the strong electrostatic repulsive barrier around most nuclei. Some cosmic rays have far higher energies than any accelerator of today can produce, but their location and occurrence cannot be predicted, making them unsuitable for systematic studies.

*So the idea that naturally occurred was how one could accelerate particles in a controlled way?*

The answer is YES and are used :

In high energy particle physics to study the internal structure of nuclei and the interactions between elementary particles.

*To see, one needs a probe with even smaller dimensions, particles have a De Broglie wavelength  $\lambda = h/p = hc/E$  (for  $10^{-15} \text{ m}$  one needs practically  $1 \text{ GeV}$ )*

In applied science are used for production of synchrotron radiation, isotopes and beams for medical uses and for research in the physics of matter

Based on

- directing beams to hit specific targets or colliding beams onto each other
- production of thin beams of synchrotron light

Particle physics

- structure of the atom, standard model, quarks, neutrinos, CP violation

Bombardment of targets used to obtain new materials with different chemical, physical and mechanical properties

Synchrotron radiation covers spectroscopy, X-ray diffraction, x-ray microscopy, crystallography of proteins. Techniques used to manufacture products for aeronautics, medicine, pharmacology, steel production, chemical, car, oil and space industries.

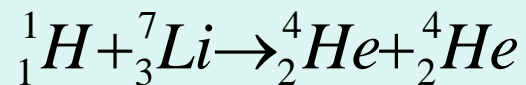
In medicine, beams are used for Positron Emission Tomography (PET), therapy of tumours, and for surgery.

Nuclear waste transmutation – convert long lived nucleides into short-lived waste

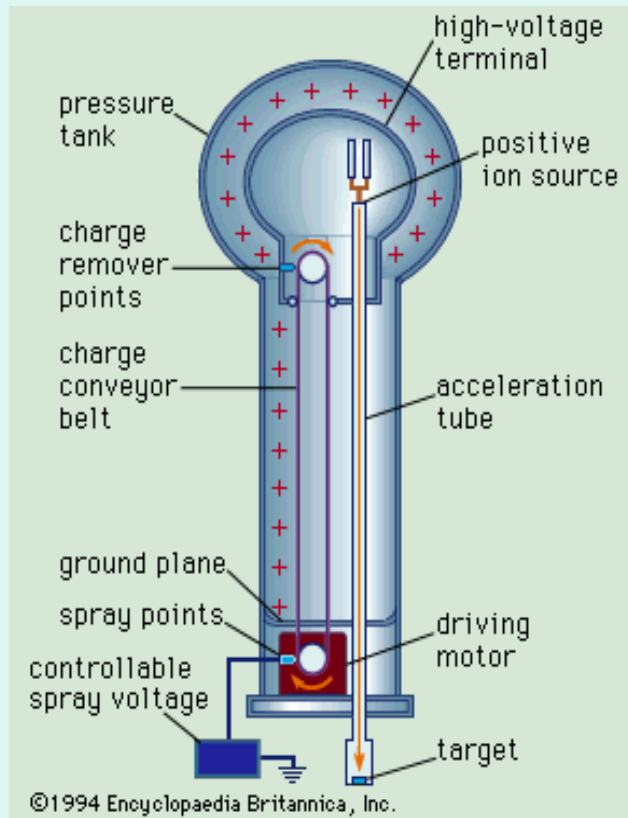
Generation of energy ( energy amplifier, heavy ion driver for fusion)

In the early 1930's, Brasch and Lange worked to use the huge potential differences available in lightning storms for accelerating particles. An insulated wire was hung across a valley in the Swiss Alps, with a conducting terminal suspended from it. Sparks several hundred feet long were seen between this terminal and an earthed one on the valley floor but the project was abandoned when Lange was fatally electrocuted.

The first accelerators were build around 1928 by Van de Graaff and separately by Cockroft and Walton 1930 that succeeded in doing the first nuclear reaction using protons at 600 keV



*This Cockcroft-Walton [voltage multiplier](#) was part of one of the early particle accelerators responsible for development of the [atomic bomb](#). Built in [1937](#) by [Philips](#) of [Eindhoven](#) it currently resides in the [National Science Museum](#) in [London, England](#).*



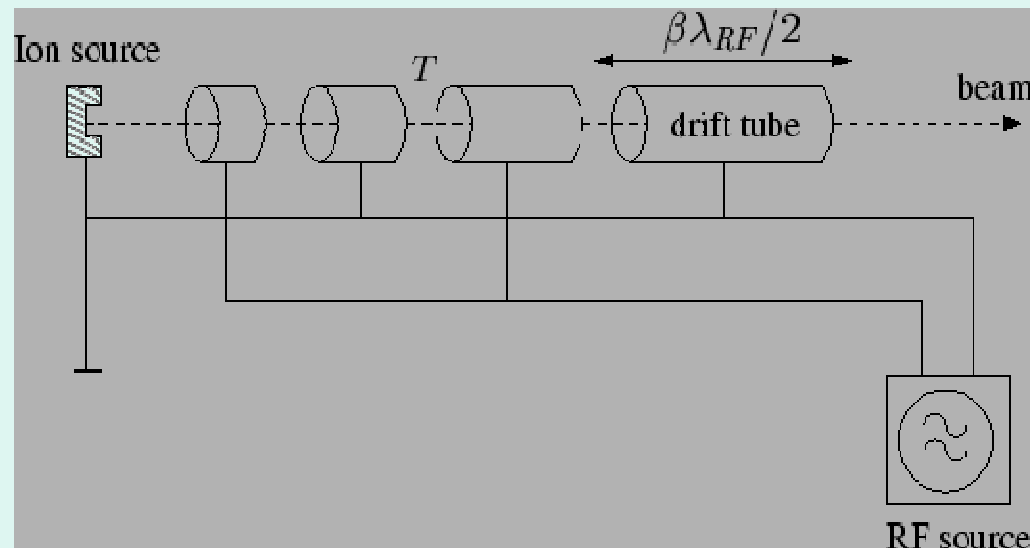
A **Van de Graaff generator** is an electrostatic machine which uses a moving belt to accumulate very high voltages on a hollow metal globe. The potential difference achieved in modern Van de Graaff generators can reach 5 megavolts. Applications for these high voltage generators include driving X-ray tubes, accelerating electrons to sterilize food and process materials, and accelerating protons for nuclear physics experiments. The Van de Graaff generator can be thought of as a constant-current source connected in parallel with a capacitor and a very large electrical resistance.

*The electrostatic accelerators can reach up to 10 MeV*

A limiting factor of direct voltage and cascade accelerators is that they expose the particle to the entire voltage at once and so are limited by the problem of electrical breakdown to a few MeV.

Ising 1924 got the idea to use a series of tubes (drift tubes) and an oscillating electric field between them.

Wideroe 1928 constructed an accelerator based on this idea i.e. the  $E$ -fields are alternating at RF cavities. To maintain the synchronism the tubes length must increase proportional to the velocity of the particle.



- ❑ A series of drift tubes alternately connected to high frequency oscillator.
- ❑ Particles accelerated in gaps, drift inside tubes .
- ❑ For constant frequency generator, drift tubes increase in length as velocity increases.
- ❑ Beam has pulsed structure.



A radio frequency linear accelerator uses a smaller but changing electric field over and over again to increase the energy of the particle. The particles pass through tubes called cavities which are alternately charged by the alternating supply

Ignoring relativity, the energy gained by the particle is equal to  $NqV$  where  $N$  is the number of cavities and  $V$  the maximum voltage of the RF supply. For example, a linac with 50 cavities with a peak voltage supply of 40 *kV* to accelerates to 2.0 *MeV*.

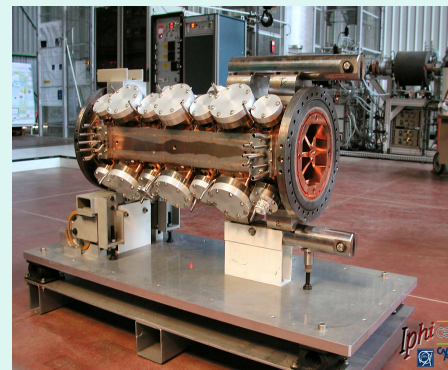
To reach very high energies, a large number of cavities are needed and so the machine becomes very long. The world's largest such machine is SLAC, the Stanford Linear Collider, a 20 *GeV* electron accelerator two miles long.

*Alvarez tanks*  
*Standing and traveling wave linacs*  
*RFQs*  
*Induction linacs*  
*Wake fields accelerators*



Image courtesy Fermilab

- ❑ Travelling wave structure: particles keep in phase with the accelerating waveform.
- ❑ Phase velocity in the waveguide is greater than  $c$  and needs to be reduced to the particle velocity with a series of irises inside the tube whose polarity changes with time.
- ❑ In order to match the phase of the particles with the polarity of the irises, the distance between the irises increases farther down the structure where the particle is moving faster. But note that electrons at 3 MeV are already at  $0.99c$ .

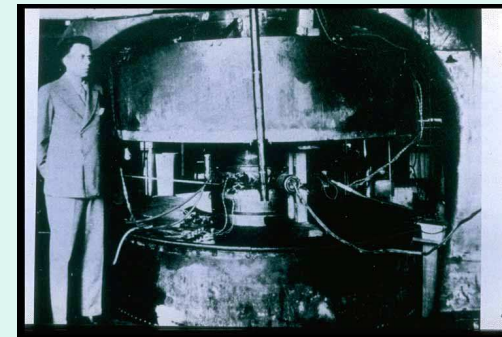


***Linacs:***  
***Avoid expensive magnets***  
***No loss of energy from synchrotron radiation***  
***However requires many structures, limited energy gain/metre***  
***Large energy increase requires a long accelerator***

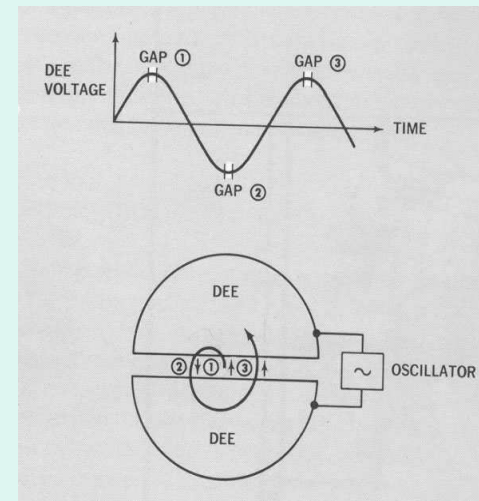
Use magnetic fields to force particles to pass through accelerating fields at regular intervals

## Cyclotrons

- Constant B field
- Constant accelerating frequency  $f$
- Spiral [trajectories](#)
- For synchronism  $f = n\omega$ , which is possible only at low energies,  $\gamma \sim 1$ .
- **Use for heavy particles** (protons, deuterons,  $\alpha$ -particles).
- **But useless for electrons**



George Lawrence and cyclotron

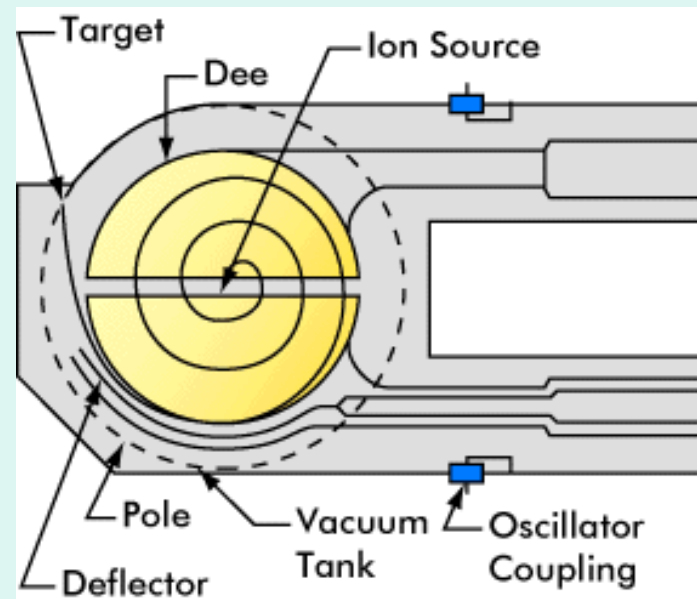
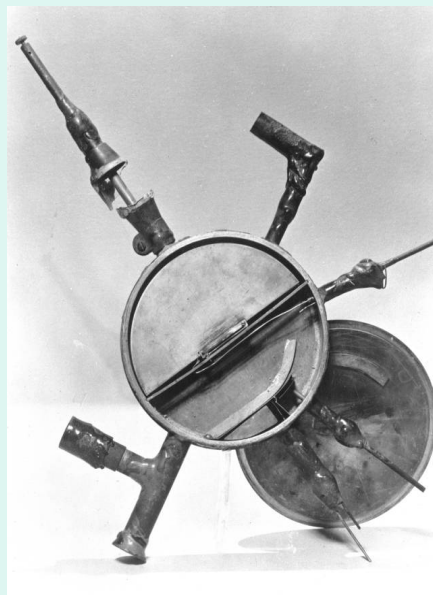


Dimensions: first 80 keV 12 cm , 1 MeV 30 cm

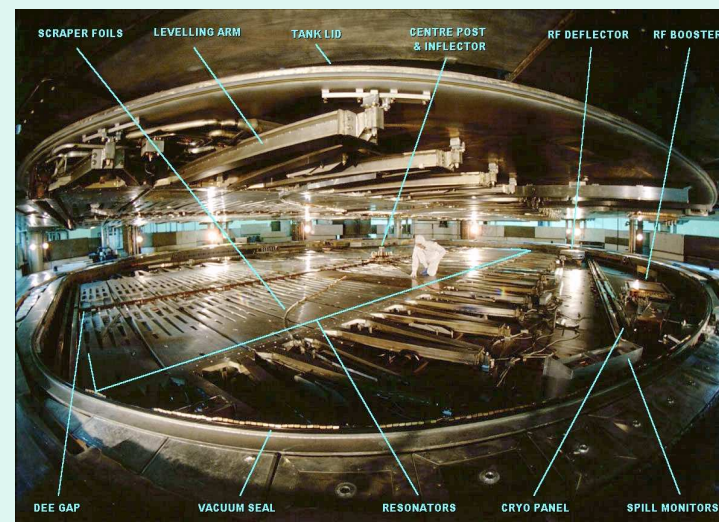
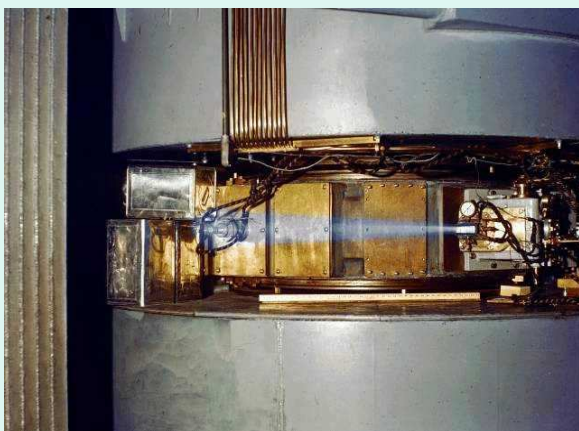
$$\rho = \left| \frac{p}{qB} \right|$$

$$\omega = \frac{qBc^2}{E} = \frac{v}{\rho}$$





Published on the web at [the Lawrence Berkely National Lab](http://www.lbl.gov) website



TRIUMF, 520 MeV, 18 m

Higher energies => relativistic effects =>  $\omega$  no longer constant.  
Particles get out of phase with accelerating fields; eventually no overall acceleration.

### **Isochronous cyclotron**

- Vary  $B$  to compensate and keep  $f$  constant.
- For stable orbits need both radial (because  $\rho$  varies) and azimuthal  $B$ -field variation
- Leads to construction difficulties.

### **Synchro-cyclotron**

- Modulate frequency  $f$  of accelerating structure instead.
- In this case, oscillations are stable (McMillan & Veksler, 1945)

Also called electron cyclotron , invented by Veksler 1944, (small, low cost)

Mainly accelerates electron up to 20 MeV / 100 MeV

Very small dimension

The Ds are replaced by a small cavity driven by a klystron or magnetron

$$\omega_0 = \frac{qB}{m_0} = \omega_{rf}$$

$$\omega_1 = \frac{qB}{2m_0} = \frac{\omega_{rf}}{2}$$

$$\omega_2 = \frac{qB}{3m_0} = \frac{\omega_{rf}}{3}$$

.....

$$\omega_n = \frac{qB}{nm_0} = \frac{\omega_{rf}}{n}$$

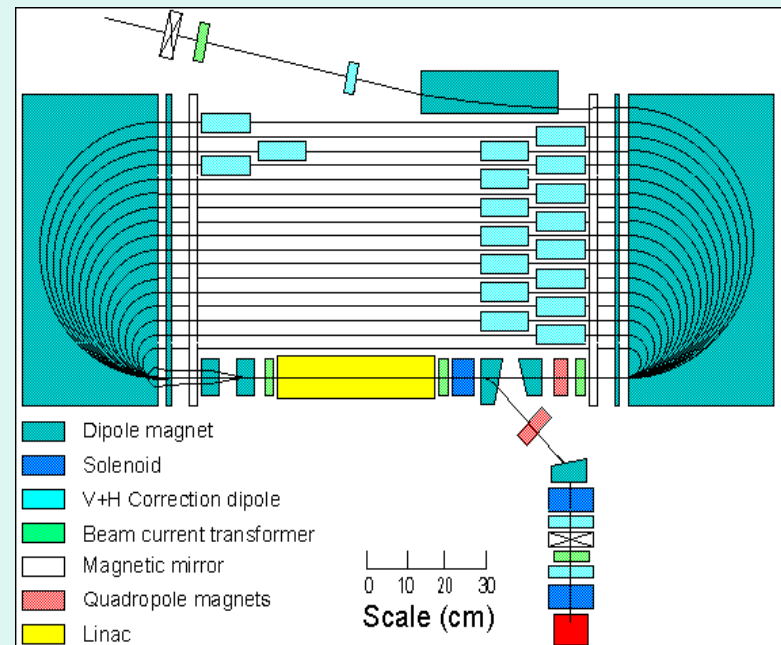
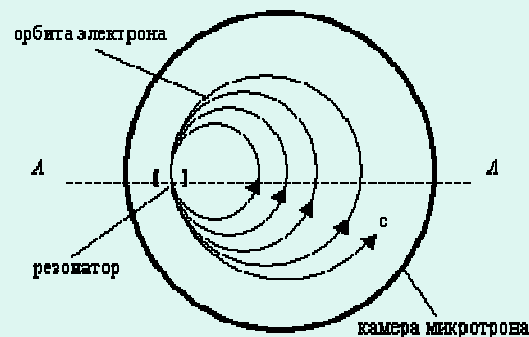
Setting the rf so that at each passage the electron gains energy equal to  $m_0c^2$  (or  $n \cdot m_0c^2$ ) the revolution frequencies remain in resonance with the higher rf – harmonics.



LBNL image library



Veksler



The electron injector for ASTRID



An old idea, dating from 1950's, given a new lease of life with the development of new magnetic alloy cavities.

Field constant in time, varies with radius according to a strict mathematical formula.

Wide aperture magnets and stable orbits.

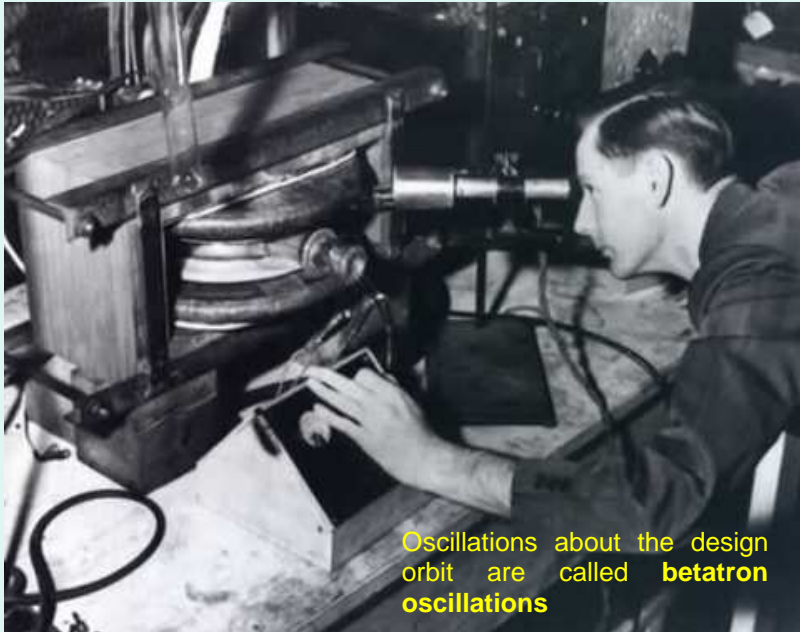
High gradient accelerating cavities combine with fixed field for **rapid acceleration**.

**Good for particles with short half-lives** (e.g. muons).



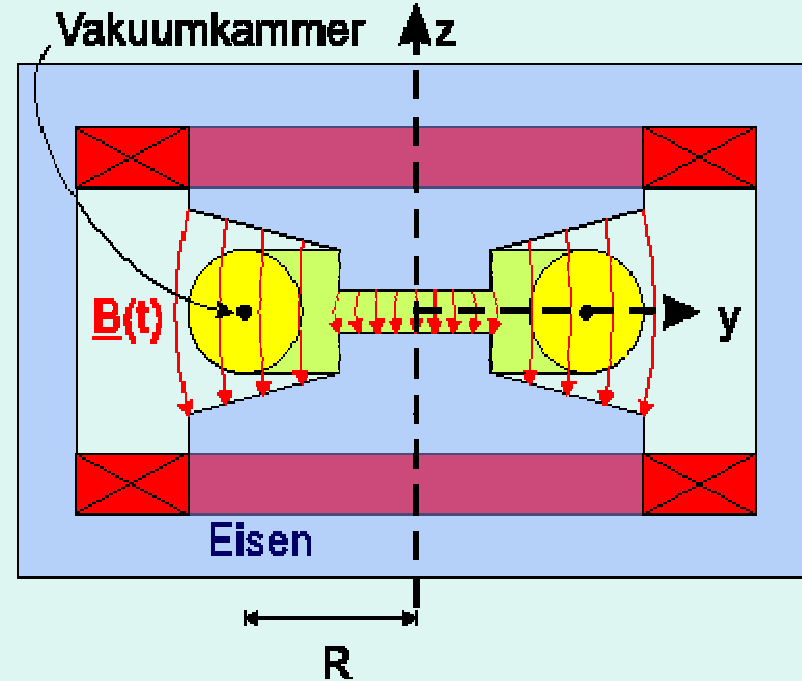
Prototype FFAG, accelerating protons from 50 keV to 500 keV, was successfully built and tested at the KEK laboratory in Japan, 2000.

Best thought of as a transformer, with a ring of electrons as the secondary coil



Oscillations about the design orbit are called **betatron oscillations**

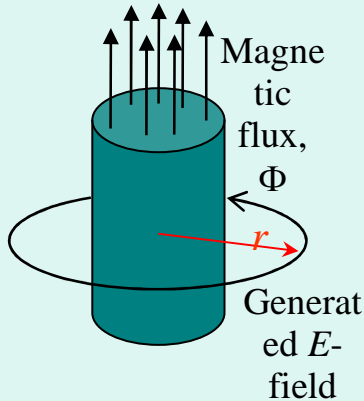
Professor Donald Kerst built the world's first magnetic induction accelerator at the University of Illinois in 1940



$B$ -field on orbit is one half of the average  $B$  over the circle. This imposes a limit on the energy that can be achieved. Nevertheless the constant radius principle is attractive for high energy circular accelerators.

$$B_g = \frac{1}{2} B_a$$

**Max energy 300 MeV** – synchrotron radiation destroys equilibrium



$$\nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Leftrightarrow \oint \vec{E} \cdot d\vec{l} = \left(-\frac{d}{dt} \iint \vec{B} \cdot d\vec{S}\right) = \int_0^{2\pi} E r d\varphi$$

$$\Rightarrow 2\pi r E = -\frac{d\Phi}{dt} \text{ but } \Phi = \pi r^2 B_a \text{ so } E = \frac{\dot{B}_a r}{2}$$



- Variation in time of  $B$ -field to match increase in energy and keep revolution radius constant.
- Particles can stay for a long time in orbit. Beams of particles need to be focused.
- Principle of frequency modulation and phase stability

## Types of synchrotrons

Boosters to accelerate particles

Storage rings: accumulate particles and keep circulating for long periods; used for high intensity beams to inject into more powerful machines or synchrotron radiation sources.

Storage ring can also accelerate particles (Like Elettra)

Colliders: two beams circulating in opposite directions, made to intersect; maximises energy in centre of mass frame.



1947 70 MeV electron synchrotron  
General Electric Co. first observation of  
SR - resulting to Nobel price.

From 1947 to 1964 many such synchrotrons were built including the 1959 Frascati electron synchrotron of 1.2 GeV

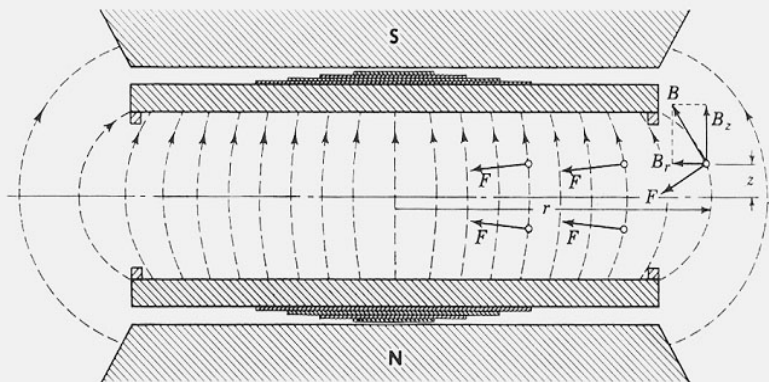


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

Simultaneously horizontal and vertical magnetic focusing.

But bulky magnets scaled with energy!

The Bevatron (6.2GeV) was a weak-focusing synchrotron — at LBNL which began operating in 1954. The **antiproton** was discovered there in 1955, resulting in the 1959 Nobel Price in physics for **Emilio Segrè** and **Owen Chamberlain**. At the time it was built, there was no known way to confine a particle beam to a narrow aperture, so the beam space was about 2 feet by 3 feet in cross section. In order to create anti-protons (mass 938 MeV) in collisions with nucleons in a stationary target while conserving both energy and momentum, a beam proton energy of slightly over 5 GeV is required. The combination of beam aperture and energy required a huge, 10,000 ton iron magnet. It was finally decommissioned in **1994**.

- Magnetic field produced by several bending magnets (*dipoles*), increases linearly with momentum.

$$B\rho = \frac{p}{e} \approx \frac{E}{ce} \text{ so } E[\text{GeV}] \approx 0.3 B[\text{T}] \rho[\text{m}]$$

- Alternating horizontal focusing (vertical defocusing) and vertical focusing (horizontal defocusing) can have a net focusing effect of focusing in both planes.
- Practical limitations for magnetic fields => high energies only at large radius
- e.g. **LHC**     **$E = 8 \text{ TeV}$ ,  $B = 10 \text{ T}$ ,  $r = 2.7 \text{ km}$**
- But **Elettra**     **$E = 2 \text{ GeV}$ ,  $B = 1.2 \text{ T}$ ,  $r = 5.5 \text{ m}$**

Electrostatic and Linear Accelerators (1928)	<i>Electrostatic Accelerators (Van de Graaff, Cockroft and Walton) Resonant accelerators (Wideroe and Lawrence-Sloan)</i>
Cyclotron (1931)	<i>Acceleration by varying electric fields and guiding by magnetic fields (Lawrence)</i>
Betatron (1940)	<i>Guiding and acceleration by magnetic fields (Kerst)</i>
Microtron and Synchrotron (1944)	<i>Invented to overcome the problems of cyclotron (relativistic limit) (Veksler and McMillan )</i>
Strong focusing Synchrotron (1950)	<i>Alternating gradient concept (N. Christofilos from his home in Greece 1950 and Snyder, Courant, Livingston -Brookhaven lab 1952)</i>
Large Accelerators (1980 )	
Synchrotron light sources (3 generations) (1980 – 2000)	
Free Electron lasers (2000)	

<b>Machine</b>	<b>RF frequency <math>f</math></b>	<b>Magnetic Field <math>B</math></b>	<b>Orbit Radius <math>\rho</math></b>	<b>Comment</b>
Cyclotron	constant	constant	increases with energy	Particles out of synch with RF; low energy beam or heavy ions
Isochronous Cyclotron	constant	varies	increases with energy	Particles in synch, but difficult to create stable orbits
Synchro-cyclotron	varies	constant	increases with energy	Stable oscillations
Synchrotron	varies	varies	constant	Flexible machine, high energies possible
FFAG	varies	constant in time, varies with radius	increases with energy	Increasingly attraction option for 21 <sup>st</sup> century designs

1840 Laws of Coulomb (Gauss), Faraday and Ampere worked quite well but Ampere's did not conserve charge...

$$\nabla E = 4\pi\rho \quad \nabla B = 0 \quad \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$

$$\nabla \times B = \frac{4\pi}{c} J \rightarrow \begin{cases} \nabla(\nabla \times B) = 0 = \frac{4\pi}{c} \nabla J \\ \nabla J + \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial \rho}{\partial t} = 0 \end{cases}$$

Maxwell solved the problem by introducing a term to enlarge the validity of Ampere's law. This led to enormous consequences for Physics:

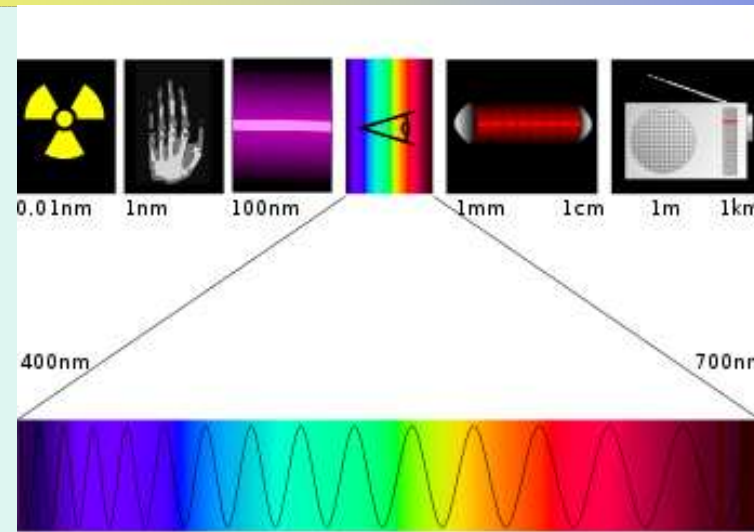
- **Set of equations describing magnetism and electricity**
- **Existence of electromagnetic field and unification with light**
- **Relativity**

$$J \rightarrow J + \frac{1}{4\pi} \frac{\partial E}{\partial t}$$

Displacement  $J_d$

$$c = 1 / \sqrt{\mu_0 \epsilon_0}$$

Thus a spatially-varying electric field generates a time-varying magnetic field and *vice versa*. Therefore, as an oscillating electric field generates an oscillating magnetic field, the magnetic field in turn generates an oscillating electric field, and so on. These oscillating fields together form a self-sustaining electromagnetic wave i.e. **Radiation**



The fields in Maxwell's equations are generated by charges and currents. Conversely, the charges and currents are affected by the fields through the Lorentz force equation:

$$\frac{d\vec{p}}{dt} = e(\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

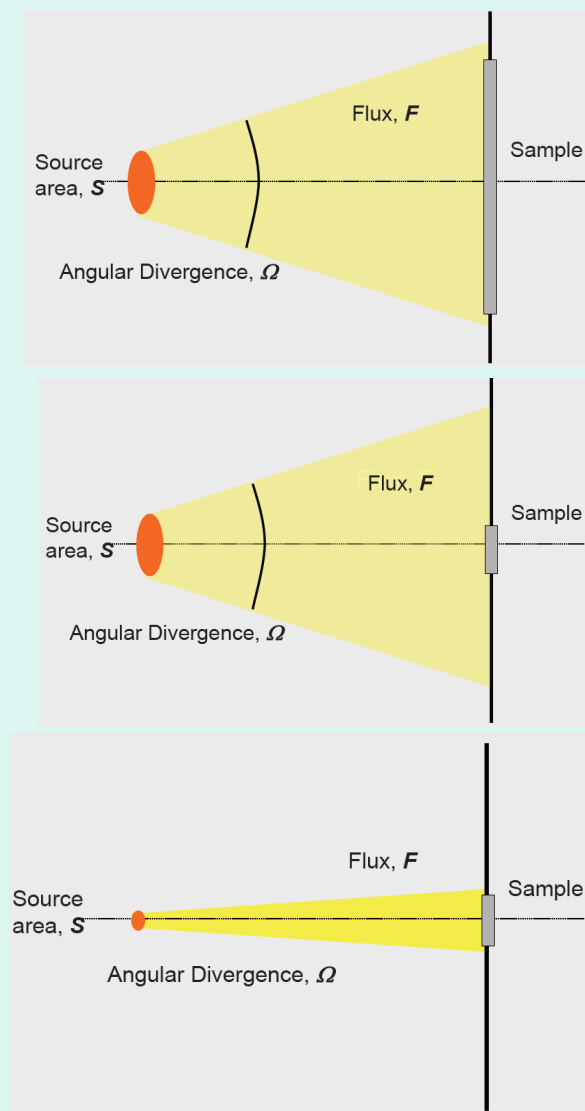
Maxwell and Lorentz fully describe the precise dynamics of the system



# ***LIGHT CHARACTERISTICS REQUESTED FROM EXPERIMENTS***

- ***HIGH BRILLIANCE***
- ***COHERENCE-Transverse & Longitudinal coherence***
- ***POLARISATION***
- ***SHORT PULSES***

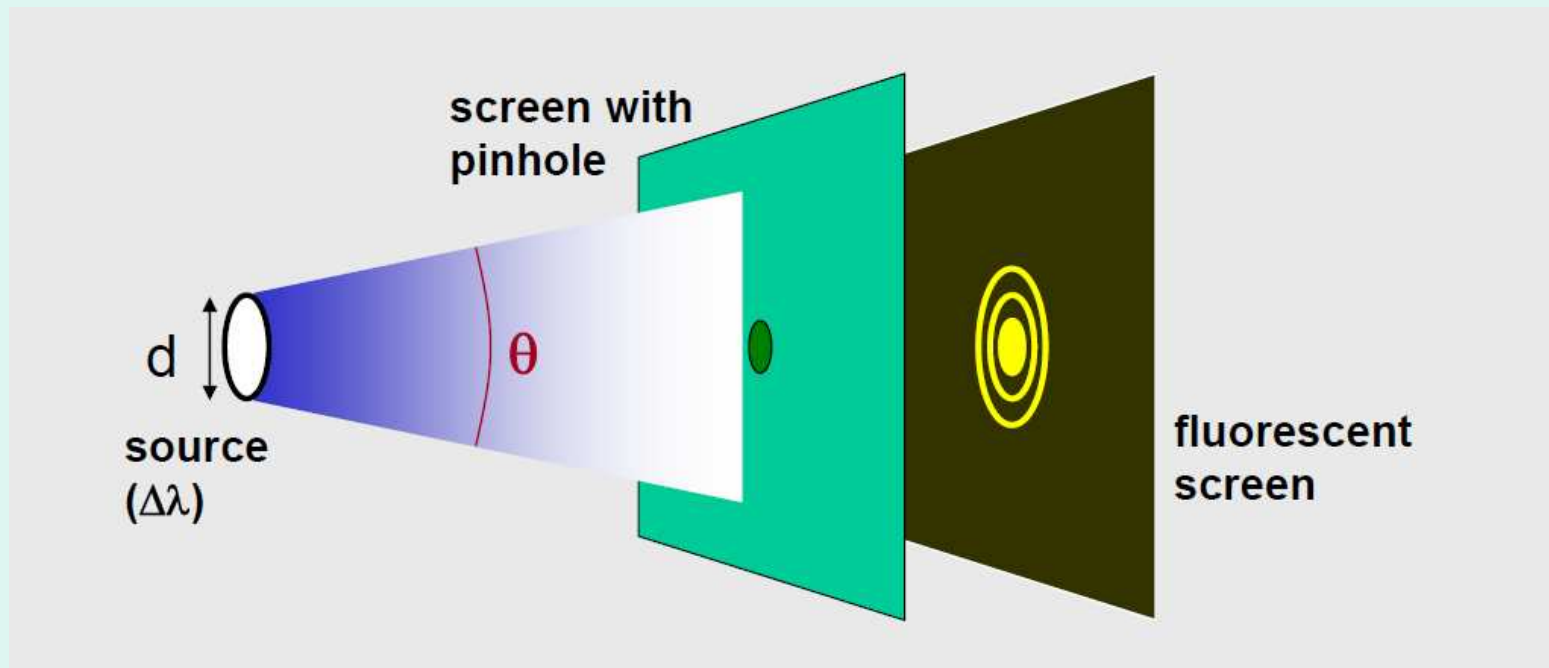




$$B \sim \frac{F}{S\Omega}$$

crystallography

Allows a wave to produce visible diffraction and interference effects

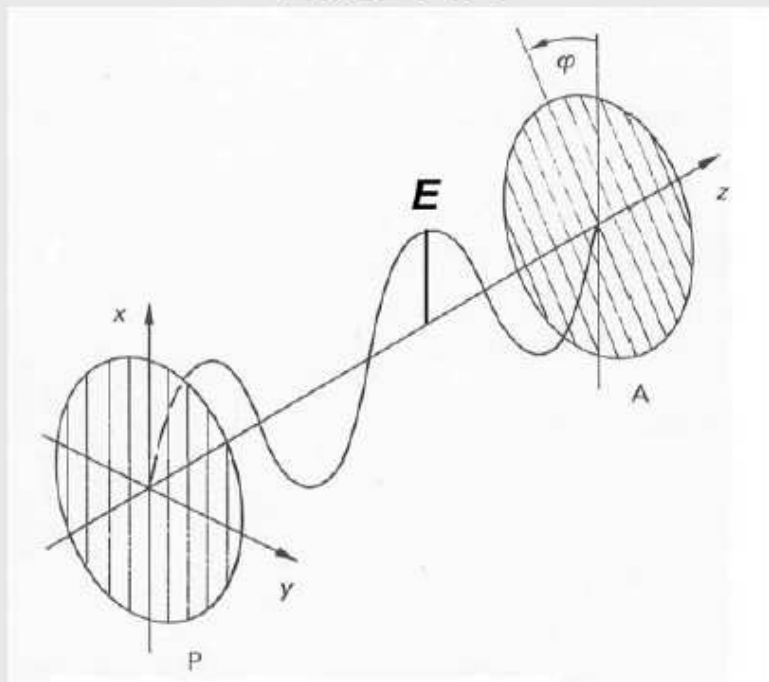


A point-like monochromatic source always creates diffraction

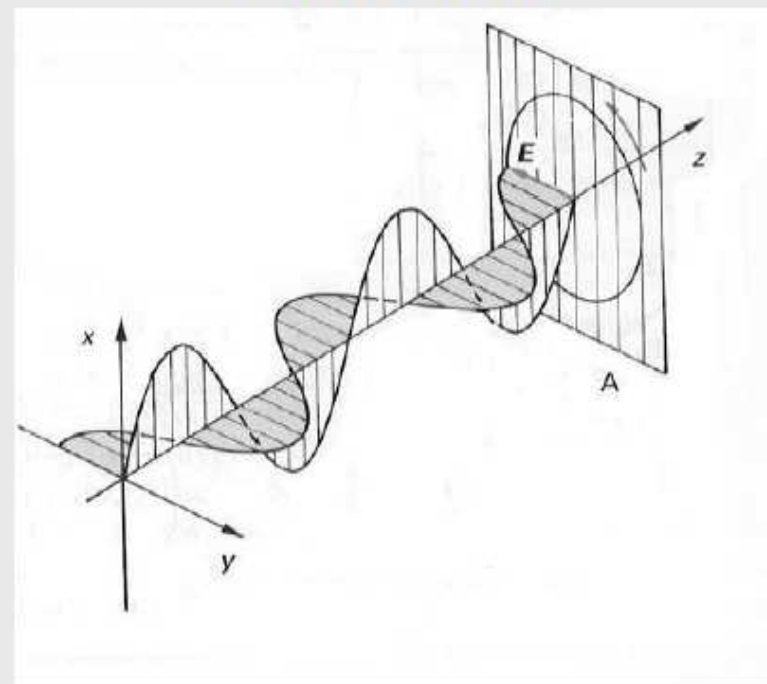
Phase contrast imaging ,holography, interferometry

Electric field vector oscillates in one plane or rotates as the wave propagates

LINEAR

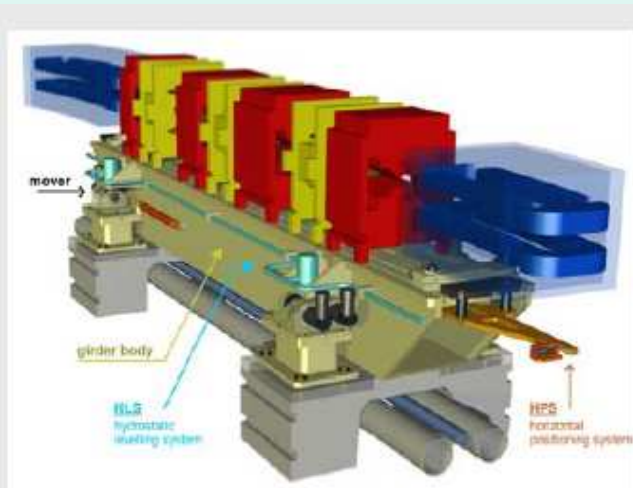


CIRCULAR

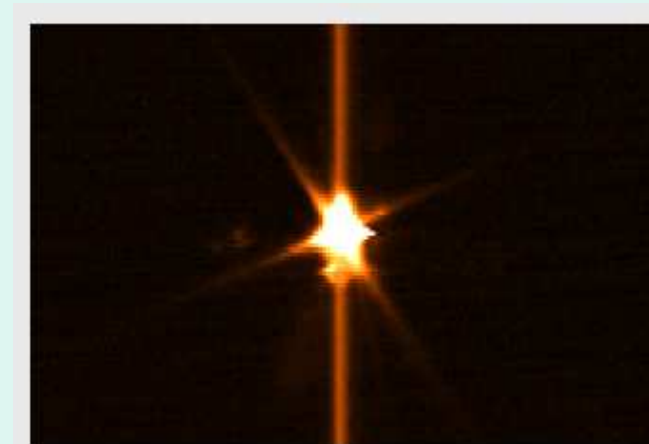
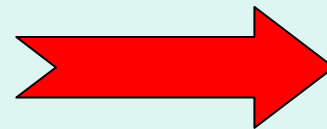


Magnetic DICHROISM -> molecule orientation

# How accelerators fit to the above requirements?



MAGNET  
STRUCTURE



LIGHT

Already Maxwell's equations (1873) made it clear that changing charge densities would radiate electromagnetic waves and Hertz demonstrated these waves in 1887.

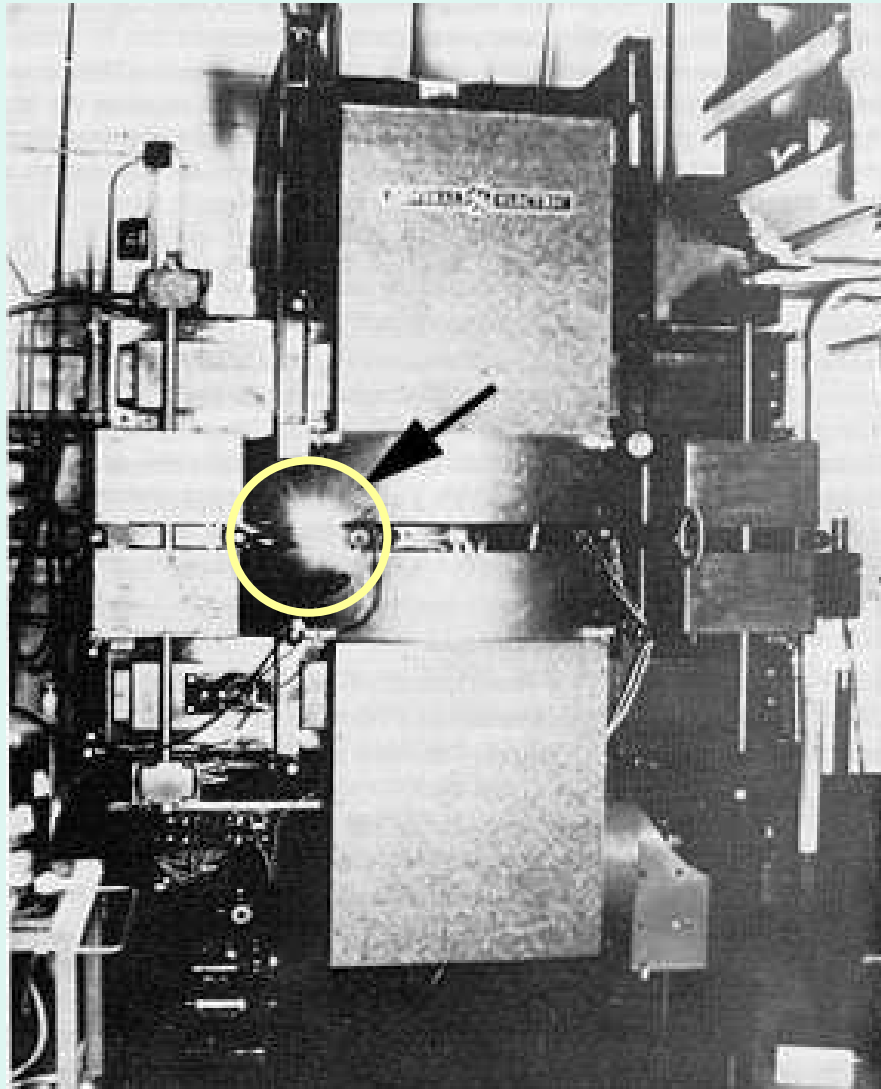
The most directly relevant development was the publication of A. Liénard's paper (1898) on the "Electric and Magnetic field produced by an electric charge concentrated at a point and in arbitrary motion", formulating the power radiated from accelerating charges. Not possible then to prove it experimentally

1944: D. Ivanenko and I. Pomeranchuk predict that this radiation may pose a limit on the maximum energy obtainable in the Betatron.

1945: J.P. Blewett, measured the energy loss due to SR in the G.E. 100 MeV Betatron and found agreement with theory but failed to detect the radiation. However he was looking at the IR part of the spectrum.

Later it was pointed out by J. Schwinger that the spectrum peaks at a much higher harmonic of the orbit frequency

27 Aprile 1947: a technician (Floyd Haber) observes for the first time “a small spot of brilliant white light” – emerging from the electrons of the G.E. 70 MeV synchrotron, however he thought that it was an electric discharge in the vacuum chamber



When charged particles, in particular electrons or positrons, are forced to move in a circular orbit, photons are emitted. At relativistic velocities (when the particles are moving at close to the speed of light) these photons are emitted in a narrow cone in the forward direction, at a tangent to the orbit. In a high energy electron or positron storage ring these photons are emitted with energies ranging from infra-red to energetic (short wavelength) X-rays. This radiation is called ***Synchrotron Radiation***

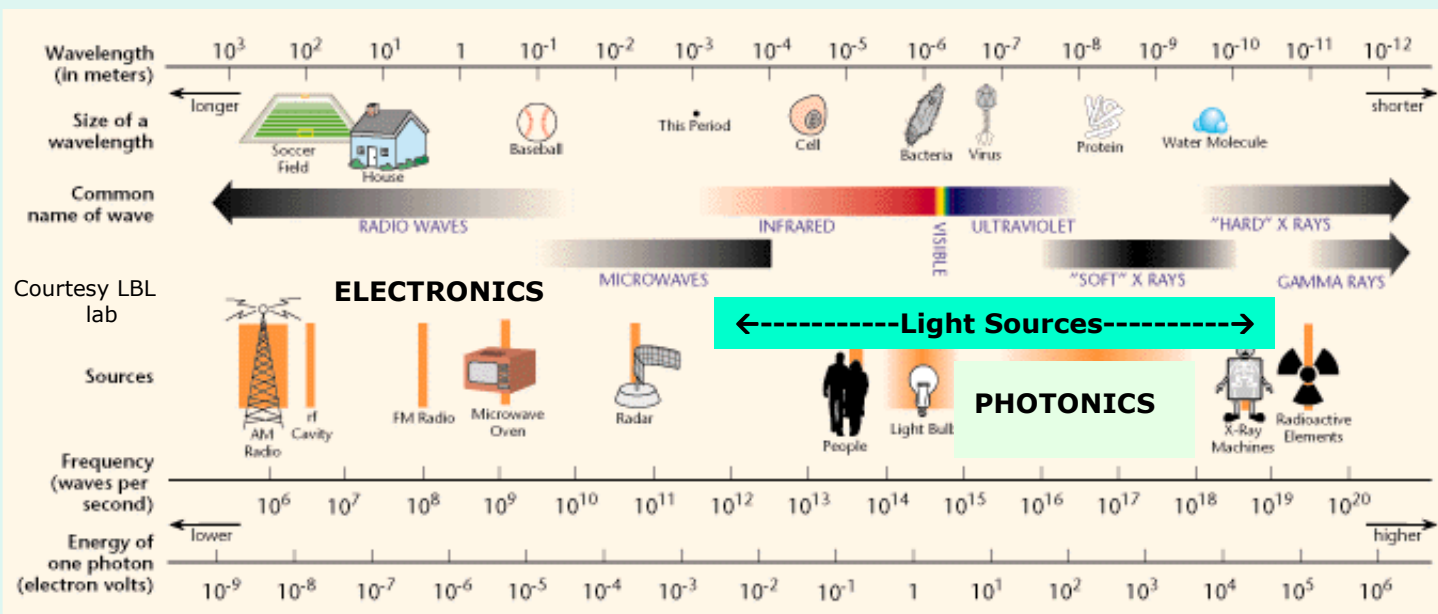
High brightness: synchrotron radiation is extremely intense (hundreds of thousands of times higher than conventional X-ray tubes) and highly collimated.

Wide energy spectrum: synchrotron radiation is emitted with a wide range of energies, allowing a beam of any energy to be produced.

Synchrotron radiation is highly polarized.

It is emitted in very short pulses, typically less than a nano-second (a billionth of a second). Elettra gives 30 ps but the FELs  $<1$  ps





Spectrum of Electromagnetic Radiation				
Region	Wavelength (Angstroms)	Wavelength (centimetres)	Frequency (Hz)	Energy (eV)
Radio	$> 10^9$	$> 10$	$< 3 \times 10^9$	$< 10^{-5}$
Microwave	$10^9 - 10^6$	$10 - 0.01$	$3 \times 10^9 - 3 \times 10^{12}$	$10^{-5} - 0.01$
Infrared	$10^6 - 7000$	$0.01 - 7 \times 10^{-5}$	$3 \times 10^{12} - 4.3 \times 10^{14}$	$0.01 - 2$
Visible	$7000 - 4000$	$7 \times 10^{-5} - 4 \times 10^{-5}$	$4.3 \times 10^{14} - 7.5 \times 10^{14}$	$2 - 3$
Ultraviolet	$4000 - 10$	$4 \times 10^{-5} - 10^{-7}$	$7.5 \times 10^{14} - 3 \times 10^{17}$	$3 - 10^3$
X-Rays	$10 - 0.1$	$10^{-7} - 10^{-9}$	$3 \times 10^{17} - 3 \times 10^{19}$	$10^3 - 10^5$
Gamma Rays	$< 0.1$	$< 10^{-9}$	$> 3 \times 10^{19}$	$> 10^5$

Calculating the EM field of a arbitrarily moving electron using the *Lienard - Wiechert* potentials finally one obtains:

$$\vec{E} = e \frac{(\vec{n} - \vec{\beta})}{\gamma^2 R^2 (1 - \vec{\beta}\vec{n})^3} + \frac{e}{c} \frac{\vec{n} \times \{ (\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}} \}}{R(1 - \vec{\beta}\vec{n})^3}$$

Near field

$$1/R^2$$

$$\vec{B} = \frac{\vec{R} \times \vec{E}}{R}$$

Far field

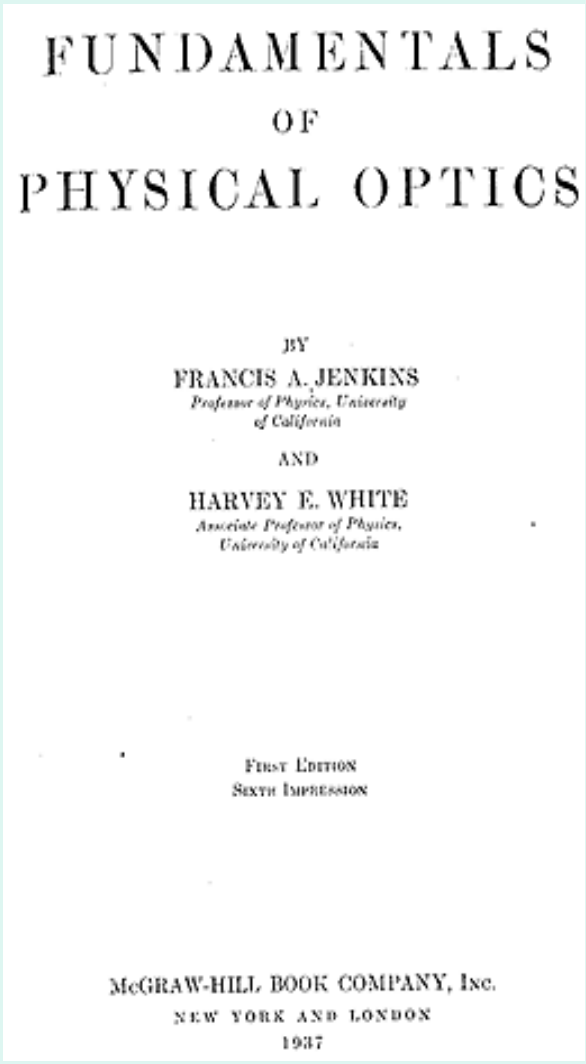
$$1/R$$

The power emitted per solid angle is given using the pointing vector and reads

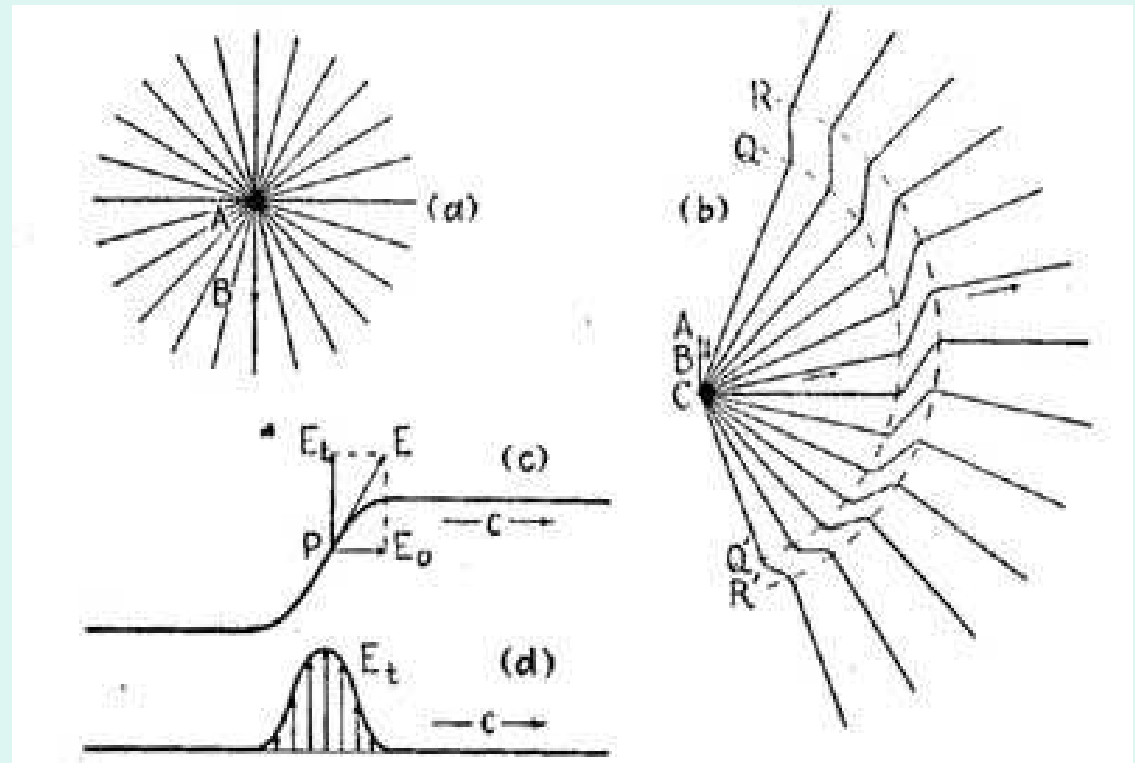
$$\vec{S} = \frac{c}{4\pi} \vec{n} |\vec{E}|^2$$

$$P = \oint S d\sigma$$

$$\frac{dP}{d\Omega} = \frac{c}{4\pi} |RE|^2$$



Disturbances cannot go faster than  $c$   
 Field lines are distorted and a longitudinal E field is created



*Liénard's* (1898) equation thus reads.

$$P = \frac{e^2}{6\pi\epsilon_0 c} \gamma^6 (\dot{\vec{\beta}}^2 - (\vec{\beta} \times \dot{\vec{\beta}})^2)$$

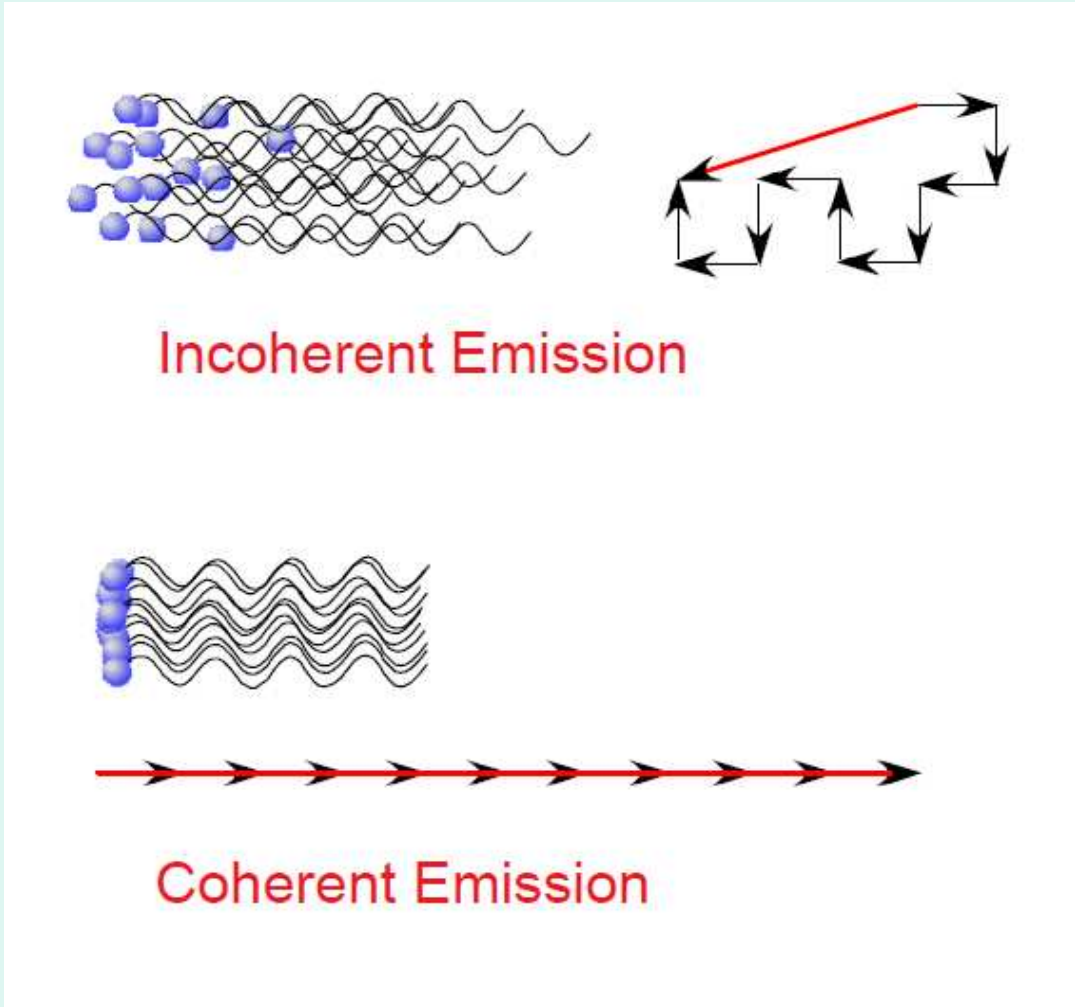
$$\left( \frac{e^2}{4\pi\epsilon_0} \right)_{mks} = (e^2)_{\text{gaussian}}$$

$$P = \frac{e^2}{6\pi\epsilon_0 c^3} \gamma^4 \dot{u}^2 \quad \text{if } \gamma \gg 1$$

$$P_{\text{multiparticle}} = [N(1 - g(\omega)) + N^2 g(\omega)] P_{\text{singleparticle}}$$

$$g(\omega) = \left| \int_{-\infty}^{\infty} e^{\frac{-i\omega z}{c}} S(z) dz \right|^2$$

*A pulse of electrons will emit coherently at wave lengths comparable or higher than its length*



→ N

→ N<sup>2</sup>

First assume linear motion

$$\left( \vec{\beta} \times \dot{\vec{\beta}} \right) = 0$$

$$\vec{p} = m_0 \gamma c \vec{\beta}$$

$$\frac{d\vec{p}}{dt} = m_0 \gamma^3 \vec{u}$$

$$\frac{d\gamma}{dt} = \gamma^3 \frac{\vec{u} \cdot \dot{\vec{u}}}{c^2}$$

$$P = \frac{e^2}{6\pi\epsilon_0 m^2 c^3} \left( \frac{d\vec{p}}{dt} \right)^2$$

Since the rate of change of momentum should equal the gain in energy per unit distance  $dE/dx$  (i.e. MeV/m), the ratio of power radiated to power supplied by external source is:

$$\frac{P}{dE/dt} = \frac{e^2 / mc^2}{6\pi\epsilon_0 m c^2} \left( \frac{dE}{dx} \right)$$

$$\frac{P}{dE/dt} = \frac{r_0}{mc^2} \left( \frac{dE}{dx} \right)$$

And shows that radiation loss is unimportant in a Linac unless the gain in energy is  $mc^2 = 0.511$  MeV in a  $r_0$  distance ( $1.28 \cdot 10^{-15}$  m) or  $2 \times 10^{14}$  MeV/m (typical gains are 10 MeV/m)

*Circumstances change dramatically when circular /“bend” motion is involved like in circular accelerators : (due to  $\gamma^4$ )*

$$P = \frac{2}{3} \frac{e^2}{4\pi\epsilon_0 m^2 c^3} \gamma^2 \left( \frac{d\vec{p}}{dt} \right)^2$$

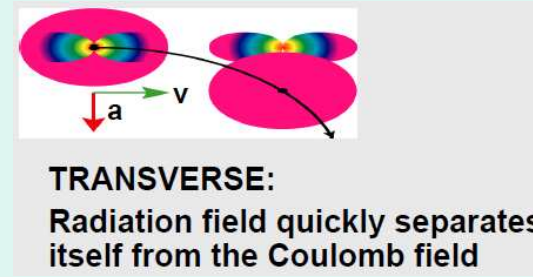
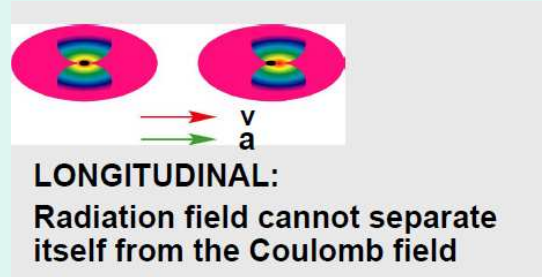
$$\left( \vec{\beta} \times \dot{\vec{\beta}} \right) = \beta \dot{\beta} \quad \frac{d\vec{p}}{dt} = \gamma \omega |\vec{p}| \quad \omega = c\beta / \rho$$

$$P = \frac{2}{3} \frac{e^2 c}{4\pi\epsilon_0} \frac{\gamma^4 \beta^4}{\rho^2}$$

*And the energy loss per revolution is given by:*

$$\delta E \equiv U_0 = \frac{2\pi\rho}{\beta c} P = \frac{e^2}{3\epsilon_0} \frac{\gamma^4 \beta^3}{\rho}$$

$$U_0 (keV) = 88.5 \frac{E^4 (GeV)}{\rho (m)} = 26.5 E^3 (GeV) B(T) \quad P(kW) = U_0 (keV) I(A)$$



So the dominant effect comes from transverse acceleration, as the deflection of a charged particle in a bending magnet

