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School on Synchrotron and Free-Electron-Laser Sources and their Multidisciplinary Applications

26 April - 7 May, 2010

Interaction of X-Rays with Matter:
Coherence and Time Structure

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Interaction of X-Rays with Matter: Coherence and Time Structure

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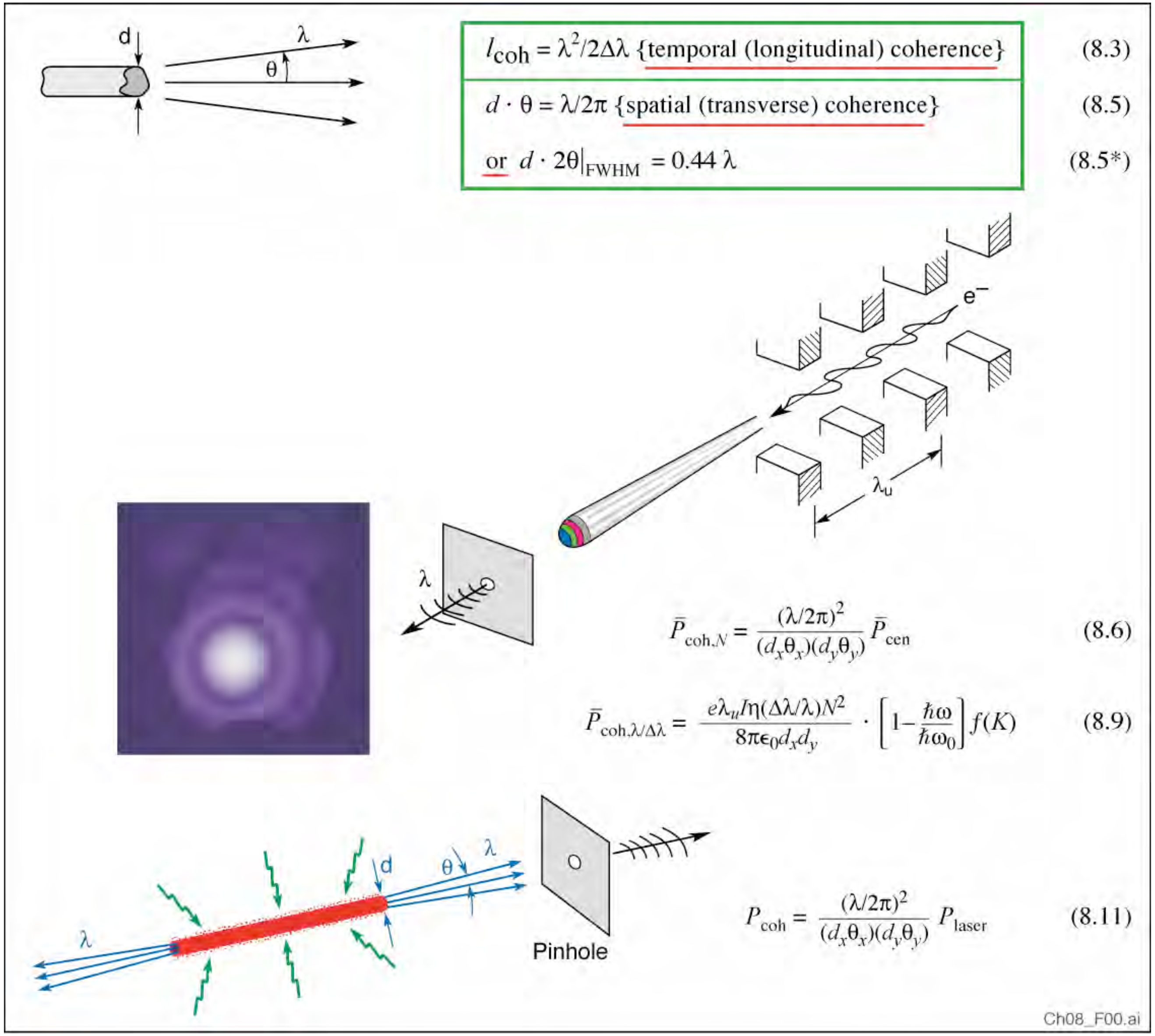
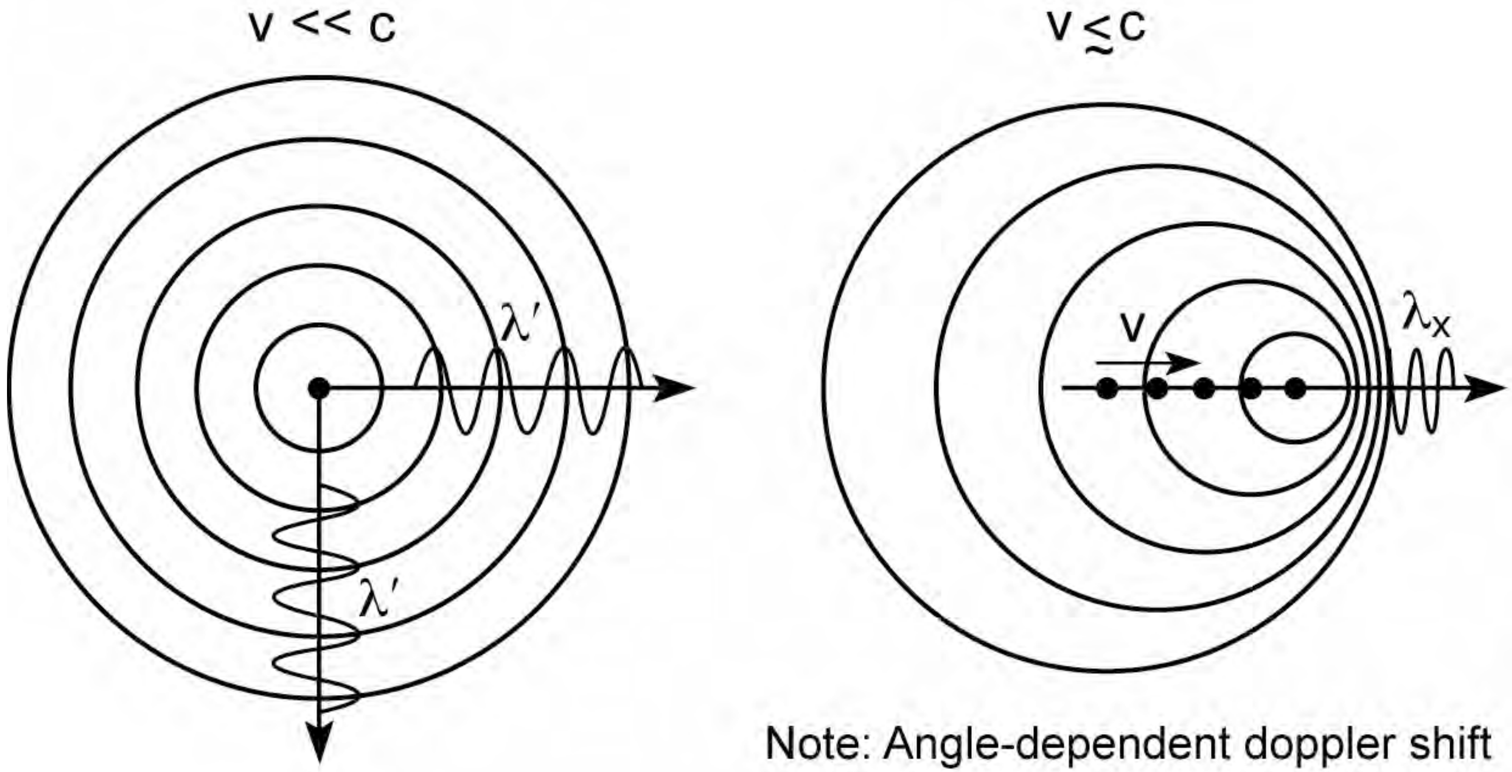
Coherence at short wavelengths

Chapter 8



Synchrotron radiation from relativistic electrons



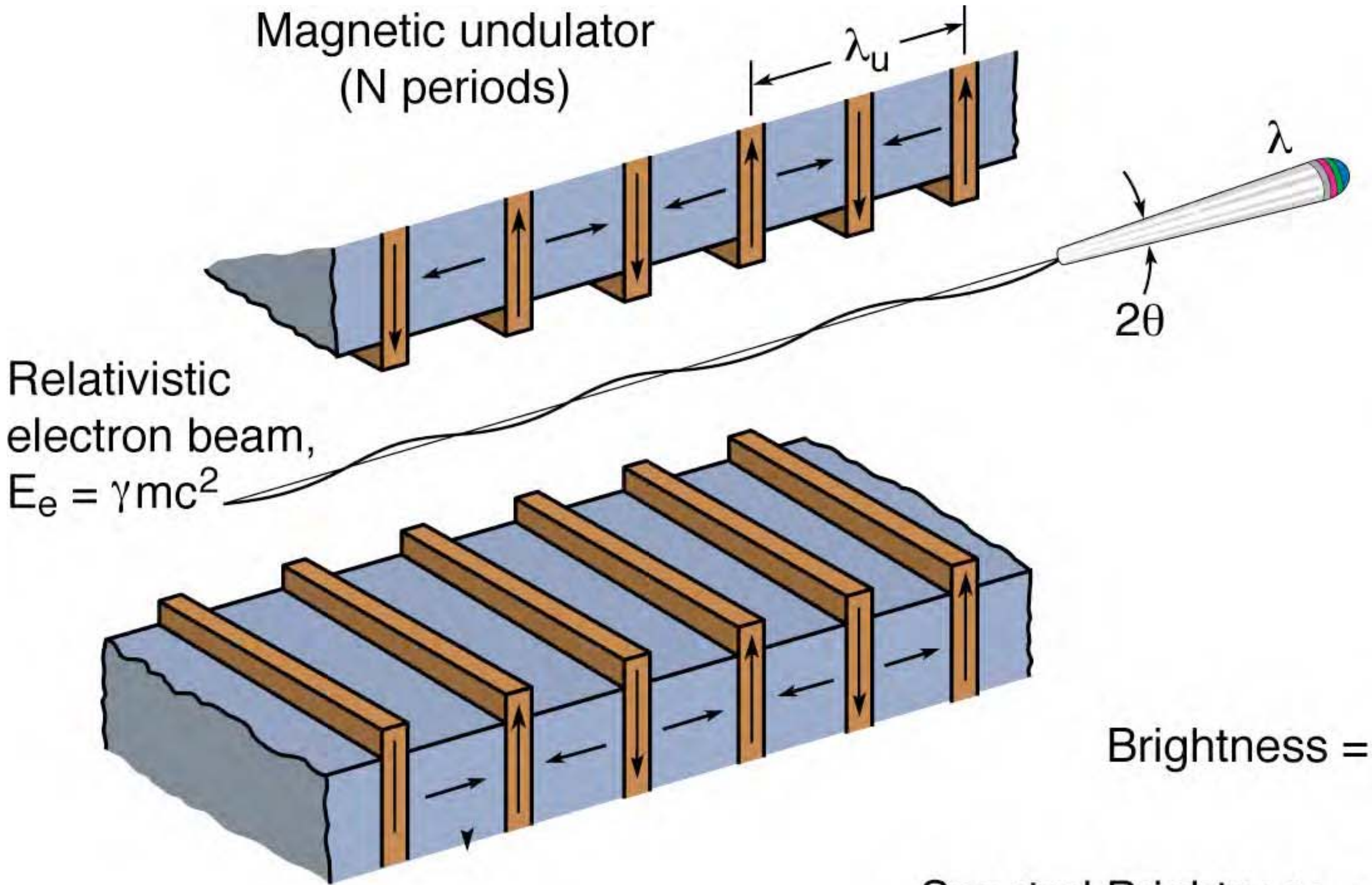
Note: Angle-dependent doppler shift

$$\lambda = \lambda' \left(1 - \frac{v}{c}\cos\theta\right) \qquad \lambda = \lambda' \gamma \left(1 - \frac{v}{c}\cos\theta\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Ch05_F09_Apr2008.ai

Undulator radiation from a small electron beam radiating into a narrow forward cone, is very bright



$$\lambda \simeq \frac{\lambda_u}{2\gamma^2}$$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

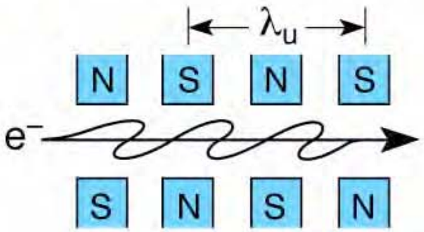
$$\text{Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)}$$

$$\text{Spectral Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)\,(\Delta\lambda/\lambda)}$$

Ch05_F08_Apr08.ai

Undulator radiation

Laboratory Frame of Reference

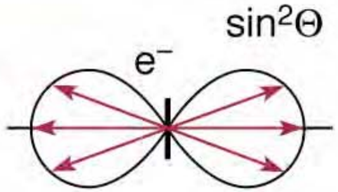


$E = \gamma mc^2$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

N = # periods

Frame of Moving e⁻



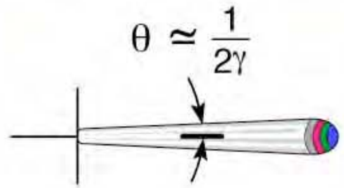
e^- radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta\lambda'} \simeq N$$

Frame of Observer



Doppler shortened wavelength on axis:

$$\lambda = \lambda'\gamma(1 - \beta\cos\theta)$$

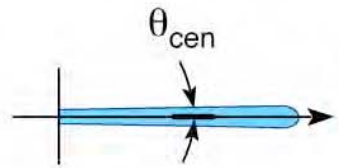
$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

where $K = eB_0\lambda_u/2\pi mc$

Following Monochromator



For $\frac{\Delta\lambda}{\lambda} \simeq \frac{1}{N}$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

typically

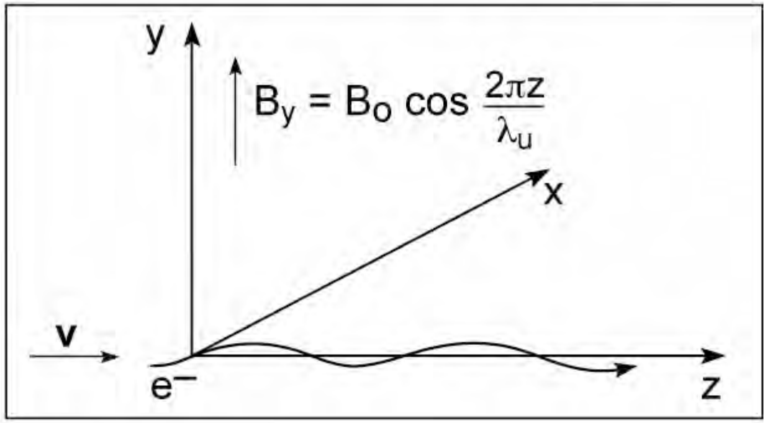
$$\theta_{cen} \simeq 40 \text{ rad}$$

Ch05_LG186_Apr2010.ai

Determining the power radiated: the equation of motion of an electron in an undulator

Magnetic fields in the periodic undulator cause the electrons to oscillate and thus radiate. These magnetic fields also slow the electrons axial (z) velocity somewhat, reducing both the Lorentz contraction and the Doppler shift, so that the observed radiation wavelength is not quite so short. The force equation for an electron is

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (5.16)$$



where $\mathbf{p} = \gamma m\mathbf{v}$ is the momentum. The radiated fields are relatively weak so that

$$\frac{d\mathbf{p}}{dt} \simeq -e(\mathbf{v} \times \mathbf{B})$$

Taking to first order $v \simeq v_z$, motion in the x-direction is

$$m\gamma \frac{dv_x}{dt} = +ev_z B_y$$

$$v_x = \frac{Kc}{\gamma} \sin\left(\frac{2\pi z}{\lambda_u}\right) \qquad (5.19)$$

$$K \equiv \frac{eB_0\lambda_u}{2\pi mc} = 0.9337 B_0(\mathrm{T})\lambda_u(\mathrm{cm}) \qquad (5.18)$$

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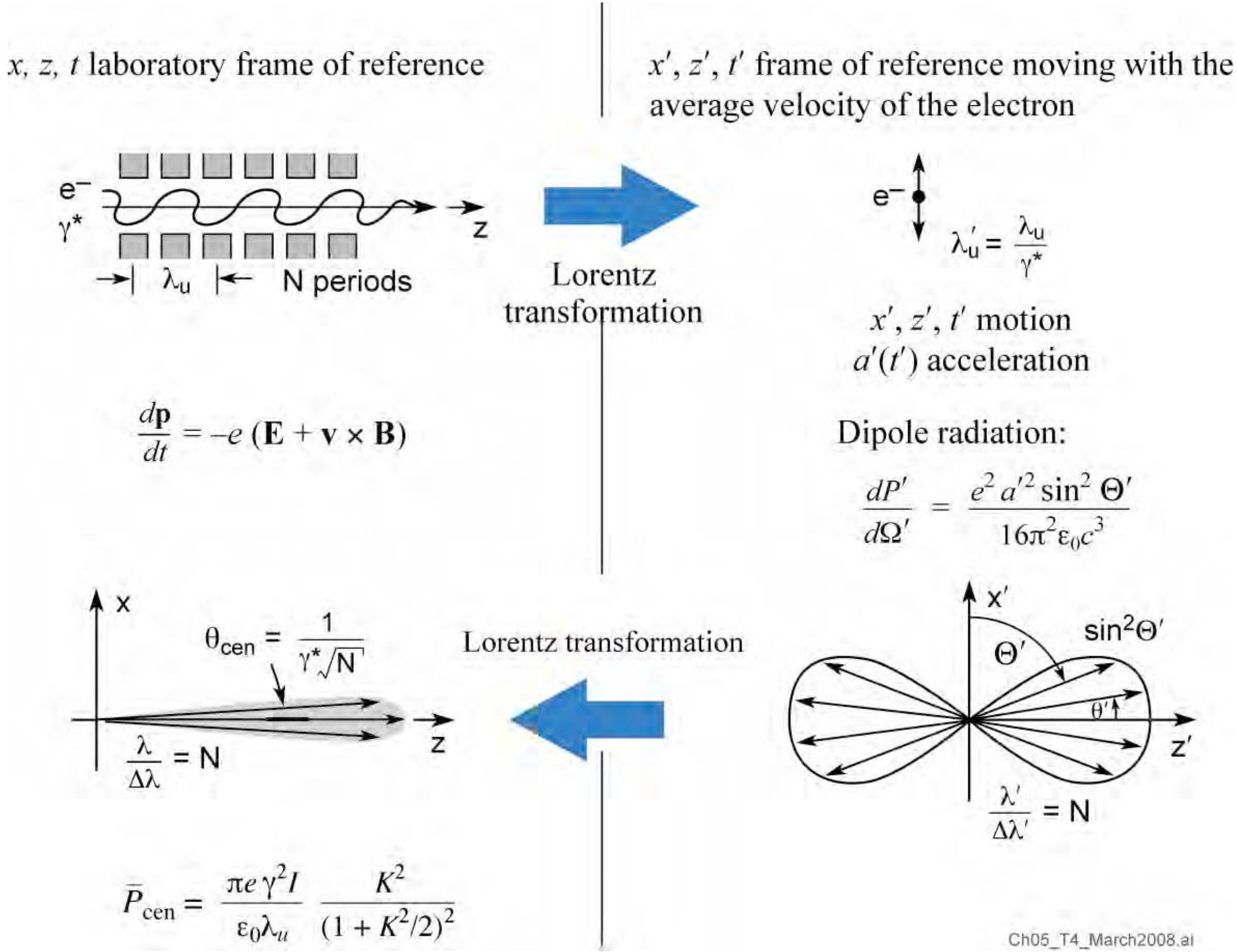
Calculating power in the central radiation cone: using the well known "dipole radiation" formula by transforming to the frame of reference moving with the electrons



Undulator radiated power in the central cone

$$\lambda_x = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

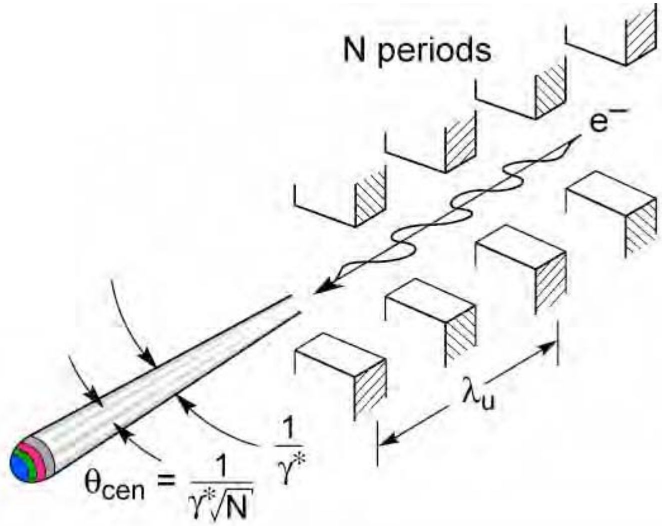
$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{\left(1 + \frac{K^2}{2}\right)^2} f(K)$$

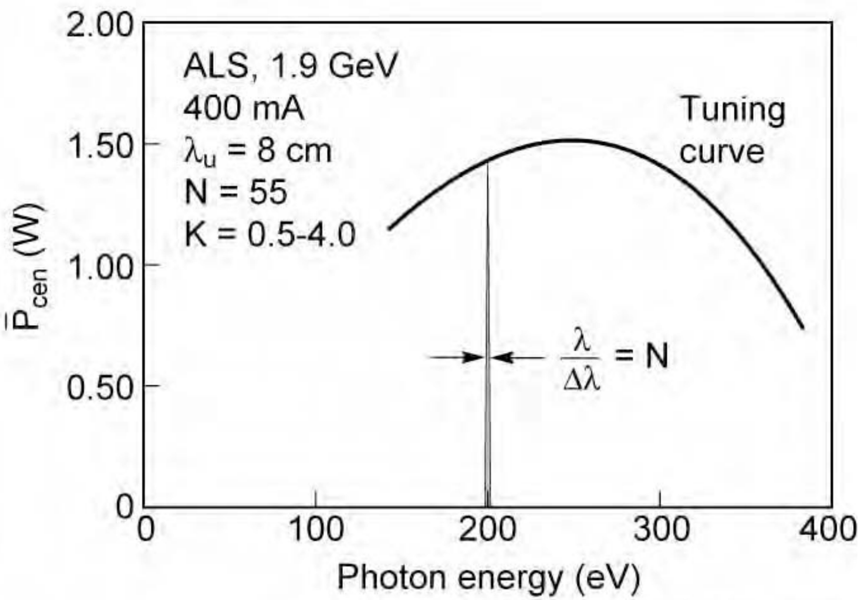
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

$$K = \frac{eB_0\lambda_u}{2\pi m_0 c}$$

$$\gamma^* = \gamma / \sqrt{1 + \frac{K^2}{2}}$$





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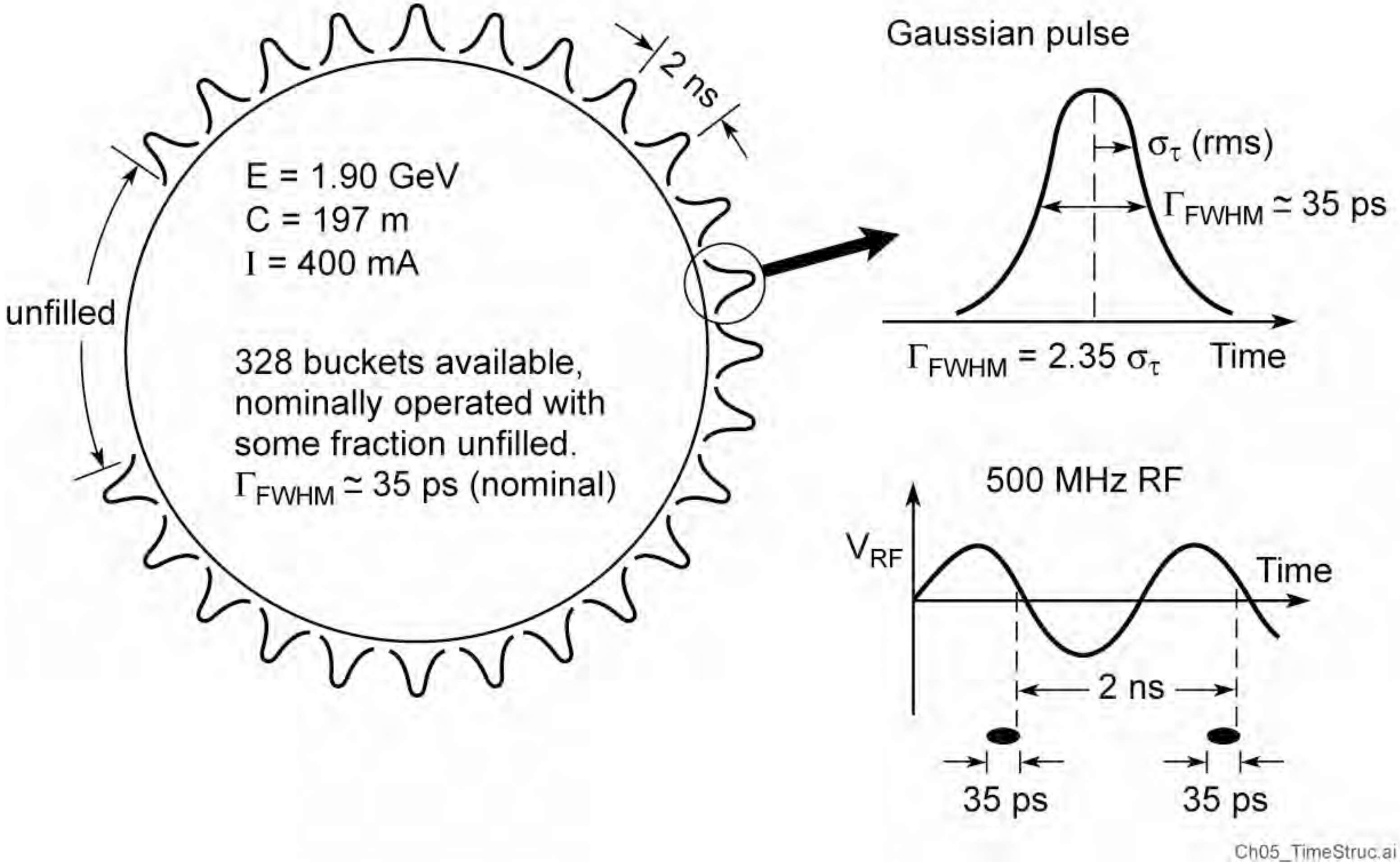
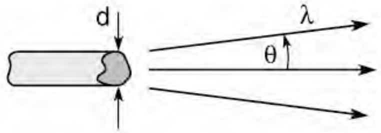
Time structure of synchrotron radiation

The axial electric field within the RF cavity, used to replenish lost (radiated) energy, forms a potential well "bucket" system that forces electrons into axial electron "bunches". This leads to a time structure in the emitted radiation.



Coherence at short wavelengths

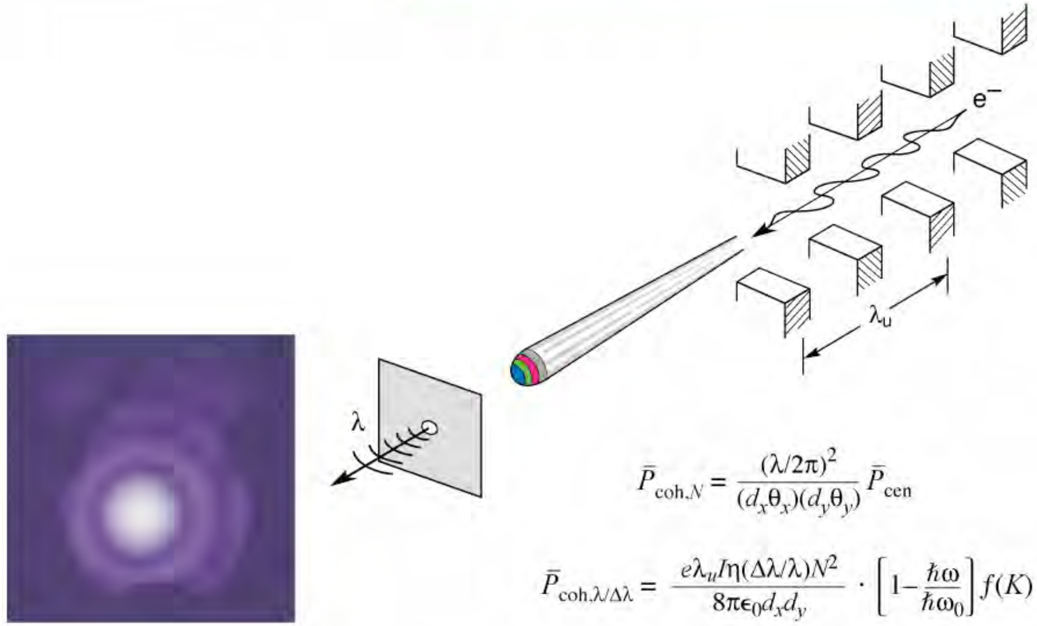
Chapter 8



$$l_{coh} = \lambda^2/2\Delta\lambda \ \{\text{temporal (longitudinal) coherence}\} \quad (8.3)$$

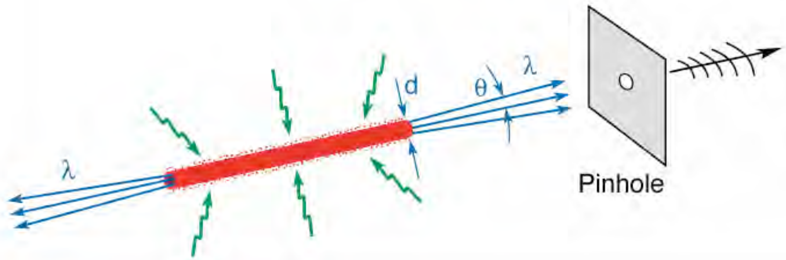
$$d \cdot \theta = \lambda/2\pi \ \{\text{spatial (transverse) coherence}\} \quad (8.5)$$

$$\text{or} \ \ d \cdot 2\theta|_{FWHM} = 0.44\,\lambda \quad (8.5^*)$$



$$\bar{P}_{coh,N} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}\bar{P}_{cen} \quad (8.6)$$

$$\bar{P}_{coh,\lambda/\Delta\lambda} = \frac{e\lambda_u I\eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left[1 - \frac{\hbar\omega}{\hbar\omega_0}\right] f(K) \quad (8.9)$$

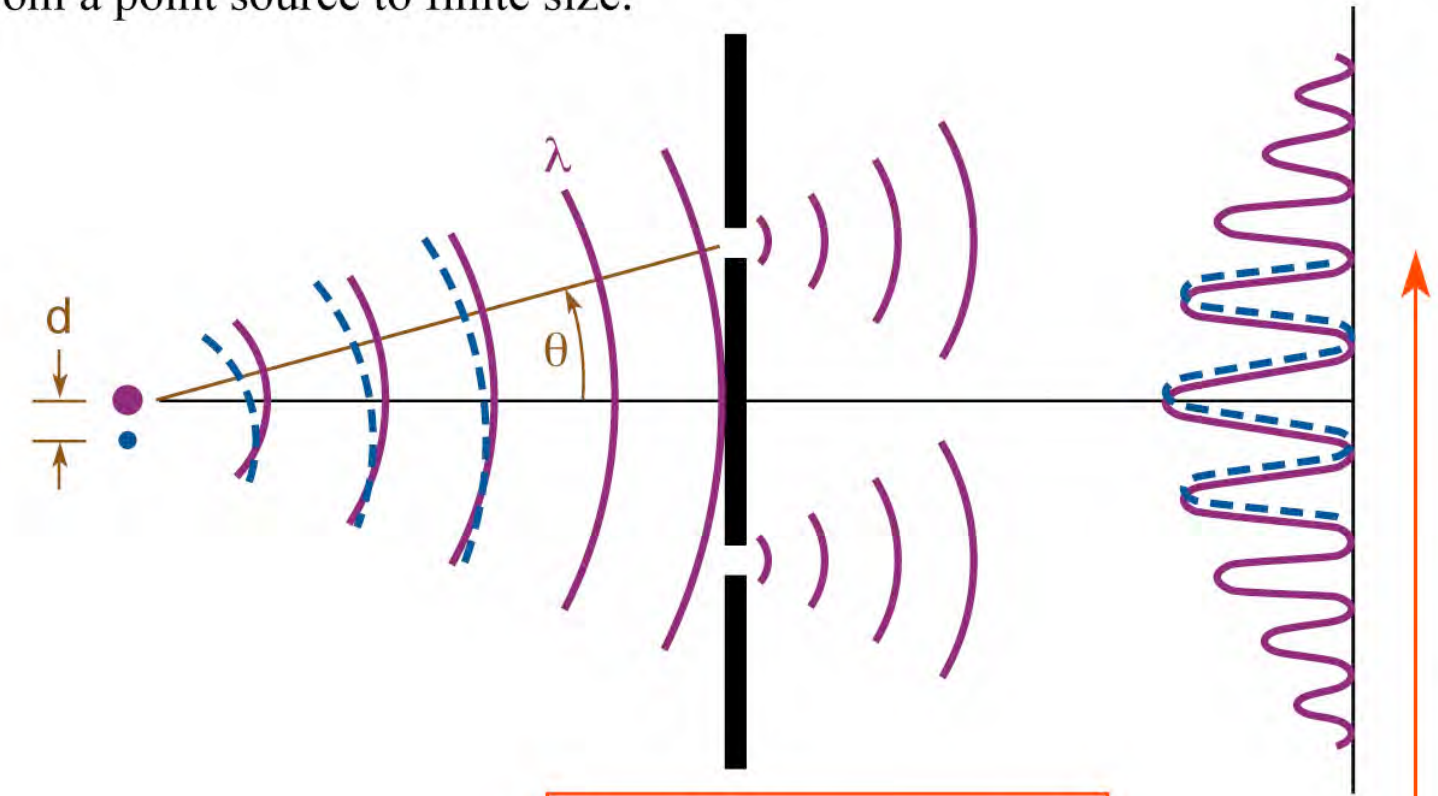


$$P_{coh} = \frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)} P_{laser} \quad (8.11)$$

Ch08_F00.ai

Young's double slit experiment: spatial coherence and the persistence of fringes

Persistence of fringes as the source grows from a point source to finite size.

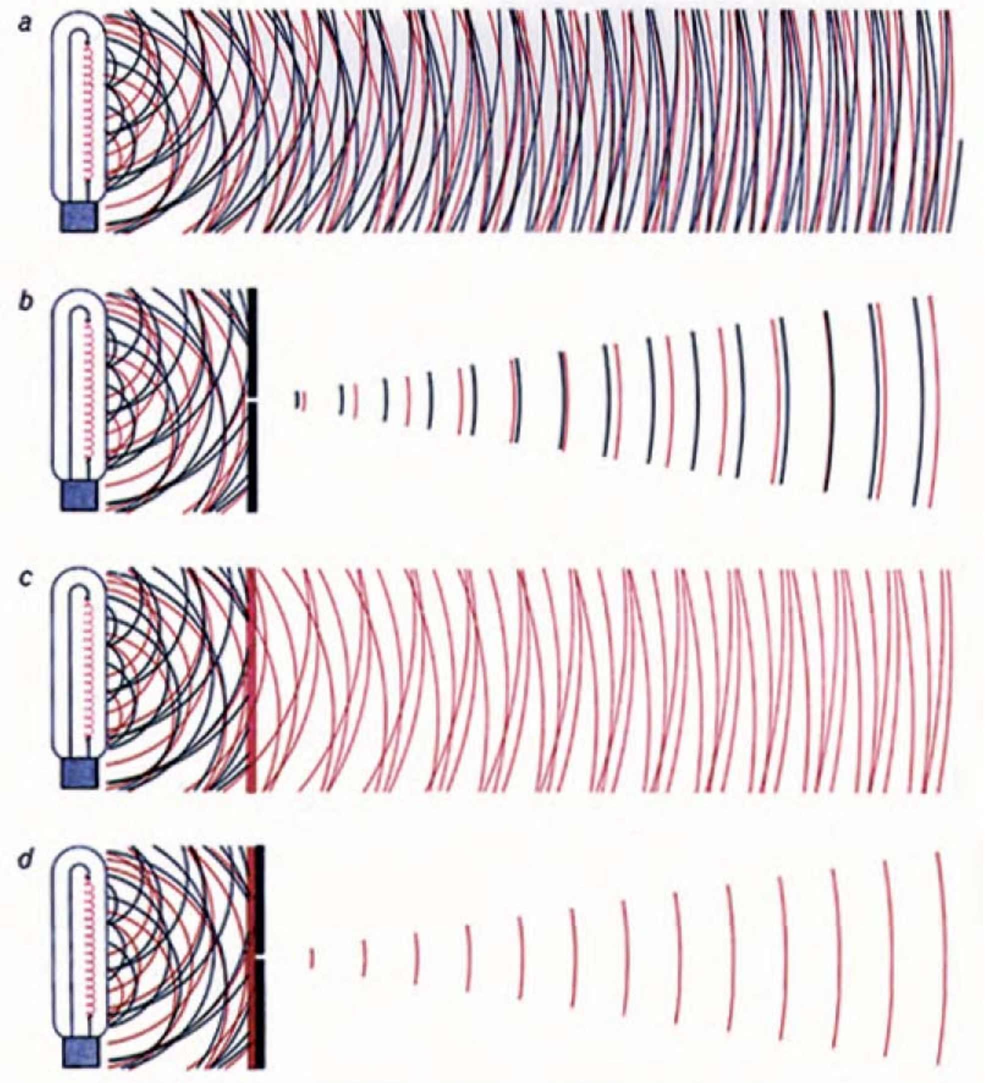


$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

CH08_YoungsExprmt_v3.ai

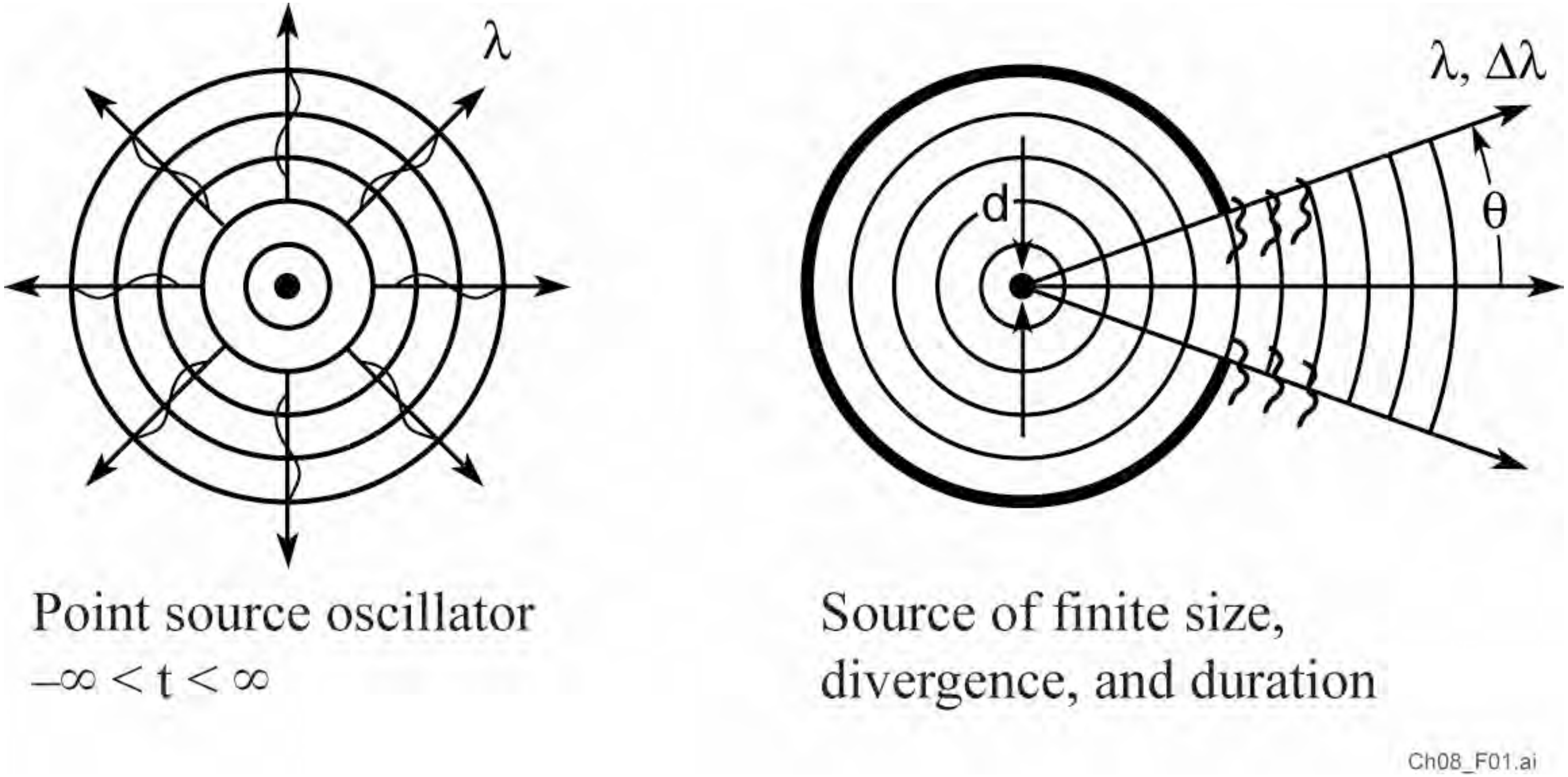
Spatial and spectral filtering to produce coherent radiation



Courtesy of A. Schawlow, Stanford.

Ch08_F08.ai

Coherence, partial coherence and incoherence



Spatial and temporal coherence

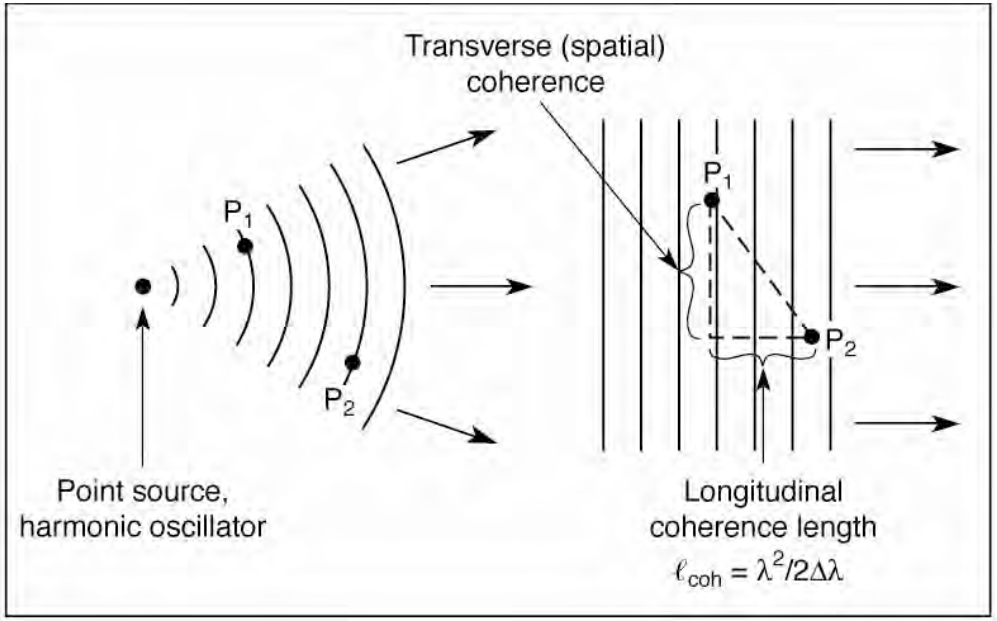
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t+\tau)E_2^*(t)\rangle \qquad (8.1)$$

Normalize degree of spatial coherence (complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}} \qquad (8.12)$$

A high degree of coherence ($\mu \to 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \to 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



Longitudinal (temporal) coherence length

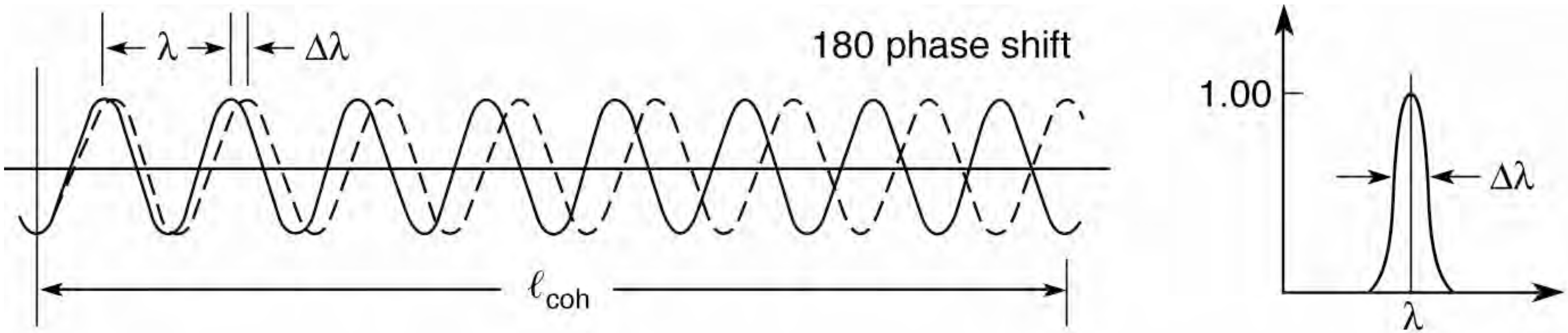
$$\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda} \qquad (8.3)$$

Full spatial (transverse) coherence

$$d \cdot \theta = \lambda/2\pi \qquad (8.5)$$

Ch08_Eq1_12_F2.ai

Spectral bandwidth and longitudinal coherence length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})(\lambda + \Delta\lambda)$$

Equating the two

$$N = \lambda/2\Delta\lambda$$

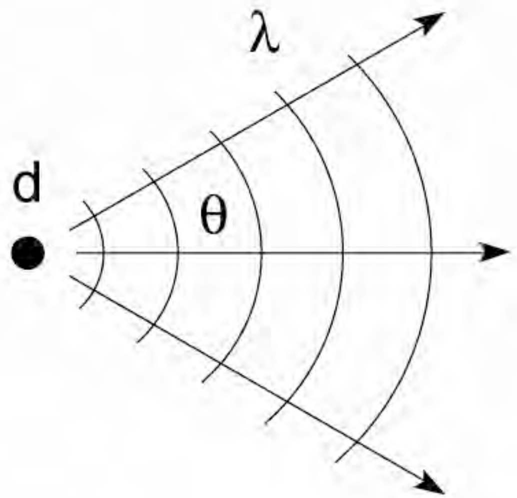
so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

Ch08_F03.ai

A practical interpretation of spatial coherence

- Associate spatial coherence with a spherical wavefront.
- A spherical wavefront implies a point source.
- How small is a "point source"?



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size "d" you can resolve, with wavelength λ and half angle θ, is

$$d \cdot \theta = \frac{\lambda}{2\pi}$$

Ch08_XBL 915-6740A.ai

Partially coherent radiation approaches uncertainty principle limits

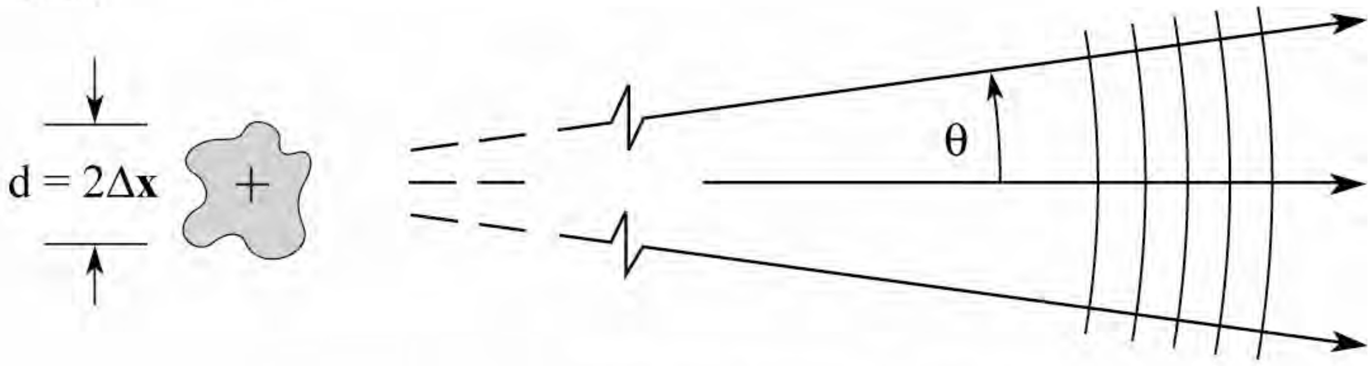
$\Delta \mathbf{x} \cdot \Delta \mathbf{p} \geq \hbar/2$ (8.4)

Standard deviations of Gaussian distributed functions (Tipler, 1978, pp. 174-189)

$\Delta \mathbf{x} \cdot \hbar \Delta \mathbf{k} \geq \hbar/2$

$\Delta \mathbf{x} \cdot \mathbf{k} \Delta \theta \geq 1/2$

$2\Delta \mathbf{x} \cdot \Delta \theta \geq \lambda/2\pi$

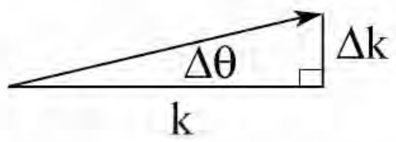


Spherical wavefronts occur in the limiting case

Note:

$\Delta \mathbf{p} = \hbar \Delta k$

$\Delta k = k \Delta \theta$



$$\boxed{d \cdot \theta = \lambda/2\pi}$$

(spatially coherent)

$\left.\right\} \frac{1}{\sqrt{e}}$ quantities

or

$(d \cdot 2\theta)_{FWHM} \simeq \lambda/2$ } FWHM quantities

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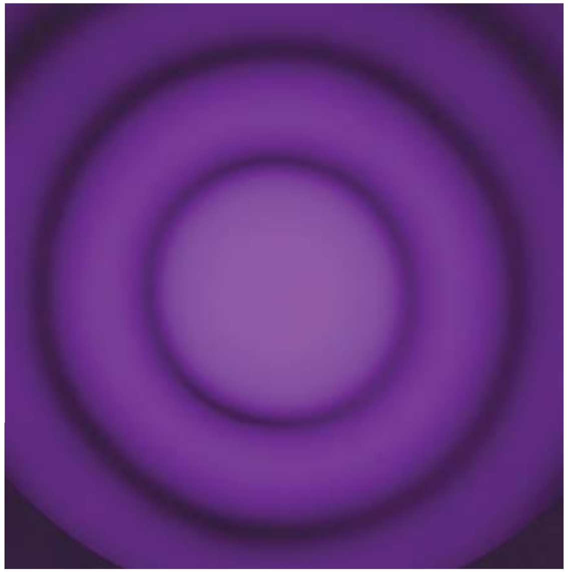
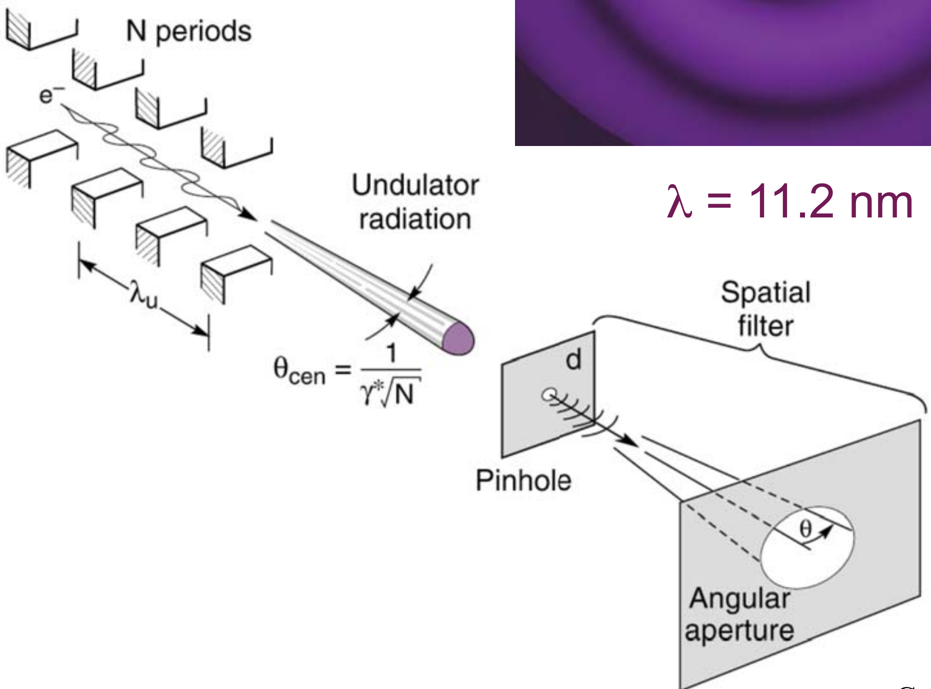
Spatially coherent x-rays: spatially filtered undulator radiation



λ = 11.2 nm

λ = 13.4 nm

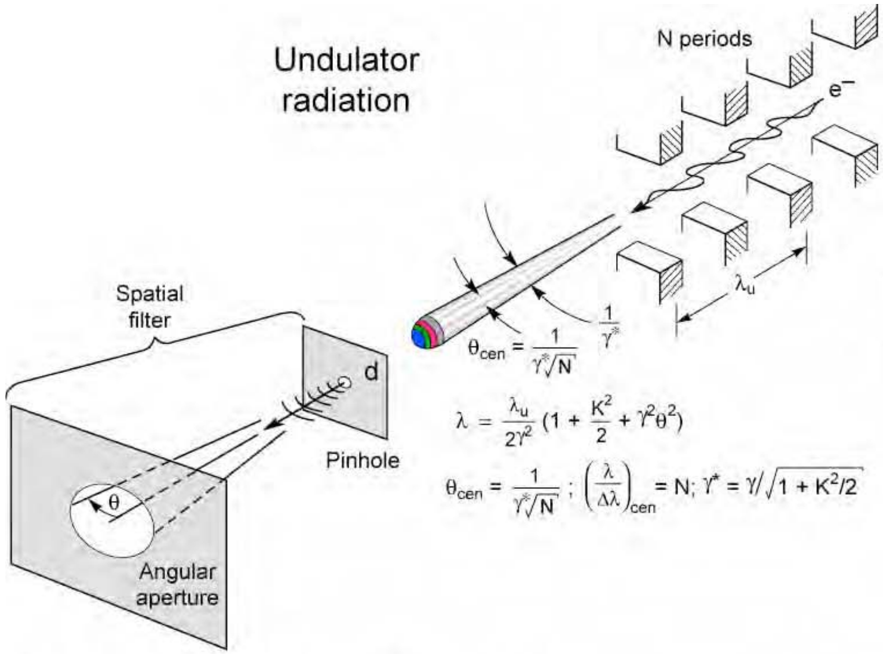


1 μm^{D} pinhole

25 mm wide CCD at 410 mm

Courtesy of Patrick Naulleau, LBNL.

Spatially filtered undulator radiation

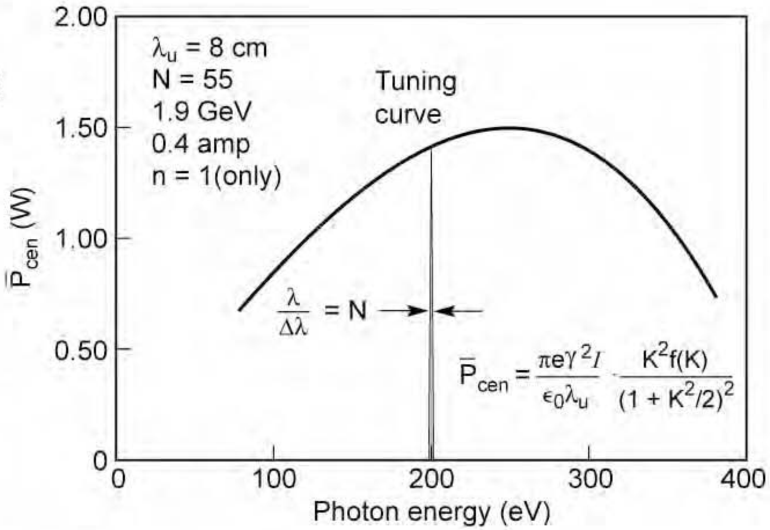


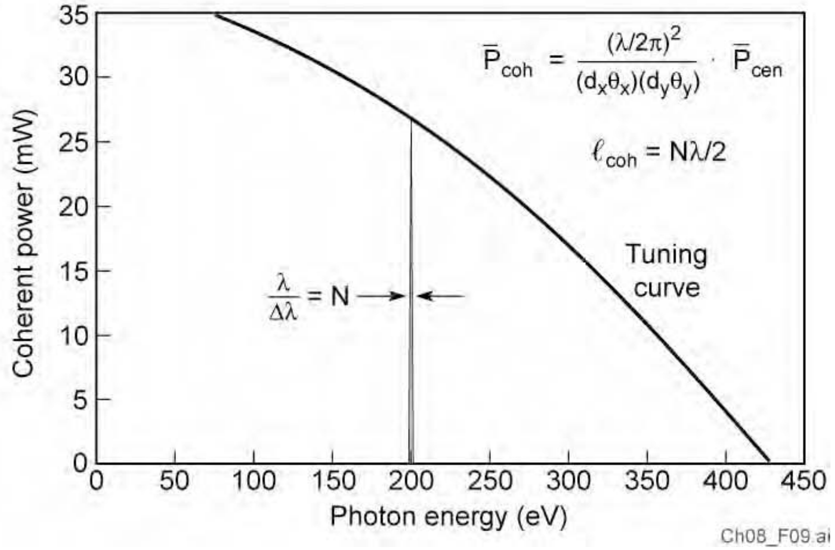
Using a pinhole-aperture spatial filter, passing only radiation that satisfies $d \cdot \theta = \lambda/2\pi$

$$\bar{P}_{coh,N} = \left(\frac{\lambda/2\pi}{d_x\theta_x}\right)\left(\frac{\lambda/2\pi}{d_y\theta_y}\right)\bar{P}_{cen} \qquad (8.6)$$

$$\bar{P}_{coh,N} = \frac{e\lambda_u I N}{8\pi\epsilon_0 d_x d_y}\left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0) \qquad (8.9)$$

for $d_x = 2\sigma_x$, $d_y = 2\sigma_y$, $\theta_{Tx} \rightarrow \theta_x$, $\theta_{Ty} \rightarrow \theta_y$, and $\sigma'^2 << \theta_{cen}^2$.





Spatial and spectral filtering of undulator radiation

In addition to the pinhole – angular aperture for spatial filtering and spatial coherence, add a monochromator for narrowed bandwidth and increased temporal coherence:

$$\bar{P}_{\text{coh},\lambda/\Delta\lambda} = \underbrace{\eta}_{\text{beamline efficiency}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y,\theta_y)}}_{\text{spatial filtering}} \cdot N \underbrace{\frac{\Delta\lambda}{\lambda}}_{\text{spectral filtering}} \cdot \bar{P}_{\text{cen}} \qquad (8.10a)$$

Coherent power ($\bar{P}_{\text{coh},\lambda/\Delta\lambda}$); Undulator radiated power ($\bar{P}_{\text{cen}}$)

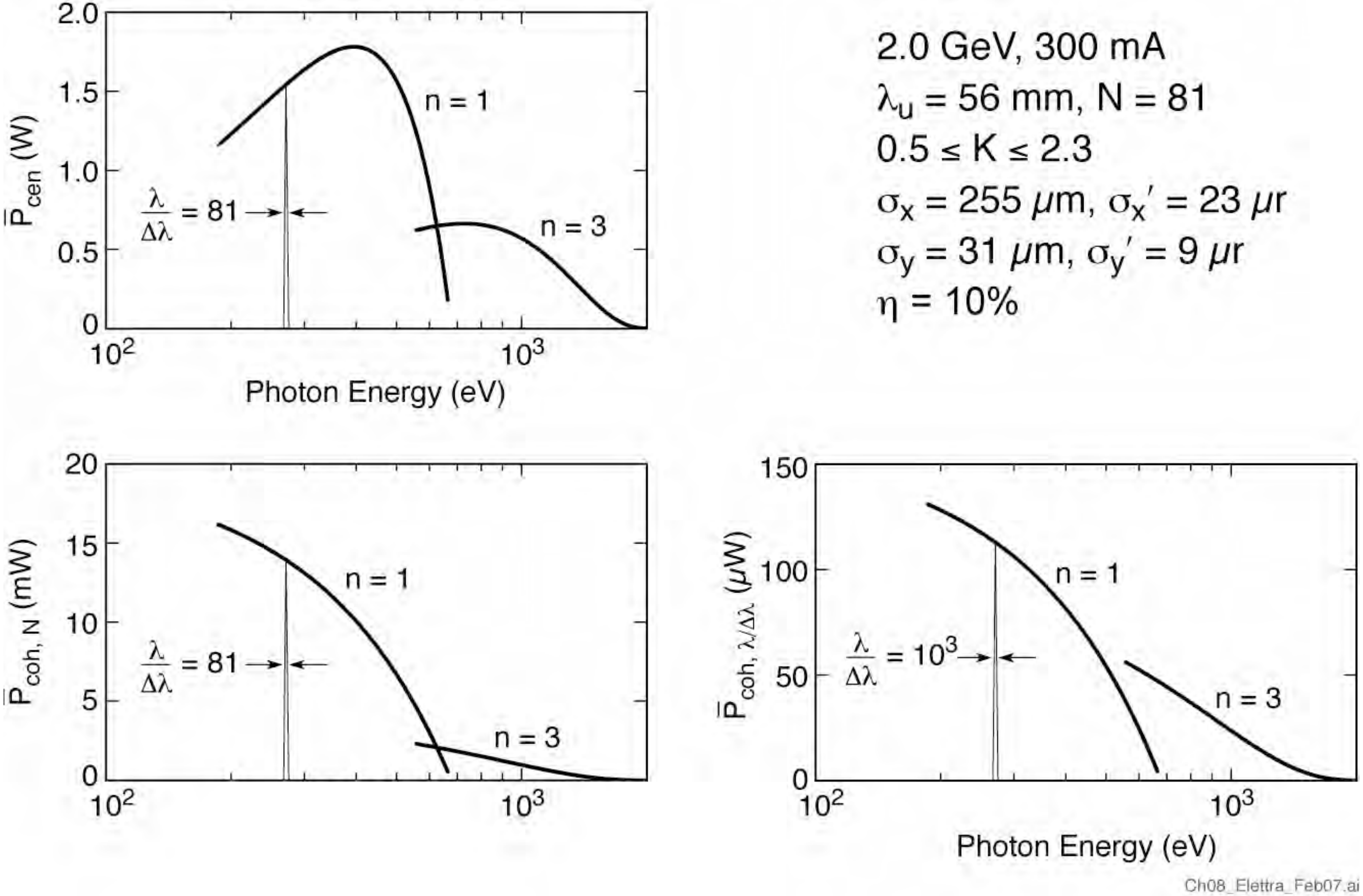
which for $\sigma'^2_{x,y} << \theta^2_{cen}$ (the undulator condition) gives the spatially and temporally coherent power ($d \cdot \theta = \lambda/2\pi$; $l_{\text{coh}} = \dfrac{\lambda^2}{2\,\Delta\lambda}$)

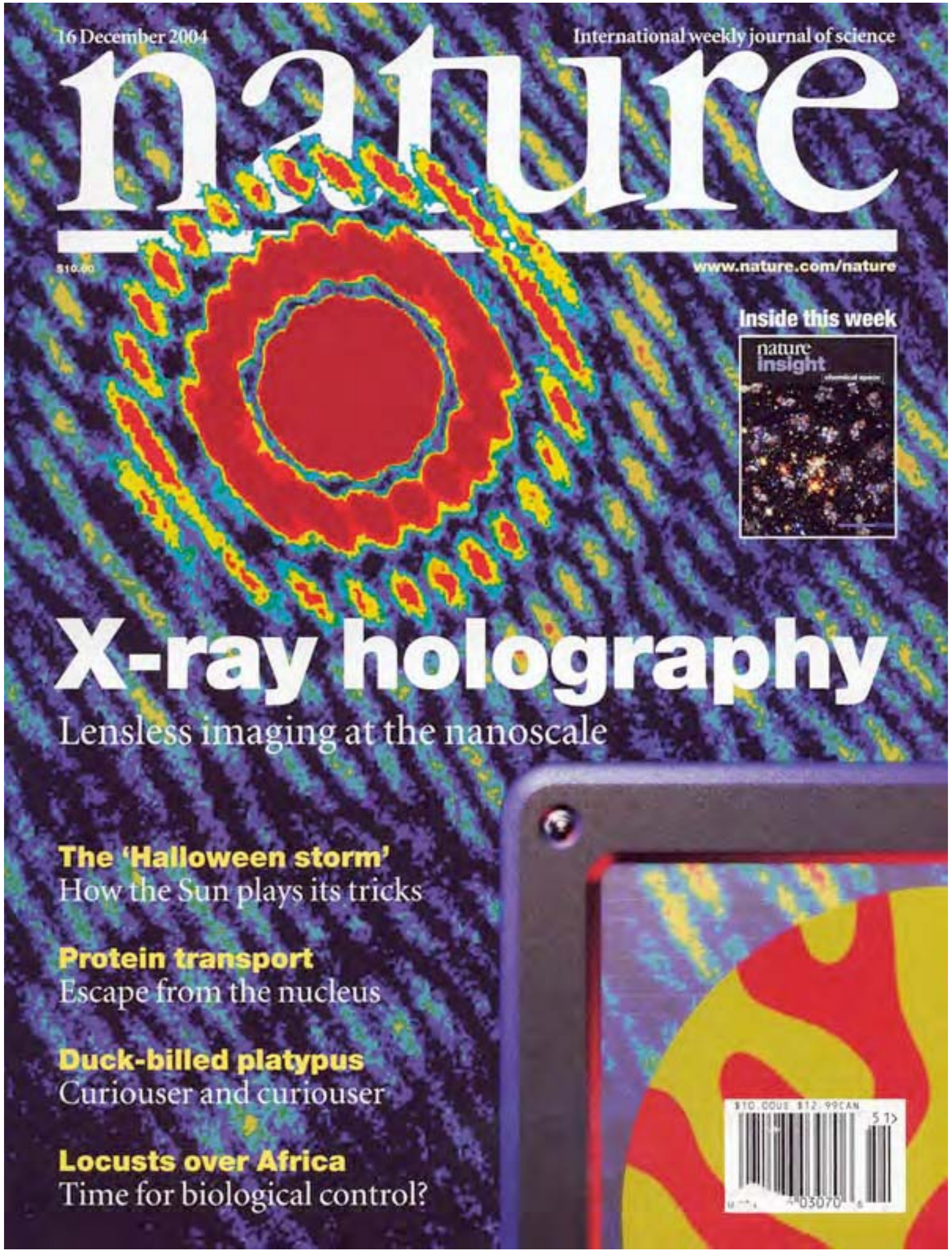
$$\bar{P}_{\text{coh},\lambda/\Delta\lambda} = \frac{e\lambda_u I \eta(\Delta\lambda/\lambda)N^2}{8\pi\epsilon_0 d_x d_y} \cdot \left(1 - \frac{\hbar\omega}{\hbar\omega_0}\right) f(\hbar\omega/\hbar\omega_0) \qquad (8.10c)$$

which we note scales as N^2.

Ch08_SpatialSpectral_Apr2010.ai

Coherent power at Elettra

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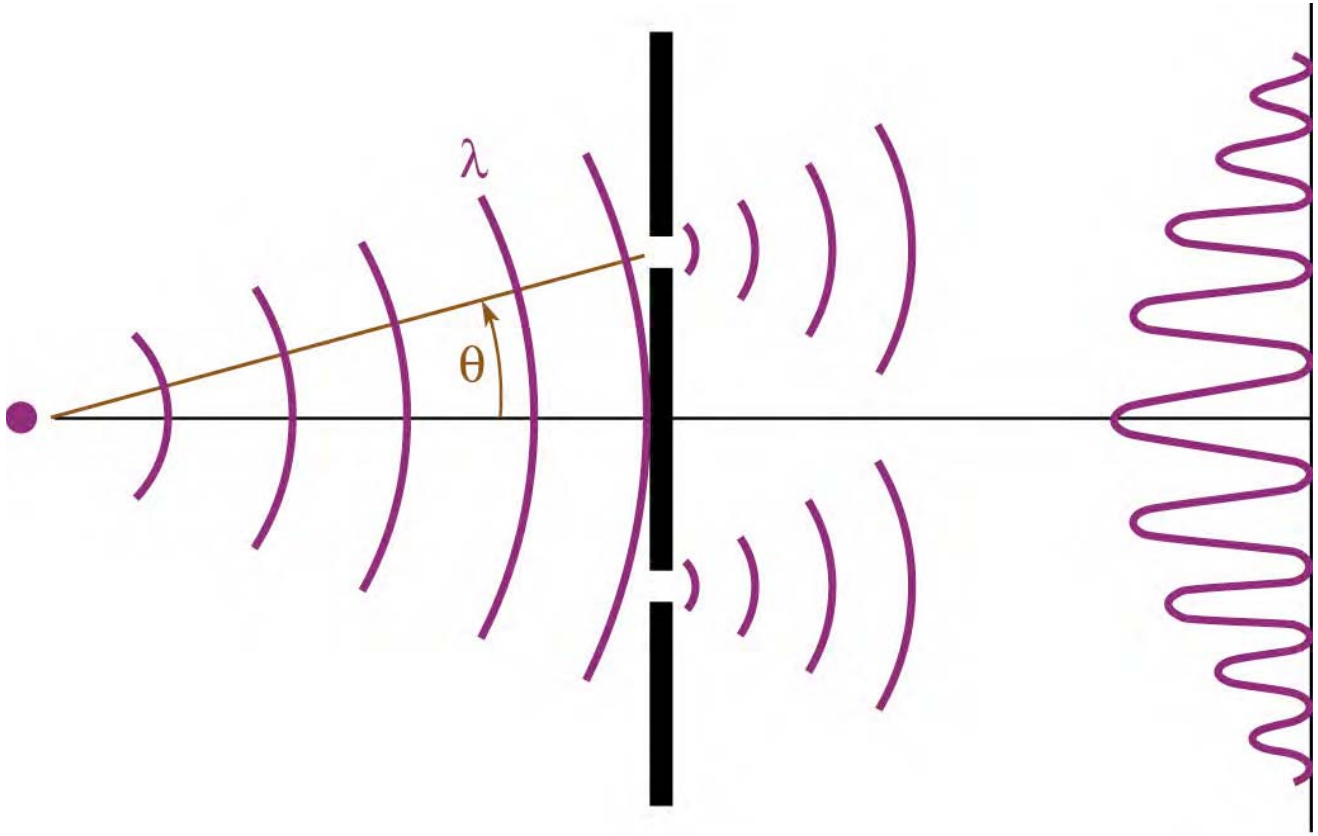
Curiouser and curiouser
Locusts over Africa
Time for biological control?

Undulators, FELs and coherence

- Spatial coherence
- Temporal coherence
- Partial coherence
- Full coherence
- Spatial filtering
- Uncorrelated emitters
- Correlated emitters
- True phase coherence and mode control
- Lasers, amplified spontaneous emission (ASE) and mode control
- Undulator radiation
- SASE FEL fsec and asec x-rays
- Seeded FEL true phase coherent x-rays

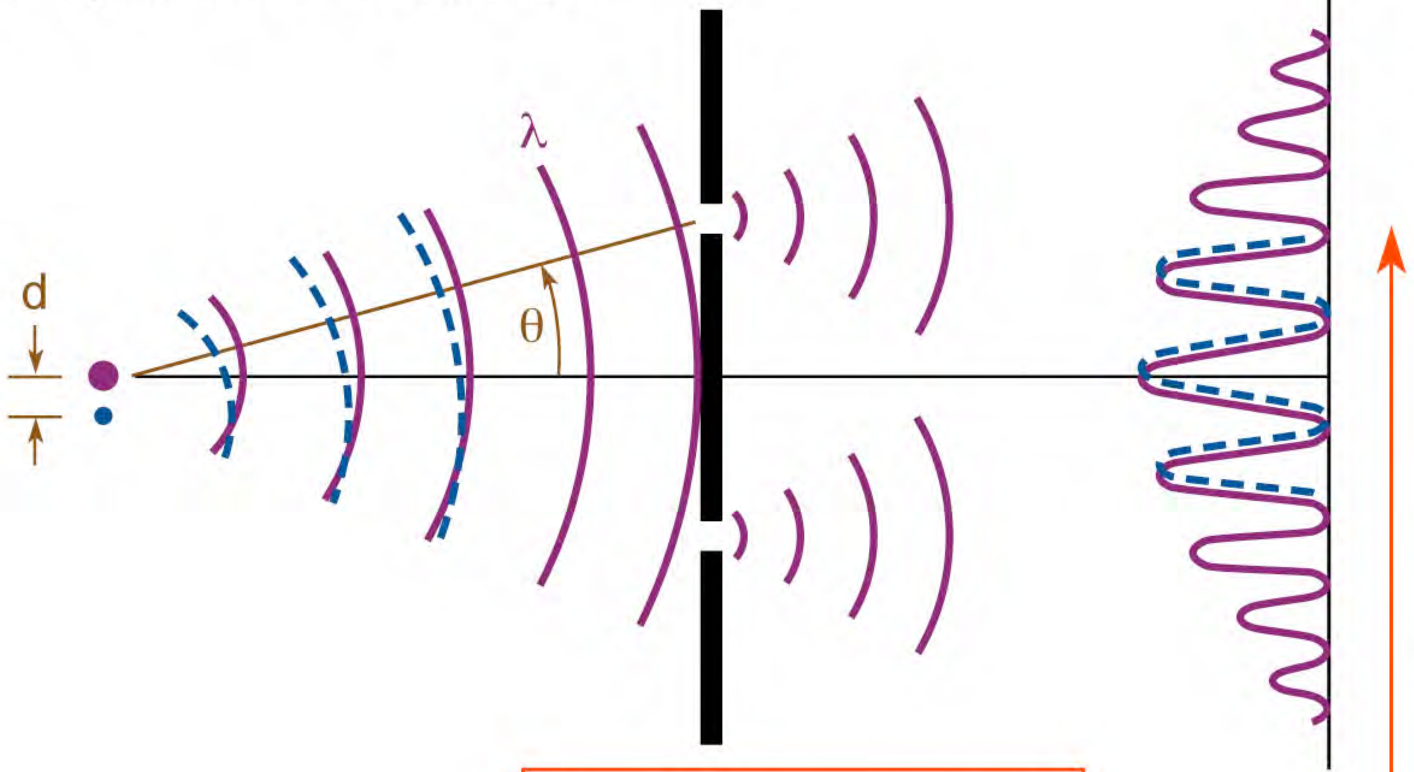
Young's double slit experiment: spatial coherence and the persistence of fringes



YoungsExprmt.ai

Young's double slit experiment: spatial coherence and the persistence of fringes

Persistence of fringes as the source grows
from a point source to finite size.

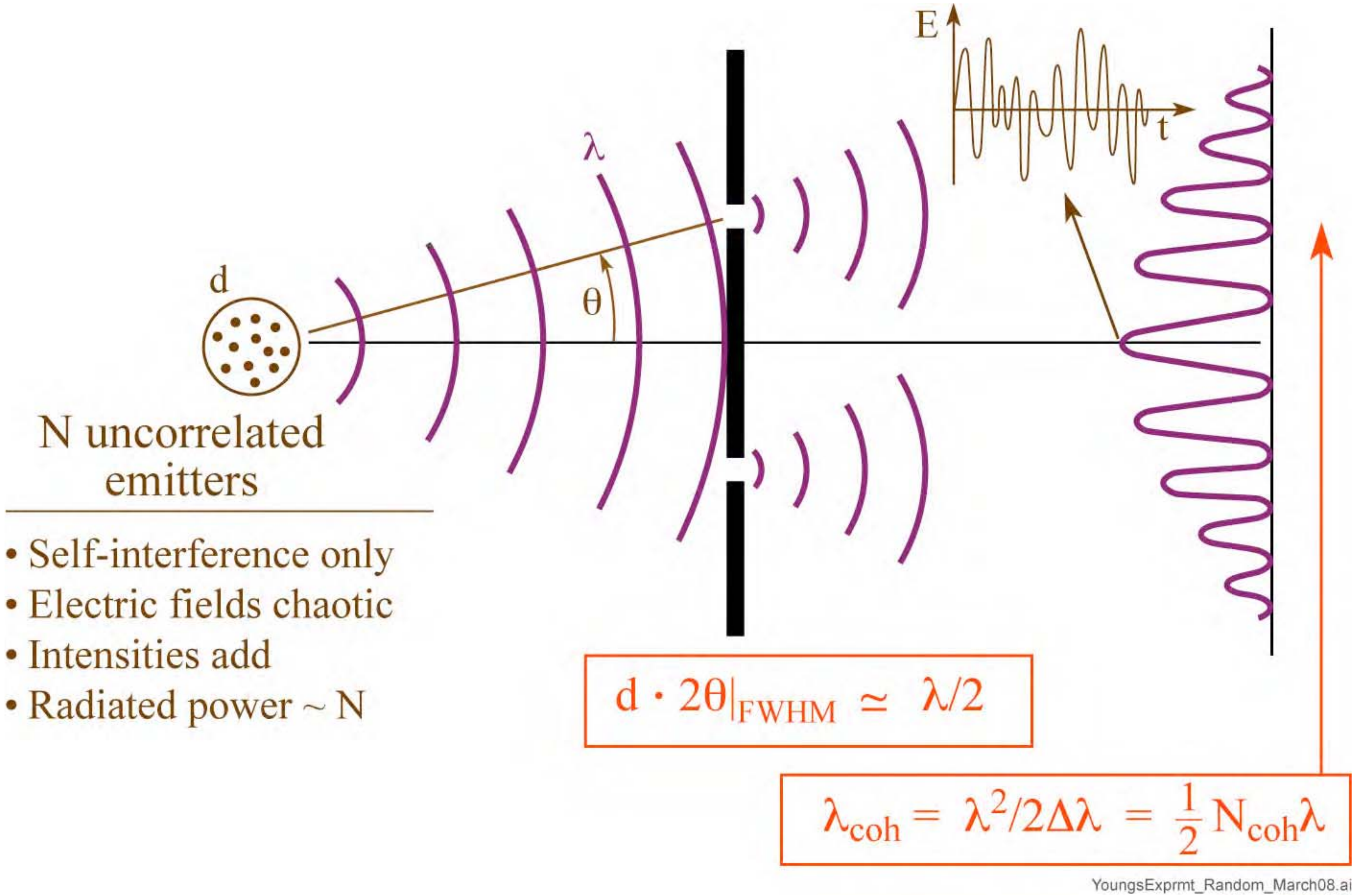


$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

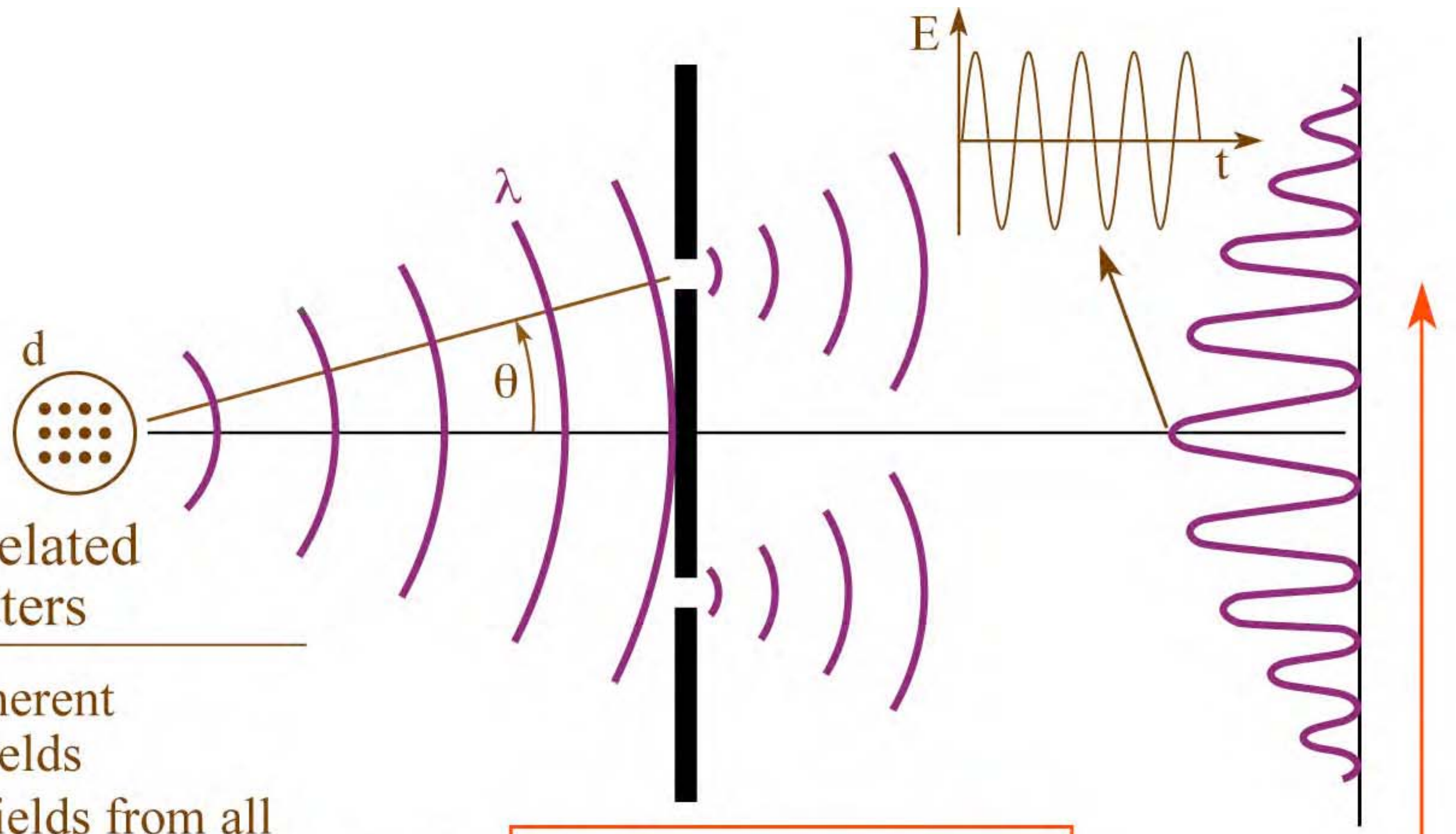
$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

CH08_YoungsExprmt_v3.ai

Young's double slit experiment with random emitters: Young did not have a laser



Young's double slit experiment with phase coherent emitters (some lasers, or properly seeded FELs)



N correlated emitters

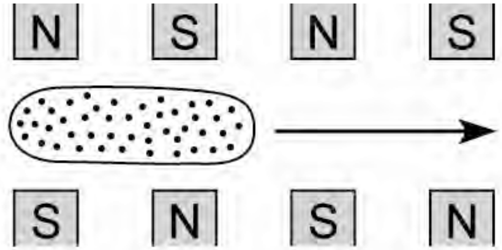
- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power ~ N^2
- New phase sensitive probing of matter possible

$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

YoungsExprmt_PhaseCoh_March08.ai

How do these concepts apply to undulators and FELs?

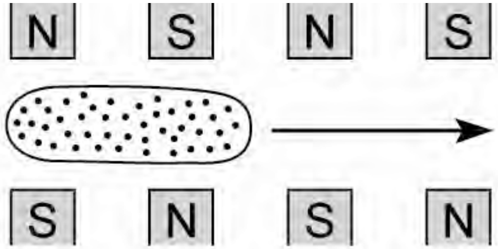


$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.

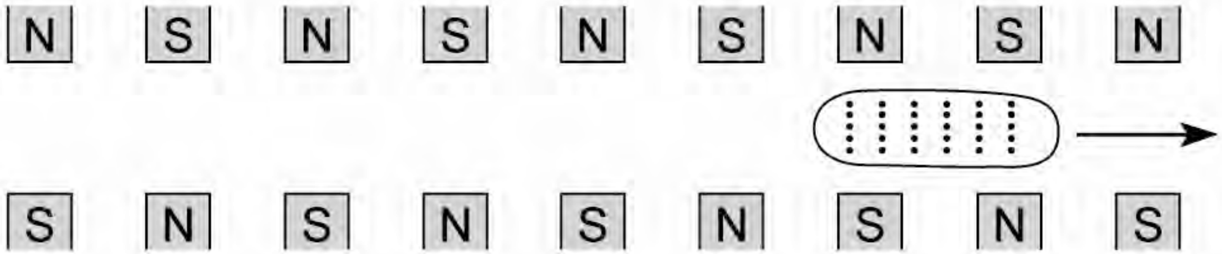
UndulatorsAndFELs1.ai

Undulators and FELs



$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.

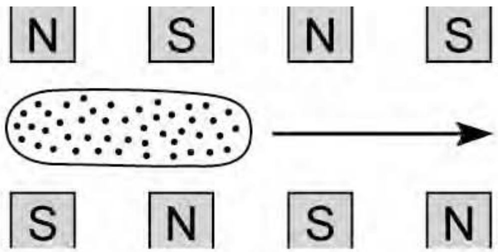


Free Electron Laser (FEL) – very long undulator, electrons are "microbunched" by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power ~ N^2

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

UndulatorsAndFELs2.ai

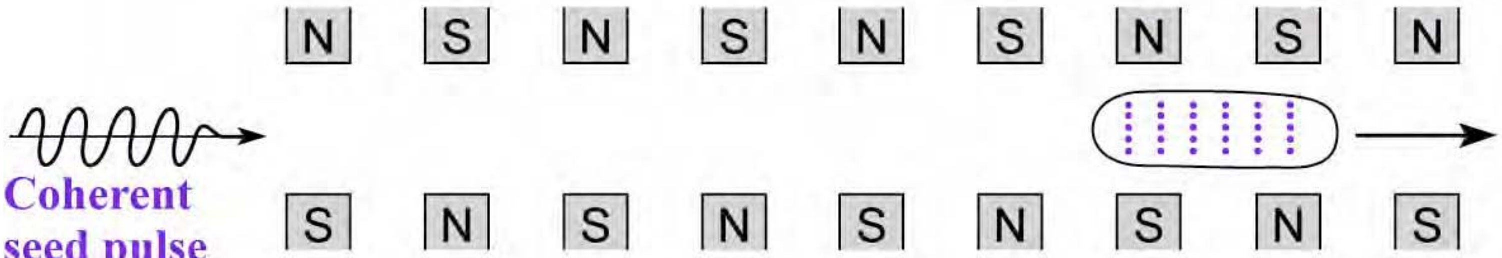
Seeded FEL



$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.

Better electron distribution throughout bunch, resulting in better coherence properties.

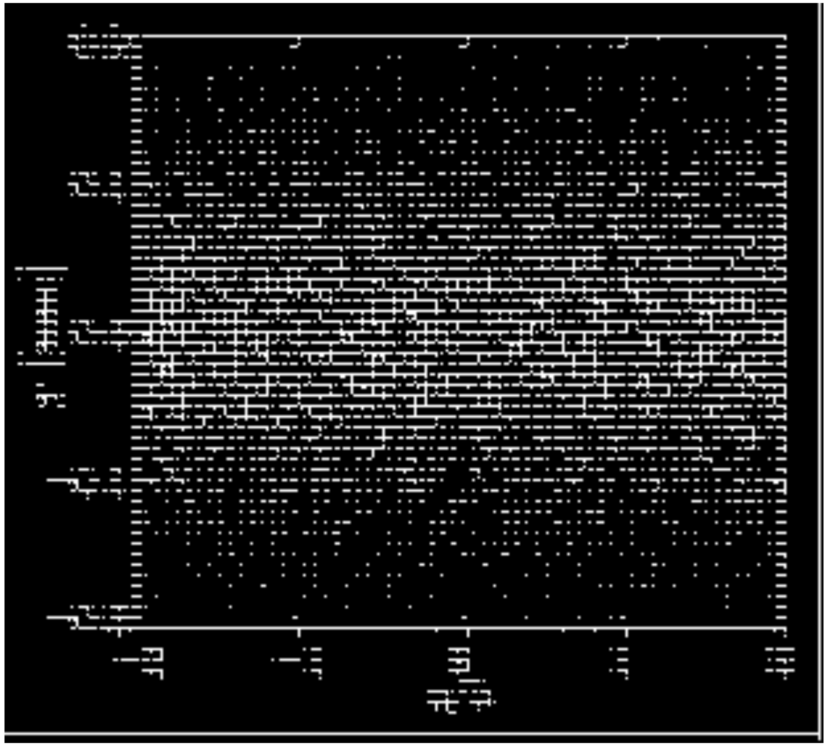


Free Electron Laser (FEL) – very long undulator, electrons are "microbunched" by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power ~ N^2

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

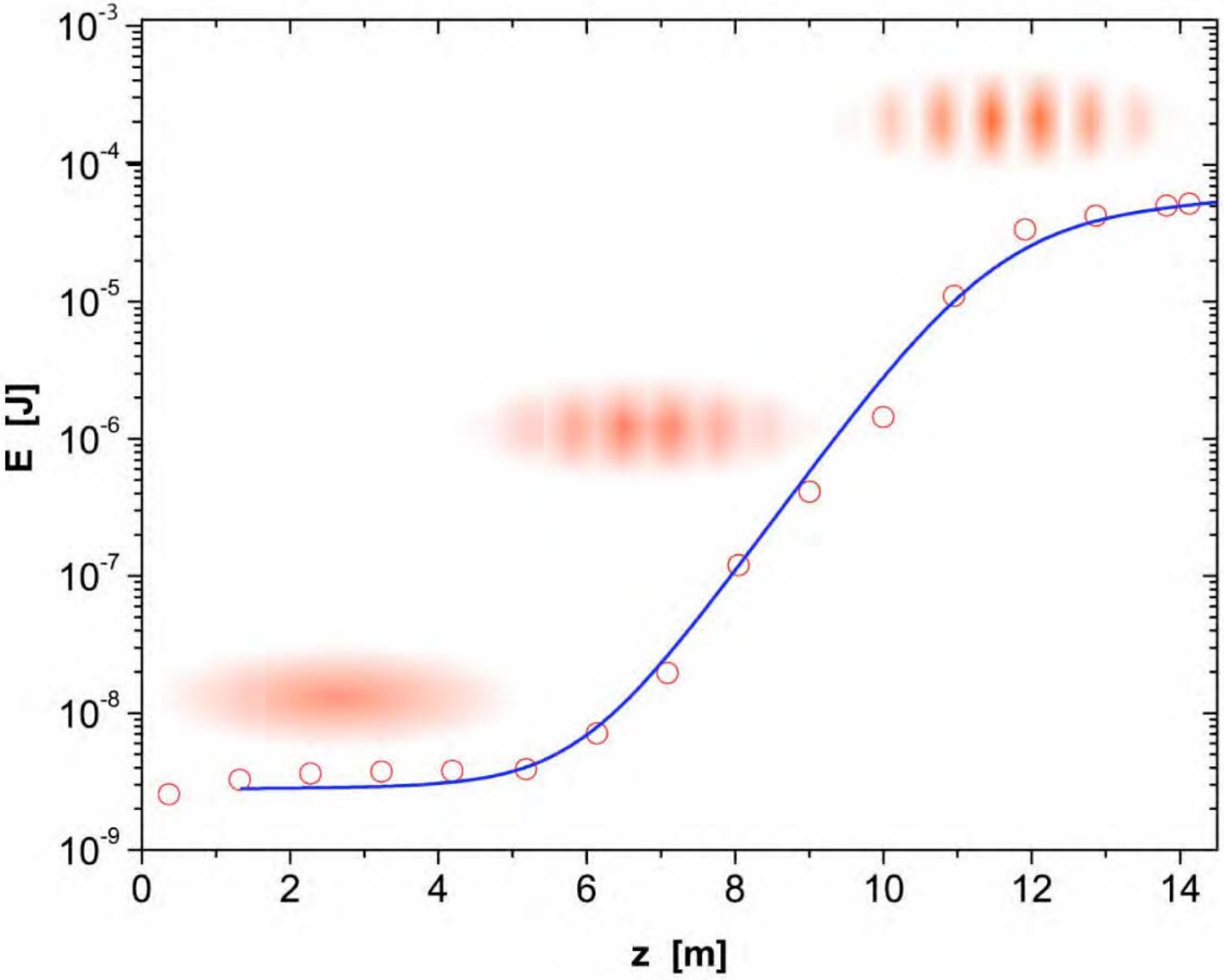
Seeded_FEL.ai

FEL Microbunching



Courtesy of Sven Reiche, UCLA, now SLS

Gain and saturation in an FEL



Courtesy of K-J. Kim

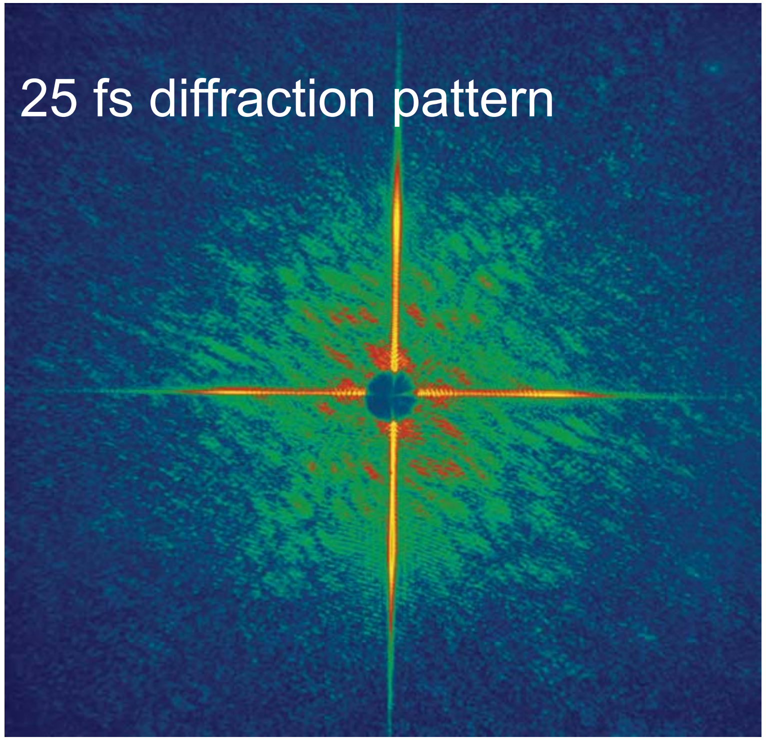
Gain_Saturation_FEL_graph.ai

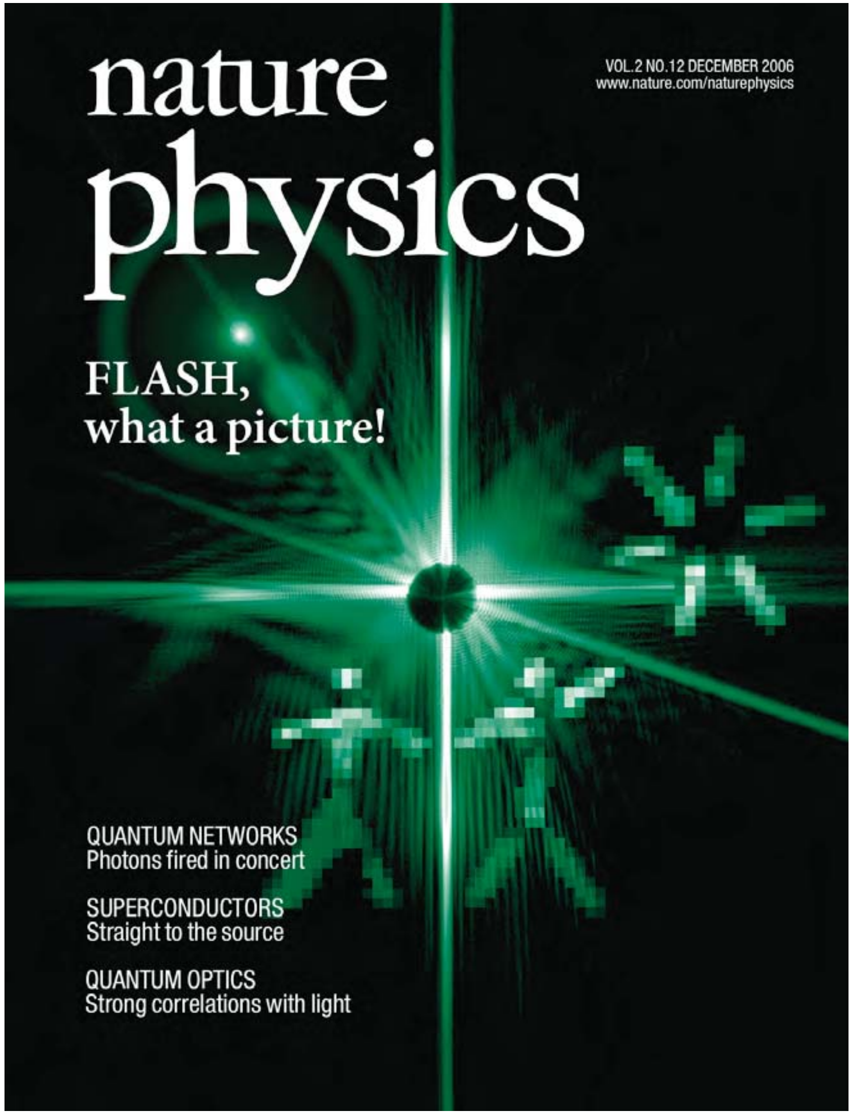
FLASH EUV/soft x-ray FEL at DESY Lab, Hamburg

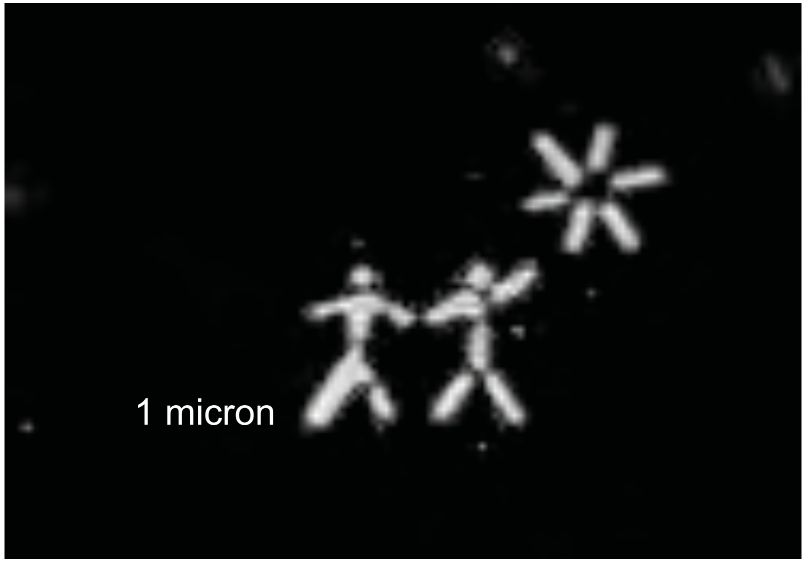
6.5-32 nm wavelength in 1st harmonic
20 fsec, 10^{12} photons per pulse

Courtesy of Henry Chapman (LLNL, now Hamburg) and Stefano Marchesini (LLNL, now LBL).

Coherent x-ray diffractive imaging with the FLASH free-electron laser (FEL) in Hamburg, Germany







Chapman et al, *Nature Phys* **2** 839 (2006)

The Linear Coherent Light Source (LCLS), an x-ray FEL at Stanford

Free Electron Lasers

Parameters	Flash FEL (Hamburg)	LCLS (Stanford, 2010)	European XFEL (Hamburg, Schenefeld; 2014)
E_e	230/1000 MeV	13.6 GeV	17.5 GeV
γ	450/2000	26,600	35,000
λ_u	2.73 cm	3 cm	5 cm
N	500/1100	3700	4000
L_u	30 m	112 m	200 m
$\hbar\omega$	50-200 eV	1-10 keV	4-12 keV
$\lambda/\Delta\lambda$	100	350	1000
e^-/bunch	10^9	6×10^9 (1 nC)	6×10^9
$\Delta\tau$	25 fsec	160 fsec	100 fsec
$\dot{\mathcal{F}}$	3×10^{12} ph/pulse	2×10^{12} ph/pulse	$10^{12} - 10^{14}$ ph/pulse
rep rate	1 Hz	120 Hz	10 Hz
$\hat{I}$	1.3 kA	3.4 kA	
$\hat{P}$	0.3 GW	8 GW	20-100 GW
$\hat{B}$	1×10^{28}	1×10^{33}	5×10^{33}
L	260 m	5 km	3.4 km

FreeElectronLasers.ai

References

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