



**The Abdus Salam  
International Centre for Theoretical Physics**



**1936-32**

**Advanced School on Synchrotron and Free Electron Laser Sources  
and their Multidisciplinary Applications**

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**Angle Resolved photoemission spectroscopy.**

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Italy*

# Angle resolved photoemission: Interacting electrons

## Part III

Papers: S. Hüfner et al., J. Electron Spectroscopy Rel. Phenom. 100 (1999)  
A. Damascelli et al., Rev. Modern Phys. (2005)  
F. Reinert et al., New J. Phys. 7 (2005)  
X.J. Zhou et al., J. Electron Spectroscopy Rel. Phenom. 126 (2002)

Thanks to: A. Damascelli, X. Shen, R. Claessen, Ph. Hofmann and E. Rotemberg  
from whose I have taken slides and figures

# Outline

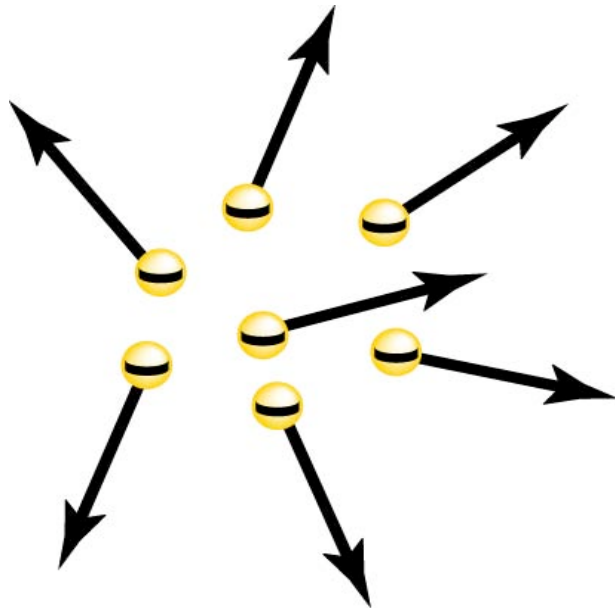
- Interacting electrons: many-body physics
- Single particle spectral density function  $A(k, \omega)$
- The self-energy
- The “kinky” physics: electron-phonon interactions
- 1D System: Luttinger liquid. Spinon and holon dispersion
- Mott-Hubbard insulator
- Fullerenes

# Interaction effects between electrons: “Many-body Physics”

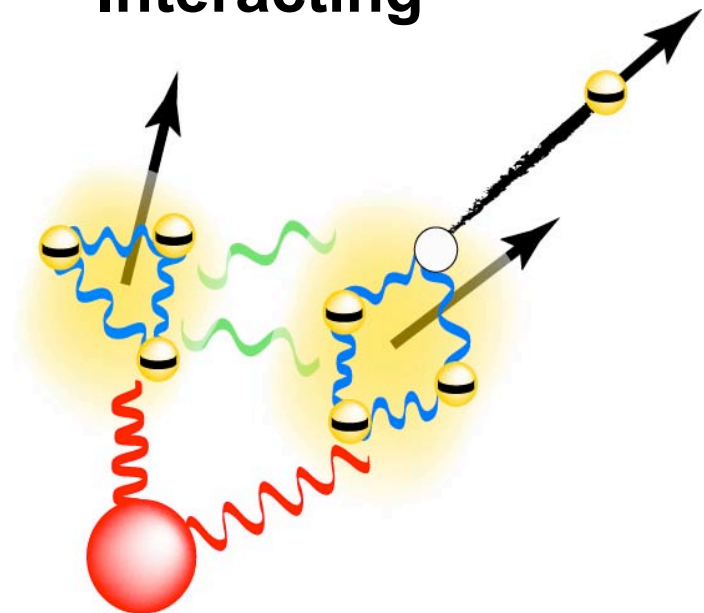
**Many body effects** are due to the interactions between the **electrons** and each other, or with **excitations inside the crystal** :

- 1) A “many-body” problem: intrinsically hard to calculate and understand
- 2) Responsible for many surprising phenomena:  
*Superconductivity, Magnetism, Density Waves, ....*

**Non-Interacting**



**Interacting**



## Interacting Electrons: many-body physics

$$\sum_{j=1}^N A(\mathbf{r}_j) \cdot \mathbf{p}_j = \mathbf{A} \cdot \mathbf{p}$$

$$I(E_{\text{kin}}) = \frac{2e^2\pi}{m^2\hbar} \sum_f \left| \langle N, f | \mathbf{A} \cdot \mathbf{p} | N, i \rangle \right|^2 \rho(E_f^N) \delta(E_f^N - E_i^N - h\nu)$$

$$I(E_{\text{kin}}) = \sum_f |M_{fi}|^2 \rho(E_f^N) \delta(E_f^N - E_i^N - h\nu)$$

The final state relevant for photoemission must contain a free electron with wave vector  $\mathbf{k}$  and energy  $E_{\text{kin}}$ .

$$I(E_{\text{kin}}) = \frac{2e^2\pi}{m^2\hbar} \sum_x \left| \langle \mathbf{k}, N-1, x | \mathbf{A} \cdot \mathbf{p} | N, i \rangle \right|^2 \rho(E_x^{N-1}) \rho(E_{\text{kin}}) \delta(E_{\text{kin}} + E_x^{N-1} - E_i^N - h\nu)$$

Now the sum is over all the possible final excited states  $x$  of the  $(N-1)$ -electrons system left behind by the photoelectron. The essential step in simplifying this expression consists in the factorization of the final state wave function  $|\mathbf{k}, N-1, x\rangle$  as the product of the photoelectron  $\phi_{\mathbf{k}}$  and the  $\phi_x(N-1)$  electrons wave functions.

In second quantization  $|\mathbf{k}, N-1, \mathbf{x}\rangle \approx \phi_{\mathbf{k}} \phi_{\mathbf{x}} (N-1) = c_{\mathbf{k}}^+ |N-1, \mathbf{x}\rangle$

This involves two assumptions:

- 1) **Sudden approx.:** The photoelectron decouples immediately from the photohole left behind and carries no information on the relaxation of the (N-1) system
- 2) We neglect inelastic losses of the photoelectron on its travel inside the crystal

$$I(E_{\text{kin}}) = \frac{2e^2\pi}{m^2\hbar} \sum_{j=1}^N |M_{\mathbf{k},\mathbf{k}j}|^2 \sum_{\mathbf{x}} |\langle N-1, \mathbf{x} | c_{\mathbf{k}j} | N, \mathbf{i} \rangle|^2 \rho(E_{\mathbf{x}}^{N-1}) \rho(E_{\text{kin}}) \delta(E_{\text{kin}} + E_{\mathbf{x}}^{N-1} - E_{\mathbf{i}}^N - \hbar\nu)$$

where

$$|N, \mathbf{i}\rangle = \frac{1}{N} c_{\mathbf{k}j}^+ c_{\mathbf{k}j} |N, \mathbf{i}\rangle \quad M_{\mathbf{k},\mathbf{k}j} = \frac{1}{N} c_{\mathbf{k}} A(\mathbf{r}_j) p_j c_{\mathbf{k}j}^+ = \frac{1}{N} \langle \mathbf{k} | A(\mathbf{r}_j) p_j | \mathbf{k}j \rangle$$

## Transition probability for interacting electrons

$$I(E_{\text{kin}}) = \frac{2e^2\pi}{m^2\hbar} \sum_{j=1}^N |M_{k,kj}|^2 \sum_x | \langle N-1, x | c_{kj} | N, i \rangle |^2 \rho(E_x^{N-1}) \rho(E_{\text{kin}}) \delta(E_{\text{kin}} + E_x^{N-1} - E_i^N - h\nu)$$

$c_{kj}$  destroys an electron with momentum  $kj$  from the initial state  $|N, i\rangle$

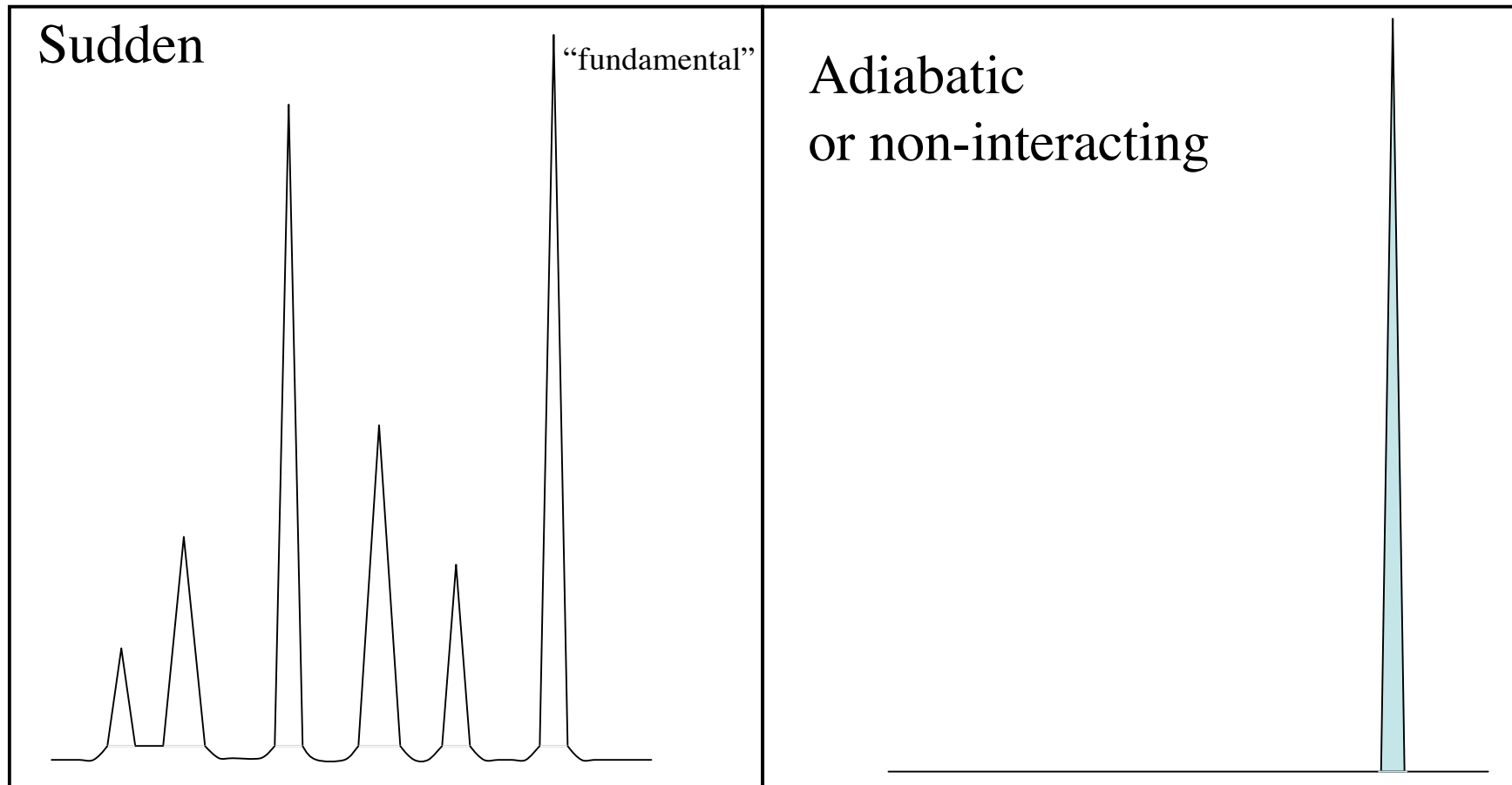
$|N-1, x\rangle$  is an eigenstate of the (N-1) Hamiltonian,  
while the (N-1) wave function  $c_{kj}|N, i\rangle$  is not

We can write  $E_i^N - E_x^{N-1} = (E_i^N - E_0^{N-1}) - (E_x^{N-1} - E_0^{N-1}) = \epsilon_0^i - \Delta\epsilon_x$

**Interpretation:** The photon absorption suddenly creates an (N-1)-electron state  $c_{kj}|N, i\rangle$  that is not an eigenstate of the (N-1) Hamiltonian (frozen state). The spectrum is the projection of this frozen states over the “fully relaxed” eigenstates  $|N-1, x\rangle$  of the (N-1) Hamiltonian. We call “fundamental peak” (or “elastic peak”, or “coherent peak”) the transition leaving the (N-1)-system in the ground state  $|N-1, 0\rangle$  that correspond to a photoelectron with kinetic energy  $E_{\text{kin}} = (E_i^N - E_0^{N-1}) + h\nu = \epsilon_0^i + h\nu$

The spectrum also exhibits peaks at lower kinetic energies by quantities  $-\Delta\epsilon_x$  when the system is left in an excited  $|N-1, x\rangle$  state.

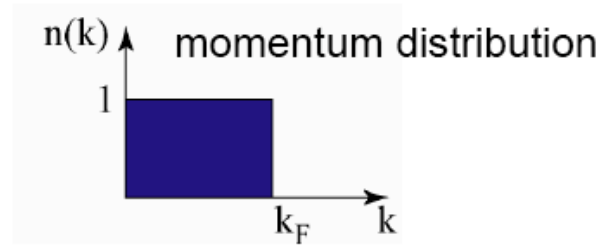
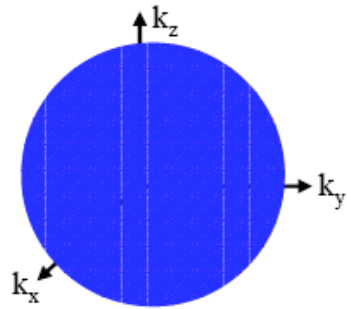
# Interacting electrons



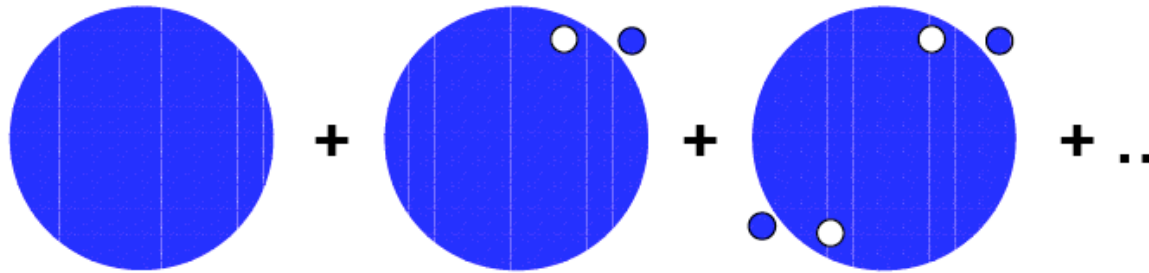
Kinetic Energy  $\longrightarrow$



### non-interacting system



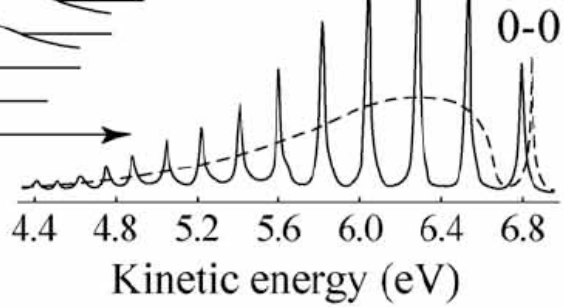
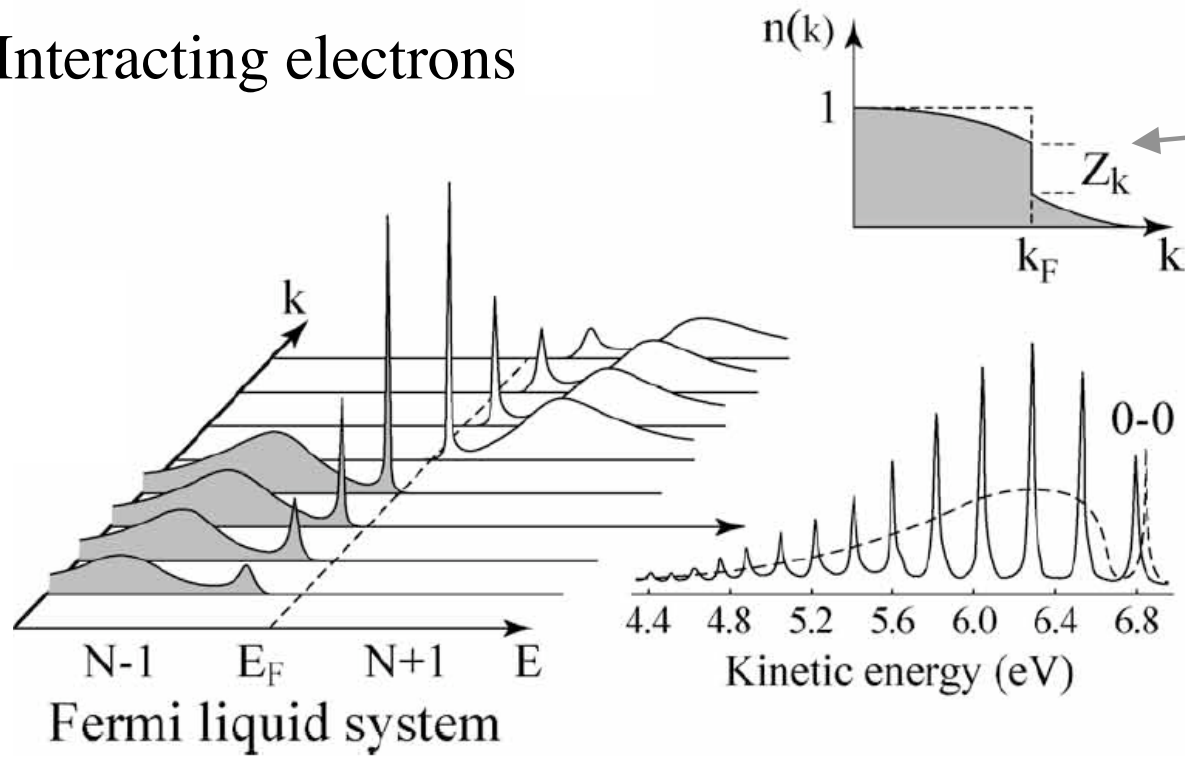
### interacting system



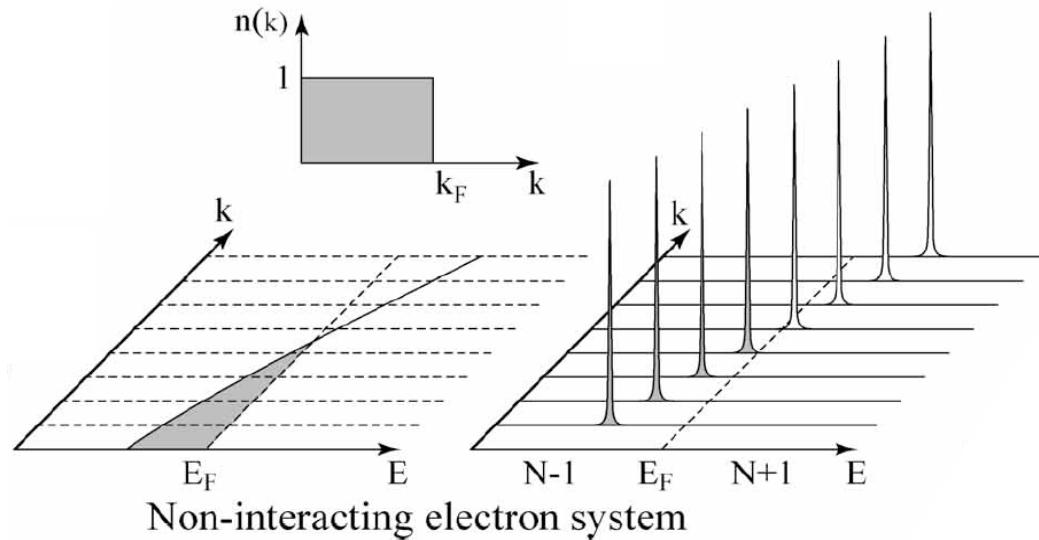
quasiparticle weight  $Z < 1$

1:1 mapping

# Interacting electrons



Still no information on the lifetime of the states (width of the peaks)



## A bit of math and the quasi-particles

It is useful to introduce the **one-electron removal Green function** formalism:

$$G(\mathbf{k}j, \omega_j) = \sum_x \frac{|\langle N-1, x | c_{\mathbf{k}j} | N, i \rangle|^2}{\omega_j - E_x^{N-1} + E_i^N - i\eta} \quad \eta \text{ can be infinitesimally small}$$

$c_{\mathbf{k}j}$  destroys an electron with momentum  $\mathbf{k}j$  and energy  $\omega_j$  from the initial state  $|N, i\rangle$

... and the corresponding **spectral density function**  $A(\mathbf{k}j, \omega_j) = (1/\pi) \text{Im} G(\mathbf{k}j, \omega_j)$

In the limit  $\eta \rightarrow 0$ :

$$A(\mathbf{k}j, \omega_j) = \frac{1}{\pi} \sum_x |\langle N-1, x | c_{\mathbf{k}j} | N, i \rangle|^2 \delta(\omega_j - E_x^{N-1} + E_i^N) = \frac{1}{\pi} \text{Im} G(\mathbf{k}j, \omega_j)$$

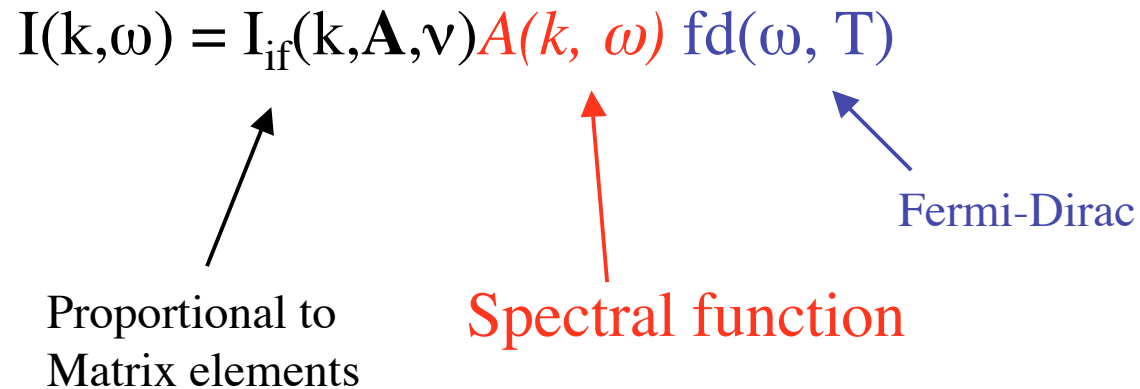
Comparing to:

$$I(E_{\text{kin}}) = \frac{2e^2\pi}{m^2\hbar} \sum_{j=1}^N |M_{\mathbf{k}, \mathbf{k}j}|^2 \sum_x |\langle N-1, x | c_{\mathbf{k}j} | N, i \rangle|^2 \rho(E_x^{N-1}) \rho(E_{\text{kin}}) \delta(E_{\text{kin}} + E_x^{N-1} - E_i^N - \hbar\nu)$$

$$\text{We have } I(E_{\text{kin}}) = \frac{2e^2\pi^2}{m^2\hbar} \sum_{j=1}^N |M_{\mathbf{k}, \mathbf{k}j}|^2 A(\mathbf{k}j, \omega_j)$$

Where  $A(\mathbf{k}j, \omega_j) \neq 0$  only when  $\omega_j = \hbar\nu - E_{\text{kin}} = |E_i^N - E_x^{N-1}|$

Rewriting in terms of the electron binding energy  $\omega$ , considering the momentum conservation and including the Fermi-Dirac distribution:

$$I(k, \omega) = I_{if}(k, \mathbf{A}, \nu) A(k, \omega) \text{fd}(\omega, T)$$


Proportional to  
Matrix elements

Spectral function

Fermi-Dirac

This is the most important result: in the sudden approx. the photoemission spectrum is proportional to the single particle spectral density function  $A(k, \omega)$

This relationship has been obtained in the limit  $\eta \rightarrow 0$  ( $\eta$  is the peak width), i.e. the peaks are Dirac's  $\delta$ . It can be extended to "real systems" where the width  $\Gamma$  is finite.

$$G(\mathbf{k}, \omega) = \sum_x \frac{|\langle N-1, x | c_{\mathbf{k}} | N, i \rangle|^2}{\omega - E_x^{N-1} + E_i^N - i\Gamma}$$

$$A(\mathbf{k}, \omega) = \frac{1}{\pi} \text{Im} G(\mathbf{k}, \omega) = \frac{\Gamma}{\pi} \sum_x \frac{|\langle N-1, x | c_{\mathbf{k}} | N, i \rangle|^2}{(\omega - E_x^{N-1} + E_i^N)^2 + \Gamma^2}$$

**Quasi-particle:** when one hole is added adiabatically forming an (N-1)-electron system the coulomb interaction is screened by the formation of an electron cloud around the hole. At equilibrium, the **hole+the screening cloud is a quasi-particle.**

An eigenstate  $|N-1, x\rangle$  of the (N-1)-electron system can be obtained by adding a quasi-particle to the N-electron system (quasi-particle state).

If instead we suddenly simply add a bare hole of momentum  $\mathbf{k}$  (or we remove an electron of momentum  $\mathbf{k}$ ) we obtain the state  $c_{\mathbf{k}} |N, i\rangle$ , that in general is not an eigenstate of the (N-1)-system, but it will have a finite overlap with the corresponding quasi-particle state.

**The spectral density function  $A(\mathbf{k}, \omega)$  gives the probability that the original system plus the bare hole will be found in an exact eigenstate of the (N-1)-system**

**non-interacting**  
electrons

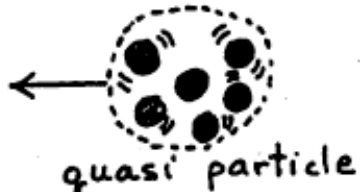
charge  $-e$   
spin  $1/2$



band structure

$$\epsilon_0(\mathbf{k})$$

**interacting**  
electrons



charge, spin  
and "dressing"



quasiparticle band structure

$$\epsilon(\mathbf{k}) = \epsilon_0(\mathbf{k}) + \Sigma(\mathbf{k}, \epsilon)$$

self-energy

$$G(\mathbf{k}, \omega) = \frac{\overbrace{|\langle N-1, i | c_{\mathbf{k}} | N, i \rangle|^2}^{Z_{\mathbf{k}}}}{\omega - \varepsilon(\mathbf{k}) - i\Gamma} + \sum_{x \neq i} \frac{|\langle N-1, x | c_{\mathbf{k}} | N, i \rangle|^2}{\omega - \varepsilon_x(\mathbf{k}) - i\Gamma} = G_{\text{coh}}(\mathbf{k}, \omega) + G_{\text{incoh}}(\mathbf{k}, \omega)$$

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} G(\mathbf{k}, \omega) = \frac{\Gamma}{\pi} \frac{|\langle N-1, i | c_{\mathbf{k}} | N, i \rangle|^2}{(\omega - \varepsilon(\mathbf{k}))^2 + \Gamma^2} + \frac{\Gamma}{\pi} \sum_{x \neq i} \frac{|\langle N-1, x | c_{\mathbf{k}} | N, i \rangle|^2}{(\omega - \varepsilon_x(\mathbf{k}))^2 + \Gamma^2}$$

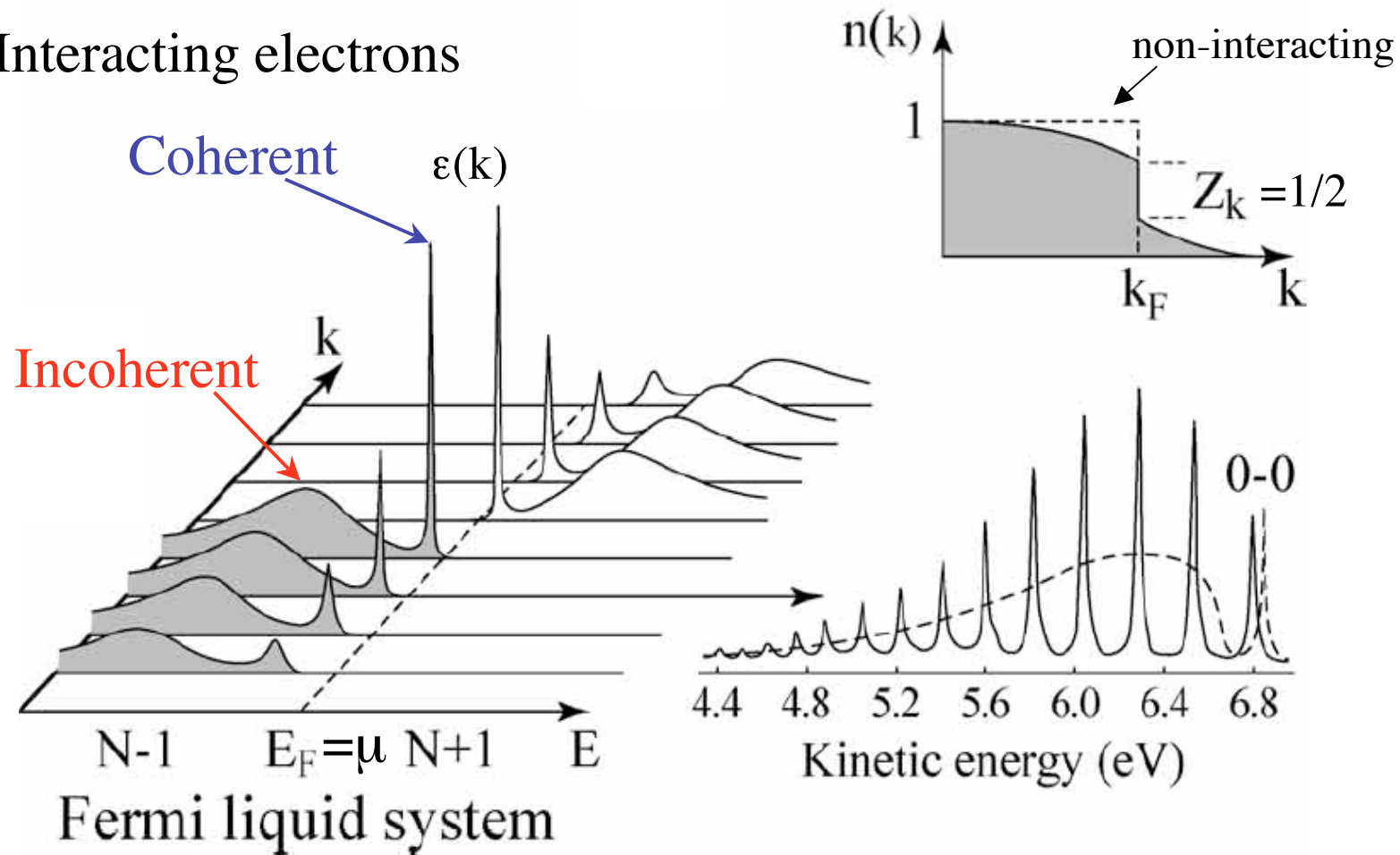
$$= A(\mathbf{k}, \omega)_{\text{coh.}} + A(\mathbf{k}, \omega)_{\text{incoh}}$$

Where  $\varepsilon(\mathbf{k})$  is the quasi-particle energy referred to the Fermi level  $\mu=0$

As  $\varepsilon(\mathbf{k}) \rightarrow \mu$ ,  $\Gamma \propto (\varepsilon - \mu)^2 \rightarrow 0$

The quasi-particle is well-defined only at (or very close to) the Fermi level, where its lifetime  $1/\Gamma \rightarrow \infty$

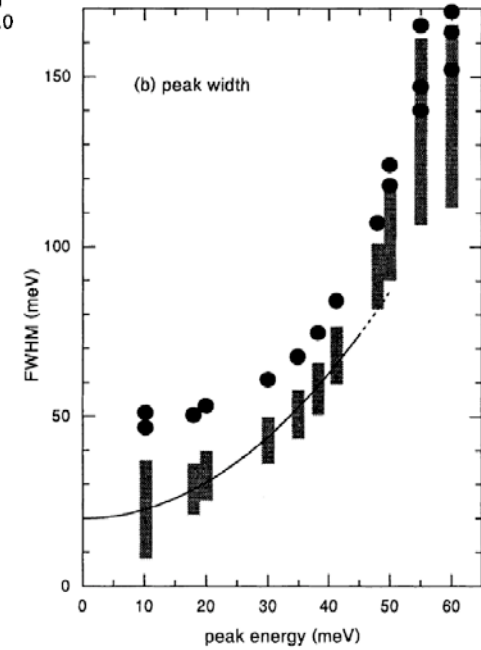
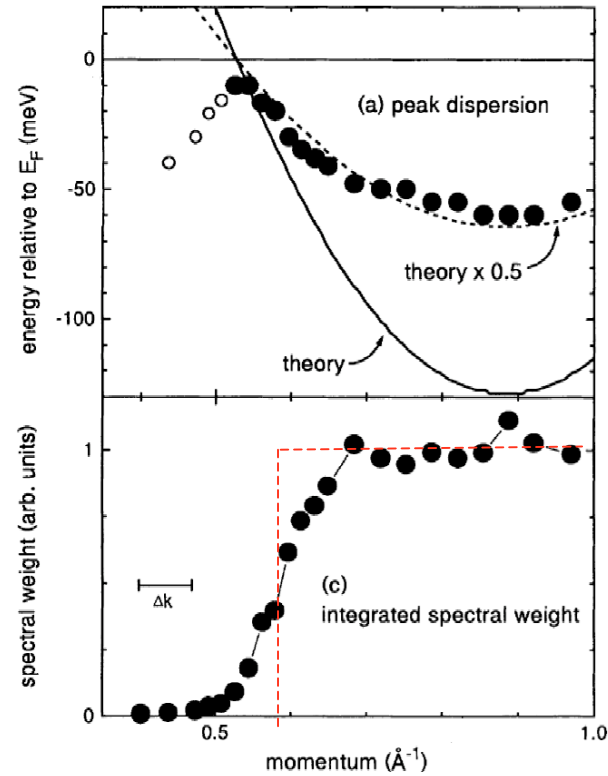
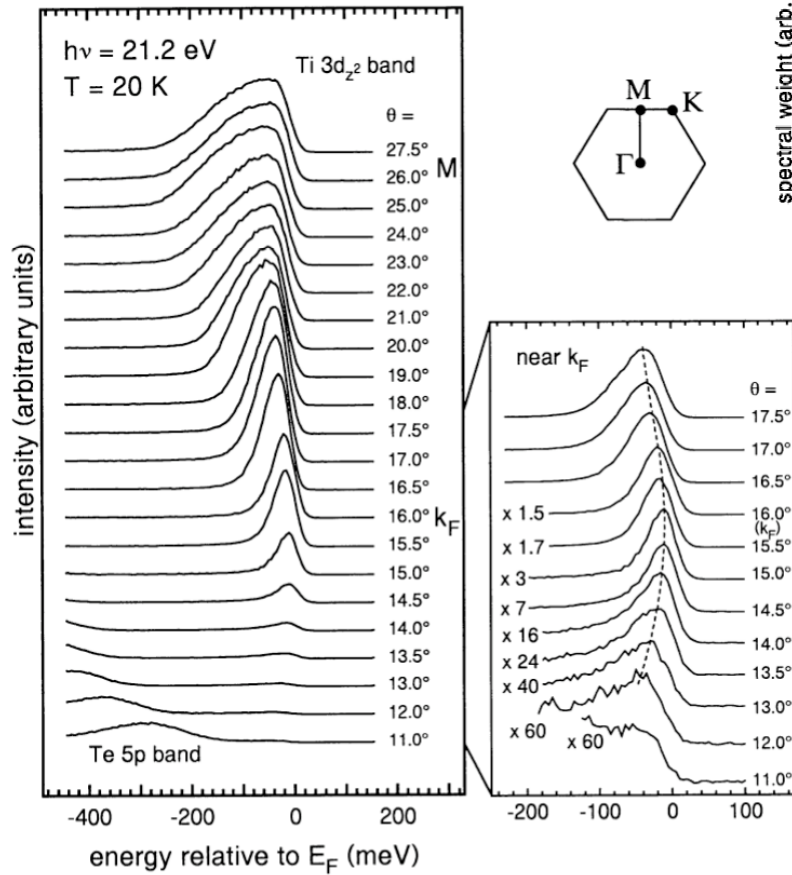
# Interacting electrons



$$A(k, \omega) = \frac{\Gamma}{\pi} \frac{Z_k}{(\omega - \epsilon(k))^2 + \Gamma^2} + \frac{\Gamma}{\pi} \sum_{x \neq i} \frac{|\langle N-1, x | c_k | N, i \rangle|^2}{(\omega - \epsilon_x(k))^2 + \Gamma^2} = A(k, \omega)_{\text{coh.}} + A(k, \omega)_{\text{incoh}}$$



# Example: $\text{TiTe}_2$



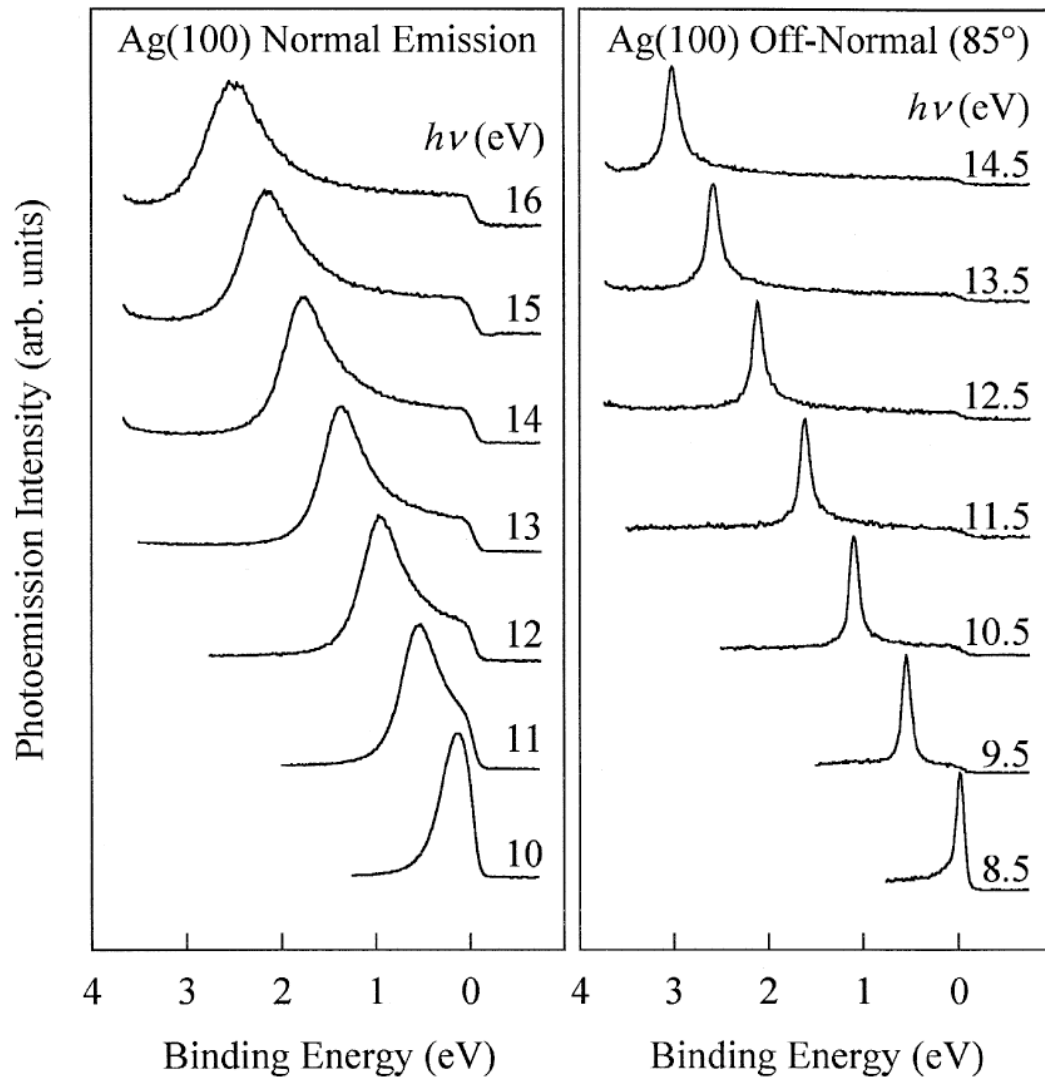
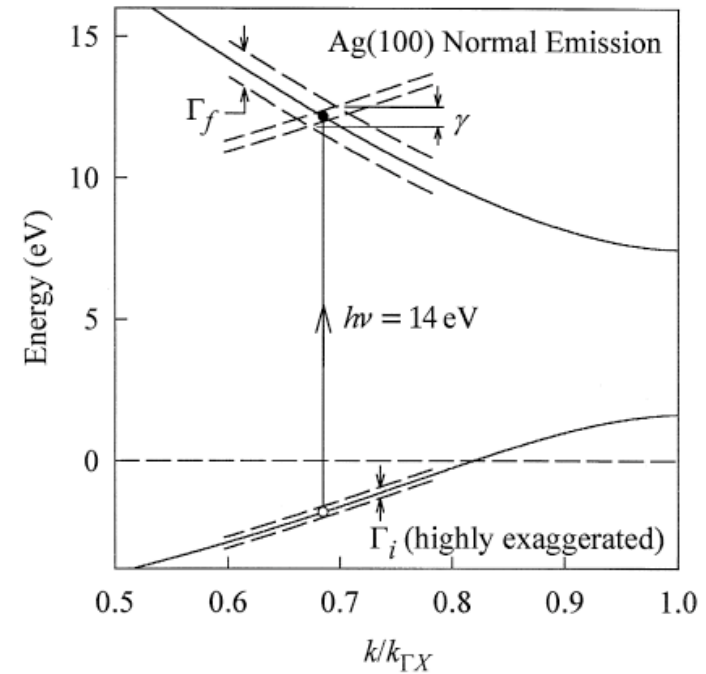


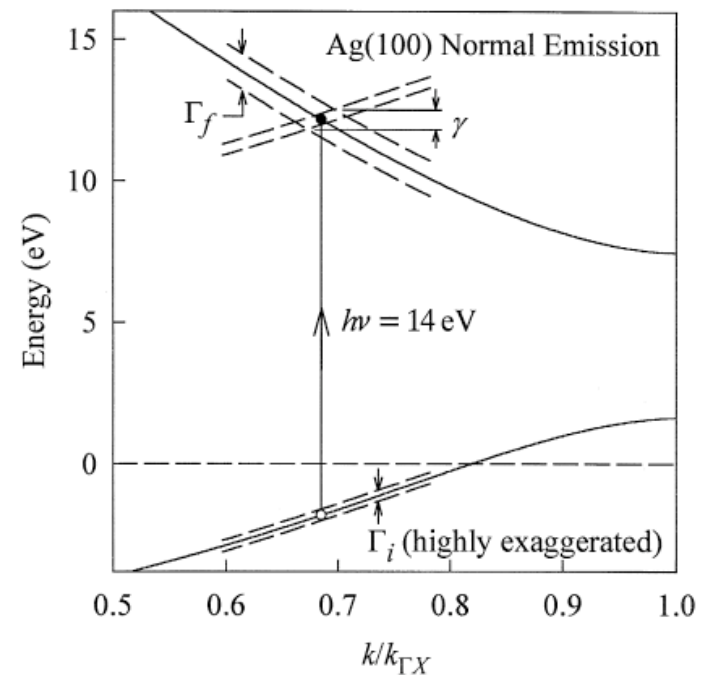
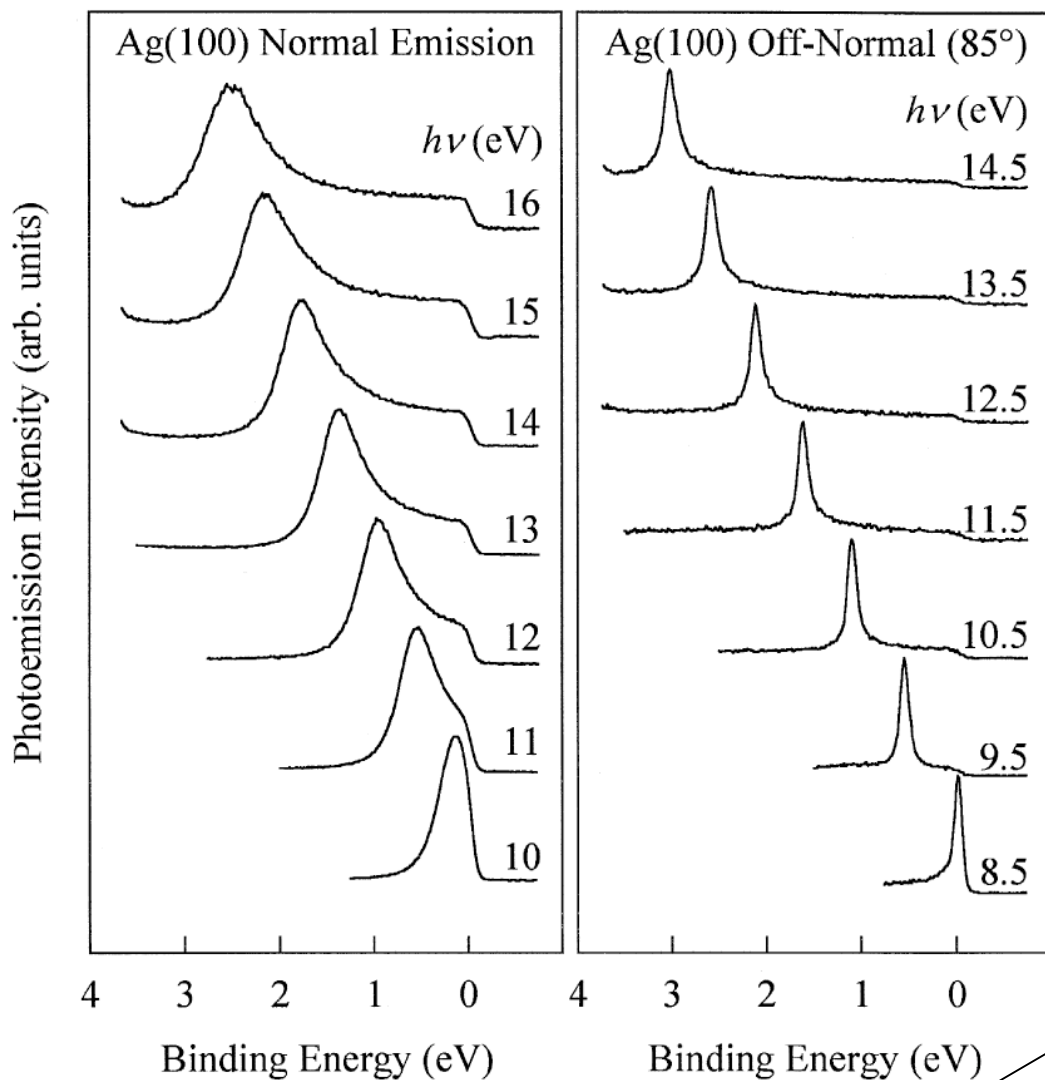
Fig. 1. Angle resolved photoemission spectra taken with a normal emission geometry (left panel) and a grazing emission geometry (right panel). The photon energies are indicated in the figure.



The total width  $\gamma$  (overlapping region) depends on the slopes, or group velocities.

$\gamma$  is dominated by  $\Gamma_f$  (much larger than  $\Gamma_i$ ).

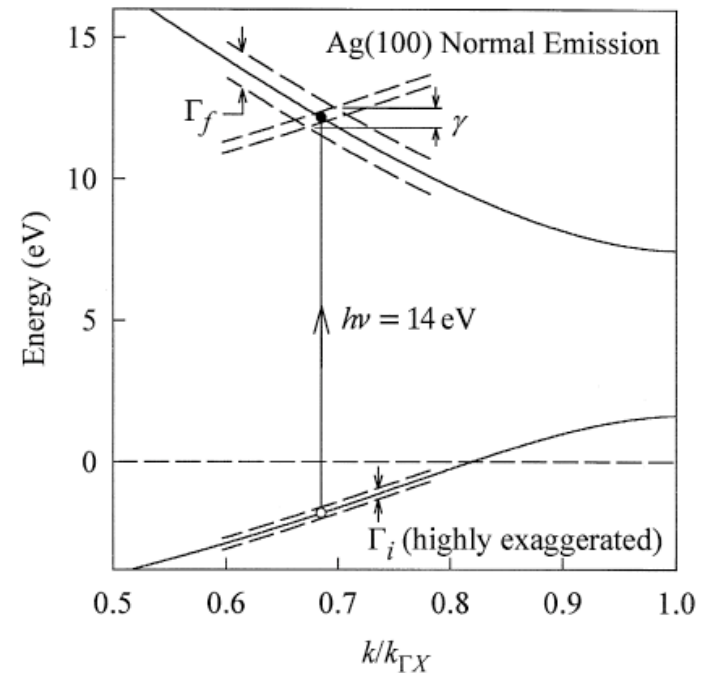
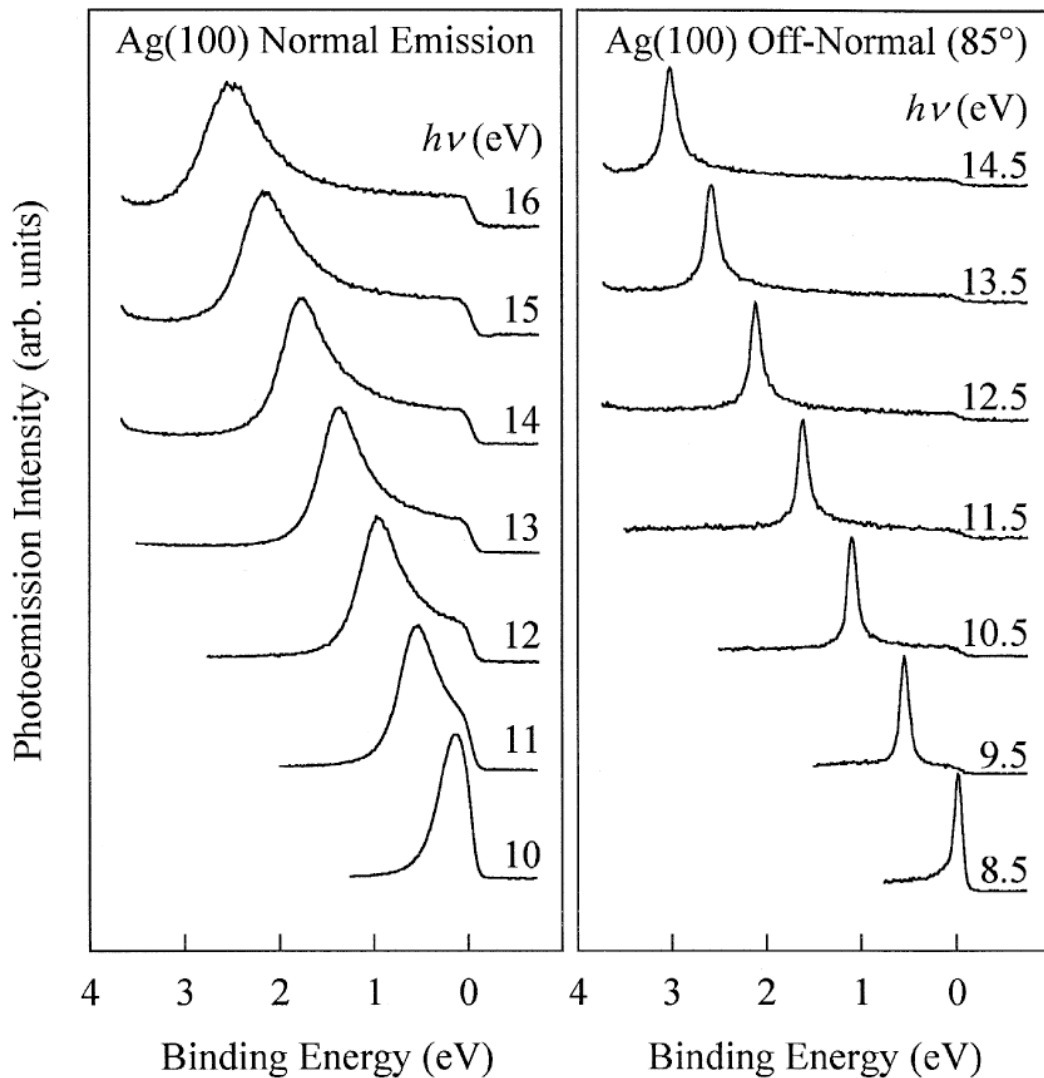
Difficult to obtain the true quasiparticle inverse lifetime.



$$\gamma = \frac{\frac{\Gamma_i}{|v_{i\perp}|} + \frac{\Gamma_f}{|v_{f\perp}|}}{\left| \frac{1}{v_{i\perp}} - \frac{1}{v_{f\perp}} \right|}$$

Normal emission

Interesting when  $v_i = v_f$  and  $v_i/v_f = 0$



When  $v_i/v_f=0$   $\gamma = C \cdot \Gamma_i$

$$C = \left| 1 - \frac{mv_{i\parallel} \sin^2 \theta}{\hbar k_{\parallel}} \right|^{-1}$$

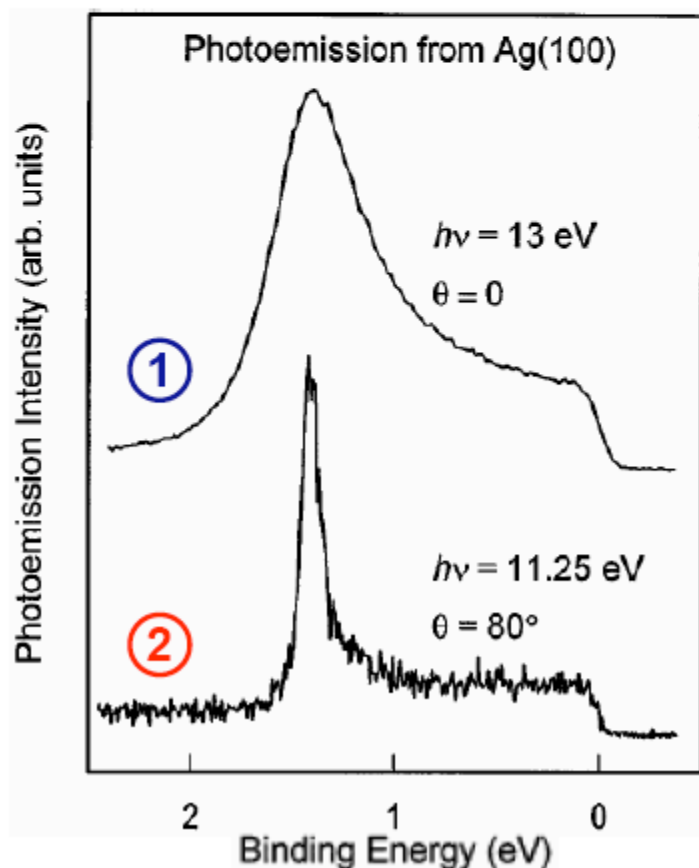
$$\frac{\Gamma_i}{|v_{i\perp}|} + \frac{\Gamma_f}{|v_{f\perp}|}$$

Grazing emission  $\longrightarrow \gamma = \left| \frac{1}{v_{i\perp}} \left( 1 - \frac{mv_{i\parallel} \sin^2 \theta}{\hbar k_{\parallel}} \right) - \frac{1}{v_{f\perp}} \left( 1 - \frac{mv_{f\parallel} \sin^2 \theta}{\hbar k_{\parallel}} \right) \right|$

# ARPES: FWHM and Inverse Lifetime

FWHM of an ARPES peak } 
$$\Gamma = \frac{\frac{\Gamma_i}{|v_{i\perp}|} + \frac{\Gamma_f}{|v_{f\perp}|}}{\left| \frac{1}{v_{i\perp}} \left[ 1 - \frac{mv_{i\parallel} \sin^2 \vartheta}{\hbar k_{\parallel}} \right] - \frac{1}{v_{f\perp}} \left[ 1 - \frac{mv_{f\parallel} \sin^2 \vartheta}{\hbar k_{\parallel}} \right] \right|}$$

Hansen *et al.*, PRL 80, 1766 (1998)



① if  $E_i \simeq E_F$

$\rightarrow \Gamma_i \rightarrow 0 \rightarrow \Gamma \propto \Gamma_f$

② if  $|v_{i\perp}| \simeq 0$

$\rightarrow \Gamma = \frac{\Gamma_i}{\left| 1 - \frac{mv_{i\parallel} \sin^2 \vartheta}{\hbar k_{\parallel}} \right|} \equiv C \Gamma_i$

if  $v_{i\parallel} < 0$ , large;  $\theta$  large;  $k_{\parallel}$  small

$\rightarrow C < 1$ , and  $\Gamma < \Gamma_i$

# The self-energy

It is useful to express the effects of the electron interactions in terms of the “electron self energy” defined as:

$$\Sigma(\mathbf{k}, \omega) = \Sigma_1(\mathbf{k}, \omega) + i\Sigma_2(\mathbf{k}, \omega)$$

$\Sigma(\mathbf{k}, \omega)$  : the “*self-energy*” - captures the effects of interactions

$$\Gamma = \Sigma_2$$

$$\varepsilon(\mathbf{k}) = \varepsilon_0(\mathbf{k}) + \Sigma_1(\mathbf{k}, \omega) = Z_{\mathbf{k}} \varepsilon_0(\mathbf{k})$$

$$Z_{\mathbf{k}} = \left( 1 - \frac{\partial \Sigma_1}{\partial \omega} \right)_{\omega = \varepsilon_0(\mathbf{k})}^{-1}$$

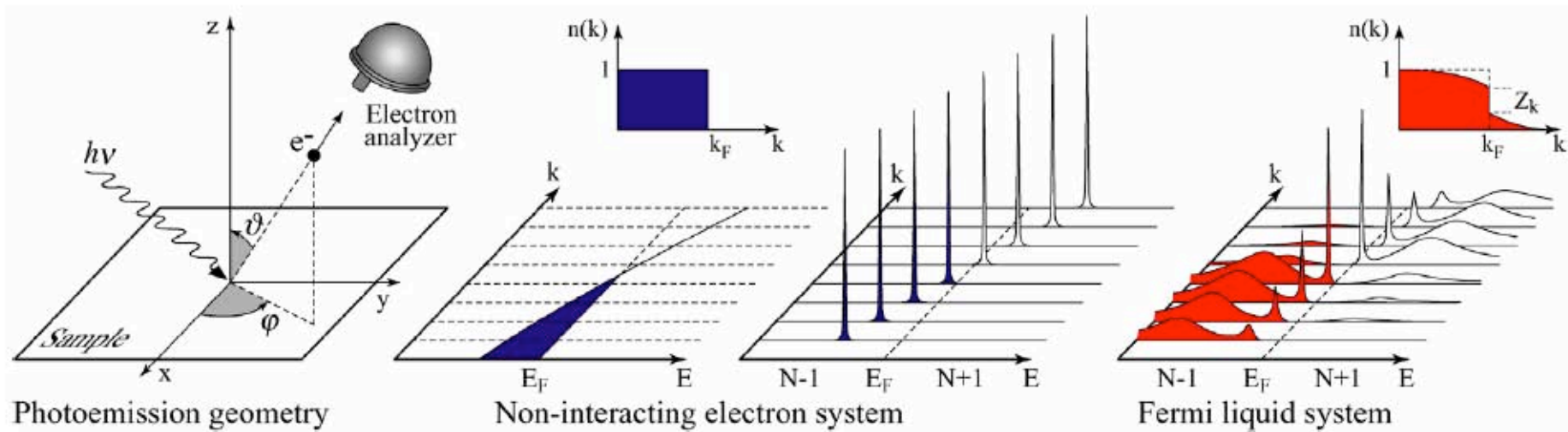
... rewriting  $G(\mathbf{k}, \omega)$  and  $A(\mathbf{k}, \omega)$ :

$$G(\mathbf{k}, \omega) = \frac{Z_{\mathbf{k}}}{\omega - [\varepsilon_0(\mathbf{k}) + \Sigma(\mathbf{k}, \omega)]} + G_{inch.}(\mathbf{k}, \omega)$$

$$A(\mathbf{k}, \omega) = \frac{\Sigma_2}{\pi} \frac{Z_{\mathbf{k}}}{[\omega - \varepsilon_0(\mathbf{k}) - \Sigma_1]^2 + \Sigma_2^2} + A_{inch.}(\mathbf{k}, \omega)$$

$$\lambda = \left( -\frac{\partial \Sigma_1}{\partial \omega} \right)_{\omega = E_F}$$

Coupling constant



### non-interacting

$$A(\vec{k}, \omega) = -\frac{1}{\pi} \text{Im} \frac{1}{\omega - \varepsilon_{\vec{k}} + 0^+}$$

$$= \delta(\omega - \varepsilon_{\vec{k}})$$

### Fermi liquid

$$A(\vec{k}, \omega) = -\frac{1}{\pi} \text{Im} \frac{1}{\omega - \varepsilon_{\vec{k}} - \Sigma(\vec{k}, \omega) + 0^+}$$

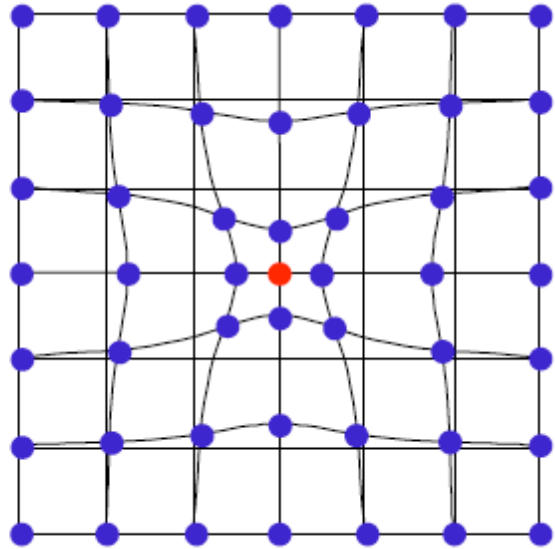
$$= -\frac{1}{\pi} \text{Im} \frac{Z_{\vec{k}}}{\omega - E_{\vec{k}} - i\Gamma_{\vec{k}}} + A_{inc}$$

→ energy renormalization:  $E_{\vec{k}} = \varepsilon_{\vec{k}} + \text{Re} \Sigma = Z_k \cdot \varepsilon_{\vec{k}}$

→ lifetime broadening:  $\Gamma_{\vec{k}} = \text{Im} \Sigma$

→ quasiparticle weight:  $Z_{\vec{k}} = (1 - \frac{\partial \Sigma}{\partial \omega})^{-1} \leq 1$

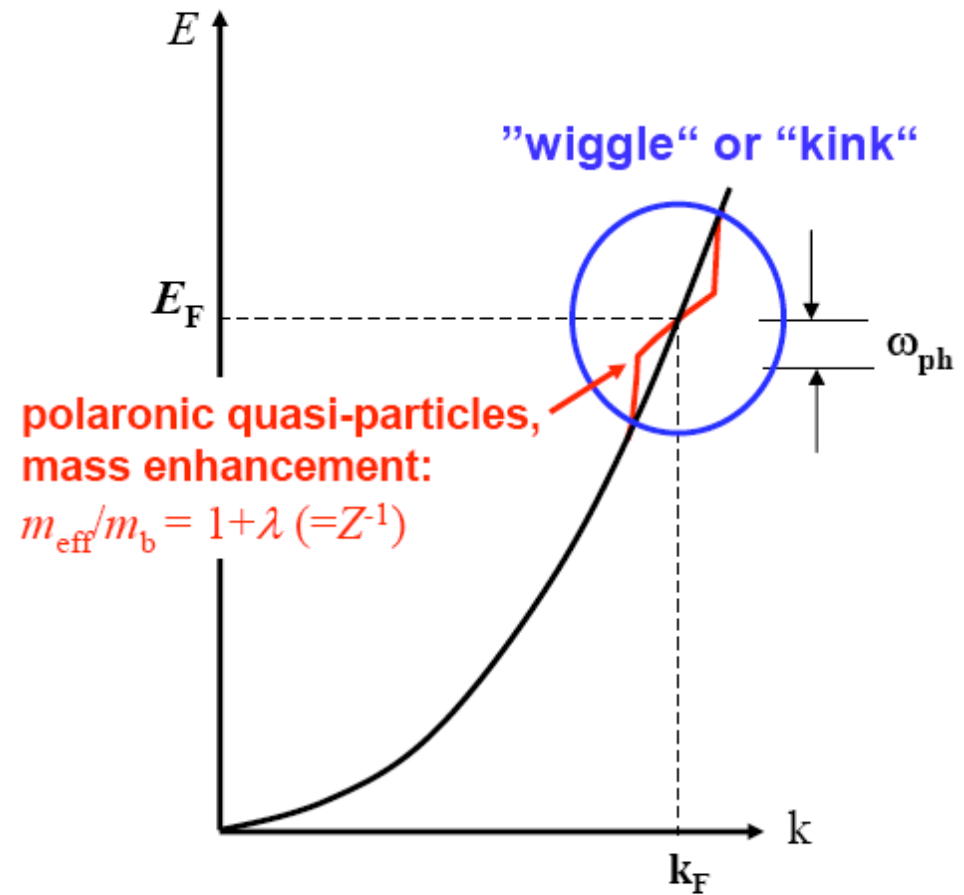
# The “Kinky” physics



- Clear-cut case of a quasiparticle picture
- The quasiparticle mass near  $E_F$  is renormalized and the density of state increased



electron + (dynamical) lattice polarization  
= **polaronic quasiparticle**





# Electron-phonon coupling

$$A(k, \omega) = \frac{\Sigma_2}{\pi} \frac{Z_k}{[\omega - \varepsilon_0(k) - \Sigma_1]^2 + \Sigma_2^2} + A_{inch.}(k, \omega)$$

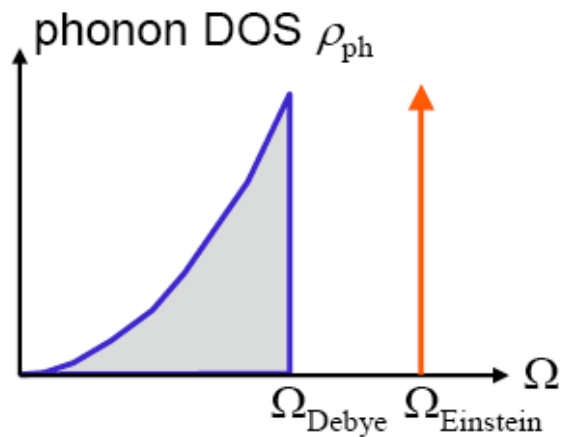
Spectral function:  
excitation spectrum

## electronic self-energy

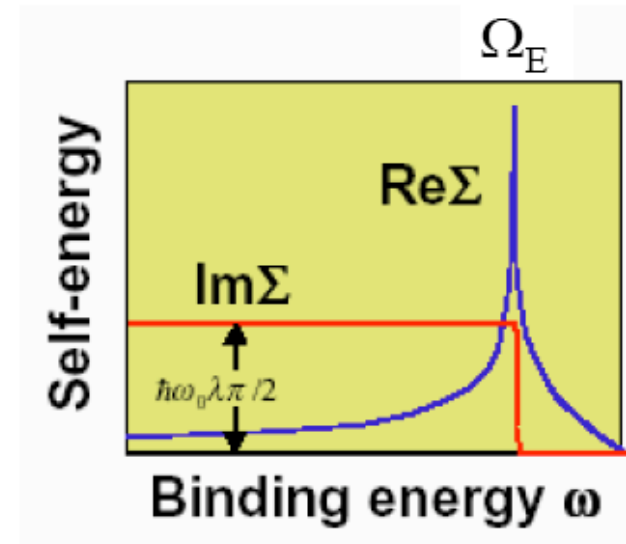
$$\text{Im}\Sigma(\omega) \propto \int_0^\omega \alpha^2 F(\Omega) d\Omega = \lambda \int_0^\omega \rho_{ph}(\Omega) d\Omega$$

$\lambda$  -  
electron-phonon coupling constant

$\text{Re}\Sigma(\omega)$  from Kramers-Kronig relation

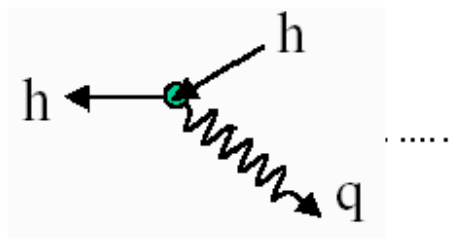


example: Einstein model:



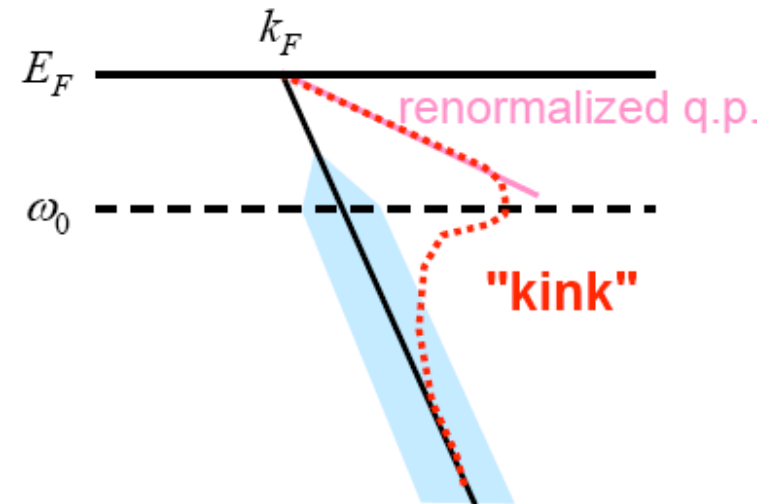
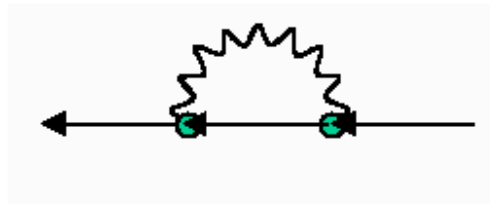
$$|\varepsilon_{\vec{k}}^- - E_F| > \omega_0$$

hole emits (decays into)  
*real* phonon

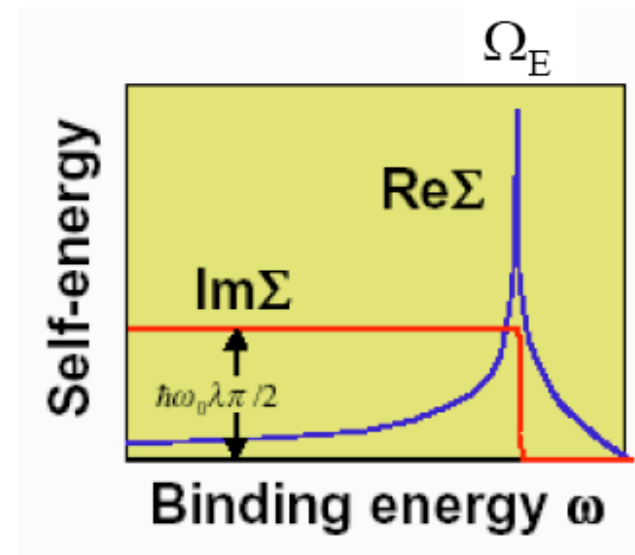


$$|\varepsilon_{\vec{k}}^- - E_F| < \omega_0$$

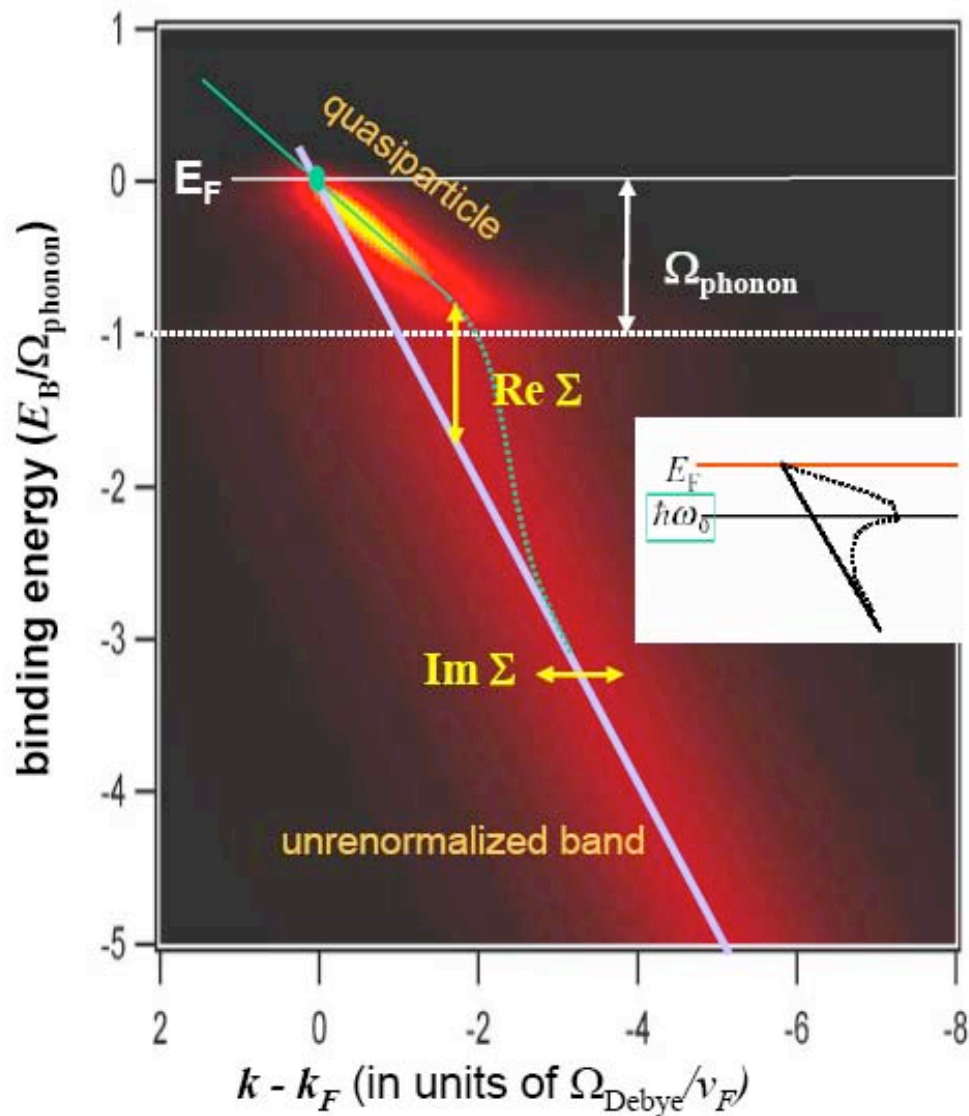
hole emits and reabsorbs phonon,  
dressed with cloud of *virtual* phonons



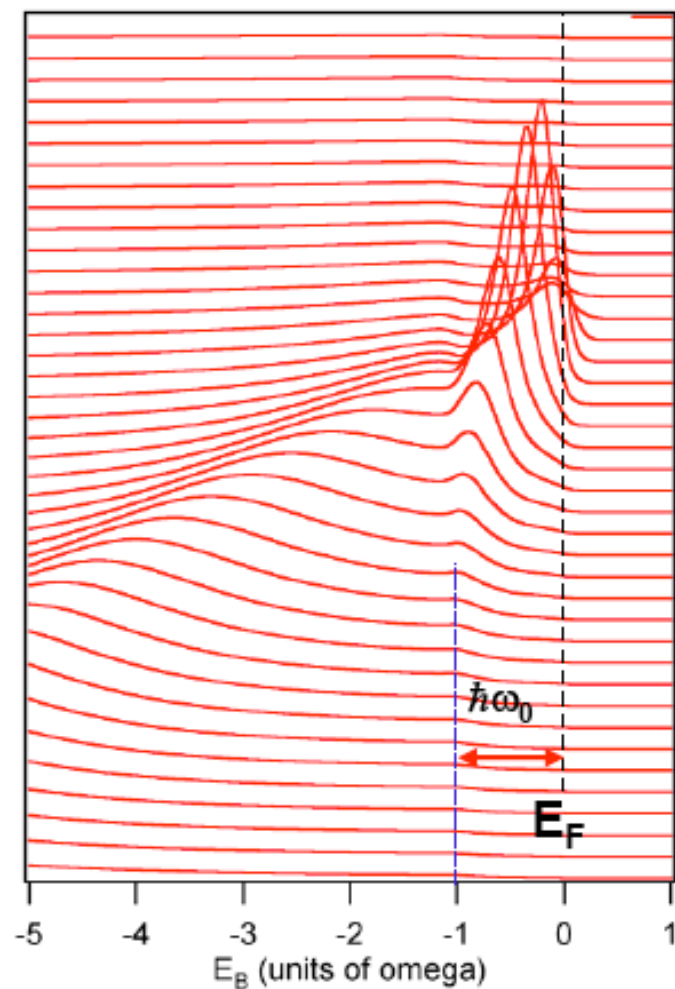
example: Einstein model:



## Debye Model ( $\lambda = 1$ )



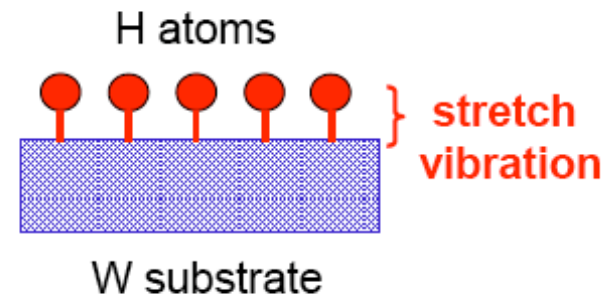
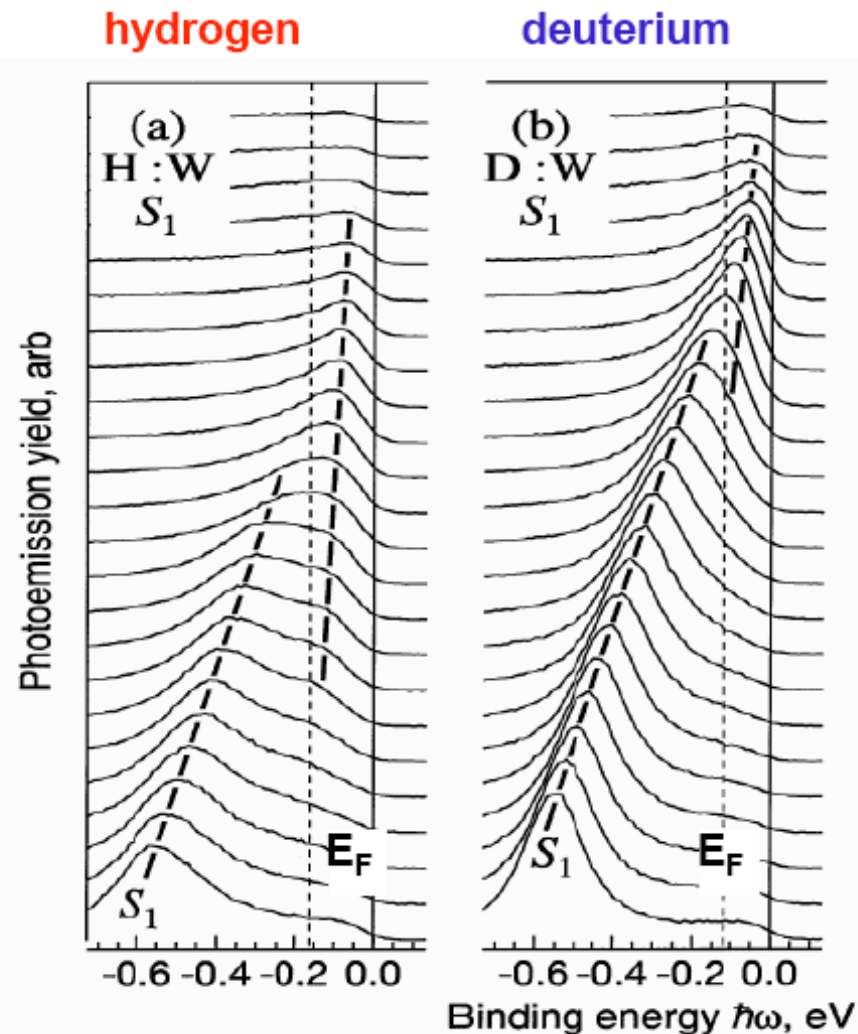
## theoretical energy distribution curves (EDCs)



# Coupling to adsorbate vibrations on a surface

## hydrogen adsorbed on W(110) – an Einstein-type system

*E. Rotenberg, J. Schäfer et al., PRL 84, 2925 (2000)*

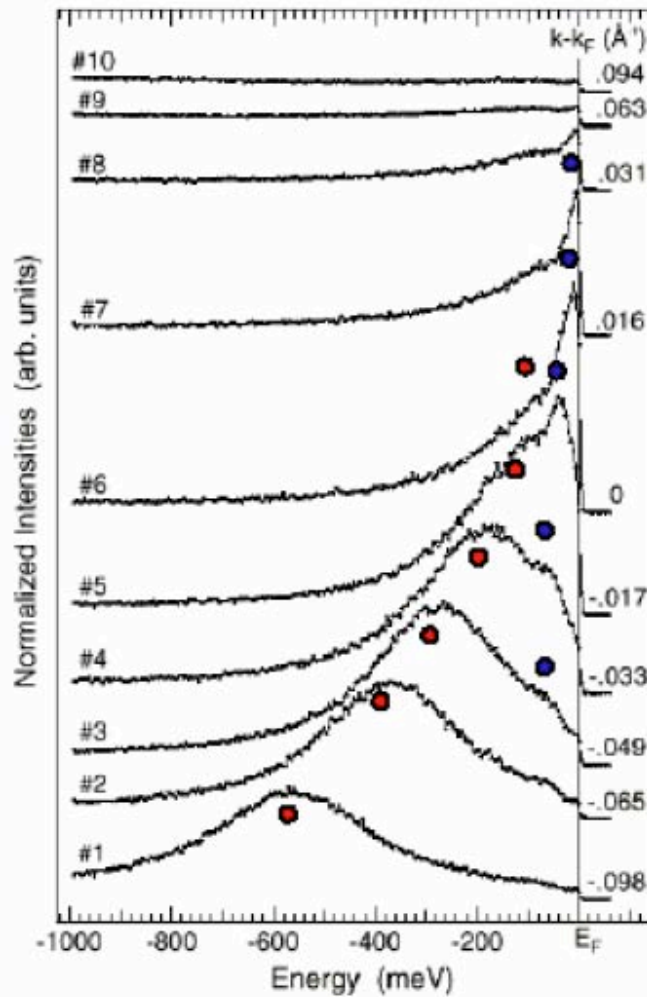


quantitative analysis:

- energy scale:  $\omega_0 = 160$  meV for H,  
115 meV for D (isotope effect  $1/\sqrt{2}$ )
- coupling constant:  $\lambda = 1.4$  for H/W(110)
- **significant mass renormalization**
- **isotope effect H  $\rightarrow$  D**  
( $\omega_0 \sim 1/\sqrt{M}$ )

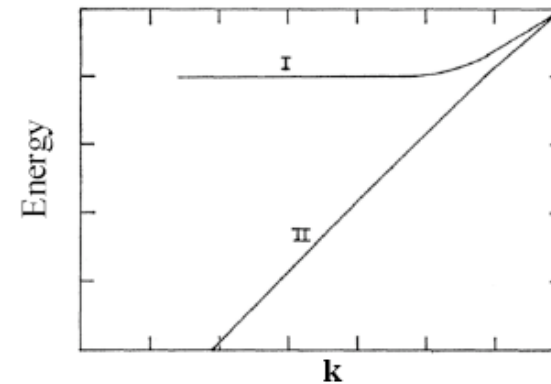
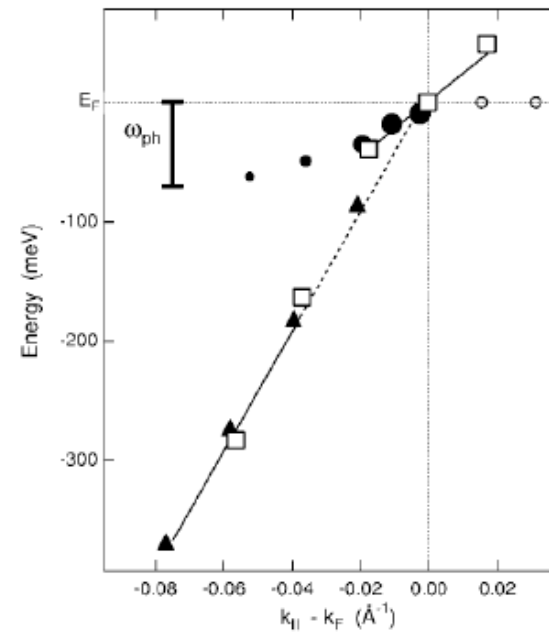
# Intrinsic electron-phonon coupling on a surface: Debye model

Be(0001) T=12K



Hengsberger et al, PRL 83,592 (1999).

Theory and Experiment



S. Engelsberg and J.R. Schrieffer, PR 131, 993 (1963)

# How to get $\lambda$ ?: real part of $\Sigma$

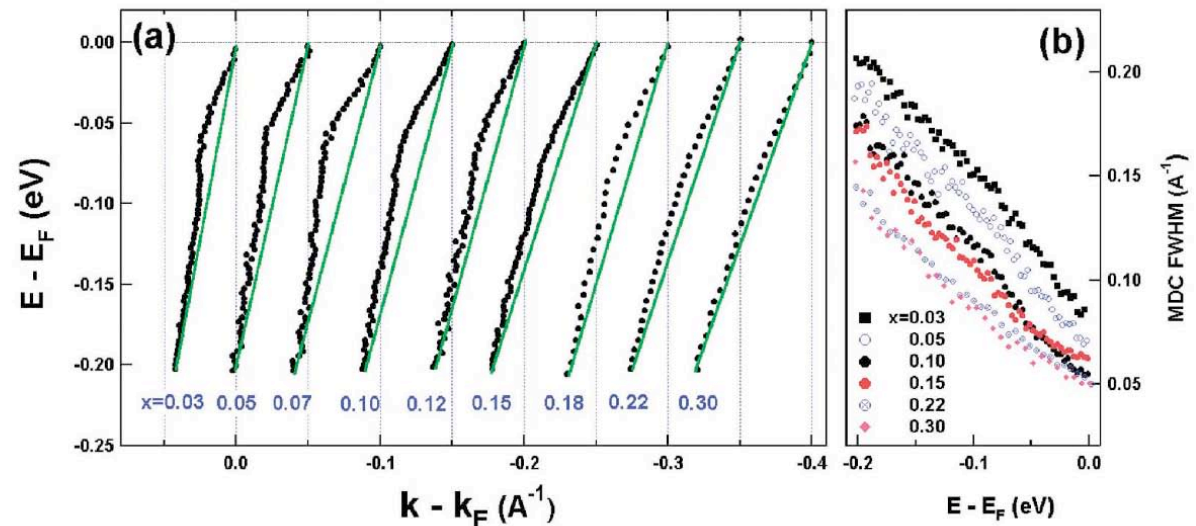
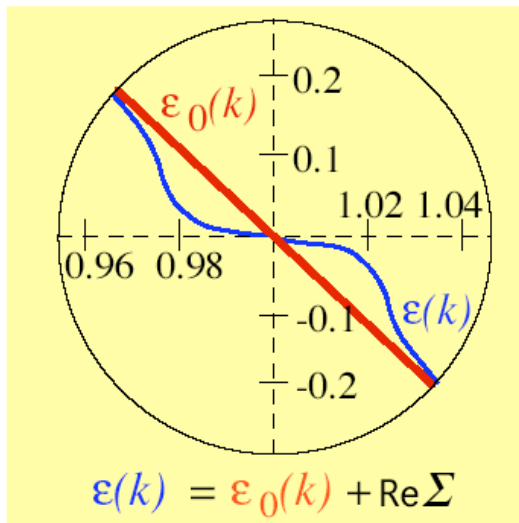


Fig. 13. Dispersion (a) and MDC width (FWHM)(b) of LSCO samples ( $x=0.03\sim 0.30$ ) measured along the nodal direction, as determined from fitting MDCs. For clarity, the dispersion in (a) is offset horizontally along the momentum axis. The green lines in (a) connect the points in dispersion at  $E_F$  and 0.2 eV which approximately represents the bare band; they also serve as guides to the eye to identify the kink in dispersion. The MDC width (b) shows an overall decrease with increasing doping. A slight drop in MDC width is discernible at a binding energy of  $\sim 80$  meV, particularly obvious for lower doping samples.

$$\lambda = \left. \frac{d\Re\Sigma}{d\epsilon} \right|_{\epsilon_F}$$

but this may also be caused by something else....

- yields directly  $\lambda$  when measured at 0 K.
- the effect has to be large in order to be observable
- the un-renormalized dispersion has to be known



# How to get $\lambda$ ?: imag. part of $\Sigma$

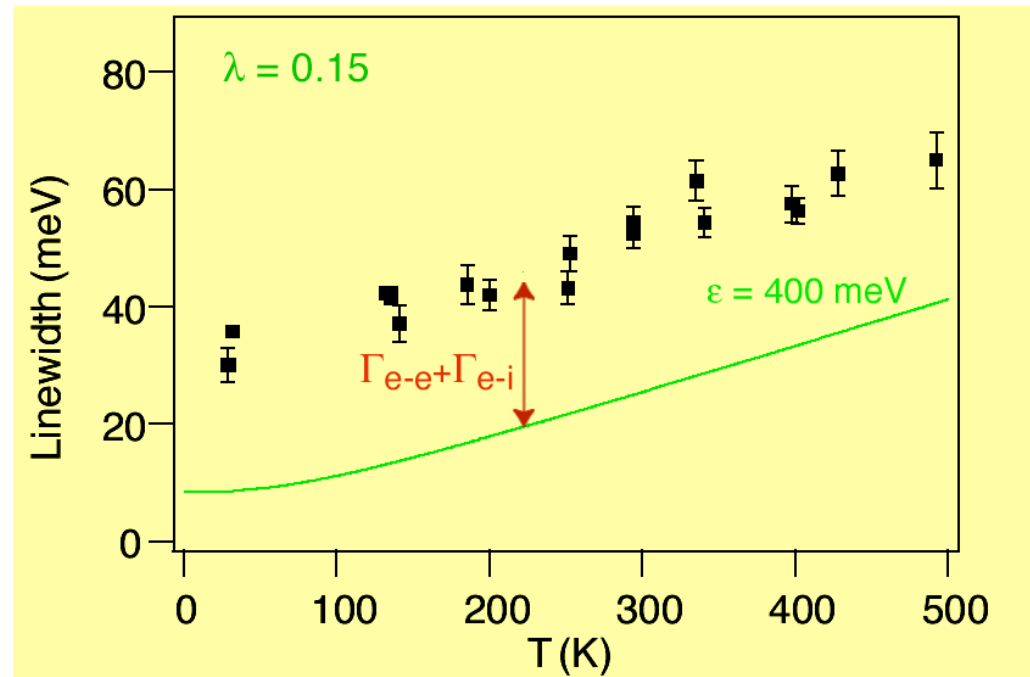
$$\text{EDC linewidth } \Gamma = \Gamma_{e-e}(T) + \Gamma_{e-i}(T) + \Gamma_{e-ph}(T)$$

$$\Gamma_{e-ph}(T) = 2\pi\hbar \int_0^{\omega_m} d\omega' \alpha^2 F(\omega') [1 - f(\omega - \omega') + 2n(\omega') + f(\omega + \omega')] \simeq 2\pi\lambda k_B T$$

- In most cases, the temperature dependence of the lifetime is dominated by the electron-phonon coupling.
- The T-dependence of the linewidth can be used to extract  $\lambda$ .
- The T-dependence is **independent of binding energy** (not too close to the Fermi level).
- several practical problems when the binding energy is very close to the Fermi level

$$\alpha^2 F(\omega) = \lambda \left(\frac{\omega}{\omega_D}\right)^2, \omega < \omega_D$$

$$\alpha^2 F(\omega) = 0, \omega > \omega_D$$



# How to get $\lambda$ ?: imag. part of $\Sigma$

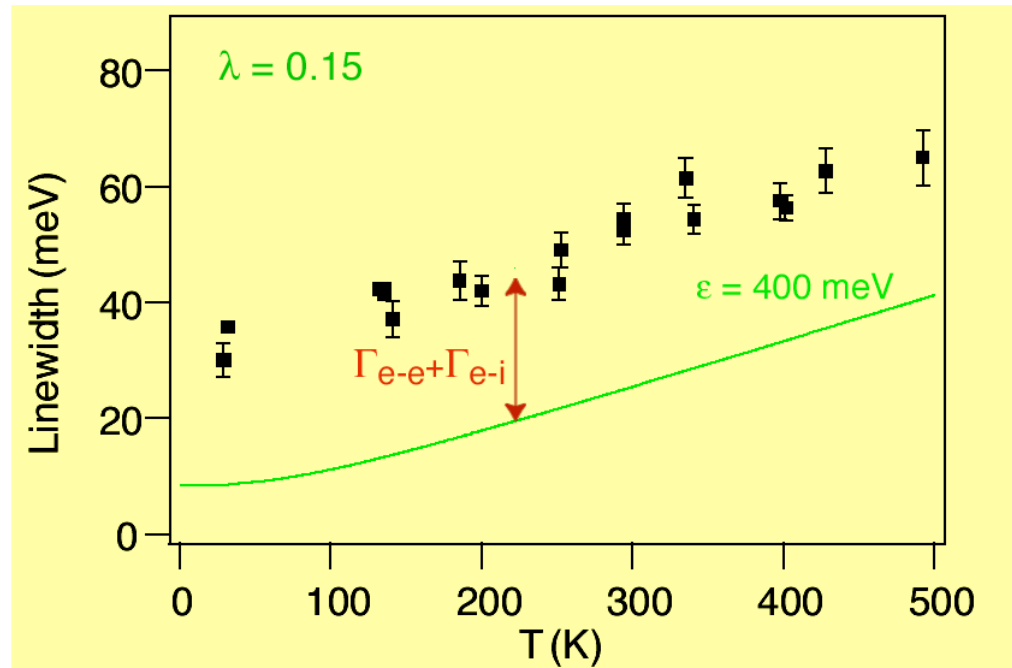
$$\text{EDC linewidth } \Gamma = \Gamma_{e-e}(\cancel{T}) + \Gamma_{e-i}(\cancel{T}) + \Gamma_{e-ph}(T)$$

$$\Gamma_{e-ph}(T) = 2\pi\hbar \int_0^{\omega_m} d\omega' \alpha^2 F(\omega') [1 - f(\omega - \omega') + 2n(\omega') + f(\omega + \omega')] \simeq 2\pi\lambda k_B T$$

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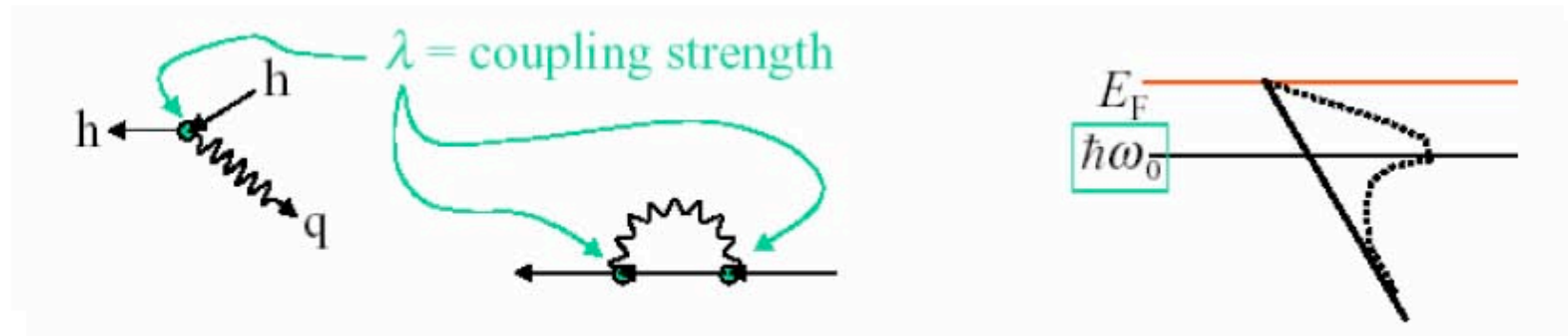
$$\alpha^2 F(\omega) = 0, \omega > \omega_D$$



B.A. McDougall, T. Balasubramanian and E. Jensen PRB 51, 13891 (1995).



# Electron-phonon coupling constant and superconductivity

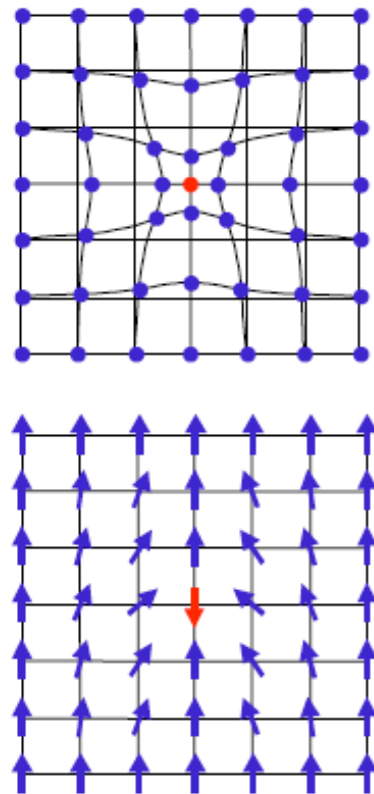
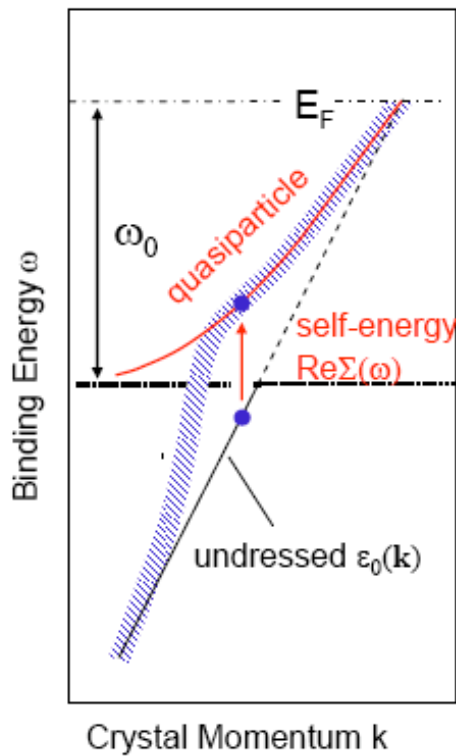


## Strong-coupling theory of superconductivity

	Matl	$\lambda$	$\omega_0$ [K]	$T_c(\text{theory})$	$T_c(\text{expt})$
	W	0.28	390	0.01	0.012
	Mo	0.41	460	0.84	0.92
	Be	0.24	1000	<b>0.02</b>	0.026
3-D	Hg	1	72	3.56	4.16
	Pb	1.12	105	6.25	7.19
	Nb	0.82	277	9.20	9.22
	Pd	0.15	274	0.00	not SC
	PdH	0.75	475	12.67	10
2-D	H:W	0.5-1.4	1932	<b>50-150</b>	?
	Be surf	1.15	1000	<b>70.21</b>	?

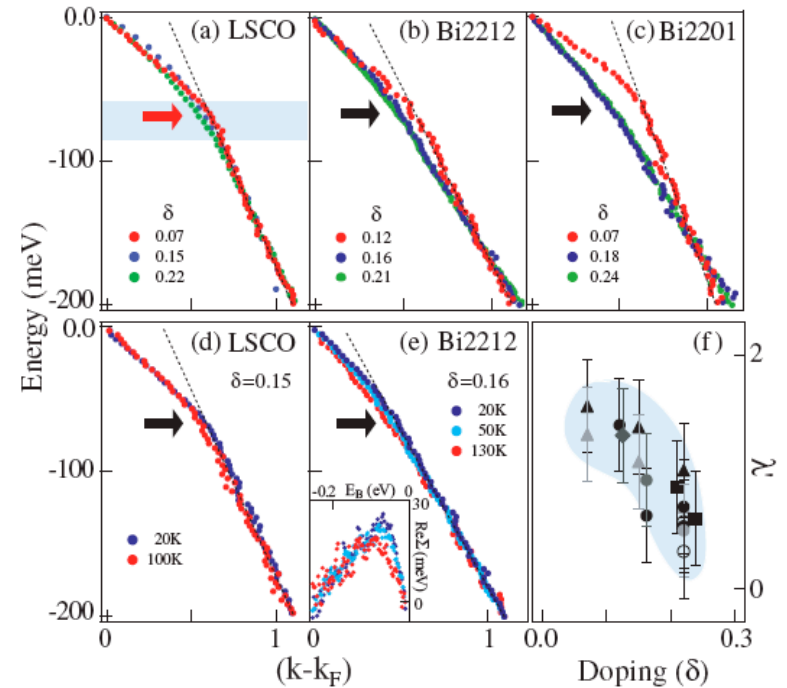
$$T_c \propto \omega_0 \exp\left(-\frac{1}{\lambda}\right)$$

# Coupling to other bosons



e-ph

e-spin

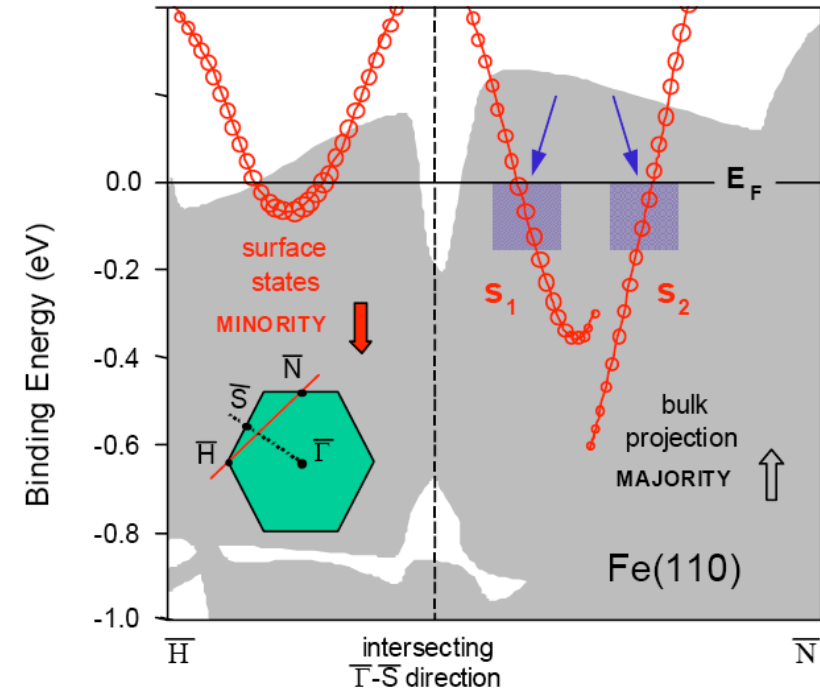
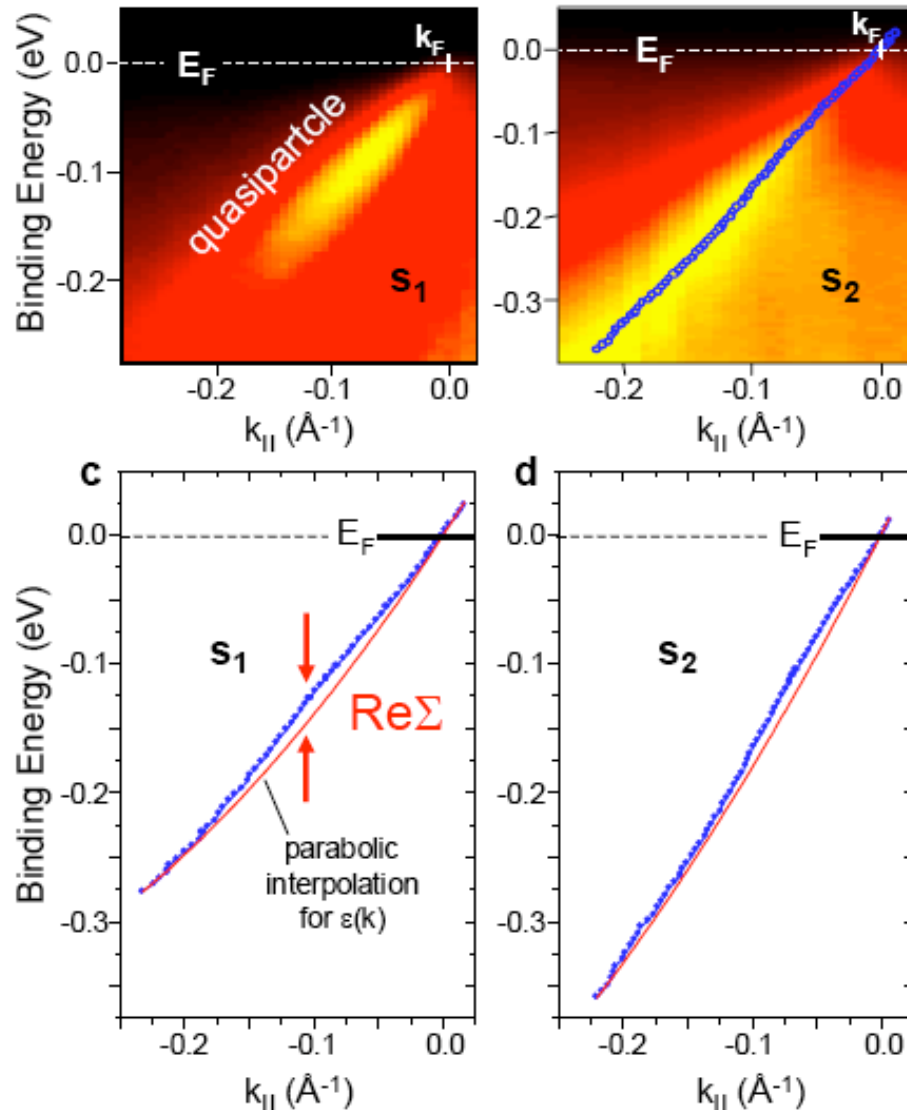


formation of quasiparticles by coupling to spin excitations ?

**kink observed in the cuprates :**

- Cooper-pairing mediated by spin fluctuations ?
- problem: energy scales for phonons and magnetic modes very similar

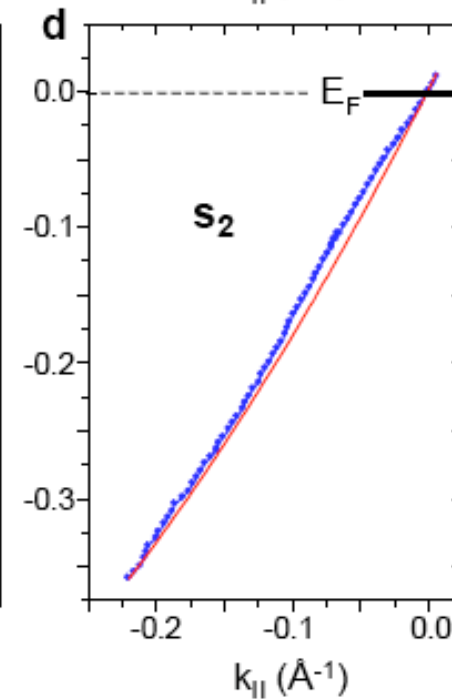
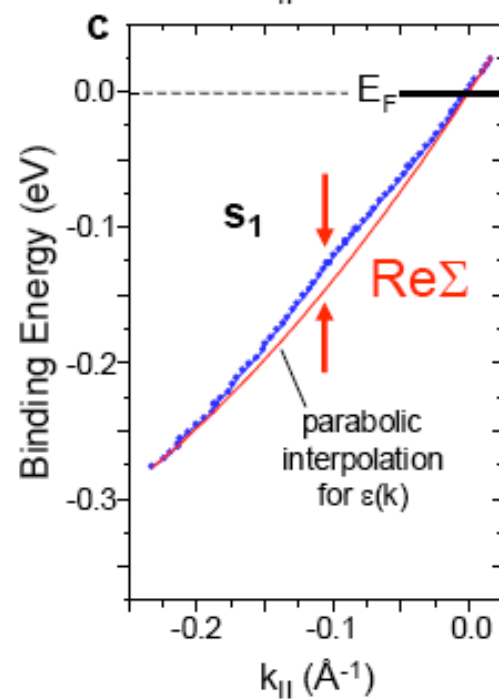
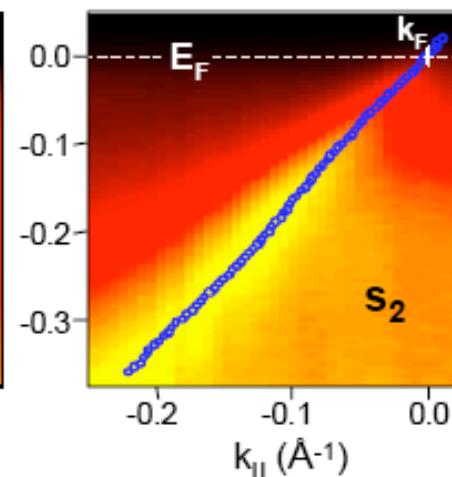
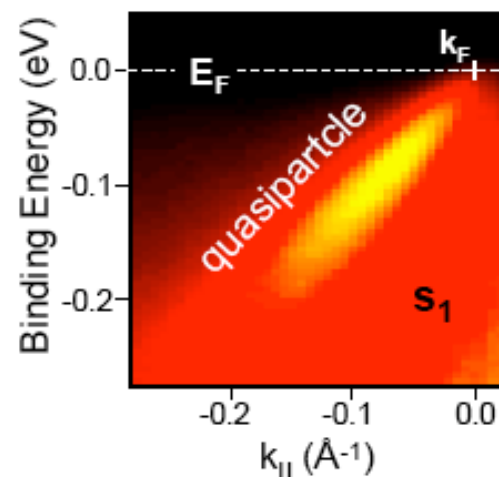
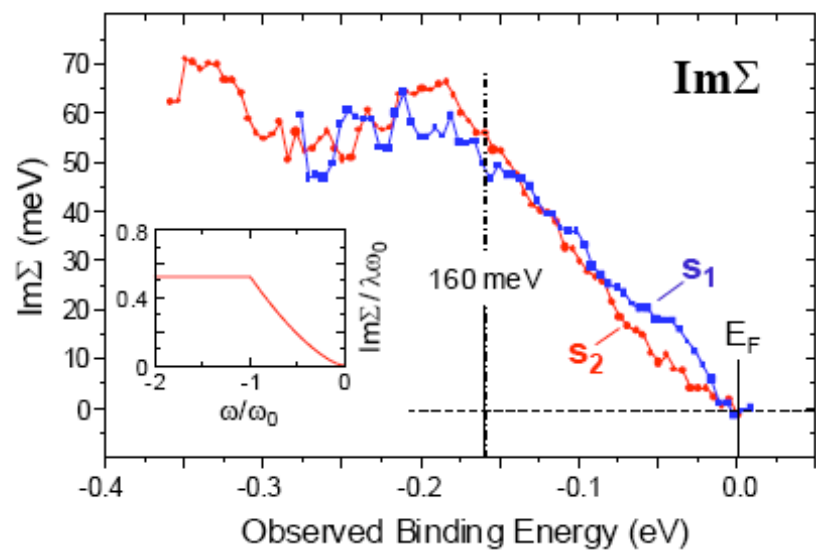
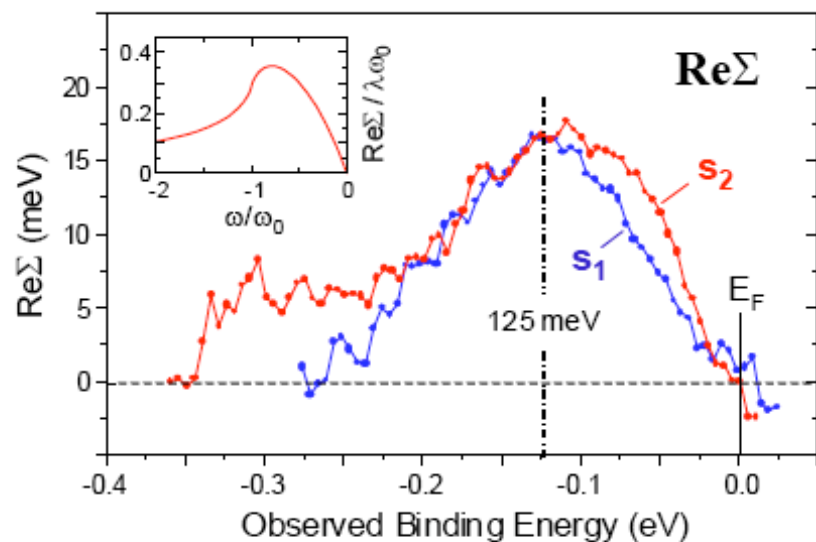
# Fe(001) surface state: dispersion and lifetime width



**self-energy  $\Sigma(\omega)$  derived from momentum distribution curves (MDCs)**

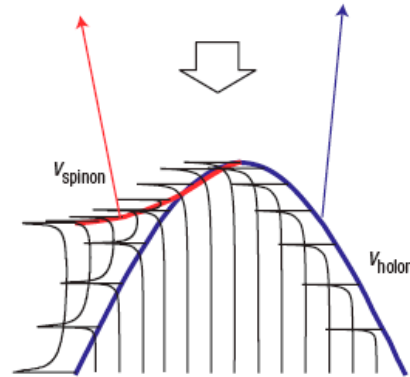
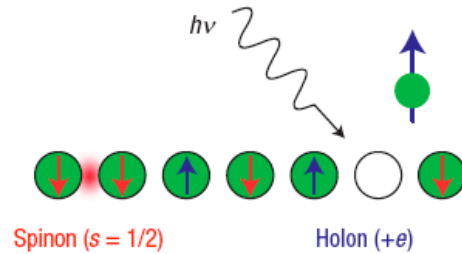
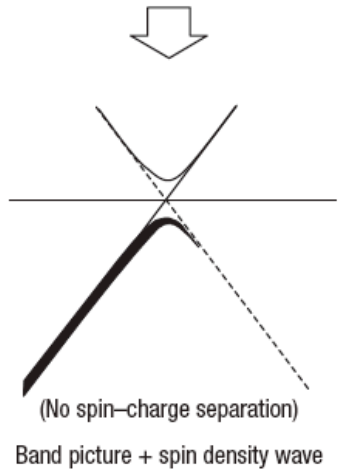
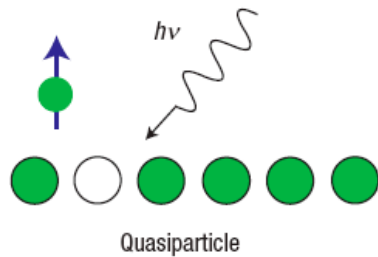
peak maximum  $\rightarrow \epsilon_k + \text{Re} \Sigma(\omega)$

**renormalization @ 100-200 meV**  
 $(\gg \omega_{\text{Debye}} \sim 30 \text{ meV})$

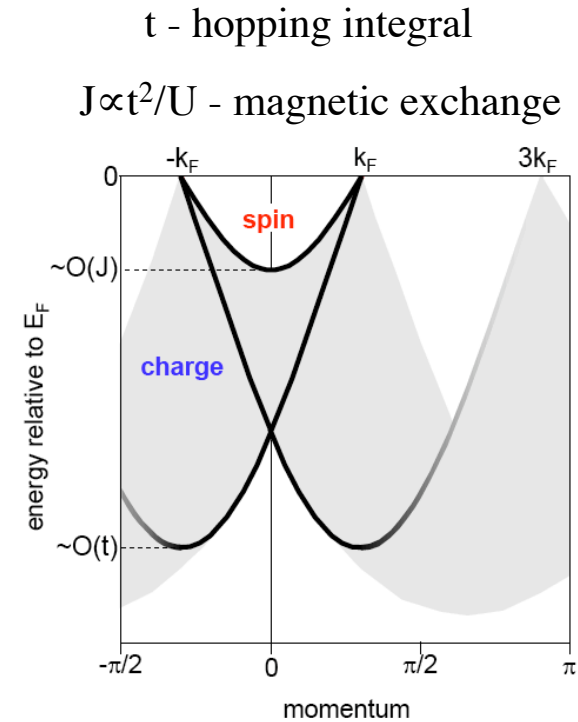
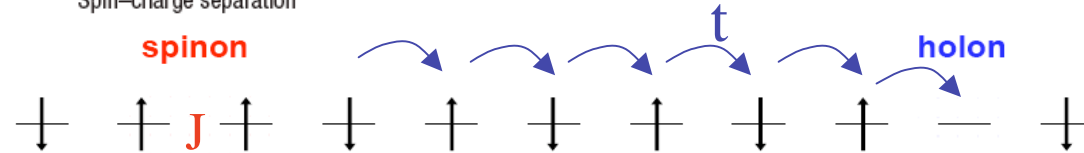


# 1D System: breakdown of the Fermi liquid

## Spinon and holon dispersion



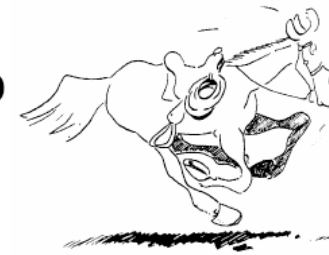
Spin-charge separation



$U \gg t$   
strong coupling

spin-charge separation in 1D

*K. Penc et al. (1996): tJ-model*  
*J.M.P. Carmelo et al. (2002 / 2003): Bethe ansatz*  
*E. Jeckelmann et al. (2003): dynamical DMRG*



# Distinct spinon and holon dispersions in photoemission spectral functions from one-dimensional SrCuO<sub>2</sub>

B.J. Kim et al., Nature Physics 2 (2006)

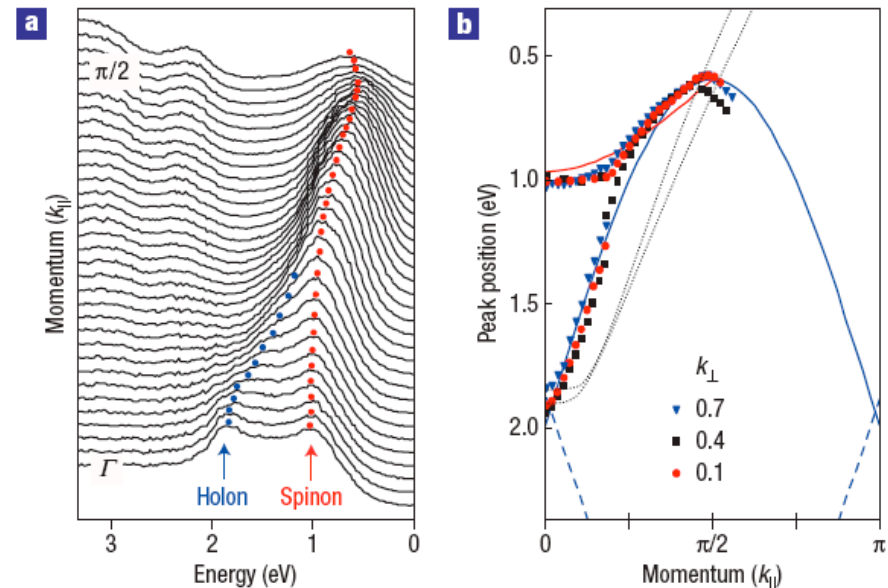
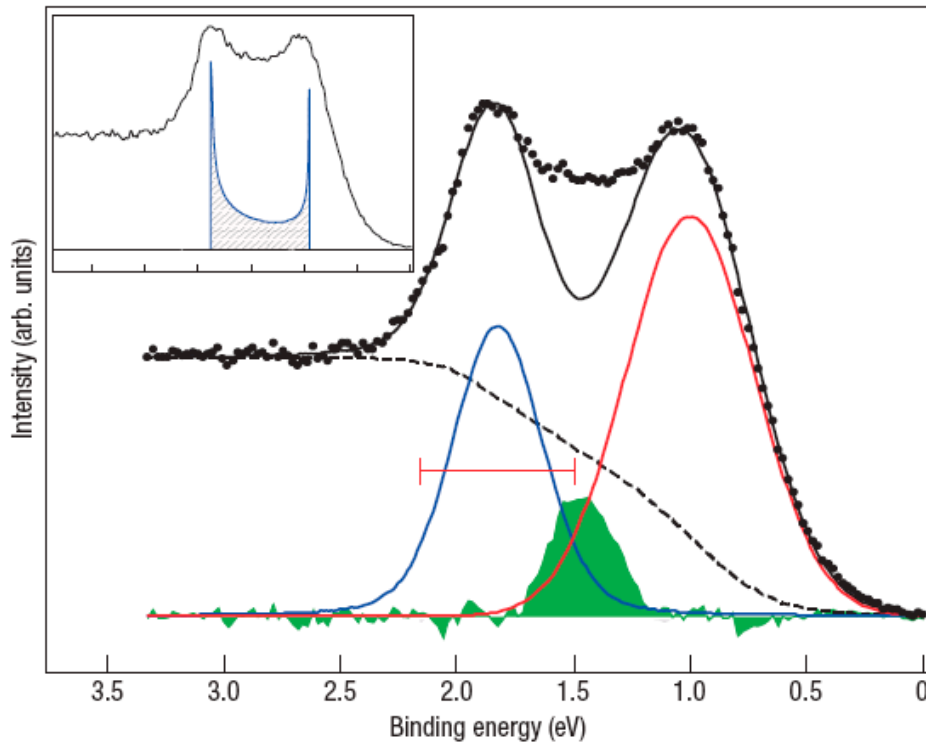
$$H = t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$t$  – hopping integral

$U$  – local Coulomb energy

$J \propto t^2/U$  – magnetic exchange energy

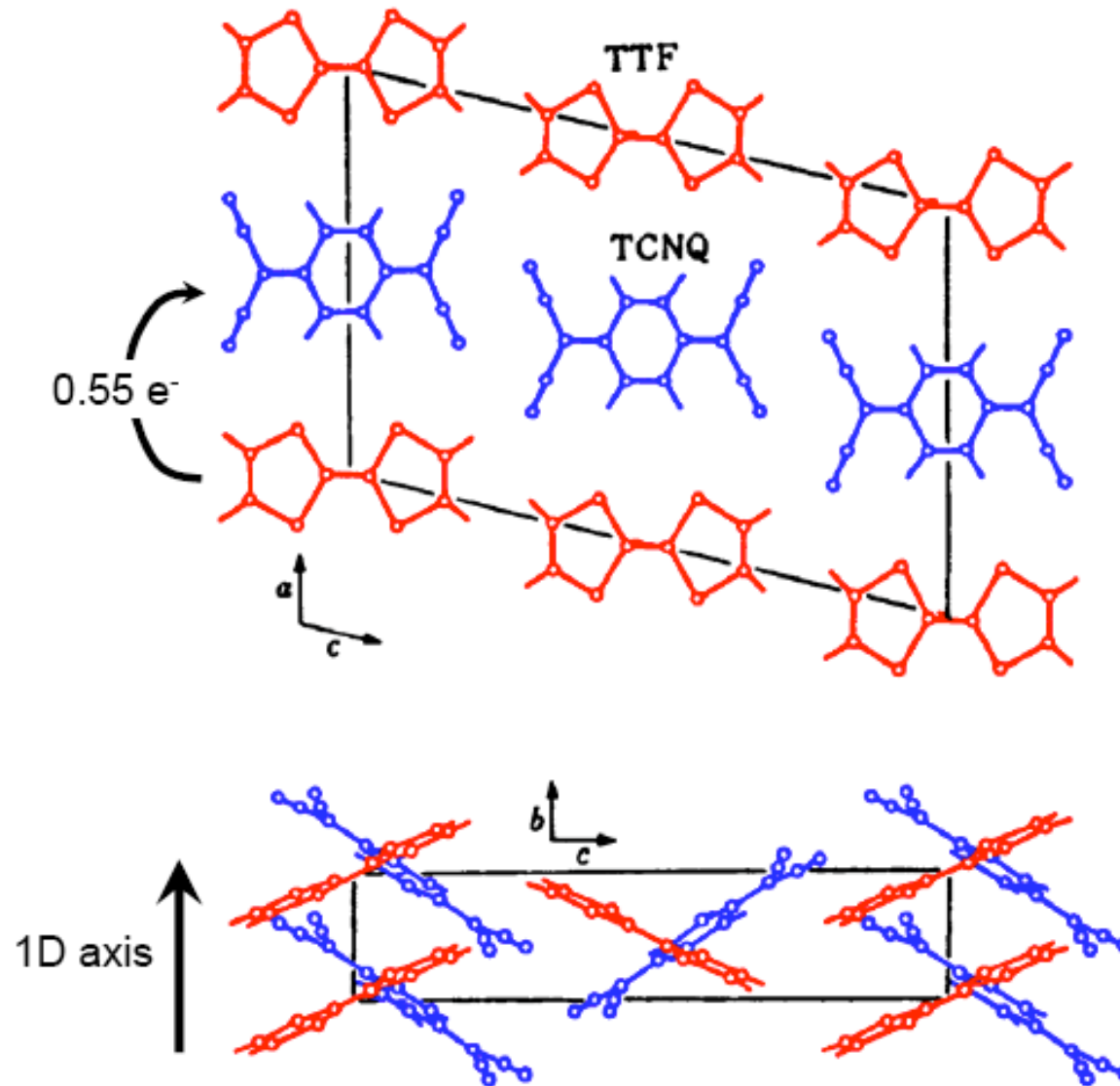
strong coupling  $U \gg t$



**Figure 3** EDCs and dispersions. **a**, EDCs for  $k_{\parallel}$  between  $\Gamma$  and  $0.6\pi$  at  $k_{\perp} = 0.7 \text{ \AA}^{-1}$ . Each EDC is curve-fitted to find the peak positions. Two peaks were used in the region between  $k_{\parallel} = \Gamma$  and  $\pi/4$ , whereas a single peak was used for the rest. The results are marked by red and blue circles. **b**, Experimental (symbols) and theoretical (solid and dashed lines) dispersions. The theoretical dispersions are obtained by taking the nearest-neighbour hopping of  $t = 0.65 \text{ eV}$  and the exchange coupling of  $J = 0.23 \text{ eV}$ . Dispersions from the band theory are also shown as two dotted lines<sup>22</sup>.



# TTF-TCNQ: an organic 1D metal



strongly anisotropic conductivity  
 $\sigma_b/\sigma_a \approx \sigma_b/\sigma_c \sim 1000$

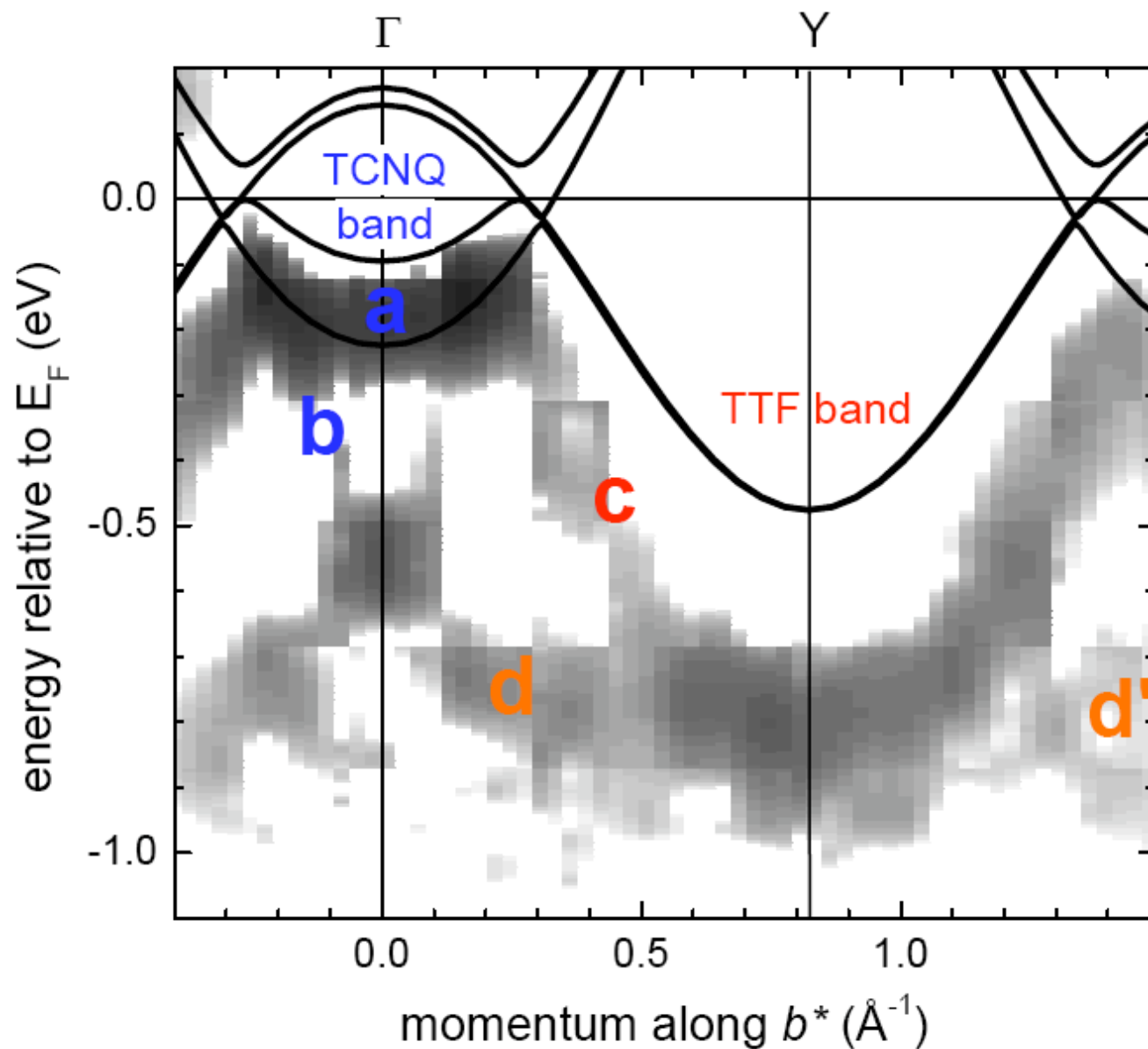
Peierls instability @  $T_p = 54$  K,  
incommensurate CDWs on  
TCNQ stacks

### electronic correlation effects:

- $4k_F$ -fluctuations (on TTF stacks ?)  
observed up to 300 K
- enhanced magnetic susceptibility
- enhanced NMR relaxation rate

⇒ estimate for Hubbard model  
parameters:

$$U \sim 4t \sim 1 \text{ eV}$$



$-\frac{d^2I}{dE^2}$  vs.  $k$

experimental band width  
doubled relative to theory

→ **molecular surface  
relaxation**

*PRB 67, 125402 (2003)*

spectral structure (**d**) not  
explained by band theory

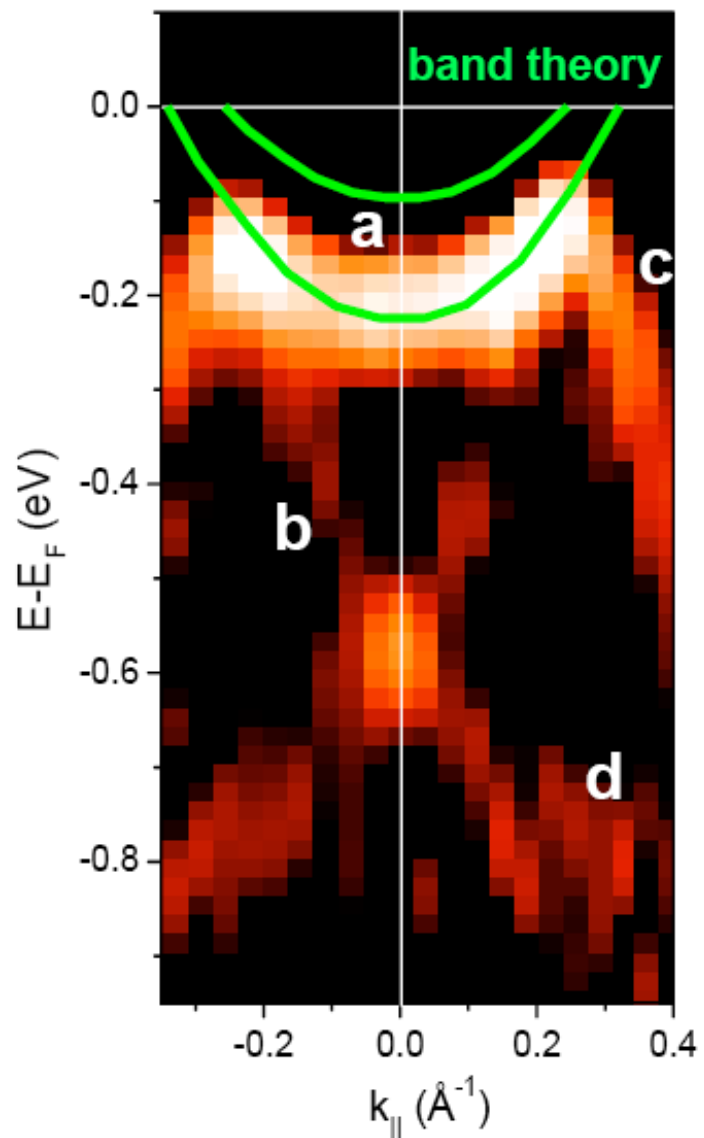
→ **1D correlation effects**

*PRL 88, 096402 (2002);*

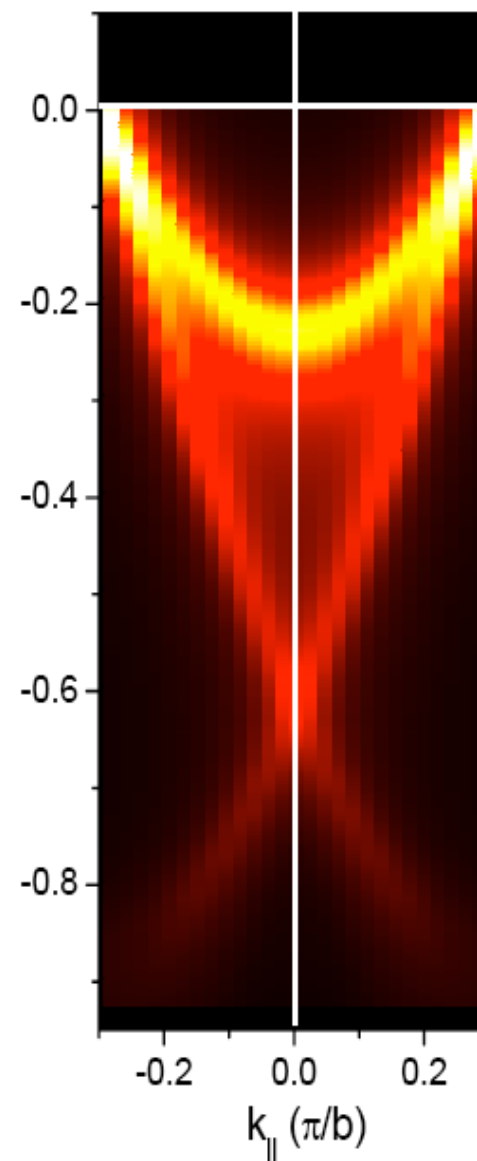
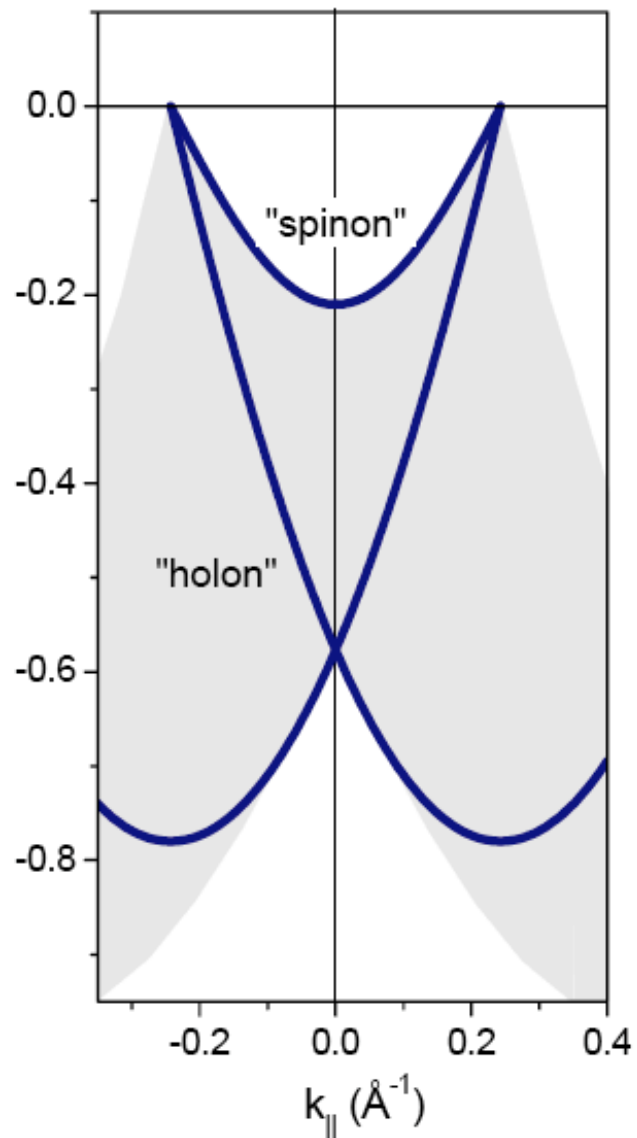
*PRB 68, 125111 (2003)*



### photoemission



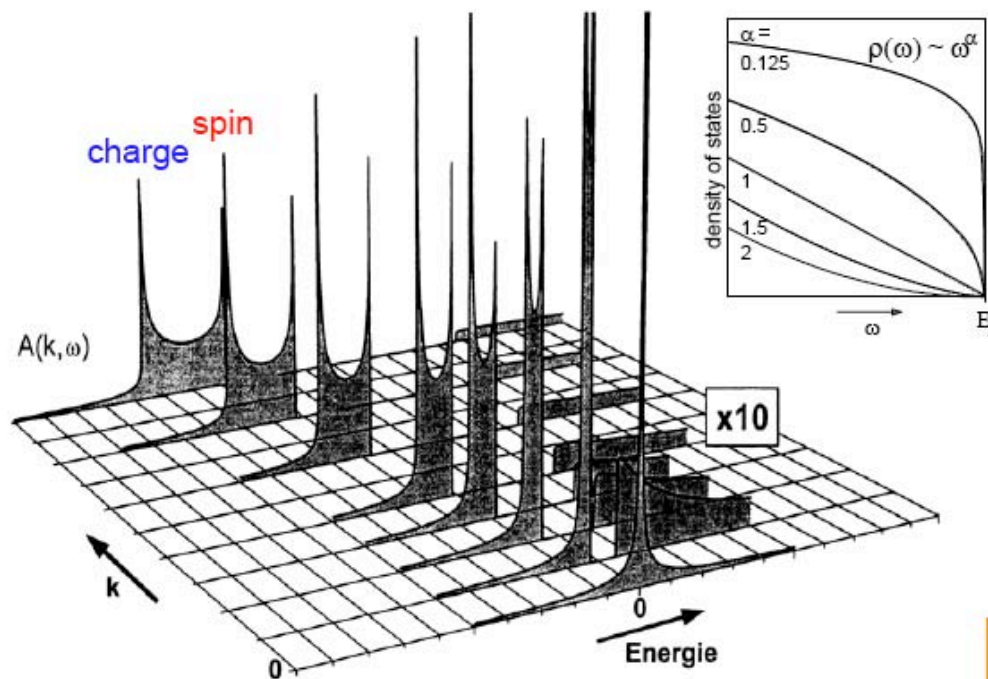
### theory



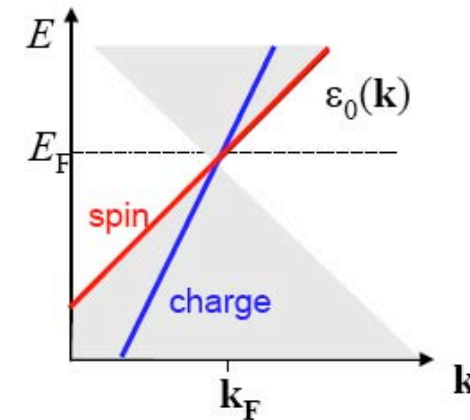
# 1D System: breakdown of the Fermi liquid

## The Tomonaga-Luttinger liquid

Tomonaga-Luttinger model:  
spectral function  $A(\mathbf{k}, \omega)$



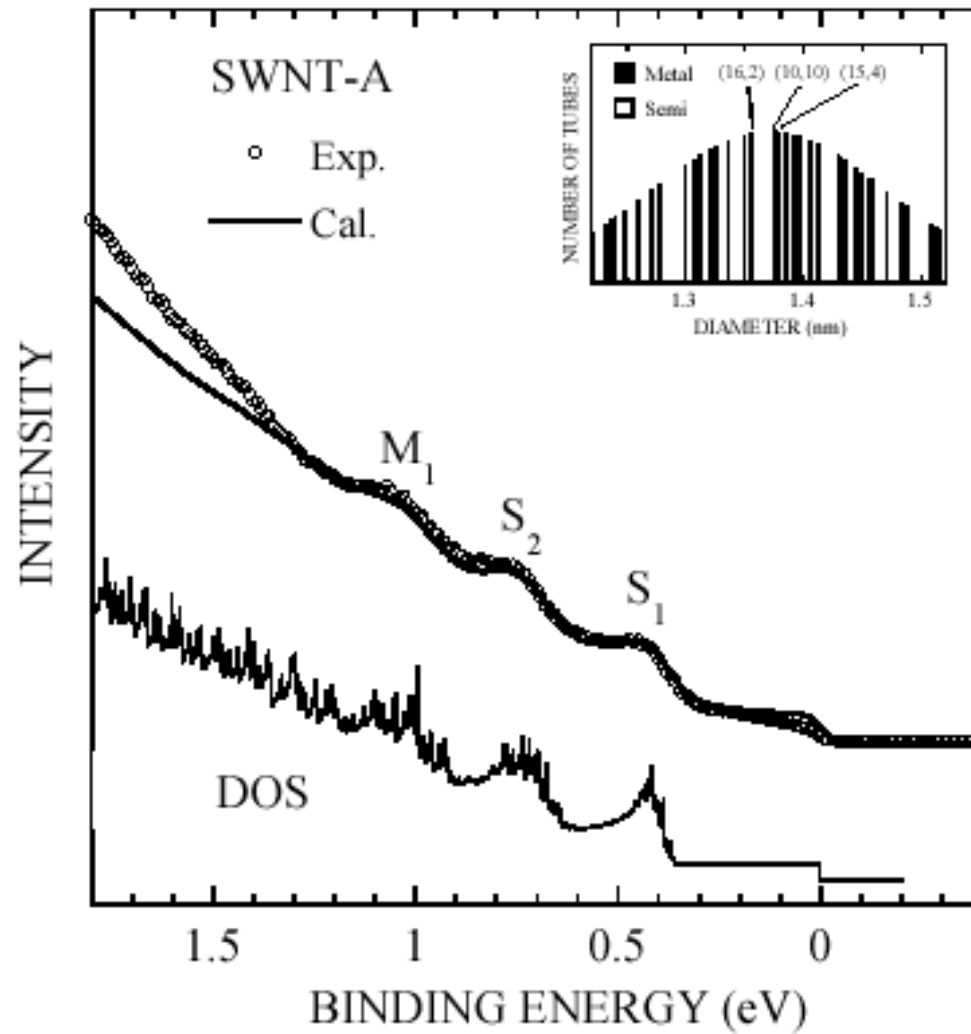
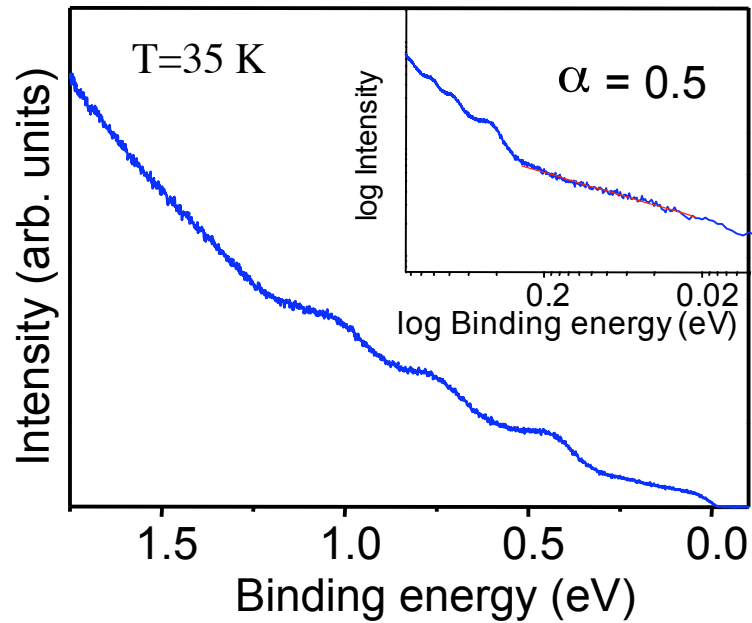
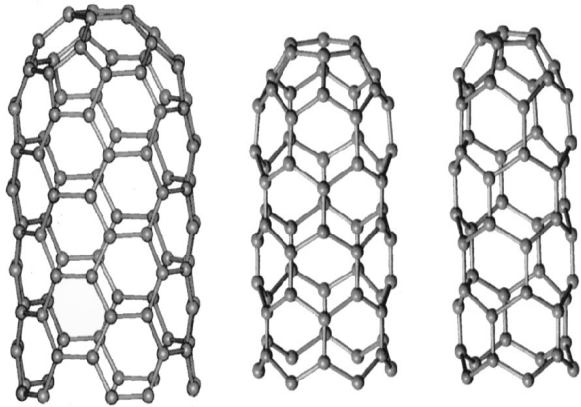
*Voit (1995)*  
*Schönhammer and Meden (1995)*



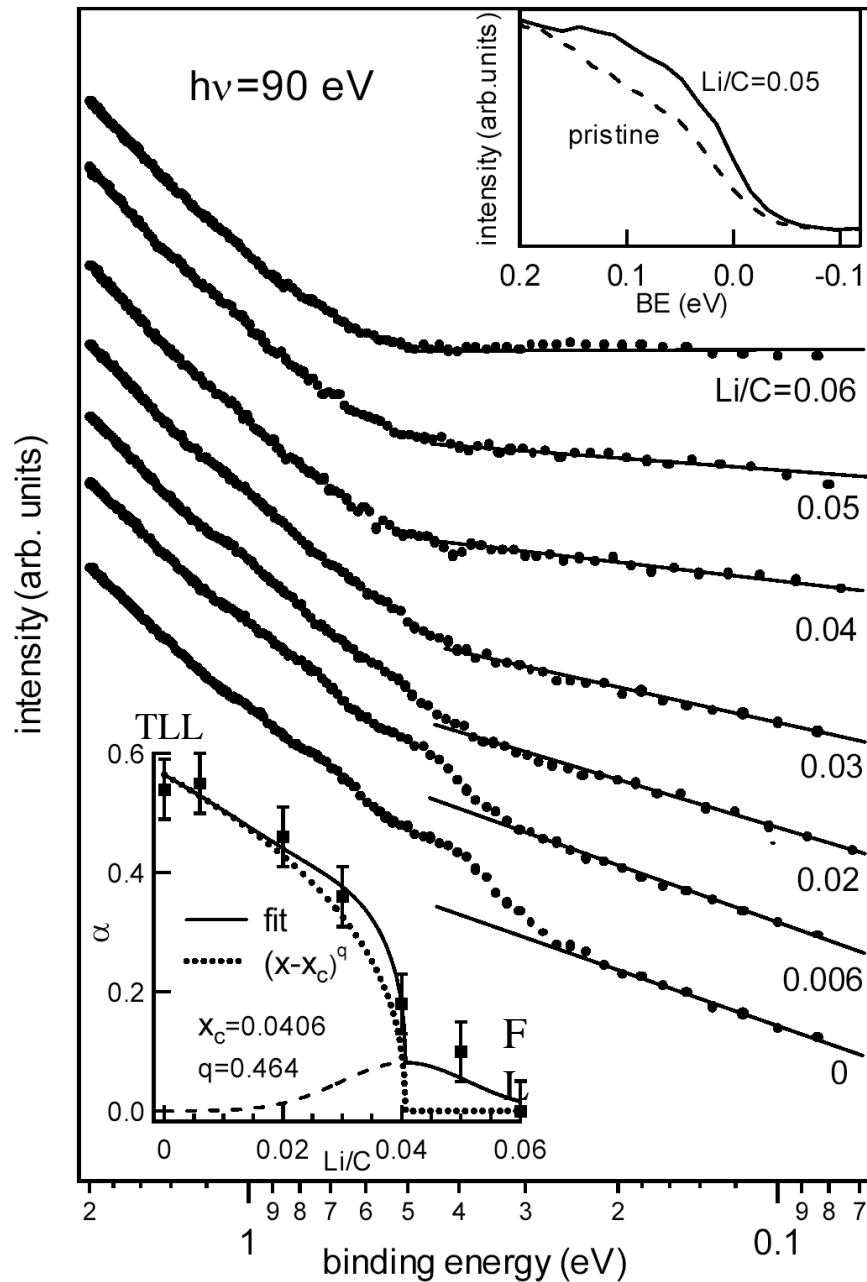
- spin-charge “separation”
- failure of the quasiparticle concept
- power law onset in DOS (no Fermi edge !)

generic low-energy physics  
of interacting 1D metals:  
**LUTTINGER LIQUID**

# SWCNTs: an example of Tomonaga-Luttinger liquid



log-log plot



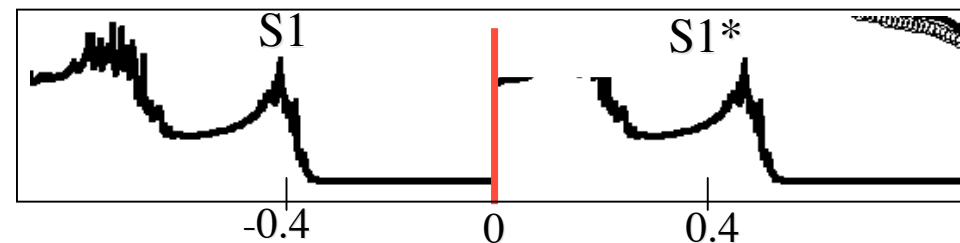
## Transition from a Tomonaga-Luttinger liquid to a Fermi liquid

$n(E) \propto |E-E_F|^\alpha$ , where  $\alpha=(g+g^{-1}-2)/8$  depends on the size of the Coulomb interaction and  $g$  is the Luttinger parameter

$\alpha \sim 0.53 \pm 0.05$  for pristine SWCNTs in agreement with other estimations

$\alpha=0$  Fermi liquid

R. Larciprete, S. Lizzit, L. Petaccia, A. Goldoni, PRB 71, (2005)



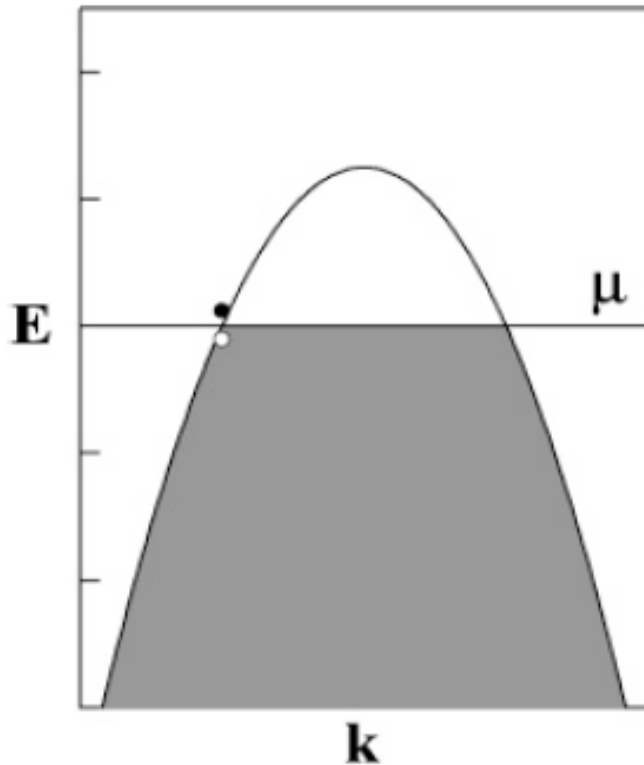
# Mott-Hubbard insulator

## Non-interacting Limit

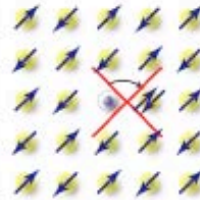


- metallic
- bandwidth  $\sim 8t \sim 3 \text{ eV}$

## *Metal*

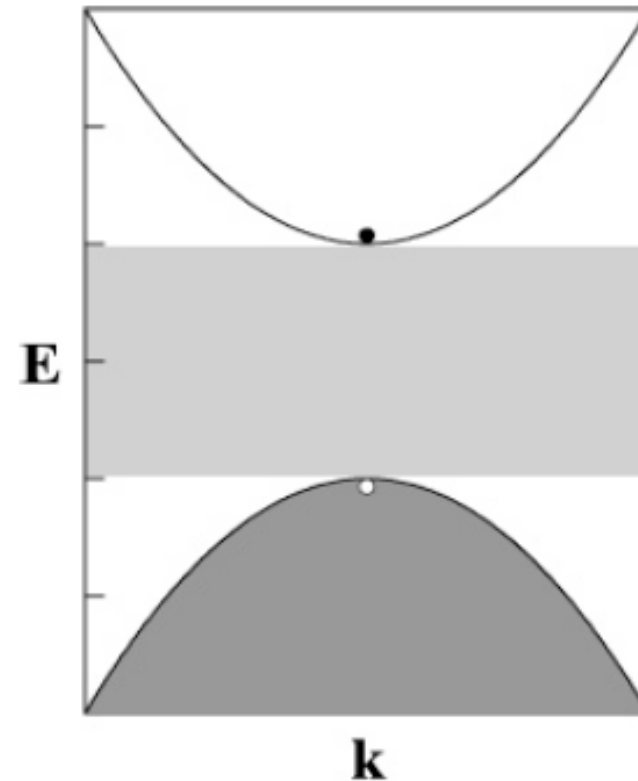


## Strongly Interacting (Mott) Limit

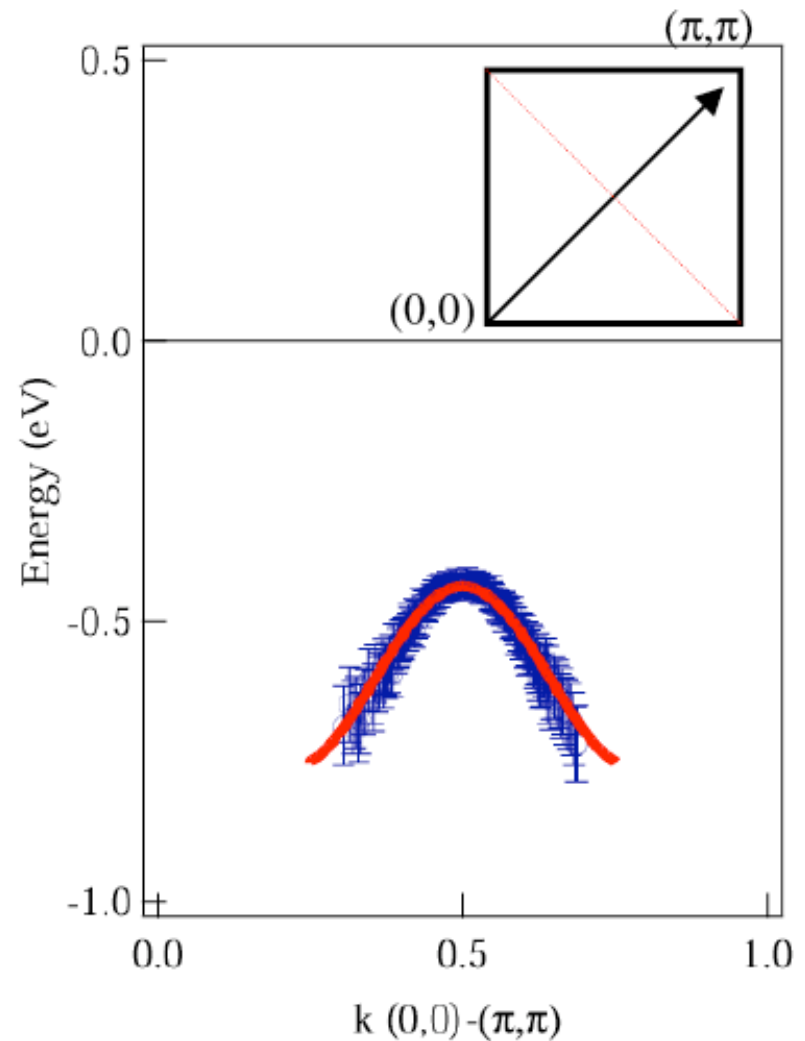
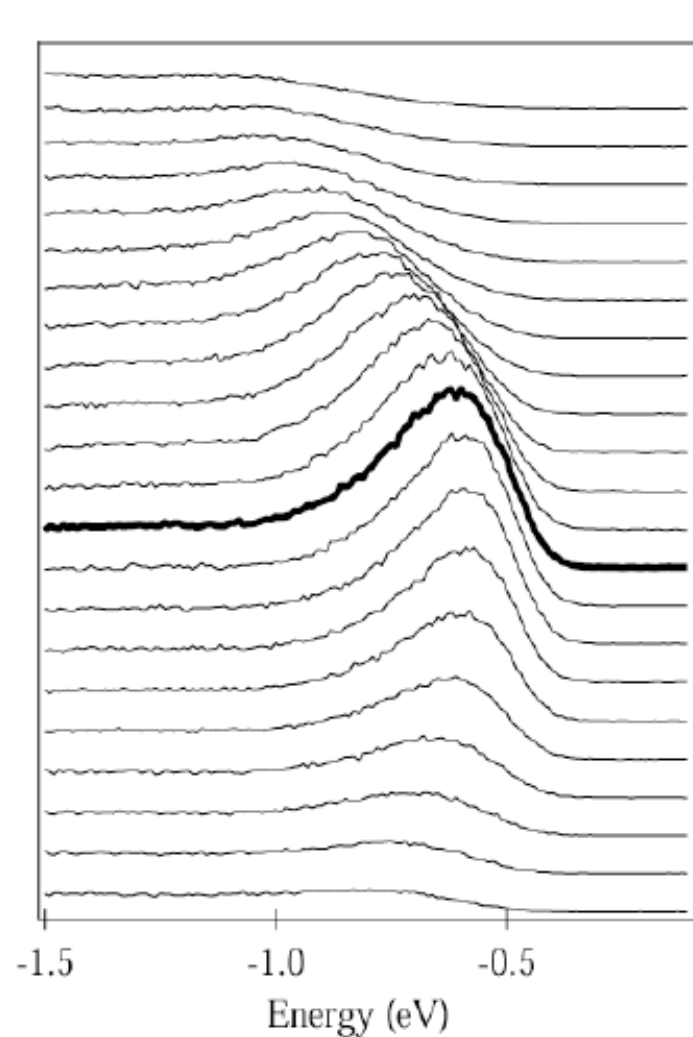


- insulating ( $\Delta \sim 2 \text{ eV}$ )
- antiferromagnetic
- bandwidth  $\sim 2J \sim 0.3 \text{ eV}$

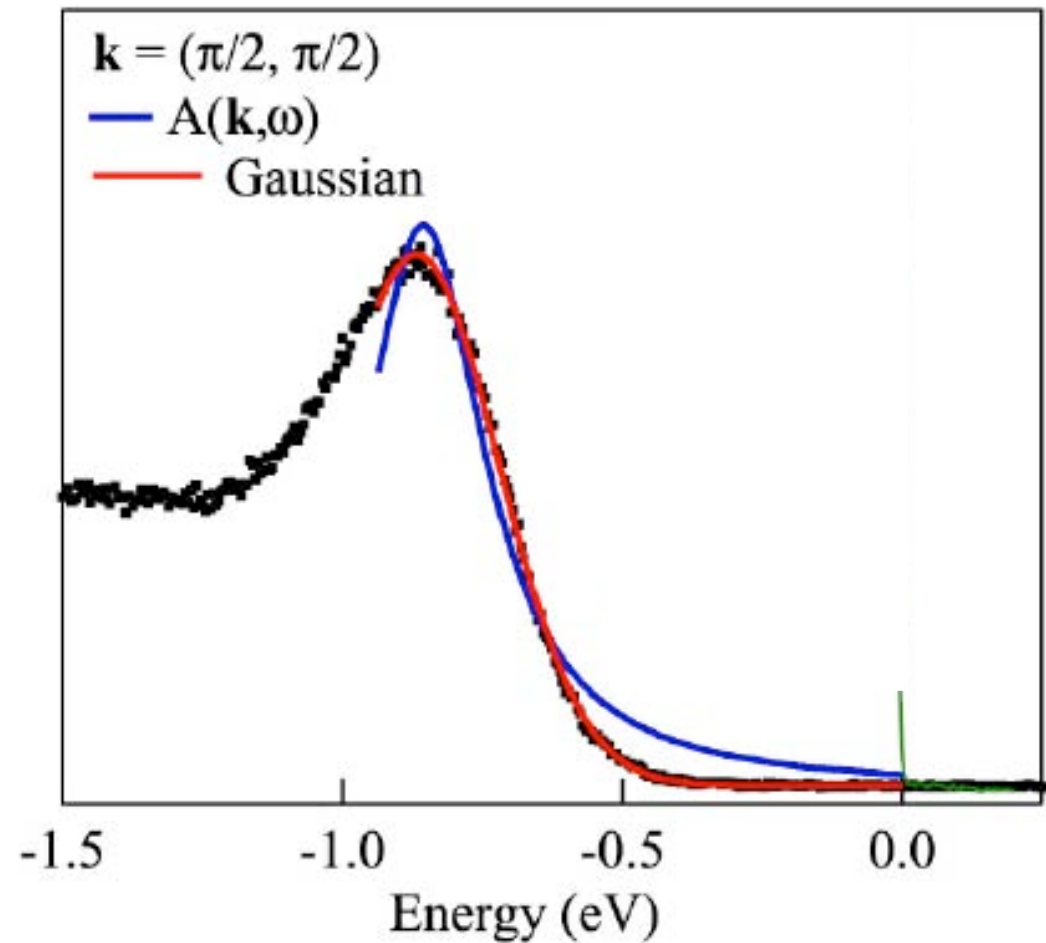
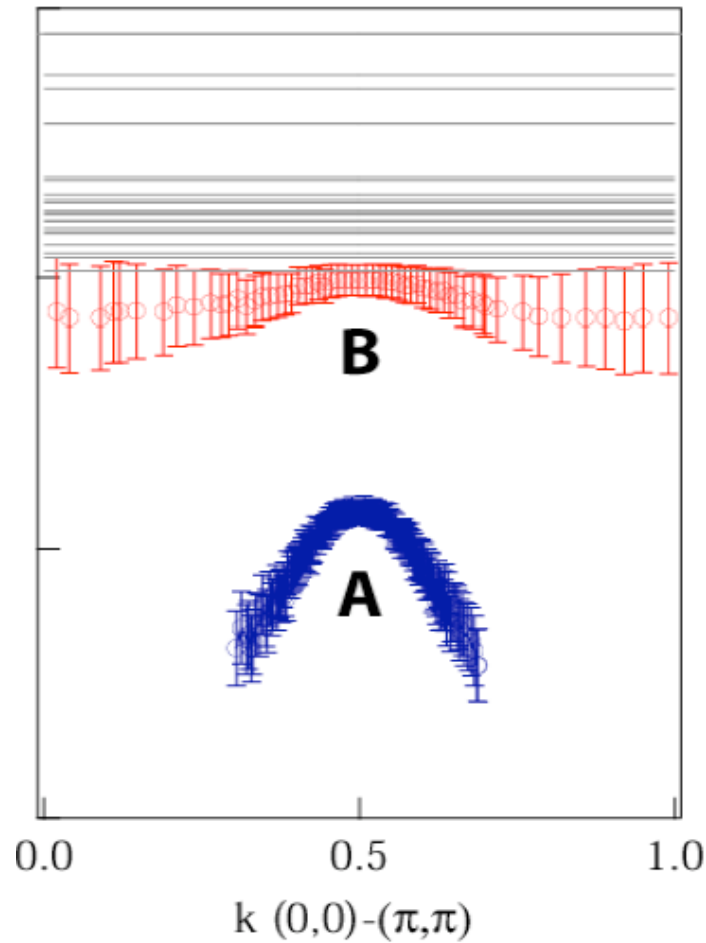
## *Insulator*



# $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$ : at $x=0$ Mott-Hubbard insulator



- Dispersion of peak intensity follows  $t$ - $J$  model well
- Suggests quasiparticle excitations from  $t$ - $J$  model are appropriate
- However, this is not the end of the story....

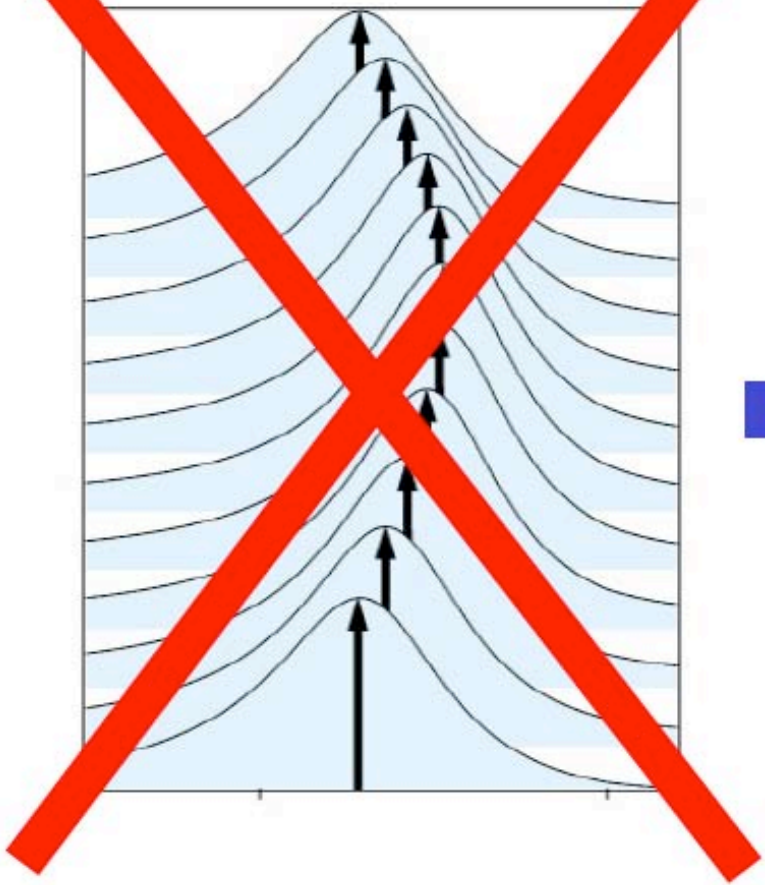


$$A(\mathbf{k}, \omega) \propto \frac{\text{Im}\Sigma(\mathbf{k}, \omega)}{[\omega - \varepsilon_{\mathbf{k}} - \text{Re}\Sigma(\mathbf{k}, \omega)]^2 + [\text{Im}\Sigma(\mathbf{k}, \omega)]^2}$$

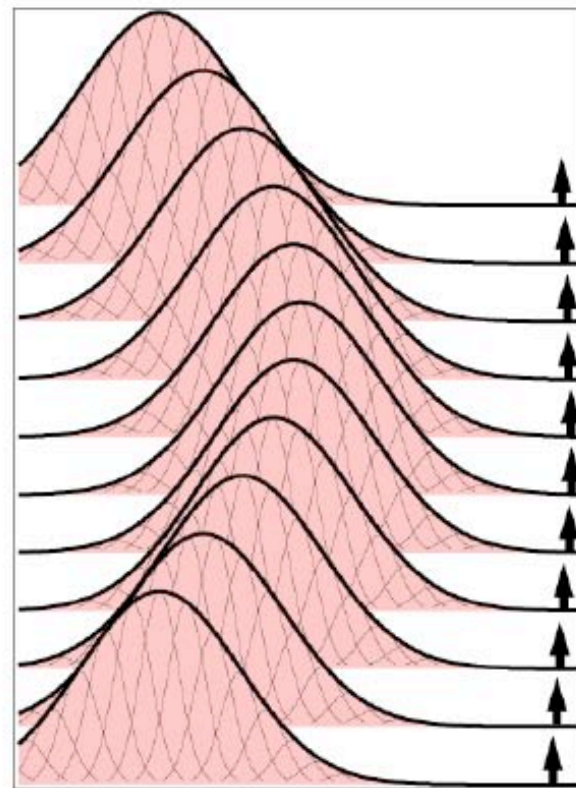
- Spectral function formalism **FAILS** to qualitatively describe lineshape
- Linewidth far too broad



~~“Weak Coupling” / Quasiparticle Limit~~



Strong Coupling  
 $Z \ll 1$  Limit

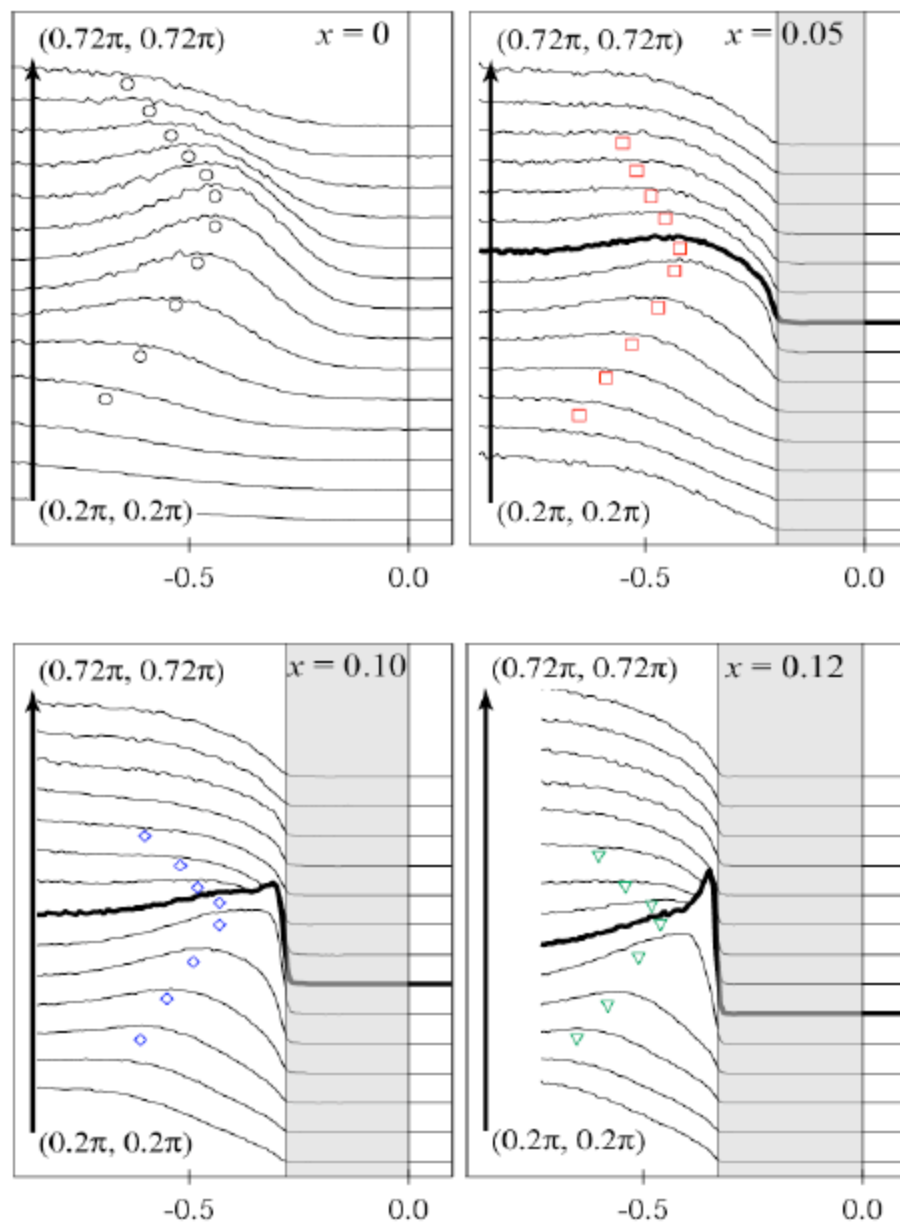
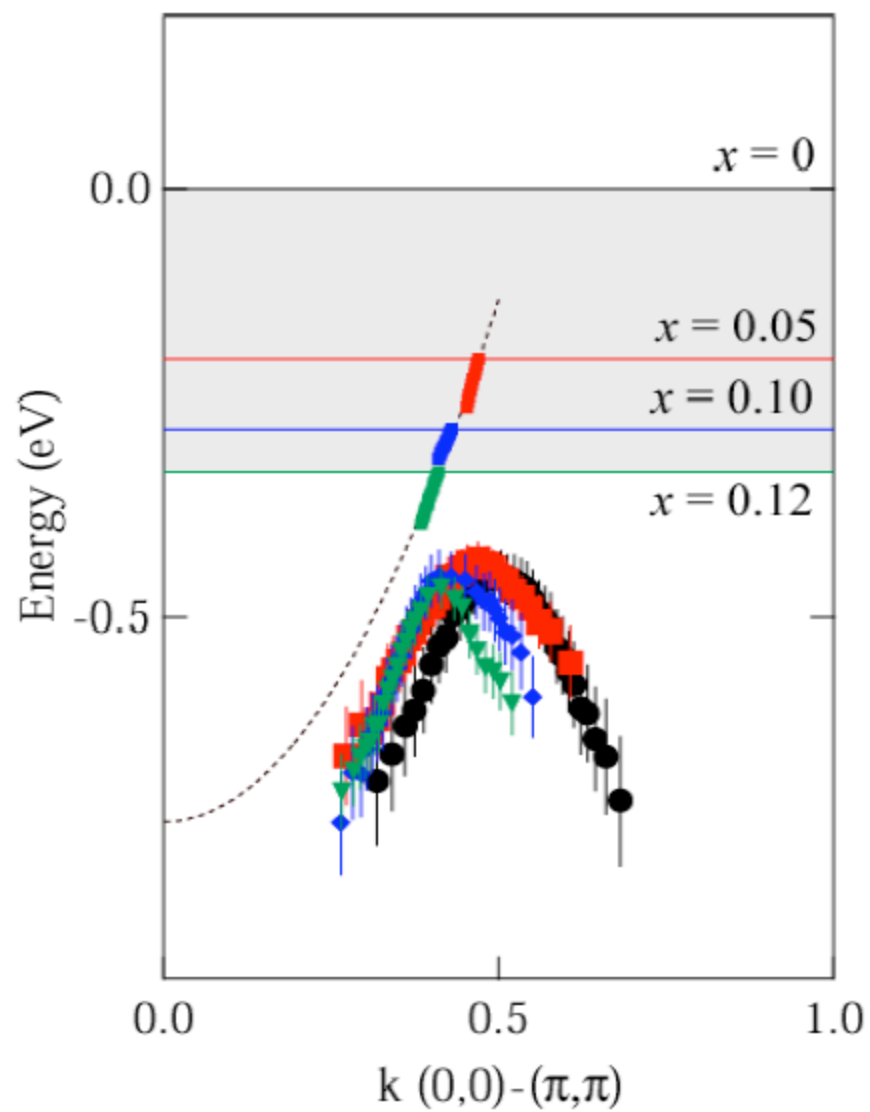


Polaron formation

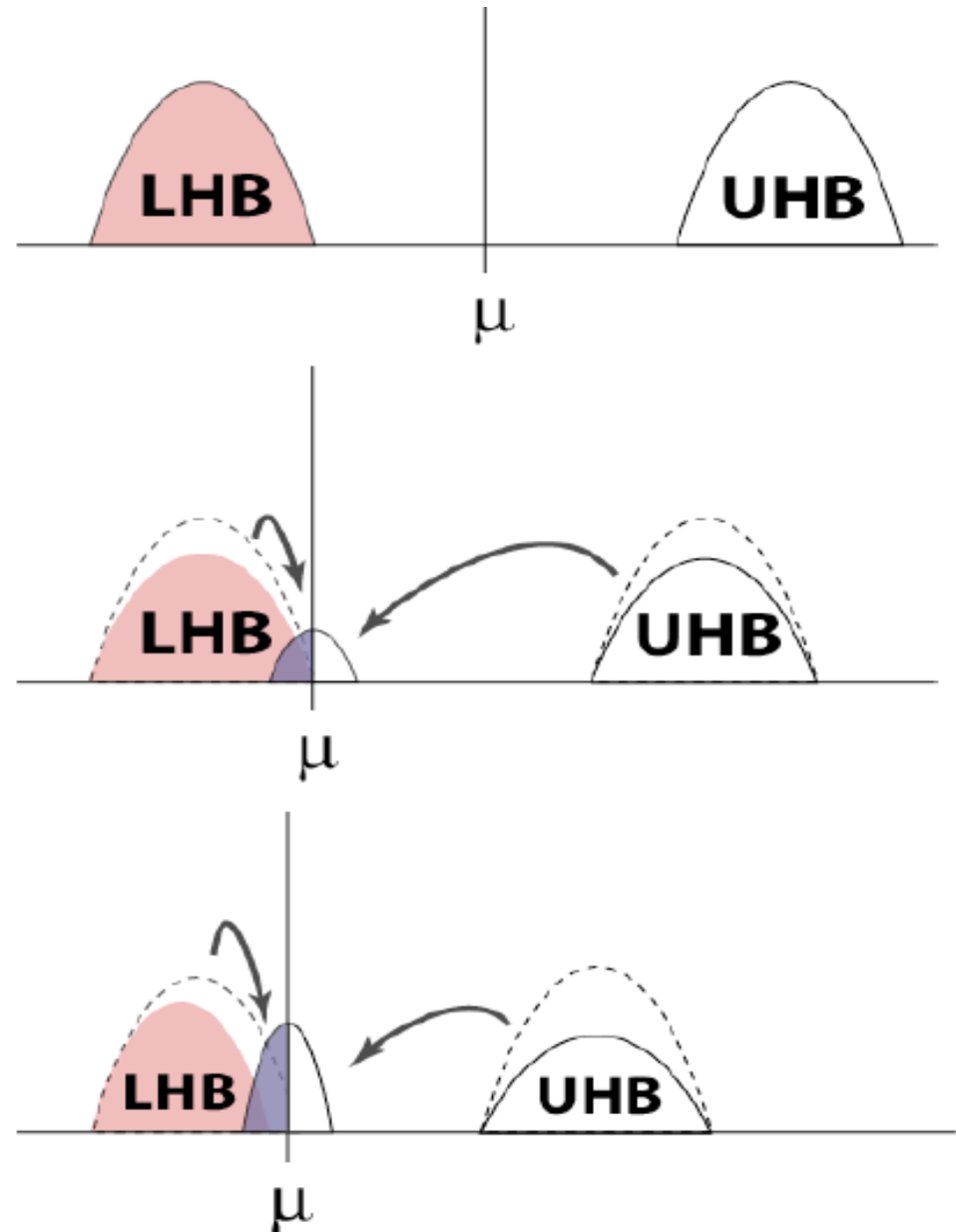
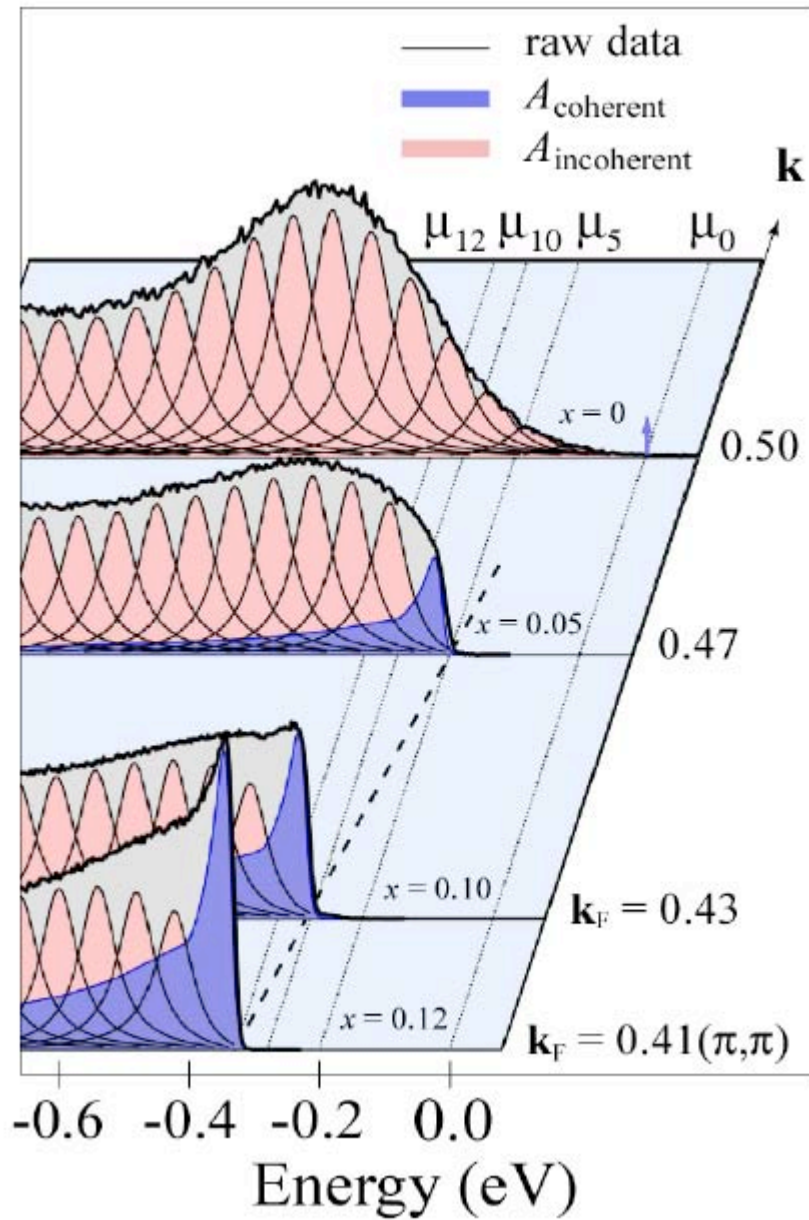
- Consistent with extreme width
- Explains anomalous lineshape
- Justifies positions of chemical potential
- Vast majority of spectral weight in *incoherent* transitions



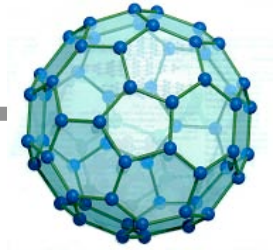
# $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$ : doping dependence



# Ca<sub>2-x</sub>Na<sub>x</sub>CuO<sub>2</sub>Cl<sub>2</sub>: doping dependence



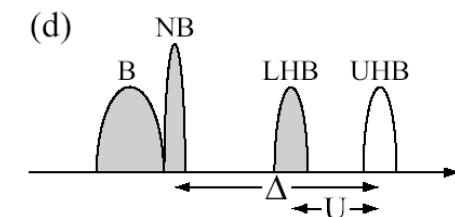
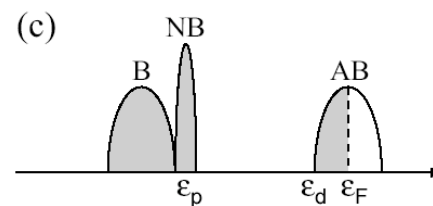
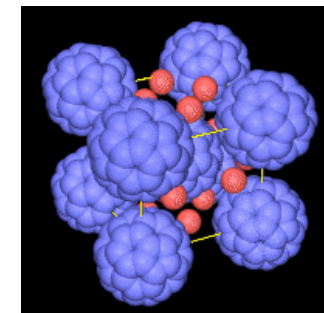
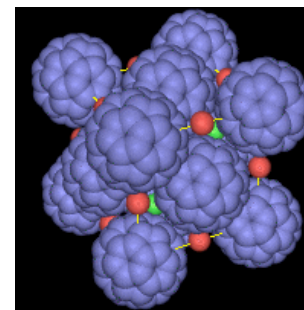
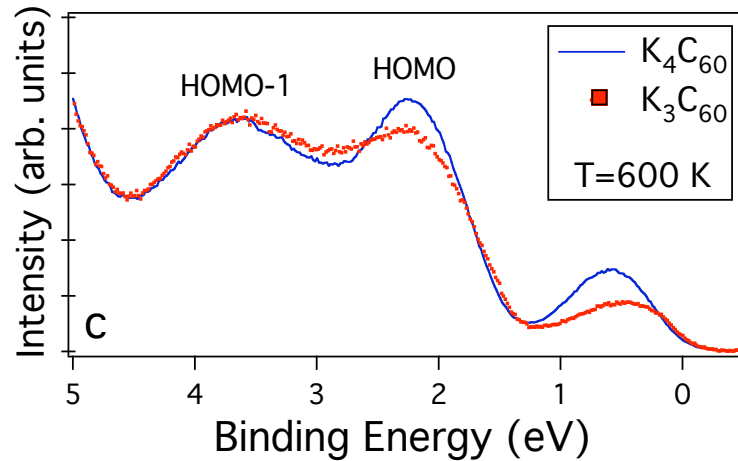
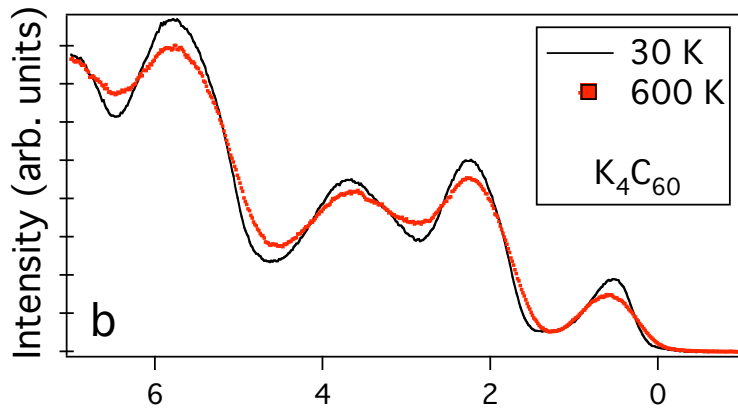
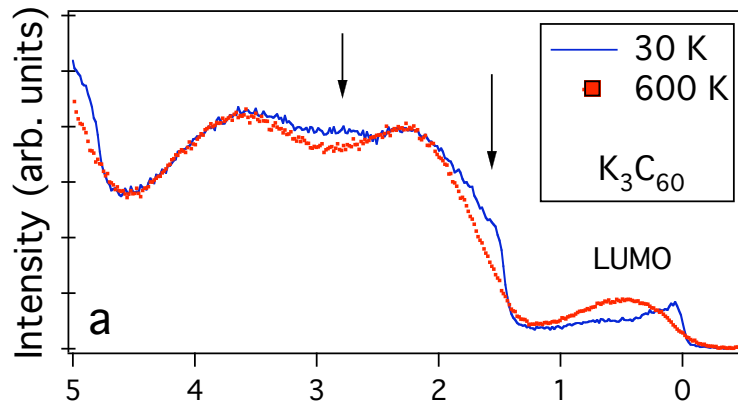
# Fullerenes



Strongly correlated metal

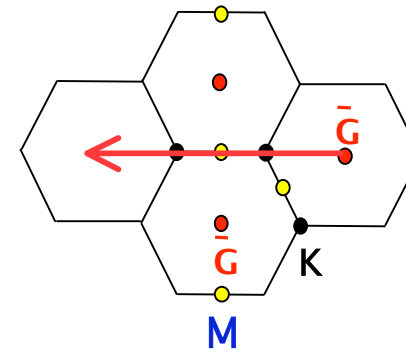
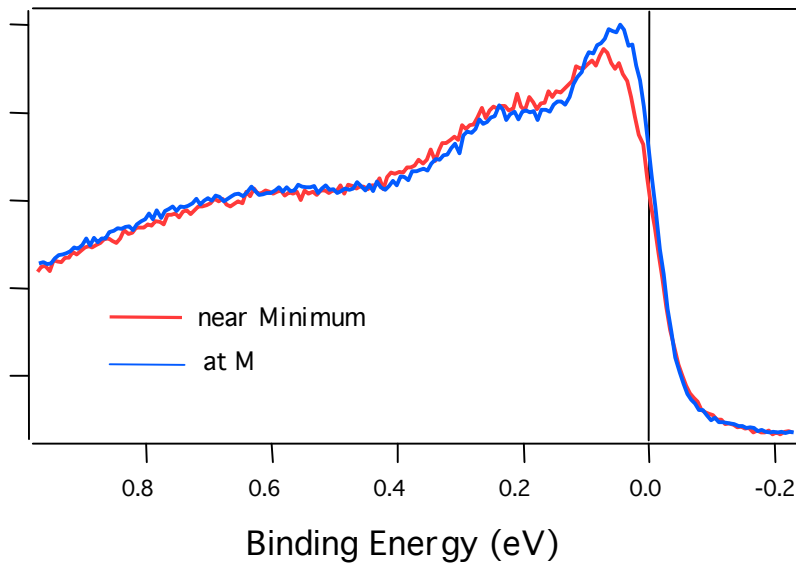
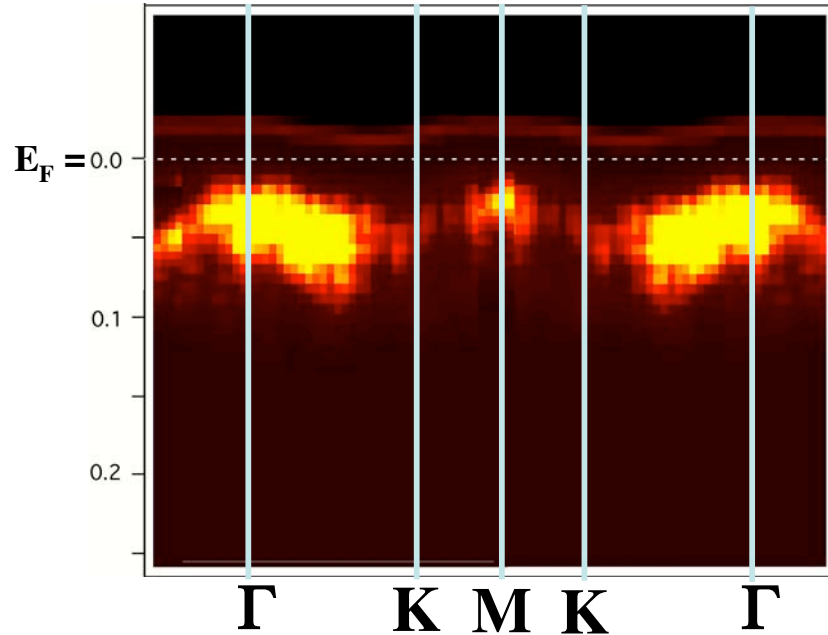
**Strong correlations  $U \sim 1 - 1.3$  eV**  
**Small band dispersion  $W \sim 0.5 - 0.6$  eV**  
**Orbital degeneracy (HOMO 5 fold, LUMO 3 fold)**  
**Small Fermi energy  $\sim 0.25$  eV**  
**Phonon spectrum up to 0.2 eV**  
**Jahn-Teller distortions in charged  $C_{60}^{n-}$**   
 **$E_{JT} \sim 0.03-0.18$  eV for  $C_{60}^{n-}$**

Mott-Hubbard insulator



# Band dispersion of $K_3C_{60}(111)$

Dispersion  $< 100$  meV



Spectrum dominated by phonon and plasmon excitations; quasi-particle coherent peak confined near  $E_F$

