



**The Abdus Salam
International Centre for Theoretical Physics**



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**School on Synchrotron and Free-Electron-Laser Sources and their
Multidisciplinary Applications**

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Inelastic x-ray scattering: principles

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Inelastic x-ray scattering: principles



Filippo Bencivenga



OUTLINE

Introduction

High resolution inelastic x-ray scattering (IXS)

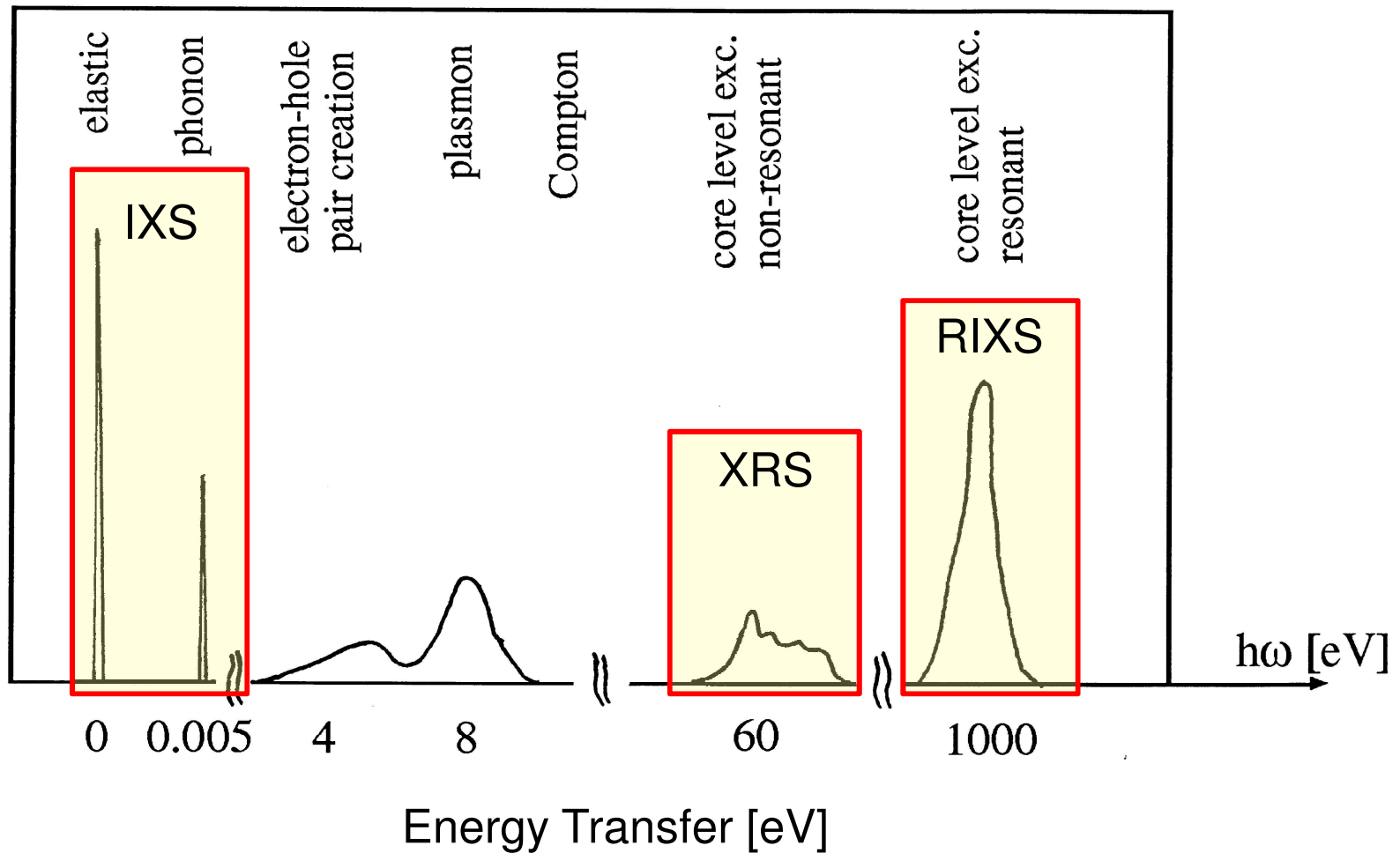
- Collective atomic dynamics
- Neutrons vs. X-rays
- Basic theory and instrumentation
- Experimental highlights

Inelastic x-ray “Raman” scattering (XRS)

- Experimental/theoretical aspects
- Scattering vs. absorption spectroscopy
- Experimental highlights

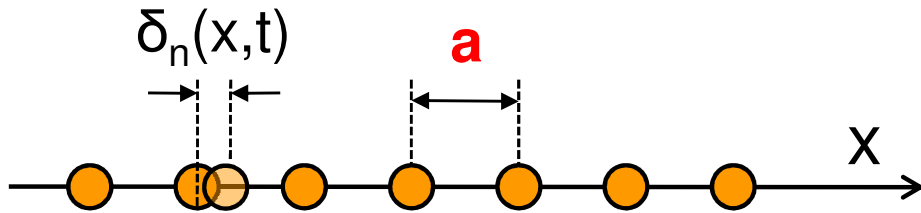
Resonant inelastic x-ray scattering (RIXS)

Introduction: inelastic X-ray spectrum



IXS: collective atomic dynamics

The simpler case



Information:

- Interatomic Structure (a)
- Interaction Potential (β)

$$U = -\beta x^2$$

Phonons



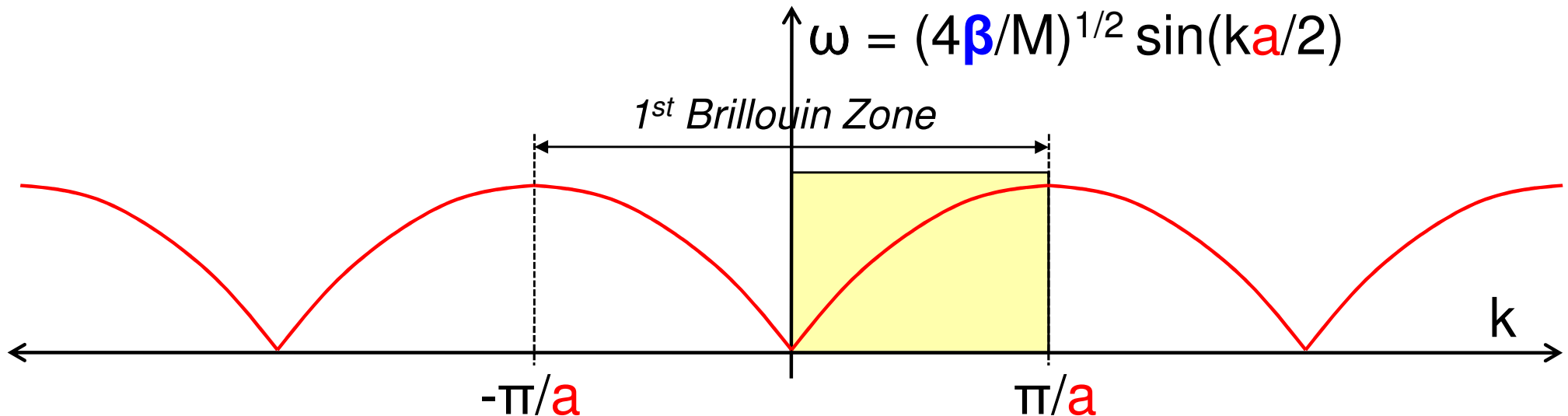
Eigenstates of vibrational field

$$\delta_n(x,t) = \delta_{n,0} \exp[i(kx - \omega t)]$$

$\omega(k)$

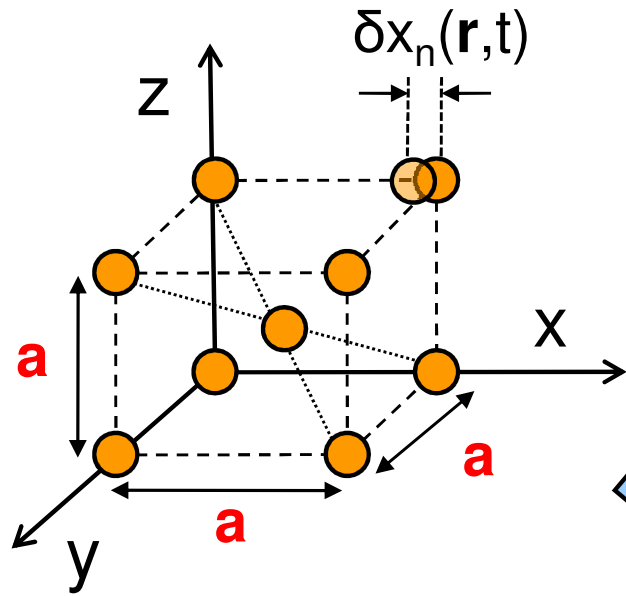


Dispersion relation



IXS: collective atomic dynamics

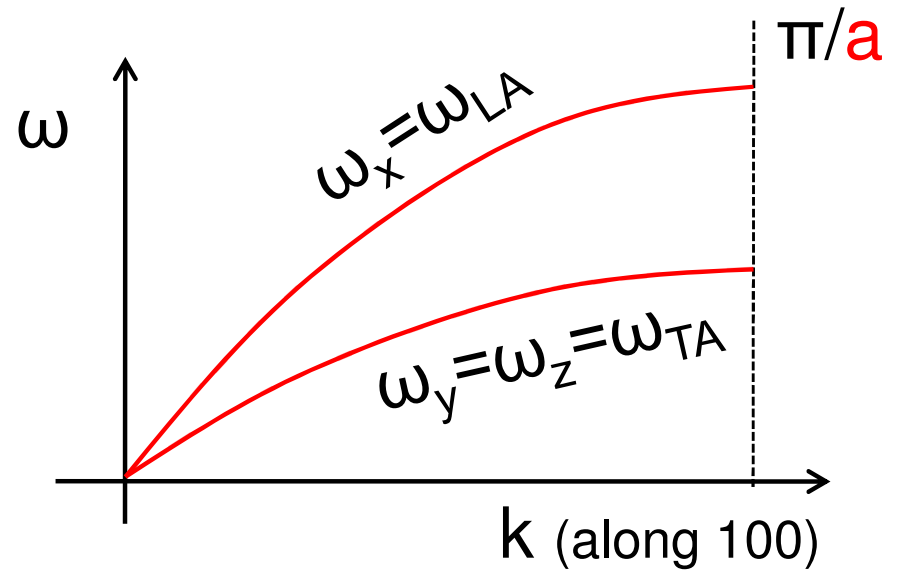
One step forward: 3D lattice



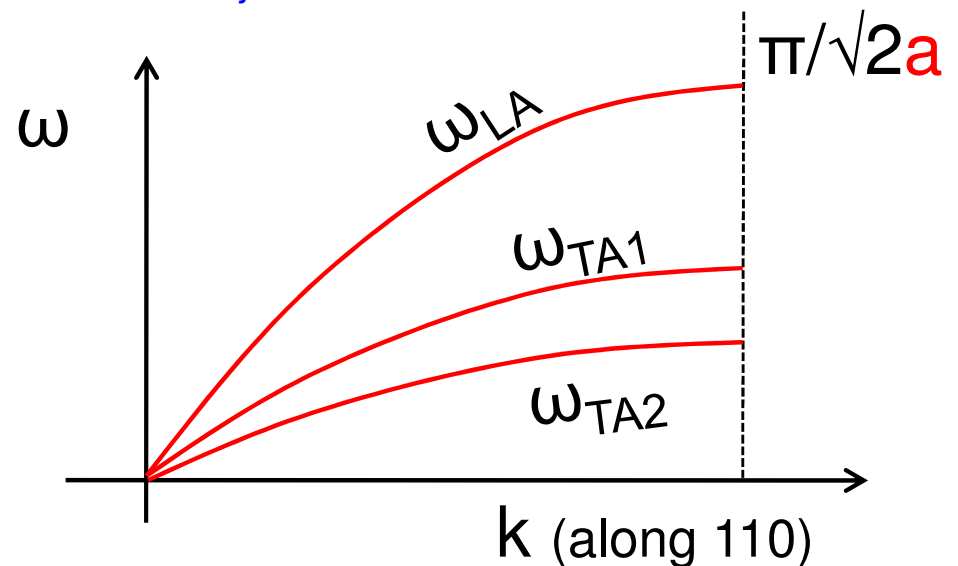
$$U = -\beta |\mathbf{r}|^2 \quad \mathbf{r} = (x, y, z)$$

Information:

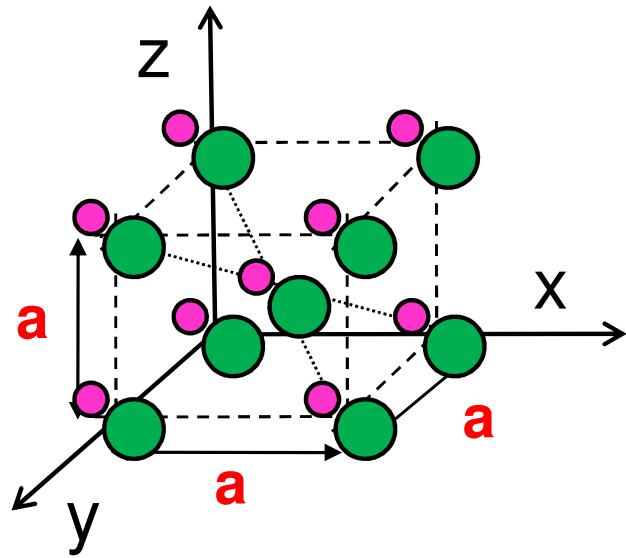
- Interatomic Structure (\mathbf{a})
- Interaction Potential (β)
- Anisotropy (elasticity: \mathbf{c}_{ij})



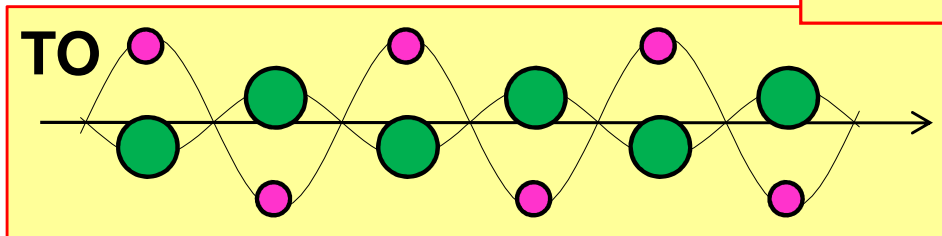
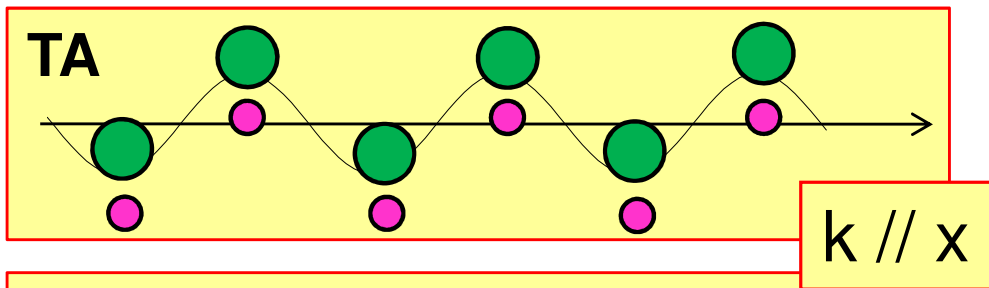
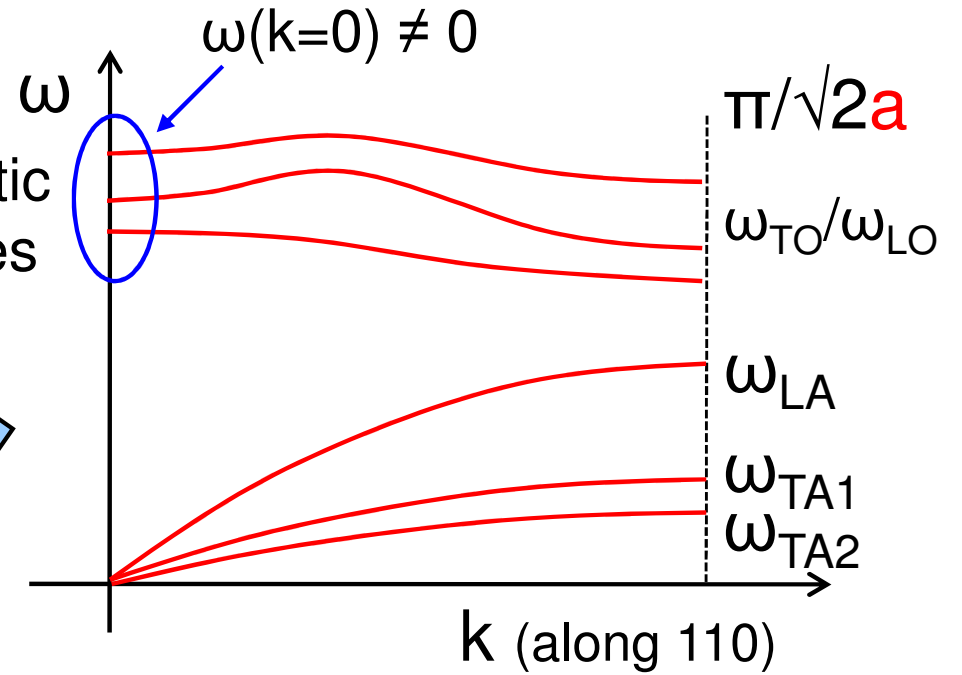
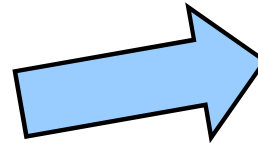
$$\omega = (\mathbf{c}_{ij}/\rho)^{1/2} \sin(k\mathbf{a}^*)$$



IXS: collective atomic dynamics



3N-3 optic branches

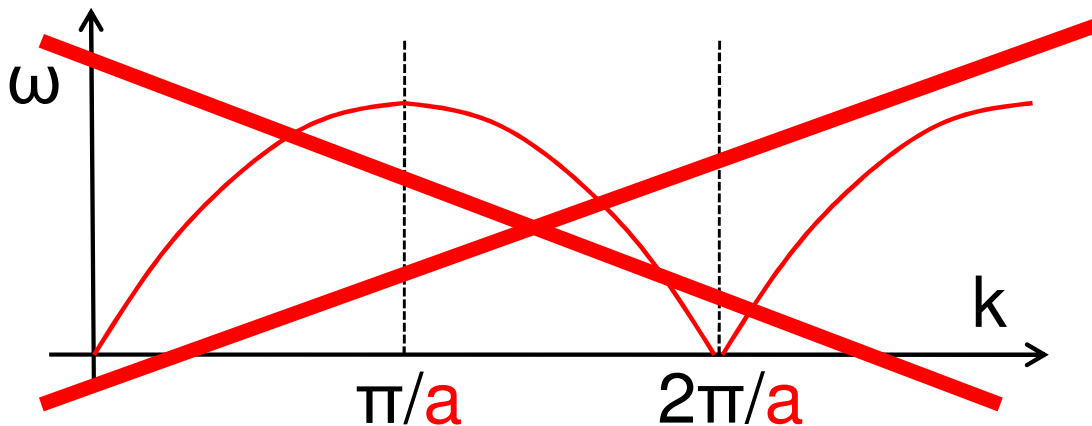
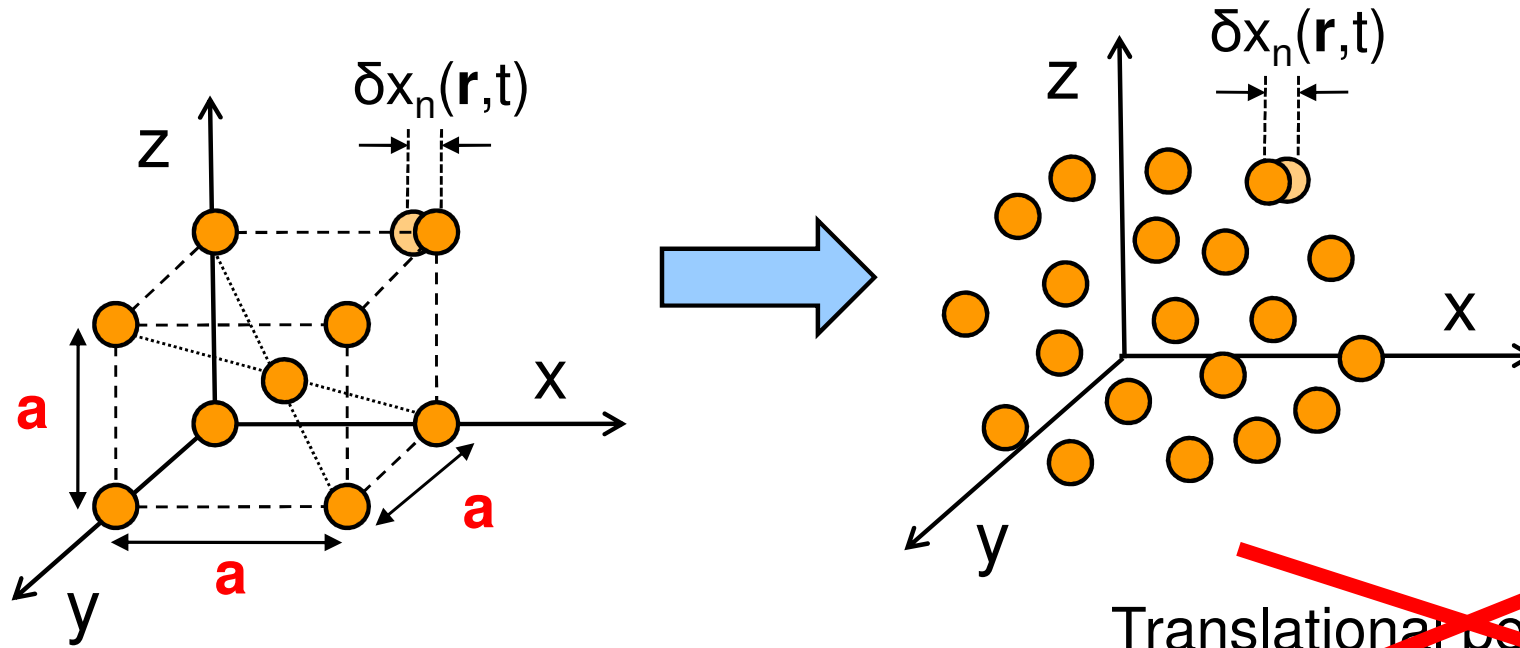


Information:

- Interatomic Structure (a)
- Interaction Potential (β)
- Anisotropy (elasticity: c_{ij})
- Intramolecular vibrations

IXS: collective atomic dynamics

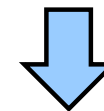
The most complex case: disordered systems



~~Translational periodicity (a)~~



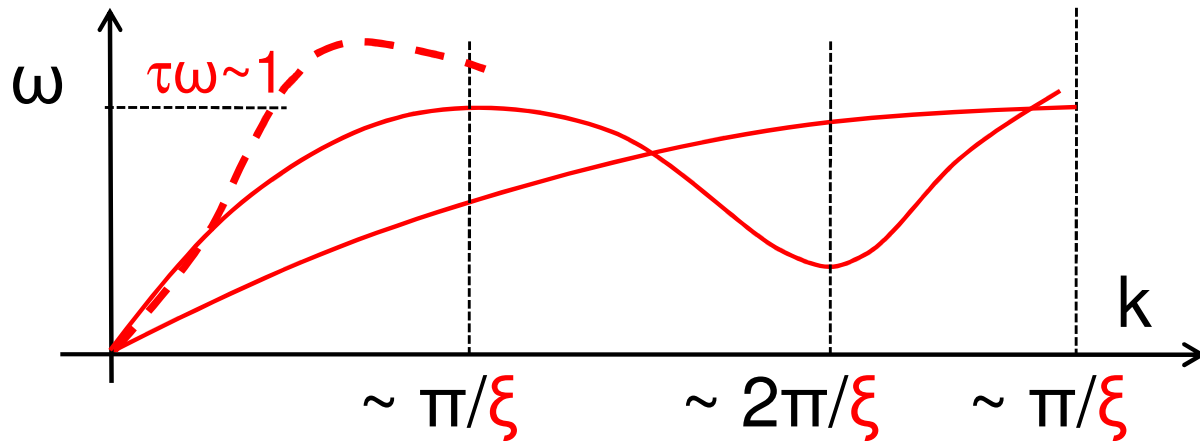
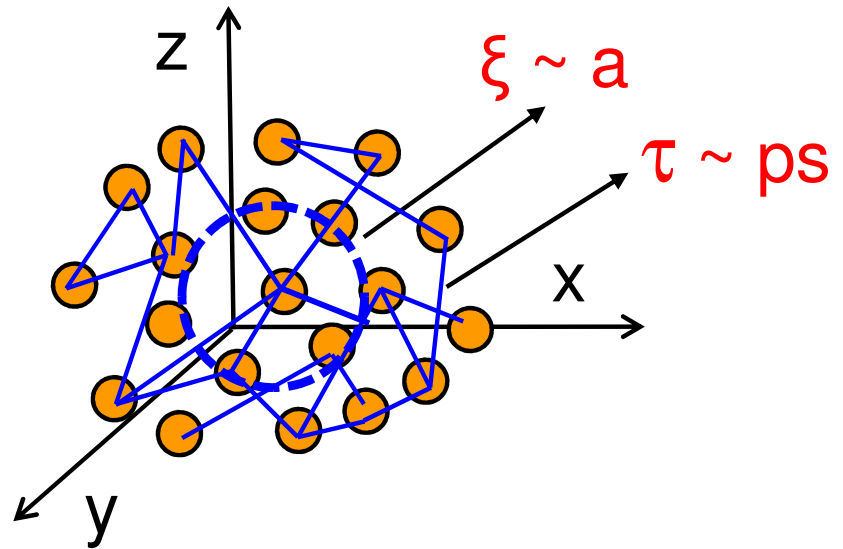
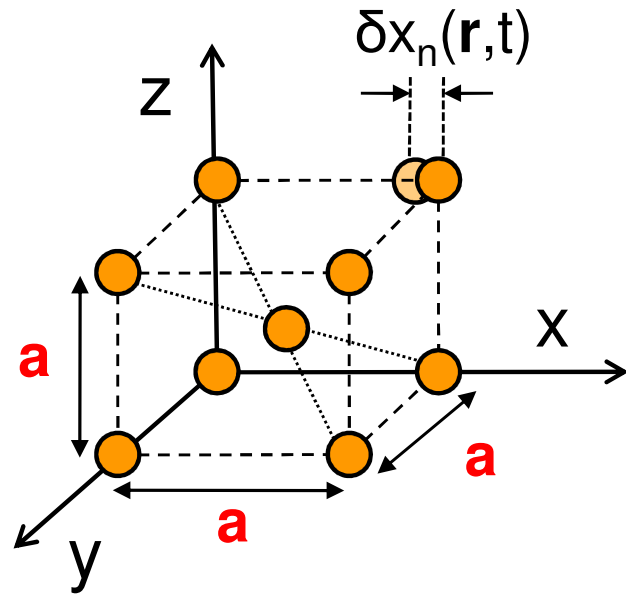
~~Brillouin Zones~~



~~Eigenstates $\sim \exp[i(kx - \omega t)]$~~

IXS: collective atomic dynamics

The most complex case: disordered systems

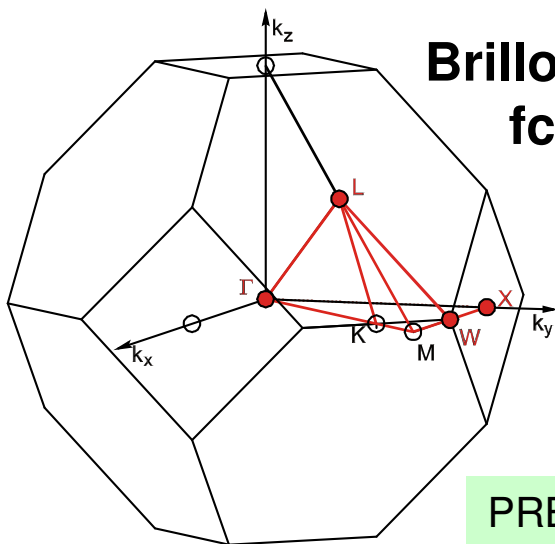
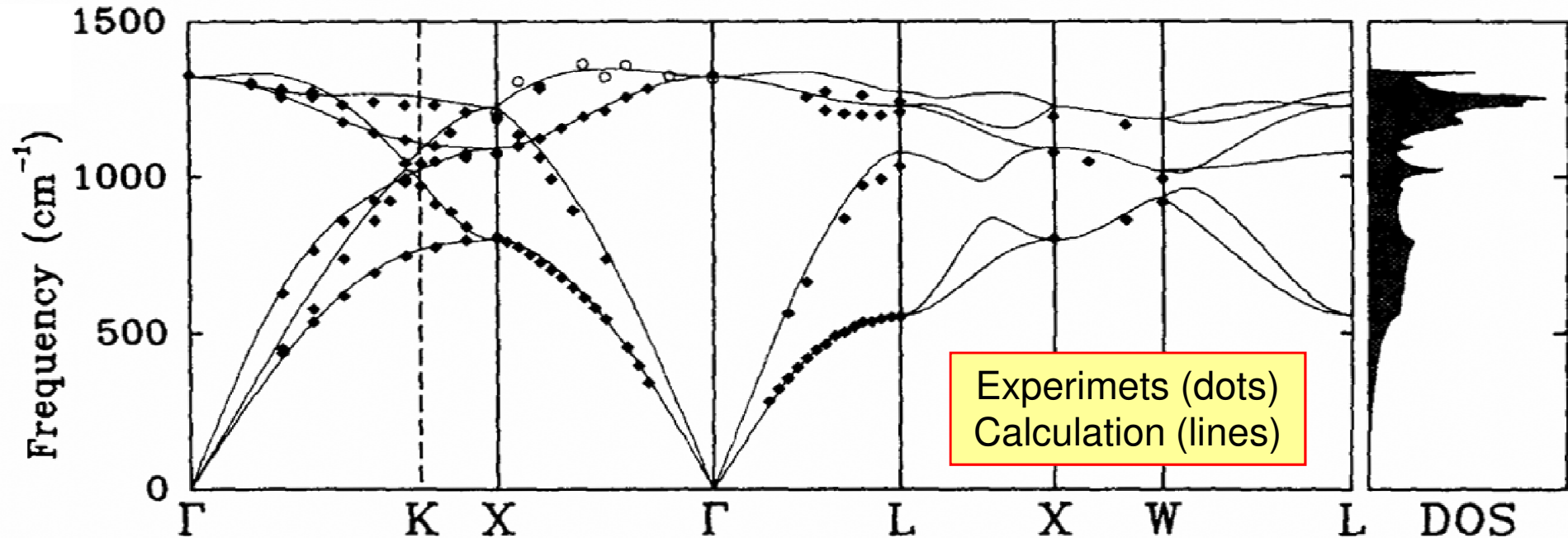


Information:

- Interatomic Structure (ξ)
- Interaction potential
- Topological disorder
- Relaxation processes
- “Lineshape” (anharmonicity, etc...)

An example...

Diamond: fcc symmetry + 2 C atoms each lattice site @ $(\frac{1}{4}\frac{1}{4}\frac{1}{4})$

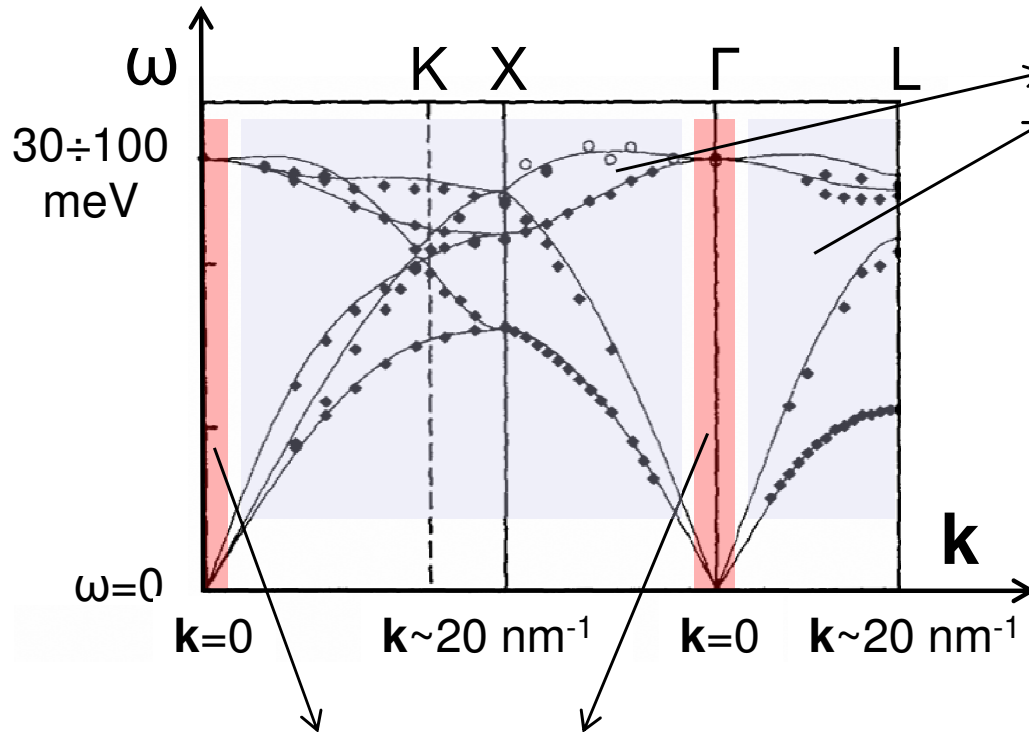


PRB 48, 3156 (1993)

Information:

- Structure and Elasticity (sound velocities)
- Interaction Potential and Anharmonicity
- Dynamical Instabilities (phonon softening)
- Phonon-Electron coupling
- Thermodynamics (c_V , λ , Θ_D , S_D , etc ...)

How can we measure Atomic Dynamics?



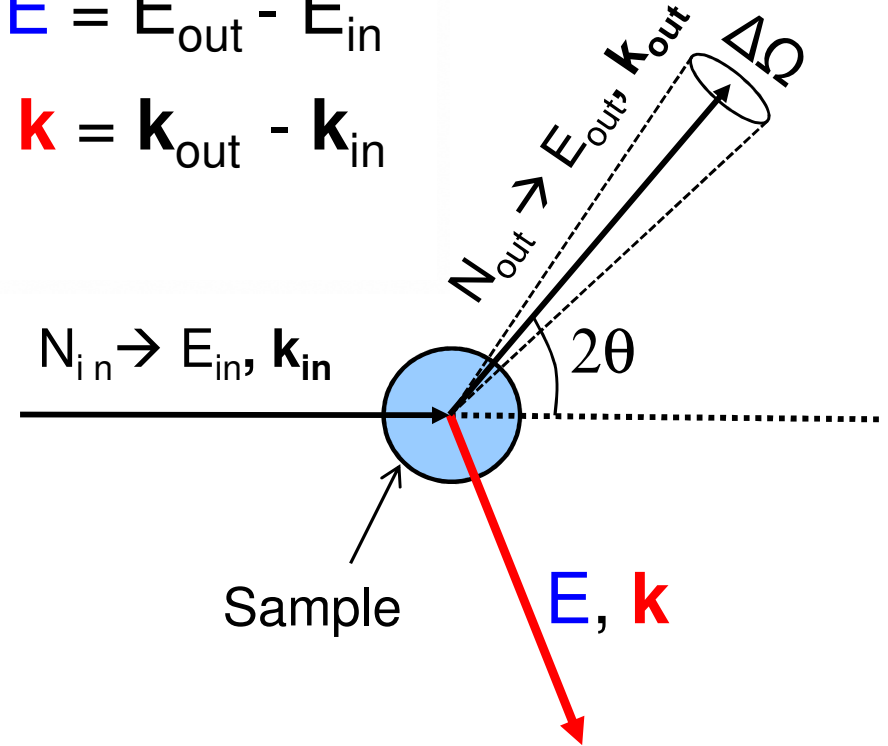
- Probe wavelength ($2\pi/|\mathbf{k}|$) < 0.1 nm
- Probe energy (E) $> 30\div 100$ meV

- Inelastic Light Scattering (Brillouin & Raman)
- Ultrasonics
- Transient Grating
- Etc ...

Inelastic scattering:

$$E = E_{out} - E_{in}$$

$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$



Cross section: $\frac{\partial^2 \sigma}{\partial \Omega \partial E} \sim N_{out}/N_{in}$

Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \Rightarrow E_{in} = 82 \text{ meV}$$

$$\lambda_{in} = 1 \text{ \AA} \Rightarrow E_{in} = 12.4 \text{ keV}$$

$$E > 4 \text{ meV} \rightarrow \Delta E/E_{in} = 0.05$$

$$E > 4 \text{ meV} \rightarrow \Delta E/E_{in} = 3 \cdot 10^{-7}$$

Moderate energy resolution

Very high energy resolution

100 INS instruments

4 IXS instruments

+

Spin sensitive

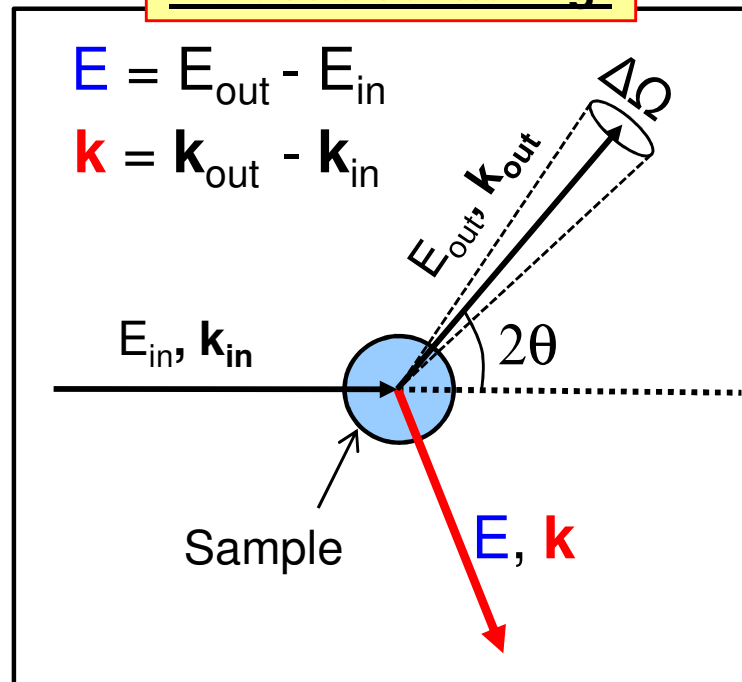
Better contrast

“Older” technique

Inelastic scattering:

$$E = E_{out} - E_{in}$$

$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$



Why X-rays?

Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \rightarrow E_{in} = 82 \text{ meV}$$

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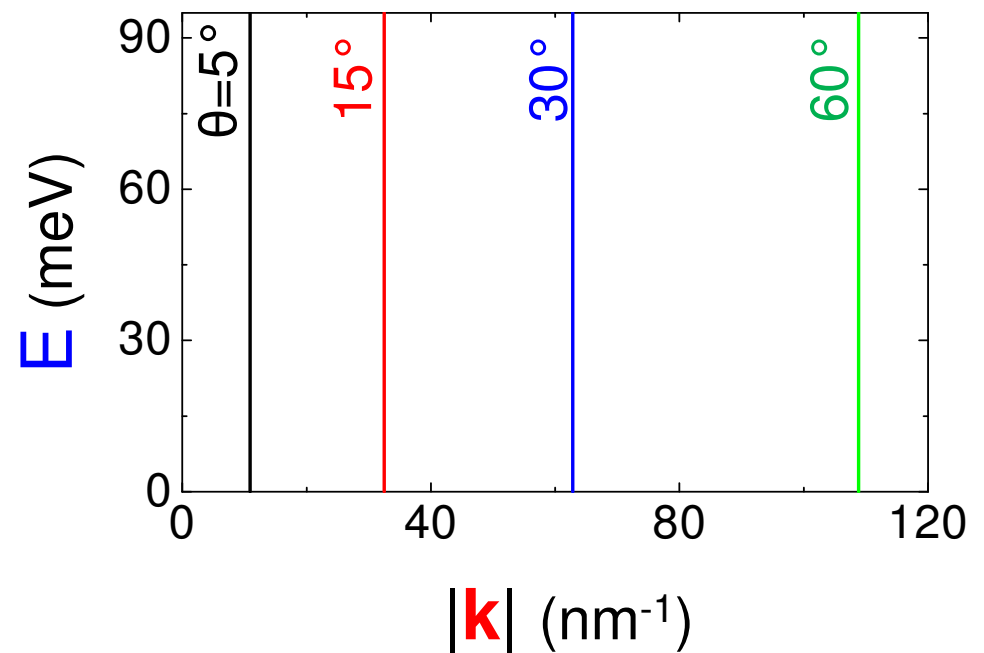
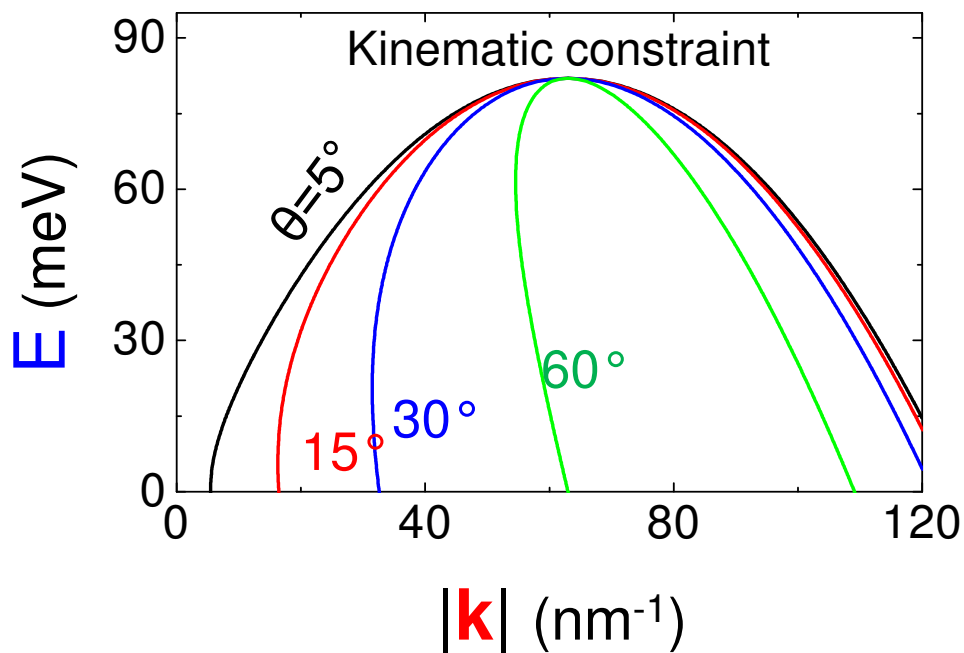
$$E_{out} \neq E_{in}$$

$$E = E_{out} - E_{in} \quad \& \quad \mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$E_{out} \approx E_{in}$$

$$\frac{|\mathbf{k}|^2}{2|\mathbf{k}_{in}|^2} = 1 - E/E_{in} + \cos(2\theta)(1 - 2E/E_{in})^{1/2}$$

$$|\mathbf{k}| = 2|\mathbf{k}_{in}|\sin(\theta)$$



Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \quad \Rightarrow \quad E_{in} = 82 \text{ meV}$$

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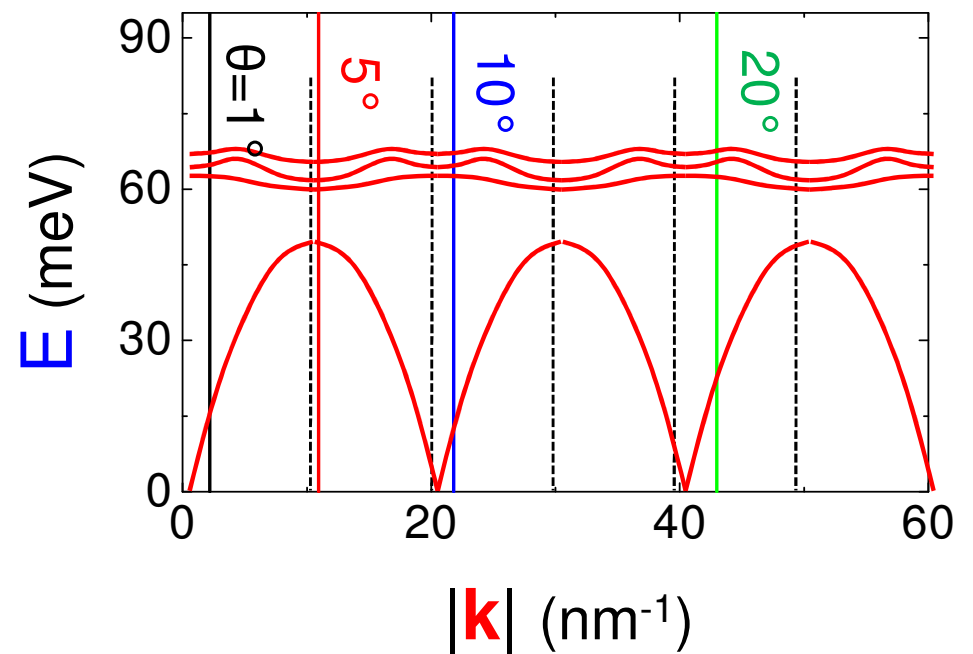
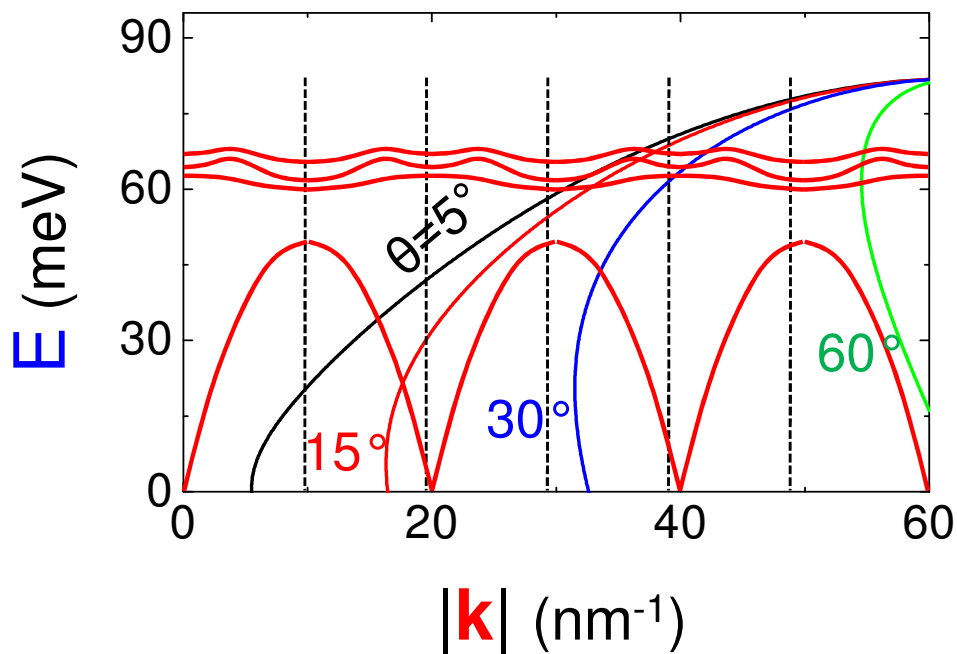
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Neutrons

vs.

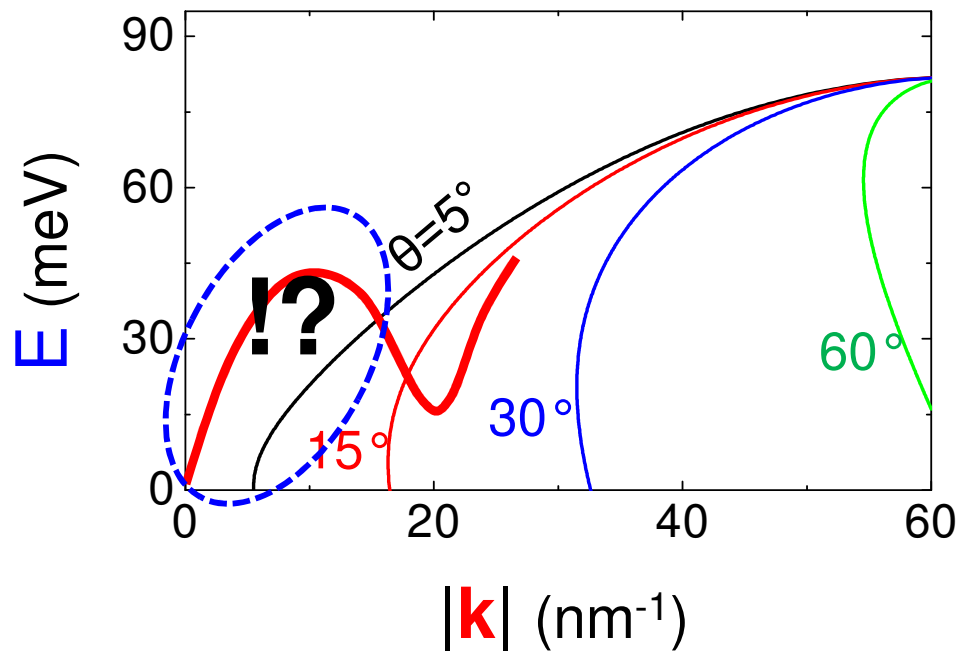
X-rays

Inelastic excitations in disordered systems

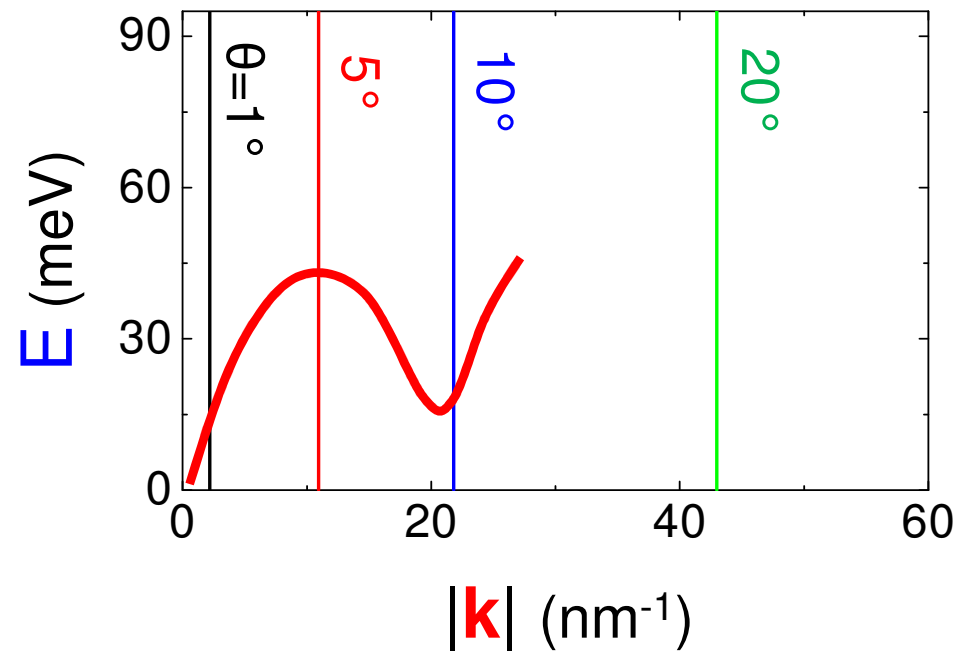
Neutrons

X-rays

$\lambda_{in} = 1 \text{ \AA}$ $E_{in} = 82 \text{ meV}$



$\lambda_{in} = 1 \text{ \AA}$ $E_{in} = 12.4 \text{ keV}$



Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \Rightarrow E_{in} = 82 \text{ meV}$$

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Moderate energy resolution

Very high energy resolution

100 INS instruments

3 IXS instruments

+

Spin sensitive

Better contrast

“Older” technique

No kinematical constraints
(Disordered systems)

Small beams
(small samples: high pressure, exotic materials, etc...)

Why X-rays?

No incoherent cross section

Basic theoretical aspects

$$H_{\text{int}} = (e/m_e c) \sum_j \left[(e/2c) \mathbf{A}_j \cdot \mathbf{A}_j + \mathbf{A}_j \cdot \mathbf{p}_j + \text{magnetic} \right]$$

\mathbf{A} is the vector potential of electromagnetic field

\mathbf{p} is the momentum operator of the electrons

j is the summation over all electrons of the system

1st order perturbation theory

$\mathbf{A} \cdot \mathbf{A}$ term \rightarrow one photon (non-resonant) scattering

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\varepsilon}_{\text{in}} \cdot \boldsymbol{\varepsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_{\text{out}} + E_{\text{in}})$$

Basic theoretical aspects

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_i P_i |\langle I | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_I)$$

The key assumption:

Adiabatic approximation \rightarrow $|I\rangle = |I_n\rangle |I_e\rangle$ and $|F\rangle = |F_n\rangle |F_e\rangle$

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = \underbrace{r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}})}_{\text{Thomson scattering cross section}} \underbrace{F(|\mathbf{k}|)^2}_{\text{Molecular form factor } (|I_e\rangle, |F_e\rangle)} \underbrace{S(\mathbf{k}, E)}_{\text{Dynamic structure factor}}$$

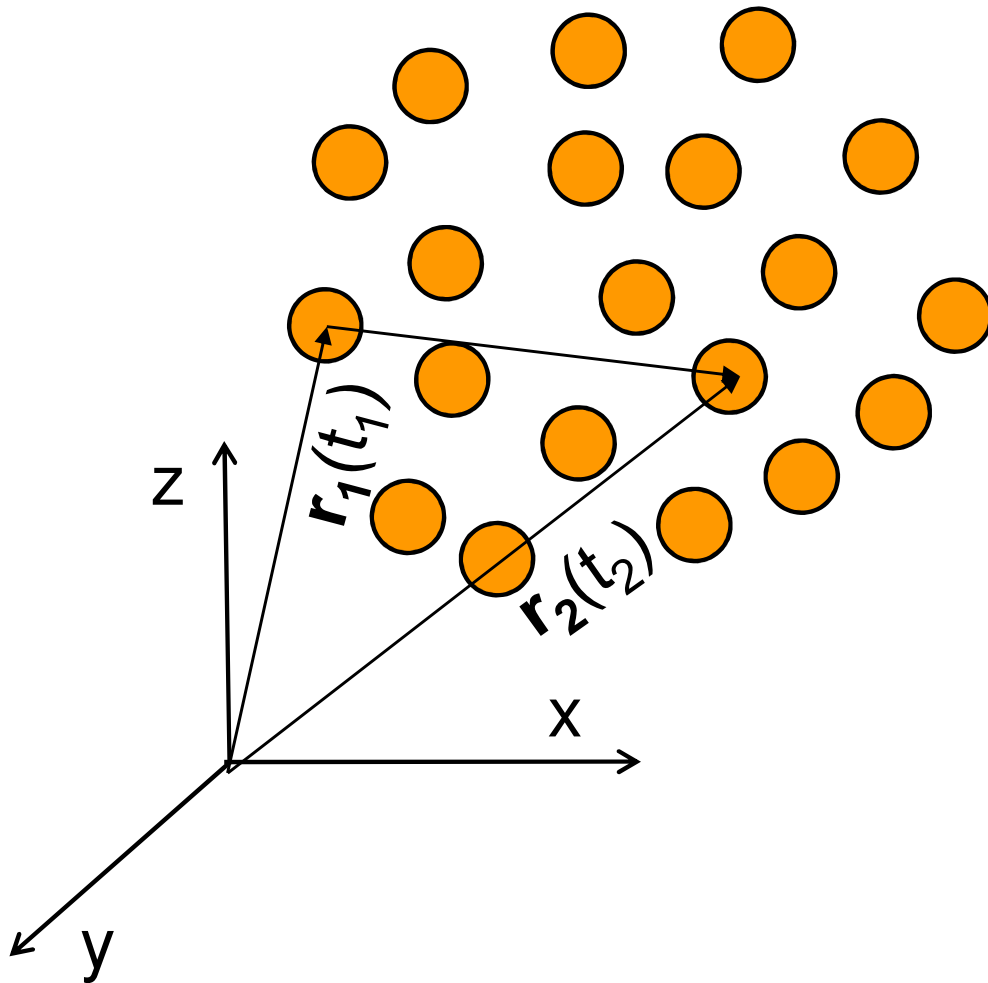
Thomson scattering
cross section

Molecular form factor ($|I_e\rangle, |F_e\rangle$)

Dynamic structure factor

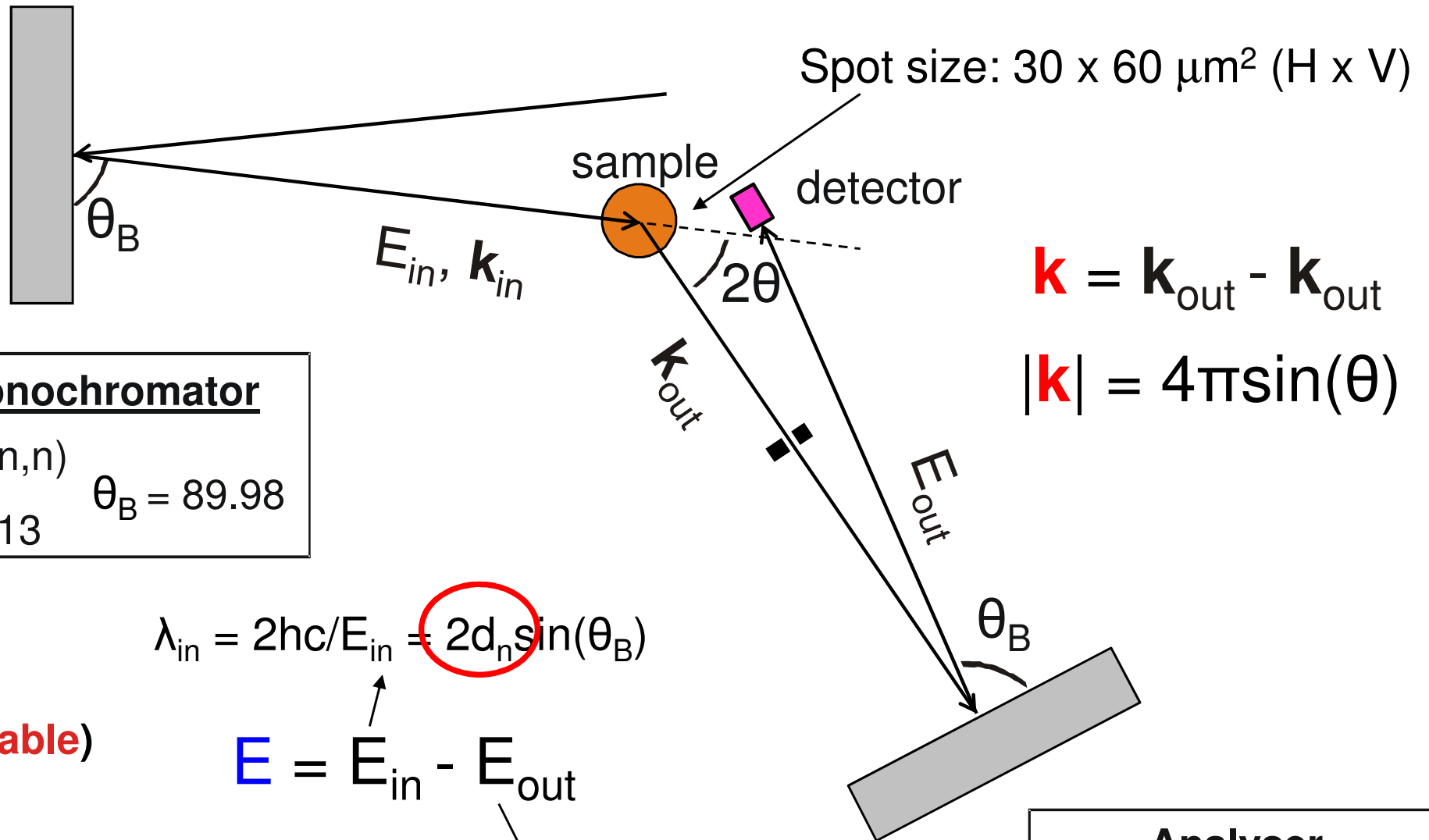
The dynamic structure factor

$S(\mathbf{k}, E)$ is the **SPACE** and **TIME** Fourier transform of $G(\mathbf{r}, t)$



$G(\mathbf{r}, t)$ is the probability to find two distinct particles at positions $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$, separated by the distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ and the time interval $t = t_2 - t_1$.

Basic IXS instrumentation



$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$|\mathbf{k}| = 4\pi \sin(\theta)$$

Monochromator
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

$$\lambda_{in} = 2hc/E_{in} = 2d_n \sin(\theta_B)$$

λ_{in} (**tunable**)

$$E = E_{in} - E_{out}$$

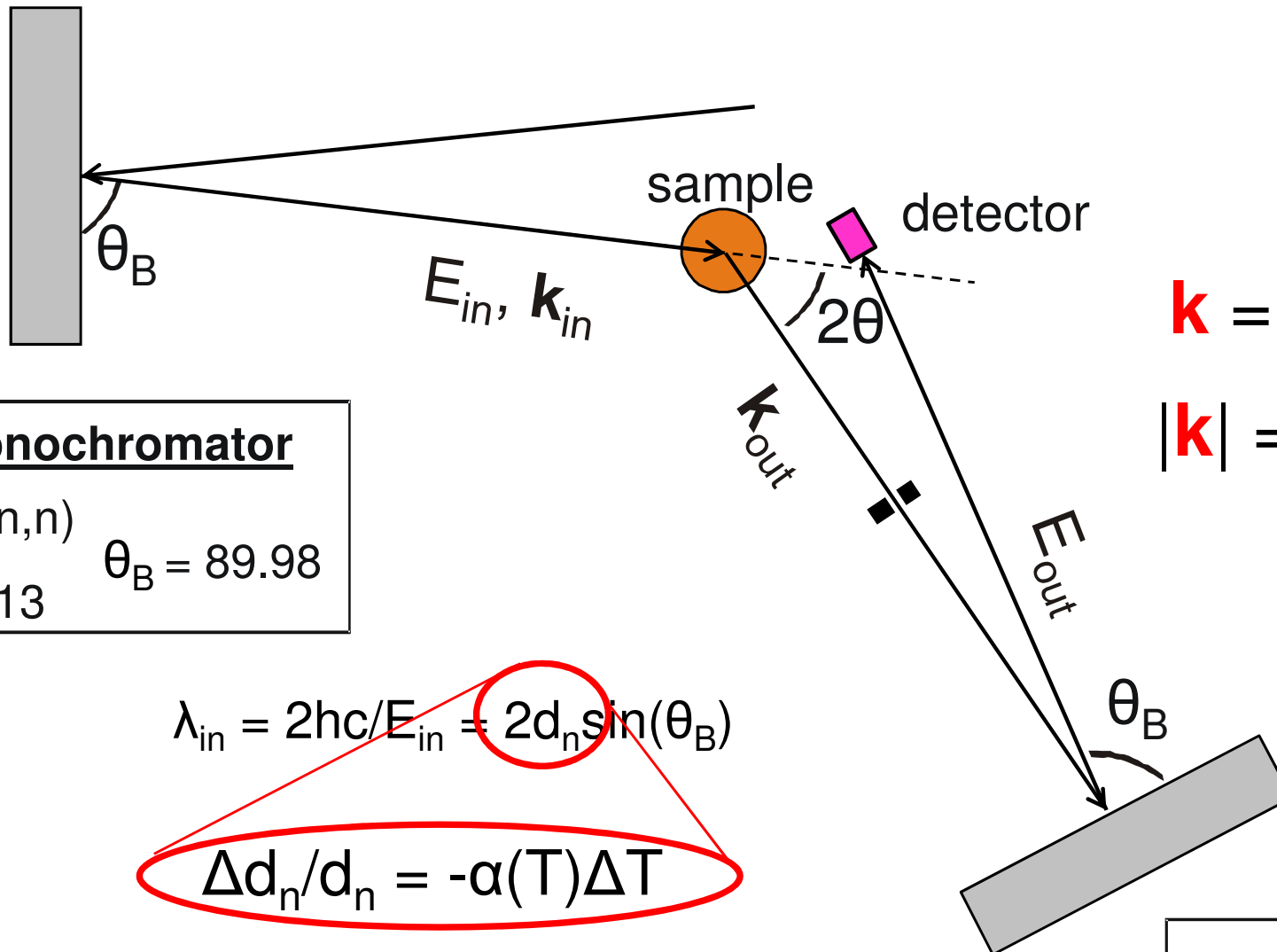
λ_{out} (**constant**)

$$\lambda_{out} = 2hc/E_{out} = 2d_n \sin(\theta_B)$$

$$\Delta E/E = \Delta \lambda_{in}/\lambda_{in} \sim \cot(\theta_B)$$

Analyser
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

Basic IXS instrumentation



$$\mathbf{k} = \mathbf{k}_{\text{out}} - \mathbf{k}_{\text{in}}$$

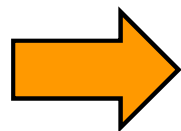
$$|\mathbf{k}| = 4\pi \sin(\theta)$$

Monochromator
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

$$\lambda_{\text{in}} = 2hc/E_{\text{in}} = 2d_n \sin(\theta_B)$$

$$\Delta d_n/d_n = -\alpha(T)\Delta T$$

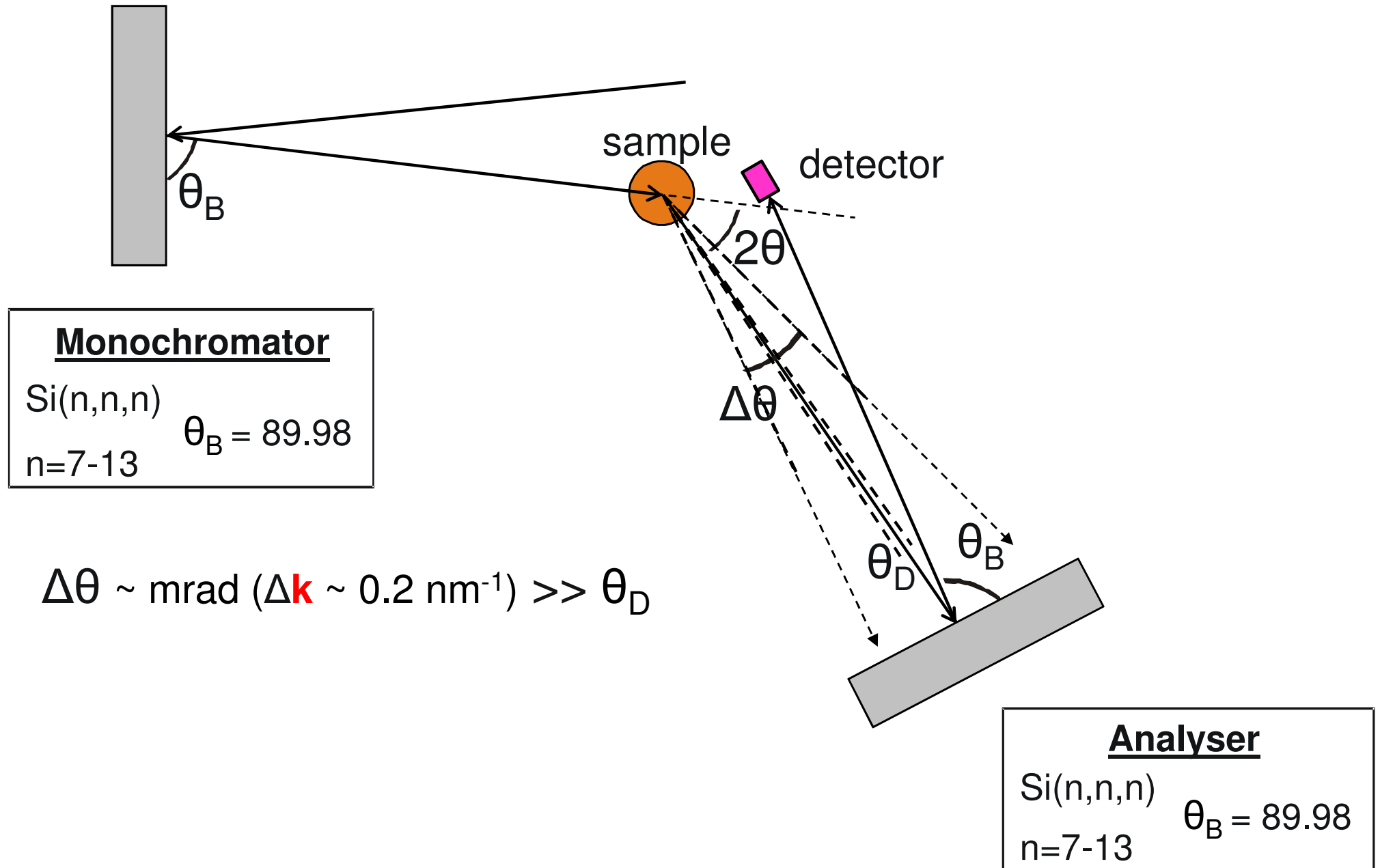
$\alpha \sim 2.58 \cdot 10^{-6}$
 $\Delta T \sim \text{mK}$



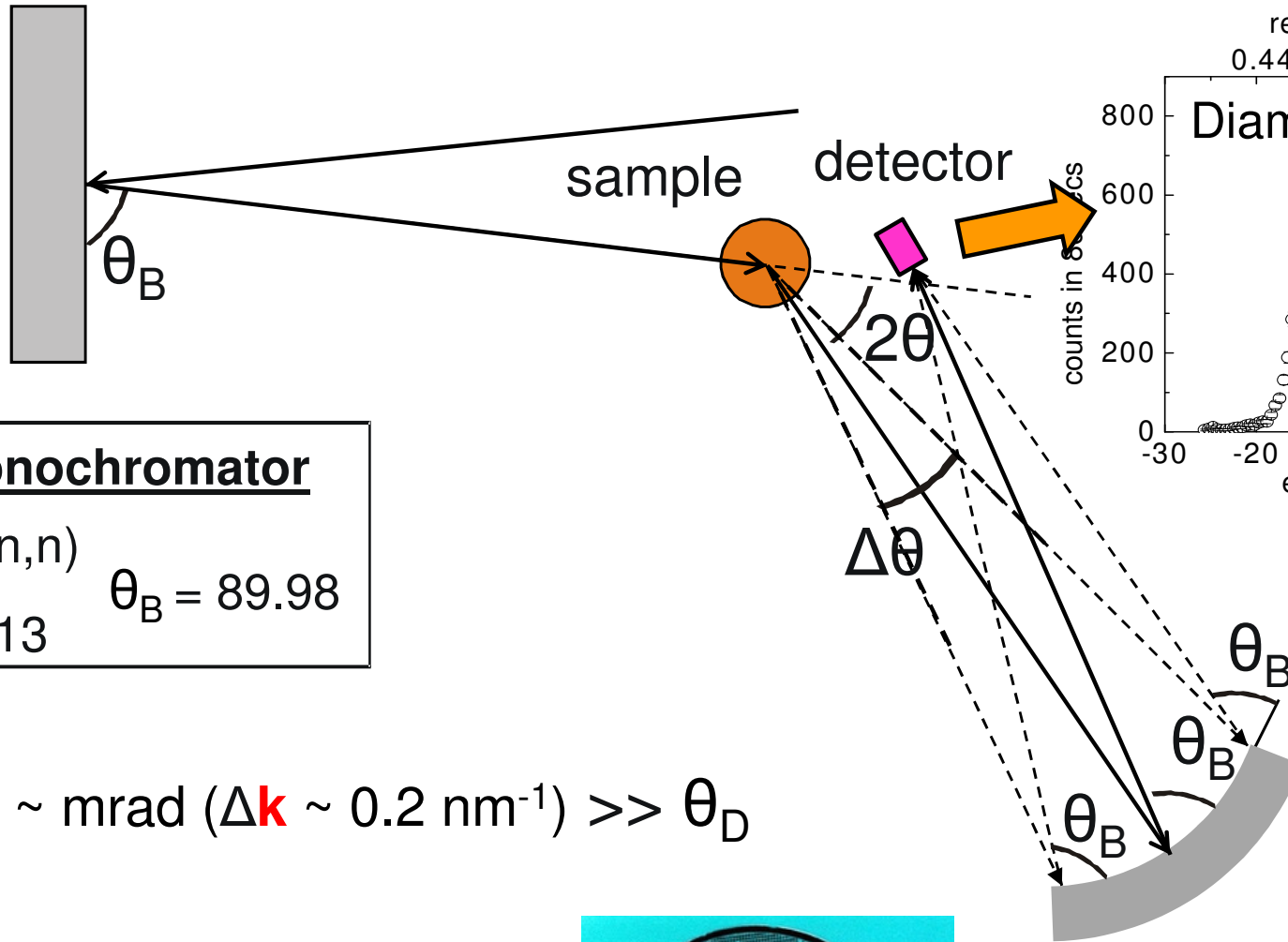
$(\Delta E/E)_{\text{MAX}} \sim 2 \cdot 10^{-8}$
 $\rightarrow < 1 \text{ meV @ } 27 \text{ keV}$

Analyser
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

Basic IXS instrumentation

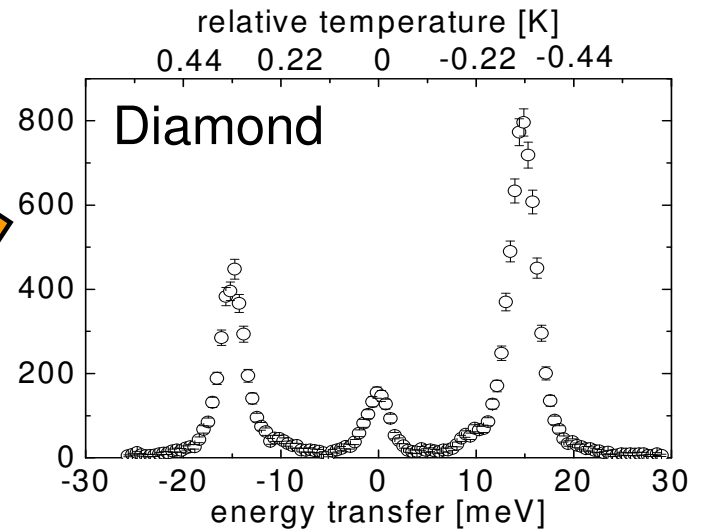


Basic IXS instrumentation



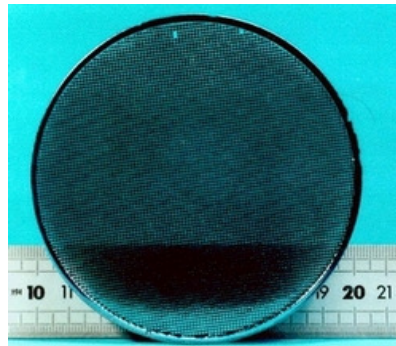
Monochromator
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

$\Delta\theta \sim \text{mrad}$ ($\Delta\mathbf{k} \sim 0.2 \text{ nm}^{-1}$) $\gg \theta_D$



≈ 12000 flat Si “perfect” single crystals ($0.6 \times 0.6 \text{ mm}^2$) that approximate a spherical surface

NIM 111, 181 (1996)



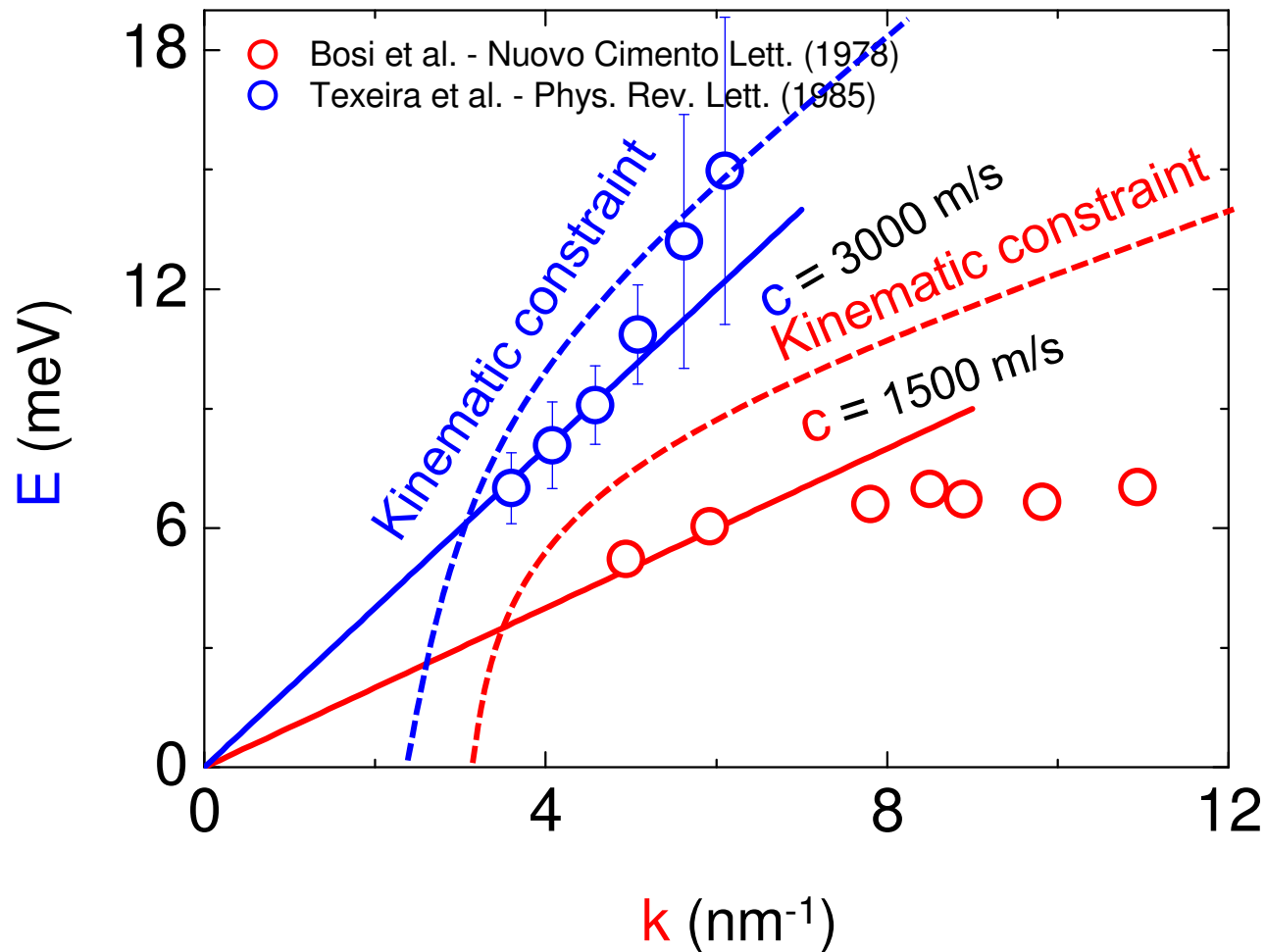
Analyser
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering (D_2O):

2 experiments, 2 results: why?



A possible interpretation:

- High frequency mode \rightarrow **D**
- Low frequency mode \rightarrow **O**

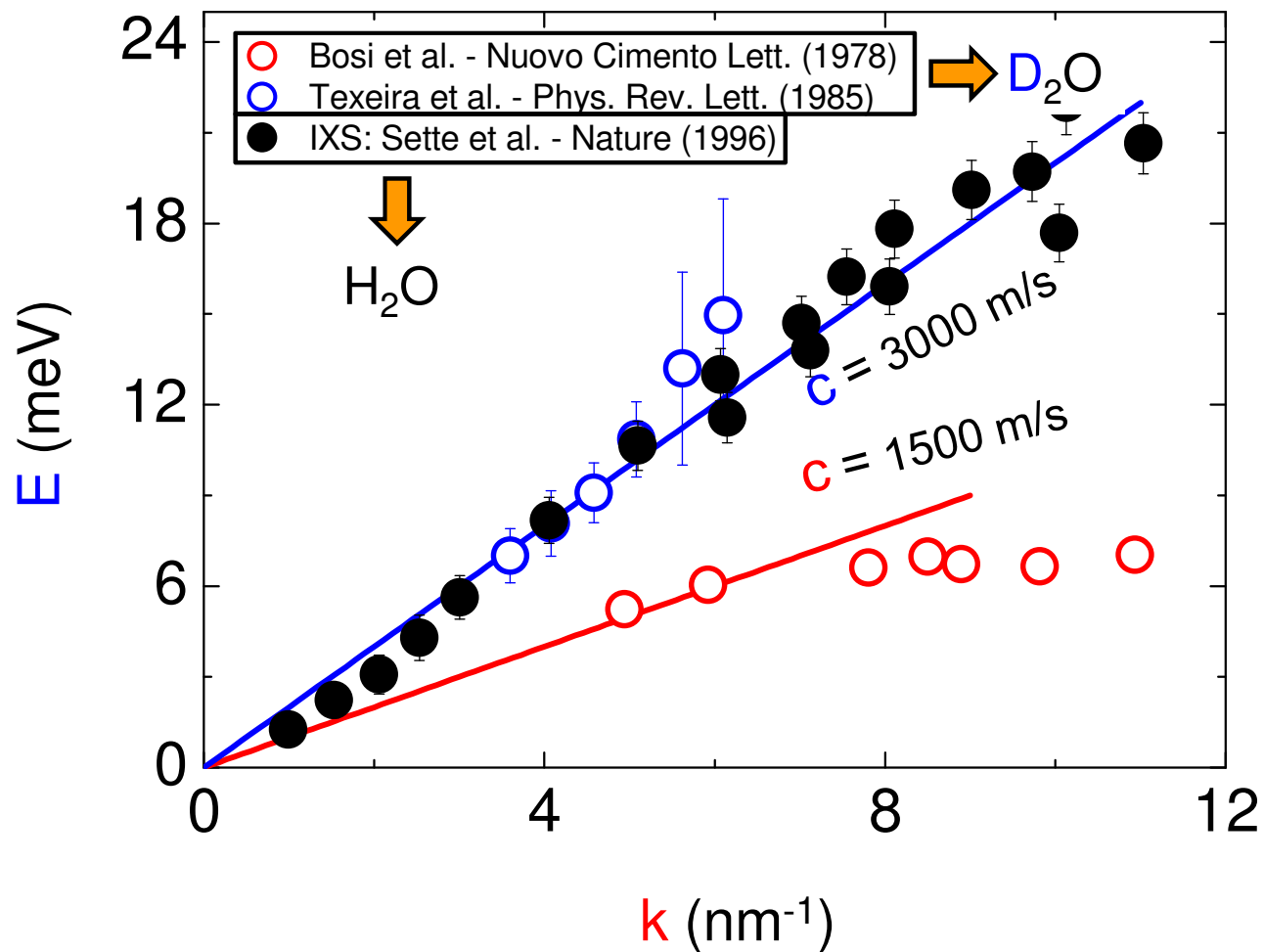


$$\Omega_{\text{HF}}/\Omega_{\text{LF}} \sim (m_{\text{O}}/m_{\text{D}})^{1/2} \sim 2$$

Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering vs.
Inelastic X-ray Scattering (H_2O vs. D_2O)



A possible interpretation:

- High frequency mode → **D**
- Low frequency mode → **O**



$$\Omega_{HF}/\Omega_{LF} \sim (m_O/m_D)^{1/2} \sim 2$$



High frequency mode:

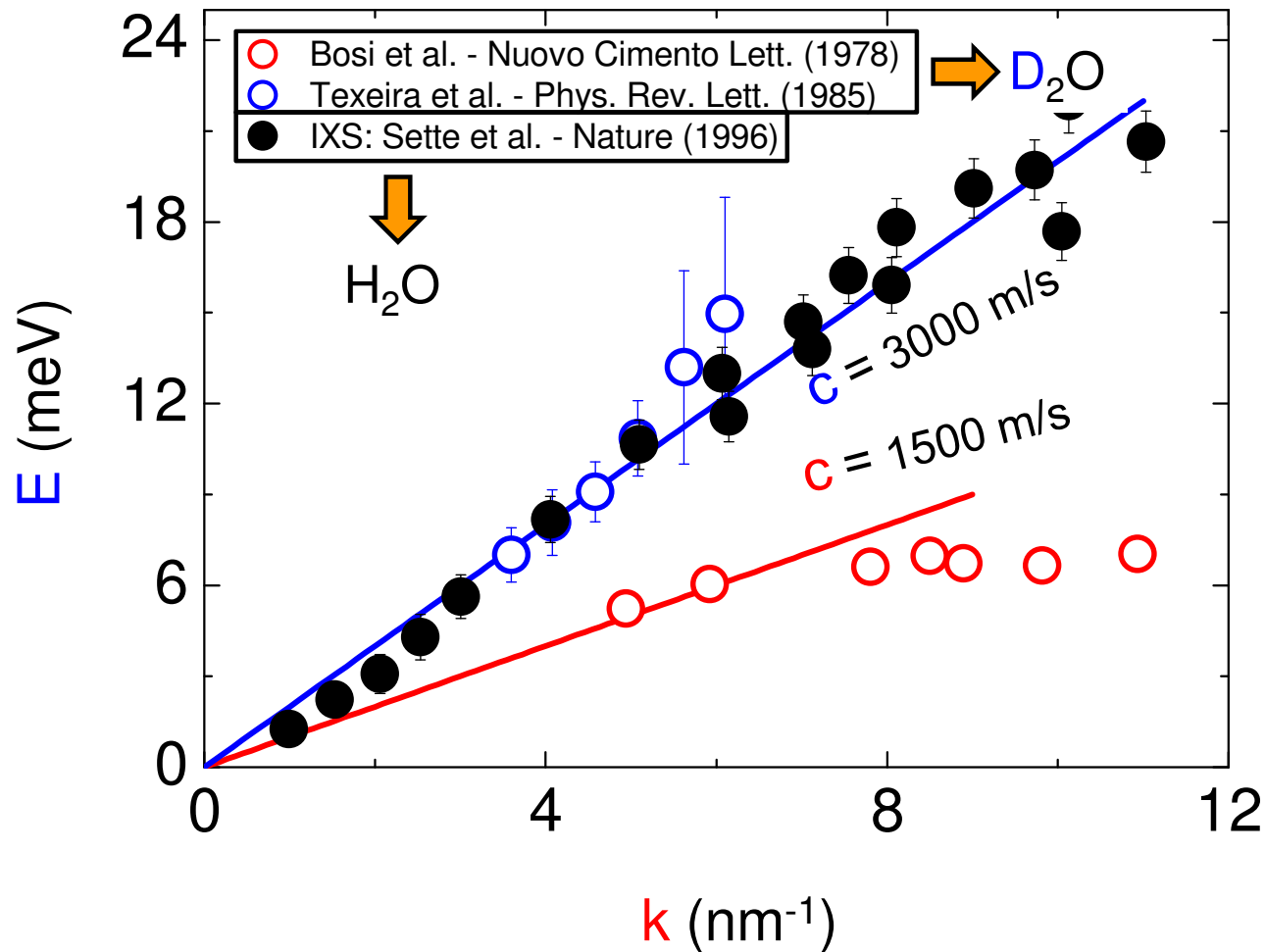
Expected for H_2O ...

$$\Omega_{IXS}/\Omega_{INS} \sim (m_H/m_D)^{1/2} \sim 1.4$$

Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering vs.
Inelastic X-ray Scattering (H_2O vs. D_2O)



A possible interpretation:

- High frequency mode → **D**
- Low frequency mode → **O**



$$\Omega_{HF}/\Omega_{LF} \sim (m_O/m_D)^{1/2} \sim 2$$



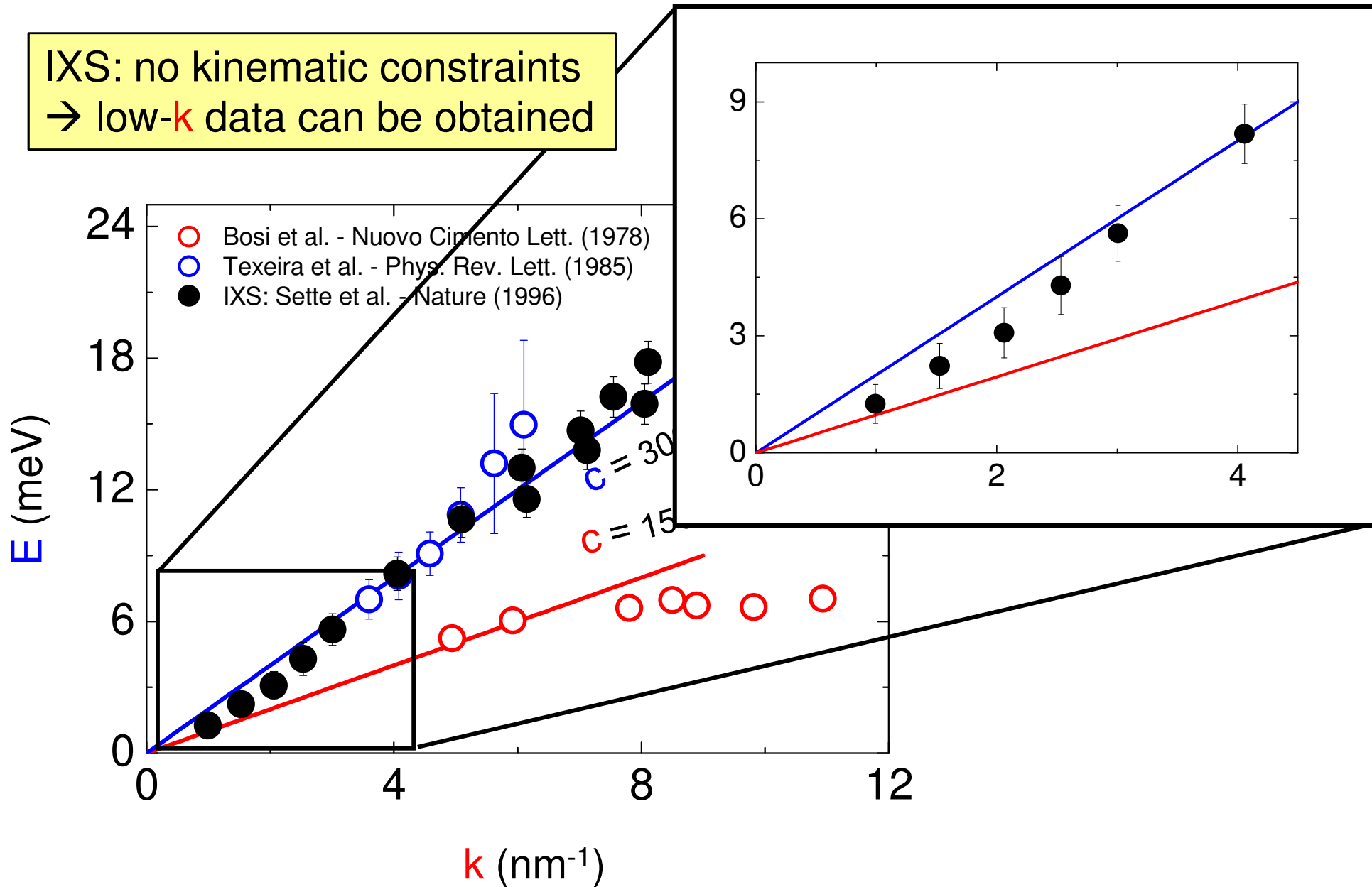
High frequency mode:

but... $\Omega_{IXS} = \Omega_{INS}$

Experimental highlights (1)

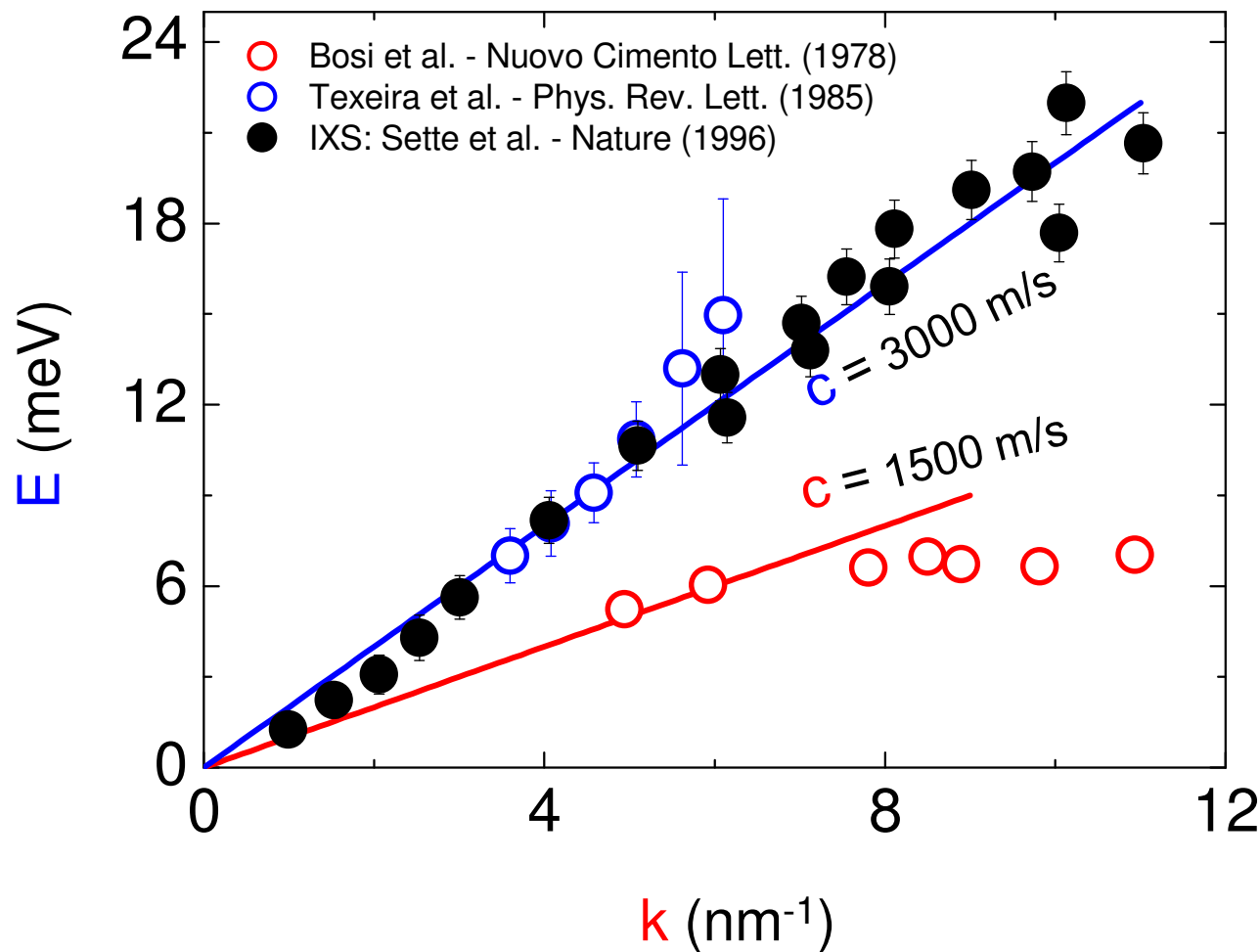
Collective dynamics in water

IXS: no kinematic constraints
→ low- k data can be obtained

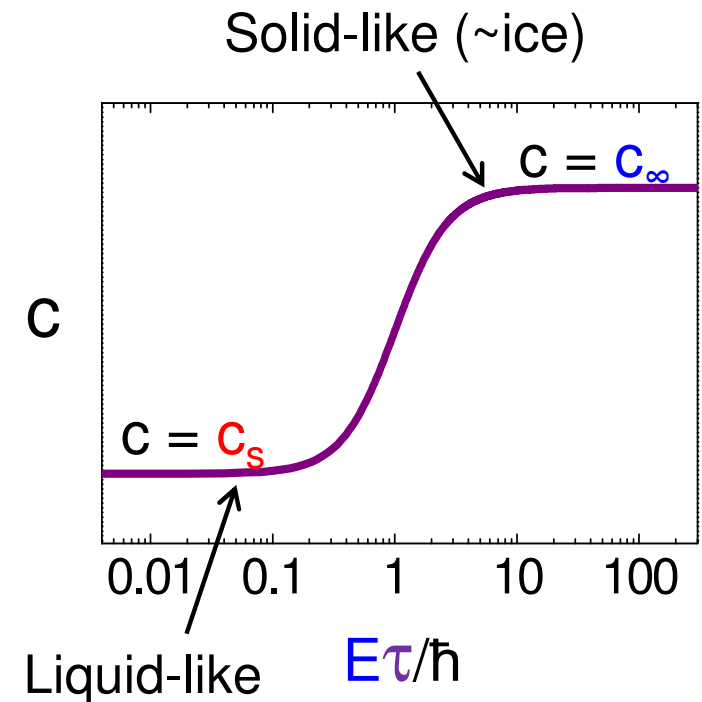


Experimental highlights (1)

Collective dynamics in water



Viscoelasticity:
The sound velocity ($c = E/\hbar k$) is not constant but depends on $E\tau/\hbar$

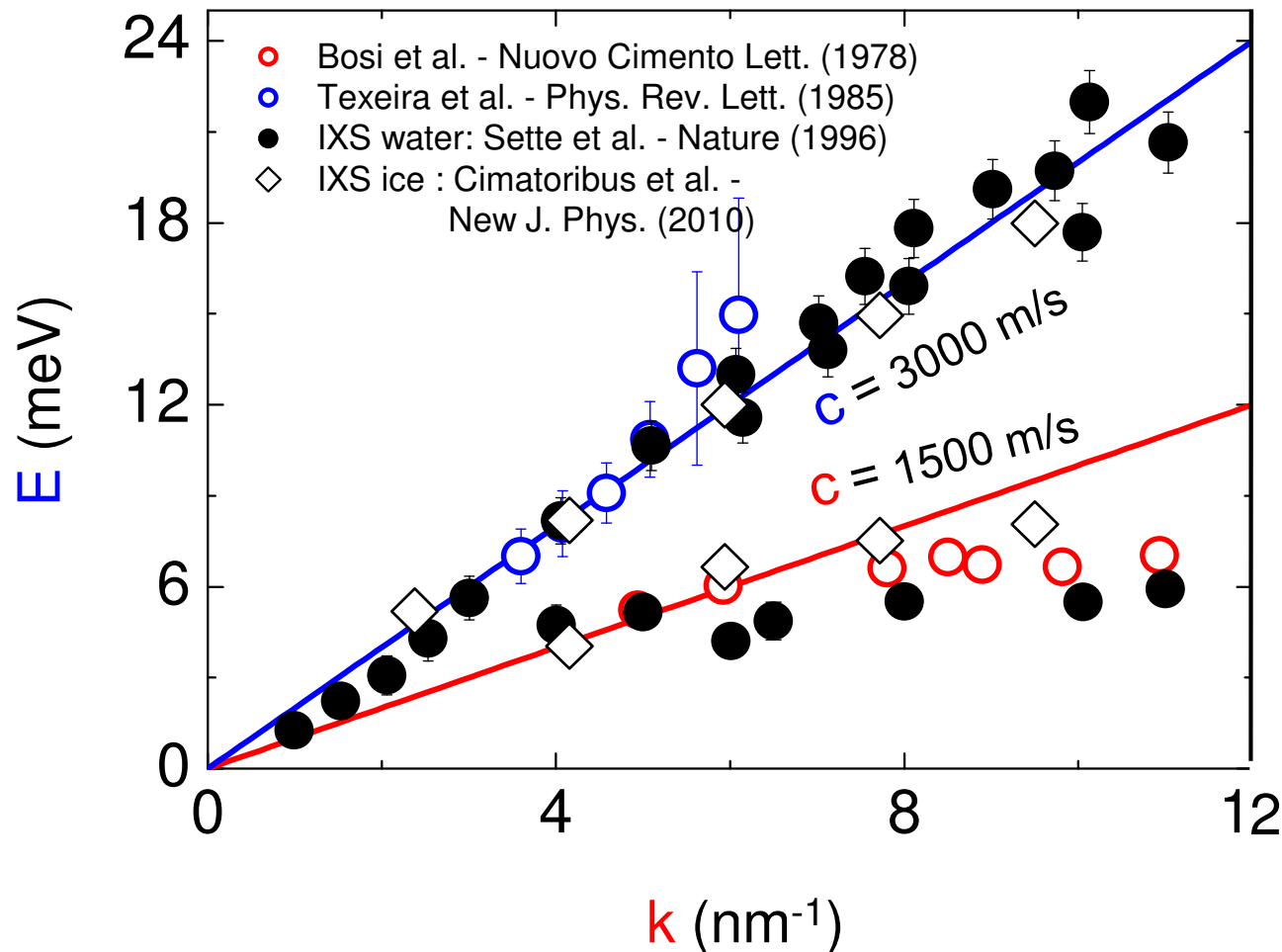


Experimental highlights (1)

Collective dynamics in water

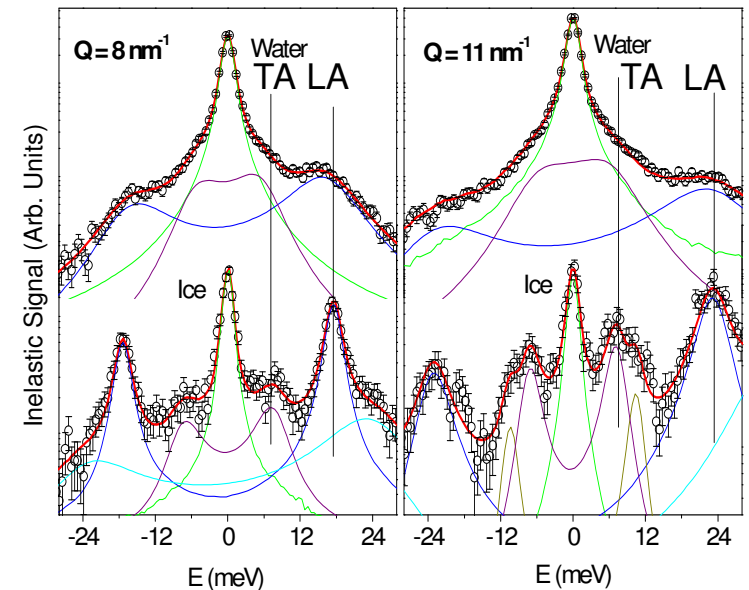
Low frequency mode:

Transverse-like sound propagation in the elastic (solid-like) limit: $E\tau/\hbar \gg 1$



Viscoelasticity:

The sound velocity ($c = E/\hbar k$) is not constant but depends on $E\tau/\hbar$



PRL 79, 1678 (1997)

PRE 71, 011501 (2005)

Experimental highlights (2)

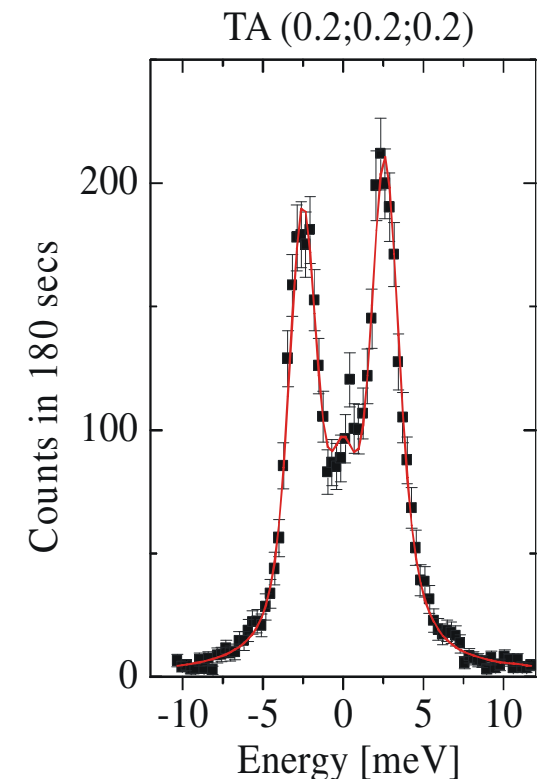
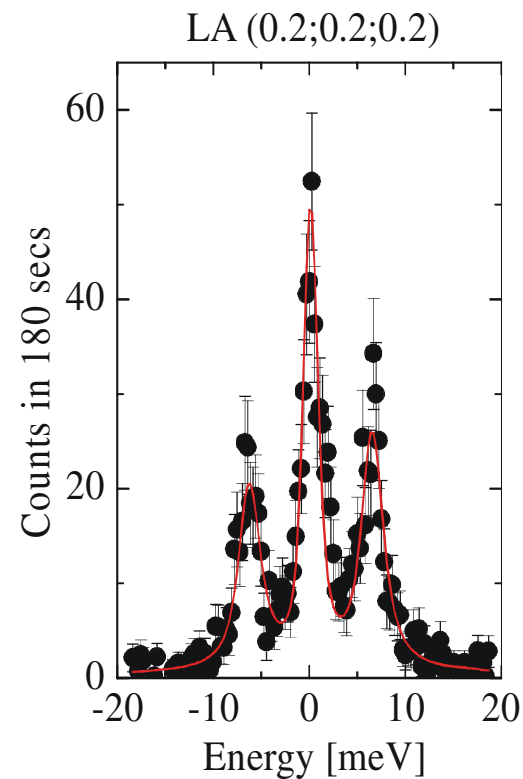
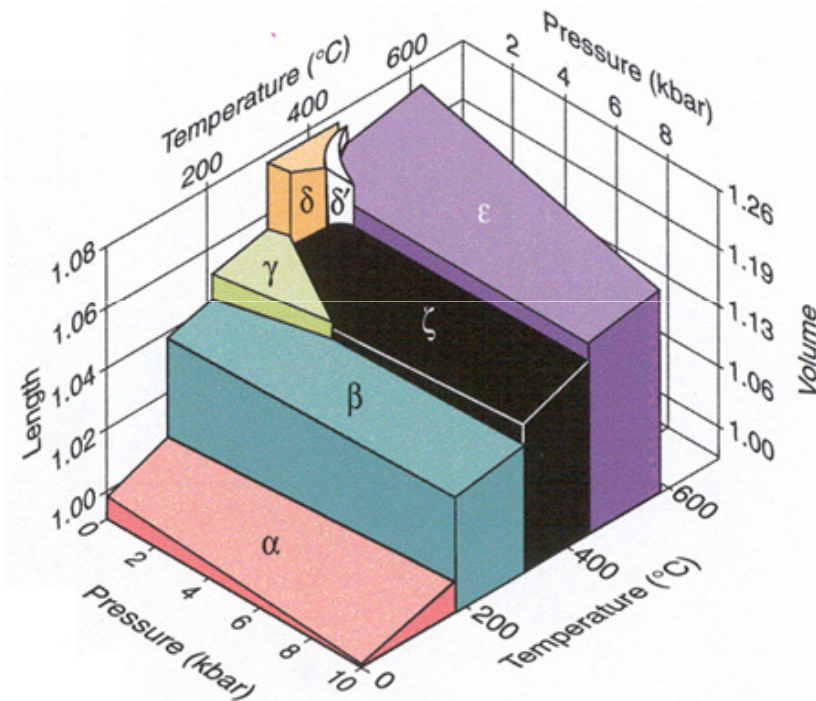
Phonon dispersions in plutonium

Plutonium is one of the most fascinating and exotic elements:

- Multitude of unusual properties
- Central role of 5f electrons

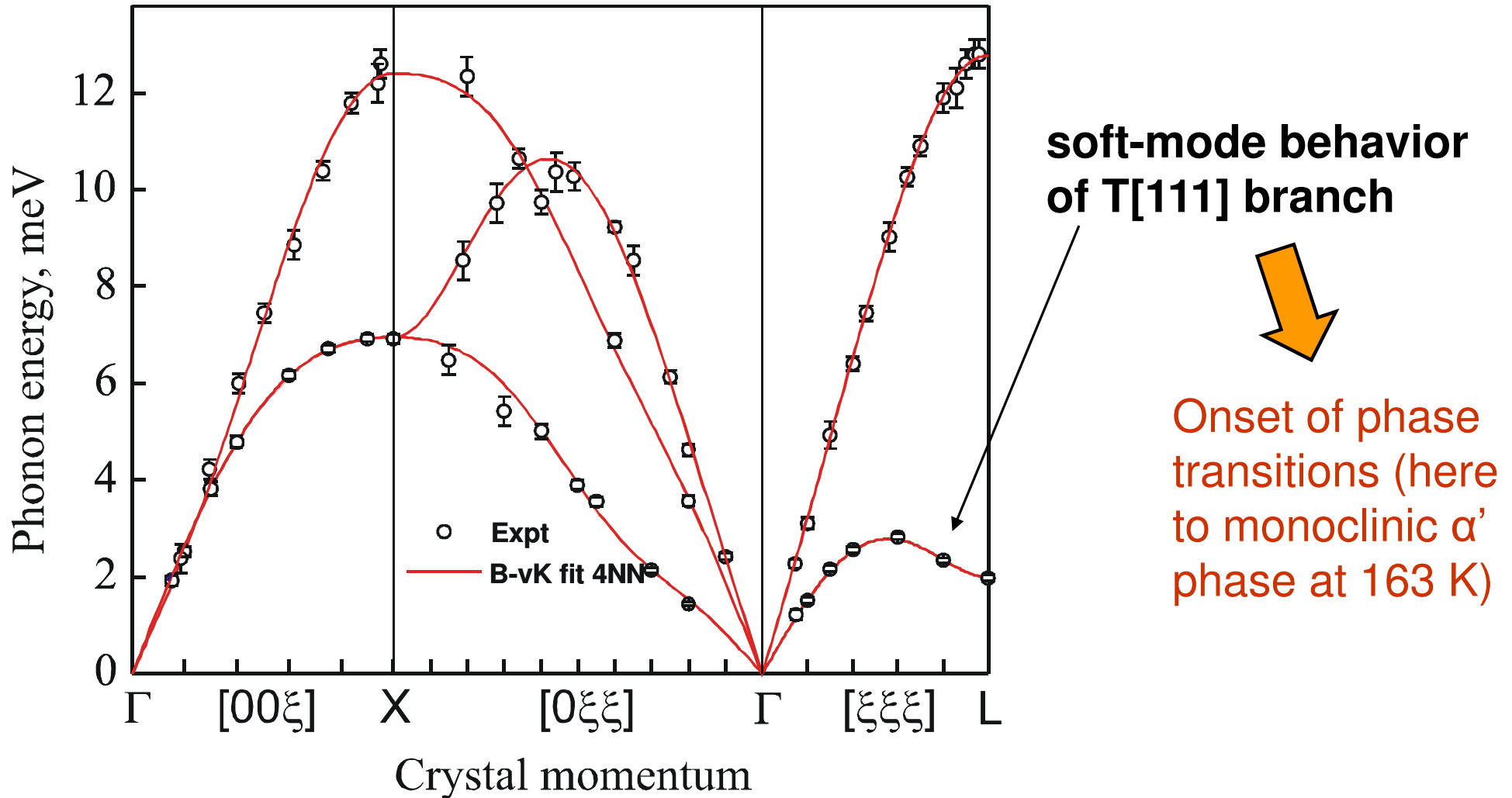
ID28 at ESRF

- Energy resolution: 1.8 meV
- Beam size: 20 x 60 μm^2 (HxV)
- Grain size: $\sim 80 \mu\text{m}^2$
- On-line diffraction analysis



Experimental highlights (2)

Phonon dispersions in plutonium



- **Born-von Karman force constant model fit** (fourth nearest neighbors)

Experimental highlights (2)

Phonon dispersions in plutonium

Close to Γ -point: $E = \mathbf{V}q/\hbar$



$$V_L[100] = (C_{11}/\rho)^{1/2}$$

$$V_T[100] = (C_{44}/\rho)^{1/2}$$

$$V_L[110] = ([C_{11}+C_{12}+2C_{44}]/\rho)^{1/2}$$

$$V_{T1}[110] = ([C_{11} - C_{12}] / 2\rho)^{1/2}$$

$$V_{T2}[110] = (C_{44}/\rho)^{1/2}$$

$$V_L[111] = [C_{11}+2C_{12}+4C_{44}]/3\rho)^{1/2}$$

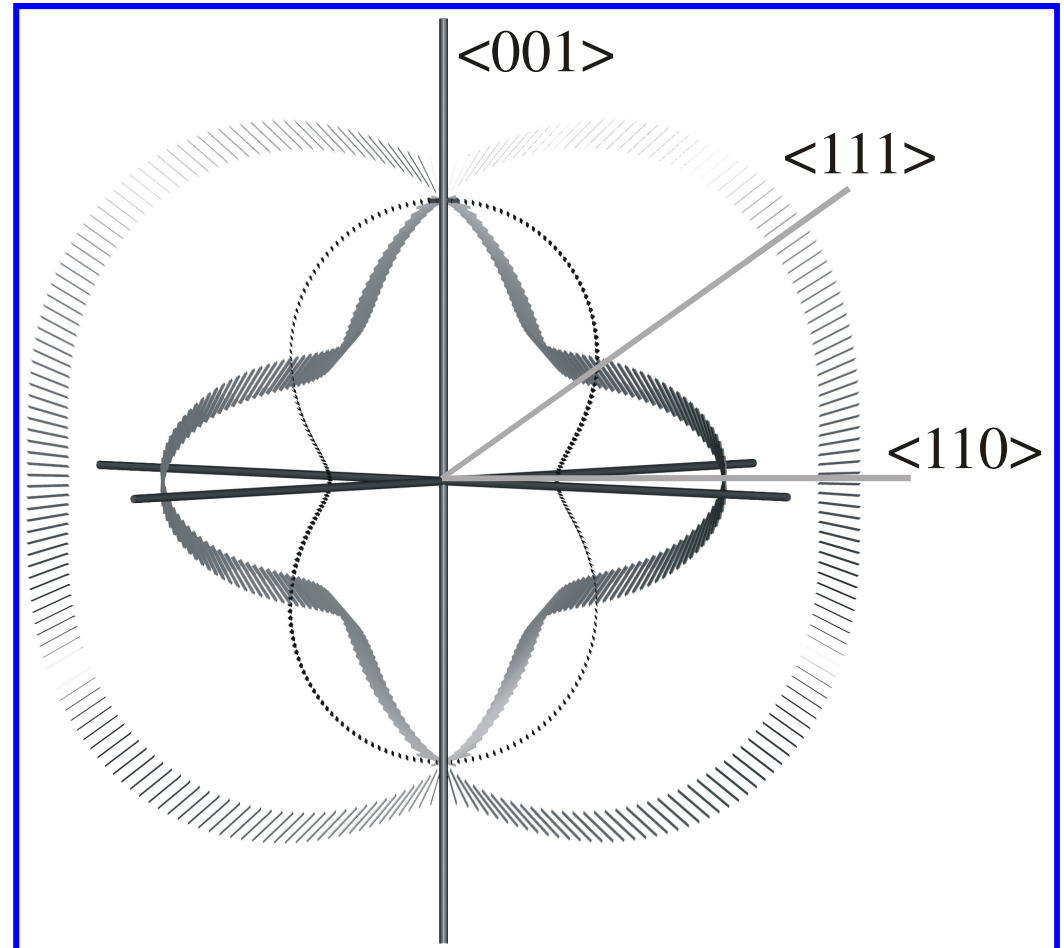
$$V_T[111] = ([C_{11}-C_{12}+C_{44}]/3\rho)^{1/2}$$



$$C_{11} = 35.3 \pm 1.4 \text{ GPa}$$

$$C_{12} = 25.5 \pm 1.5 \text{ GPa}$$

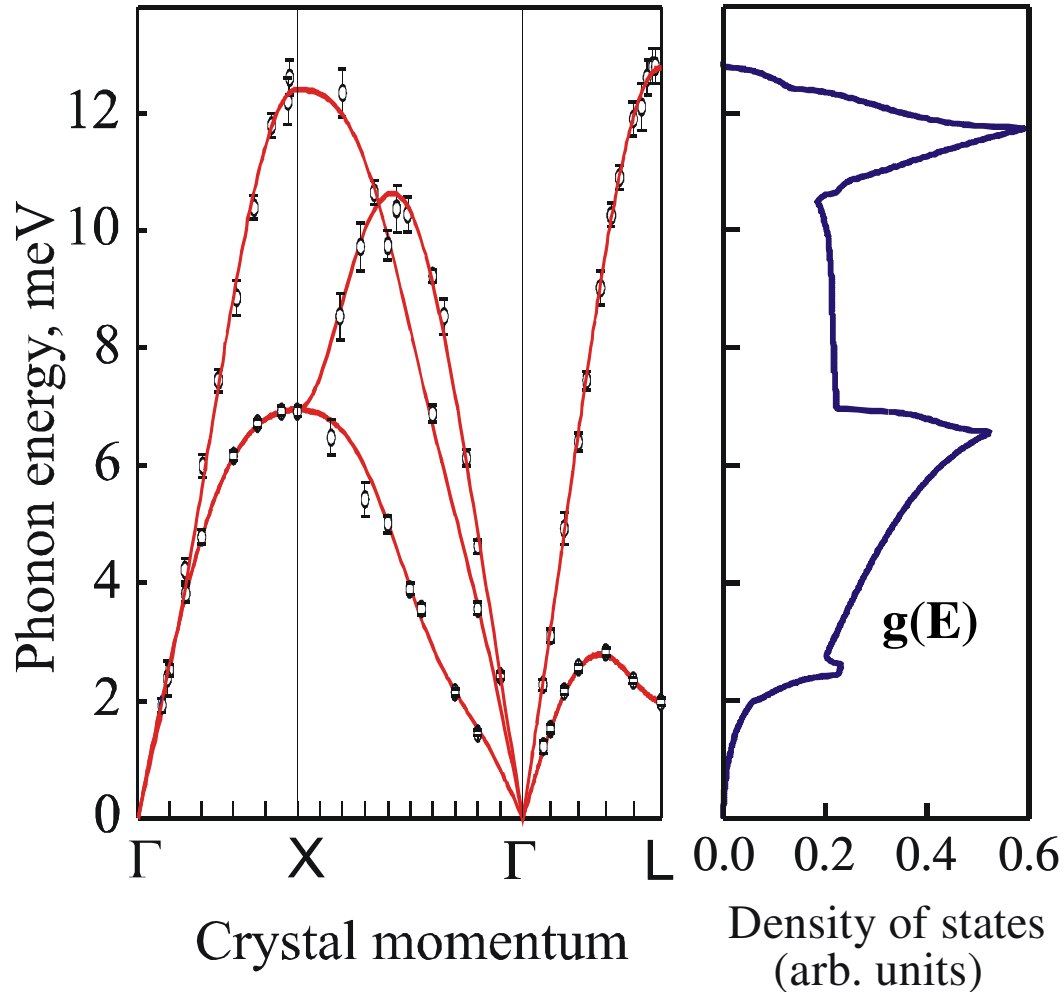
$$C_{44} = 30.5 \pm 1.1 \text{ GPa}$$



**highest elastic anisotropy
of all known fcc metals**

Experimental highlights (2)

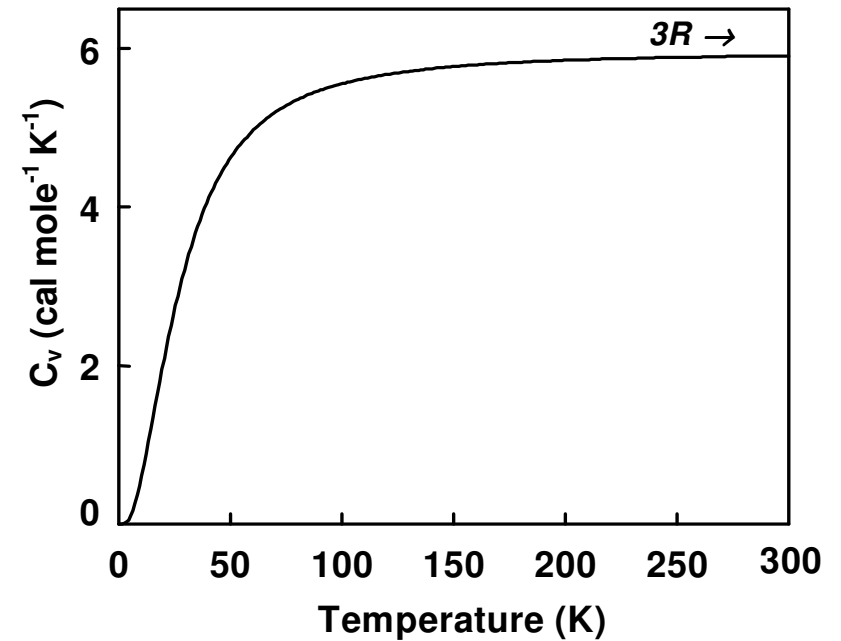
Phonon dispersions in plutonium



• **Born-von Karman fit**

Specific heat:

$$C_v = 3Nk_B \int_0^{E_{\max}} \left(\frac{E}{k_B T} \right)^2 \frac{\exp(E/k_B T) g(E) dE}{(\exp(E/k_B T) - 1)^2}$$

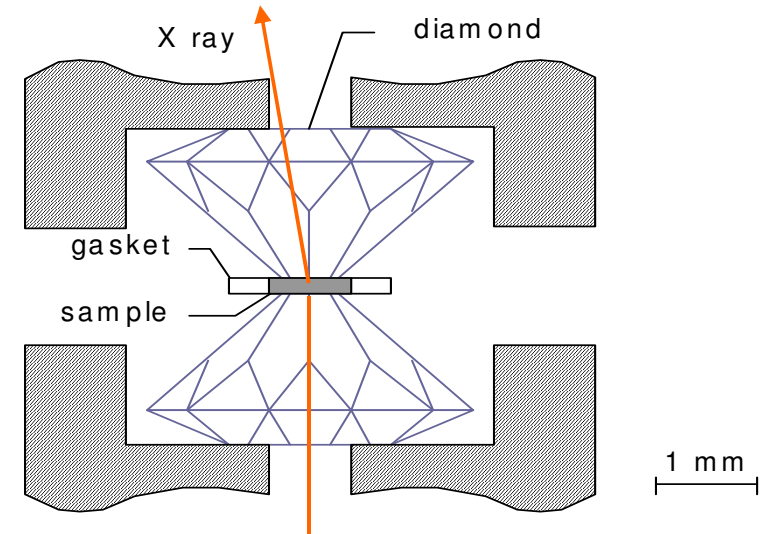
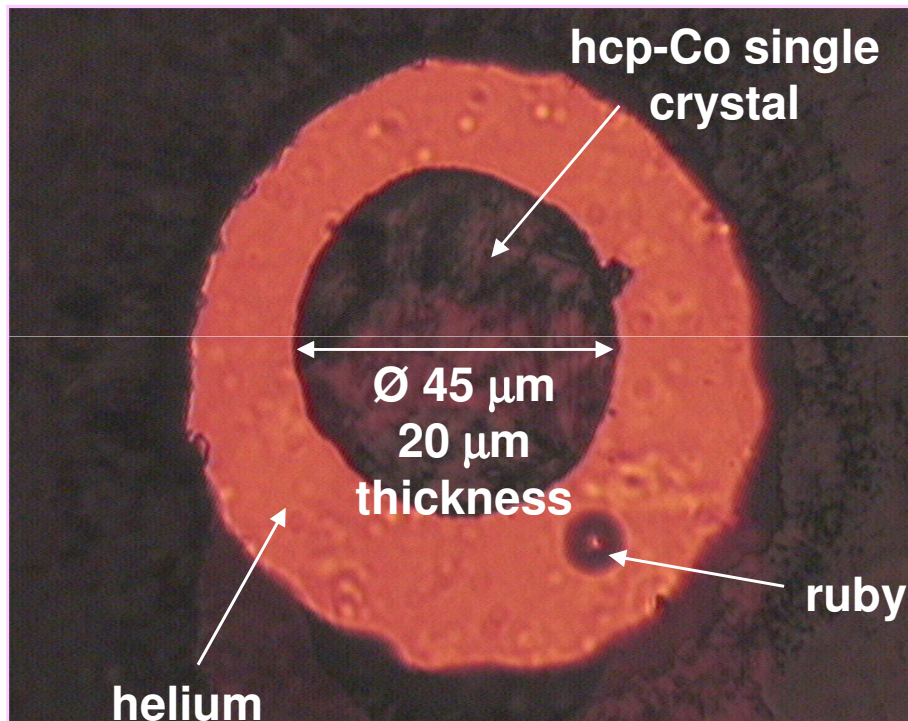


Experimental highlights (3)

Elasticity at high pressure

Elasticity of hcp-metals under very high pressure (up to 1 Mbar):

- Geophysical interest (Earth core)
- DAC sample environment + IXS



hcp-structure:

5 independent elastic moduli

$$V_{L[001]} = (C_{33}/\rho)^{1/2}$$

$$V_{L[100]} = (C_{11}/\rho)^{1/2}$$

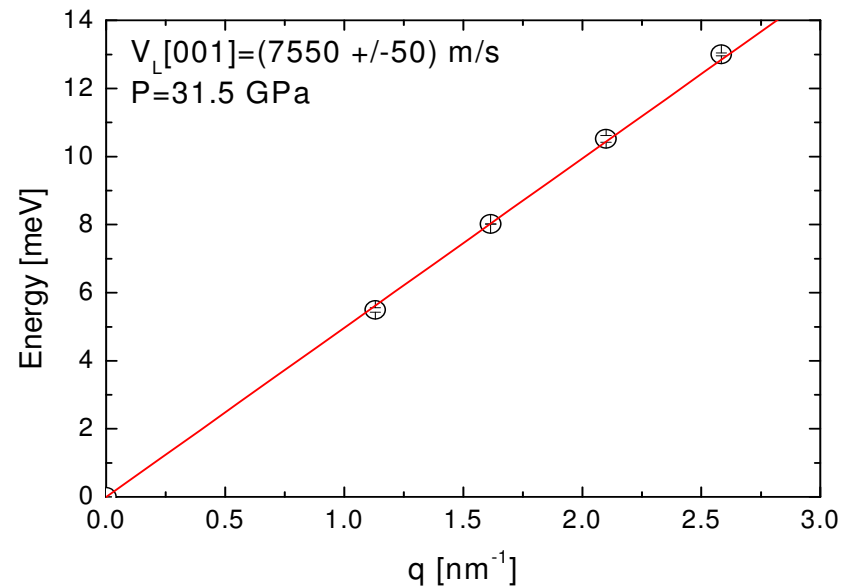
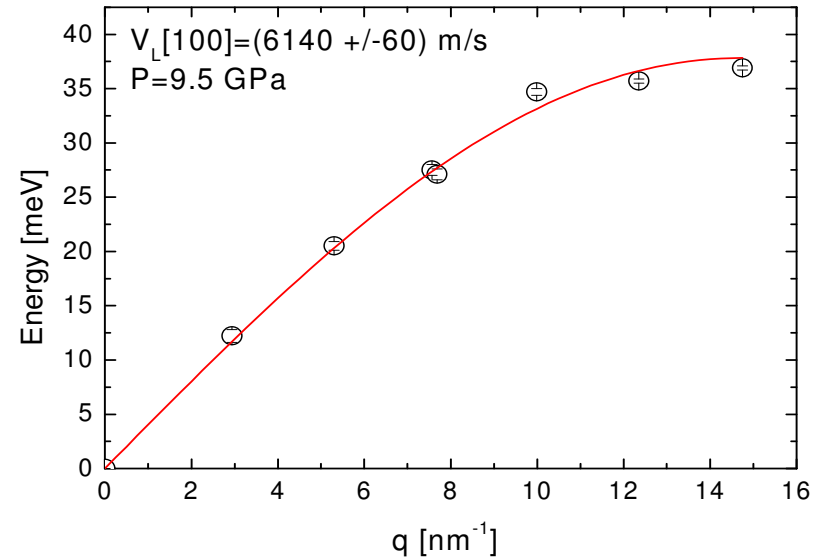
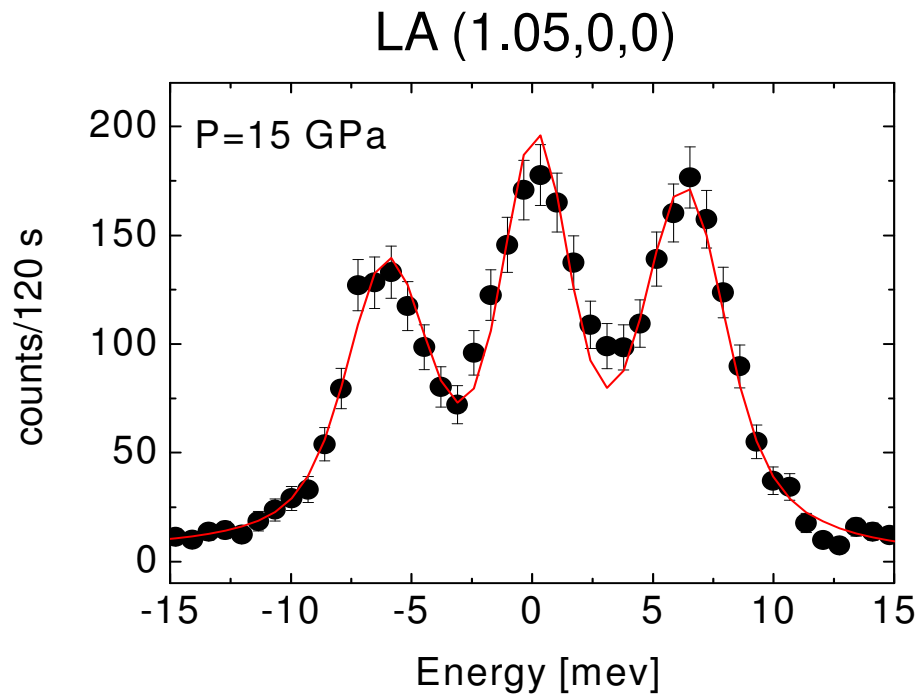
$$V_{T1[110]} = ([C_{11} - C_{12}]/2\rho)^{1/2}$$

$$V_{T2[110]} = (C_{44}/\rho)^{1/2}$$

$$V_{QL[101]} = f(C_{ij}, \rho) \rightarrow C_{13}$$

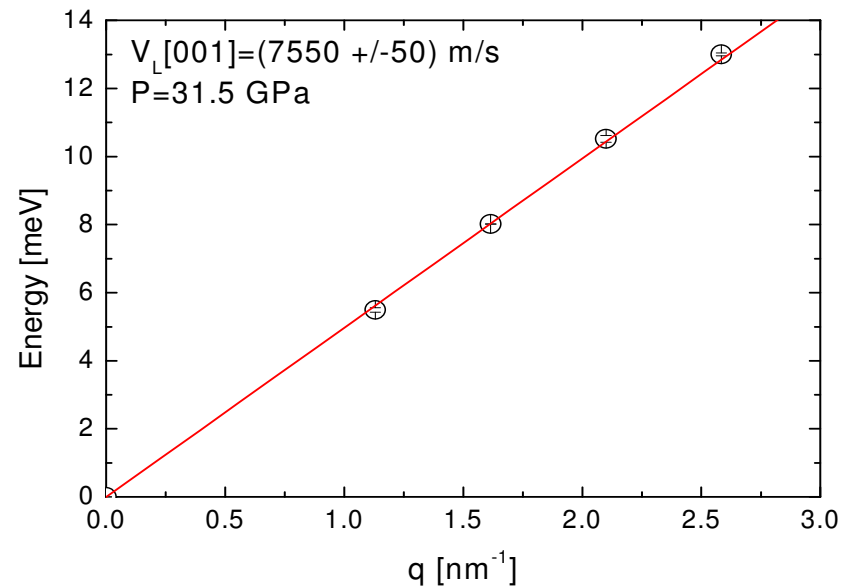
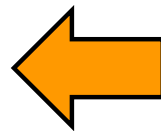
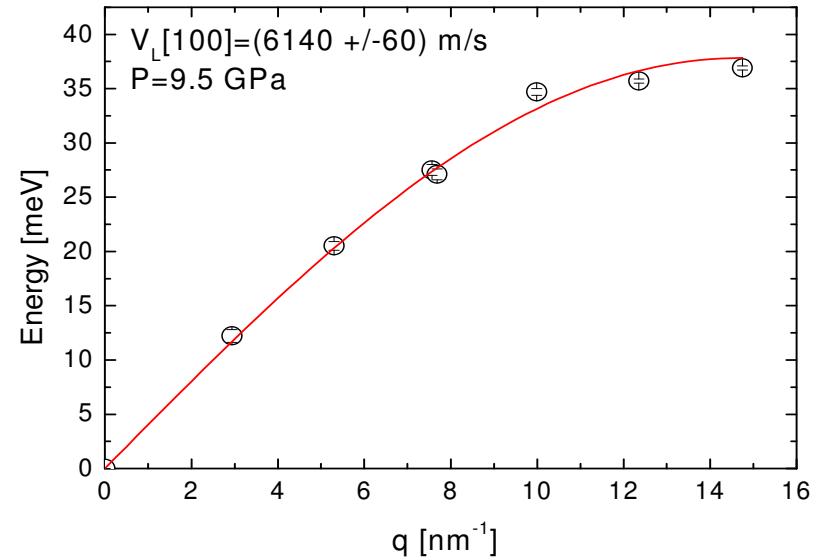
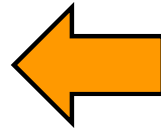
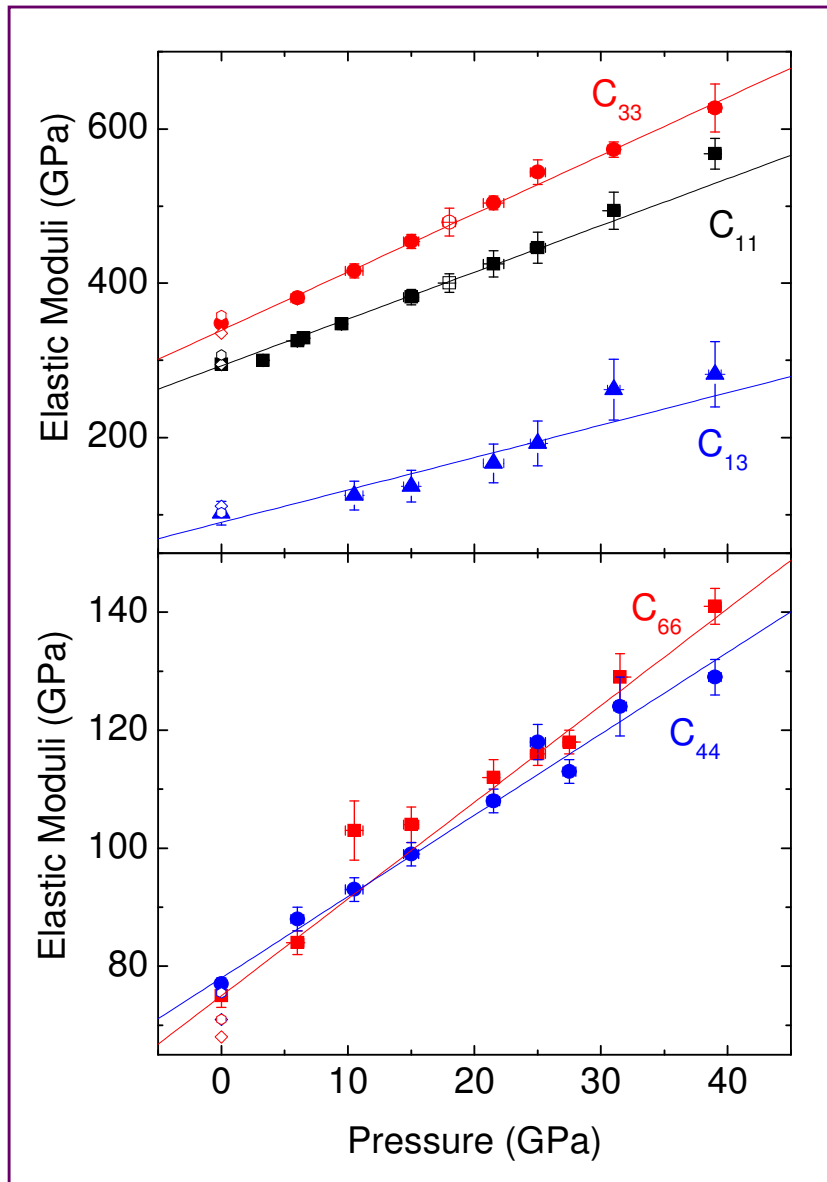
Experimental highlights (3)

Elasticity at high pressure



Experimental highlights (3)

Elasticity at high pressure

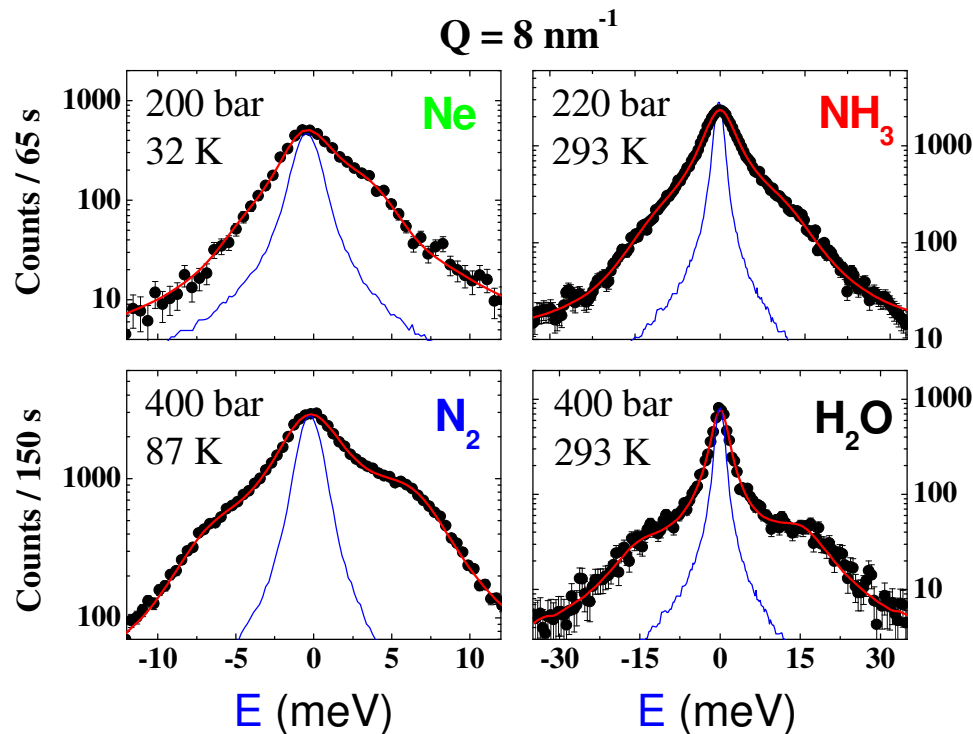


Experimental highlights (4)

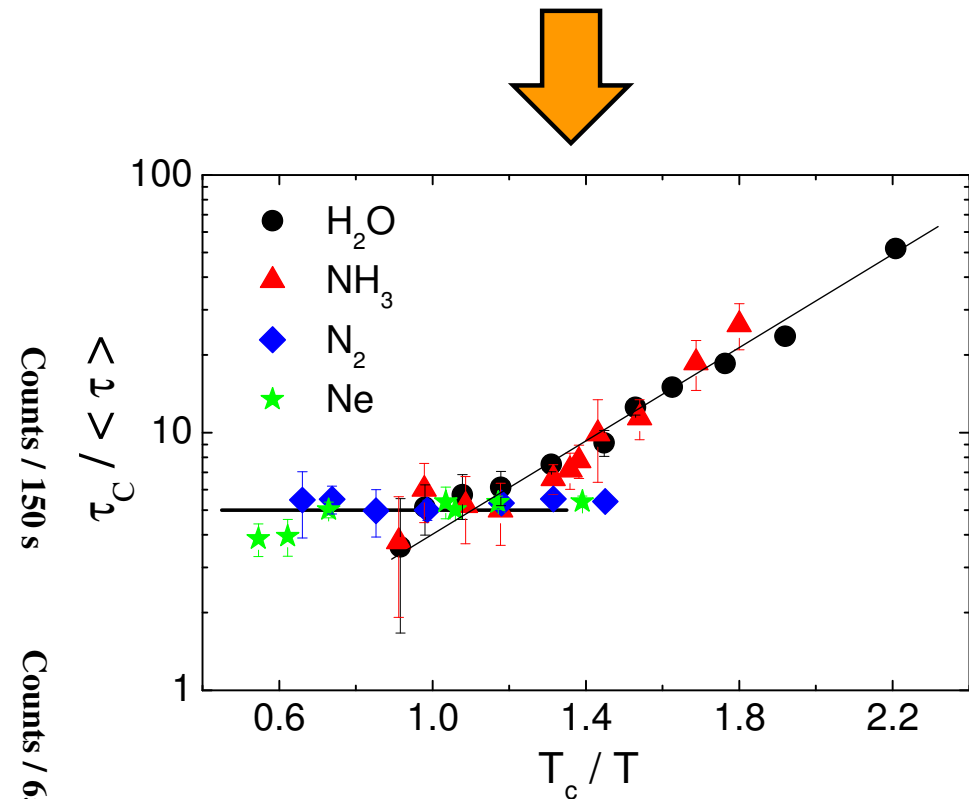
Liquids and supercritical fluids

ID28 and ID16 at ESRF

- Energy resolution: 1.5 meV
- Moderate pressure (< 500 bar)
- Various temperatures (20÷800 K)



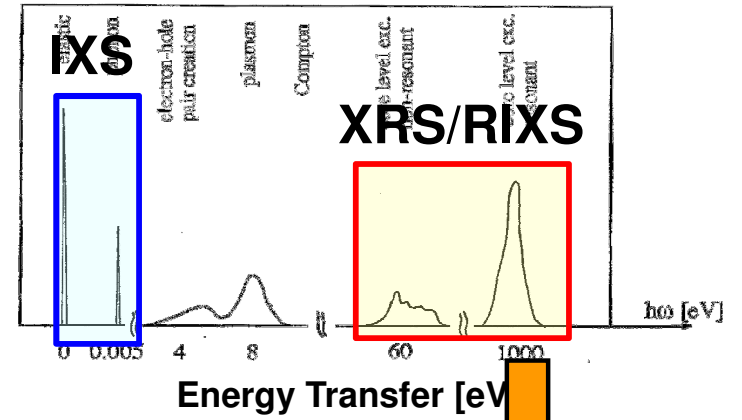
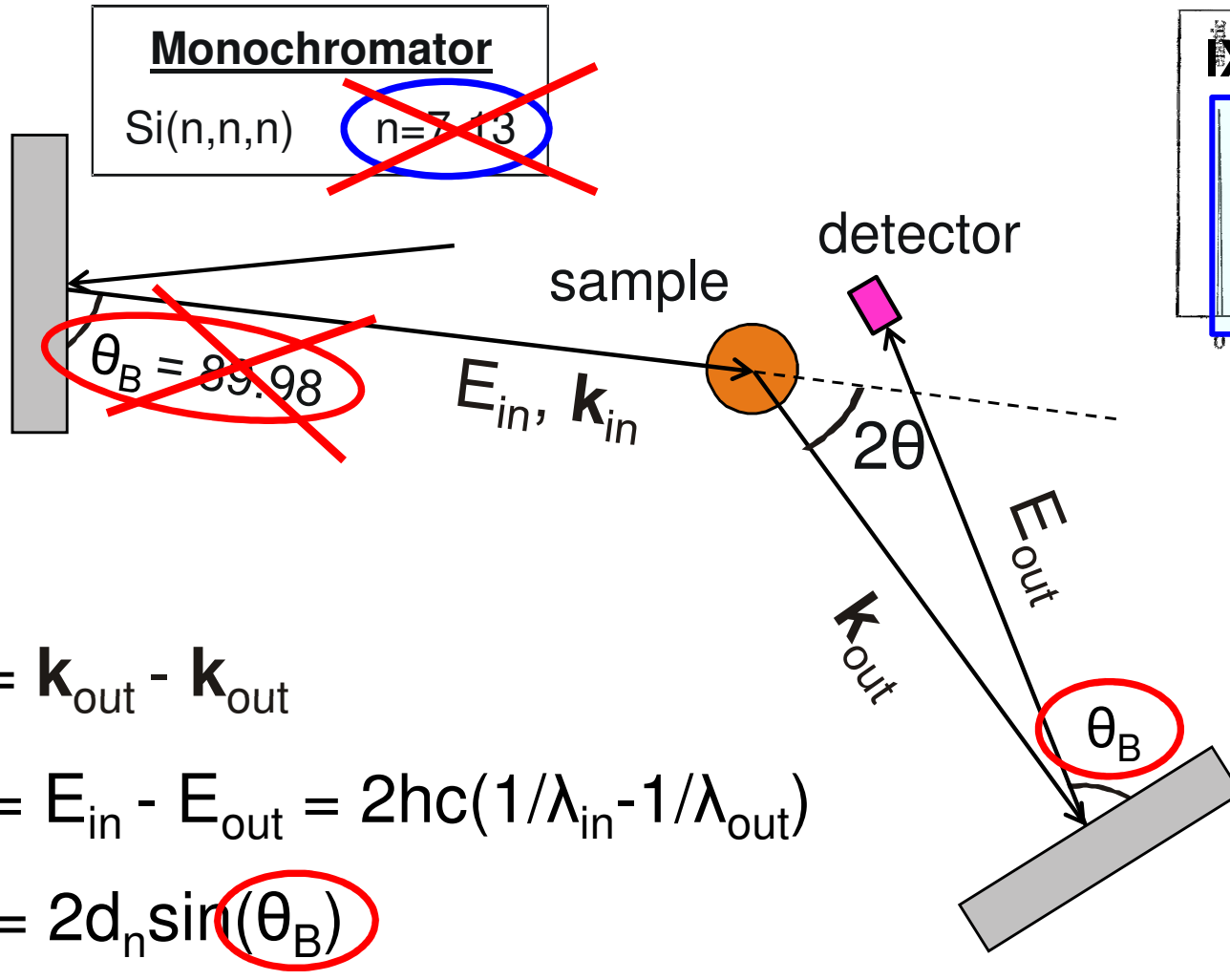
Viscoelastic data analysis:



Leading dynamics in fluids:

- $T > T_c \rightarrow$ collisions
- $T < T_c \rightarrow$ bond's lifetime

Basic instrumentation



$\Delta E \sim eV$

$$\mathbf{k} = \mathbf{k}_{in} - \mathbf{k}_{out}$$

$$E = E_{in} - E_{out} = 2hc(1/\lambda_{in} - 1/\lambda_{out})$$

$$\lambda = 2d_n \sin(\theta_B)$$

Analyser
Si(n,n,n) n=7-13

~~backscattering~~ + ~~high order reflections~~ = ~~$\Delta E \sim meV$~~

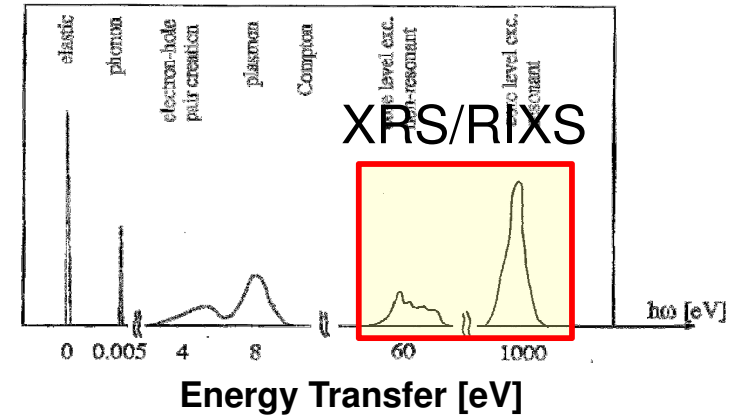
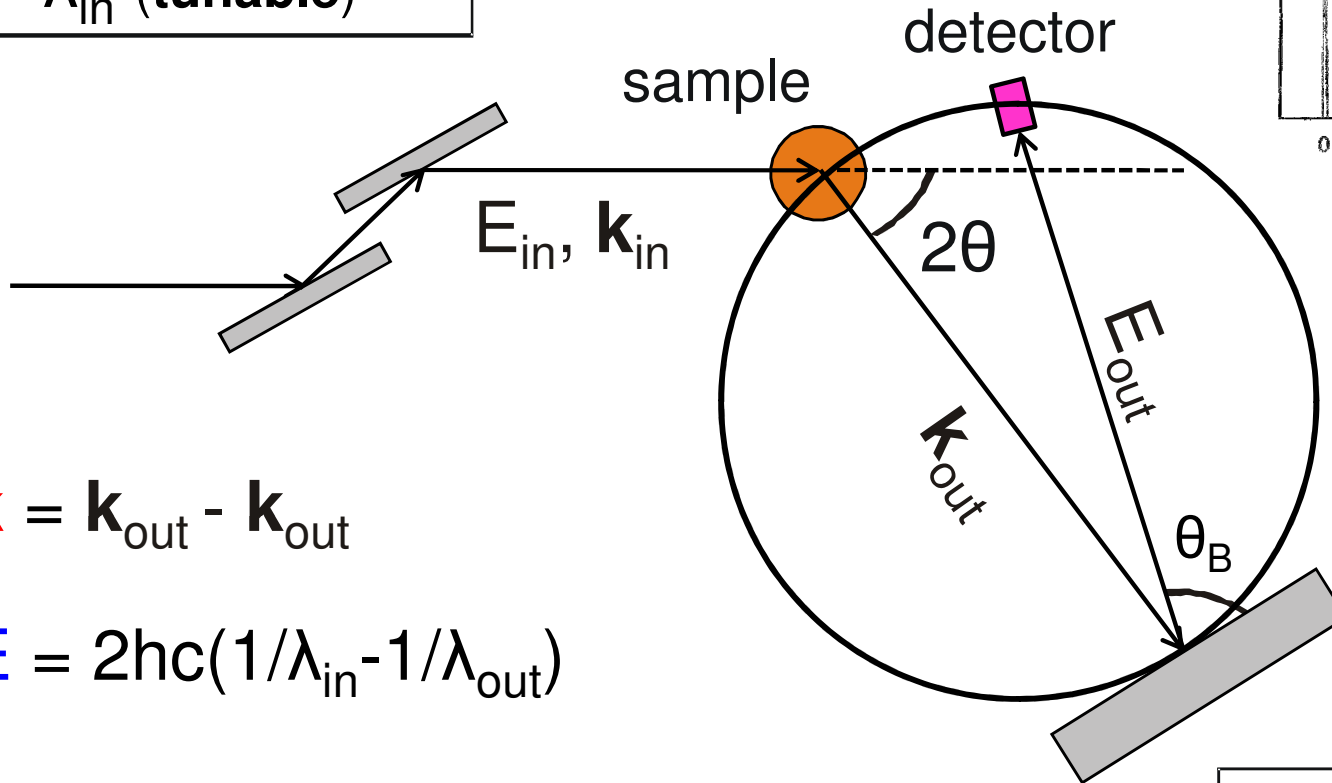
Basic instrumentation

XRS / RIXS

Monochromator

Si(1,1,1); (2,2,0); ...

λ_{in} (tunable)



Rowland circle spectrometer (1 m)

$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$E = 2hc(1/\lambda_{in} - 1/\lambda_{out})$$

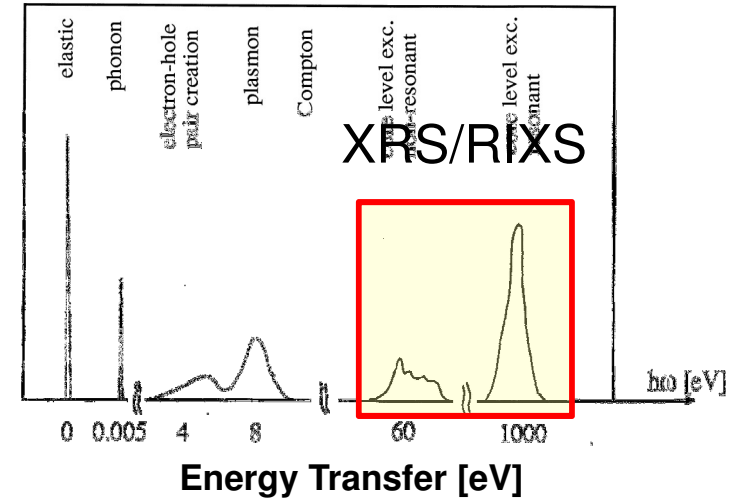
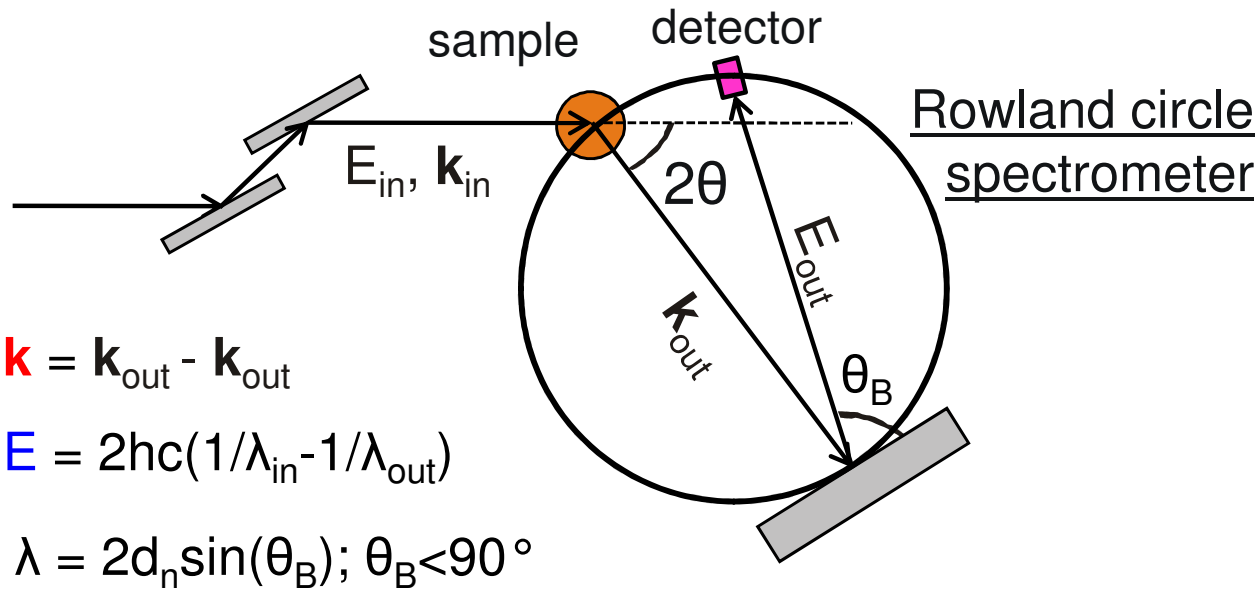
$$\lambda = 2d_n \sin(\theta_B); \theta_B < 90^\circ$$

Analyser

Si(h,k,l) λ_{out} (tunable)

Basic instrumentation

XRS / RIXS



Scanning strategy

1. E_{out} fixed, scanning E_{in}
2. E_{in} fixed, scanning E_{out}
(rotating crystal and follow with the detector)
3. Scanning E_{in} and E_{out} keeping E constant

IXS, XRS, RIXS

RIXS

RIXS

Basic theoretical aspects

$$H_{\text{int}} = (e/m_e c) \sum_j [(e/2c) \mathbf{A}_j \cdot \mathbf{A}_j + \mathbf{A}_j \cdot \mathbf{p}_j]$$

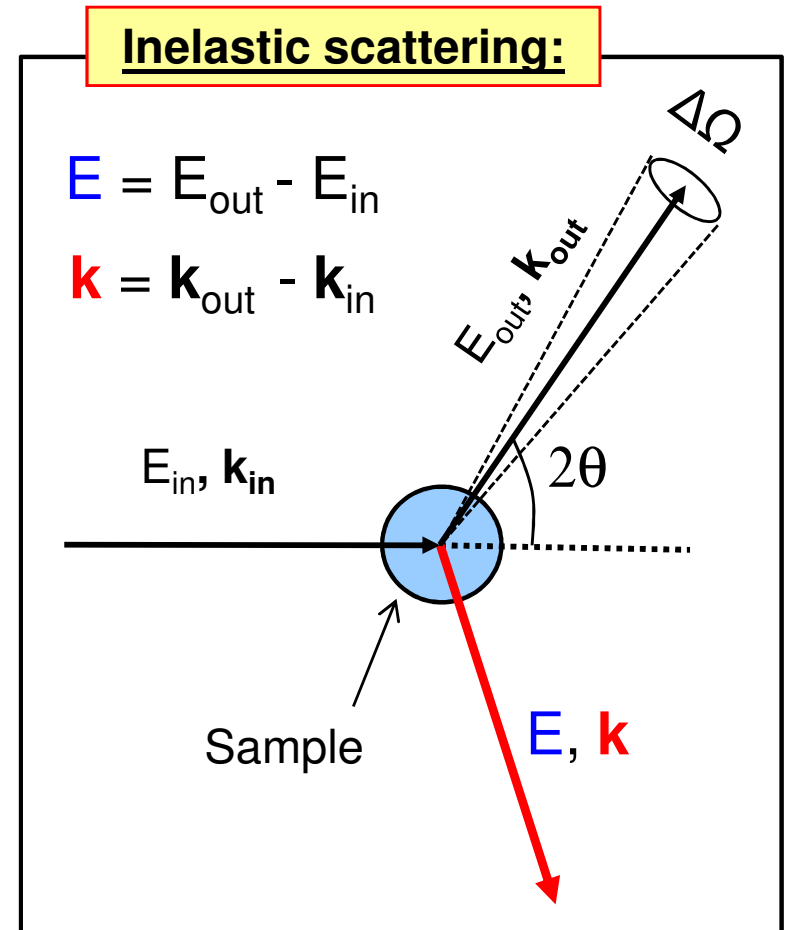
A: vector potential of electromagnetic field

P: momentum operator of the electrons

j : summation over all electrons of the system

$\mathbf{A} \cdot \mathbf{A} \rightarrow$ non-resonant scattering (example: IXS)

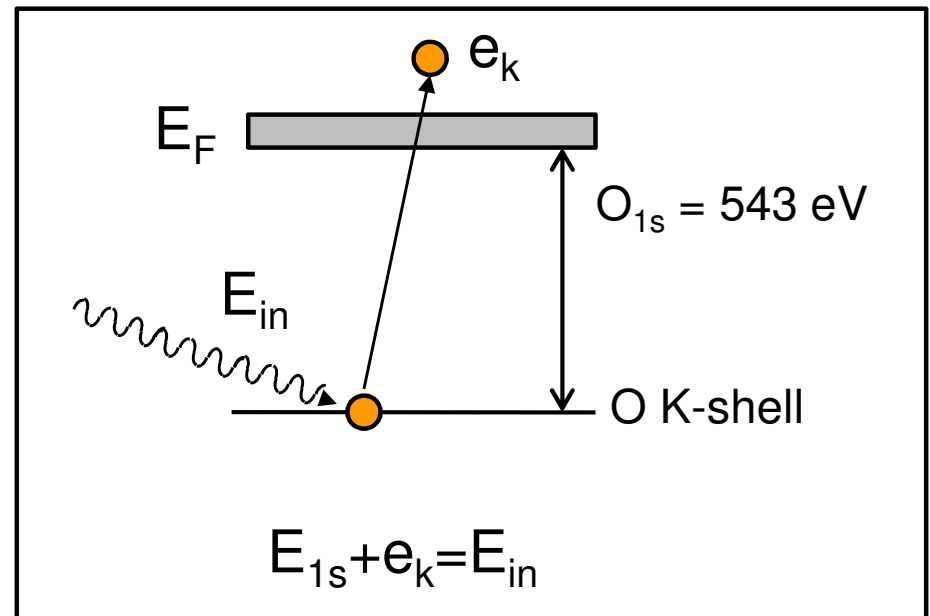
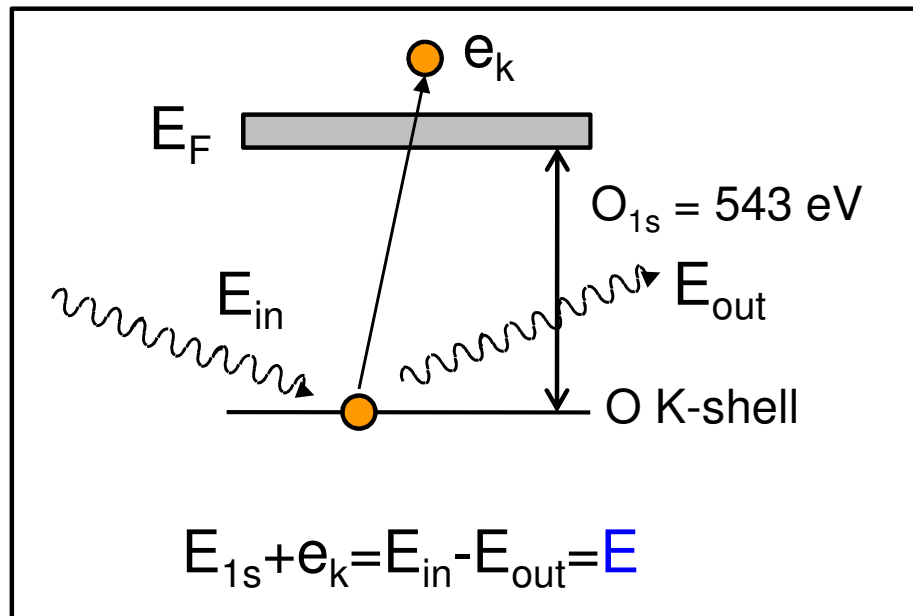
$\mathbf{A} \cdot \mathbf{p} \rightarrow$ resonant scattering, absorption followed by emission



Basic theoretical aspects

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_l)$$



X-ray absorption cross section (dipolar approximation):

$$\frac{\partial \sigma}{\partial E_{\text{in}}} = 4\pi^2 \alpha E_{\text{in}} \sum_l P_l |\langle l | \boldsymbol{\epsilon}_{\text{in}} \cdot \mathbf{r}_j | F \rangle|^2 \delta(E_{\text{in}} - E_F + E_l)$$

Basic theoretical aspects

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_l)$$

↓

$$\mathbf{k} \cdot \mathbf{r}_j \ll 1 \rightarrow e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 + i\mathbf{k} \cdot \mathbf{r}_j$$

$\mathbf{k} \cdot \mathbf{r}_j \ll 1 \rightarrow$ Dipolar regime: identical to photon absorption, where:

- i) The momentum transfer (\mathbf{k}) plays the role of the photon polarization vector ($\boldsymbol{\epsilon}_{\text{in}}$)
- ii) The energy transfer (E) plays the role of the incident energy (E_{in})

X-ray absorption cross section (dipolar approximation):

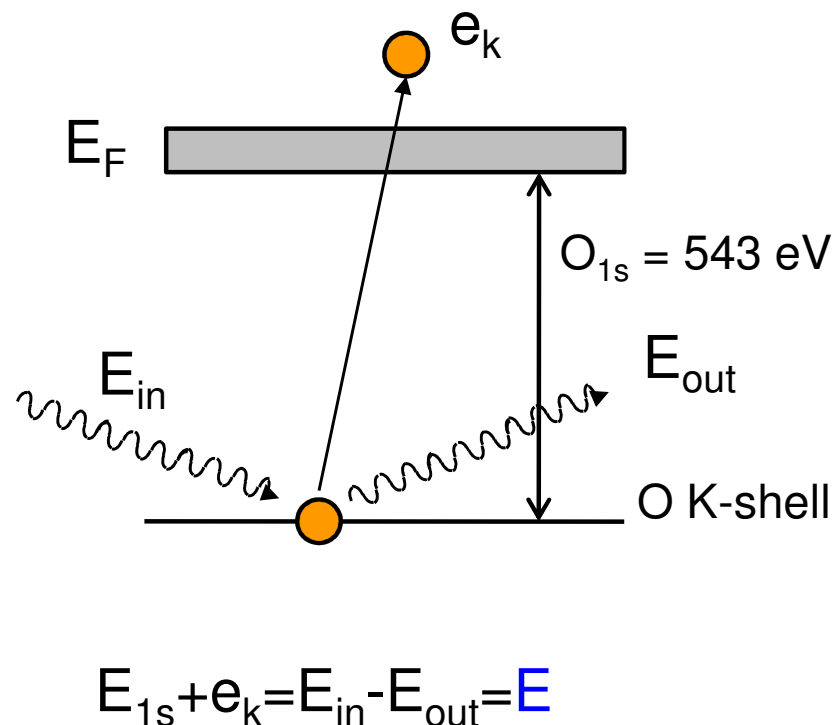
$$\frac{\partial \sigma}{\partial E_{\text{in}}} = 4\pi^2 \alpha E_{\text{in}} \sum_l P_l |\langle l | \boldsymbol{\epsilon}_{\text{in}} \cdot \mathbf{r}_j | F \rangle|^2 \delta(E_{\text{in}} - E_F + E_l)$$

Basic theoretical aspects

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_F P_F |\langle I | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_I)$$

X-ray Raman Scattering (XRS)



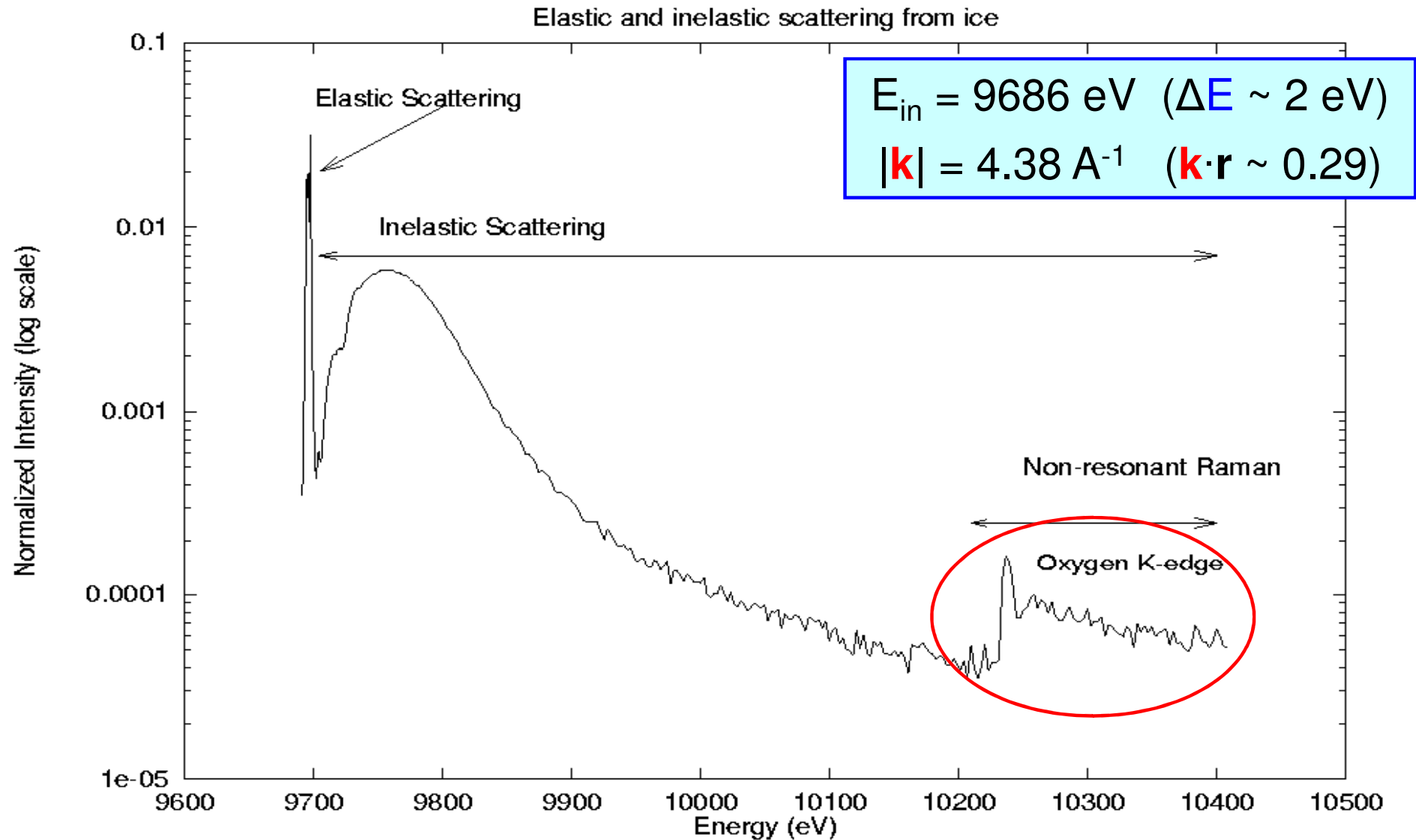
Motivation: element-selective probe for local atomic structure

XRS is alternative to:

- Neutron scattering (with isotopic substitution)
- X-ray (anomalous) scattering
- XANES and EXAFS

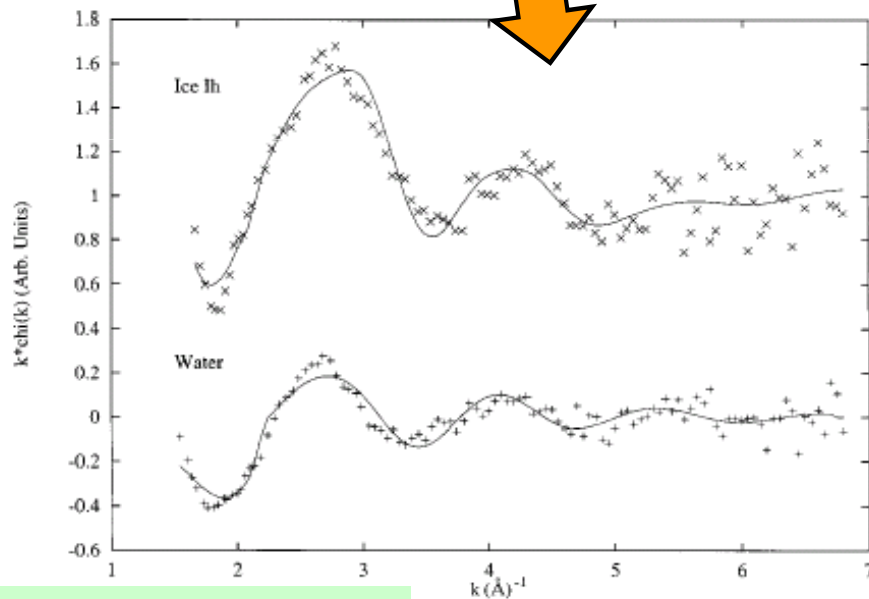
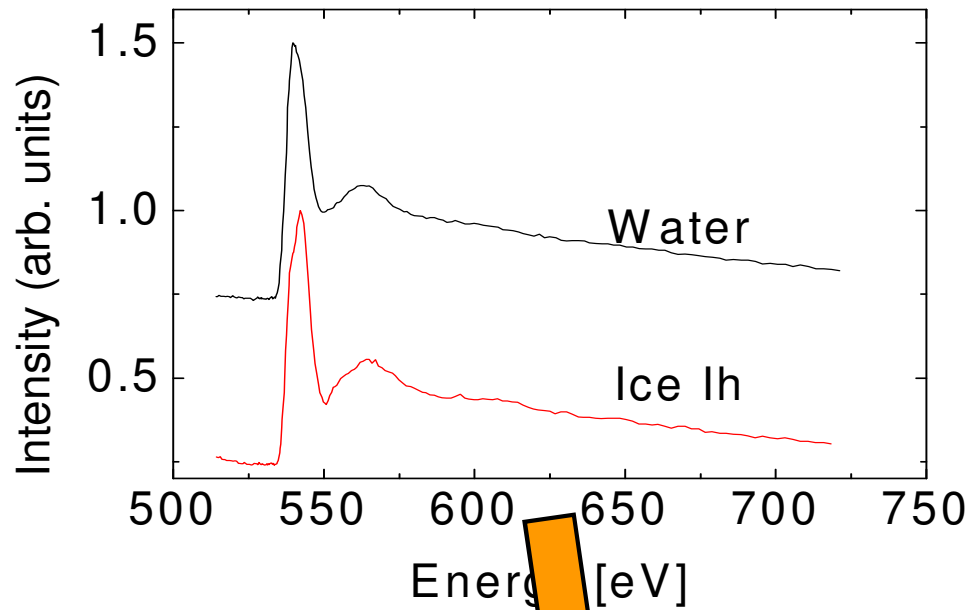
Experimental highlights (XRS)

XRS from O K-edge in water and ice



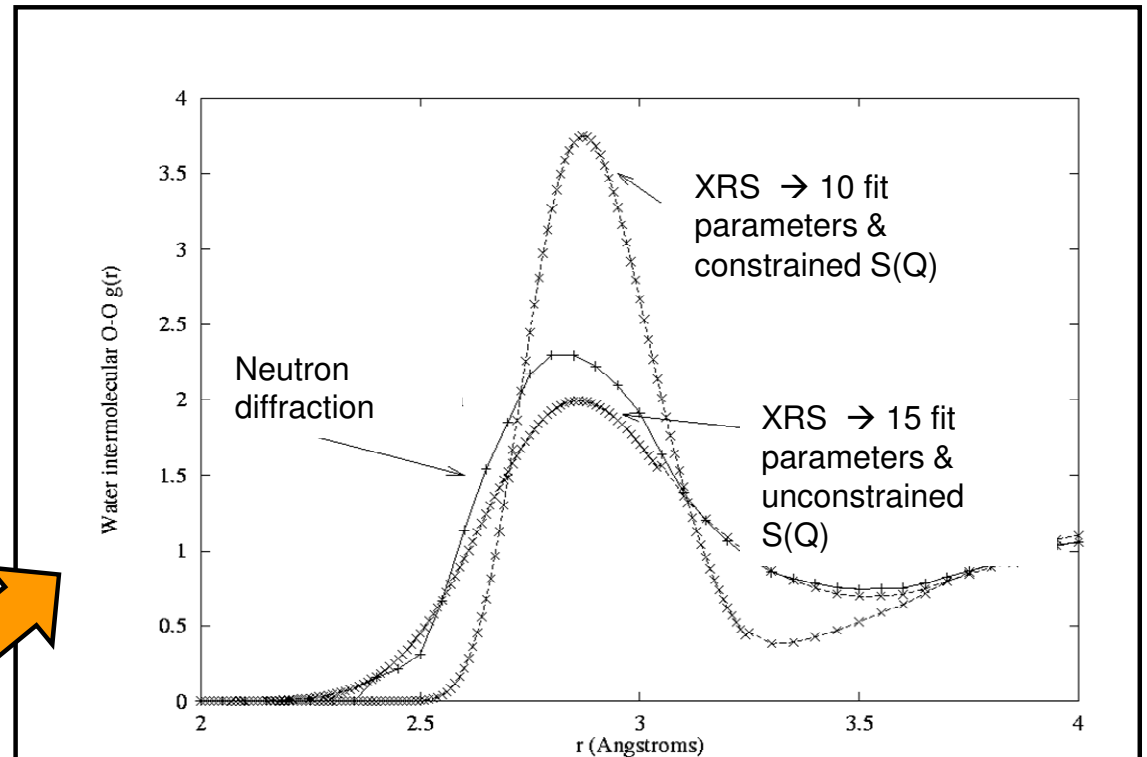
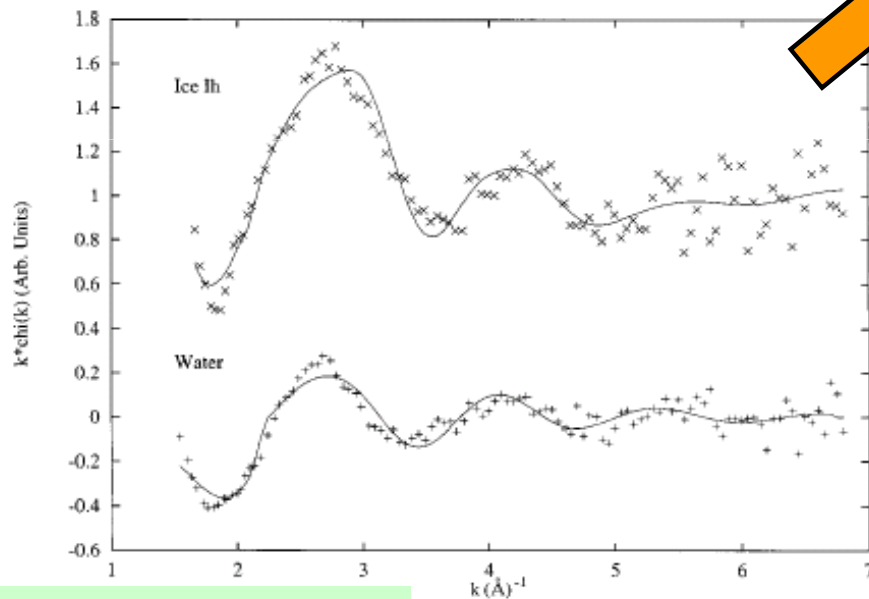
Experimental highlights (XRS)

XRS from O K-edge in water and ice (EXAFS)



Experimental highlights (XRS)

XRS from O K-edge in water and ice (EXAFS)



XRS:

O-O distance: 2.87 \AA
Coordination: 4 - 7

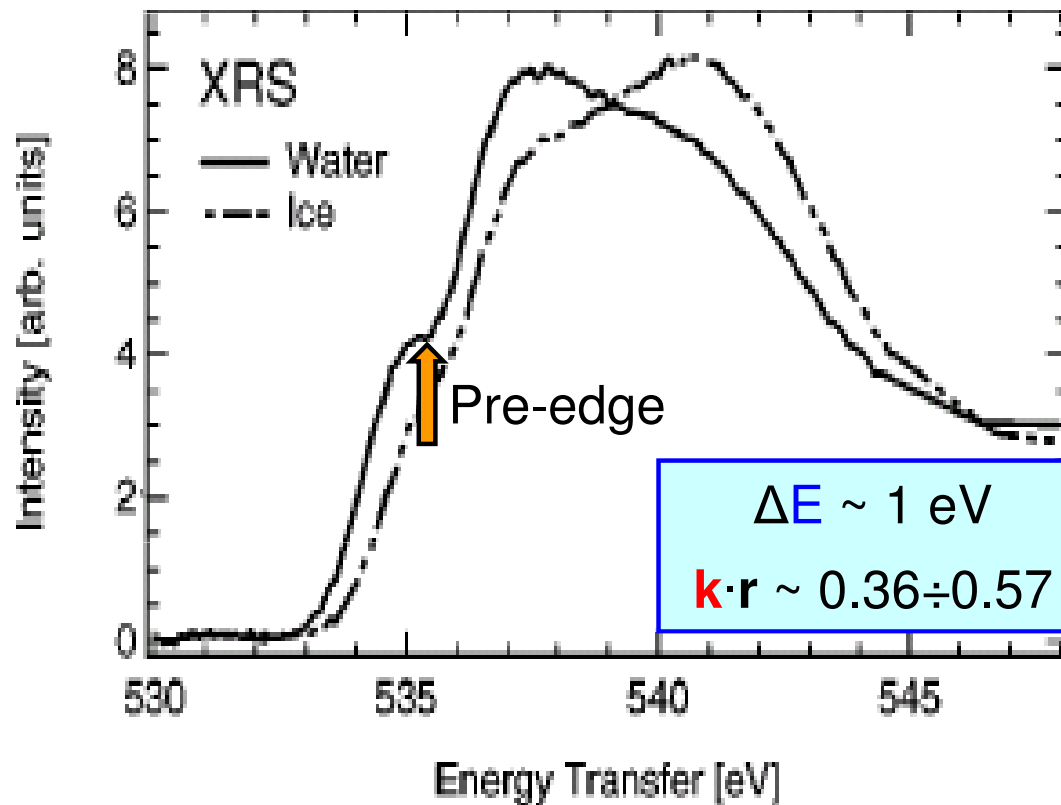
Neutrons:

O-O distance: 2.85 \AA
Coordination: 4.4

Experimental highlights (XRS)

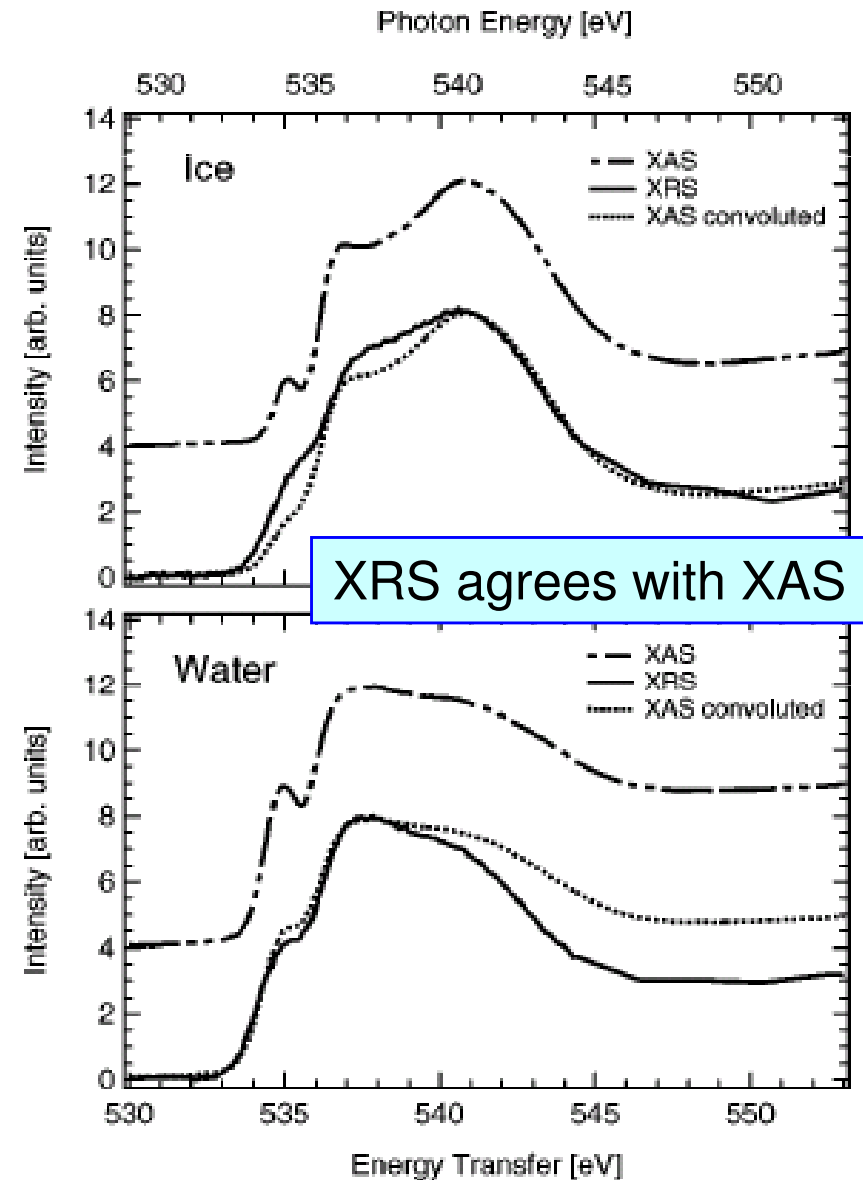
XRS from O K-edge in water and ice (XANES)

XANES sensitive to the number and “type” of hydrogen bonds (HB)



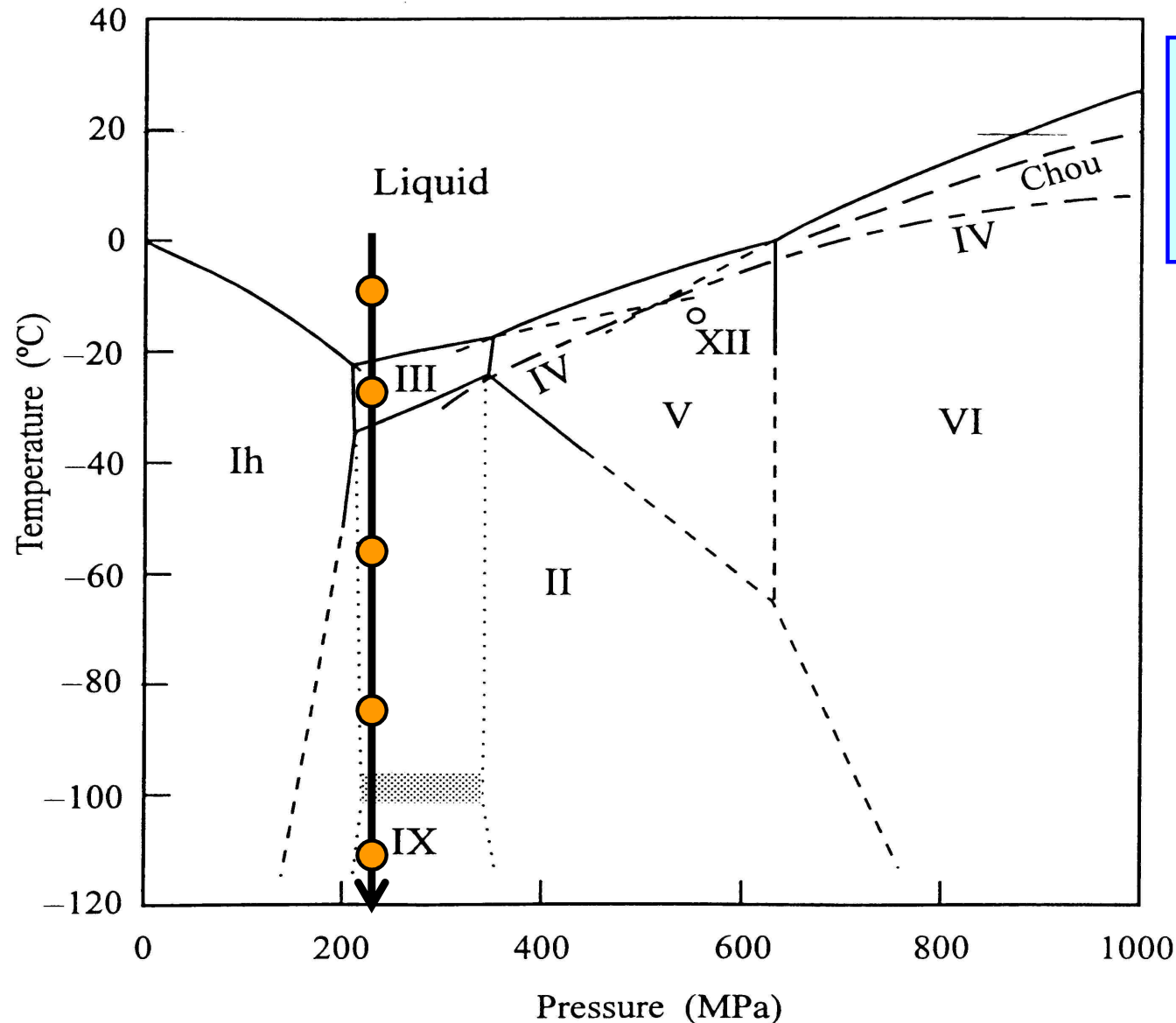
- Pre-edge indicates a large number (~ 70%) of distorted or broken HB (supported by calculation)

PRB 66, 092107 (2002)



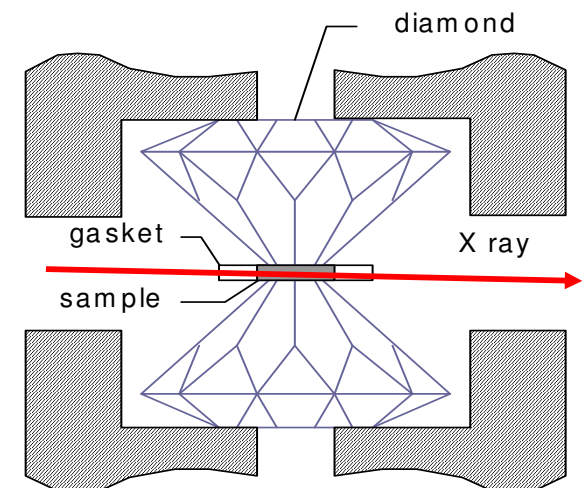
Experimental highlights (XRS)

XRS from O K-edge in ice under high pressure



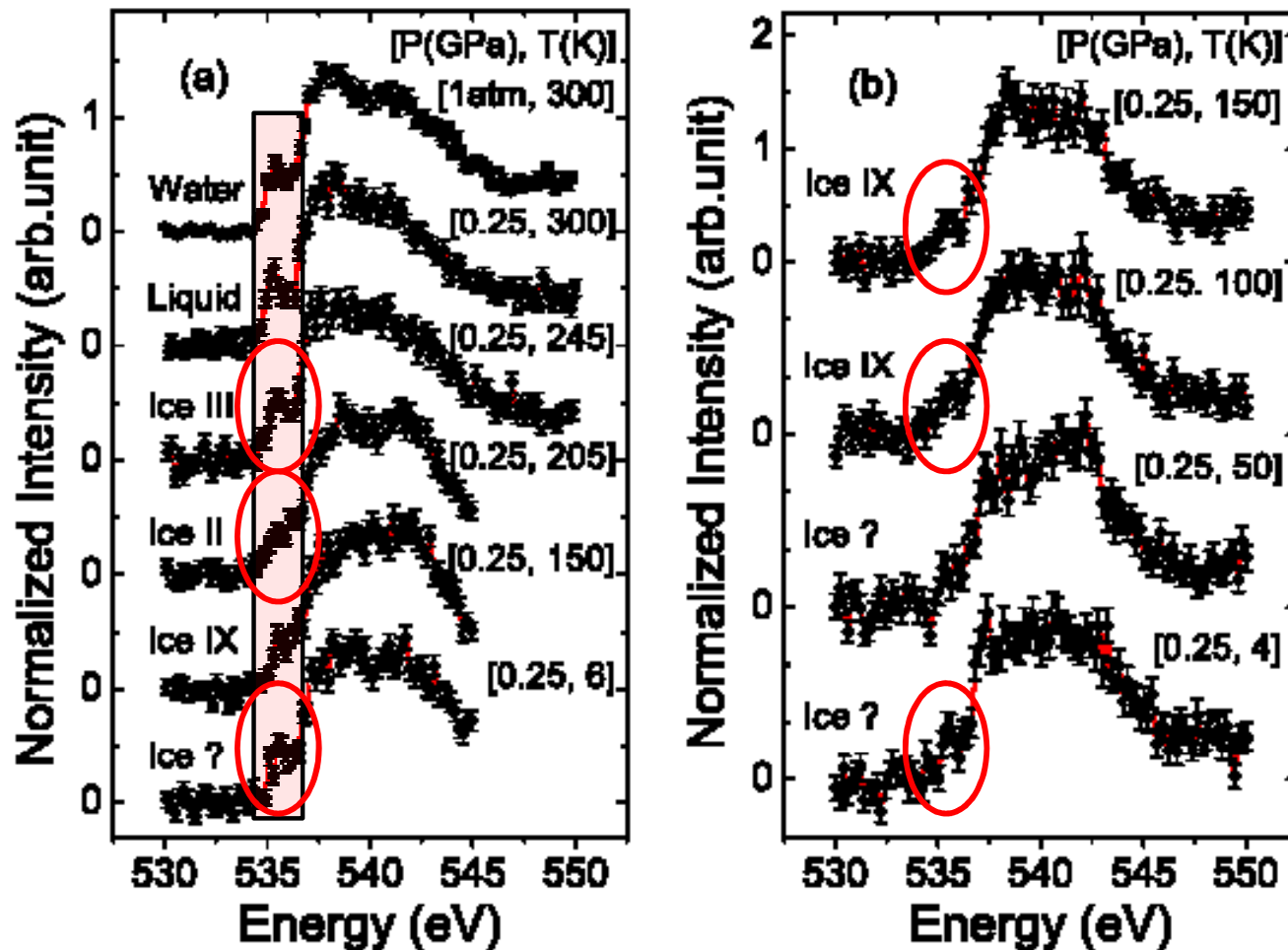
$E_{\text{out}} = 9885 \text{ eV}$
($\Delta E \sim 0.2 \div 0.3 \text{ eV}$)
 $|\mathbf{k}| \sim 3 \text{ \AA}^{-1}$ ($\mathbf{k} \cdot \mathbf{r} \sim 0.2$)

DAC sample environment:
hard x-ray needed



Experimental highlights (XRS)

XRS from O K-edge in ice under high pressure



- Slight increase of pre-edge with P (larger HB distortion)
- Increasing order of HB from liquid \rightarrow Ice III (tetrahedral) \rightarrow Ice II / IX
- New pre-edge increase @ low-T: new Ice phase?

Observation of spectral changes:

Need of much better statistics and theory to extract quantitative information

XRS in summary

Soft x-ray spectroscopy in the hard x-ray regime

Advantages

- “simpler” sample environment (high pressure/temperature, etc...)
- bulk sensitive
- indicated for studying (bulk) Oxygen and Carbon

Drawbacks

- “weak probe” (practically limited to $Z < 14$)
- limited quality for structural analysis (EXAFS), reasonable quality in the XANES region

Exploit information in the near-edge region

Basic theoretical aspects (RIXS)

$$H_{\text{int}} = (e/m_e c) \sum_j [(e/2c) \mathbf{A}_j \cdot \mathbf{A}_j + \mathbf{A}_j \cdot \mathbf{p}_j]$$

A: vector potential of electromagnetic field

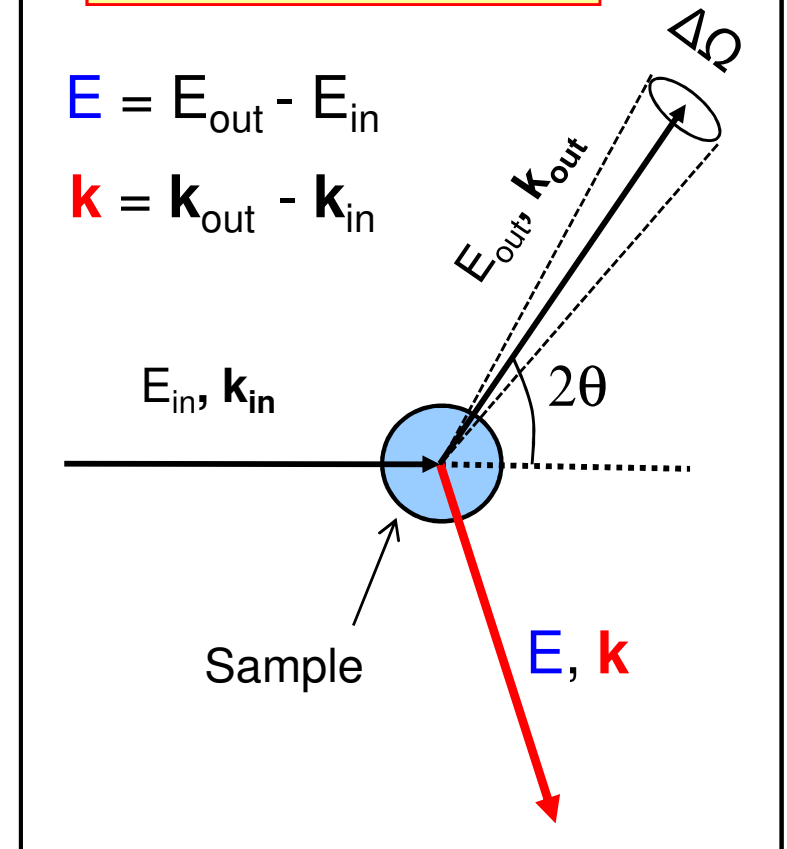
P: momentum operator of the electrons

j : summation over all electrons of the system

Inelastic scattering:

$$\mathbf{E} = E_{\text{out}} - E_{\text{in}}$$

$$\mathbf{k} = \mathbf{k}_{\text{out}} - \mathbf{k}_{\text{in}}$$



$\mathbf{A} \cdot \mathbf{A} \rightarrow$ non-resonant scattering (IXS - XRS)

$\mathbf{A} \cdot \mathbf{p} \rightarrow$ resonant scattering, absorption followed by emission

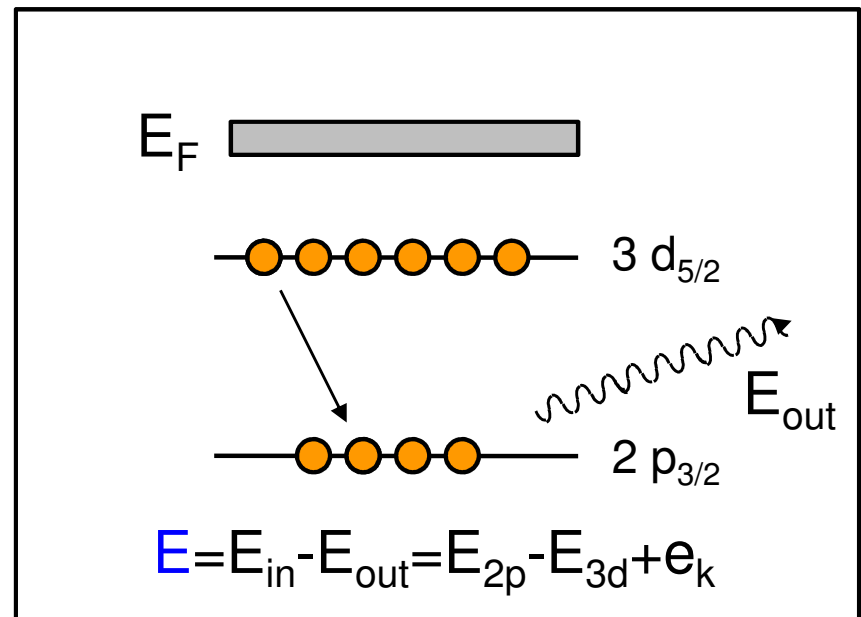
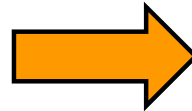
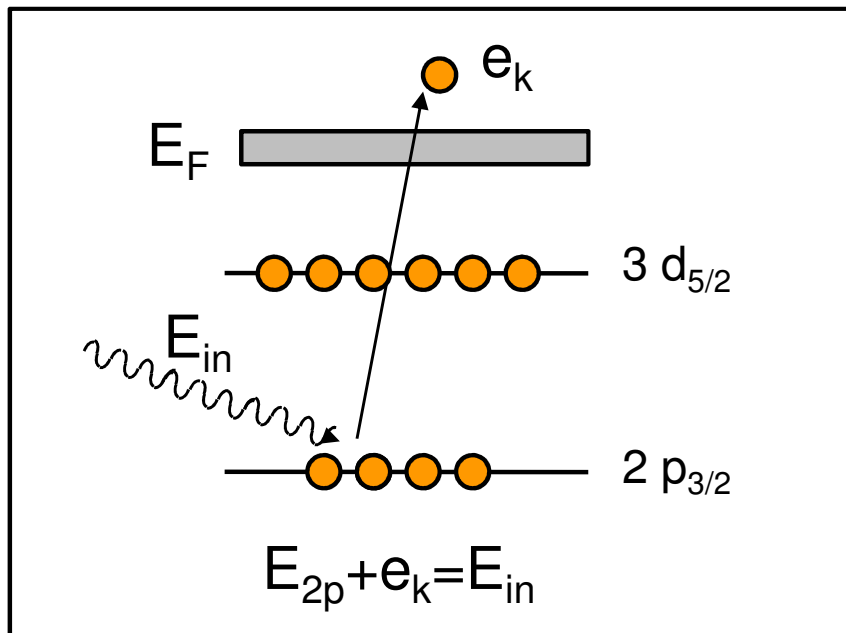
Basic theoretical aspects (RIXS)

Absorption Resonant IXS cross section: **Emission**

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} \sim \sum_F \left| \sum_n \frac{\langle I | C_k | N \rangle \langle N | C_k^\dagger | F \rangle}{E_N - E_I - E_{in} - i\Gamma_N} \right|^2 \delta(E - E_F + E_I)$$

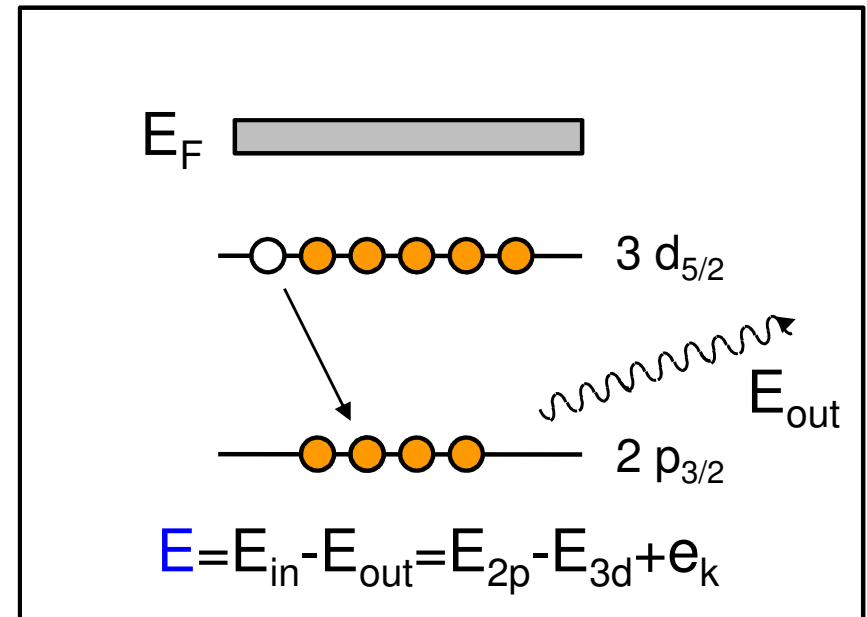
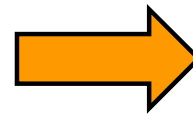
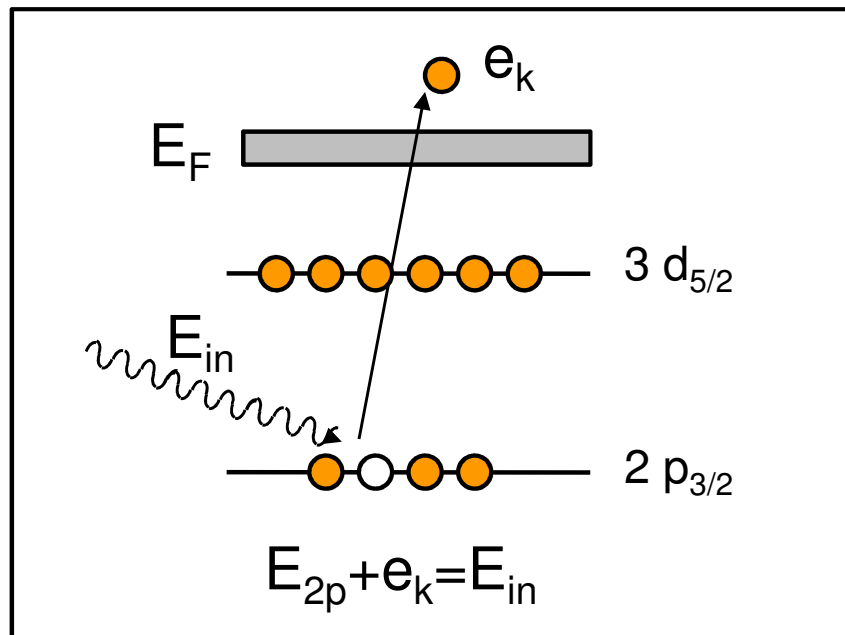
$$C_k = \sum_j (\boldsymbol{\varepsilon} \cdot \mathbf{p}_j) \exp\{i\mathbf{k} \cdot \mathbf{r}_j\}$$

Resonant denominator

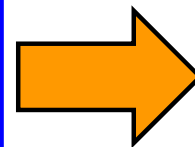


Basic theoretical aspects (RIXS)

Final states for XAS are intermediate for RIXS (resolution $\sim \Gamma_N$)

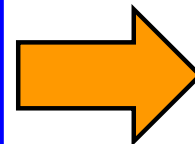


Motivation: Final state core-hole lifetime $<$ energy separation of the multiplet families



RIXS allows the separation of different excitation channels, which are obscured in XAS

Motivation: Keeping E fixed and tuning E_{in} through edge (CFS)



Resonant enhancement of intermediate states