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International Centre for Theoretical Physics*



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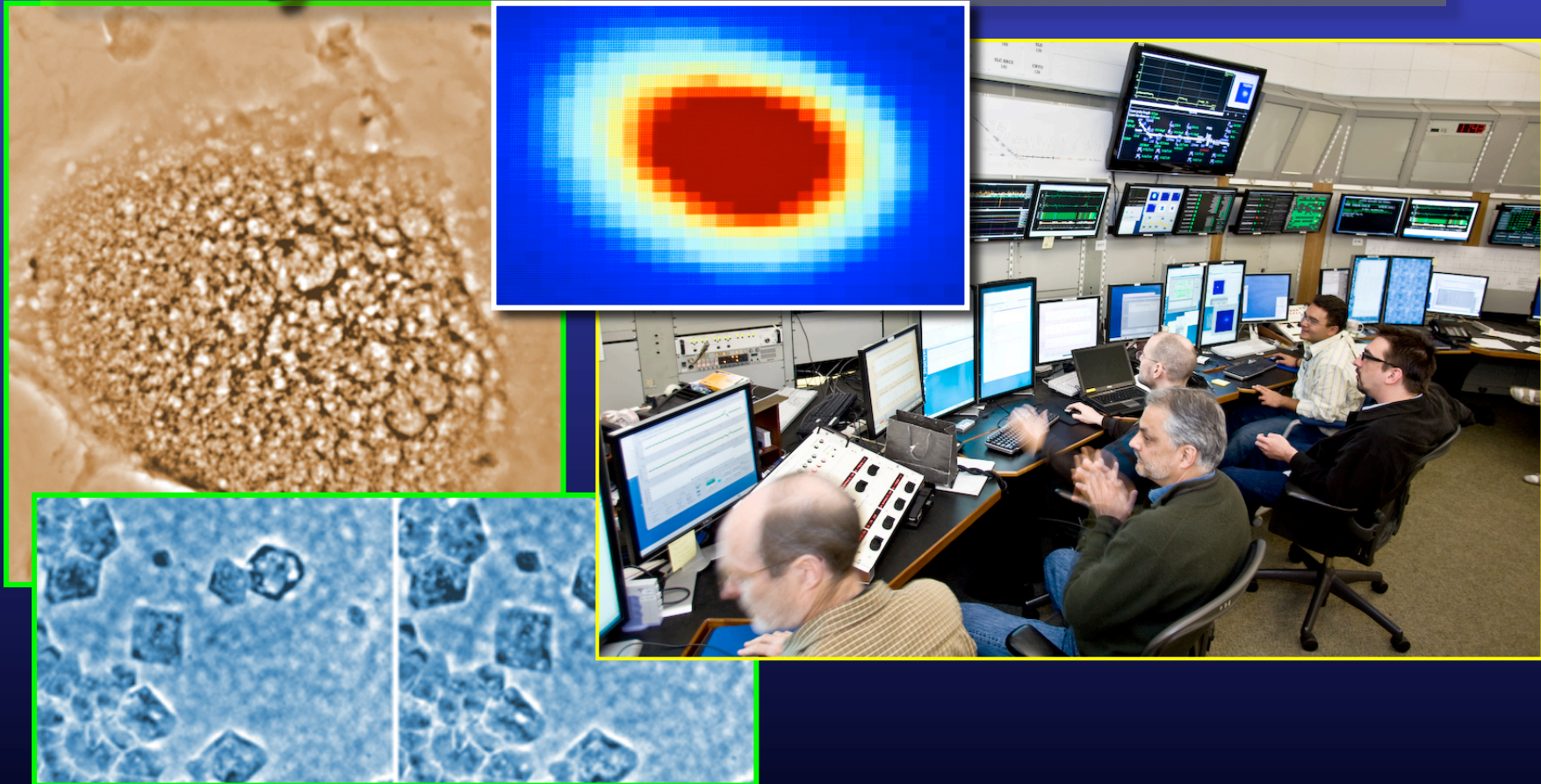
**School on Synchrotron and Free-Electron-Laser Sources and their
Multidisciplinary Applications**

26 April - 7 May, 2010

Fundamentals of Synchrotron Radiation

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Fundamentals of Synchrotron Radiation



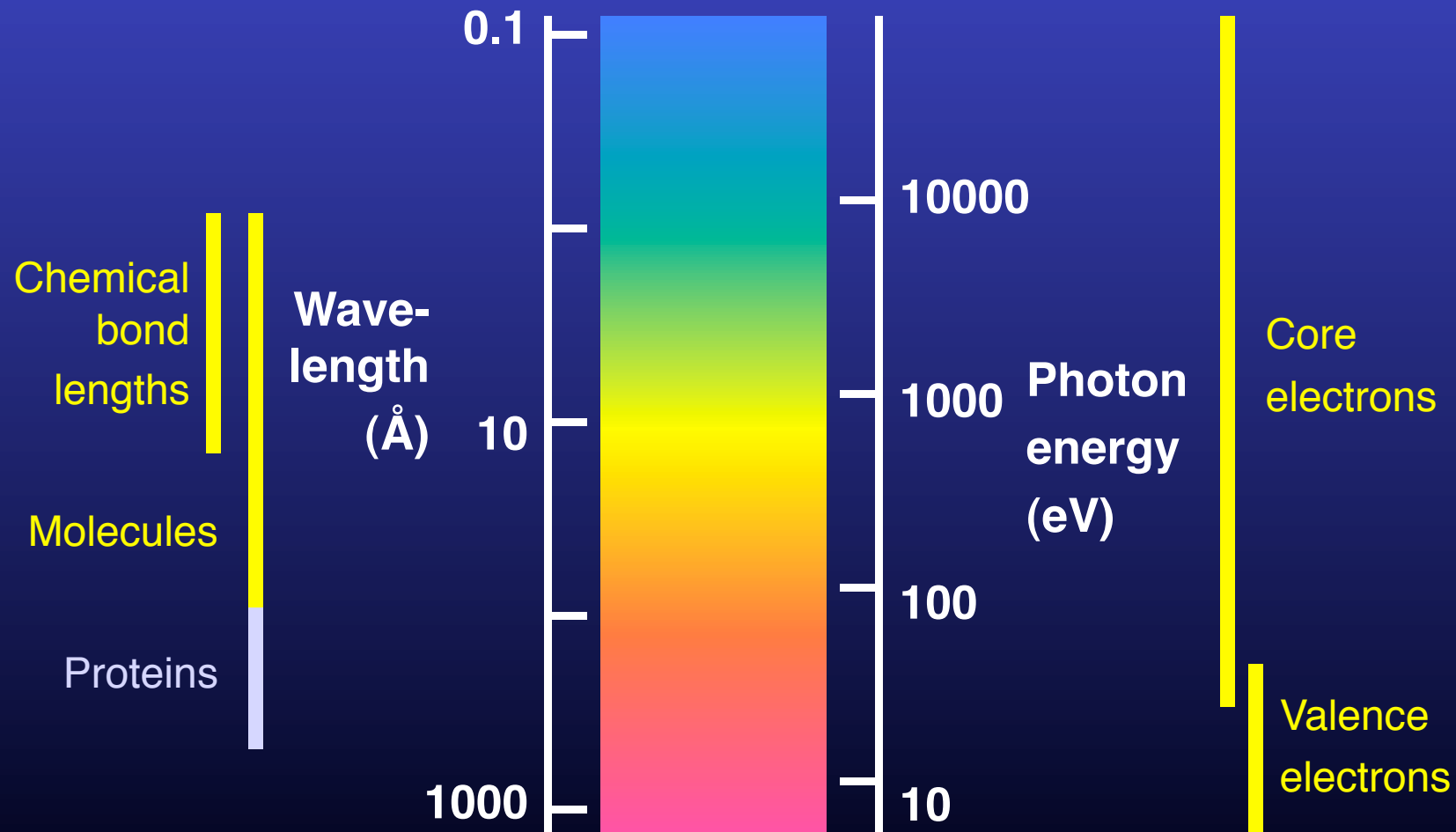
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Ecole Polytechnique Fédérale de Lausanne (EPFL)

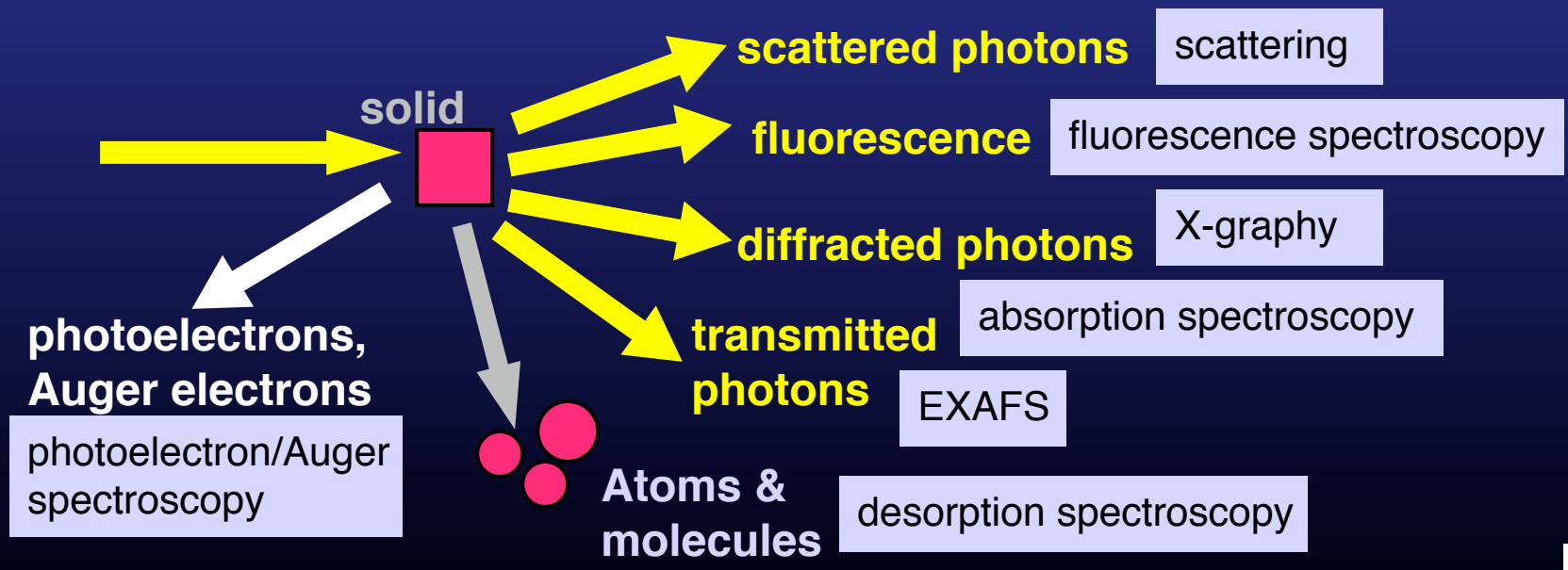
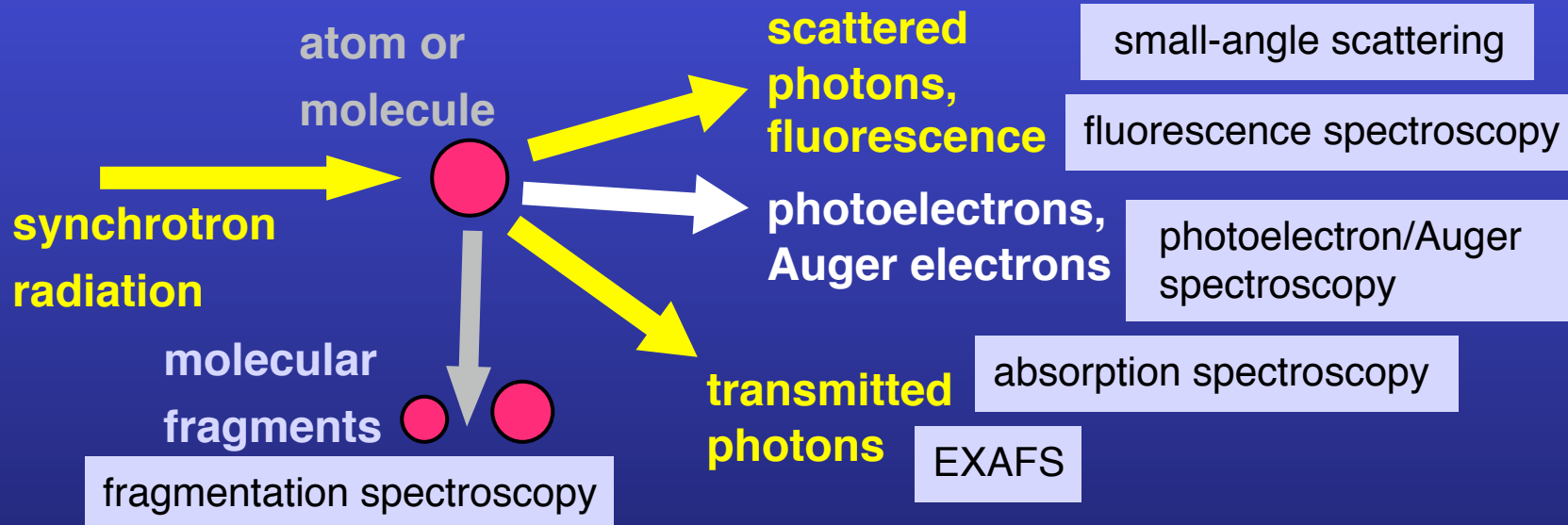
Outline:

- **Why do we need x-rays?**
- **How to build an excellent x-ray source using Einstein's relativity:**
 - 3.5 minute presentation
 - 9.5 minute presentation
 - Everything else
- **History**
- **Coherence: a revolution in radiology**
- **Future: from storage rings to free electron lasers**

Why x-rays and ultraviolet?



Synchrotron x-rays:
Many different interactions
↓
Many different applications



SO, WE DO NEED SYNCHROTRONS: BUT HOW DO THEY WORK? THE "BEACH OPTION PROGRAM":

START!

STEP A (3.5 minutes): how are x-ray produced?

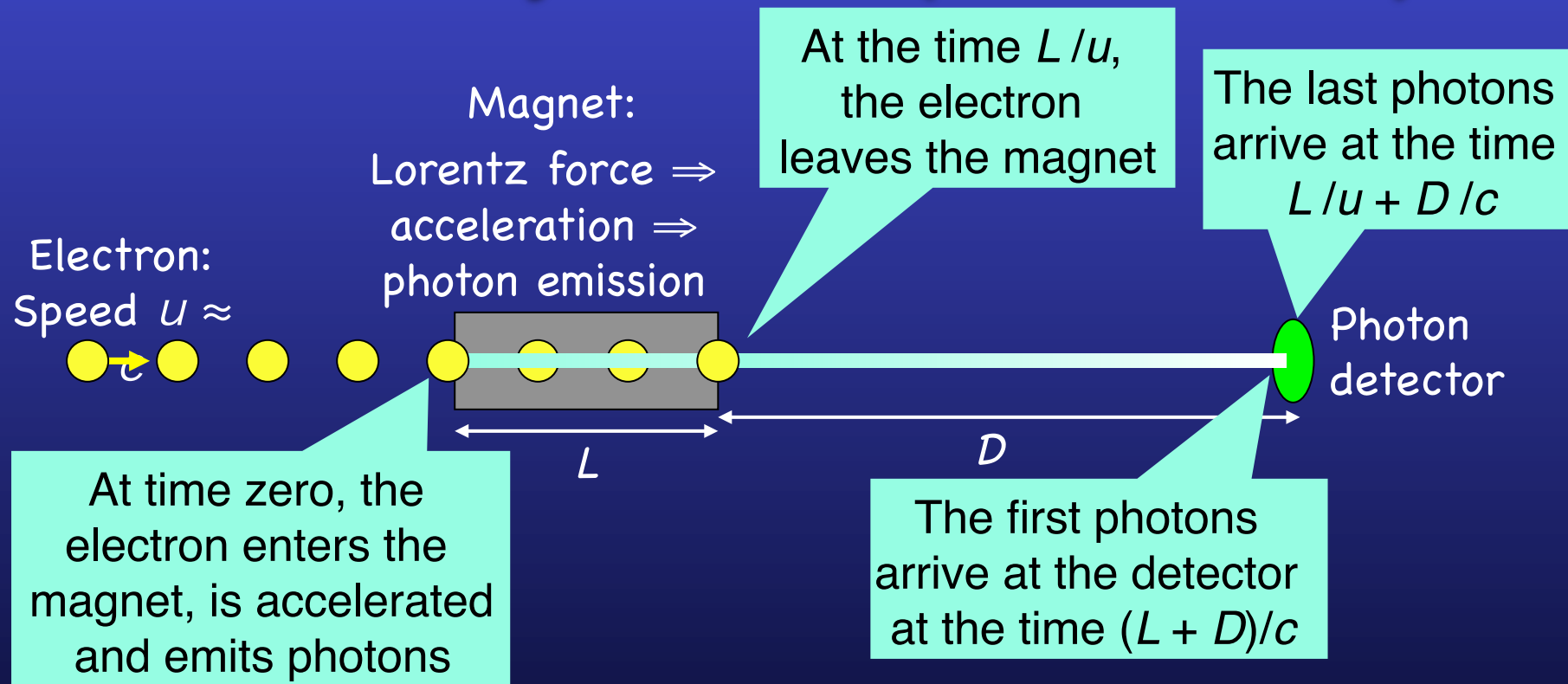
NEXT OPTIONS: (1) go to the beach for the day, or
(2) go to step B

STEP B (9.5 minutes): how to get
collimation? And, again, how are x-ray produced?

NEXT OPTIONS: (1) go to the beach for the day, or
(2) go to step C

STEP C (the rest of the time... maybe more):
(almost) everything else about synchrotrons

Synchrotron light in 3.5 minutes for lazy students (and teachers):



Photon pulse duration: $\Delta t = L/u - L/c = (L/u) (1 - u/c)$

Characteristic frequency: $\nu = 1/\Delta t = u/[L(1 - u/c)] = u \gamma^2 (1 + u/c)/L$

For $u \approx c$, $(1 + u/c) \approx 2$ and $\nu \approx 2c\gamma^2/L$

For $L = 0.1$ m and $\gamma = 4000$, $\nu \approx 10^{17}$ s⁻¹ -- x-rays!

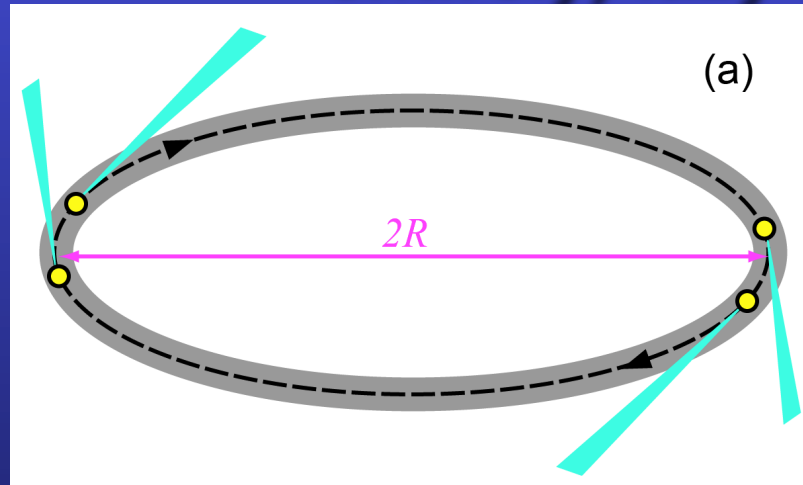
$$\gamma^2 = 1/(1 - u^2/c^2)$$

And now, we can
go to the beach
for the day!



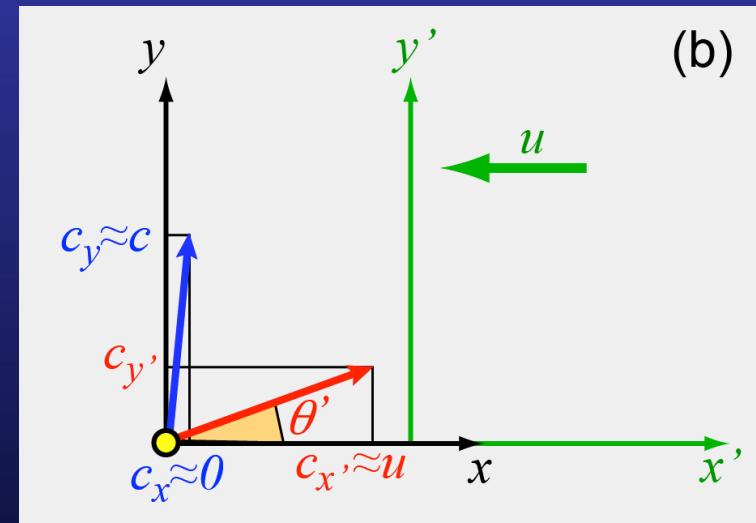
...Well, not really, Maya (sorry)!!!

Synchrotron light in 9.5 minutes for (not entirely) lazy students (and teachers):



Electrons circulating at a speed $u \approx c$ in a storage ring emit photons in a narrow angular cone, like a "flashlight": why?

Answer: RELATIVITY

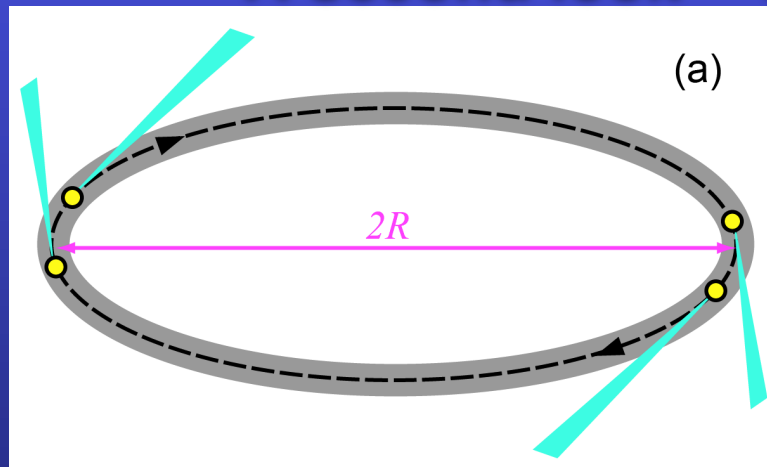


But in the laboratory frame the emission shrinks to a narrow cone

Seen in the electron reference frame, the photons are emitted in a wide angular range

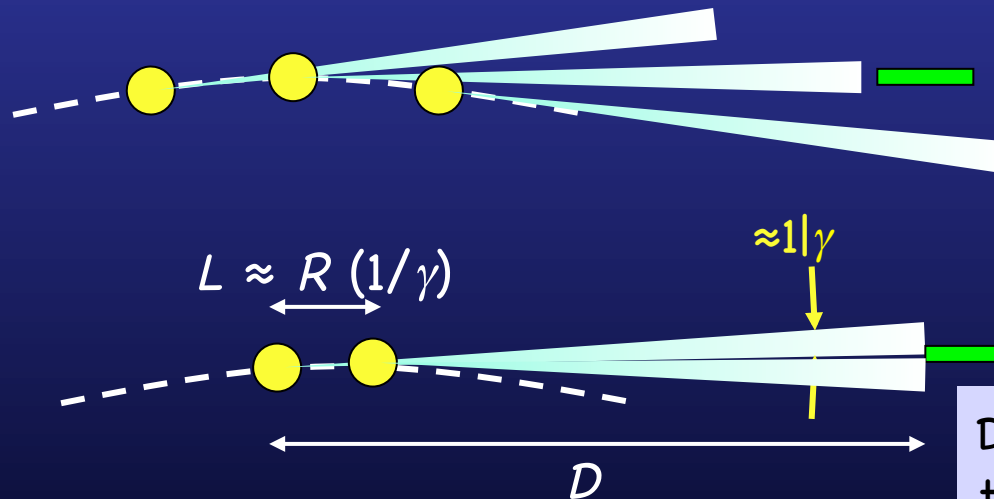
Take a photon emitted (blue arrow) in a near-transverse direction in the (black) electron frame, with velocity components $c_x \approx 0$, $c_y \approx c$. In the (green) laboratory frame the velocity (red arrow) components become $c'_x \approx u$, $c'_y \approx (c^2 - u^2)^{1/2} = c/\gamma$. The angle θ' is $\approx c'_y/c = 1/\gamma$ -- very narrow!!!

A second look -- the emission is x-rays: why?



Seen from the side of the ring, each electron looks like an oscillating charge in an antenna, emitting photons with a frequency $2\pi R/c$ -- in the radio wave range.

What shifts the emission to the x-ray range? RELATIVITY AGAIN!



A torchlight-electron illuminates a **small-area detector** once per turn around the ring for a short time Δt

Photons start to be detected at the time D/c

Detection ends at the time $L/u + (D - L)/c$

Photon pulse duration:

$$\Delta t = L/u + (D - L)/c - D/c = L/u - L/c = (L/u) (1 - u/c) = (L/u) \gamma^2 / (1 + u/c)$$

For $u \approx c$, $(1 + u/c) \approx 2$ and $\Delta t \approx L/(2c\gamma^2) \approx R/(2c\gamma^3)$.

Characteristic frequency $\nu = 1/\Delta t \approx 2c\gamma^3/R$ -- again, x-rays

And now, we
can really go to
the beach!

Yuppe!

Grrrrrrr!



Sorry again, Maya & Nadia, not yet!!!

Fireplaces and torchlights :



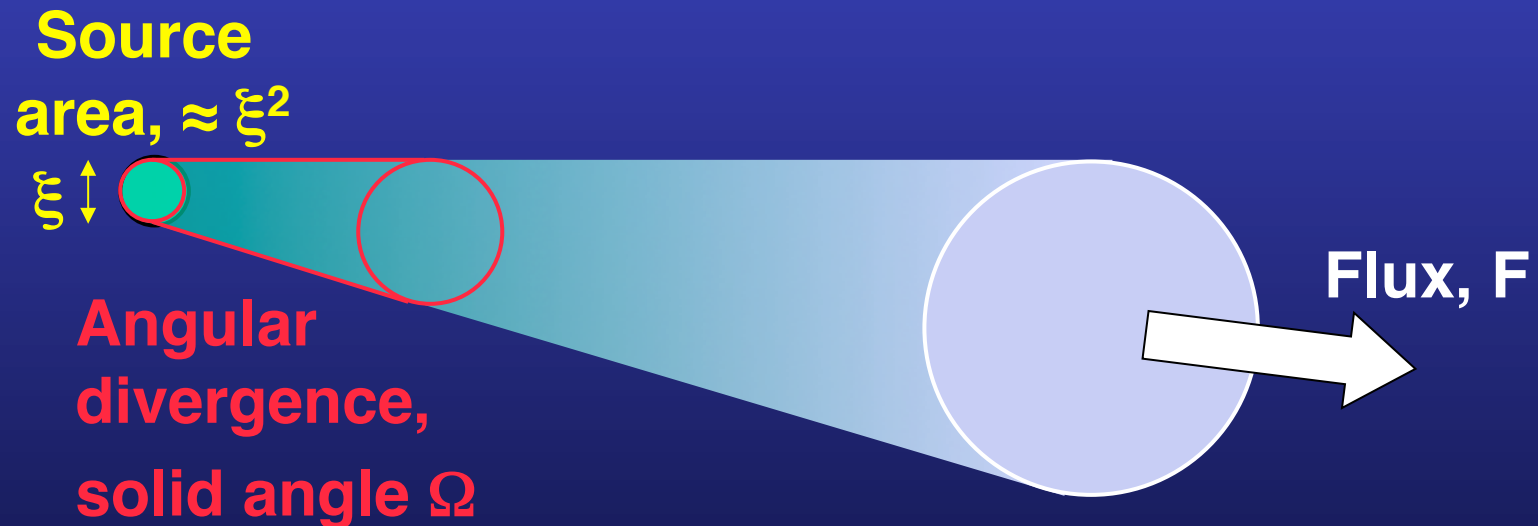
A fireplace is not very effective in "illuminating" a specific target: its emitted power is spread in all directions



A torchlight is much more effective: it is a small-size source with emission concentrated within a narrow angular spread

**This can be expressed
using the "brightness"**

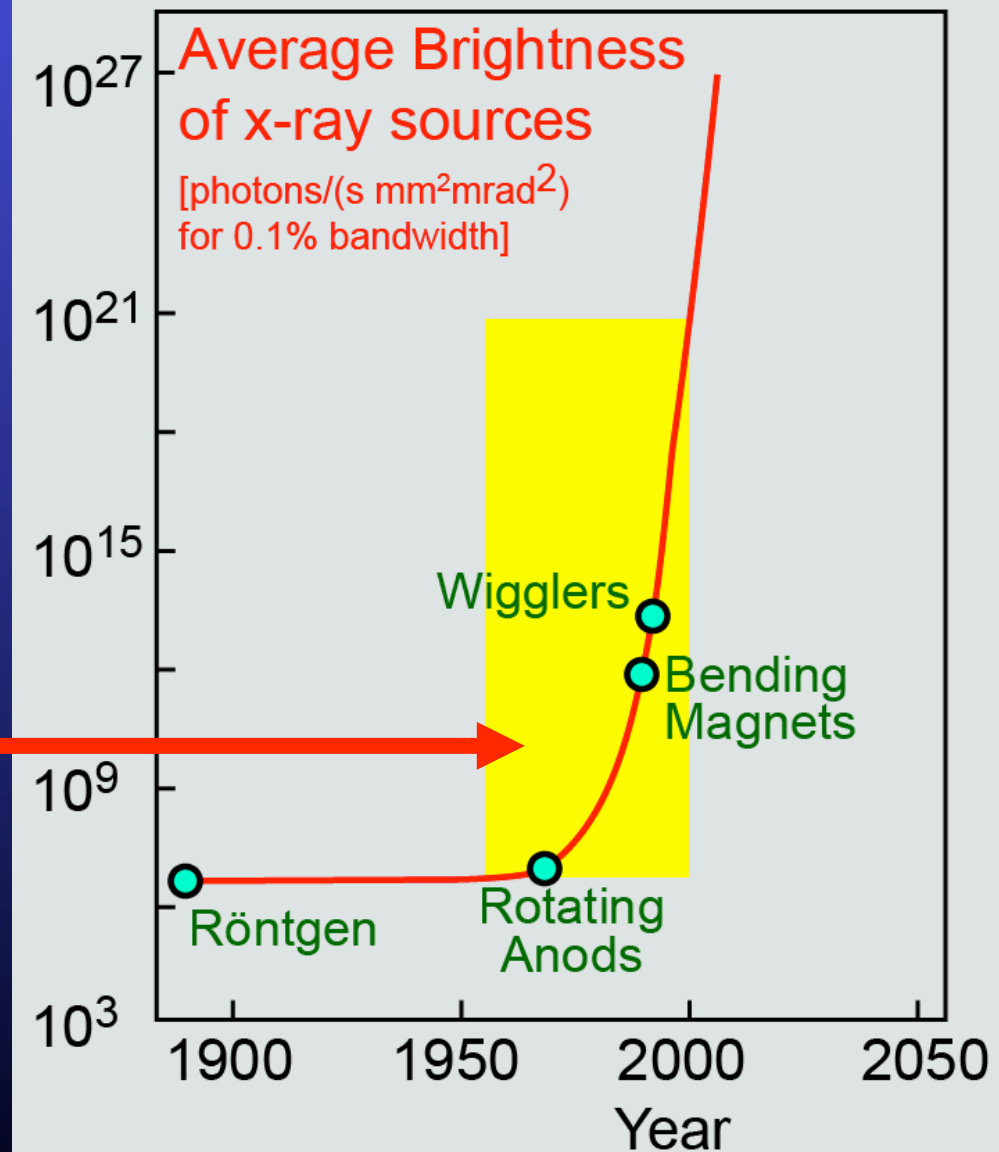
The “brightness” (or brilliance) of a source of light :



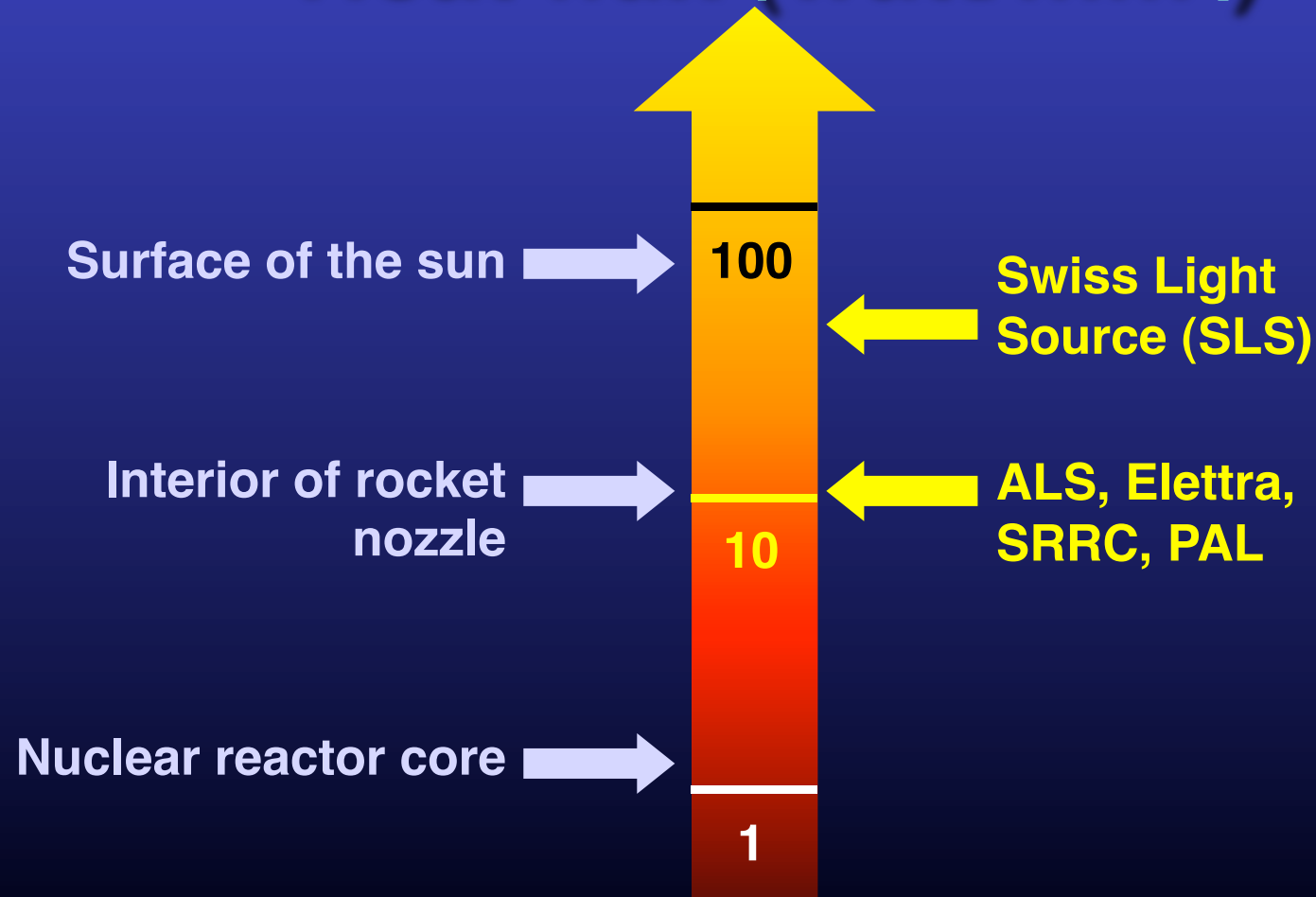
$$\text{Brightness} = \text{constant} \frac{F}{\xi^2 \Omega}$$

The historical growth in x-ray brightness/brilliance

Between 1955 and 2000, the brightness increased by more than 15 orders of magnitude... whereas the top power of computing increased "only" by 6-7 orders of magnitude



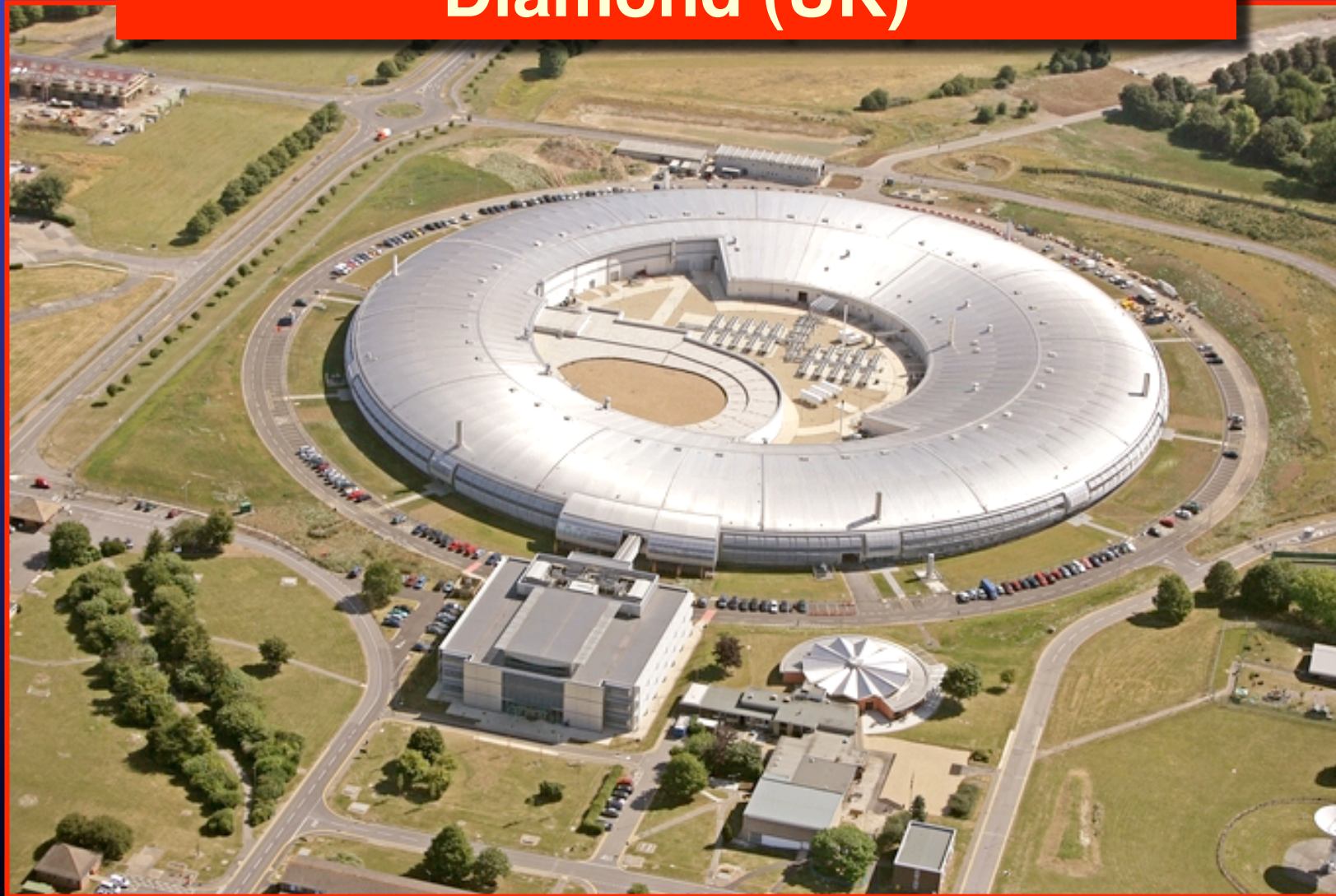
Heat flux (watt/mm²)



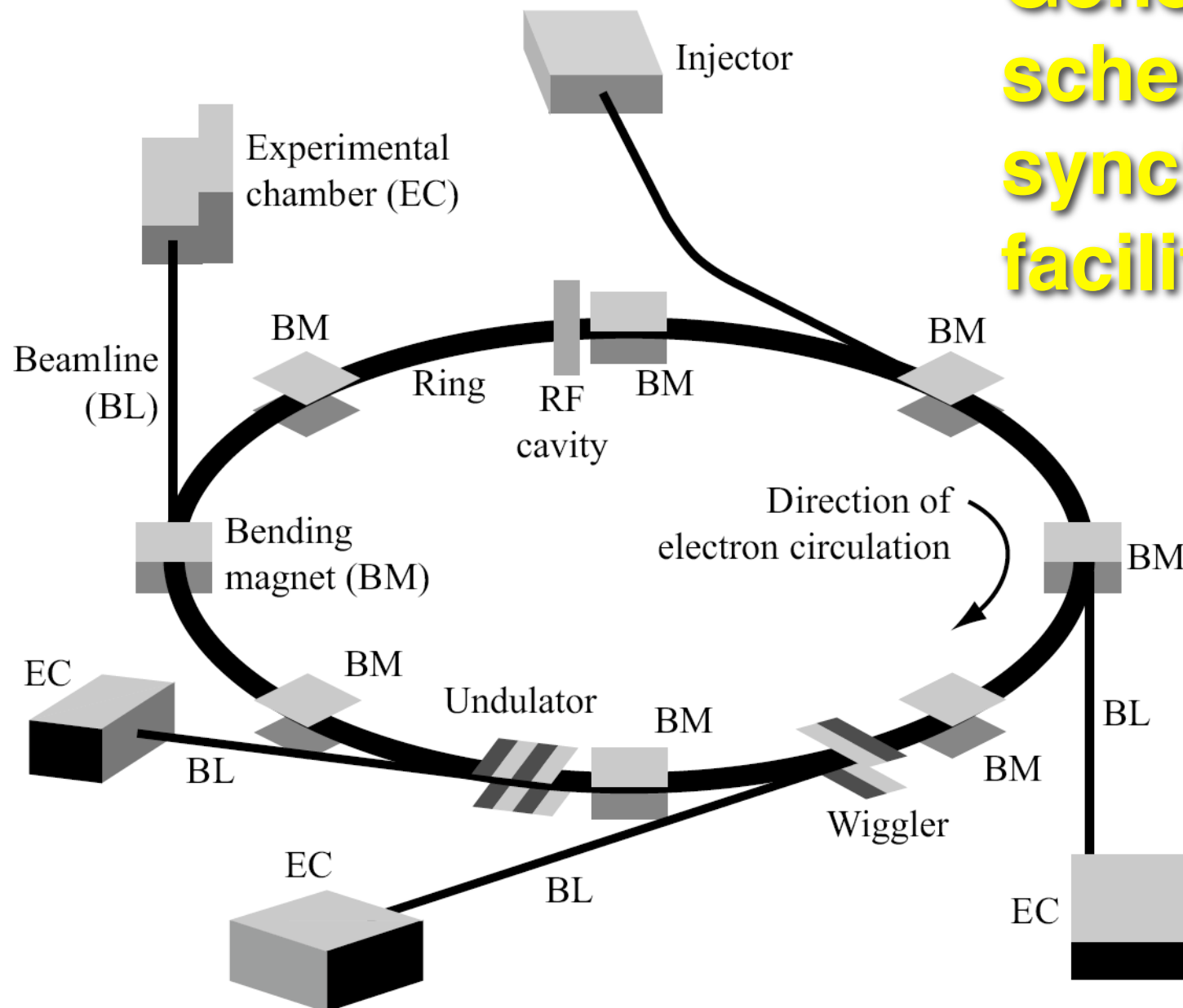
What produces the high brightness of a synchrotron source?

- Free electrons can emit more power than electrons bound in a solid because the power does not damage their environment \Rightarrow **high flux**
- The control of the electron beam trajectories in a synchrotron source (storage ring) is very sophisticated, producing a small transverse beam cross section \Rightarrow **small photon source size**
- Relativity drastically reduces the **angular divergence** of the emitted synchrotron radiation

A real synchrotron facility: Diamond (UK)

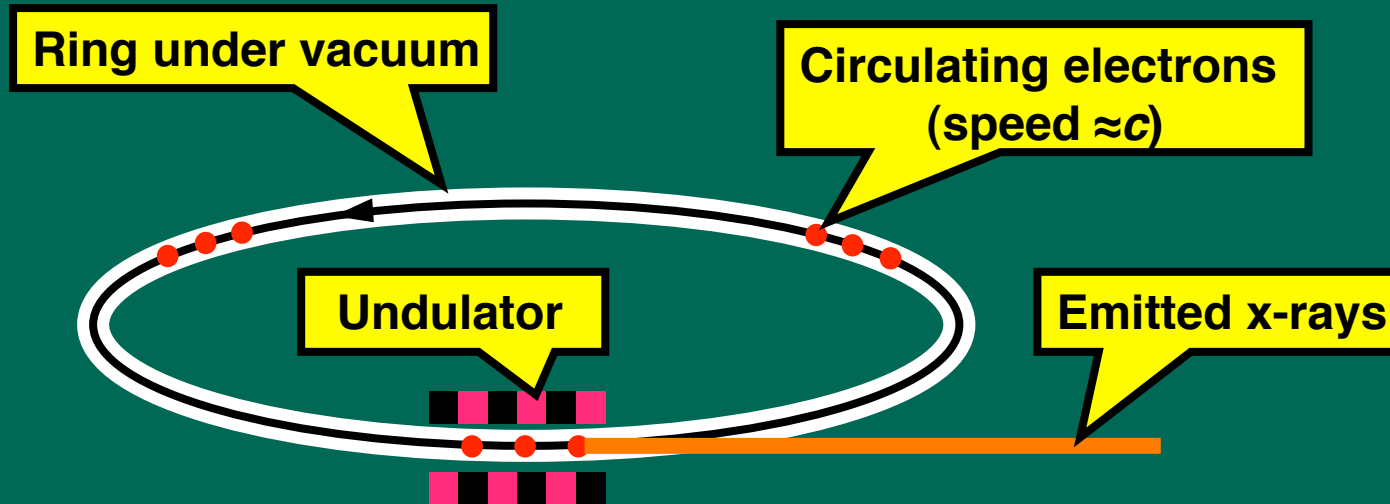


General scheme of a synchrotron facility



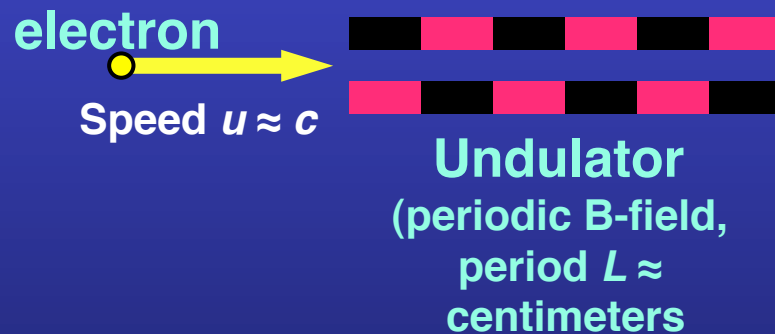
Objective: building a very bright x-ray source.

Solution: relativity!!



- The undulator (periodic magnet array) period determines the emitted wavelength. This period is **shortened by the relativistic “Lorentz contraction”** giving x-ray wavelengths
- The emitted x-rays are “**projected ahead**” by the motion of their sources (the electrons), and therefore collimated. Relativity enhances the effect

Objective: building a very bright x-ray source using an “undulator” and relativity



In the undulator (laboratory) frame, the electron moves at speed $\approx c$

The period L is Lorentz-contracted becoming $\approx L/\gamma$

In the electron frame:

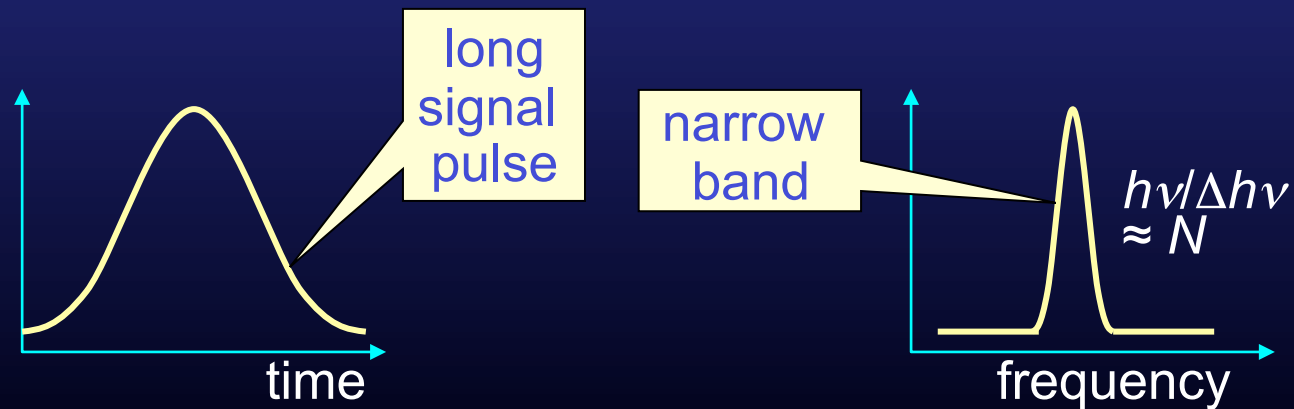
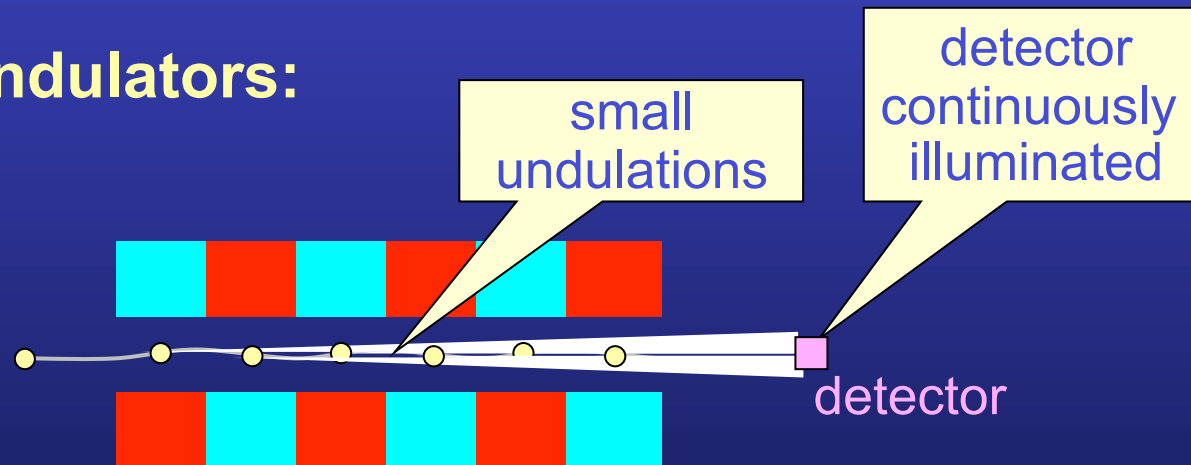


The periodic B-field is accompanied by a perpendicular periodic E-field. Moving at a speed $\approx c$ towards the electron, the undulator looks like an electromagnetic wave with wavelength L/γ . Synchrotron radiation is produced by the elastic scattering of this wave by the electron.

Back to the laboratory frame, the wavelength L/γ emitted by the moving electron is Doppler-shifted by a factor $\approx 2\gamma$, becoming $L/2\gamma^2$. The “macroscopic” undulator period is transformed into x-ray wavelengths!

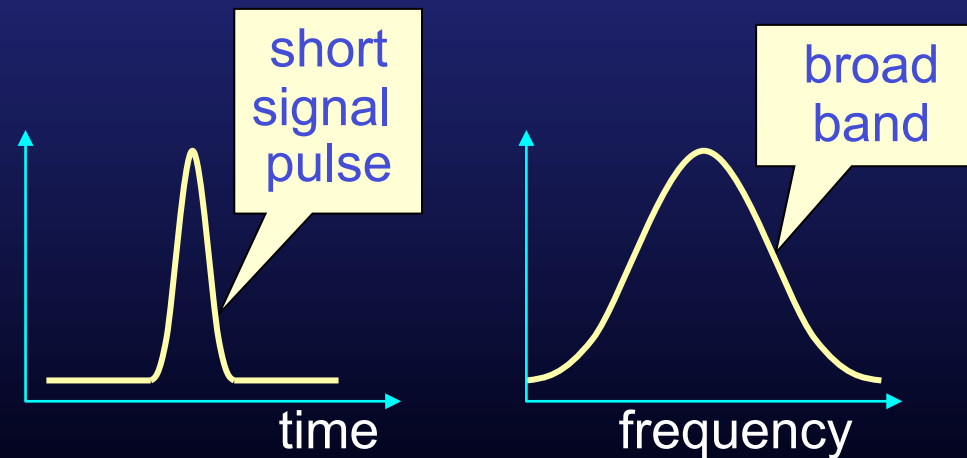
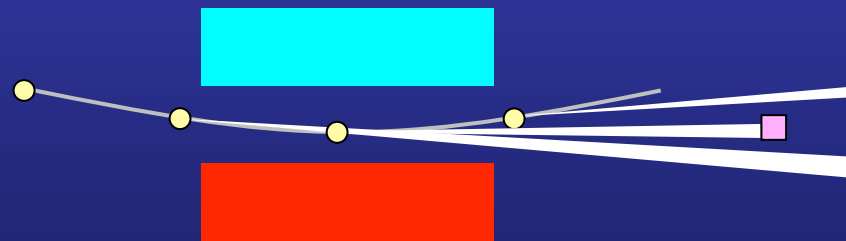
3 types of sources:

1. Undulators:



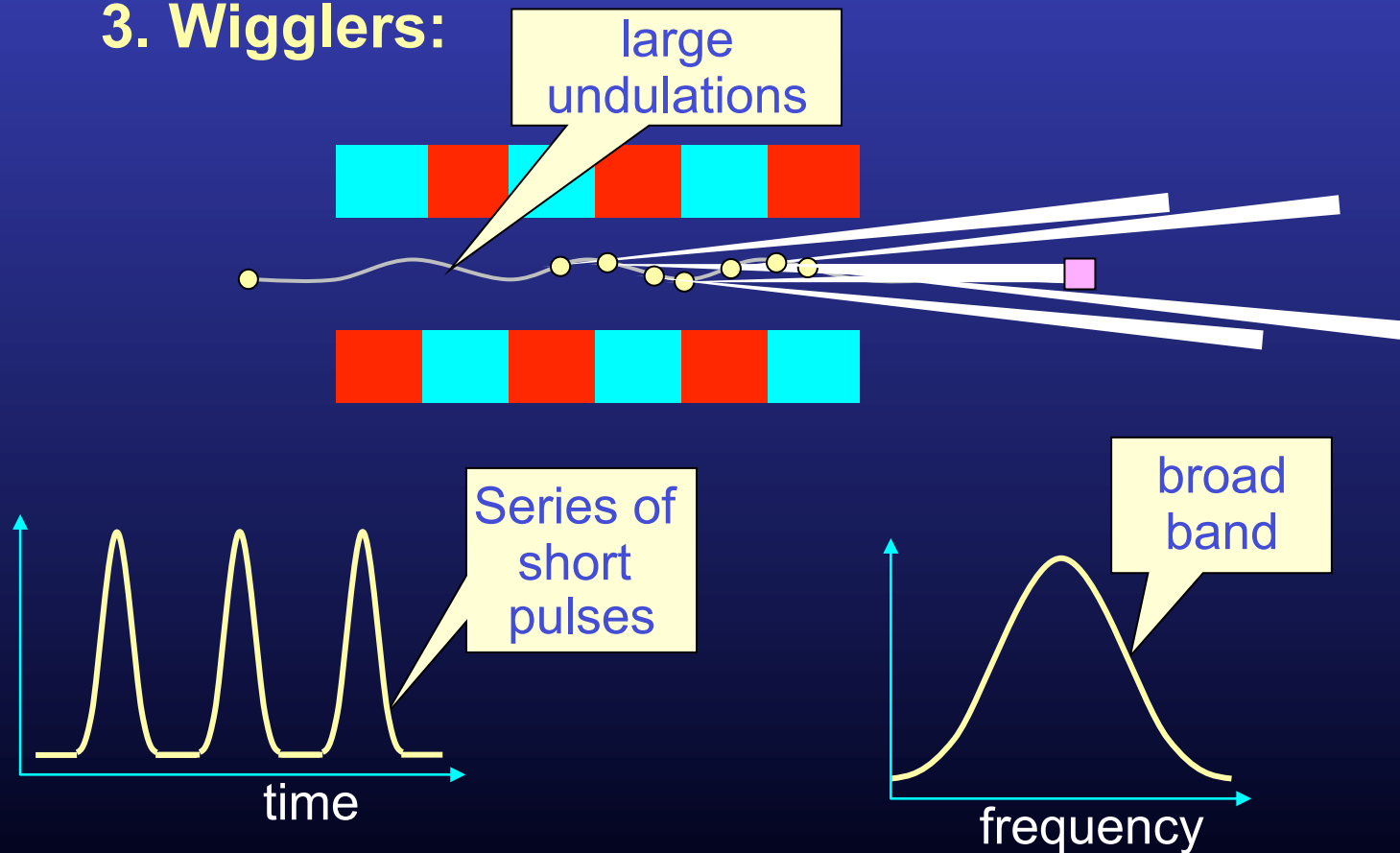
3 types of sources:

2. Bending magnets:



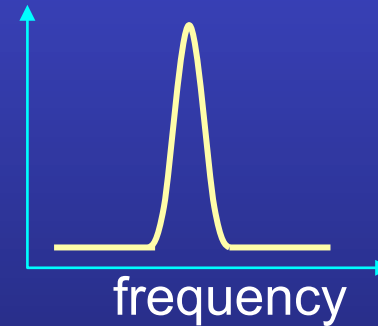
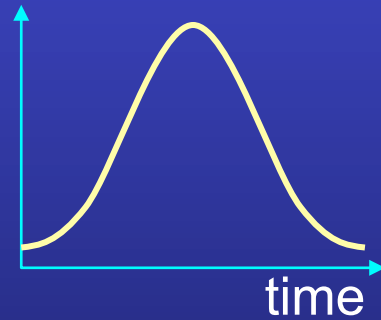
3 types of sources:

3. Wigglers:

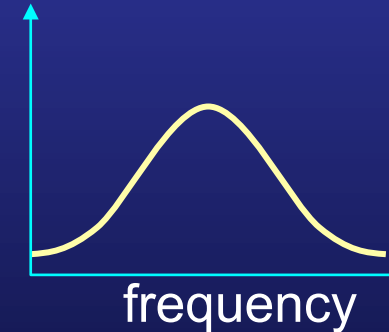
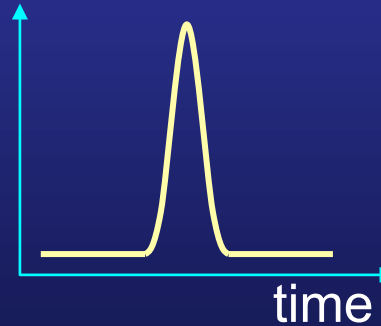


3 types of sources - summary:

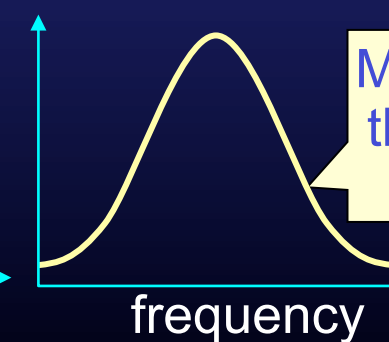
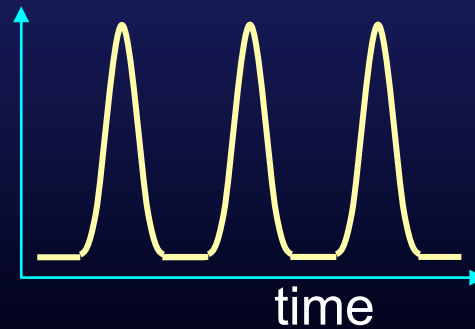
Undulators:



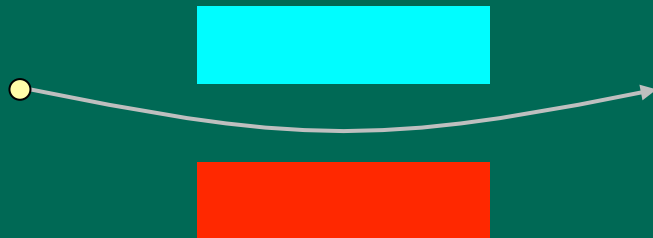
Bending magnets:



Wigglers:



Bending magnet emission spectrum:

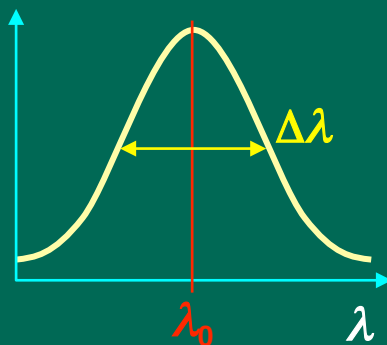
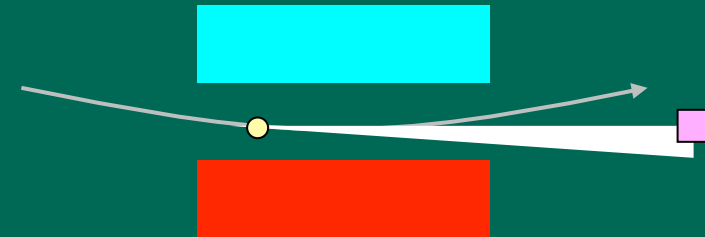


The (relativistic) rotation frequency of the electron determines the (Doppler-shifted) central wavelength:

$$\lambda_0 = (1/2\gamma^2)(2\pi cm_0/e)(1/B)$$

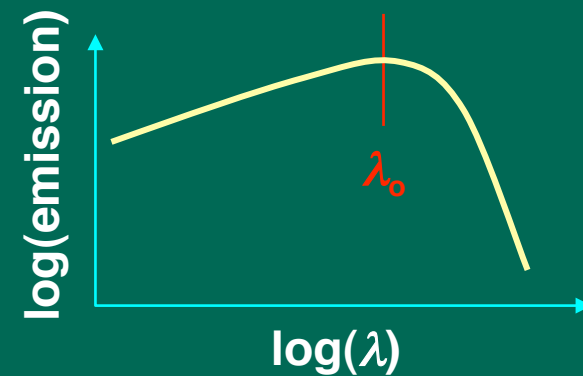
The “sweep time” δt of the emitted light cone determines the frequency spread $\delta\nu$ and the wavelength bandwidth:

$$\Delta\lambda / \lambda_0 = 1$$



A peak centered at λ_c with width $\Delta\lambda$: is this really the well-known synchrotron spectrum?

YES -- see the log-log plot:



Undulator emission spectrum:

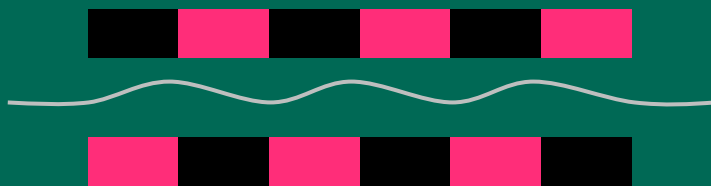
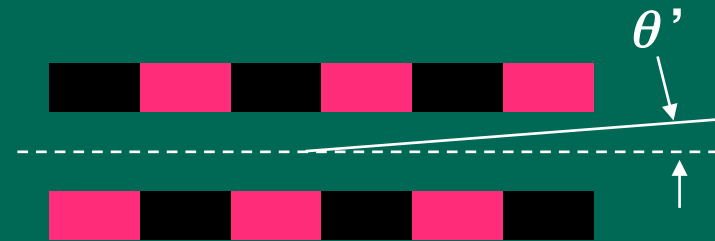


$L = \text{period}$

Central wavelength: $L/2\gamma^2$

First correction: out of axis, the Doppler factor is not $2\gamma^2$ but changes with θ'

Central wavelength: $(L/2\gamma^2)/(1 + 2\gamma^2\theta'^2)$

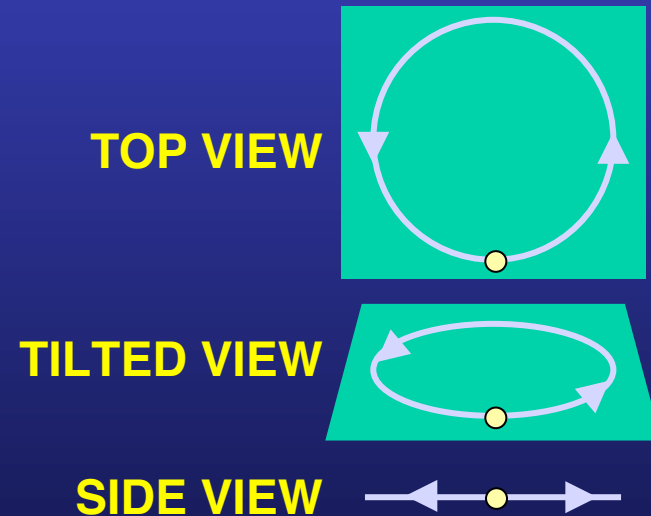


Second correction: stronger B-field means stronger undulations and less on-axis electron speed. This changes γ so that:

Central wavelength: $(L/2\gamma^2)/(1 + aB^2)$

Synchrotron light polarization:

Electron in a storage ring:



Polarization:
Linear in the plane of the ring,
elliptical out of the plane

Special (elliptical) wigglers and undulators can provide elliptically polarized light with high intensity