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Workshop on Localization Phenomena in Novel Phases of Condensed Matter

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Dark Solitons: A Model of Impurity-Phonon Interactions in Quantum Liquids

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Dark solitons: a model of impurity-phonon interactions in quantum liquids

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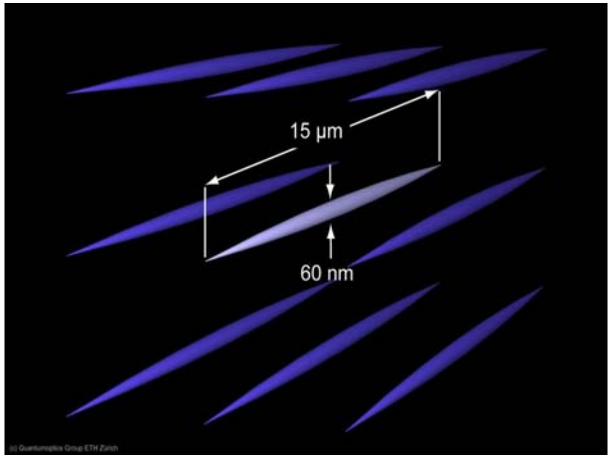
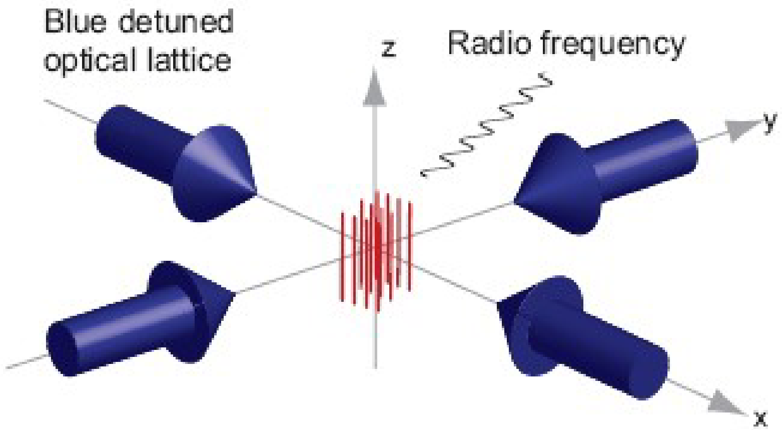
Phys. Rev. Lett. 2009, 2010

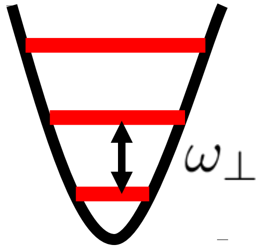
EPSRC

Experiments in 1D





Tight transverse confinement



1 dimensional regime

$$\hbar\omega_\perp \gg \mu, T$$

1D and quasi 1D

Exact Analysis of an Interacting Bose Gas. I. The General Solution and the Ground State

Elliott H. Lieb and Werner Liniger

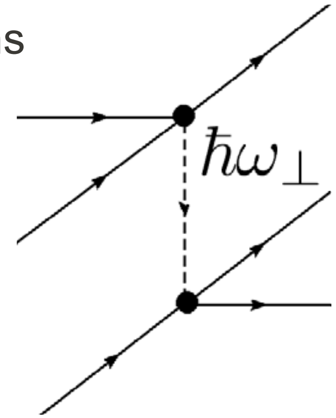
$$\hat{H} = \int dx \, \frac{\hbar^2}{2m} \partial_x \hat{\psi}^\dagger \partial_x \hat{\psi} + \frac{g}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}$$

$$g = 2\omega_\perp a > 0$$

Olshanii '99

Virtual transitions to higher transverse states – 3-body interactions

Muryshev et al. 2002   Mazets, Schumm, Schmiedmayer 2008



$$\hat{H}_3 = \frac{\alpha}{6} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \hat{\psi} \qquad \alpha = -\left(6 \ln \frac{4}{3}\right) \frac{g^2}{\omega_\perp}$$

Mean Field, Semiclassics

Weakly interacting bosons: small interactions g , high density n

$$\gamma = \frac{mg}{\hbar^2 n} << 1 \qquad K = \frac{\pi}{\sqrt{\gamma}} = \frac{\pi n}{mc} >> 1$$

Semiclassical Gross-Pitaevskii description for fields $\psi^\dagger(x,t), \psi(x,t)$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\partial_x^2\psi - \mu\psi + (g|\psi|^2 + \alpha|\psi|^4)\psi$$

Chemical potential $\mu = gn + \alpha n^2/2$

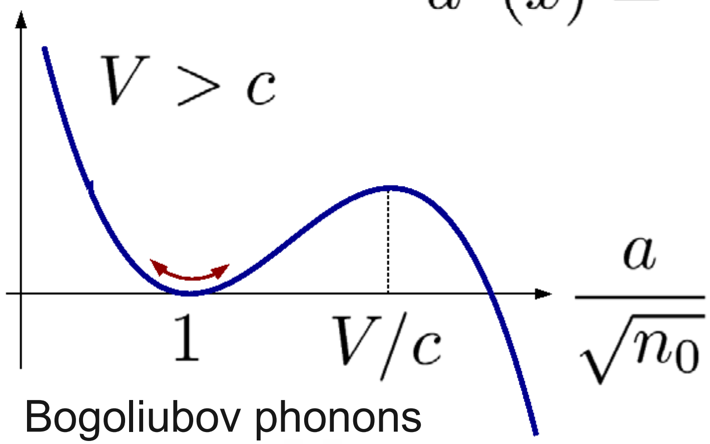
Excitations: phonons, solitons

$$\psi(x,t) = \psi(x - Vt) = a(x - Vt)e^{i\phi(x-Vt)}$$

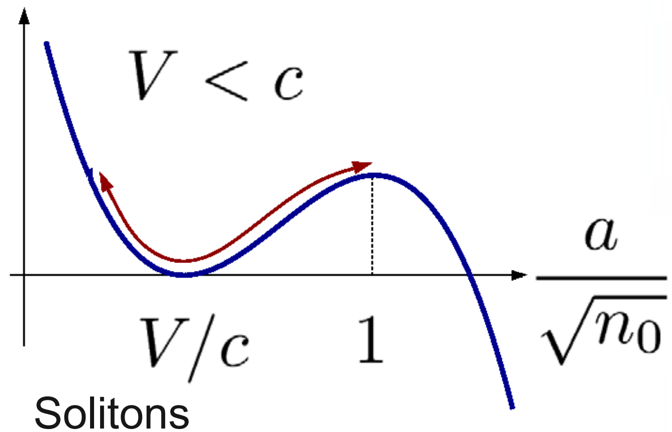
density, velocity at ∞

Continuity $\quad n(x)(\partial_x\phi(x) - mV) = mn_0(u_0 - V)$

$$a''(x) = -\frac{dV_{\text{eff}}(a)}{da}$$

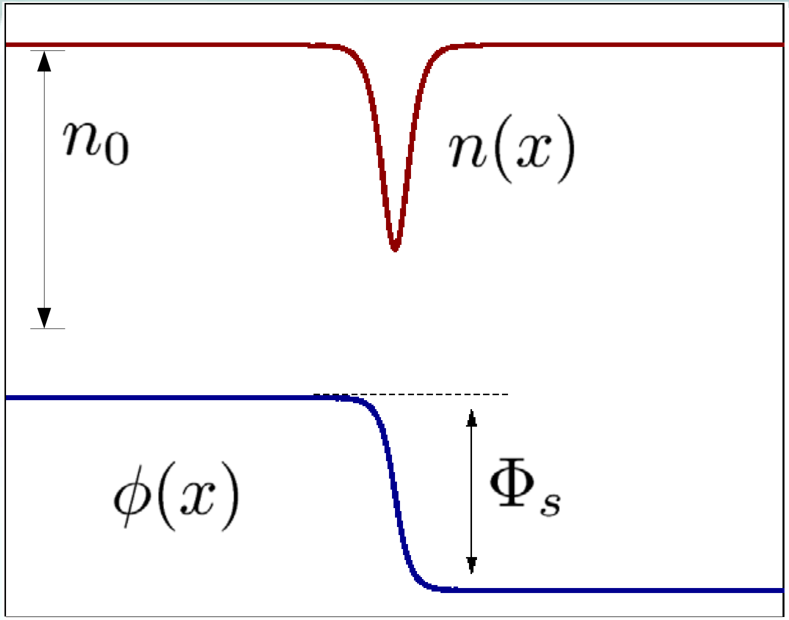


Bogoliubov phonons



Solitons

Dark Soliton



Phase drop Φ_s → Velocity $V_s(\Phi_s)$

Number of expelled particles

$$N_s(\Phi_s) = \int (n_0 - n(x))dx$$

Momentum $P_s(\Phi_s) = n_0\Phi_s - mN_sV_s$

Energy $E_s(\Phi_s)$

$$V_s \frac{\partial P_s}{\partial \Phi_s} = \frac{\partial E_s}{\partial \Phi_s}$$

$$V_s\left(\frac{\partial P_s}{\partial n_0} - \Phi_s\right) = \frac{\partial E_s}{\partial n_0} - mc^2 N_s$$

Pelinovsky, Kivshar, Afanasjev '96

Soliton in integrable GP $\alpha = 0$

$$\frac{V}{c} = \cos\frac{\Phi_s}{2} \qquad N_s = \frac{2n_0}{mc}\sin\frac{\Phi_s}{2} >> 1$$

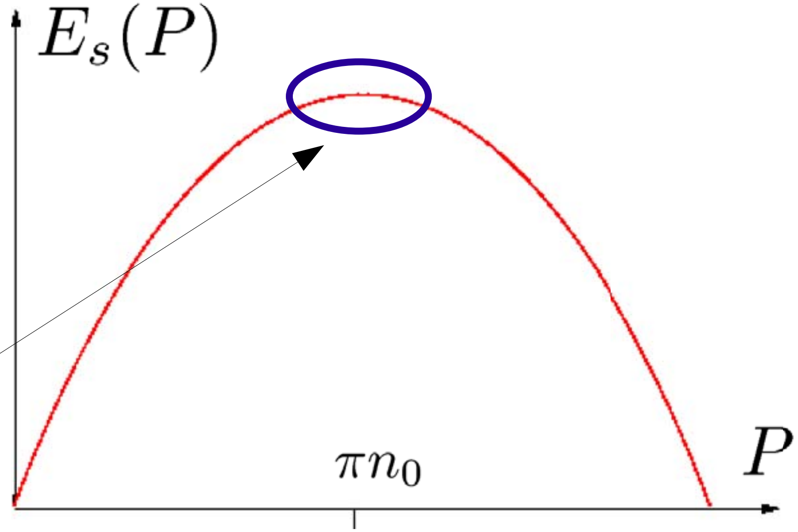
Momentum $P_s = n_0(\Phi_s - \sin\Phi_s)$

Energy

$$E_s = \frac{4}{3}cn_0\left|\sin\frac{\Phi_s}{2}\right|^3 \gg \mu$$

Effective mass

$$M_s = -2mN_s$$



Quantum Theory - Bethe Ansatz

$$\Psi(x_1, x_2, \ldots, x_N) \sim \sum a(P) e^{i k_{P_i} x_i}$$

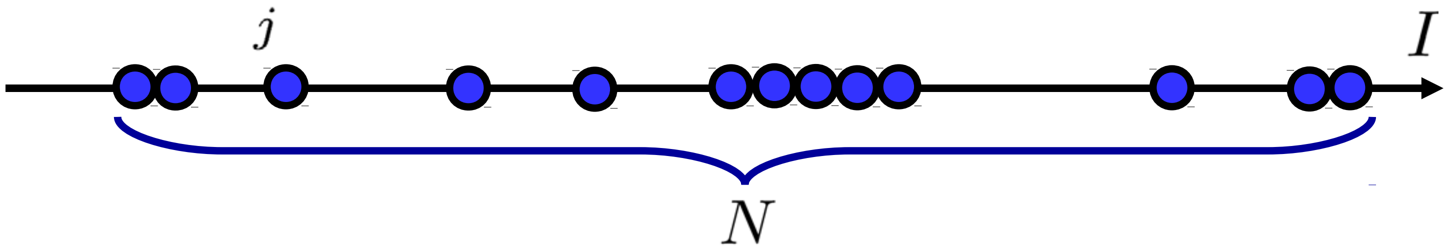
partial waves

$$\sim \prod_{l<j} e^{-i\theta(k_l - k_j)}$$

phase shift

$$L k_i + 2 \sum_j \theta(k_i - k_j) = 2\pi I_i$$

quantum numbers (integers)



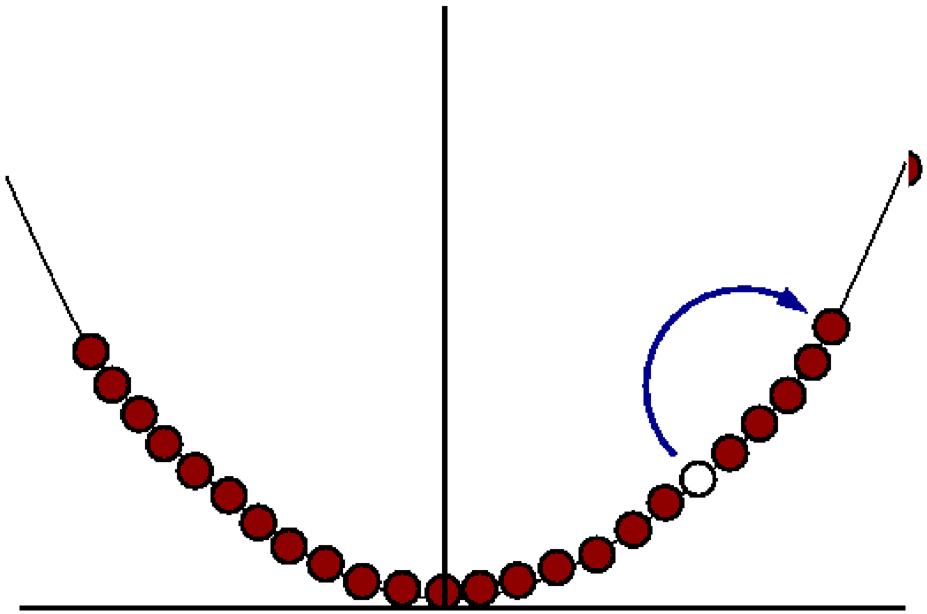
$$E = \sum k_j^2 / 2m \qquad P = \sum k_j$$

Excitations

Lieb '63

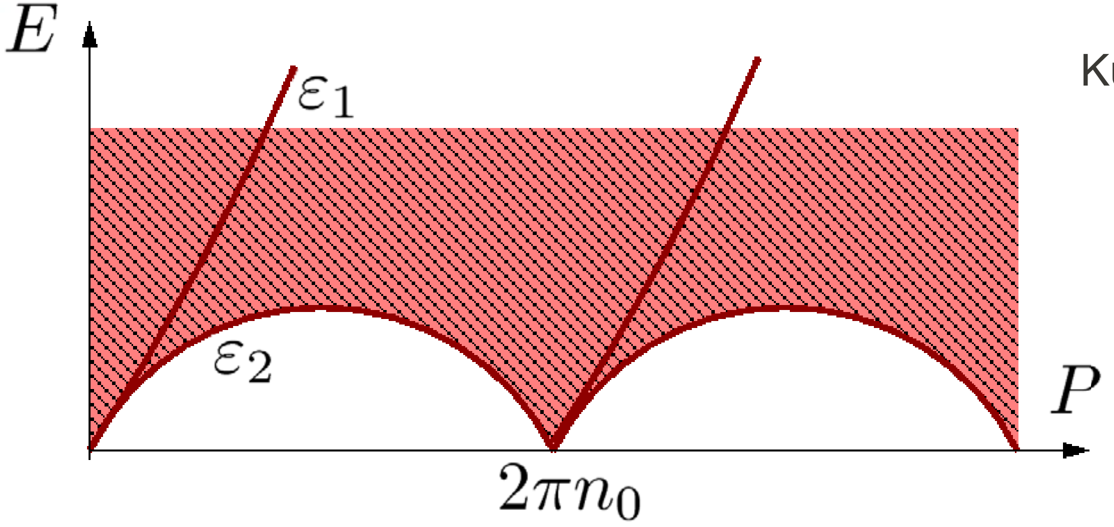
Ground state

Lieb I (“particles”) $\varepsilon_1(P)$



Lieb II (“holes”) $\varepsilon_2(P)$

Quantum phonons and solitons



Kulish, Manakov, Faddeev 1976

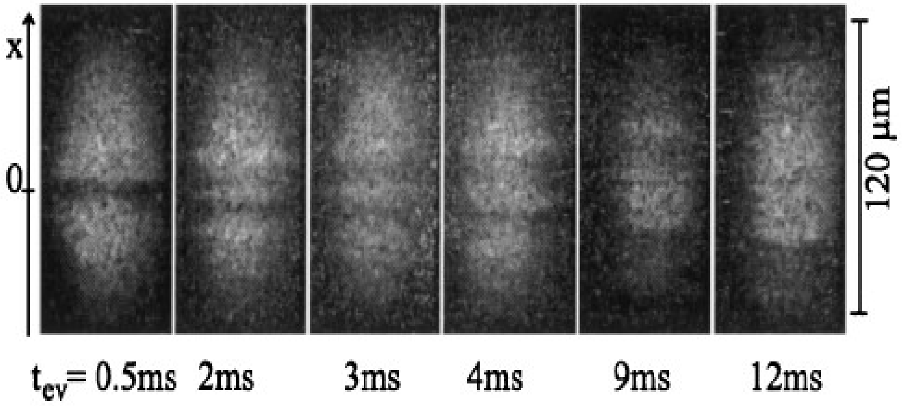
$$\varepsilon_2(p) \to E_s(p)$$

as $\gamma \to 0 \quad K \to \infty$

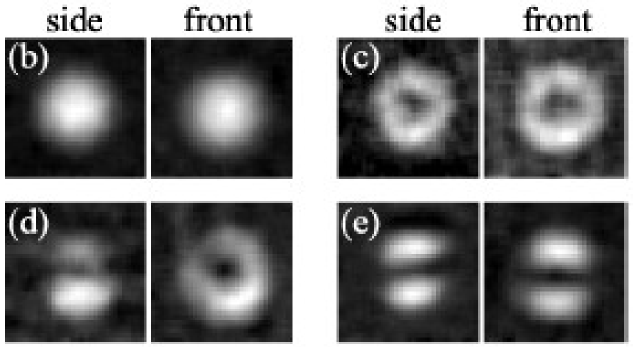
lower bound – ground state for given momentum

- What about non-integrable case?

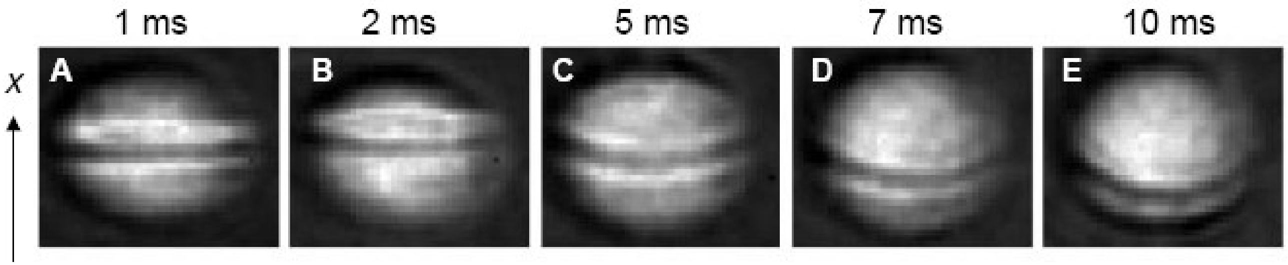
Observation of dark solitons in Cold Atomic Gases



Sengstock, et al. 1999



Cornell, et al. 2001

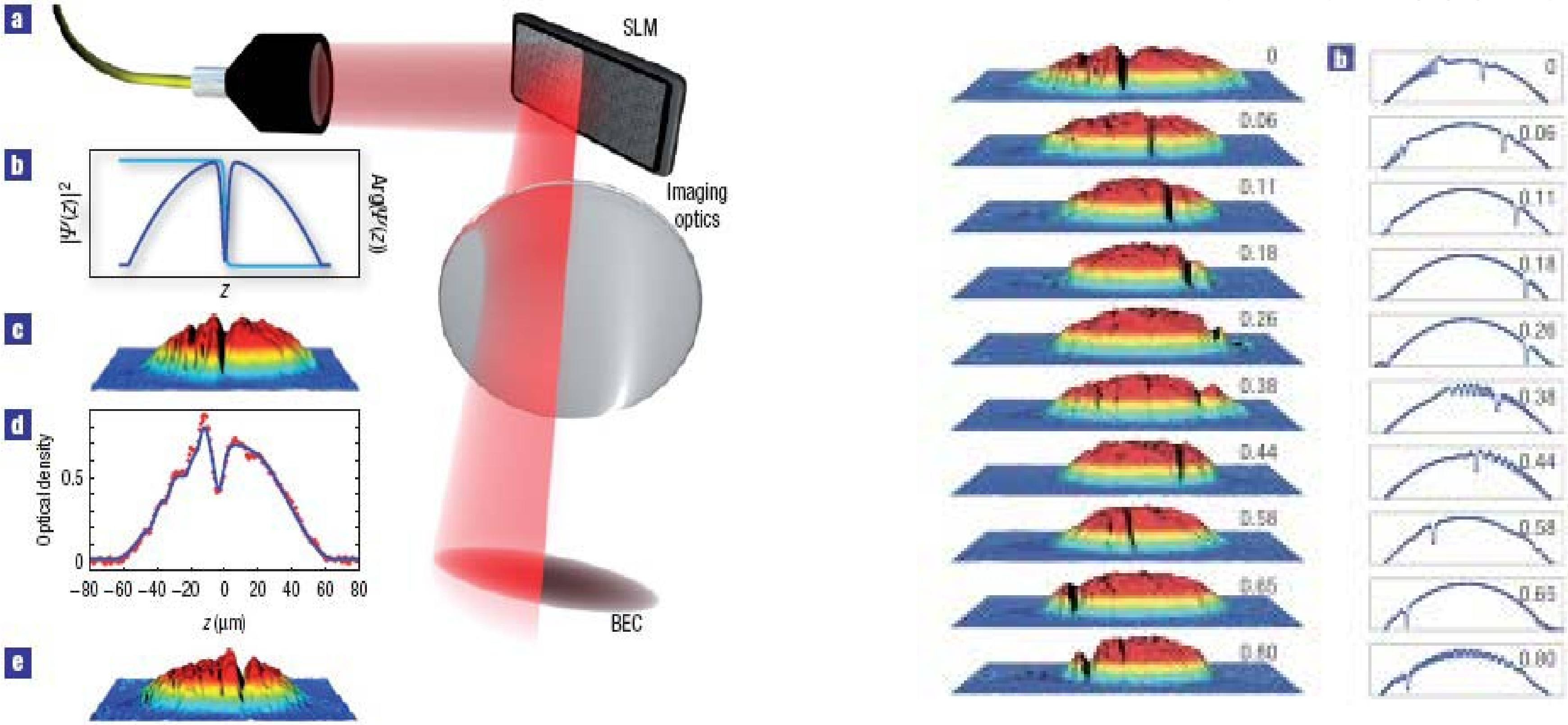


Phillips, et al. 2000

Phase imprinting

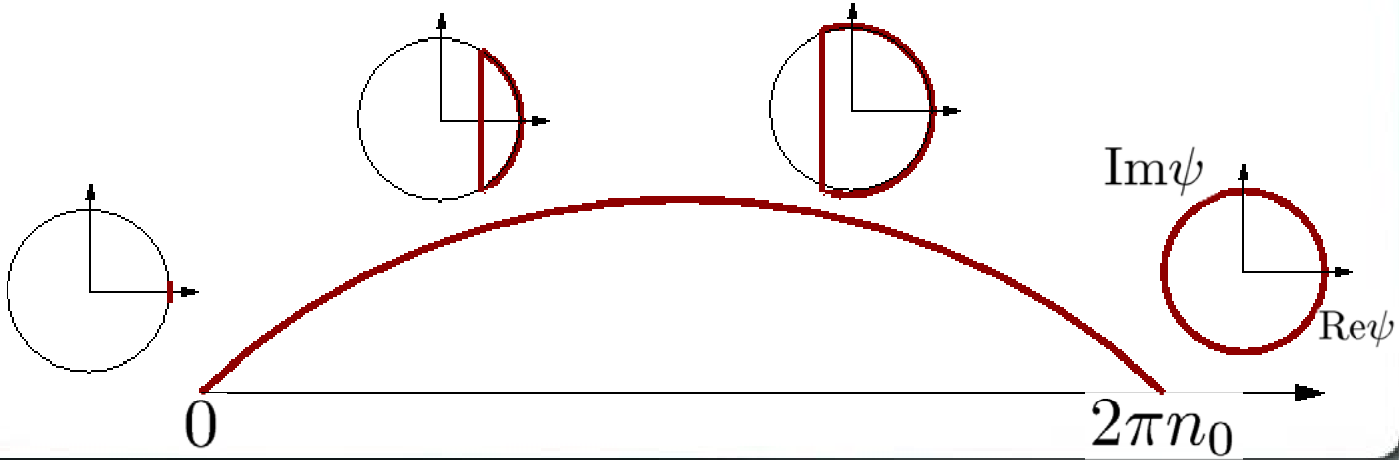
C. Becker *et al.*, Nature Physics **4**, 496 (2008)

Lifetime ~ 300ms



Dissipative dynamics of DS

- Are dark solitons stable at finite T?
- What is the mechanism of DS decay?
- Integrable vs. Non-integrable
- Supercurrent relaxation via activation of DS:



Solitons and phonons: kinematics

One phonon processes

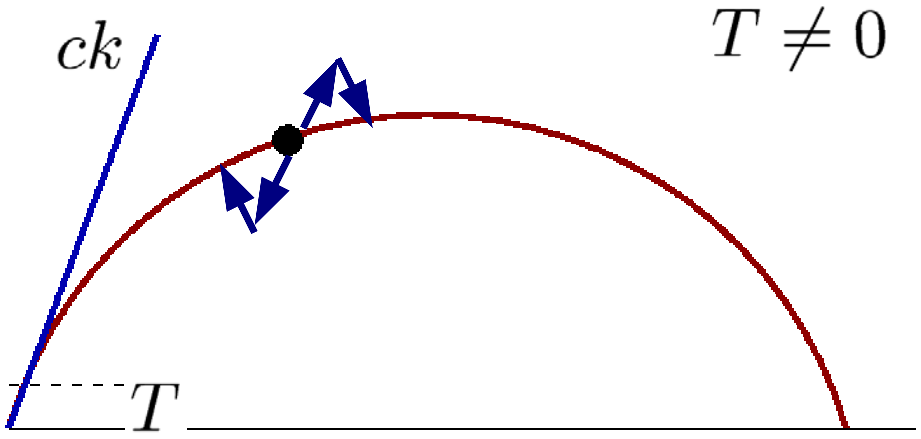
$$\Delta E \sim c\Delta P - V(P)\Delta P$$

$$V(P) = P/M^* \ll c$$

- Landau criterion

$T = 0$ no dissipation

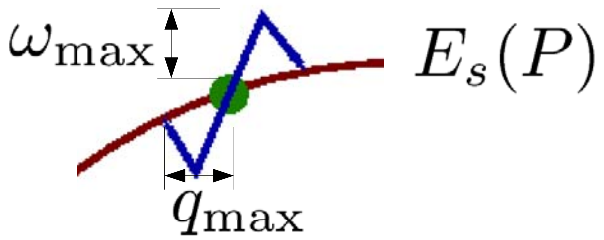
Two phonon processes (Raman scattering)



$$\dot{P} = -\kappa V$$ + noise

Momentum transfer rate

$$\omega_{\max} = c q_{\max} \sim T \qquad \ll E_s$$



$$\kappa \sim \int_0^T dq\; q\; q\; q^{2d-1} \sim T^{2d+2}$$

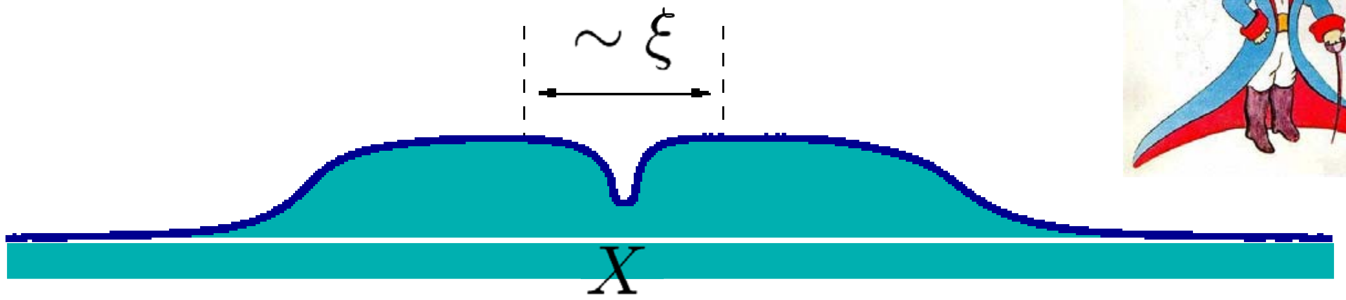
In 3 dimensions $\kappa \sim T^8$ — phase space — Landau & Khalatnikov '49 He^3 - He^4

In 1 dimension $\kappa \sim (\,?\,)\, \dfrac{T^4}{c^4 \hbar^3 n^2}$ — Castro-Neto & Fisher '96

depends crucially on amplitude of soliton – phonon interactions

Soliton – phonon coupling

Soliton is a tiny object $\sim \xi = 1/mc$ localised at X



Coupling to density: local change of the background

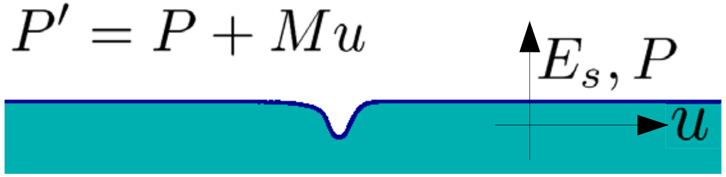
$$E_s(P, n_0 + \rho(X,t)) = E_s(P, n_0) + \frac{\partial E_s}{\partial n}\rho + \frac{1}{2}\frac{\partial^2 E_s}{\partial n^2}\rho^2$$

Galilean invariance

Baym & Ebner '67

In the co-moving frame: $E_s(P, n)$

$E'_s, P' = P + Mu$



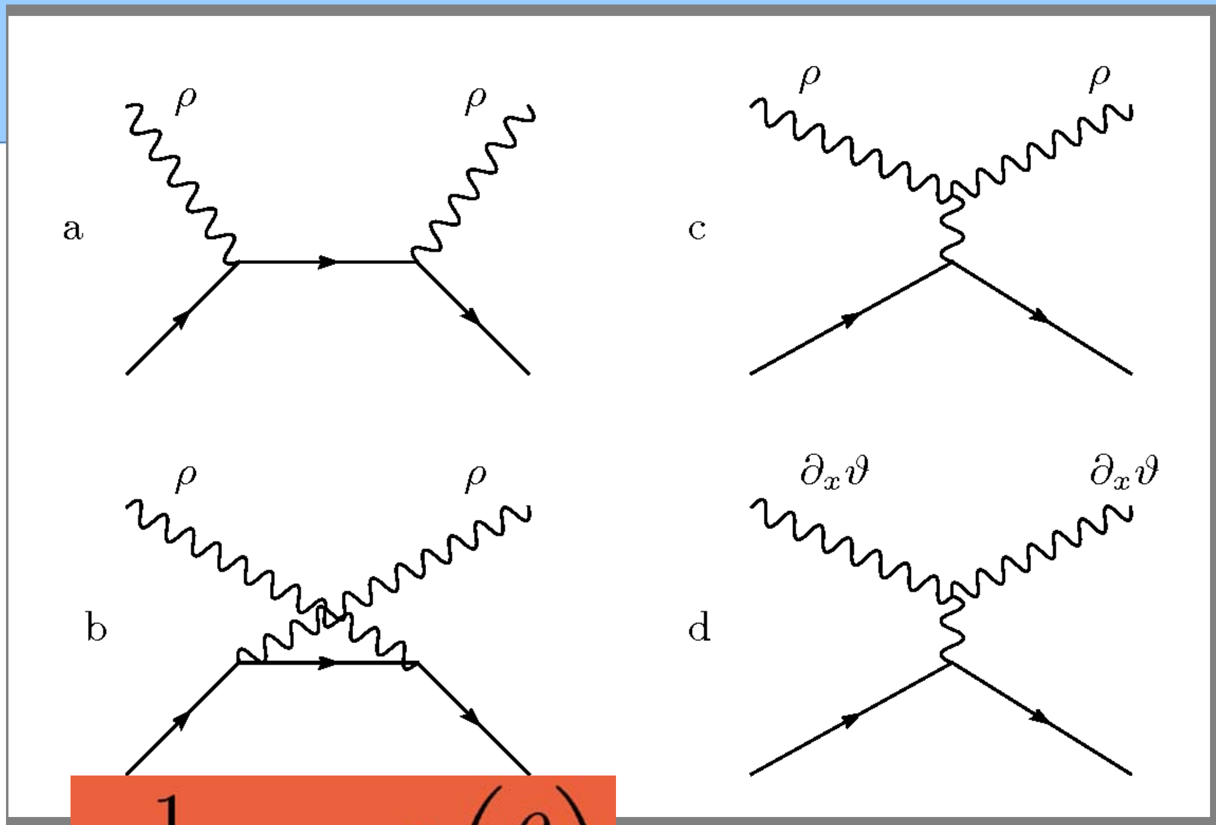
Galilean transformation:

$$E'_s = E_s(P, n) + Pu + \frac{Mu^2}{2} =$$
$$= E_s(P' - Mu, n) + P'u - \frac{Mu^2}{2}$$

Coupling to local (super) velocity field $u = \frac{\hbar}{m}\partial_x \phi \to u(X, t)$

NB: For DS the bare mass M is zero

Coupling with phonons:

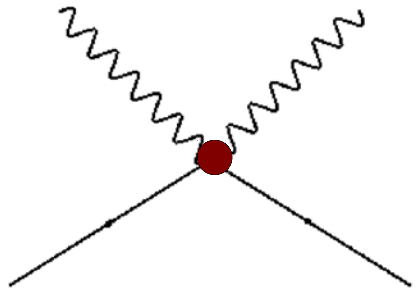


$$E_s(P, n+\rho,$$

Linear coupling

Quadratic coupling

$$+\frac{1}{2}(\rho, u)\hat{\Gamma}\begin{pmatrix}\rho \\ u\end{pmatrix}$$



Can be eliminated in favour of

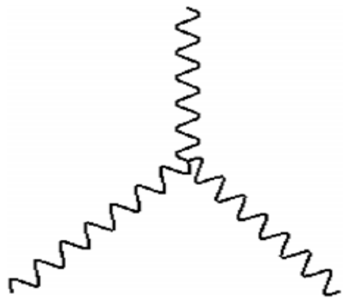
Phonons

Phase-density representation for slow $\lambda \gg \xi$ fields

$$\psi(x,t) = \sqrt{n + \rho(x,t)}\, e^{i\phi(x,t)}$$

$$L_{\text{ph}} = \int dx \left(-\rho\partial_t\phi - \frac{mc^2}{2n}\rho^2 - \frac{n_0+\rho}{2m}(\partial_x\phi)^2 \right)$$

nonlinear term



Gauge transformation

$$\rho(x,t) \to \rho(x,t) - R(t)\,\delta(x-X)$$

$$\phi(x,t) \to \phi(x,t) - F(t)\,\theta(x-X)$$

$$P \to P + mRu(X,t) + F\rho(X,t)$$

Use soliton identities + anharmonic terms of phonons

$$R = N_s, \; F = \Phi_s$$

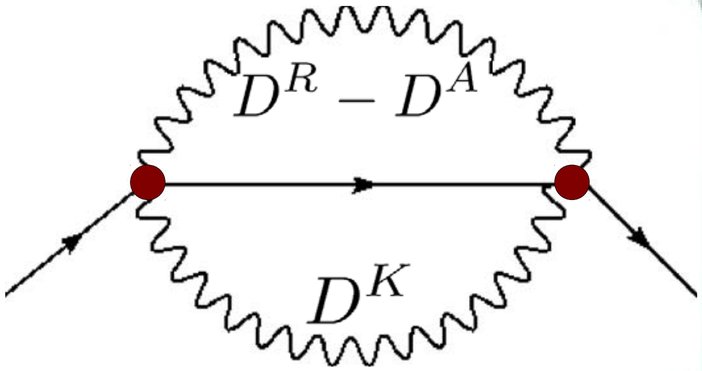
eliminate linear couplings:

$$L_{\mathrm{s-ph}} = \frac{1}{2}\Gamma_\rho\big[\rho(X)\big]^2 + \frac{1}{2}\Gamma_u\big[u(X)\big]^2$$

$$+\dot{N}_s\phi + \dot{\Phi}_s\theta$$

Integrating out phonons

$$\dot{P} = -\kappa V$$



$$\kappa = \mathrm{Tr}\left[\Gamma\left(D^A - D^R\right)\Gamma D^K\right]$$

$$= \frac{1}{4}\left(\Gamma_\rho - \frac{m^2c^2}{n^2}\Gamma_u\right)^2 \int \frac{dq}{2\pi}\, q\,\Pi(q, qV)$$

Quantum interference – possible cancellation

$$\Pi(q,\omega) = \frac{n^2}{4m^2c^3}\left(q^2 - \frac{\omega^2}{c^2}\right)\left(\coth\frac{cq-\omega}{4T} - \coth\frac{cq+\omega}{4T}\right)$$

Scattering amplitude

$$\Gamma_\rho - \frac{m^2c^2}{n^2}\Gamma_u \sim \left(N_s - \frac{1}{g}\frac{\partial E_s}{\partial n_0}\right) + \left(\frac{\partial E_s}{\partial n_0}\frac{\partial N_s}{\partial V_s} - \frac{\partial E_s}{\partial V_s}\frac{\partial N_s}{\partial n_0}\right)$$

Integrable case

$$\frac{\partial E_s}{\partial \mu} = \frac{1}{g}\frac{\partial E_s}{\partial n_0} = N_s$$

$$M_s \sim m N_s$$

The amplitude of the Raman process is identically zero!

Small α - perturbation theory

$$\Gamma_\rho = \frac{c}{n_0}\left(1 - \frac{2}{3}\frac{\alpha n_0^2}{mc^2}\right) \; ; \quad \Gamma_u = \frac{n_0}{c}\left(1 + \frac{2}{9}\frac{\alpha n_0^2}{mc^2}\right)$$

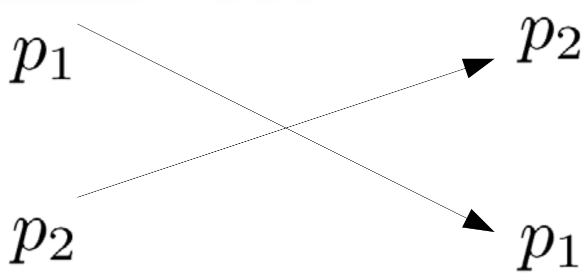
Integrability

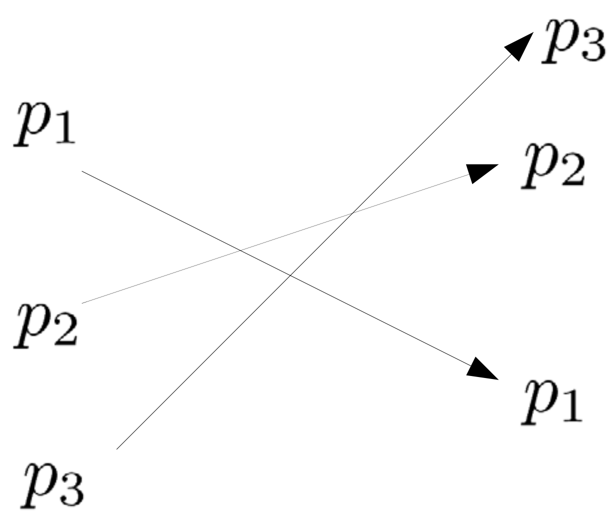
Mechanism of relaxation?

~~3-body collisions~~

STUDIES OF NON LINEAR PROBLEMS

E. FERMI, J. PASTA, and S. ULAM
Document LA-1940 (May 1955).



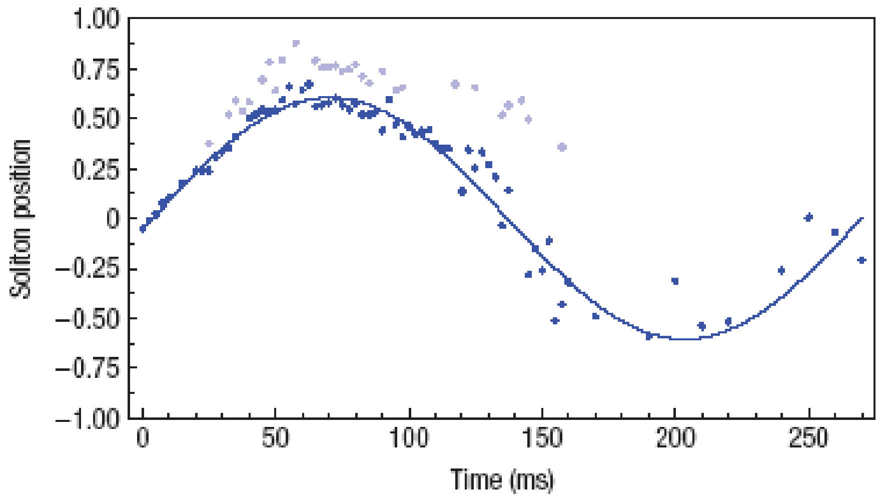


No dissipation

No decay of supercurrents

Even at $T \neq 0$

Quantum Viscosity of Dark Soliton



$$\kappa(T) = \frac{1024\pi^3}{1215} \frac{\alpha^2 n_0^4}{\hbar\, c^2} \left(\frac{T}{\mu}\right)^4$$

$$\tau = \frac{M_s}{\hbar\kappa} \sim 200\text{ms}$$

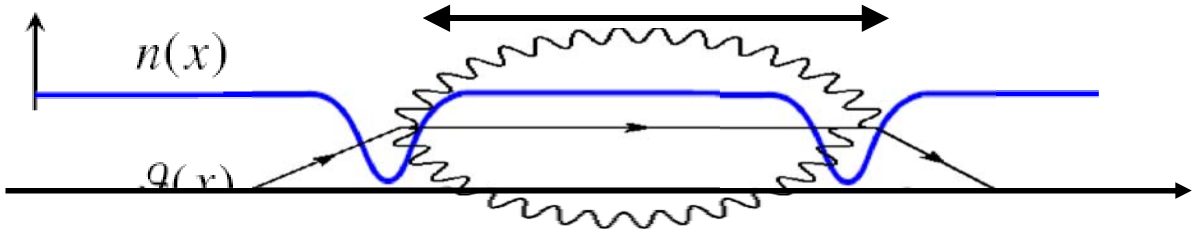
for Hamburg experiment

Soliton-Soliton Interactions

Coherent friction:

$$F(V,X) = -\frac{1}{4}\left(\Gamma_\rho - \Gamma_u \frac{c^2}{n_0^2}\right)^2 \sum_{|q|\lesssim mc} e^{iqX} q\,\Pi(q,qV)$$

Phonon coherence length $L_T = \hbar c/T$



Affects centre of mass motion

superradiation

$$M_s \dot{V}_i = -\kappa(T) \sum_j \frac{V_i + V_j}{2} f\left(\frac{X_i - X_j}{L_T}\right)$$

1D fermions with spin ½

(Gaudin '67, Young '67)

$$H_{\text{int}} = u \int dx\, \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Spin -1 excitation above a (metastable) ferromagnetic state

$$\kappa = 0 \quad \Rightarrow \quad \frac{m}{m^*} = \frac{\pi p_F}{m} \frac{1}{\frac{\partial}{\partial n}(\mu - \mu_d)} + \frac{p_F^2}{m} \frac{\frac{\partial^2}{\partial n^2}(\mu - \mu_d)}{\left(\frac{\partial}{\partial n}(\mu - \mu_d)\right)^2}$$

$$\mu - \mu_d = \frac{p_F u}{2\pi} \left[\left(\frac{2p_F}{mu} + \frac{mu}{2p_F} \right) \arctan \frac{2p_F}{mu} - 1 \right]$$

Castella, Zotos '93

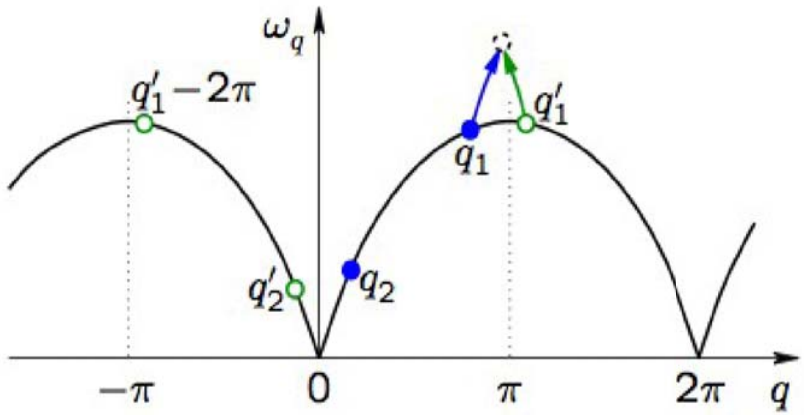
$$\frac{m}{m^*} = \frac{\pi}{2 \arctan^2 \frac{2p_F}{mu}} \left(\arctan \frac{2p_F}{mu} - \frac{\frac{2p_F}{mu}}{1 + \left(\frac{2p_F}{mu}\right)^2} \right)$$

Equilibration of a one-dimensional Wigner crystal

K. A. Matveev,[1] A. V. Andreev,[2] and M. Pustilnik[3]

At strong but finite repulsion, the particles can deviate from their respective lattice sites, $x_l = la + u_l$, but the relative change of interparticle distance remains small, $|u_l - u_{l'}| \ll |l - l'|a$. To leading order in the deviations u_l the Hamiltonian (1) takes the form

$$H_0 = \sum_l \frac{p_l^2}{2m} + \frac{1}{4}\sum_{l,l'} V_{l-l'}^{(2)} (u_l - u_{l'})^2, \qquad (2)$$



In the integrable case of inverse-square interactions the relaxation rate vanishes.

Perturbation around integrability

Weakly interacting bosons $\mu = gn \qquad \mu_d = Gn$

$$\kappa = \frac{16\pi^3}{15}\frac{T^4}{c^4\hbar^3 n^2}\left(\frac{G}{g}\right)^2\left(\frac{mG}{Mg} - 1\right)^2 \qquad M^* = M$$

$$\mathrm{Rb}^{87} \quad G/g \sim 1.05$$

Strong interactions $m \neq M \qquad g, G \to \infty$

$$\kappa = \frac{16\pi^3}{15}\frac{T^4}{c^4\hbar^3 n^2}\left[\left(\frac{m}{M}\right)^{1/3} - \frac{M}{m}\right]^2$$

$$m = M \qquad g, G \gg \hbar^2 n/m$$

$$\kappa \sim \frac{\pi^3 T^4}{c^4\hbar^3 n^2}\left(\frac{\hbar^2 n}{gm}\right)^4\left(1 - \frac{g}{G}\right)^2$$

Concluding remarks

- Dark solitons and mobile impurities – model to study nonequilibrium many body quantum dynamics.
- Coupling to phonons: $E_s(P, n)$ and Galilean invariance
- Mechanism of dissipation: Raman scattering of phonons
- Sensitive to parameters. No dissipation at integrability $(T \neq 0)$
- Can be applied to study excitation dynamics in nearly integrable systems. Lifetime of quasiparticles
- Experiments....