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Workshop on Localization Phenomena in Novel Phases of Condensed Matter

17 - 23 May 2010

Localization Phenomena in the Quantum Kicked Rotor

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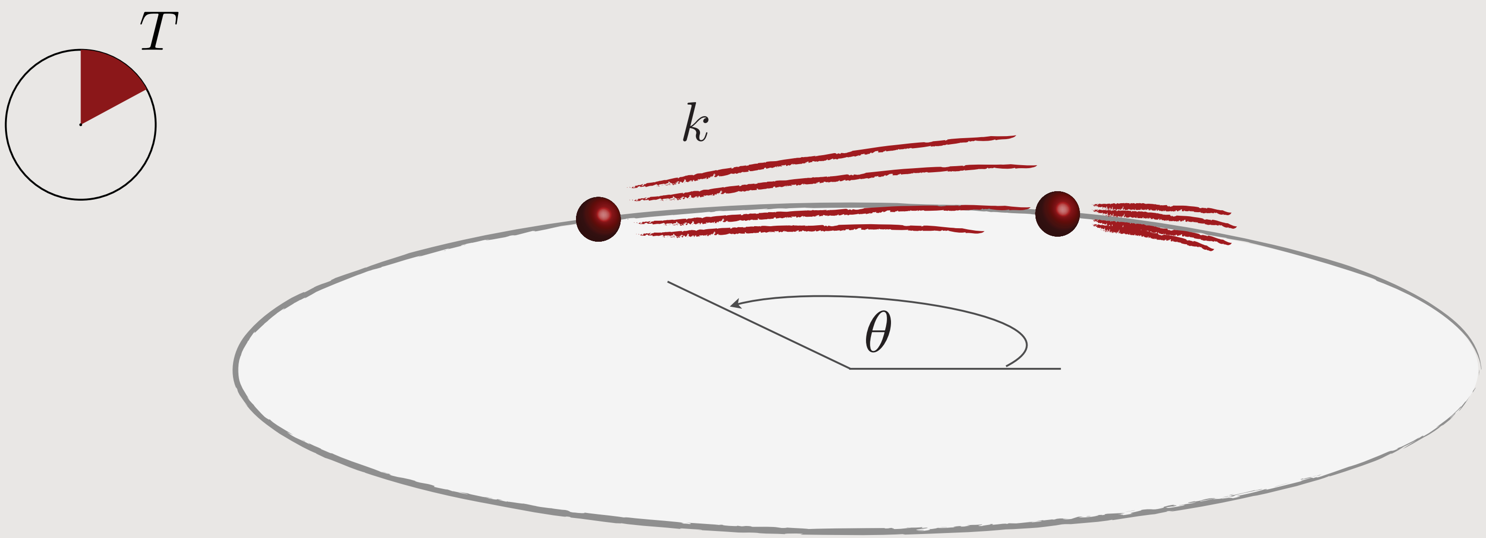

$$\hat{H}(t) = \frac{\hat{l}^2}{2} + k\cos(\hat{\theta})\sum_m \delta(t - mT)$$

Chirikov, 79

QKR is:

1. a paradigmatic model system of quantum chaos

2. experimentally realizable

3. a system of rich phenomenology

Atom Optics Realization of the Quantum δ-Kicked Rotor

F. L. Moore,* J. C. Robinson, C. F. Bharucha, Bala Sundaram, and M. G. Raizen

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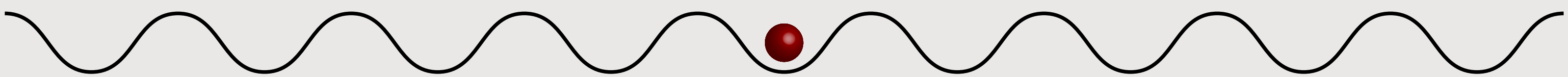
(Received 21 July 1995)

We report the first direct experimental realization of the quantum δ-kicked rotor. Our system consists of a dilute sample of ultracold sodium atoms in a periodic standing wave of near-resonant light that is pulsed on periodically in time to approximate a series of delta functions. Momentum spread of the atoms increases diffusively with every pulse until the "quantum break time" after which exponentially localized distributions are observed. Quantum resonances are found for specific values of the pulse period.



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field theory approach to the quantum kicked rotor

Trieste May 18th, 2010

Alexander Altland & Chushun Tian, Cologne University

- kicked rotor
- and its field theory

arXiv:1001.4322, NJP 12 43043 (2010)

kicked rotor

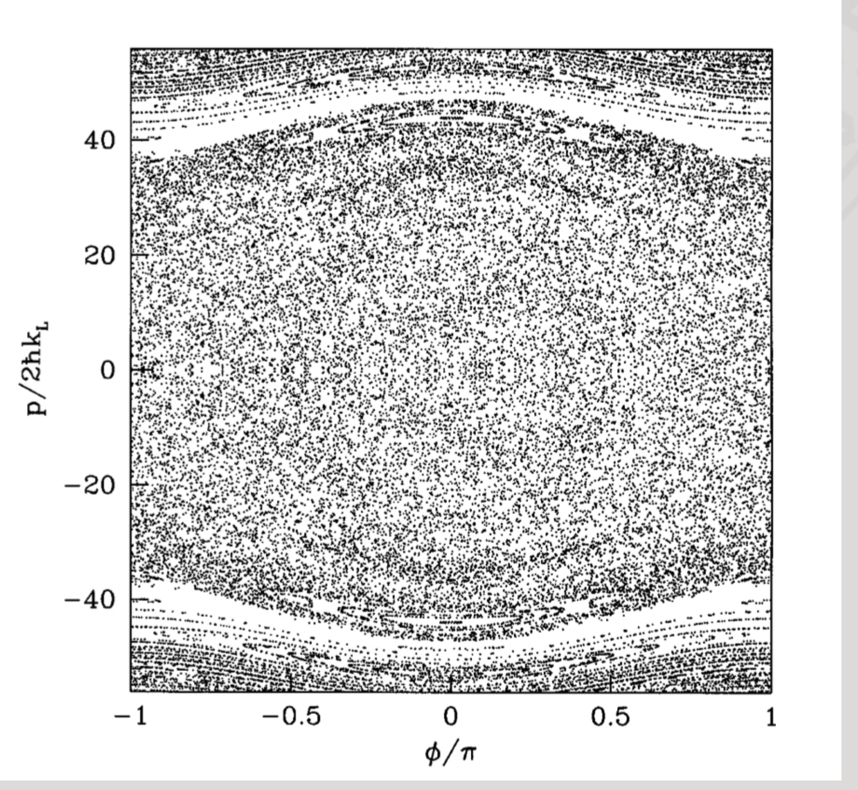
$$\boxed{\hat{H}(t) = \frac{\hat{l}^2}{2} + k\cos(\hat{\theta})\sum_m \delta(t - mT)}$$

Chirikov, 79

- classical system characterized by a single parameter $K = kT$.
- quantum system sensitive to value of $\tilde{h} \equiv \hbar T$.
- here:

$\tilde{h} \ll K$ (semiclassical dynamics)

$K \gg 1$ (chaotic dynamics)



standard map: analogies to disordered systems

standard map ← Fishman et al. 82 → disordered quantum systems

standard map	disordered quantum systems
chaotic scattering	impurity scattering
correlations in angular momentum space	velocity correlations

- Rechester-White (1980) corrections ($1/K$ corrections to diffusion constant)
- ϵ -classics, reentrant classical behaviour at $\tilde{h} = 4\pi + \epsilon$ (Fishman, 2004)
- $\tilde{h}$-resonances, absence of localization at $\tilde{h} = 4\pi p/q$ (Casati et al., 1979)
- accelerator modes, regular islands at special values of K (Karney, 1983)

standard map	disordered quantum systems
diffusion	diffusion
localization	localization

Floquet operator:

$$\boxed{\hat{U} = \exp\left(\frac{i\tilde{h}\hat{n}^2}{4}\right)\exp\left(\frac{iK\cos\hat{\theta}}{\tilde{h}}\right)\exp\left(\frac{i\tilde{h}\hat{n}^2}{4}\right)}$$

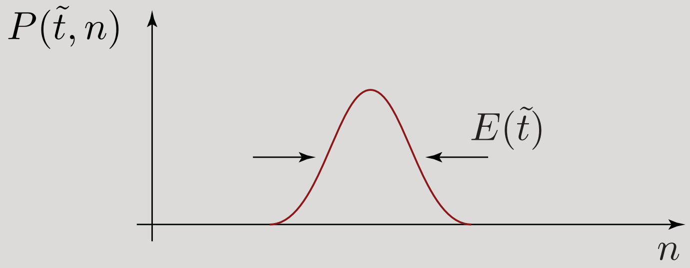
Describe system in terms of (angular momentum) correlation function

$$K_{\tilde{t}}(n) \equiv \left\langle |\langle n+n_0|(\hat{U})^{\tilde{t}}|n_0\rangle| \right\rangle_{n_0}$$

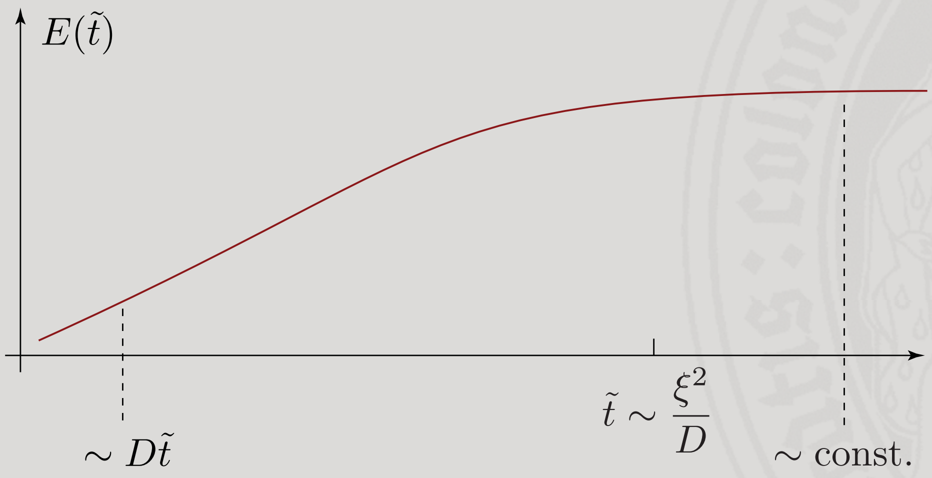
time in units of kick time

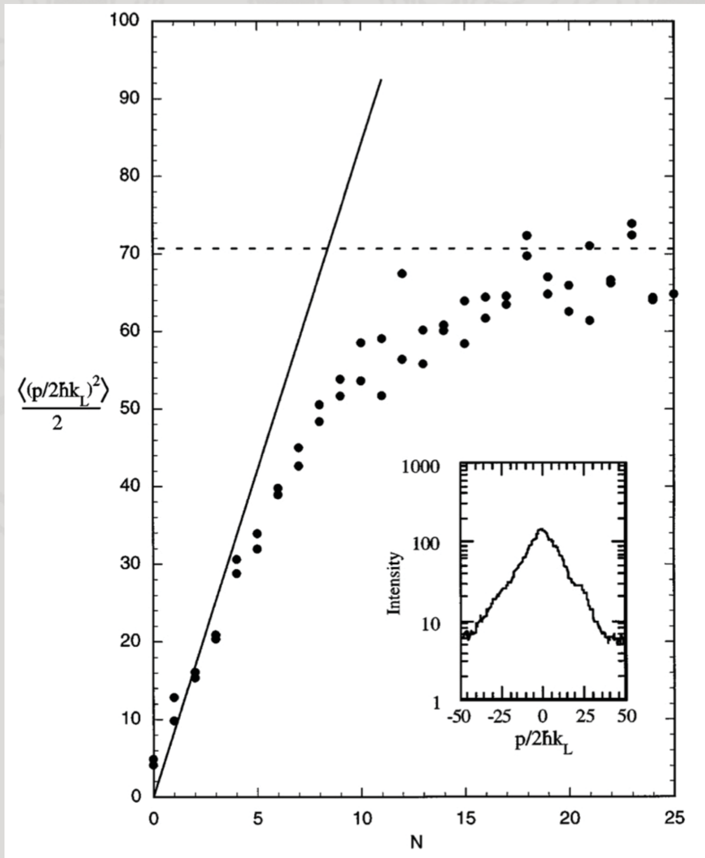
and (kinetic) energy increase

$$E(\tilde{t}) \equiv \frac{1}{2}\sum_n n^2 K_{\tilde{t}}(n)$$



Expected behavior of energy increase





$$D = \left(\frac{K}{2\tilde{h}}\right)^2$$: diffusion constant

$\xi \sim D$: localization length (Chirikov et al. 81)

Moore *et al.* 95

QKR resonances

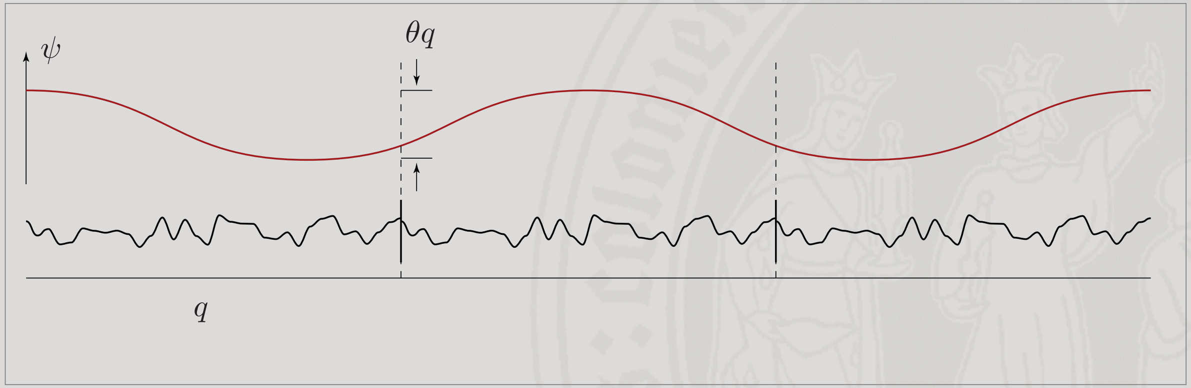
For $\tilde{h} = 4\pi\frac{p}{q}, \;\; p, q \in \mathbb{Z}$ different things happen (Izrailev & Shepelyansky, 81)

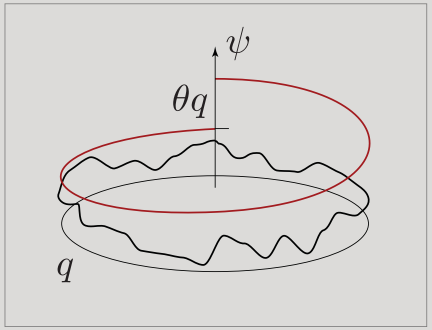
Floquet operator:

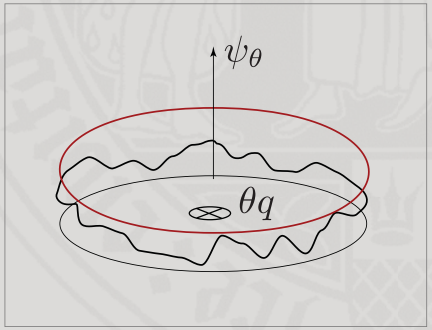
$$\boxed{\hat{U} = \exp\left(\frac{i\tilde{h}\hat{n}^2}{4}\right)\exp\left(\frac{iK\cos\hat{\theta}}{\tilde{h}}\right)\exp\left(\frac{i\tilde{h}\hat{n}^2}{4}\right)}$$

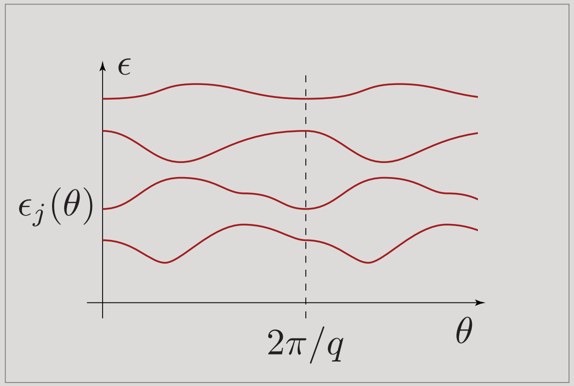
At resonant values: $[\hat{T}_q, \hat{U}] = 0$ $\quad\quad$ $\hat{T}_q|n\rangle = |n+q\rangle$: translation operator

reminder: Bloch picture of periodic quantum systems









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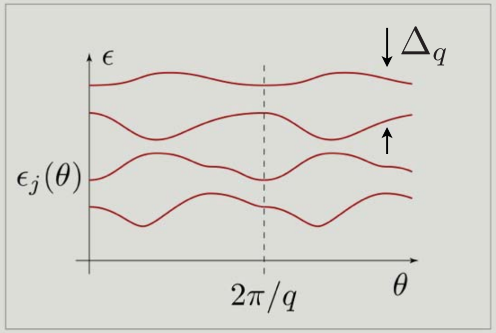
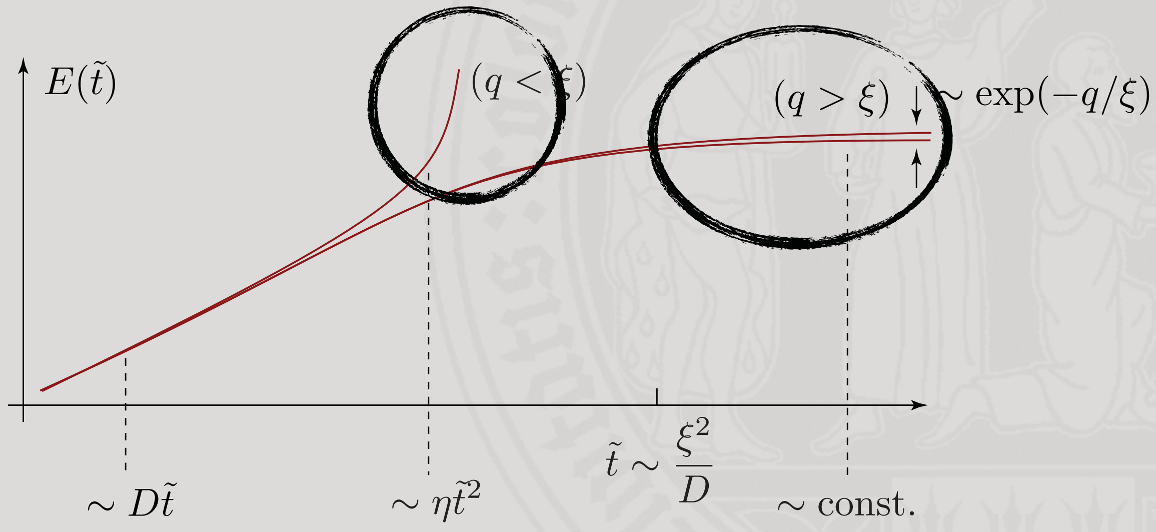
Consequences (phenomenological):



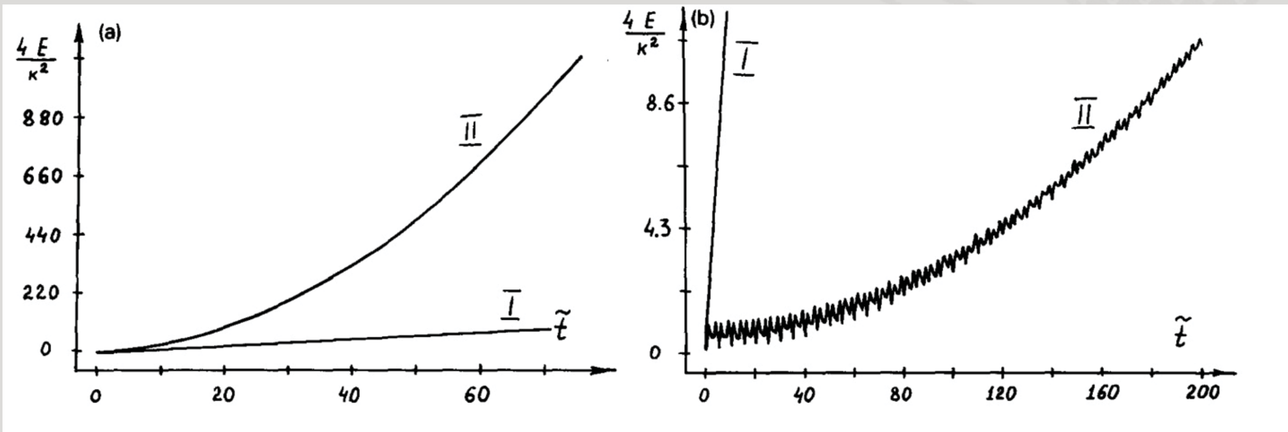
- ▷ energy expectation value: $E(\tilde{t}) = -\frac{1}{2}\sum_{n=1}^{q}\int d\theta\, \partial^2_{\theta,\theta'}\Big|_{\theta=\theta'} K_{\tilde{t}}(n,\theta,\theta')$
- ▷ short times, $\tilde{t}^{-1} > \Delta_q \equiv 2\pi/q$: diffusion, $E(\tilde{t}) \sim D\tilde{t}$
- ▷ large times, $\tilde{t}^{-1} < \Delta_q$: single level parametric correlations, $E(\tilde{t}) \sim \eta\tilde{t}^2$

QKR resonances (cont'd)

Large scale picture (Izrailev & Shepelyansky, 80)



Numerics:



field theory

prototypical field theory of the rotor, Zirnbauer, aa 96

In conclusion, the theory [1] in its present form may be expected to be valid only when a sort of band random matrix approach can be used *ad hoc*. However, it fails to take into account dynamical features which go beyond the random matrix theory description. A careful investigation of the conditions of validity of the suggested theory is needed.

Casati, Izrailev, Sokolov, 96

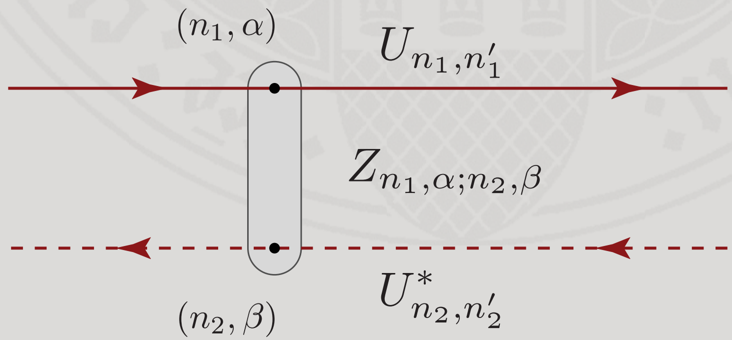
Field integral representation

Zirnbauer, aa 96:

$$K_\omega(n_1, n_2) = \int dZ\, e^{-S[Z]}(\dots),$$

$$S[Z] = -\frac{1}{2}\text{str}\,\ln(1 - Z\tilde{Z}) + \frac{1}{2}\text{str}\,\ln(1 - e^{i\omega}\hat{U}^\dagger Z \hat{U} \tilde{Z})$$

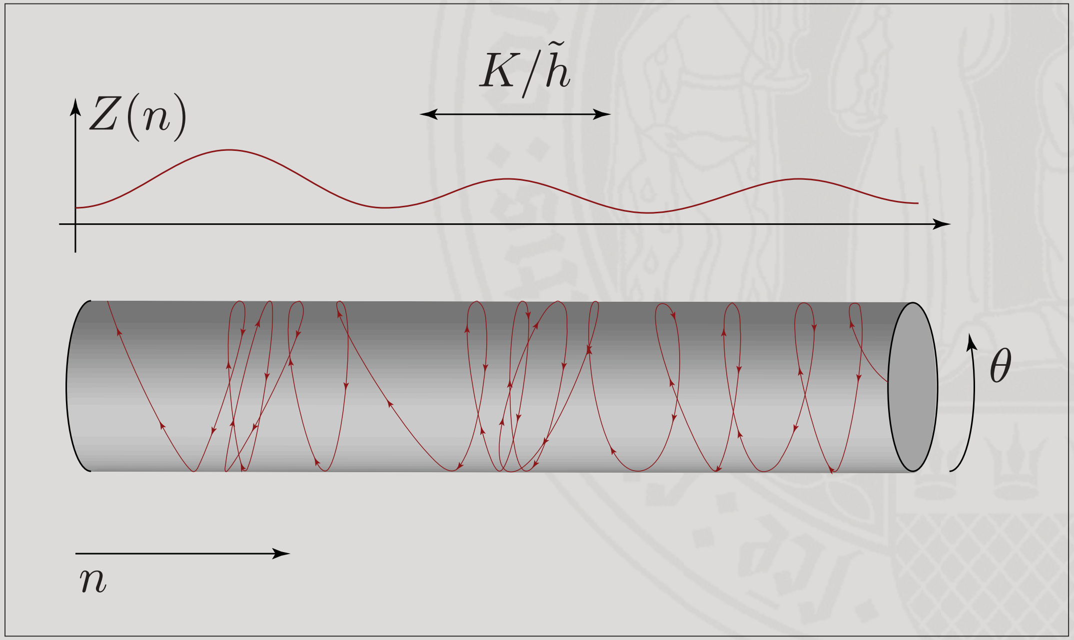
$$\begin{array}{ccc} Z_{n_1,n_2} & \longrightarrow & (UZU^\dagger)_{n_1,n_2} \\ \downarrow & & \downarrow \\ \rho_{n_1,n_2} & \longrightarrow & (U\hat{\rho}U^\dagger)_{n_1,n_2} \end{array}$$



Effective action

soft modes: fluctuations whose action can be continuously tuned to zero. For generic irrational values of $\tilde{h}$

$$Z_{n_1,\alpha;n_2,\beta} \longrightarrow Z_{\alpha,\beta}(n,\theta) \longrightarrow B_{\alpha,\beta}(n), \qquad \Delta n \gtrsim K/\tilde{h}$$



all other field configurations ($C_{\alpha,\beta}(n,\phi)$) are massive (a linear decomposition $Z = B + C$)

$$\boxed{e^{-S[B]} = \int DC\, e^{-S[B+C]}}$$

Effective low energy action

Integration over gapped modes produces

$$Q \equiv g \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} g^{-1}, \qquad g \equiv \begin{pmatrix} 1 & B \\ \tilde{B} & 1 \end{pmatrix}.$$

$$S[Q] = \frac{1}{16} \int_0^q dn \, \mathrm{str} \left(D_q (i\partial_n - [\hat{\theta}, \,]) Q (i\partial_n - [\hat{\theta}, \,]) Q - 2i\omega Q \sigma^3_{\mathrm{AR}} \right).$$

Bloch angle

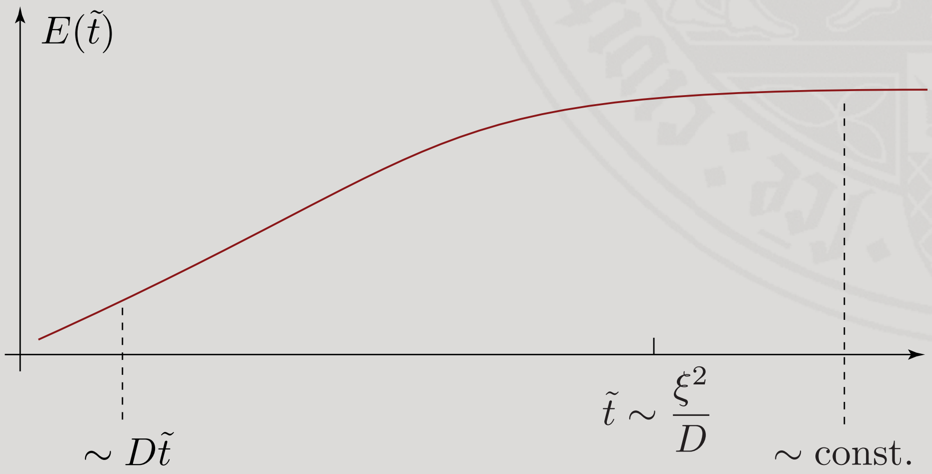
- nonlinear sigma model action on a ring subject to (Bloch gauge) flux
- similar to effective action of disordered multichannel quantum wire
- action encapsulates symmetries/system properties of the rotor
  - $D_q = D_0(1 - 2J_2(K_q) - 2J_1^2(K_q) + 2J_3^2(K_q)) + \ldots \qquad K_q \equiv \frac{2K}{\tilde{h}} \sin\left(\frac{\tilde{h}}{2}\right)$

    (Shepelyansky 87)
  - epsilon-classics symmetry (Fishman, 03)
  - resonances

off resonance $(q \gg \xi)$

Localization theory (Efetov & Larkin, 83) translated to the present context (Zirnbauer, a.a., 96):

$$E(\tilde{t}) \simeq \begin{cases} D_q\tilde{t} - \frac{4}{3\sqrt{\pi}} D_q \tilde{t}^{3/2}, & 1 \ll \tilde{t} \ll D_q, \\ D_q^2, & t \gg D_q \end{cases}$$



on resonance $(q \ll \xi)$

Action

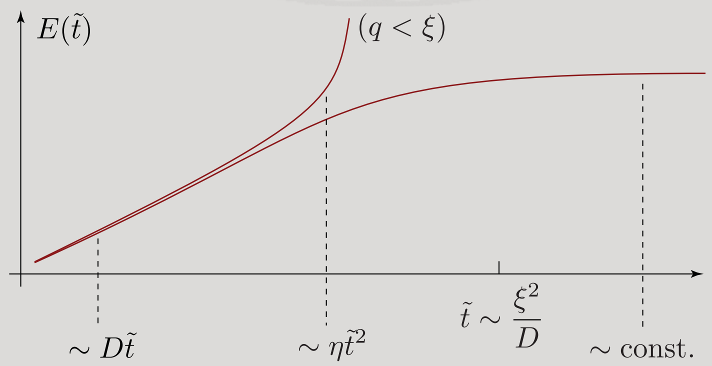
$$S[Q] = \frac{1}{16}\int_0^q dn\,\mathrm{str}\left(D_q(i\partial_n - [\hat\theta,\ ])Q(i\partial_n - [\hat\theta,\ ])Q - 2i\omega Q\sigma^3_{\mathrm{AR}}\right).$$

describes parametric correlations in mesoscopic systems (Simons & Altshuler, 93, Taniguchi & Altshuler, 93)

Result:

$$\boxed{\begin{aligned} &E(\tilde t) = (D_q q)\, F\left(\frac{\tilde t}{q}\right), \\ &F(x) = \begin{cases} x + \frac{x^3}{3}, & q^{-1} \ll x < 1 \\ x^2 + \frac{1}{3}, & x > 1. \end{cases}, \end{aligned}}$$

cf. Wojcik & Dorfman, 84



the present theory:

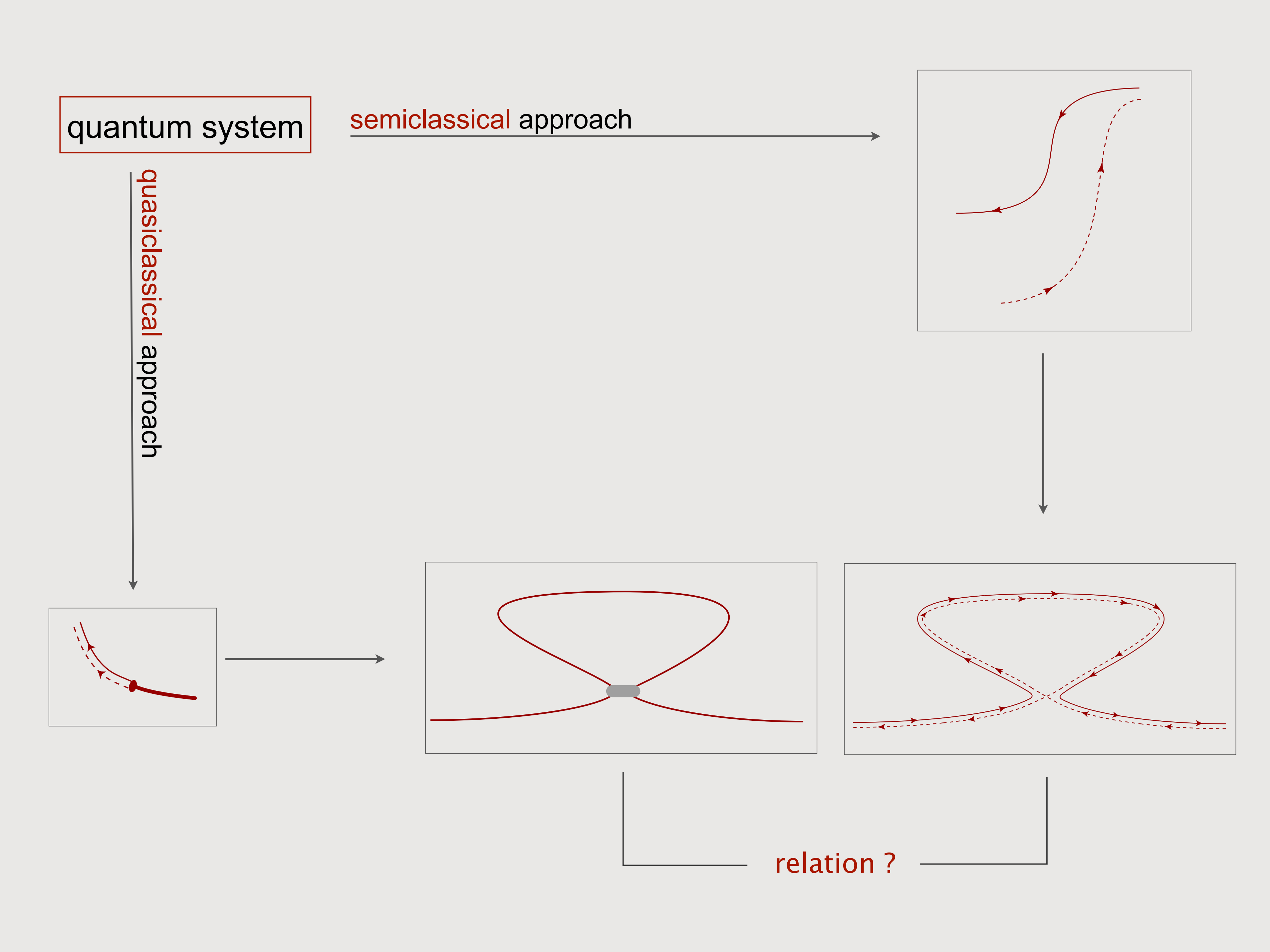
▷ analytically describes crossover ballistic motion/diffusion/localization

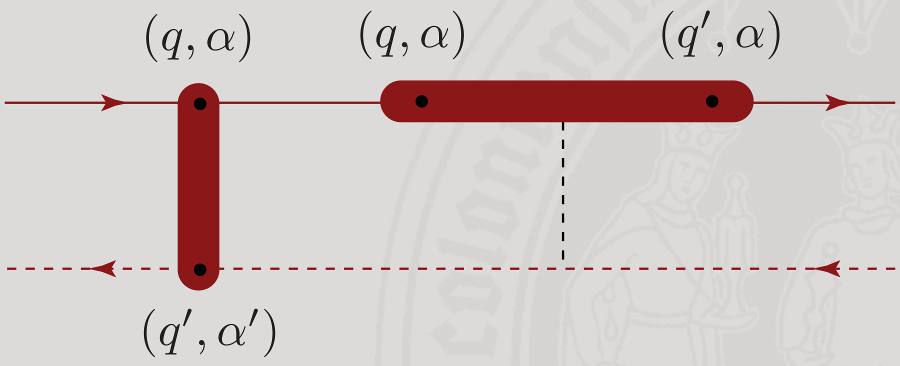
▷ knows about QKR ‘fine structure’ (Shepelyansky/Rechester White corrections to diffusion constant, epsilon classics, resonances)

▷ provides analytical results for correlation functions on and off resonance

▷ for specific parameter constellations (accelerator modes, certain arithmetic values of Planck’s constant), the construction of the theory in its present form breaks down

general ramifications


quantum system
semiclassical approach
quasiclassical approach
relation ?



	semiclassics	quasiclassics
degrees of freedom	$\bar\psi_{q\alpha}\,\psi_{q',\alpha} \to \mathcal{A}_\alpha(q,q')$	$\psi_{q\alpha'}\,\bar\psi_{q',\alpha'} \to Q_{\alpha,\alpha'}(\mathbf{x})$
formulation of theory	path integral	field integral
effective theory	path superposition (e.g. Gutzwiller double sum)	effective field theory in phase space
applications	single particle	many particle

Applications:

- ▷ field theory of standard map (Zirnbauer, aa, 96; Tian, aa, 10)
- ▷ universal spectral correlations in quantum graphs (Gnutzmann, aa, 05)
- ▷ strong Anderson localization in arrays of chaotic cavities (Brouwer, aa, 08)
- ▷ spectral gap of the Andreev billiard (Micklitz, aa, 09)
- ▷ universal spectral correlations from ballistic sigma model (Micklitz, Müller, aa, 06)

In cases of overlap, full agreement with semiclassics; non-perturbative applications

Conceptual status:

- ▷ soon after introduction (Altshuler et al., 95; Khemlnitskii & Muzykhantskii, 95): conceptual problems (‘weak localization problem’, ‘mode locking problem’, ‘zero mode problem’, ‘repetition problem’, ...)
- ▷ ... these are basically solved (personal opinion)

Effective action cont'd

quadratic integration over massive modes:

$$S_{\mathrm{fl}}[B] = \frac{1}{2}\mathrm{str}\left(\tilde{B}\left(1 - \mathrm{ad}_U \frac{1}{1 - \hat{\pi}\,\mathrm{ad}_U}\right) B\right)$$

projector on massive modes

$\mathrm{Ad}_U(B) \equiv \hat{U} B \hat{U}^\dagger$

explicit representation

$$S_{\mathrm{fl}}[B] = -\frac{D_{\mathrm{eff}}}{2}\int dn\,\mathrm{str}(B(n)\partial_n^2\tilde{B}(n))$$

$$D_{\mathrm{eff}} \simeq D^0(1 - 2J_2(x) - 2J_1^2(x) + 2J_3^2(x)), \qquad x = \frac{2K}{\tilde{h}}\sin\left(\frac{\tilde{h}}{2}\right)$$

Shepelyansky (1987)

Effective action (cont'd)

generalization beyond quadratic order

$$S[Q] = \frac{1}{16}\int dn\,\mathrm{str}\left(-D_q\partial_n Q\partial_n Q - 2i\omega Q\sigma^3_{\mathrm{ar}}\right)$$

$$Q \equiv g\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}g^{-1}, \qquad g \equiv \begin{pmatrix}1 & B\\ \tilde{B} & 1\end{pmatrix}.$$

discussion:

- ▷ effective low energy theory of disordered multi-channel quantum wire (Efetov & Larkin, 80)
- ▷ diffusion at length scales $\frac{K}{\tilde{h}} < \Delta n < \xi \equiv \frac{D_q}{2} \sim \left(\frac{K}{\tilde{h}}\right)^2$
- ▷ localization at length scales $\Delta n > \xi$
- ▷ at rational values $\tilde{h} = 4\pi\frac{p}{q}$: system effectively acquires ring topology. Circumference $\sim q$; (nonabelian) gauge flux.
- ▷ theory sensitive to ϵ-classics, reentrant classical behaviour at $\tilde{h} = 4\pi + \epsilon$

- quasiclassical methods can be powerful alternative to semiclassics
- often implemented in field theoretical framework
- good in non-perturbative settings (localization, gap formation, non-perturbative correlations)
- and not-so-good in others (repetitive dynamics, low order perturbation theory)

time

It is good to know both, semiclassical and quasiclassical concepts