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Workshop on Localization Phenomena in Novel Phases of Condensed Matter

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Topological Insulators: Magnetoelectric and Disorder Effects

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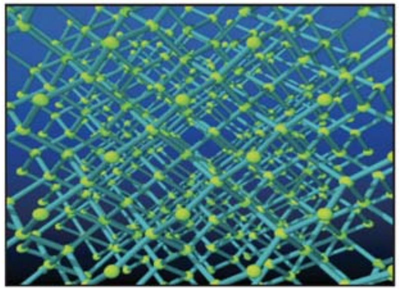
ICTP, 17 May 2010



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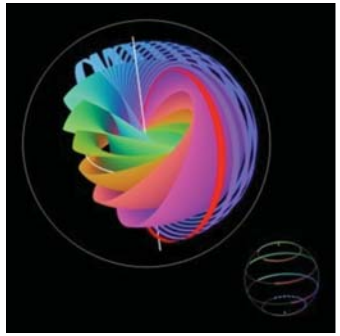

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References

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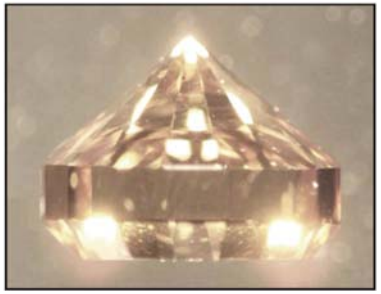
A. M. Essin, J. E. Moore, and D. Vanderbilt, PRL **102**, 146805 (2009)
B. Seradjeh, J. E. Moore, M. Franz, PRL **103**, 066402 (2009)
A. M. Essin, A. Turner, J. E. Moore, and D. Vanderbilt, 1002.0290

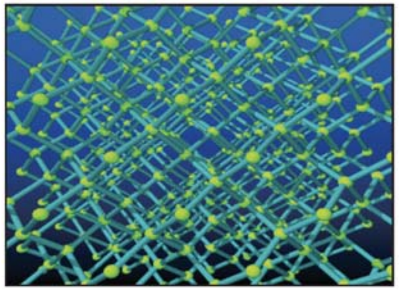
JEM, Nature **460**, 1090 (2009)
"An insulator's metallic side"
JEM, Nature **464**, 194-198 (2010)
"The birth of topological insulators"

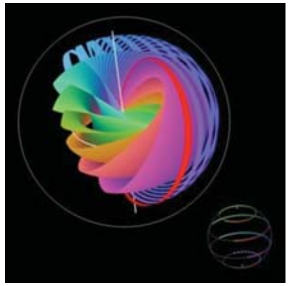
See also reviews by Hasan and Kane (RMP colloquium) and Qi and Zhang (Physics Today).

Outline

1. Band-structure picture of "topological insulators":
topological phases from spin-orbit coupling in 3D materials with no applied field.







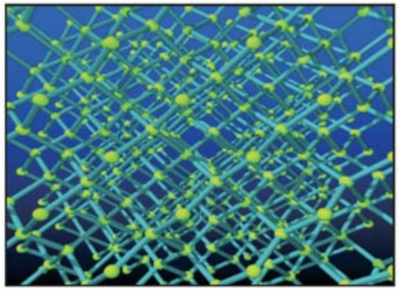
2. Redefinition A: Topological response to boundary conditions
Redefinition B: What physical response that characterizes a topological insulator?
"Axion electrodynamics" in weak electric/magnetic fields

3. What do we learn about insulators more generally?
General formula for orbital magnetoelectric coupling in solids, incl. "multiferroics"

4. Applications
Exciton condensation in thin films of topological insulators
Majorana fermions and disordered quantum criticality

Types of order

Much of condensed matter is about how different kinds of order emerge from interactions between many simple constituents.



Until 1980, all ordered phases could be understood as "symmetry breaking":

an ordered state appears at low temperature when the system spontaneously loses one of the symmetries present at high temperature.

Examples:
Crystals break the *translational* and *rotational* symmetries of free space.
The "liquid crystal" in an LCD breaks *rotational* but not *translational* symmetry.
Magnets break time-reversal symmetry and the rotational symmetry of spin space.
Superfluids break an internal symmetry of quantum mechanics.

Types of order

In 1980, the first ordered phase beyond symmetry breaking was discovered.

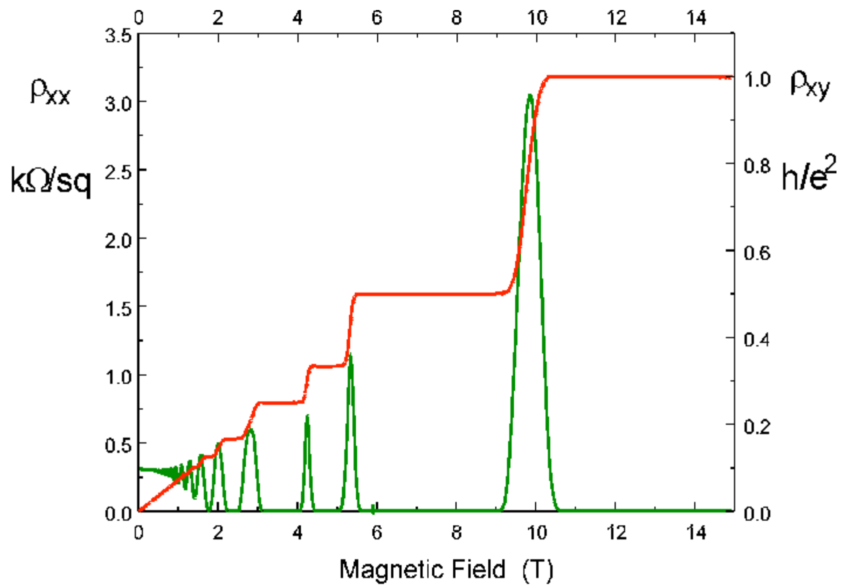
Electrons confined to a plane and in a strong magnetic field show, at low enough temperature, plateaus in the "Hall conductance":

force *I* along *x* and measure *V* along *y*

on a plateau, get

$$\sigma_{xy} = n\frac{e^2}{h}$$

at least within 1 in 10^9 or so.



What type of order causes this precise quantization?

Note I: the AC Josephson effect between superconductors similarly allows determination of *e*/*h*.

Note II: there are also *fractional* plateaus, about which more later.

Topological order

What type of order causes the precise quantization in the Integer Quantum Hall Effect (IQHE)?

Definition I:

In a topologically ordered phase, some physical response function is given by a "topological invariant".

What is a topological invariant? How does this explain the observation?

Definition II:

A topological phase is insulating but always has metallic edges/surfaces when put next to vacuum or an ordinary phase.

What does this have to do with Definition I?

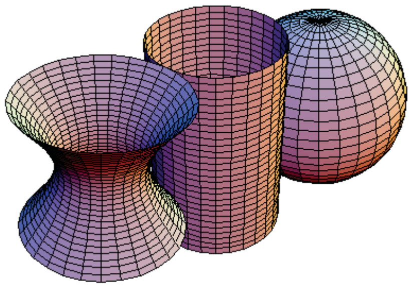
"Topological invariant" = quantity that does not change under continuous deformation

Topological invariants

Most *topological* invariants in physics arise as integrals of some *geometric* quantity.

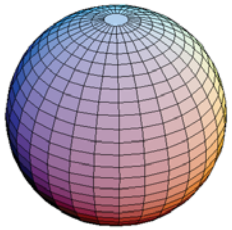
Consider a two-dimensional surface.

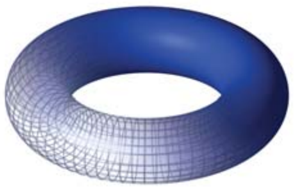
At any point on the surface, there are two radii of curvature.
We define the signed "Gaussian curvature" $\kappa = (r_1 r_2)^{-1}$



from left to right, equators have negative, 0, positive Gaussian curvature

Now consider *closed* surfaces.





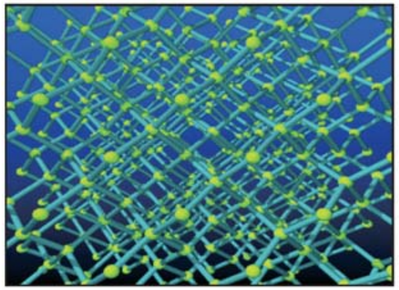
The area integral of the curvature over the whole surface is "quantized", and is a topological invariant (Gauss-Bonnet theorem).

$$\int_M \kappa \, dA = 2\pi\chi = 2\pi(2 - 2g)$$

where the "genus" g = 0 for sphere, 1 for torus, n for "n-holed torus".

Topological invariants

Good news:
for the invariants in the IQHE and topological insulators,
we need one fact about solids



Bloch's theorem:
One-electron wavefunctions in a crystal
(i.e., periodic potential) can be written

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

where *k* is "crystal momentum" and *u* is periodic (the same in every unit cell).

Crystal momentum *k* can be restricted to the Brillouin zone, a region of *k*-space with periodic boundaries.
As *k* changes, we map out an "energy band". Set of all bands = "band structure".

The Brillouin zone will play the role of the "surface" as in the previous example,

and one property of quantum mechanics, the Berry phase

which will give us the "curvature".

Berry phase

What kind of "curvature" can exist for electrons in a solid?

Consider a quantum-mechanical system in its (nondegenerate) ground state.

The adiabatic theorem in quantum mechanics implies that, if the Hamiltonian is now changed slowly, the system remains in its time-dependent ground state.

But this is actually very incomplete (Berry).

When the Hamiltonian goes around a *closed loop k(t)* in parameter space, there can be an irreducible *phase*

$$\phi = \oint \mathcal{A} \cdot d\mathbf{k}, \quad \mathcal{A} = \langle \psi_k | -i\nabla_k | \psi_k \rangle$$

relative to the initial state.

Why do we write the phase in this form?
Does it depend on the choice of reference wavefunctions?

Berry phase

Why do we write the phase in this form?
Does it depend on the choice of reference wavefunctions?

$$\phi = \oint \mathcal{A} \cdot d\mathbf{k}, \quad \mathcal{A} = \langle \psi_k | -i\nabla_k | \psi_k \rangle$$

If the ground state is non-degenerate, then the only freedom in the choice of reference functions is a local phase:

$$\psi_k \longrightarrow e^{i\chi(k)}\psi_k$$

Under this change, the "Berry connection" *A* changes by a gradient,

$$\mathcal{A} \rightarrow \mathcal{A} + \nabla_k \chi$$

$$\mathcal{F} = \nabla \times \mathcal{A}$$

just like the vector potential in electrodynamics.

So loop integrals of *A* will be gauge-invariant,
as will the *curl* of *A*, which we call the "Berry curvature".

Berry phase in solids

In a solid, the natural parameter space is electron momentum.

The change in the electron wavefunction *within the unit cell* leads to a Berry connection and Berry curvature:

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

$$\mathcal{A} = \langle u_{\mathbf{k}} | -i\nabla_k | u_{\mathbf{k}} \rangle \qquad \mathcal{F} = \nabla \times \mathcal{A}$$

We keep finding more physical properties that are determined by these quantum geometric quantities.

S. S. Chern

The first was that the integer quantum Hall effect in a 2D crystal follows from the integral of *F* (like Gauss-Bonnet!). Explicitly,

$$n = \sum_{bands} \frac{i}{2\pi} \int d^2k \left(\left\langle \frac{\partial u}{\partial k_1} \middle| \frac{\partial u}{\partial k_2} \right\rangle - \left\langle \frac{\partial u}{\partial k_2} \middle| \frac{\partial u}{\partial k_1} \right\rangle \right) \qquad \mathcal{F} = \nabla \times \mathcal{A}$$

$$\sigma_{xy} = n\frac{e^2}{h}$$

TKNN, 1982 "first Chern number"

The importance of the edge

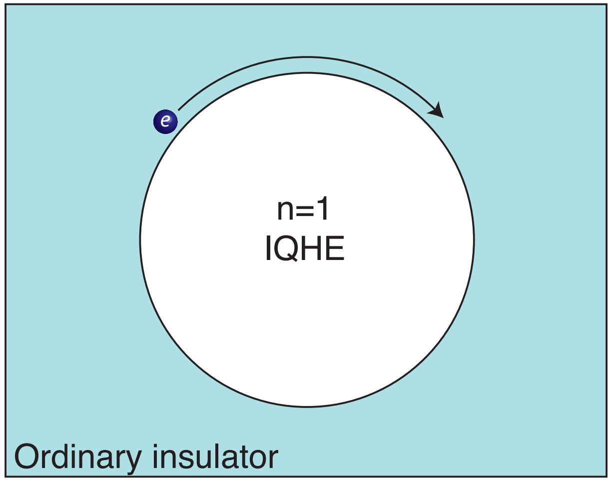
But wait a moment...

This invariant exists if we have energy bands that are either full or empty, i.e., a "band insulator".

How does an *insulator* conduct charge?

Answer: (Laughlin; Halperin)

There are *metallic edges* at the boundaries of our 2D electronic system, where the conduction occurs.



These metallic edges are "chiral" quantum wires (one-way streets). Each wire gives one conductance quantum (e^2/h).

$$\sigma_{xy} = n\frac{e^2}{h}$$

The topological invariant of the *bulk* 2D material just tells how many wires there *have* to be at the boundaries of the system.

How does the bulk topological invariant "force" an edge mode?

The importance of the edge

The topological invariant of the *bulk* 2D material just tells how many wires there *have* to be at the boundaries of the system.

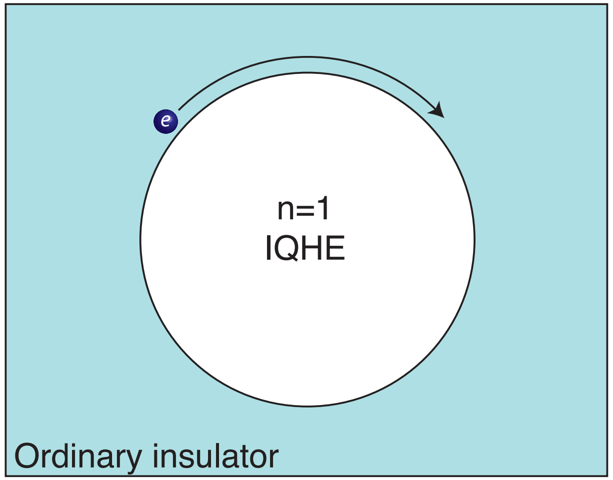
How does the bulk topological invariant "force" an edge mode?

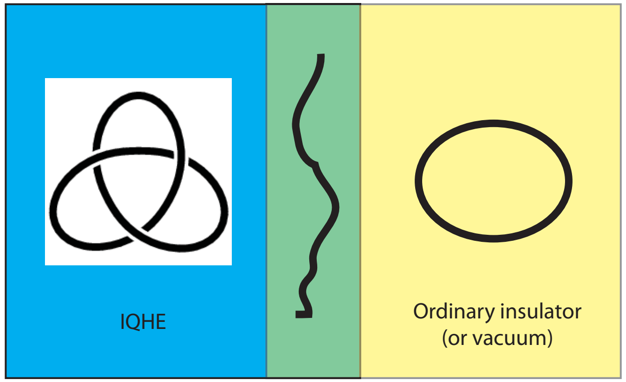
Answer:

Imagine a "smooth" edge where the system gradually evolves from IQHE to ordinary insulator. The topological invariant must change.

But the definition of our "topological invariant" means that, *if the system remains insulating* so that every band is either full or empty, the invariant cannot change.

∴ the system must not remain insulating.





(What is "knotted" are the electron wavefunctions)

The "quantum spin Hall effect"

Spin-orbit coupling appears in nearly every atom and solid. Consider the standard atomic expression

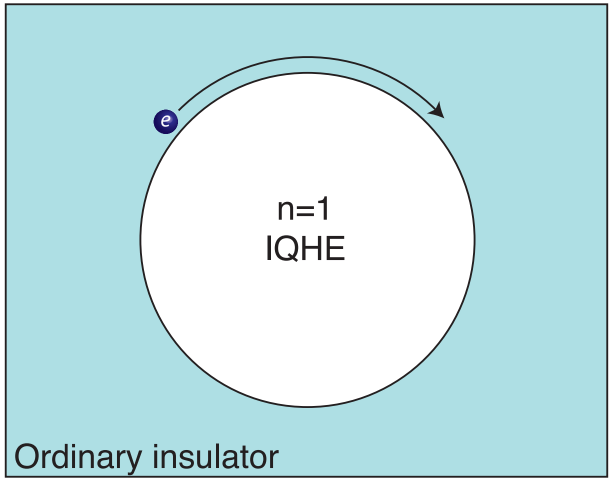
$$H_{SO} = \lambda \mathbf{L} \cdot \mathbf{S}$$

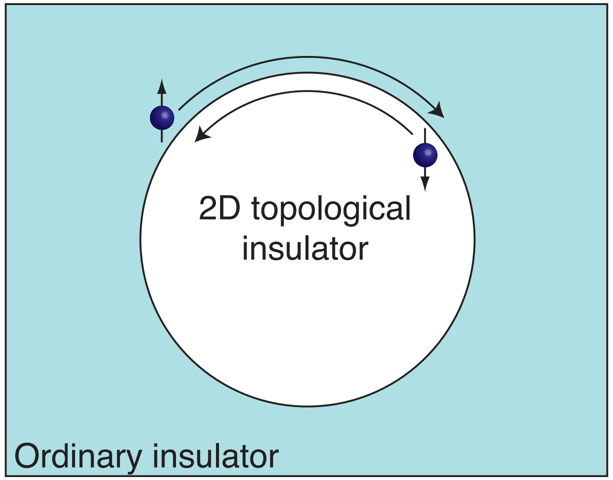
For a given spin, this term leads to a momentum-dependent force on the electron, somewhat like a magnetic field.

The spin-dependence means that the *time-reversal symmetry* of SO coupling (even) is different from a real magnetic field (odd).

It is possible to design lattice models where spin-orbit coupling has a remarkable effect: (Murakami, Nagaosa, Zhang 04; Kane, Mele 05)

spin-up and spin-down electrons are in IQHE states, with opposite "effective magnetic fields".



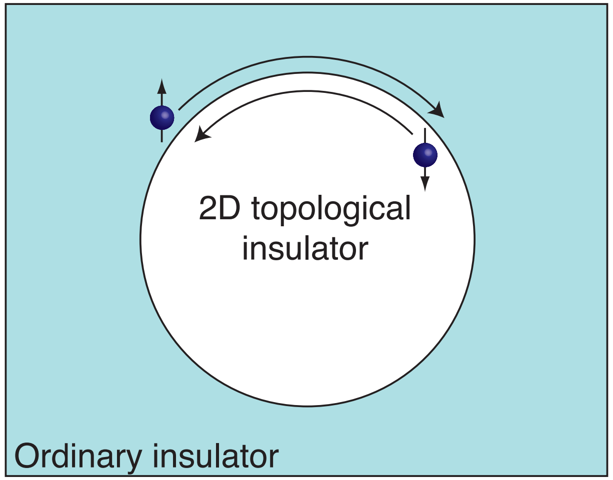


The "quantum spin Hall effect"

In this type of model, electron spin is conserved, and there can be a "spin current".

An applied electrical field causes oppositely directed Hall currents of up and down spins.

The charge current is zero, but the "spin current" is nonzero, and even quantized!



$$\mathcal{J}_j^i = \sigma_H^s \epsilon_{ijk} E_k$$

However...

1. In real solids there is no conserved direction of spin.

2. So in real solids, it was expected that "up" and "down" would always mix and the edge to disappear.

3. The theory of the above model state is just two copies of the IQHE.

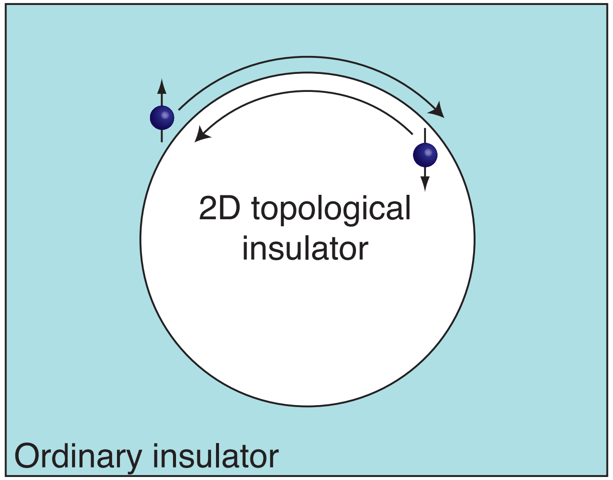
The 2D topological insulator

It was shown in 2005 (Kane and Mele) that, in real solids with all spins mixed and no "spin current", something of this physics does survive.

In a material with only spin-orbit, the "Chern number" mentioned before always vanishes.

Kane and Mele found a new topological invariant in time-reversal-invariant systems of fermions.

But it isn't an integer! It is a Chern *parity* ("odd" or "even"), or a "Z2 invariant".



Systems in the "odd" class are "2D topological insulators"

1. Where does this "odd-even" effect come from?
2. What is the Berry phase expression of the invariant?
3. How can this edge be seen?

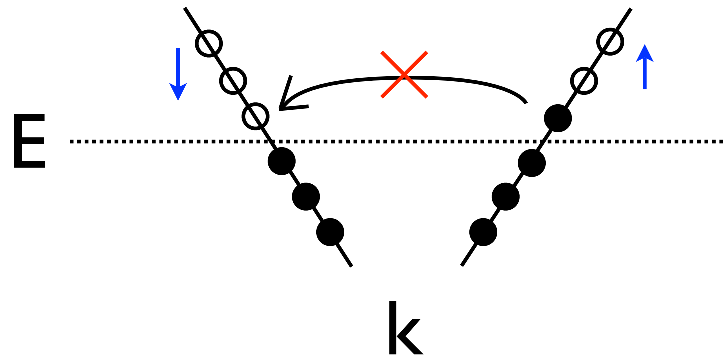
The 2D topological insulator

1. Where does this "odd-even" effect come from?

In a time-reversal-invariant system of electrons, all energy eigenstates come in degenerate pairs.

The two states in a pair cannot be mixed by any T-invariant perturbation. (disorder)

So an edge with a single Kramers pair of modes is perturbatively stable (C. Xu-JEM, C. Wu et al., 2006).



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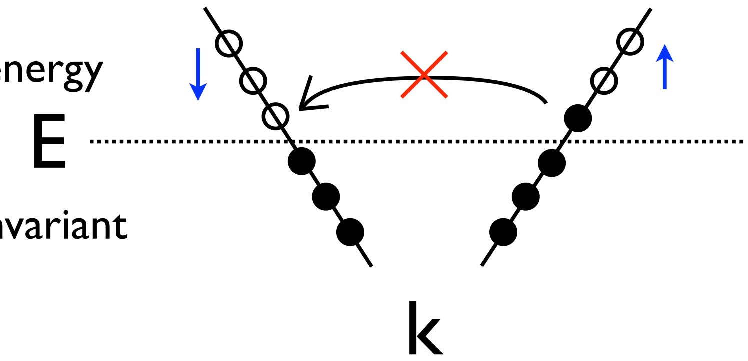

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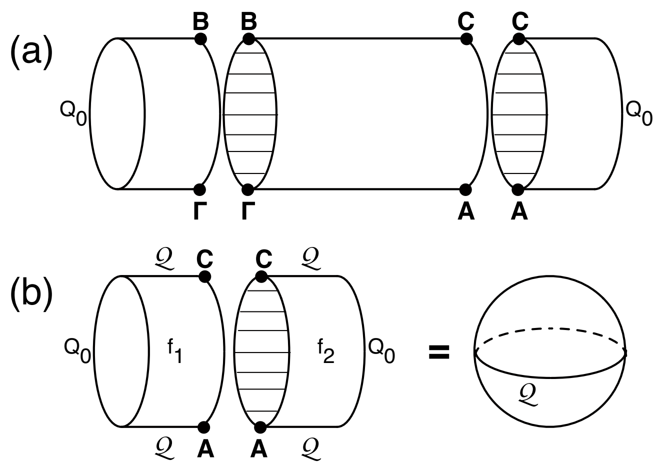
So an edge with a single Kramers pair of modes is perturbatively stable.

2. What is the Berry phase expression of the invariant?
It is an integral over *half* the Brillouin zone, plus boundary,

$$D = \frac{1}{2\pi}\left[\oint_{\partial(EBZ)} d\mathbf{k}\cdot\mathcal{A} - \int_{EBZ} d^2\mathbf{k}\,\mathcal{F}\right] \bmod 2 \qquad (1)$$



3. How can this edge be seen?

(this picture from JEM and Balents, 2007; completion is like a Wess-Zumino term in field theory)

The 2D topological insulator

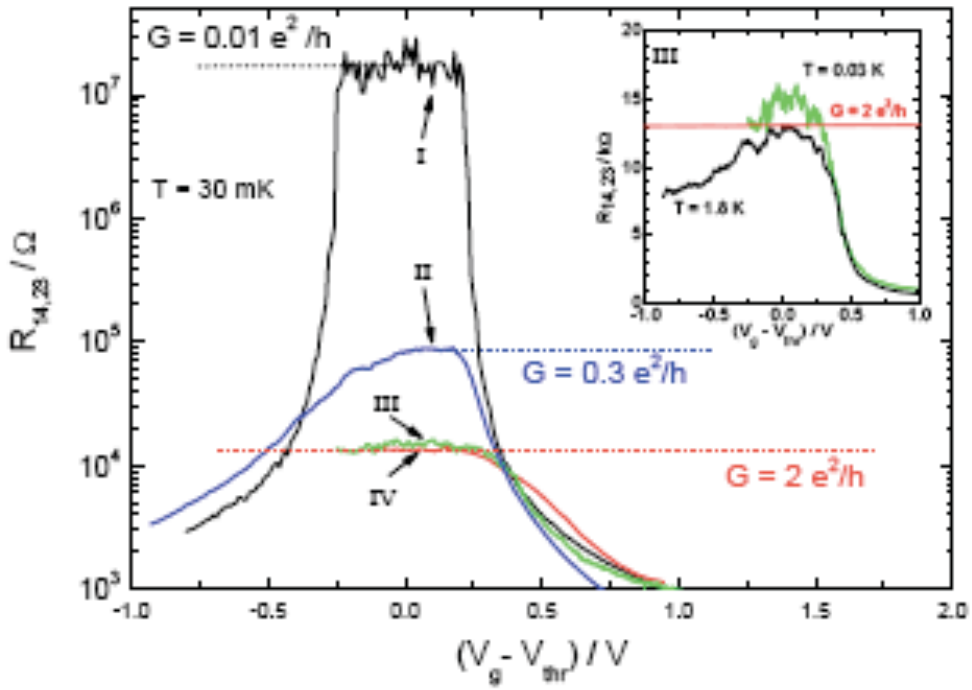
Key: the topological invariant predicts the "number of quantum wires".

While the wires are not one-way, so the Hall conductance is zero, they still contribute to the *ordinary* (two-terminal) conductance.

There should be a low-temperature *edge* conductance from one spin channel at each edge:

$$G = \frac{2e^2}{h}$$

König et al., *Science* (2007)



Laurens Molenkamp

This appears in (Hg,Cd)Te quantum wells as a quantum Hall-like plateau *in zero magnetic field.*

What about 3D?

There is no truly 3D quantum Hall effect. There are only layered versions of 2D.
(There are 3 "topological invariants", from *xy*, *yz*, and *xz* planes.)

Trying to find Kane-Mele-like invariants in 3D leads to a surprise: (JEM and Balents, 2007)

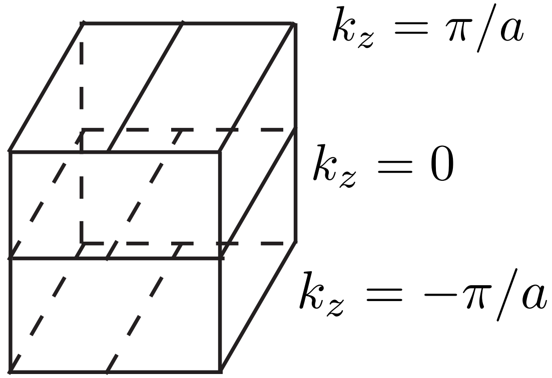
1. There are still 3 layered Z2 invariants, but there is a fourth Z2 invariant as well.
Hence there are $2^4 = 16$ different classes of band insulators in 3D.

2. The nontrivial case of the fourth invariant is fully 3D and cannot be realized in any model that doesn't mix up and down spin. (see also R. Roy, 2009)

In 3D, we can take the BZ to be a cube (with periodic boundary conditions):

think about xy planes

2 inequivalent planes look like 2D problem



3D "strong topological insulators" go from an 2D *ordinary* insulator to a 2D *topological* insulator (or vice versa) in going from $k_z=0$ to $k_z=\pm\pi/a$.

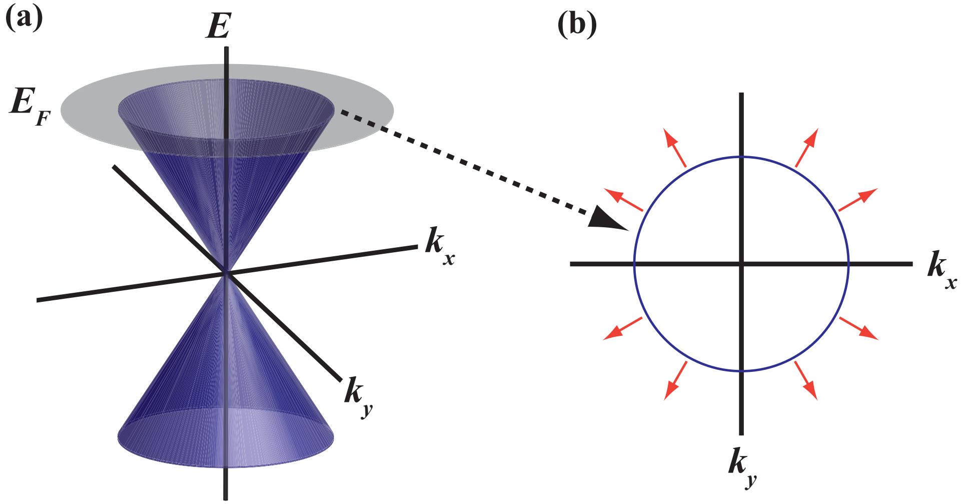
This is allowed because intermediate planes have no time-reversal constraint.

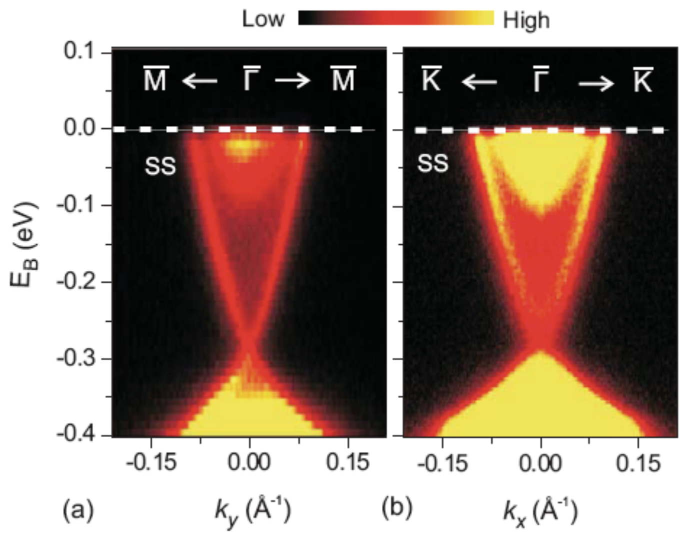
3. There should be some type of metallic surface resulting from this fourth invariant, and this is easier to picture...

Topological insulators in 3D

1. This fourth invariant gives a robust 3D "strong topological insulator" whose metallic surface state in the simplest case is a single "Dirac fermion" (Fu-Kane-Mele, 2007)

"...are equivalent to the four invariants introduced by Moore and Balents [10] using general homotopy arguments. *The power of the present approach is that it allows us to characterize the surface states on an arbitrary crystal face."* (italics mine)





2. Some fairly common 3D materials are topological insulators!

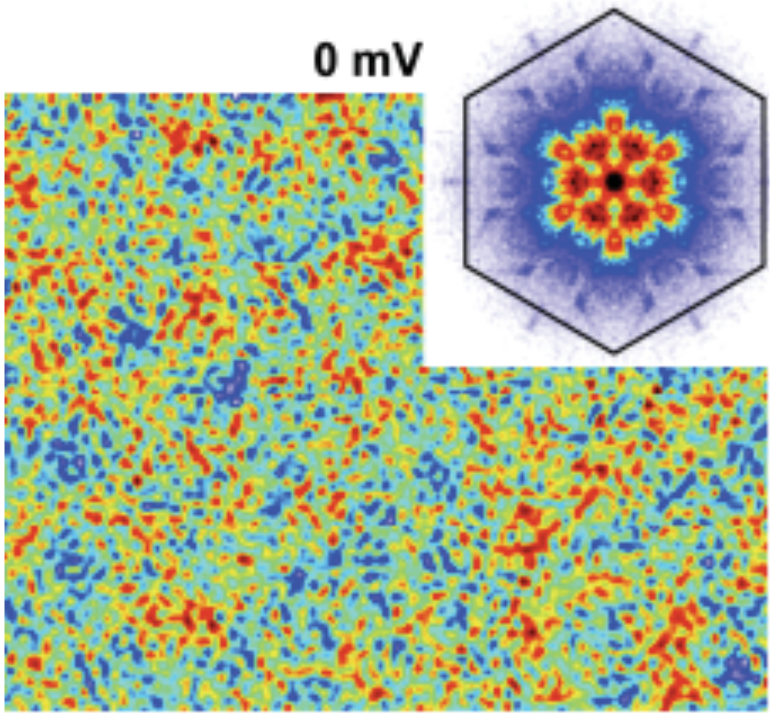
Experimental discovery by D. Hsieh, ..., Z. Hasan, Nature 2008, in Bi_xSb_{1-x}.

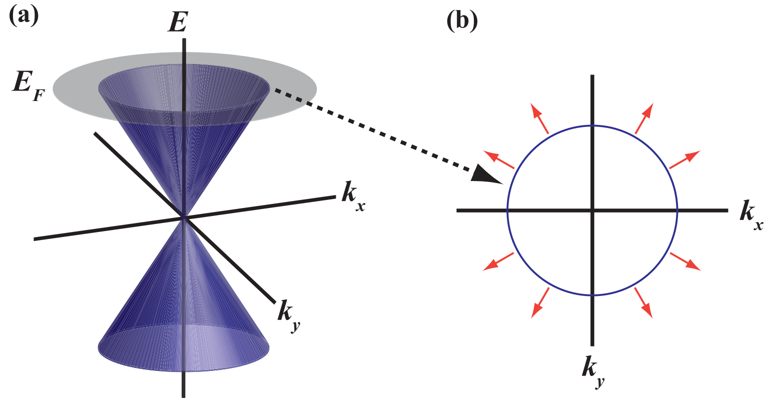
Here is shown later data by same group on Bi_2Se_3. Also Bi_2Te_3 (Chen et al.).

STM of topological insulators

The surface of a simple topological insulator like Bi_2Se_3 is "1/4 of graphene": it has the Dirac cone but no valley or spin degeneracies.

Scanning tunneling microscopy image (Roushan et al., Yazdani group, 2009)





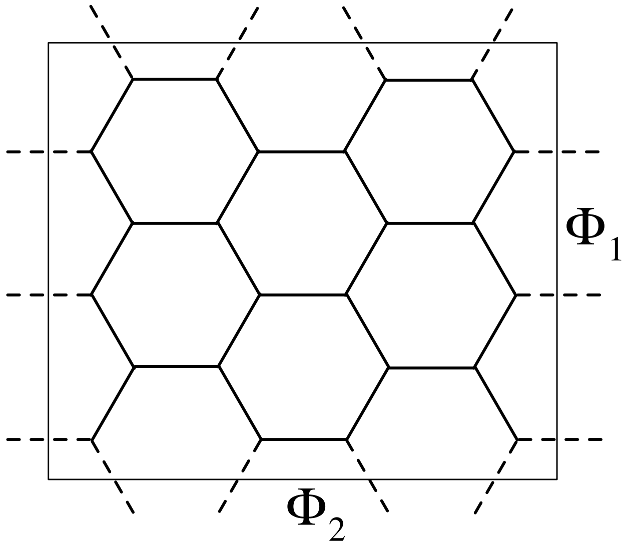
STM can see the absence of scattering within a Kramers pair (cf. analysis of superconductors using quasiparticle interference, D.-H. Lee and S. Davis).

Defining the topological insulator with disorder

Suppose that the parameters in *H* do not have exact lattice periodicity.

Imagine adding boundary phases to a finite system, or alternately considering a "supercell". Limit of large supercells -> disordered system.

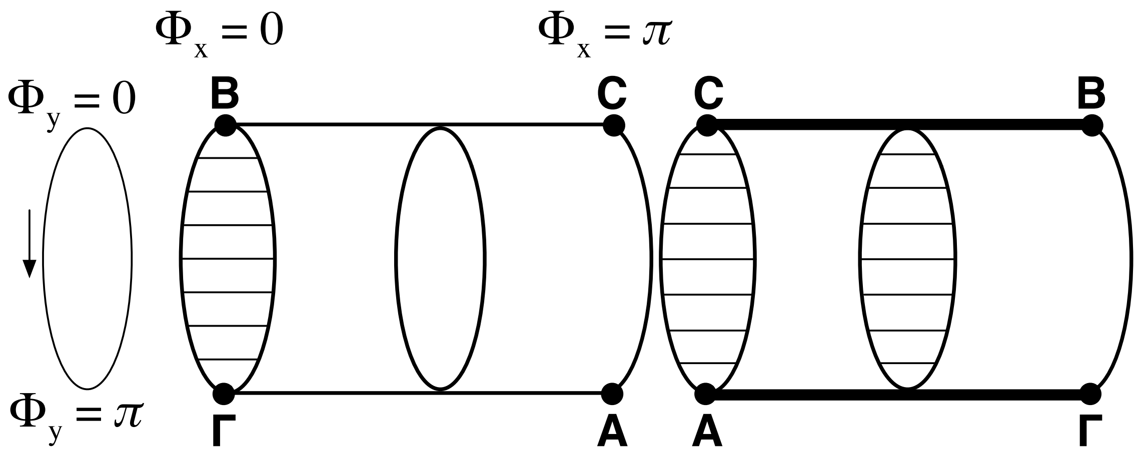
Effect of boundary phase is to shift *k*: alternate picture of topological invariant is in terms of half the (Φ_1,Φ_2) torus.



Can define Chern parities analogous to Chern numbers, and study phase diagram w/disorder

Pumping and interactions

In addition to the supercell argument, we can give a physical definition of the topological insulator in a disordered system as follows: (mathematical content is the same)

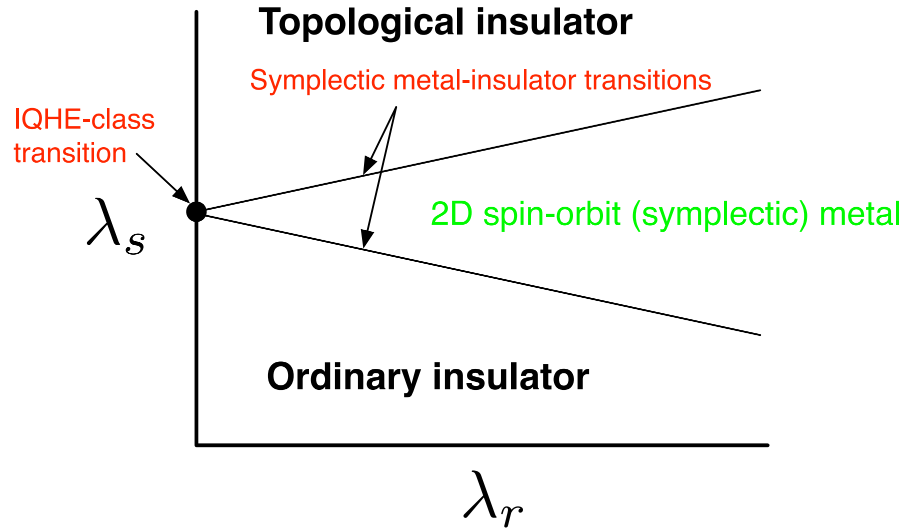


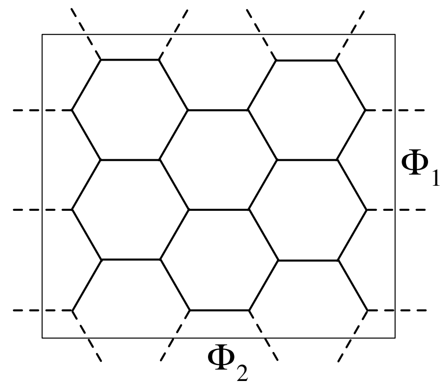
Stage I:
insert flux, breaking T

Stage II:
complete the cycle

The topological insulator with disorder

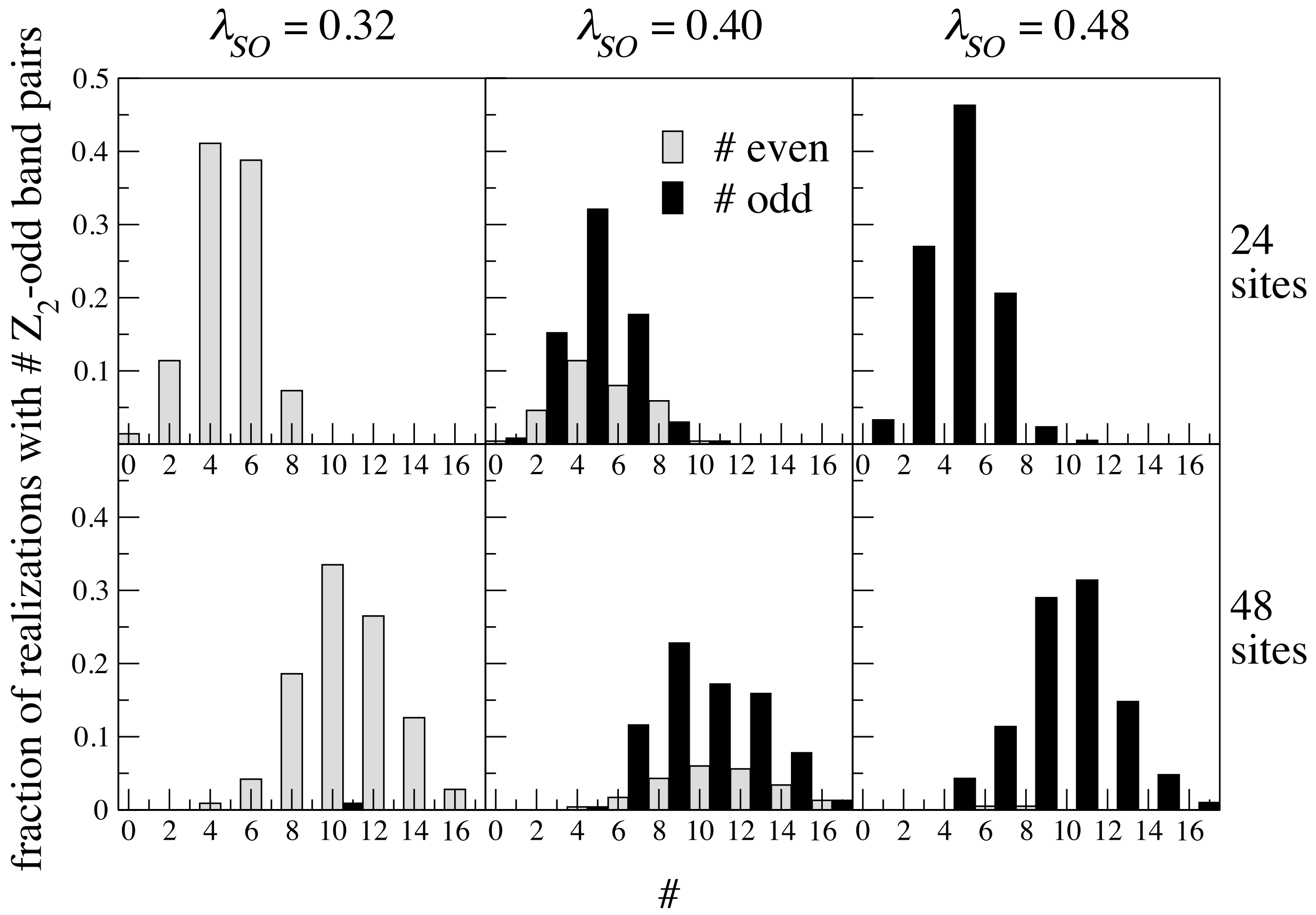
Spin-orbit *T=0* phase diagram (fix spin-independent part): instead of a point transition between ordinary and topological insulators, have a symplectic metal in between.





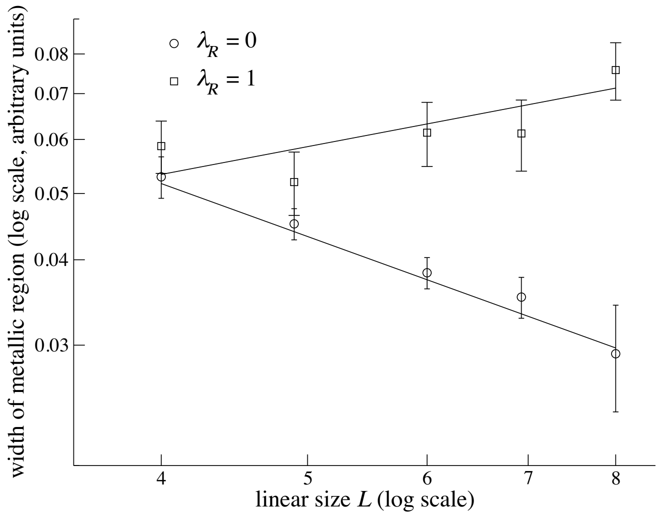
We observe this numerically using Fukui-Hatsugai algorithm (PRB 2007) to compute invariants (A. Essin and JEM, PRB 2007). See also Obuse et al., Onoda et al. for network approaches with higher accuracy->scaling exponents

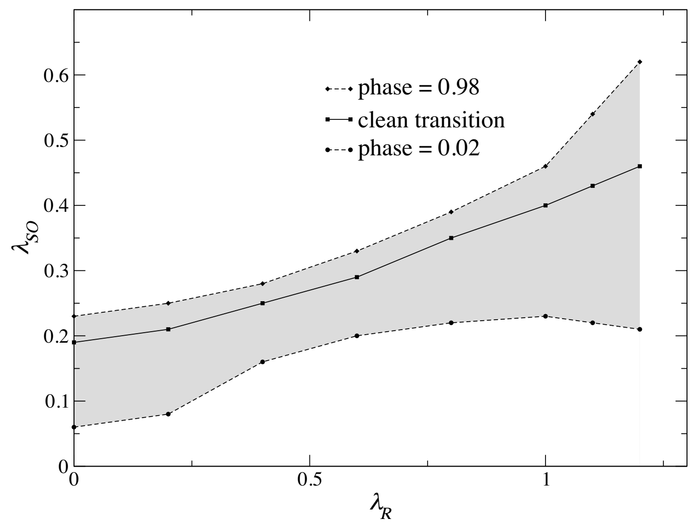
Numerical evidence



Identification of phases using Fukui-Hatsugai approach:
ordinary = even integers, topological = odd integers

Numerical evidence





Transition width

Phase diagram

Summary of 1-electron results

1. There are now at least 3 strong topological insulators that have been seen experimentally (Bi_xSb_{1-x}, Bi_2Se_3, Bi_2Te_3).

2. Their metallic surfaces exist in zero field and have the predicted form.

3. These are fairly common bulk 3D materials (and also ^{3}He B; full classifications by Ryu, Schnyder, Furusaki, and Ludwig; Kitaev).

4. The temperature over which topological behavior is observed can extend up to room temperature or so.

What's left

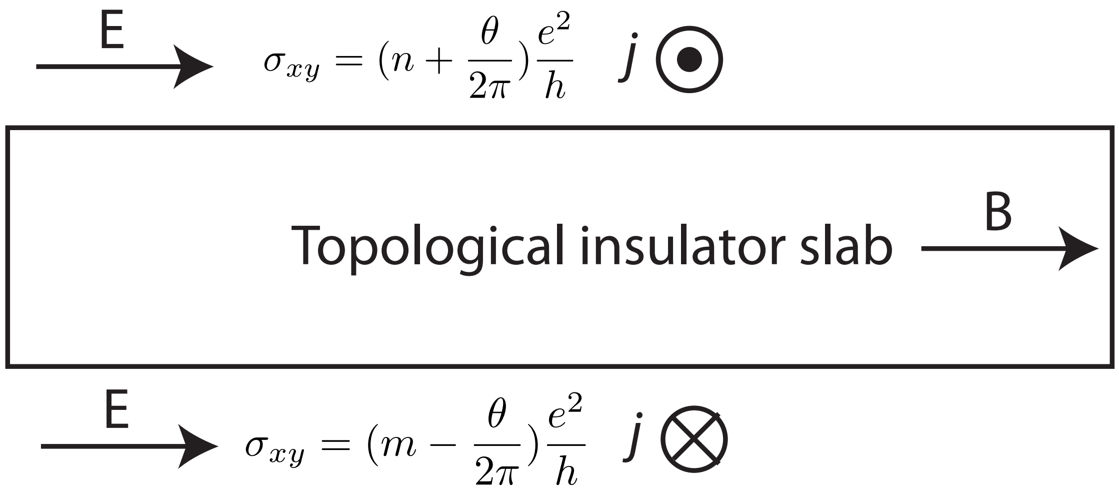
What is the physical effect or response that defines a topological insulator beyond single electrons?

What are they good for?

Magnetoelectric effect

Special feature: the metallic surface of a 3D topological insulator is very robust ("topologically protected") to perturbations that do not break time-reversal symmetry.

Adding a magnetic perturbation leads to a *half-integer* quantum Hall effect localized near the surface.



Circulating currents lead to a *quantized magnetoelectric effect* (Qi et al.) that was discussed on a macroscopic level in the 1980s as "axion electrodynamics" (cf. Wilcezk, 1987).

Magnetoelectric effect

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$$\Delta\mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B} = \frac{\theta e^2}{16\pi h}\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}.$$

Qi et al. derived a microscopic Berry-phase expression

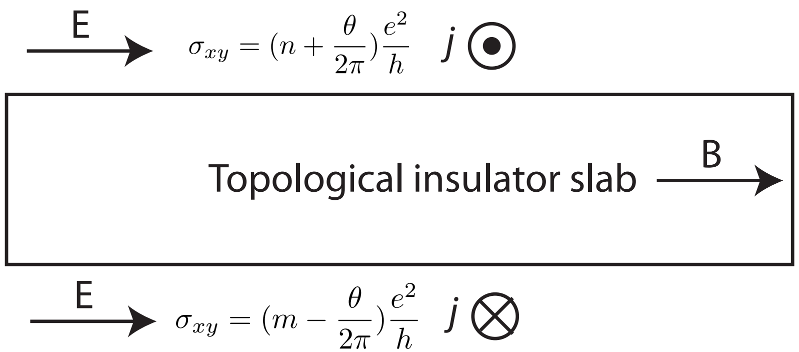
$$\theta = -\frac{1}{4\pi}\int_{\mathrm{BZ}} d^3k\ \epsilon_{ijk}\,\mathrm{Tr}[\mathcal{A}_i\partial_j\mathcal{A}_k - i\frac{2}{3}\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k]$$

which gives a new way to look at 3D topological band structures.

Initial goals: can we observe these surface QHE layers numerically?

Can we derive θ from the previous appearance of second Chern number in polarization calculations?

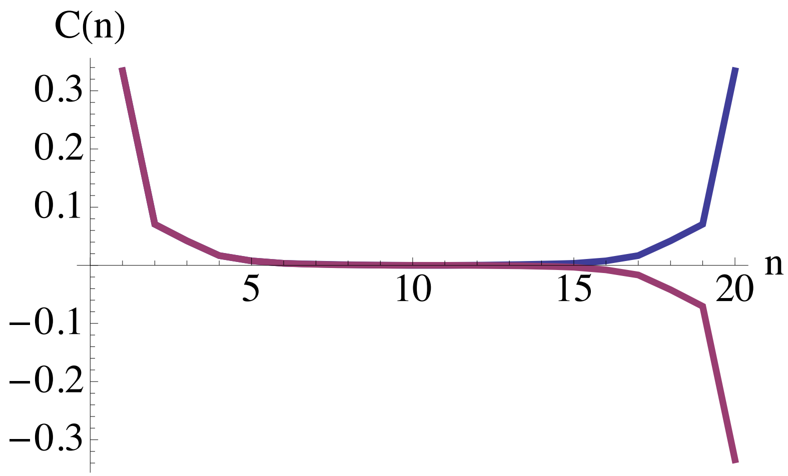
Does it generalize to the many-body case?



Magnetoelectric effect

Initial goals: can we observe these surface QHE layers numerically?

Yes: spatially resolve 2D Chern number in a slab of 3D topological insulator



See half-integer surface layers that are sensitive to surface perturbations

Can we derive θ from the previous appearance of second Chern number in polarization calculations?

Yes: The semiclassical approach to polarization in an inhomogeneous material (Xiao, Shi, Dougherty, and Niu) can be applied to our problem: a weak magnetic field is an inhomogeneity.

$$\theta = -\frac{1}{4\pi}\int_{\mathrm{BZ}} d^3k \; \epsilon_{ijk}\, \mathrm{Tr}[\mathcal{A}_i \partial_j \mathcal{A}_k - i\frac{2}{3}\mathcal{A}_i\mathcal{A}_j\mathcal{A}_k]$$

Indeed this term is present, but there are other implicit terms that may not be zero...?

Topological response

Does it generalize to the many-body case?

Many-body definition: the Chern-Simons or second Chern formula does not directly generalize. However, dP/dB *does* generalize.
a clue is that the "polarization quantum" of one charge per unit cell at the surface (King-Smith and Vanderbilt) combines nicely with the flux quantum:

$$\frac{\Delta P}{B_0} = \frac{e/\Omega}{h/e\Omega} = e^2/h.$$

So dP/dB gives a *bulk*, *many-body* test for a topological insulator.

$$\frac{e^2}{h}$$

= contact resistance in 0D or 1D
= Hall conductance quantum in 2D
= magnetoelectric polarizability in 3D

Orbital magnetoelectric polarizability

One mysterious fact about the previous result:

We indeed found the "Chern-Simons term" from the semiclassical approach.

But in that approach, it is not at all clear why this should be the only magnetoelectric term from orbital motion of electrons.

More precisely: on general symmetry grounds, it is natural to decompose the tensor into *trace* and *traceless* parts

$$\frac{\partial P^i}{\partial B^j} = \frac{\partial M_j}{\partial E_i} = \alpha^i_j = \tilde{\alpha}^i_j + \alpha_\theta \delta^i_j.$$

The traceless part can be further decomposed into symmetric and antisymmetric parts.
(The antisymmetric part is related to the "toroidal moment" in multiferroics;
cf. M. Fiebig and N. Spaldin)

But consideration of simple "molecular" models shows that even the trace part is not always equal to the Chern-Simons formula...

Orbital magnetoelectric polarizability

Computing orbital dP/dB in a fully quantum treatment reveals that there are additional terms in general. (Essin et al., 1002.0290)
For dM/dE approach and numerical tests, see Malashevich, Souza, Coh, Vanderbilt, 1002.0300.

$$\begin{aligned}
\alpha^i_j &= (\alpha_I)^i_j + \alpha_{CS}\delta^i_j \\
(\alpha_I)^i_j &= \sum_{\substack{n\,\mathrm{occ}\\ m\,\mathrm{unocc}}} \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3} \mathrm{Re}\left\{ \frac{\langle u_{n\mathbf{k}}| e\, \not{r}^i_{\mathbf{k}} |u_{m\mathbf{k}}\rangle \langle u_{m\mathbf{k}}| e(\mathbf{v}_{\mathbf{k}} \times \not{r}_{\mathbf{k}})_j - e(\not{r}_{\mathbf{k}} \times \mathbf{v}_{\mathbf{k}})_j - 2i\partial H'_{\mathbf{k}}/\partial B^j |u_{n\mathbf{k}}\rangle}{E_{n\mathbf{k}} - E_{m\mathbf{k}}} \right\} \\
\alpha_{CS} &= -\frac{e^2}{2\hbar}\epsilon_{abc} \int_{\mathrm{BZ}} \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\mathcal{A}^a \partial^b \mathcal{A}^c - \frac{2i}{3} \mathcal{A}^a \mathcal{A}^b \mathcal{A}^c \right].
\end{aligned}$$

The "ordinary part" indeed looks like a Kubo formula of electric and magnetic dipoles.

Not inconsistent with previous results:
in topological insulators, time-reversal means that only the Berry phase term survives.

There is an "ordinary part" and a "topological part", which is scalar but is the only nonzero part in TIs. But the two are not physically separable in general.
Both parts are nonzero in multiferroic materials.

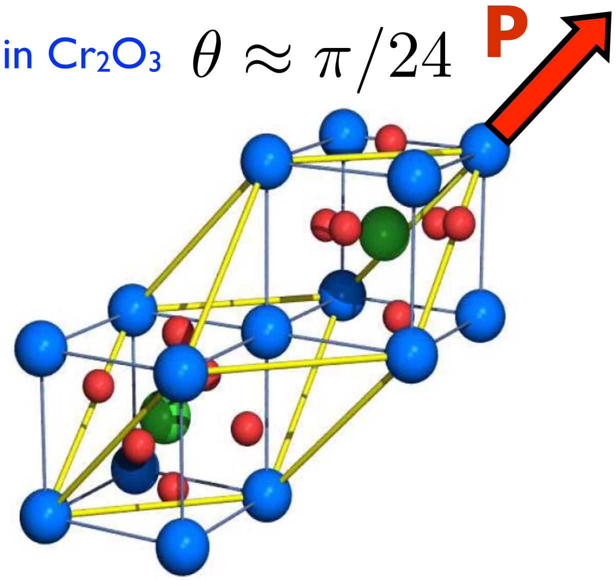
Beyond linear response

So we have a general theory for the orbital magnetoelectric response tensor in a crystal (which essentially includes the orbital "toroidal moment").

It is not a pure Berry phase in general, but *it is in topological insulators*.

Such magnetoelectric responses have been measured, e.g., in Cr_2O_3 $\theta \approx \pi/24$ (Obukhov, Hehl, et al.).

Example of the ionic "competition": $BiFeO_3$



Orbital effects should be faster and less fatiguing than magnetoelectric effects requiring ionic motion.

What about beyond linear response?

In a strong magnetic field, experiments have seen Aharonov-Bohm oscillations in nanowires (Yi Cui et al.) and Landau quantization/surface quantum Hall features (Analytis, Fisher et al.).

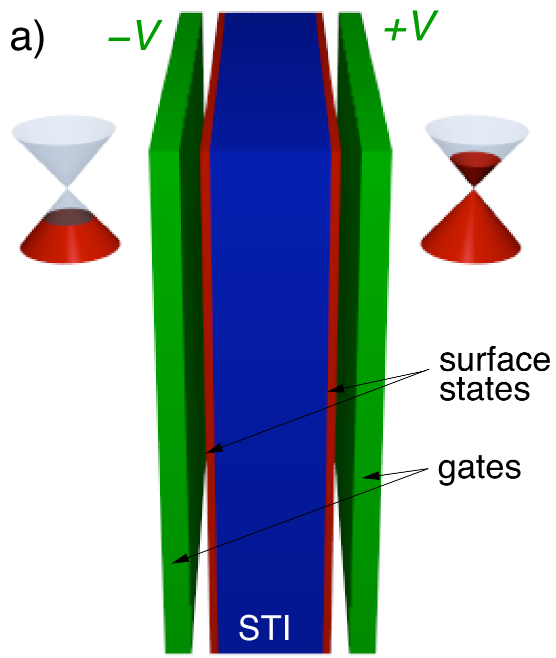
What about a strong electric field? Are there new *correlated* phases?

Correlated phases from TI surfaces

Idea of exciton condensation:

(Conventional) Superconductivity occurs when we have identical spin-up and spin-down electron Fermi surfaces and a weak attractive interaction.

Exciton condensation occurs when we have identical *electron* and *hole* Fermi surfaces and an attractive interaction between electrons and holes, i.e., Coulomb *repulsion*.



Why is this difficult? Need an applied field or some other mechanism to keep electrons and holes from recombining.

Alternately can study nonequilibrium condensation before electrons & holes recombine (Butov, Chemla; Dang et al.)

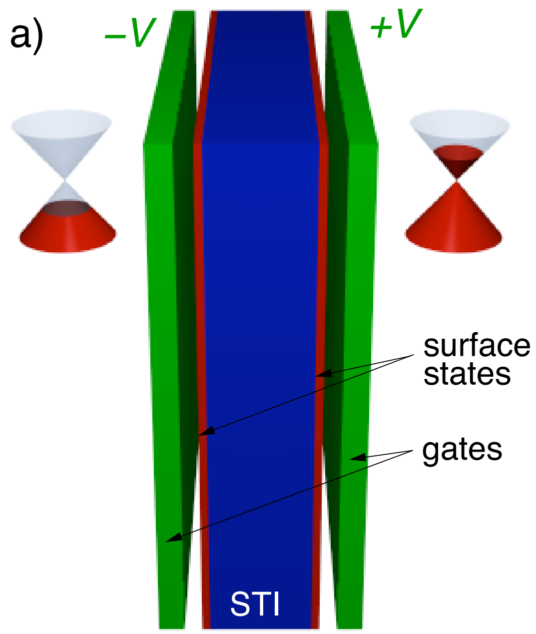
Correlated phases from TI surfaces

An "exciton condensate" can be made between holes on one surface of a 3D TI film and electrons on the other surface. This state is loosely similar to that proposed for graphene bilayers (Bistritzer, Su, MacDonald; Seradjeh, Weber, Franz), but with different topological features. (Seradjeh, JEM, Franz, PRL '09)

The Dirac dispersion enables the desired electron-hole Fermi surfaces.

Experimentally, a possible advantage is that one can use the two surfaces of a single TI, rather than having to insert a dielectric of constant thickness through two layers of graphene.

A vortex in the exciton order parameter traps a "zero-energy mode" (not a Majorana) with an offset fractional charge $\pm e/2$ relative to the zero-vortex state.



(In graphene, inter-valley mixing moves this mode away from E=0. Here, particle-hole symmetry is sufficient to stabilize it.)

Correlated phases from TI surfaces

Formally, exciton condensation is like BCS in the "particle-hole" channel: continuously connected to BEC of excitons.

$$H_{\rm MF} = H_0 + (\psi_1^\dagger M \psi_2 + {\rm h.c.}) + \frac{1}{U}{\rm Tr}(M^\dagger M),$$

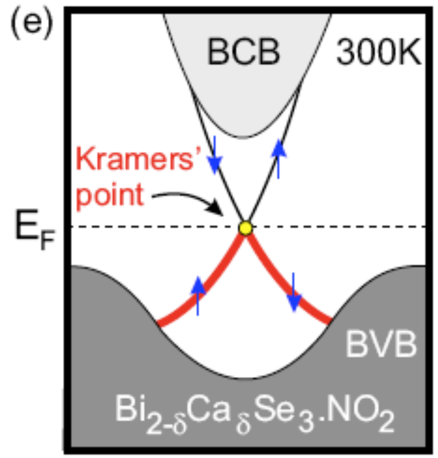
Key: unscreened interlayer Coulomb repulsion, with no tunneling.

Generated 1-particle gap in weak-coupling limit:

$$m \approx 2\sqrt{V\Lambda}e^{-\Lambda^2/UV}$$

Need large voltage V and coupling U, with chemical potentials symmetric around Dirac point.

New materials (e.g., Ca doping) allow the Dirac point to be moved out of the bulk bandgap.



Fermi-level control in crystals (Hsieh et al., 2009)

Transition temperature is of same order or higher than in graphene.

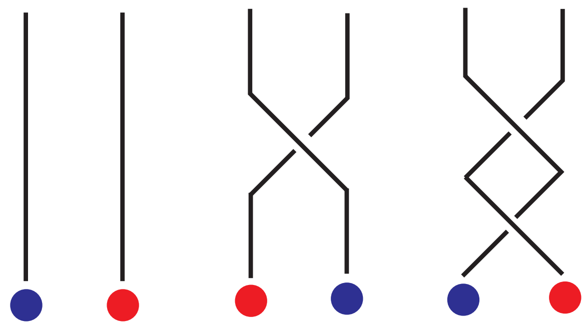
Goal: first stable exciton condensate outside quantum Hall regime.

Statistics in 2D

What makes 2D special for statistics? (Leinaas and Myrheim, 1976)

Imagine looping one particle around another to detect their statistics. In 3D, all loops are equivalent.

In 2D, but not in 3D, the result can depend on the "sense" of the looping (clockwise or counterclockwise). Exchanges are not described by the permutation group, but by the "braid group".



The effect of the exchange on the ground state need not square to 1. "Anyon" statistics: the effect of an exchange is neither +1 (bosons) or -1 (fermions), but a phase.

$$e^{i\theta}$$

Most fractional quantum Hall states, such as the Laughlin state, host "quasiparticles" with anyonic statistics.

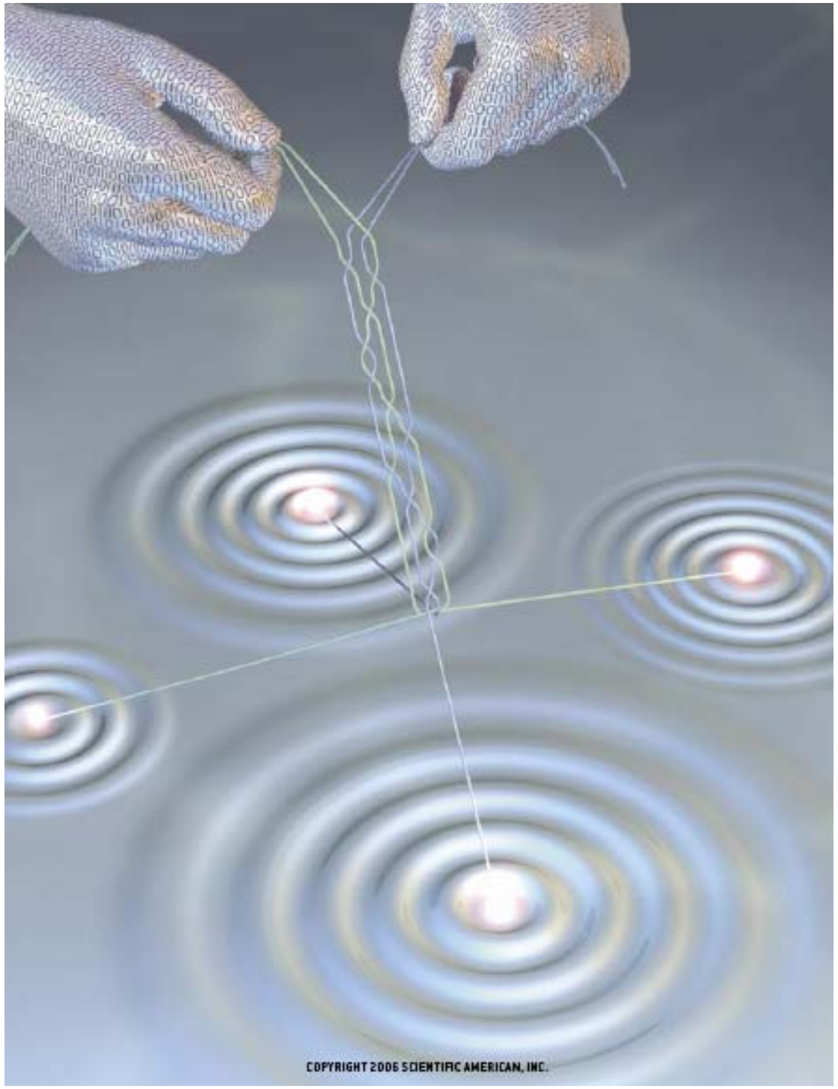
Topological quantum computing

A classical computer carries out logical operations on classical "bits".

A **quantum computer** carries out unitary transformations on "qubits" (quantum bits).

A remarkable degree of protection from errors can be obtained by implementing these via braiding of non-Abelian quasiparticles:
braiding acts as a matrix on a degenerate space of states.

The relevant quasiparticle in the Moore-Read state is a "Majorana fermion":
it is its own quasiparticle
and is "half" of a normal fermion.



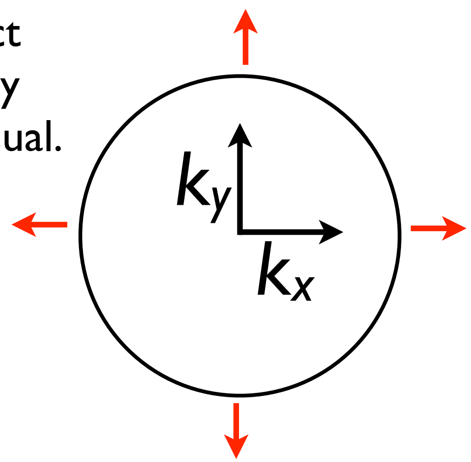
Topological insulators for topological quantum computing

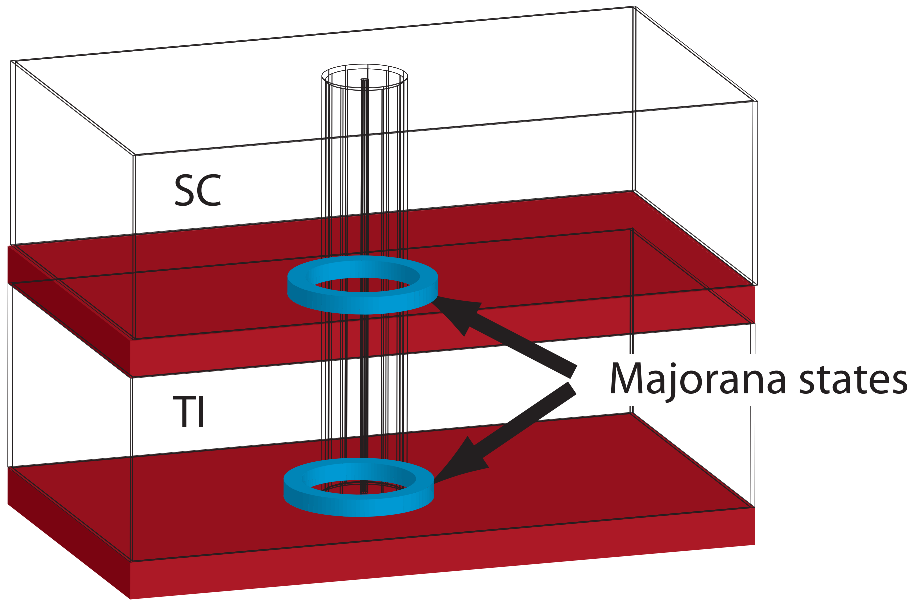
It is believed that the core of a magnetic vortex in a two-dimensional "*p+ip*" superconductor can have a Majorana fermion. (But we haven't found one yet.)
A Majorana fermion is its own antiparticle and is "half" of a (spinless) Dirac fermion.

However, a superconducting layer with this property exists at the boundary between a 3D topological insulator and an ordinary 3D superconductor (Fu and Kane, 2007).

$$H = \sum_{\mathbf{k}} (\Delta c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} + h.c.)$$

This proximity effect involves *half* as many electronic states as usual.

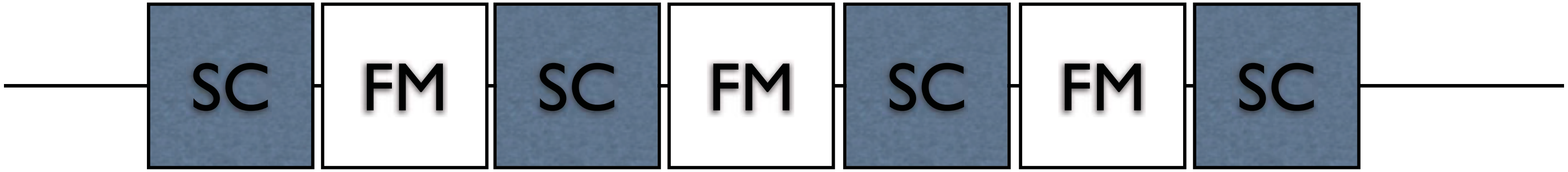




Majorana fermion chains at TI edges

(V. Shivamoggi, G. Refael, and JEM, arXiv:)

Imagine superconducting and ferromagnetic regions randomly distributed along the quantum spin Hall edge.



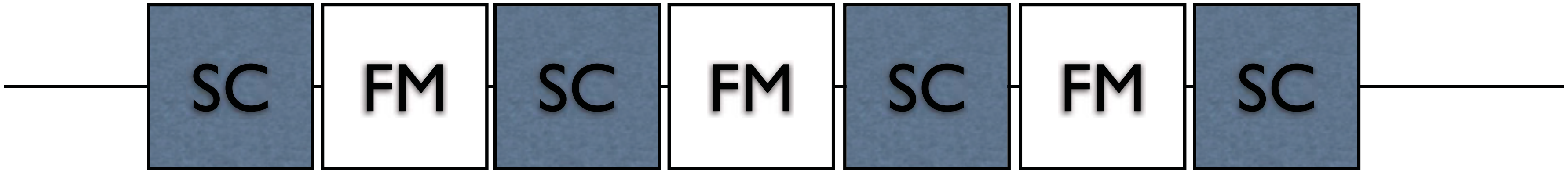
At each SC/FM boundary there is a local Majorana fermion (Kitaev; Fu and Kane; Beenakker et al.).

The Hamiltonian if the SC and FM regions are large is H=0. When tunneling becomes possible, we realize the random Majorana hopping problem,

$$H = i \sum_j t_j \gamma_j \gamma_{j+1}$$

= random quantum Ising (Bonesteel and Yang, PRL 2006)

Majorana fermion chains at TI edges



$$H = i \sum_j t_j \gamma_j \gamma_{j+1}$$

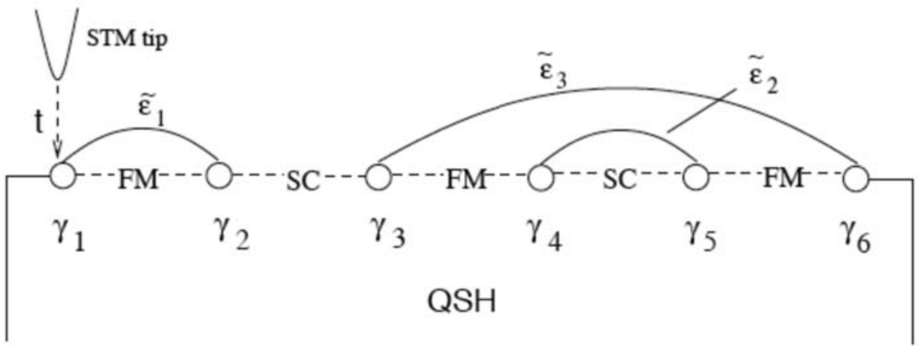
= random quantum Ising (Bonesteel and Yang, PRL 2006)

This is a random critical *c=1/2* delocalization problem with one condition on the SC and FM distributions:
(SC and FM have a duality like h and J in quantum Ising: both are U(1) because one direction of the FM just shifts the chemical potential and can be neglected.)

It is very similar to the *c=1* particle-hole symmetric Dirac chain, which maps onto the XX spin chain.

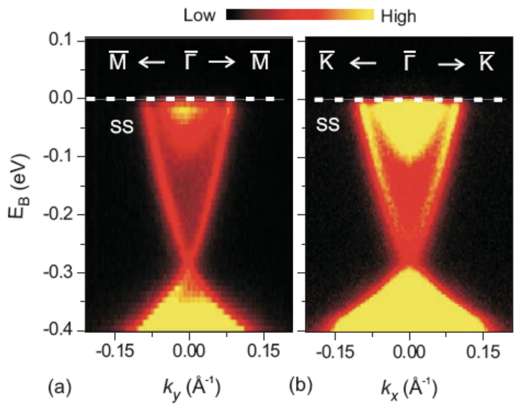
New feature: transport can be studied by RSRG:

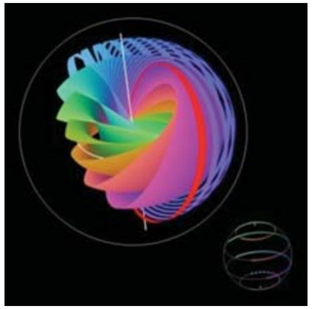
->STM spectra (peak locations and strength)



Conclusions

1. "Topological insulators" exist in two and three dimensions in zero magnetic field.





2. The 3D topological insulator generates a quantized magnetoelectric coupling

$$\Delta \mathcal{L}_{EM} = \frac{\theta e^2}{2\pi h} \mathbf{E} \cdot \mathbf{B} = \frac{\theta e^2}{16\pi h} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}. \qquad \alpha^i_j = \tilde{\alpha}^i_j + \alpha_\theta \delta^i_j.$$

that is one part of the magnetoelectric coupling in general insulators.

3. An electrical bias across a thin film leads to a "topological exciton condensate." Topological insulators may enable other correlated phases and applications.

Are there correlated topological insulators? Are there fractional 3D states?

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See also reviews by Hasan and Kane (RMP colloquium) and Qi and Zhang (Physics Today).