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Spin-Flip Scattering at Quantum Hall Transition

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Spin-flip scattering at quantum Hall transition

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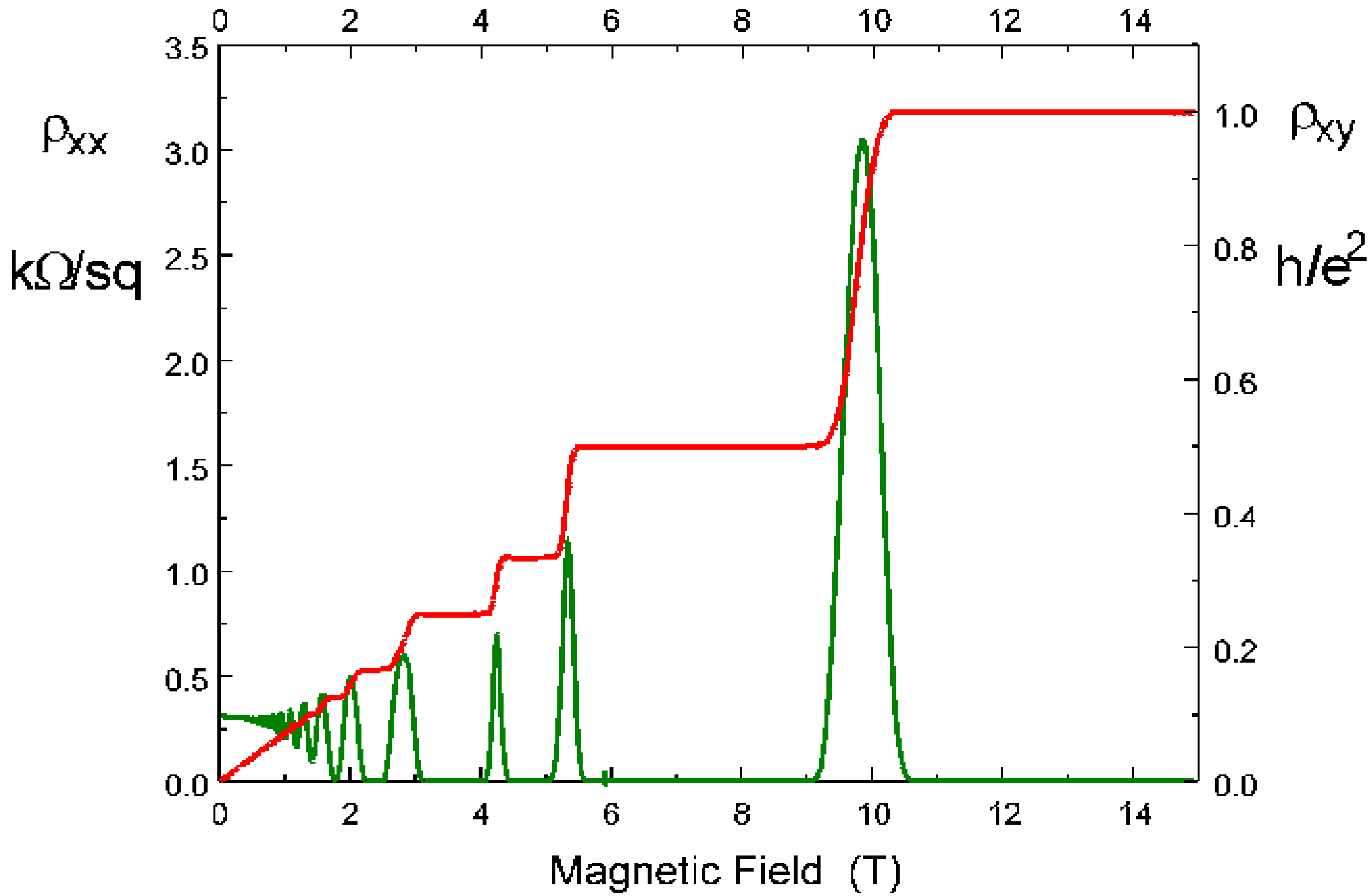
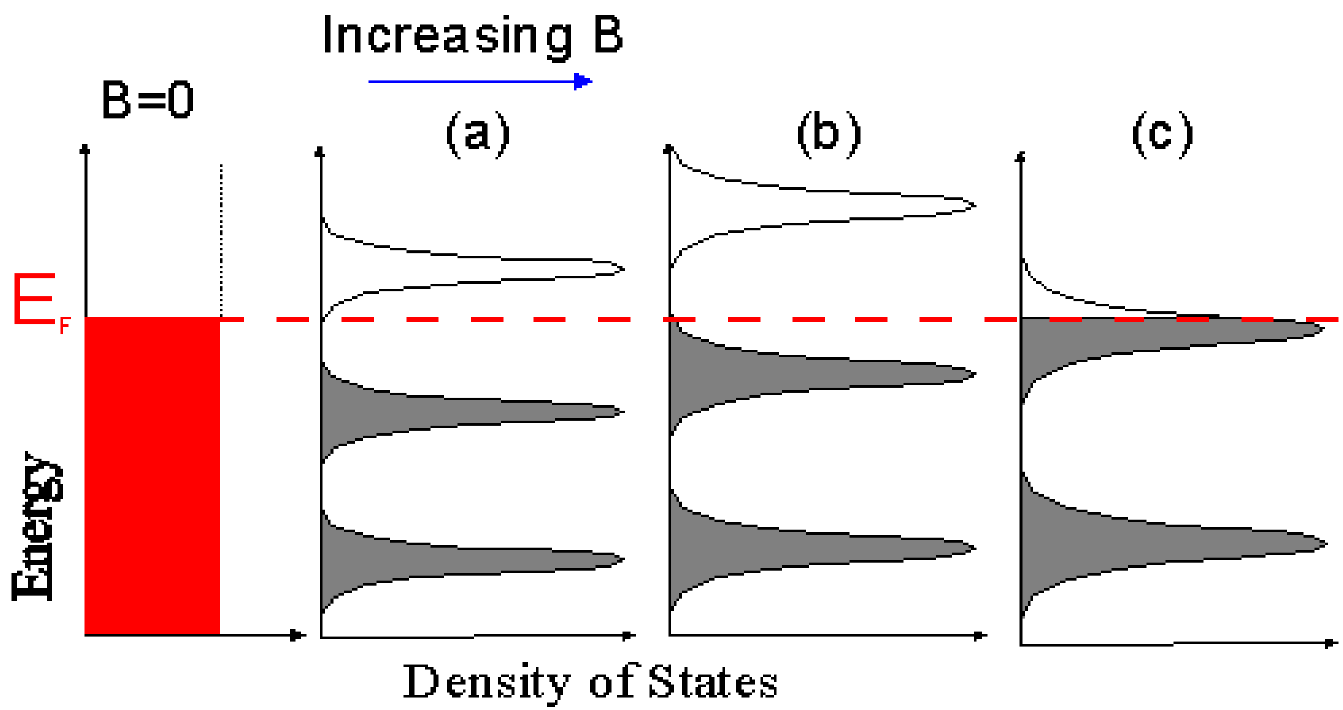
Integer quantum Hall effect (QHE)



Integer quantum Hall effect (QHE)



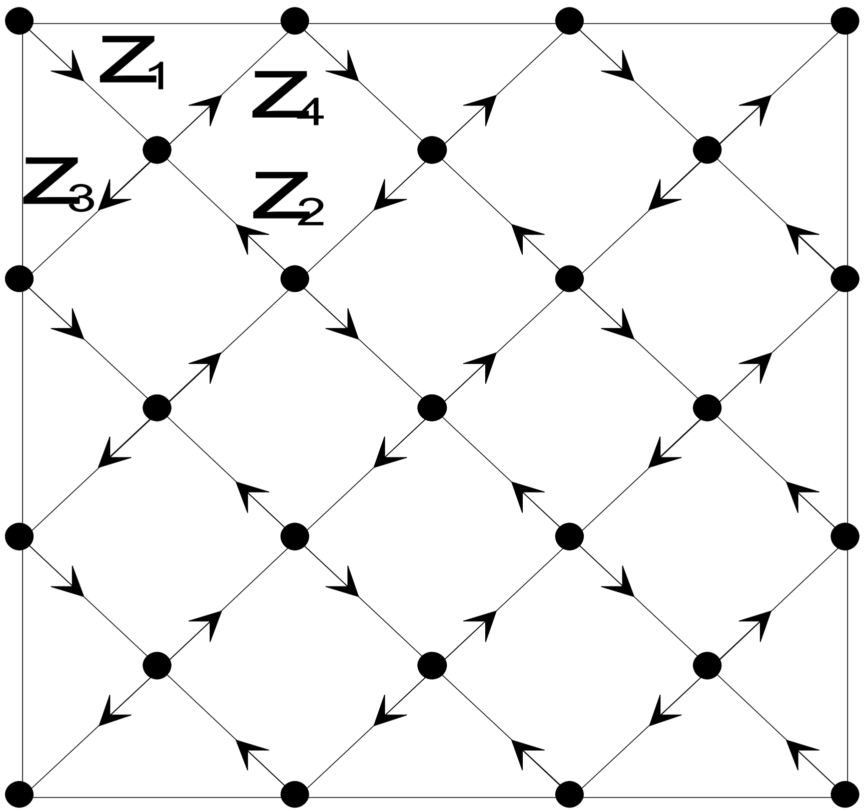
- No spin-flip scattering → two IQH transitions corresponding to Zeeman-split Landau level
- How does the picture change with spin-flip scattering?

Chalker-Coddington Model

- Electrons in strong magnetic field in smooth disorder potential
- Rapid cyclotron motion + slow chiral motion of guiding center along the quasiclassical trajectories
- Height of potential close to the Fermi-level – quantum tunneling
- Network model: chiral motion on links + quantum scattering on the nodes
- Predicts a single delocalized state at critical energy – transition between QH plateaus

Chalker-Coddington Model

Chiral network: random phases on links, quantum scattering at nodes



Scattering by nuclear spins in QHE

$$\hat{H} = \hat{H}_{IQHE}\mathbf{1_2} - \hbar g B \sigma^z / 2 + J \sum_I \vec{\sigma} \cdot \vec{\mathbf{I}}$$

I. Vagner, T. Maniv, Physica B (1995)
Y. Q. Li, J. H. Smet, in “Spin physics in semiconductors” (2008)

- Initially unpolarized nuclear spins
- Neglect Kondo correlations between electrons: $\max(T, E_z) > T_K$

- Single electron propagating through the network
- Spin-flip scattering by nuclear spins

Many-particle problem: attempt to reduce to a single particle one

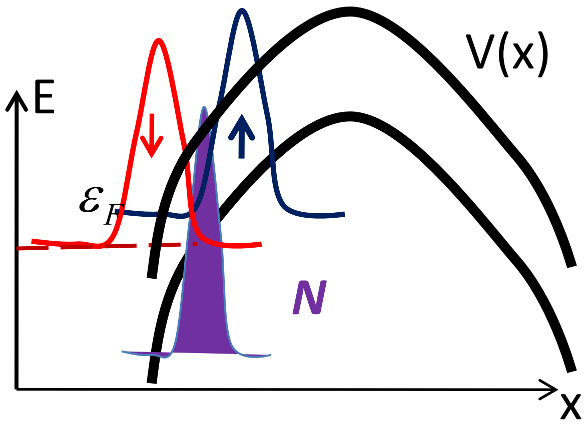
Scattering by nuclear spins in QHE

Spin-scattering is effective only close to the saddle-points of disorder potential

- Spin-scattering is approximately elastic (change of energy of the nucleus negligible)
- Finite scattering matrix element – finite overlap between the wave function of the spin-up state, spin-down state, and the nucleus

$$E_{\uparrow} = \hbar\Omega_c(n+\tfrac{1}{2}) - \hbar gB/2 + V(\mathbf{R}_{\uparrow})$$
$$E_{\downarrow} = \hbar\Omega_c(n+\tfrac{1}{2}) + \hbar gB/2 + V(\mathbf{R}_{\downarrow})$$

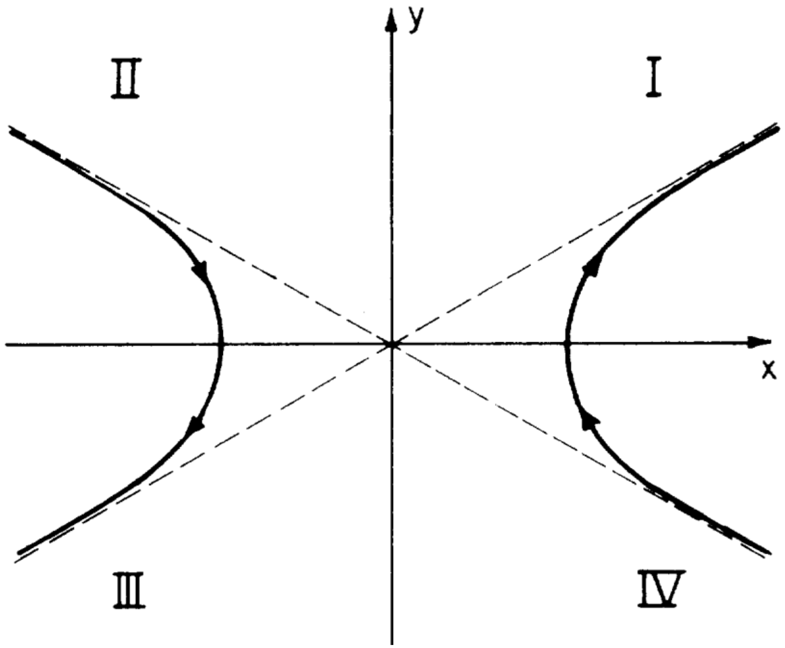
$$\left|\frac{dV}{dX}\right| \geq \frac{\hbar gB}{l_0} = \sqrt{\frac{\hbar e}{c}}\, gB^{3/2}$$



Scattering matrix at node: no spin-flip scattering

Nodes – saddle-points of potential by disorder

H. A. Fertig and B. I. Halperin, PRB (1987).



$$V_{SP}(x,y) = -U_x x^2 + U_y y^2 + V_0 \ .$$

$$T = \frac{1}{1 + \exp(-\pi\epsilon)}$$

Scattering matrix at a node with spin-flip scattering

- *Separation of fast cyclotron motion and slow center of mass motion*
- *Neglect transitions between different Landau levels by spin scattering*

Interaction with nuclear spin: $H = J\,\vec{\sigma}\cdot\vec{\mathbf{I}}$

Basis states at each node: $\left(\left|\uparrow_e\uparrow_N\right\rangle, \left|\uparrow_e\downarrow_N\right\rangle, \left|\downarrow_e\uparrow_N\right\rangle, \left|\downarrow_e\downarrow_N\right\rangle\right)$

States $\left|\uparrow_e\uparrow_N\right\rangle, \left|\downarrow_e\downarrow_N\right\rangle$ do not take part in spin-flip scattering

Spin-flip scattering of states $\left(\left|\uparrow_e\downarrow_N\right\rangle, \left|\downarrow_e\uparrow_N\right\rangle\right)$

Scattering matrix with spin scattering

Basis of singlet – triplet states

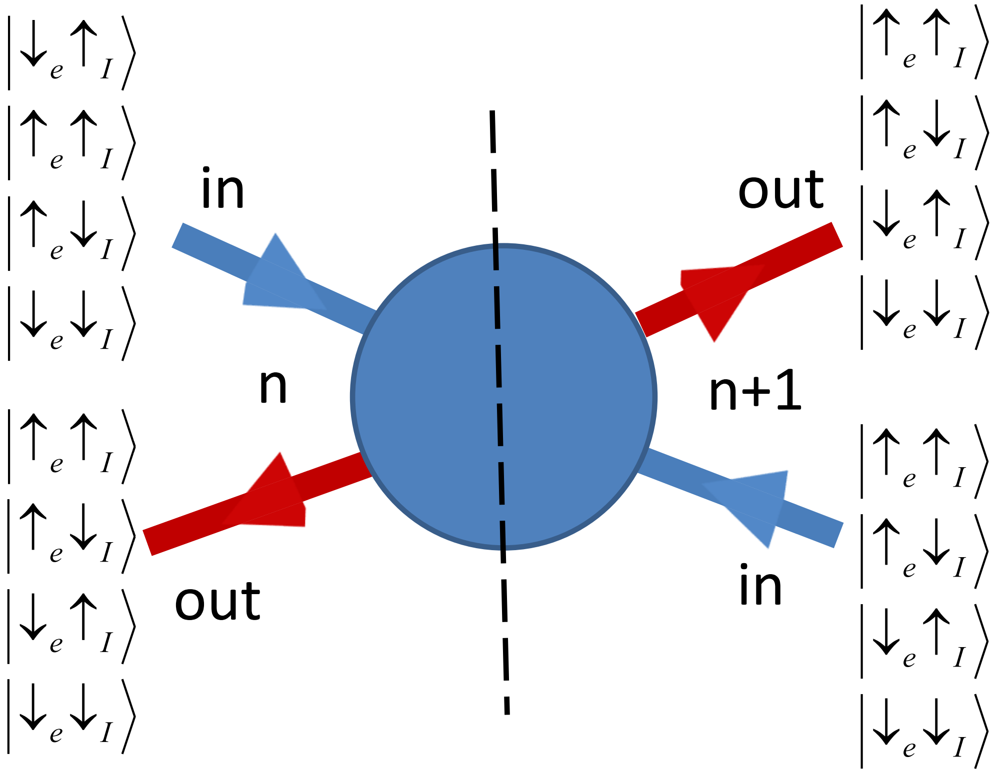
$$\begin{pmatrix} \left|\uparrow_e \downarrow_N\right\rangle \\ \left|\downarrow_e \uparrow_N\right\rangle \end{pmatrix} \to \begin{pmatrix} \Psi_{00} = \frac{1}{\sqrt{2}}\left(\left|\uparrow_e \downarrow_N\right\rangle - \left|\downarrow_e \uparrow_N\right\rangle\right) \\ \Psi_{10} = \frac{1}{\sqrt{2}}\left(\left|\uparrow_e \downarrow_N\right\rangle + \left|\downarrow_e \uparrow_N\right\rangle\right) \end{pmatrix}$$

Two-particle Schrödinger Equation

$$\left\{ \hat{H}_0 \mathbf{1_2} + J \begin{pmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} - \frac{B_0}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\} \Phi_n(X) = \mathrm{E}\Phi_n(X)$$

Exact solution: generalization of solution by Fertig and Halperin
on a two-particle wave function

Scattering matrix with spin-scattering



Exact solution in a single node

- Four eigenstates and eigenenergies

$$|\uparrow\uparrow\rangle, \quad \varepsilon_\uparrow, \quad t(\varepsilon_\uparrow)$$

$$|\downarrow\downarrow\rangle, \quad \varepsilon_\downarrow, \quad t(\varepsilon_\downarrow)$$

$$\varphi_1 = c_2|\uparrow\downarrow\rangle + c_1|\downarrow\uparrow\rangle, \quad \varepsilon_{1,} \quad t(\varepsilon_1)$$

$$\varphi_2 = c_1|\uparrow\downarrow\rangle - c_2|\downarrow\uparrow\rangle, \quad \varepsilon_2 \quad t(\varepsilon_2).$$

$$t(\varepsilon_i) = \frac{1}{\sqrt{1+\exp(-\pi\varepsilon_i)}}$$

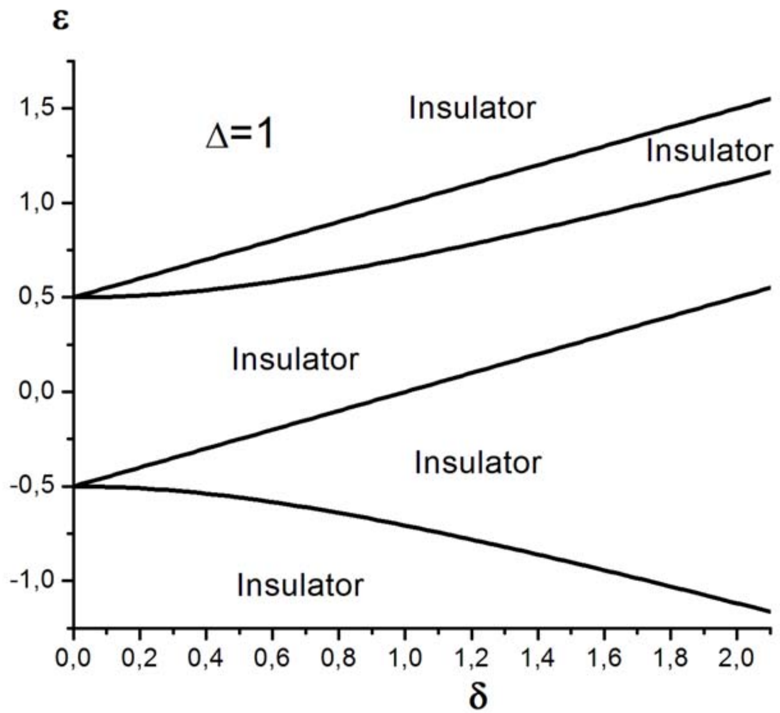
$$|\text{out}\rangle = \hat{S}|\text{in}\rangle$$

Transfer matrix

$$|n+1\rangle = \hat{T}_{n+1,n}|n\rangle$$

Critical energies

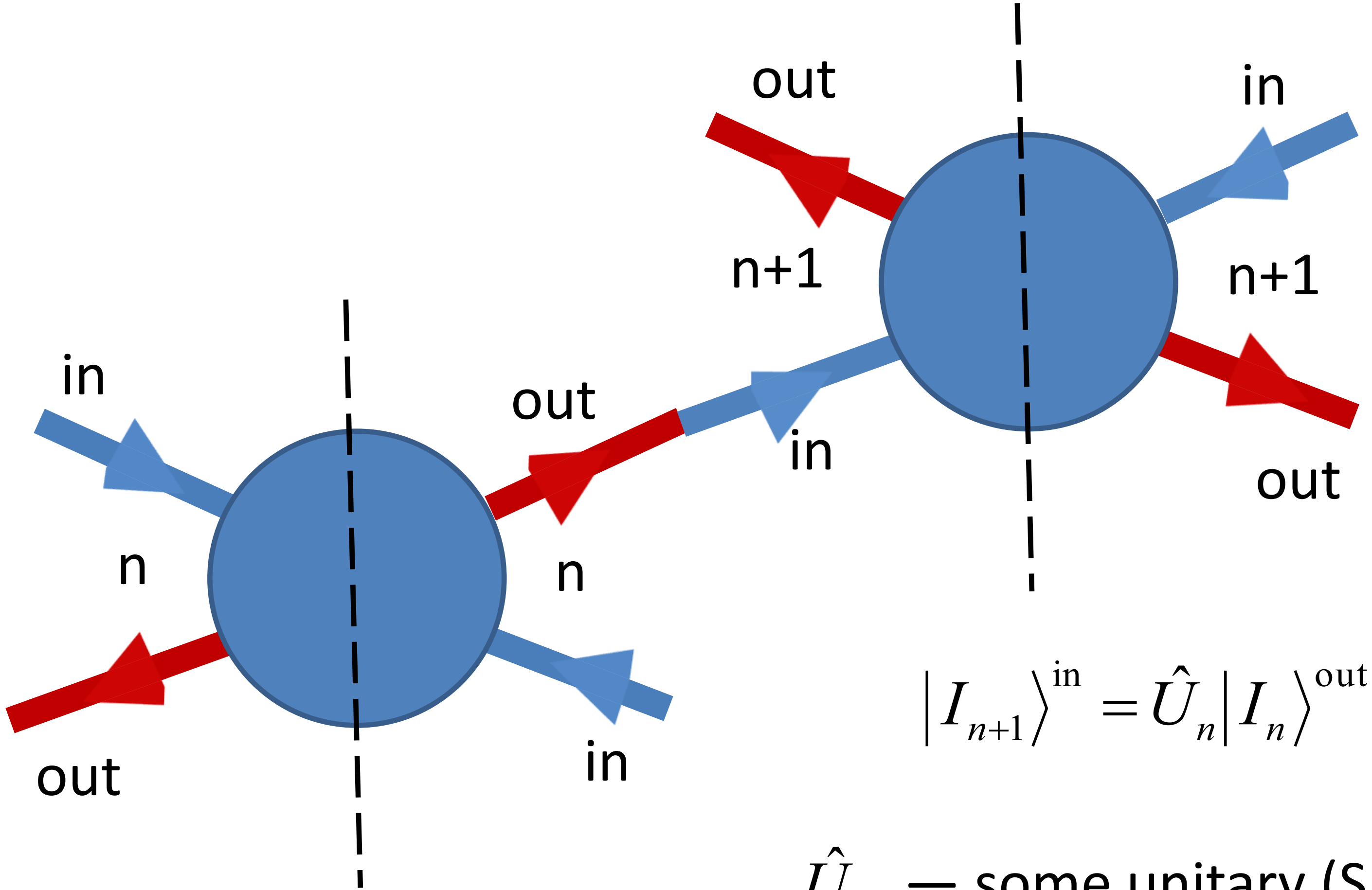
$$\left\{\hat{H}_0 \mathbf{1_2} + J\begin{pmatrix} -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix} - \frac{B_0}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\}\Phi_n(X) = \mathrm{E}\Phi_n(X)$$



Δ - dimensionless Zeeman energy

δ - dimensionless exchange coupling

Scattering matrix: many nodes



$$\left| I_{n+1} \right\rangle^{\text{in}} = \hat{U}_n \left| I_n \right\rangle^{\text{out}}$$

$\hat{U}_n$ — some unitary (SU(2)) matrix

Many-particle – Two-particle

- Many-particle wave function $\Psi(\mathbf{r},\mathbf{R_I})$
- Many-particle basis $\psi_e^{\sigma}(\mathbf{r})\prod_I \varsigma_I^{\alpha_I}$
- Spatial localization depends on $\psi_e^{\sigma}(\mathbf{r})$
- There is a unitary matrix $\hat{U}_n \in SU(2)$: $\left|\varsigma_{n+1}\right\rangle^{\text{in}} = \hat{U}_n \left|\varsigma_n\right\rangle^{\text{out}}$.
- $\hat{U}_n$ depends on the states of nuclei
- For non-polarized nuclei the matrix $\hat{U}_n$ can be chosen at random

Many-particle – Two-particle

Evolution of the wave function by propagation through a given path in the network:

$$\left\langle \psi'(\mathbf{r}_N)\prod_{\text{nodes}\, i}^{N}\varsigma'_i \middle| \psi(\mathbf{r}_1)\prod_{\text{nodes}\, i}^{N}\varsigma_i \right\rangle = \langle \psi'(\mathbf{r}_N)\varsigma'_N|\hat{\mathbf{T}}_N|\psi(\mathbf{r}_N)\varsigma_N\rangle\langle\psi(\mathbf{r}_N)|(\mathbf{e}^{\mathbf{i}\boldsymbol{\varphi}})_{N,N-1}|\psi'(\mathbf{r}_{N-1})\rangle\ldots$$
$$\langle\psi(\mathbf{r}_3)|(\mathbf{e}^{\mathbf{i}\boldsymbol{\varphi}})_{3,2}|\psi'(\mathbf{r}_2)\rangle\langle\psi'(\mathbf{r}_2)\varsigma'_2|\hat{\mathbf{T}}_{N-1}|\psi(\mathbf{r}_2)\varsigma_2\rangle\langle\psi(\mathbf{r}_2)|(\mathbf{e}^{\mathbf{i}\boldsymbol{\varphi}})_{2,1}|\psi'(\mathbf{r}_1)\rangle\langle\psi'(\mathbf{r}_1)\varsigma'_1|\hat{\mathbf{T}}_1|\psi(\mathbf{r}_1)\varsigma_1\rangle.$$

Use $\left|\varsigma_{n+1}\right\rangle^{\text{in}} = \hat{U}_n\left|\varsigma_n\right\rangle^{\text{out}}$ to replace multiple nuclear spins by an effective single nuclear spin propagating through the network.

$$\langle\psi'(\mathbf{r}_N)\varsigma'_N|\psi(\mathbf{r}_1)\varsigma_1\rangle = \langle\psi'(\mathbf{r}_N)\varsigma'_N|\hat{\mathbf{T}}_N|\psi(\mathbf{r}_N)\varsigma_N\rangle\langle\psi(\mathbf{r}_N)\varsigma_N|\hat{U}_{N-1}(\mathbf{e}^{\mathbf{i}\boldsymbol{\varphi}})_{N,N-1}|\psi'(\mathbf{r}_{N-1})\varsigma'_{N-1}\rangle$$
$$\langle\psi'(\mathbf{r}_{N-1})\varsigma'_{N-1}|\hat{\mathbf{T}}_{N-1}|\psi(\mathbf{r}_{N-1})\varsigma_{N-1}\rangle\ldots\langle\psi(\mathbf{r}_2)\varsigma_2|\hat{U}_1(\mathbf{e}^{\mathbf{i}\boldsymbol{\varphi}})_{2,1}|\psi'(\mathbf{r}_1)\varsigma'_1\rangle\langle\psi'(\mathbf{r}_1)\varsigma'_1|\hat{\mathbf{T}}_1|\psi(\mathbf{r}_1)\varsigma_1\rangle.$$

Effective model: Propagation of a bi-spinor

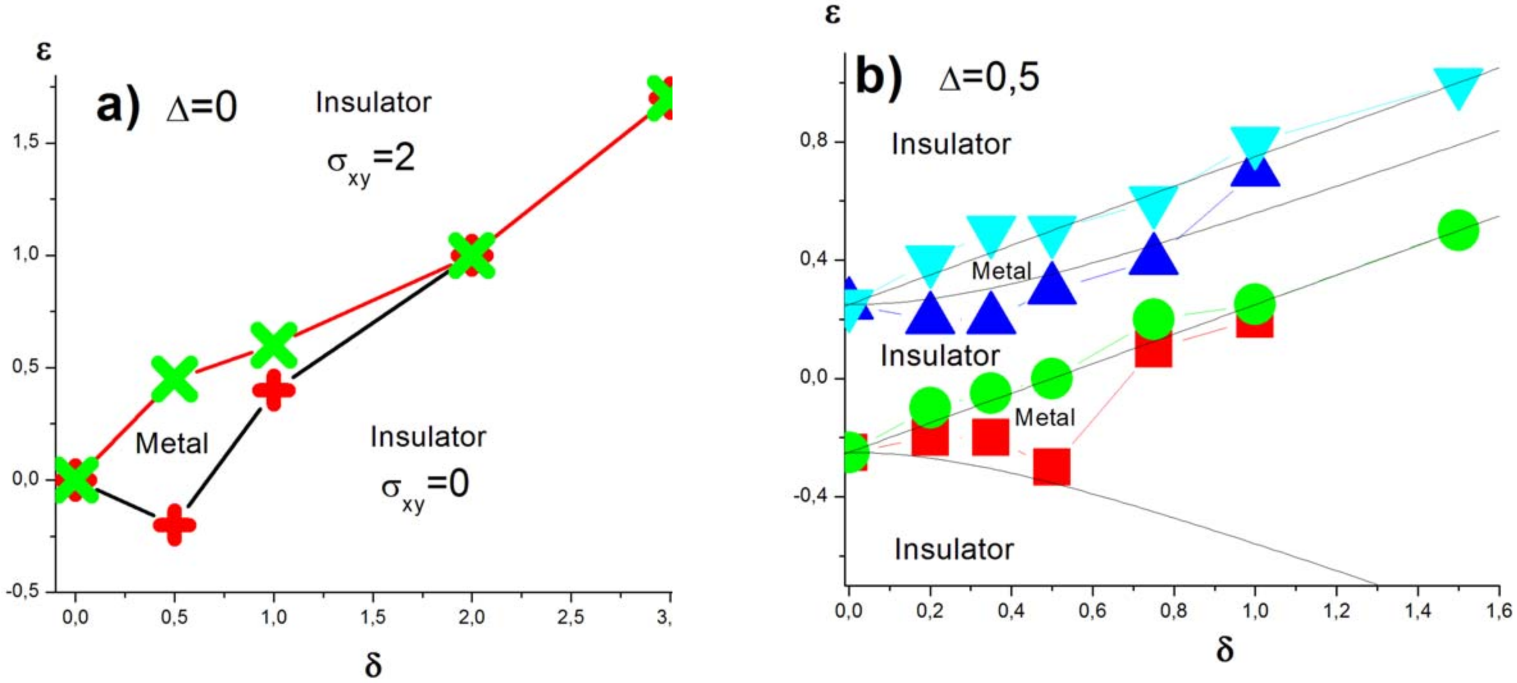
- There is a bi-spinor $|\sigma_e, I_N\rangle$ that propagates through the chiral network
- There is a random matrix $\hat{U}$ acting on a nuclear spin at each link
- Simplification: choose $\hat{U} = \hat{\sigma}^x \text{ or } \mathbf{1_2}$ with probability 1/2.

- There is a spin-dependent scattering at each node.

Result of numerical simulation:

Finite energy regions of delocalized states around critical energies

Delocalized states around quantum Hall transition



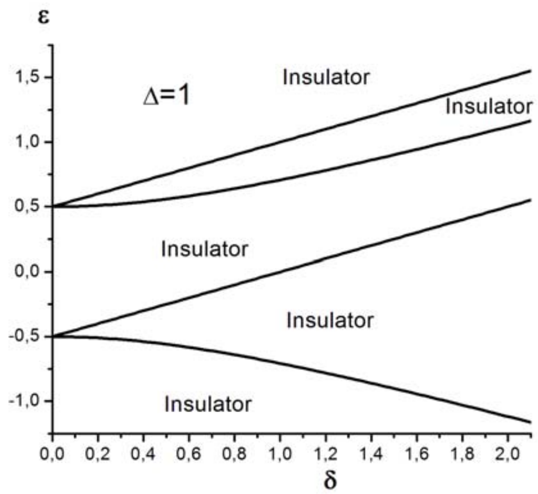
Delocalization by spin-flips on links

$$\left|\uparrow\uparrow\right\rangle,\quad \varepsilon_{\uparrow},\quad t(\varepsilon_{\uparrow})$$

$$\left|\downarrow\downarrow\right\rangle,\quad \varepsilon_{\downarrow},\quad t(\varepsilon_{\downarrow})$$

$$\varphi_1 = c_2\left|\uparrow\downarrow\right\rangle + c_1\left|\downarrow\uparrow\right\rangle,\quad \varepsilon_{1,}\quad t(\varepsilon_1)$$

$$\varphi_2 = c_1\left|\uparrow\downarrow\right\rangle - c_2\left|\downarrow\uparrow\right\rangle,\quad \varepsilon_2\quad t(\varepsilon_2).$$



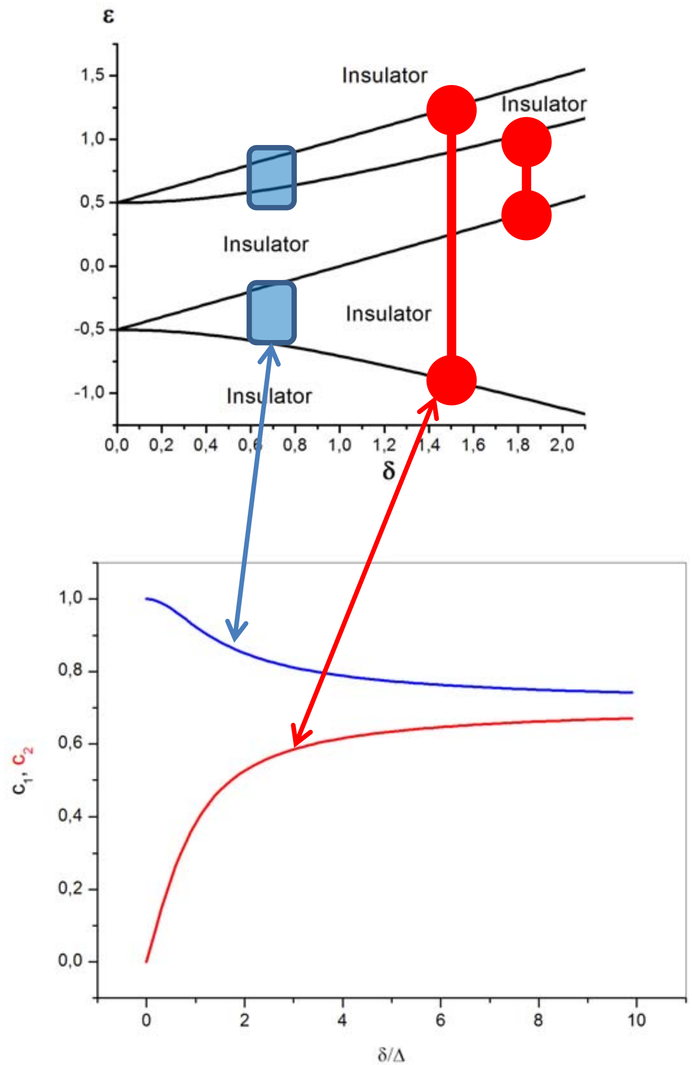
- No spin-flips on links
  - transition and reflection matrices are diagonal

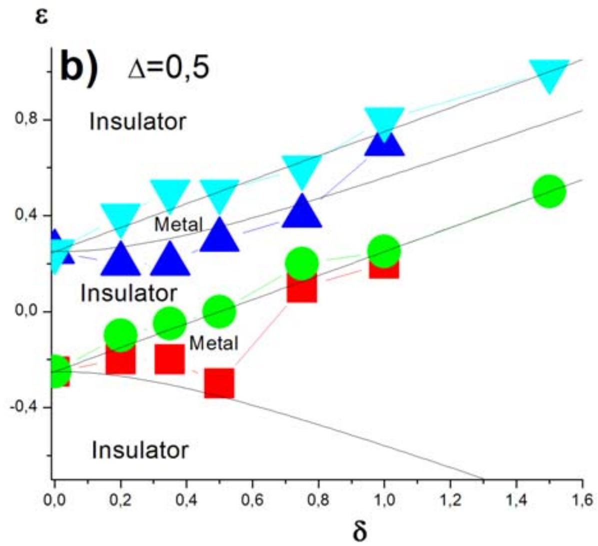
$$\hat{t} = \begin{pmatrix} t_{\uparrow\uparrow} & 0 & 0 & 0 \\ 0 & t_{\downarrow\downarrow} & 0 & 0 \\ 0 & 0 & t_1 & 0 \\ 0 & 0 & 0 & t_2 \end{pmatrix} \qquad t_i = t(\varepsilon_i)$$

- Spin flip on link
  - the states are mixed by the matrix

$$\Lambda = \begin{pmatrix} 0 & 0 & c_2 & c_1 \\ 0 & 0 & c_1 & -c_2 \\ c_2 & c_1 & 0 & 0 \\ c_1 & -c_2 & 0 & 0 \end{pmatrix}$$

Change of mixing of states with δ





- Small δ : Only the closest in energy states are mixed
- Large δ : All states are mixed

1D Version: analogy to D-class

• Suppose no spin-flip on links

• Reduce the Hilbert space $\left(\uparrow\uparrow,\uparrow\downarrow\right)^T$

• No mixing of state on links → U(1) model, two critical energies

• Matrix $\hat{\sigma}^x$ at each link (maximal mixing of states) → transfer matrix with an eigenvalue 1, all states delocalized
(analogously to class D: *J. Chalker et al. PRB 1997*).

Conclusions

- Chiral network model for a bi-spinor with spin-flip scattering at nodes and random flip of nuclear spin on links
- There are finite energy regions of delocalized (metallic) states – change of the QH phase diagram
- Model can be relevant to a number of physical situations
  - QHE with scattering by nuclear spins and magnetic impurities in semiconductors
  - QHE in mono-domain ferromagnets
  - Mixtures of cold atom gases