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# Electric Transport in Carbon Nanotubes and Graphene

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*New York*
*U.S.A.*

# Electric Transport in Nanotubes and Graphene

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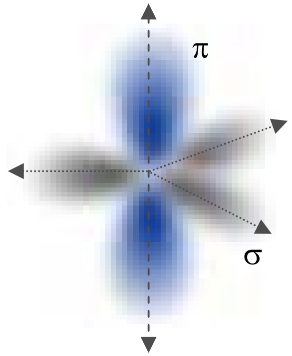
Department of Physics
Columbia University

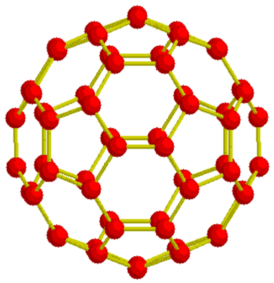
# $SP_2$ Carbon: 0-Dimension to 3-Dimension



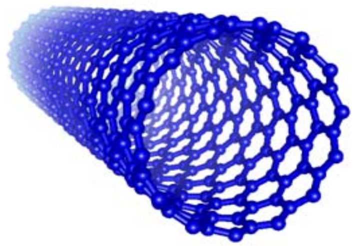

**Atomic orbital $sp_2$**

**0D** **1D** **2D** **3D**

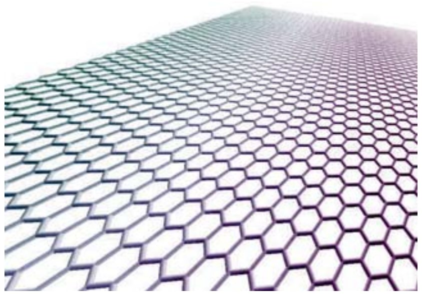
**Fullerenes ($C_{60}$)**



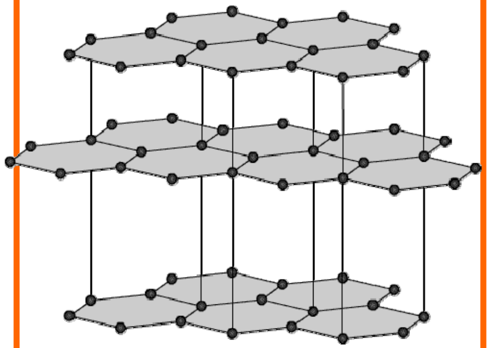
**Carbon Nanotubes**



**Graphene**

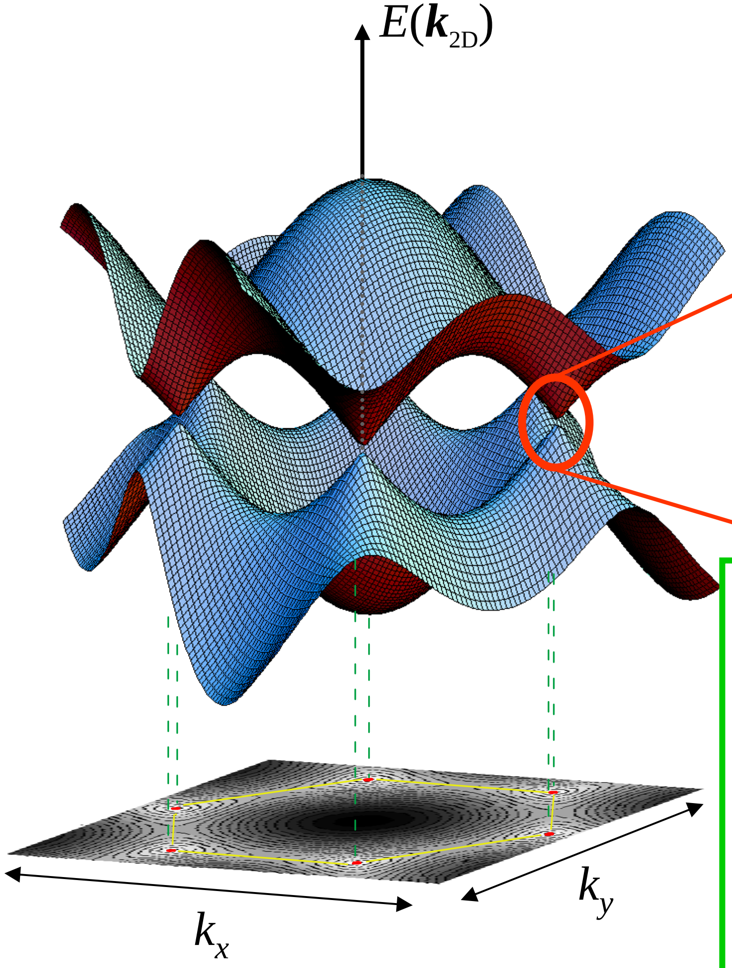


**Graphite**

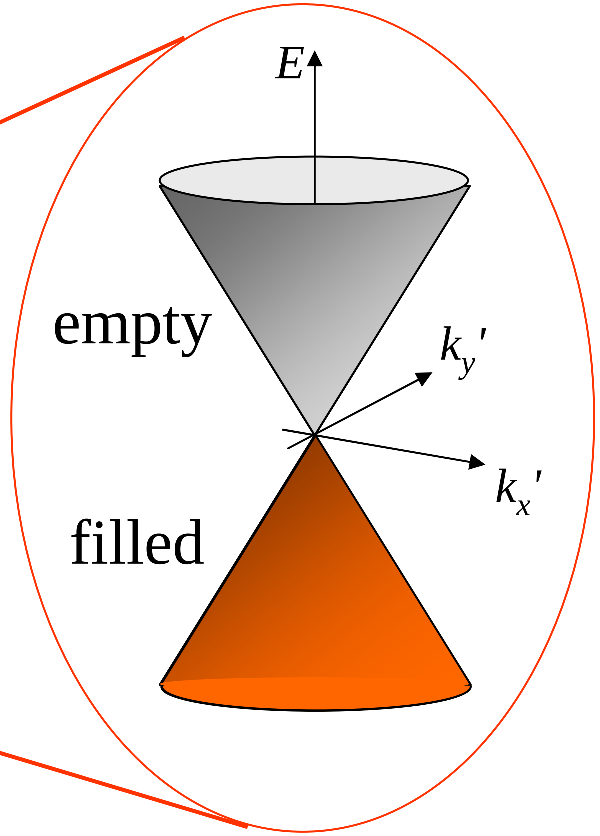


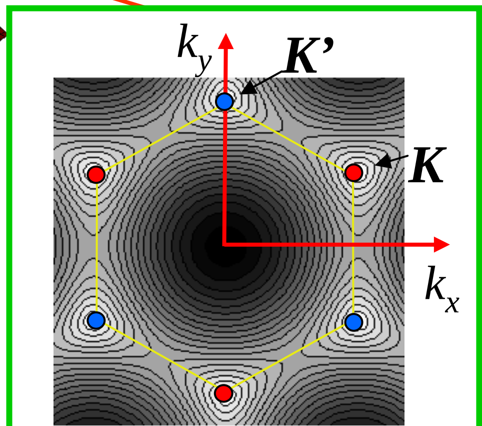
# Electronic Band Structure of Graphene

Band structure of graphene (Wallace 1947)




$$E \approx \hbar v_F \left| \vec{k}'_\perp \right|$$



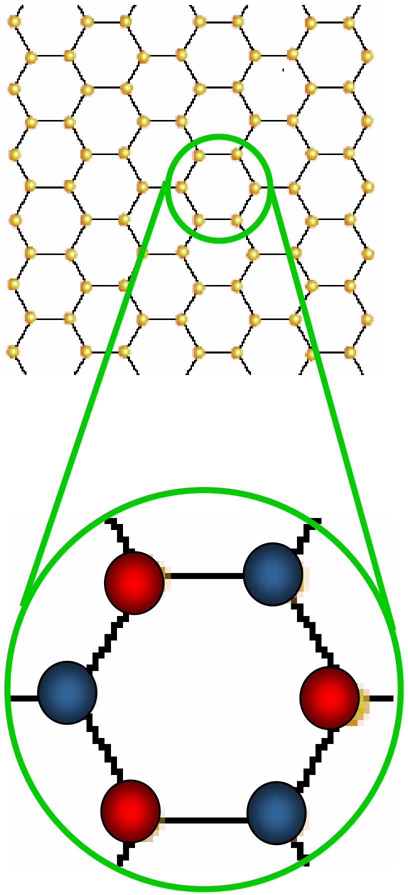




**2D Brillouin Zone**

Zero effective mass particles moving with a constant speed $v_F = c/300$

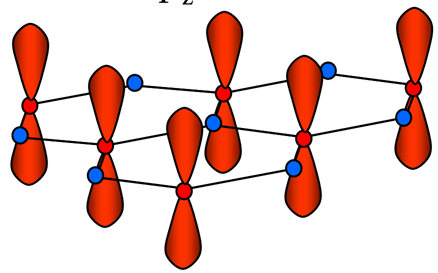
# Graphene Lattice Symmetry: Pseudo Spinor

**Graphene Lattice Structures**



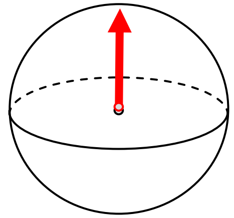
Two inequivalent lattice sites!

‘**A**’ sublattice: $p_z$ orbitals

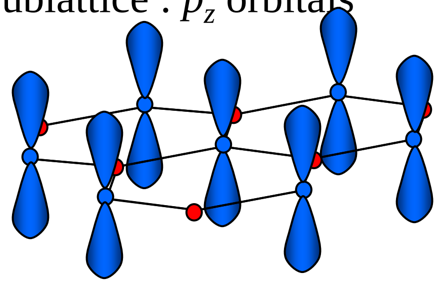


$$|A\rangle$$

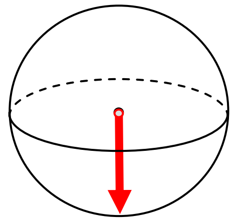
**Spinor Representation**



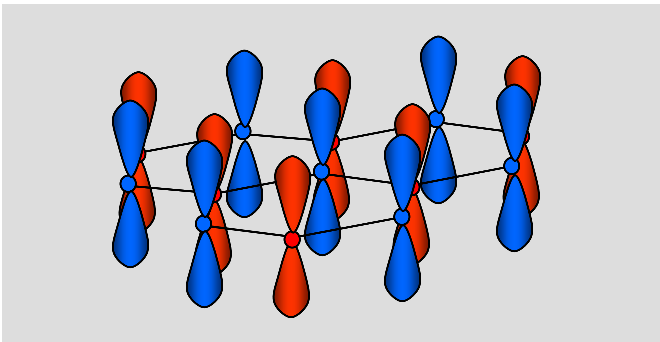
‘**B**’ sublattice : $p_z$ orbitals

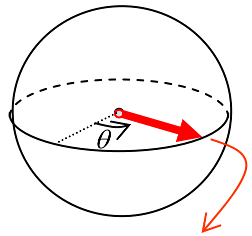


$$|B\rangle$$



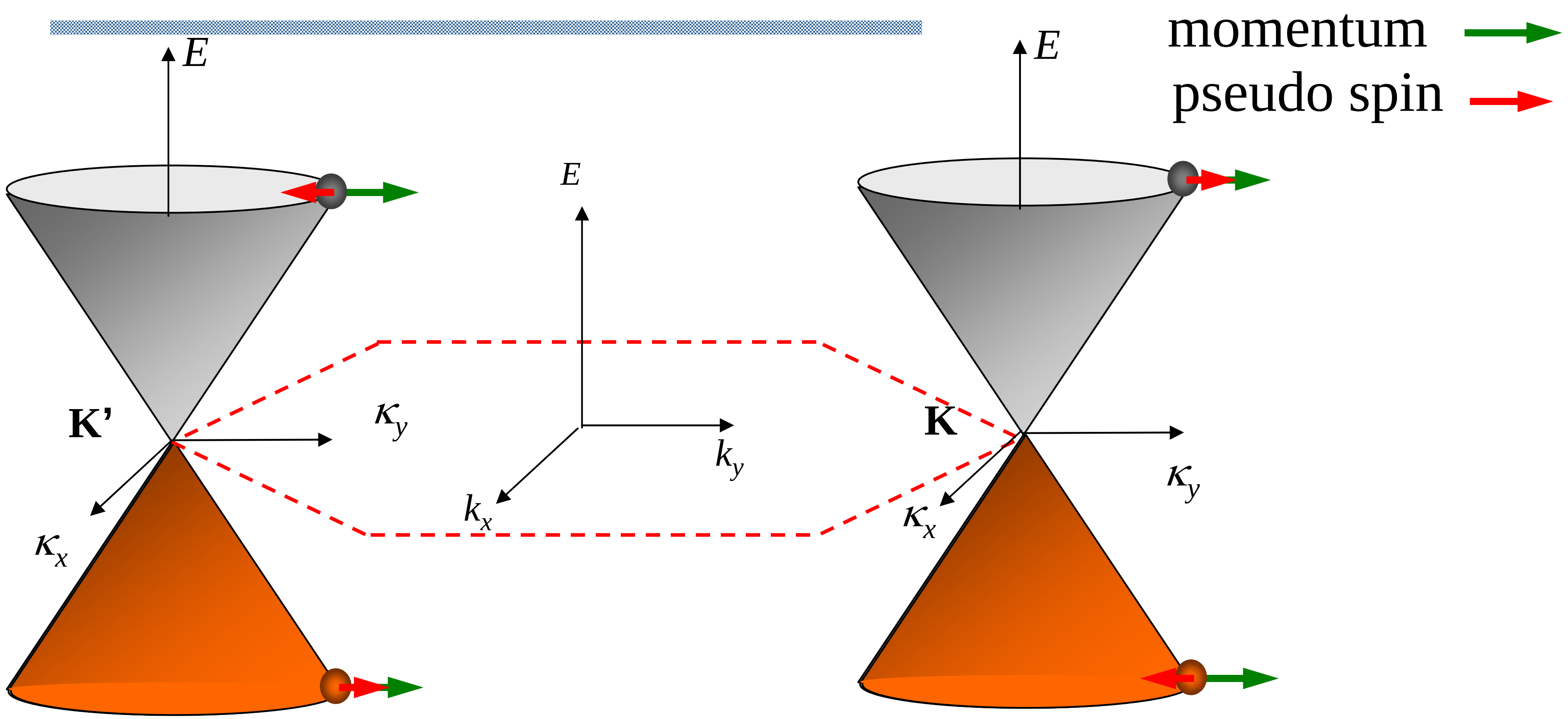
Superposition: $|A\rangle + e^{i\theta}|B\rangle$






**Pseudo spin**

# Dirac Fermions in Graphene : "Helicity"




**Effective Dirac Equations**

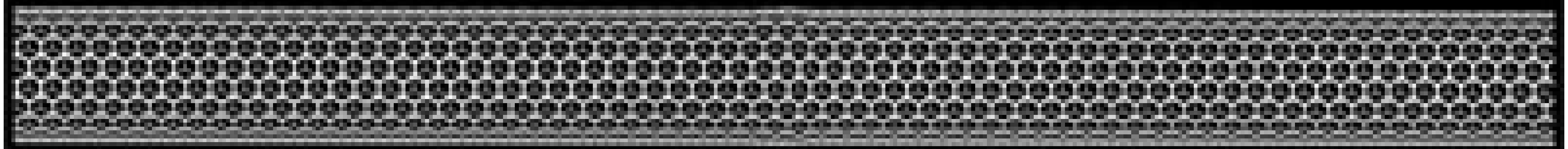
$$H_{eff} = \pm\hbar v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} = \pm\hbar v_F \vec{\sigma}\cdot\vec{k}_\perp$$

$$\left| k_\perp \right\rangle = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i\theta_\kappa} \end{pmatrix}$$

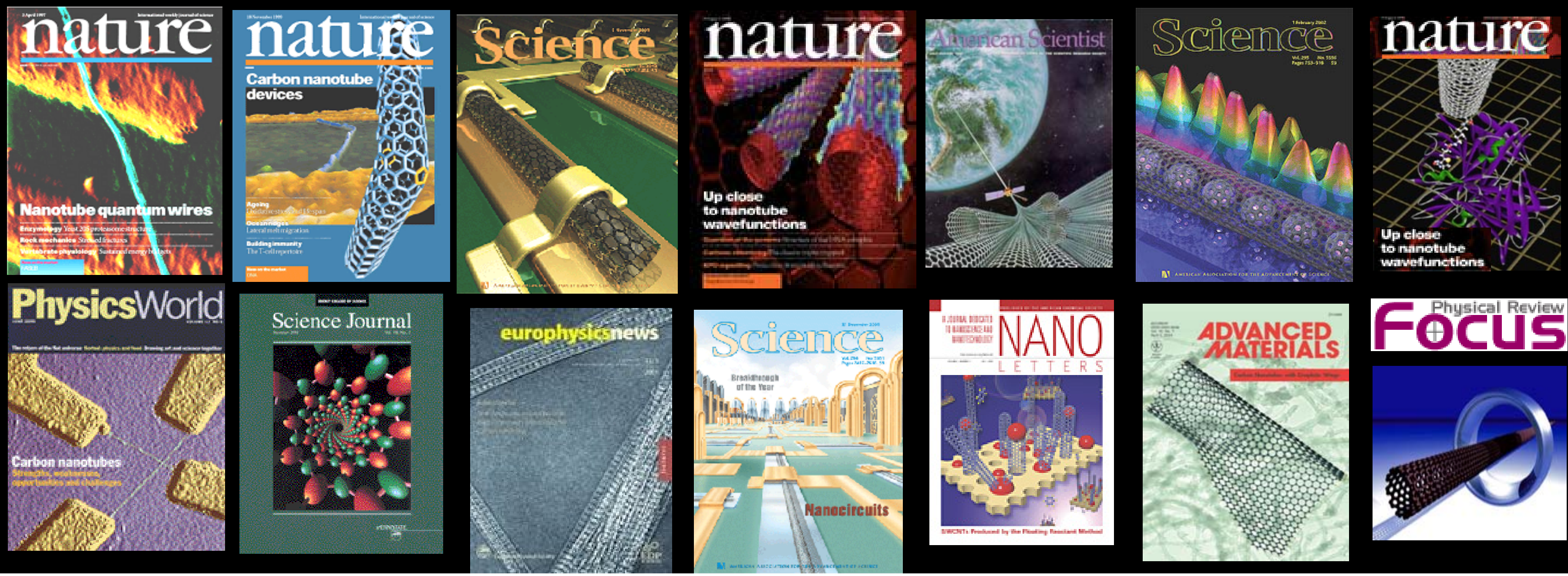
$$\theta_k = \tan^{-1}(k_y / k_x)$$

DiVincenzo and Mele, PRB (1984); T. Ando, JPSJ (1998);McEuen at al, PRL (1999)

# Single Wall Carbon Nanotube



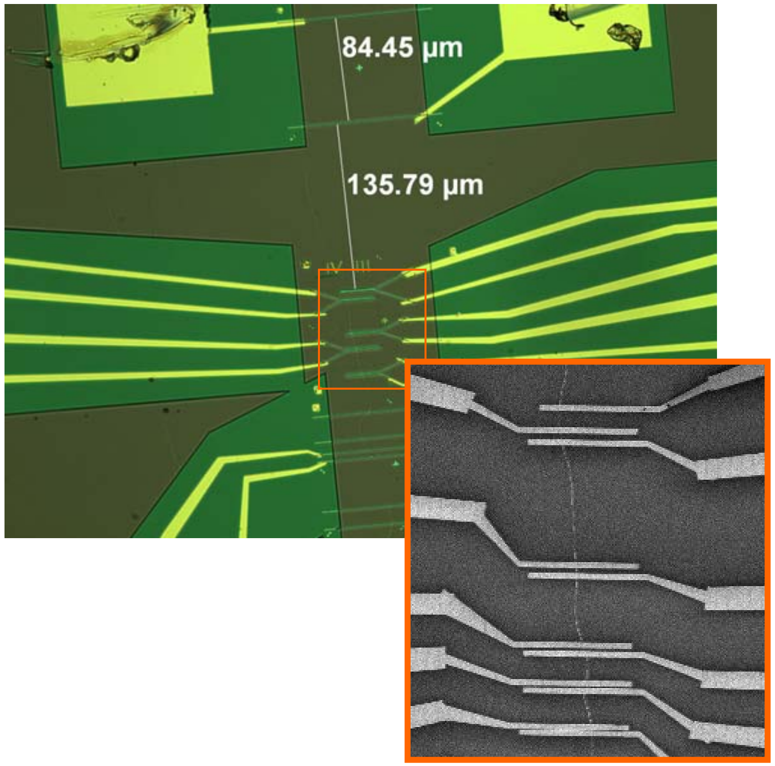
…. since 1991

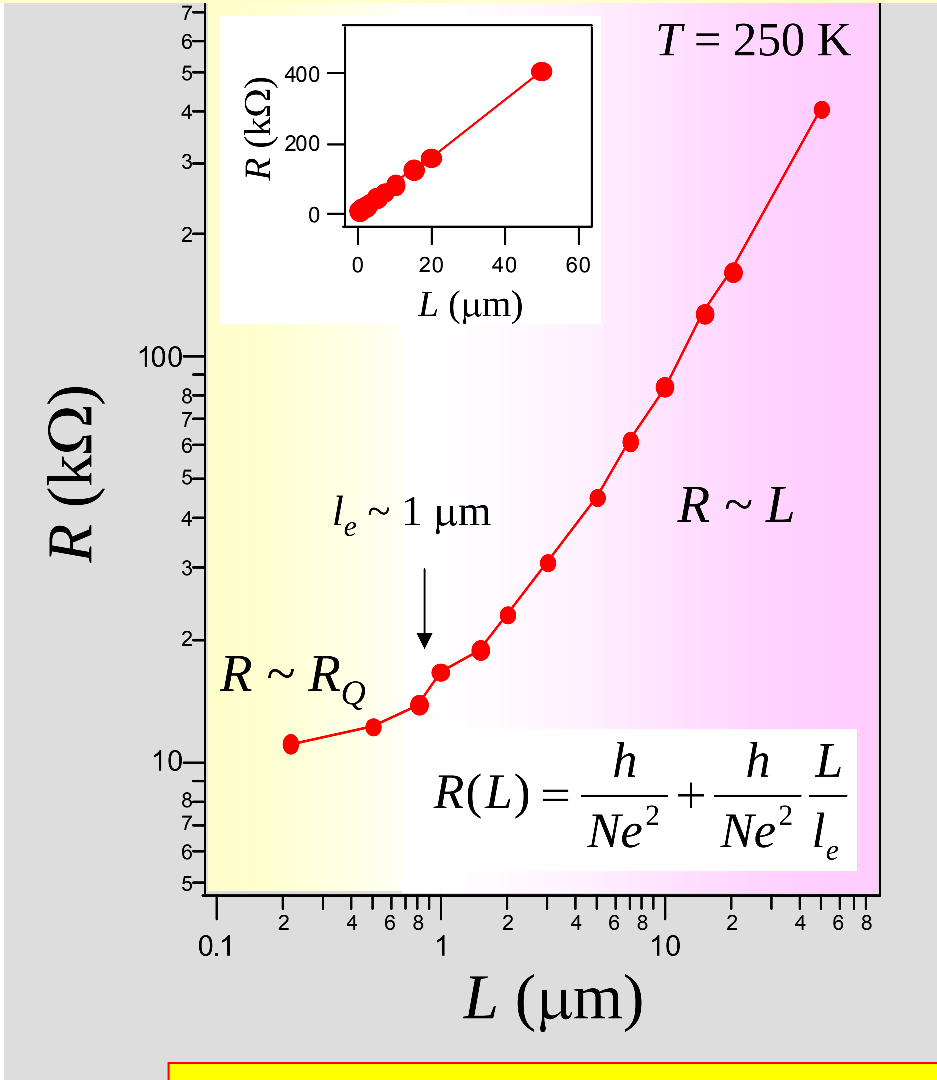


# Extremely Long Mean Free Path in Nanotubes

**See also S. Frank, P. Poncharal, Z. L. Wang, and W. A. de Heer, Science 280, 1744-1746 (1998)**

**Multi-terminal Device with Pd contact**



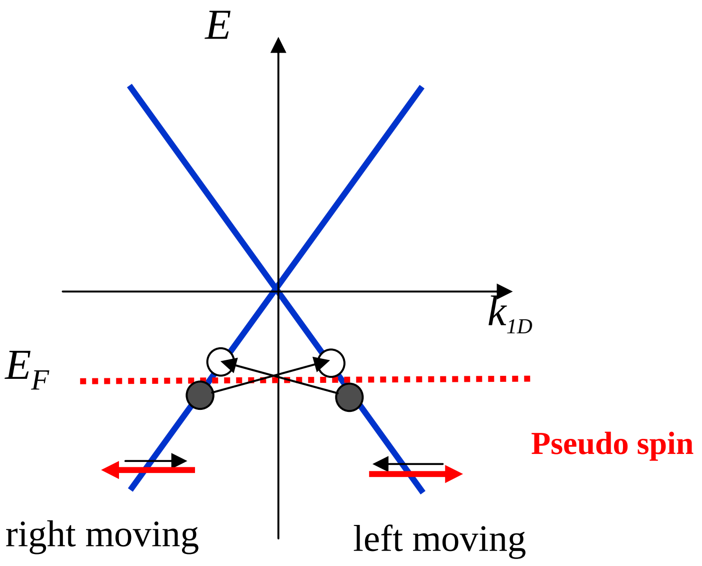

* Scaling behavior of resistance:
*R*(*L*)

M. Purewall, B. Hong, A. Ravi, B. Chnadra, J. Hone and P. Kim, PRL (2007)

**Room temperature mean free path > 0.5 µm**

# Extremely Long Mean Free Path of Nanotubes: Role of Pseudo Spin

**1D band structure of nanotubes**




$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\left\langle \vec{k}'\sigma'_{pseudo} \middle| V(\vec{r}) \middle| \vec{k}\sigma_{pseudo} \right\rangle$$

- Small momentum transfer backward scattering becomes inefficient since it requires pseudo spin flipping.

Journal of the Physical Society of Japan
Vol. 67, No. 8, August, 1998, pp. 2857–2862

**Berry's Phase and Absence of Back Scattering in Carbon Nanotubes**

Tsuneya ANDO, Takeshi NAKANISHI,[1] and Riichiro SAITO[2]

*Institute for Solid State Physics, University of Tokyo*
*7–22–1 Minato-ku, Roppongi, Tokyo 106*

[1] *The Institute of Physical and Chemical Research (RIKEN)*
*2–1 Hirosawa, Wako-shi, Saitama 351-01*

[2] *Department of Electronics Engineering*
*University of Electro-Communications, Chofugaoka, Chofu, Tokyo 182*

(Received March 16, 1998)

The absence of back scattering in carbon nanotubes is shown to be ascribed to Berry's phase which corresponds to a sign change of the wave function under a spin rotation of a neutrino-like particle in a two-dimensional graphite. Effects of trigonal warping of the bands appearing in a higher order $\boldsymbol{k}\cdot\boldsymbol{p}$ approximation are shown to give rise to a small probability of back scattering.

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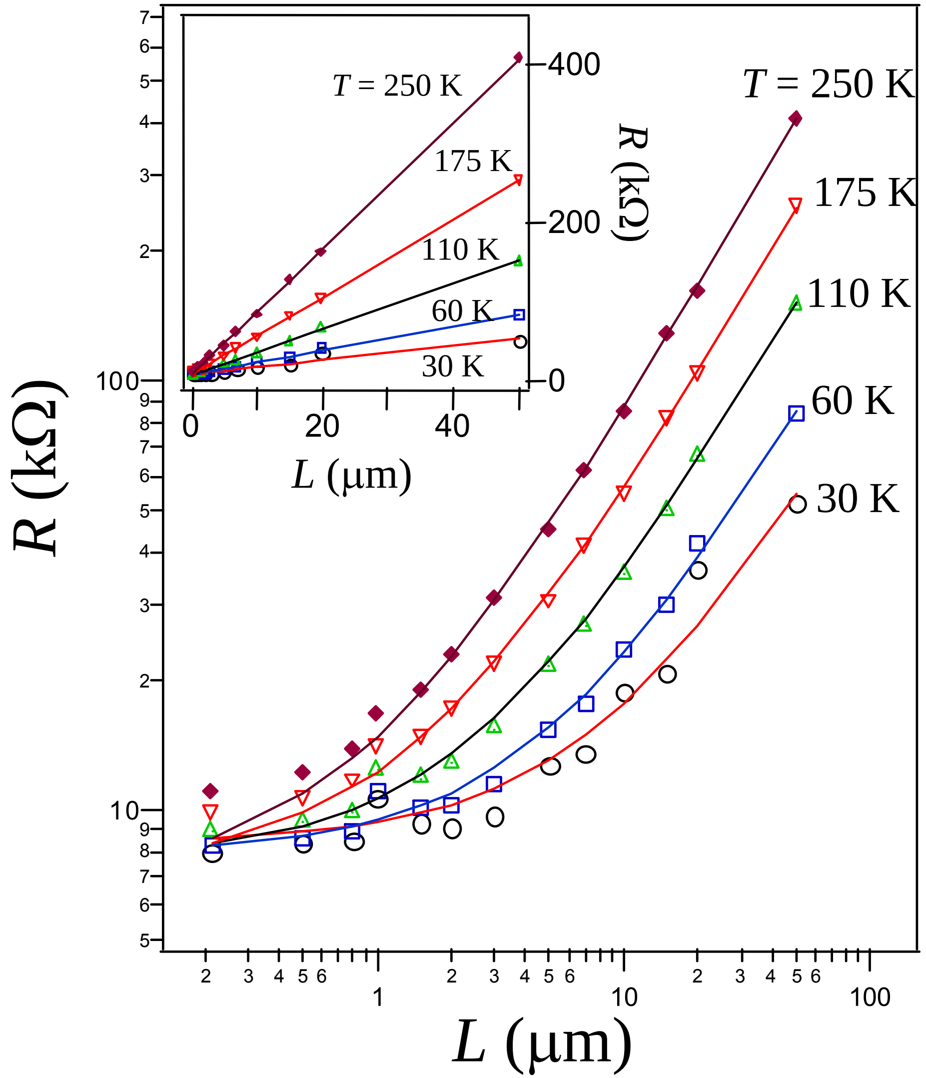
VOLUME 83, NUMBER 24 PHYSICAL REVIEW LETTERS 13 DECEMBER 1999

**Disorder, Pseudospins, and Backscattering in Carbon Nanotubes**

Paul L. McEuen, Marc Bockrath, David H. Cobden,* Young-Gui Yoon, and Steven G. Louie
*Department of Physics, University of California, and Materials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*
(Received 7 June 1999)

We address the effects of disorder on the conducting properties of metal and semiconducting carbon nanotubes. Experimentally, the mean free path is found to be much larger in metallic tubes than in doped semiconducting tubes. We show that this result can be understood theoretically if the disorder potential is long ranged. The effects of a pseudospin index that describes the internal sublattice structure of the states lead to a suppression of scattering in metallic tubes, but not in semiconducting tubes. This conclusion is supported by tight-binding calculations.

# Electron Mean Free Path of Nanotube

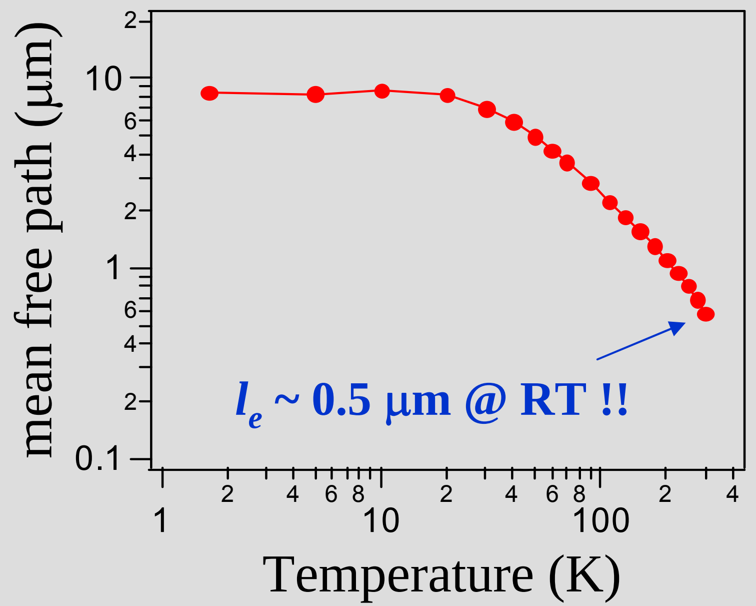



**Lines are fit to**

$$R(L) = R_c + \frac{h}{4e^2} + \frac{h}{4e^2}\frac{L}{l_e}$$

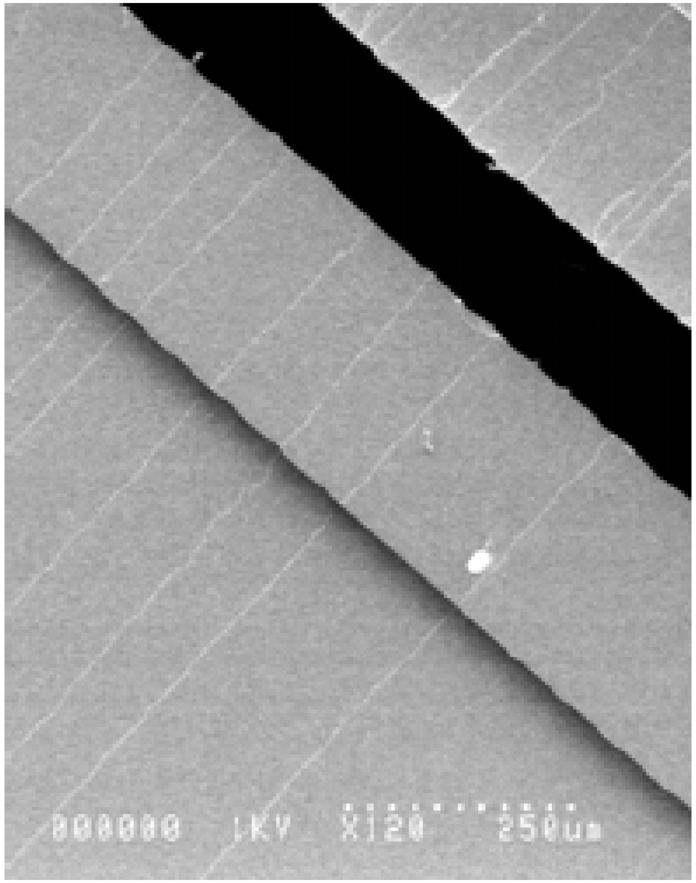
$$\frac{h}{4e^2} = 6.45\ \mathrm{k\Omega}$$

Non-ideal contact resistance $R_c < 2\ \mathrm{k\Omega}$

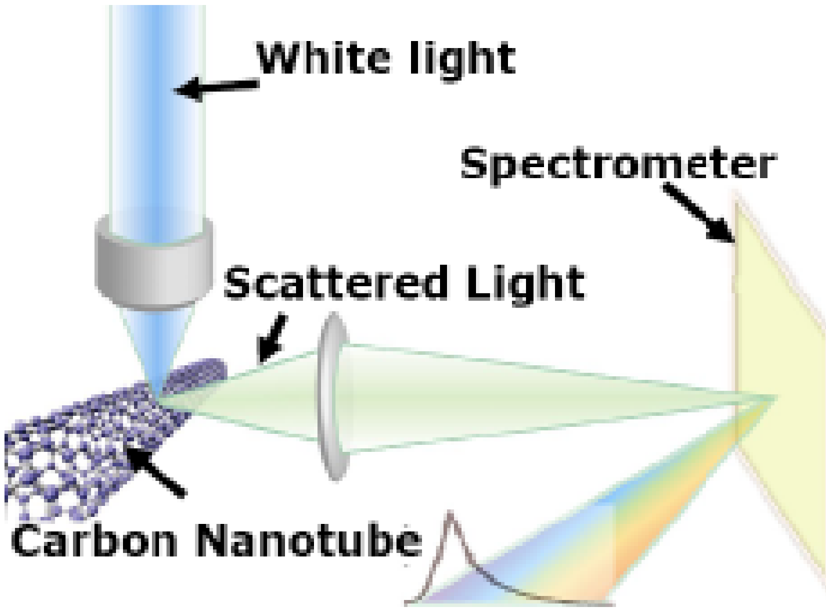


# Characterization of Nanotube Structures by Rayleigh Light Scattering

Nanotube Growth over trenches



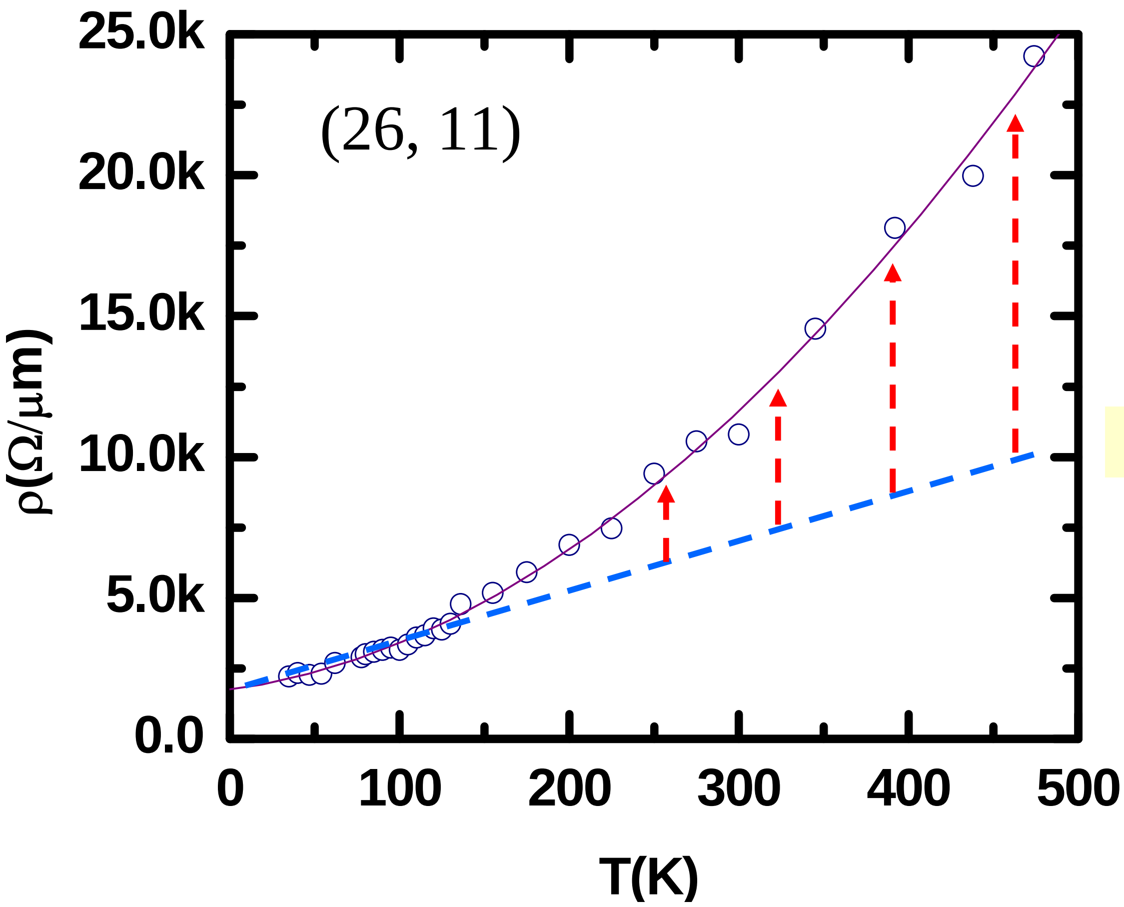

Rayleigh Scattering Characterization




**Brus, Heinz, Hone, O'Brien groups, Science <u>306</u>,1540 (2004)**

# Temperature Dependent Resistivity

B. Chandra M. Purewal, P, Kim and J. Hone




Super linear behaviors
for T > 150 K

**Scattering due to acoustic phonons**

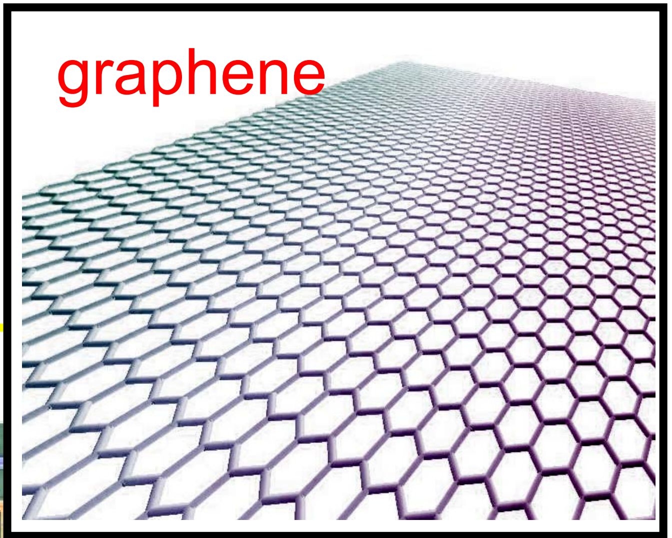
$$\frac{1}{\tau_{ac}} = 2\frac{2\pi}{\hbar}\Xi^2\left(\frac{k_B T}{2\rho v_s^2}\right)\frac{1}{h v_F}$$

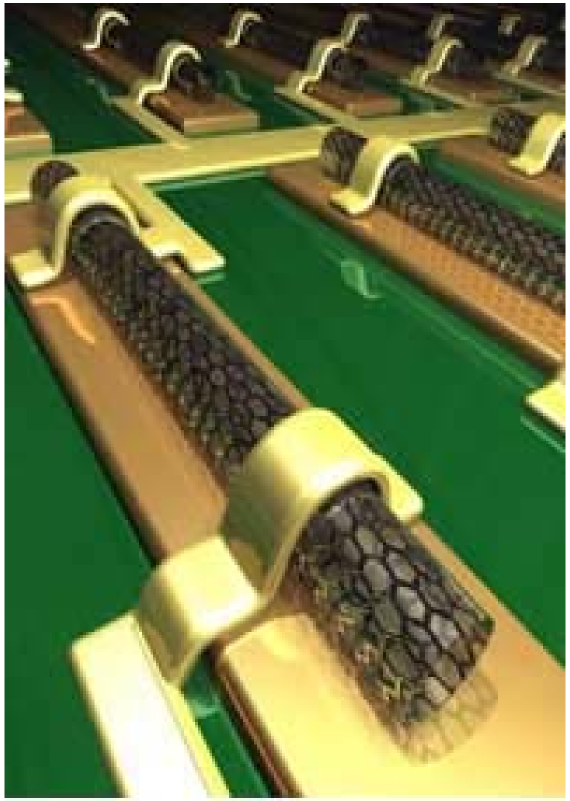
Enhanced scattering activated T > 150 K

# Carbon Electronics: Challenges

**Pros:**
High mobility
High on-off ratio (nanotubes)
High critical current density

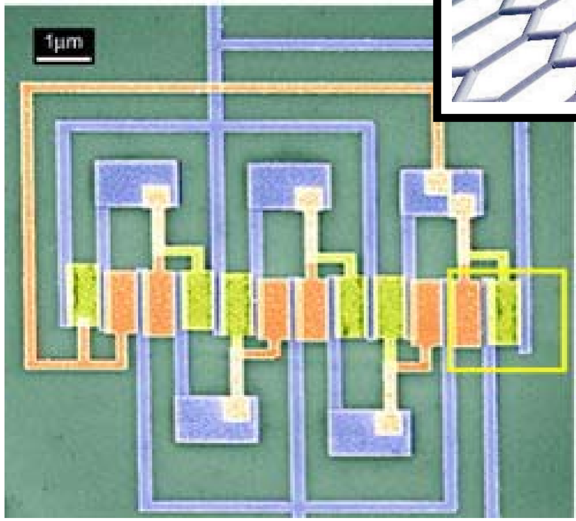
**Con:**
Controlled growth



Artistic dream (DELFT)

**IBM, Avouris group**




**Nanotube Ring Oscillators**

# Graphene Sample Preparation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We describe monocrystalline graphitic films, which are a few atoms thick but are

March 20, 2008

D.I.Y. Graphene: How to Make One-Atom-Thick Carbon Layers With Sticky Tape

Graphene, science's latest wonder material, is surprisingly easy to produce. JR Minkel explores how to make the novel substance, which is discussed in detail in Carbon Wonderland

By JR Minkel

| Slide 1 of 9

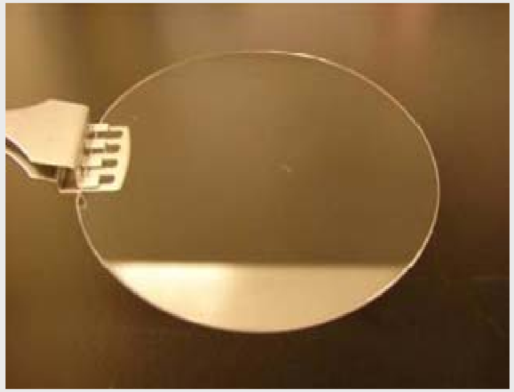
Work in a relatively clean environment (or even a clean room, if possible); stray dirt or hair wreaks havoc with graphene samples. To get that clean room vibe on the cheap, grab some goggles and rubber gloves, along with a shower cap for your head as well as two more for each foot.

Pablo Jarillo-Herrero

ENLARGE IMAGE

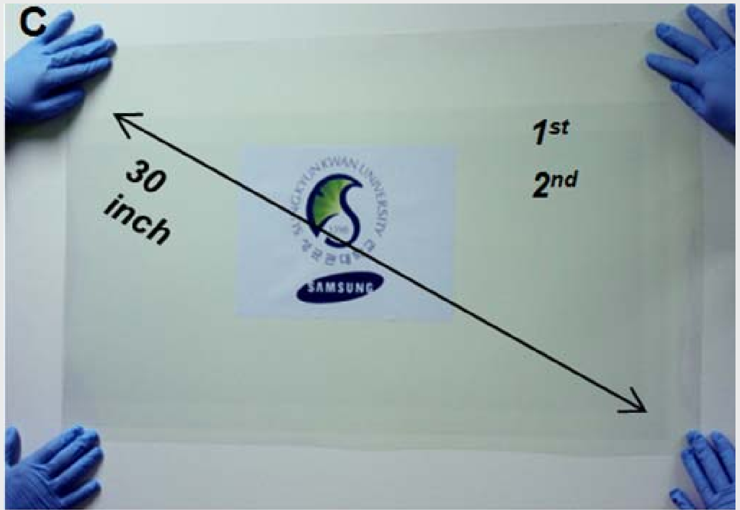
http://www.sciam.com/article.cfm?id=diy-graphene-how-to-make-carbon-layers-with-sticky-tape

## Epitaxial graphene on SiC



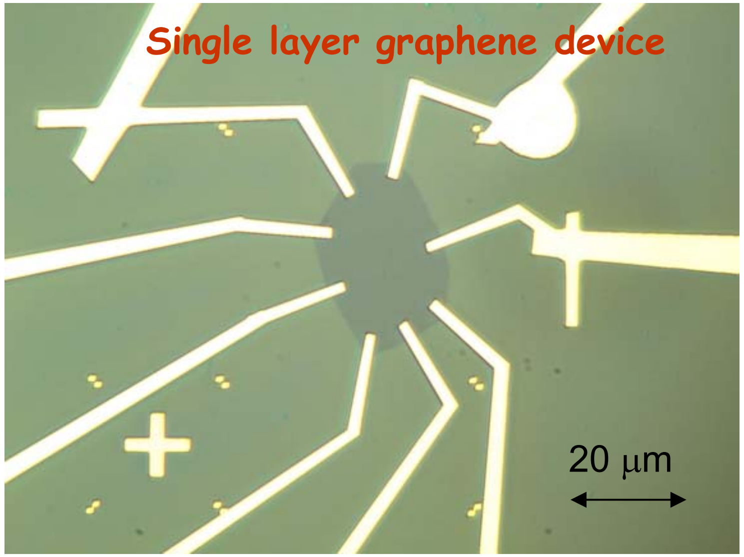
GTech, IBM, NRL, HRL, Purdue, …

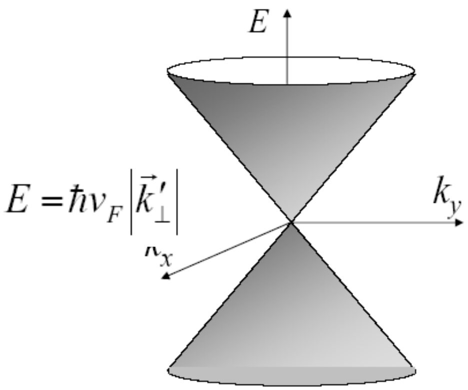
## Chemical Vapor Deposition

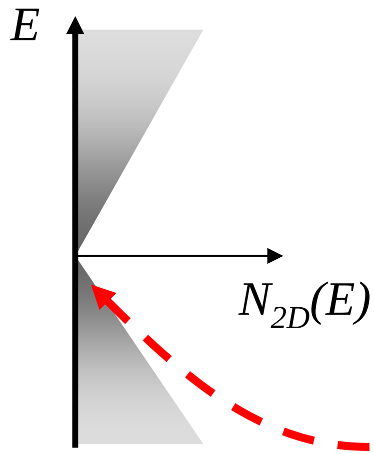



SKKU, MIT, Austin, …

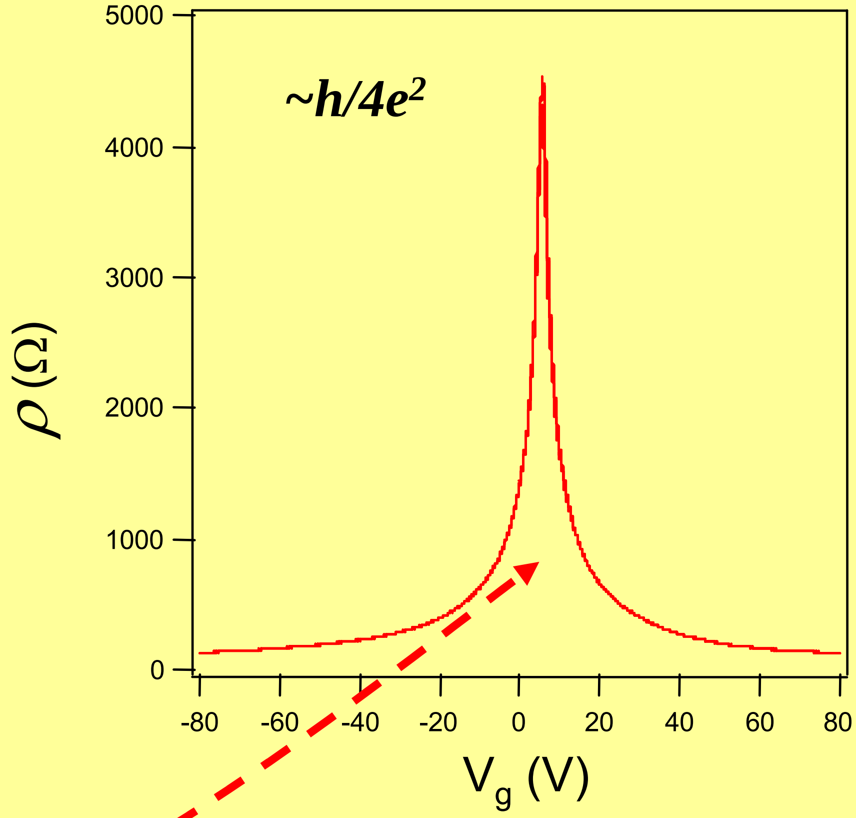
# Field Effective Transport in Single Layer Graphene




**Resistivity vs Gate Voltage**

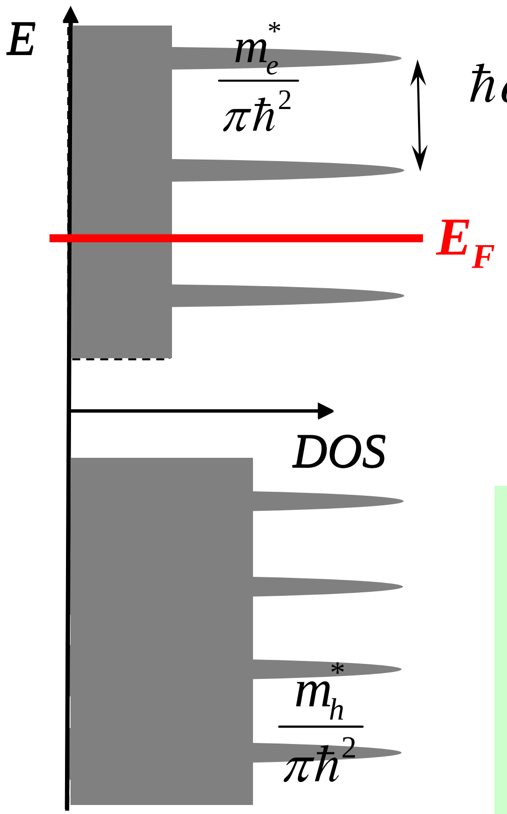



$$\rho^{-1} = e^2 v_F\, l_e\, N_{2D}$$

# 2D Gas in Quantum Limit : Conventional Case

Density of States

Landau Levels in Magnetic Field




$$\hbar\omega_c = \hbar eB / m^*$$

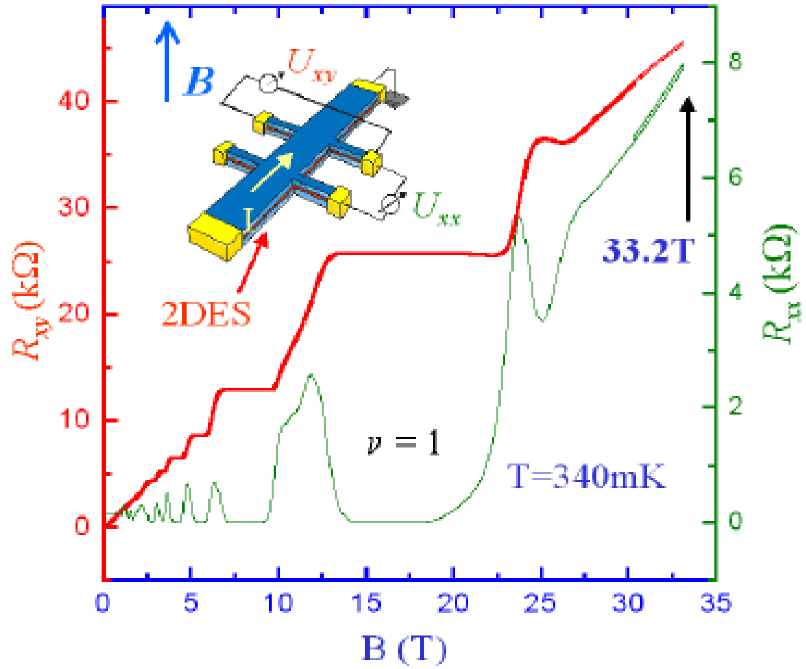
Integer Quantization:

$$R_{xy}^{-1} = \pm\nu \cdot g_s \cdot \frac{e^2}{h}$$

$\nu = 1, 2, 3 \ldots$

$g_s = 2$ (spin)

**Quantum Hall Effect in GaAs 2DEG**



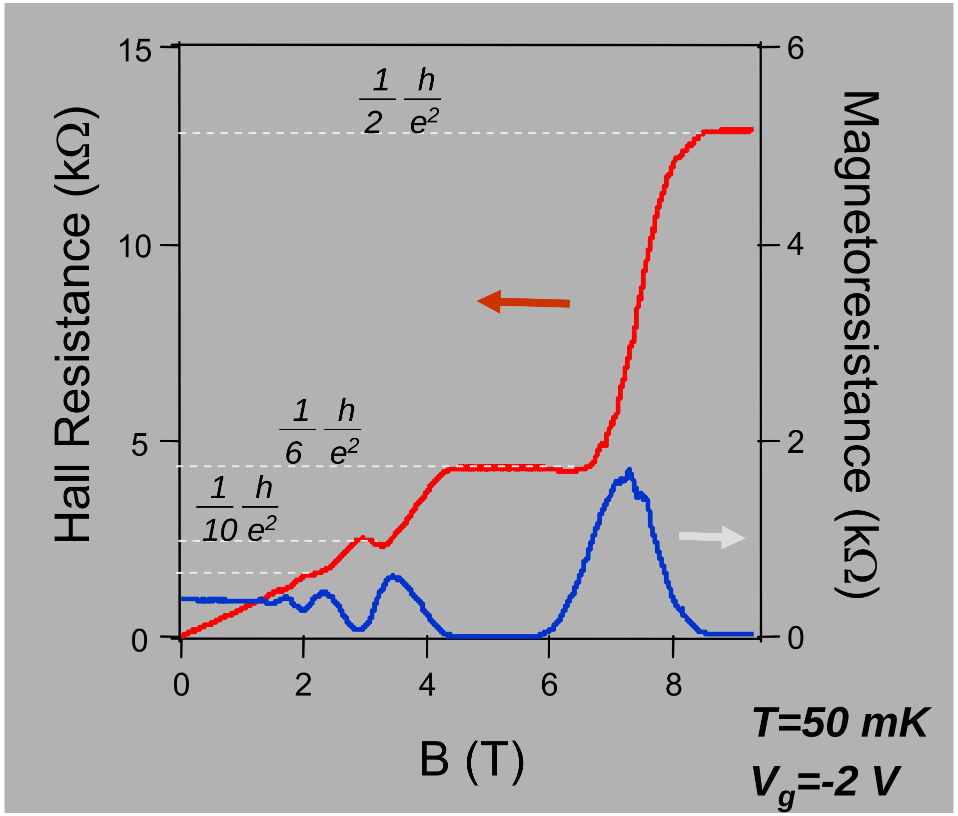

## Graphene

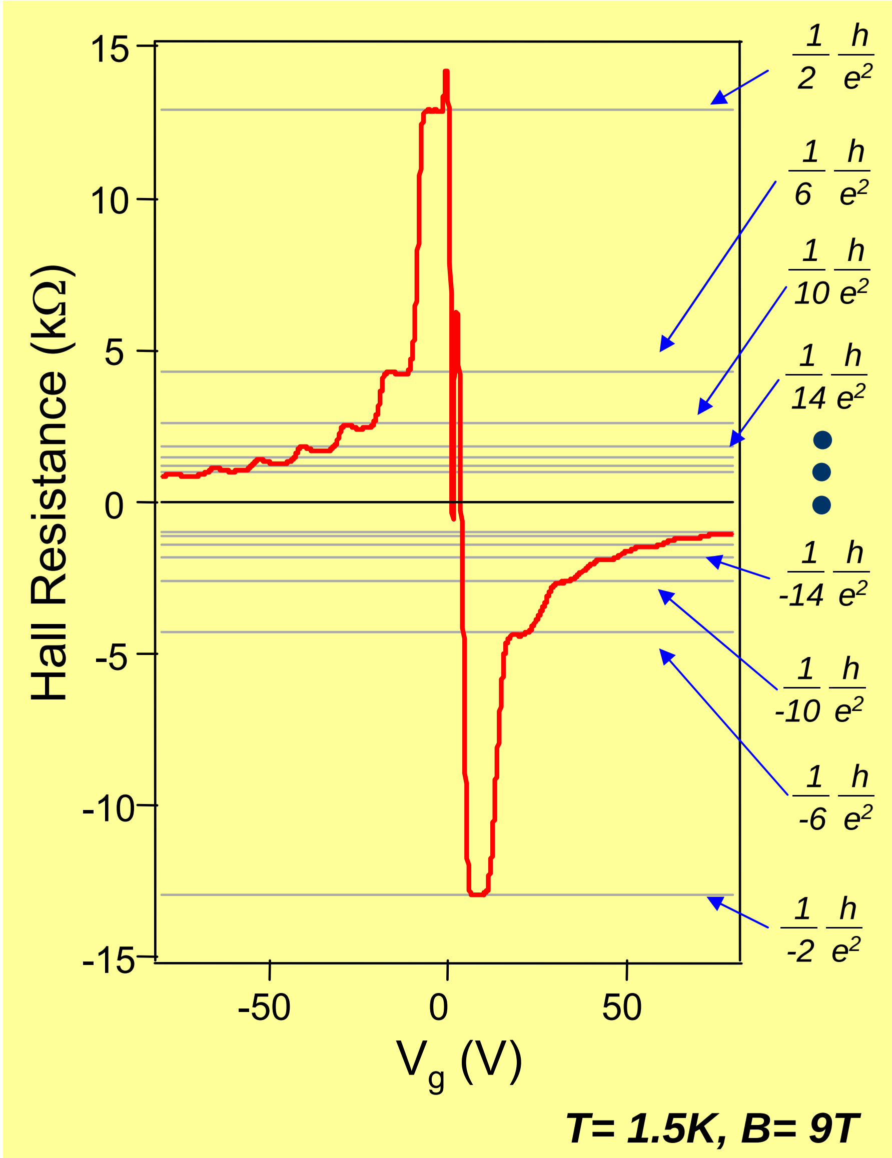
- Vanishing carrier mass near Dirac point
- Strict electron hole symmetry
- Electron hole degeneracy

$$\omega_c = \frac{eB}{m^*}$$

# Quantum Hall Effect in Graphene

Zhang *et al* (2005), Novoselov *et al.* (2005)



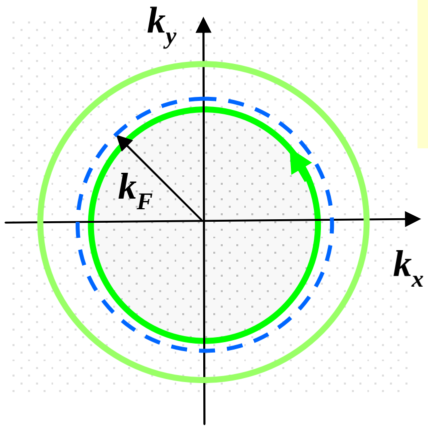




Quantization:

$$R_{xy}^{-1} = 4\left(n + \frac{1}{2}\right)\frac{e^2}{h}$$

# Berry's Phase and Magneto Oscillations

**Landau orbit near the Fermi level**



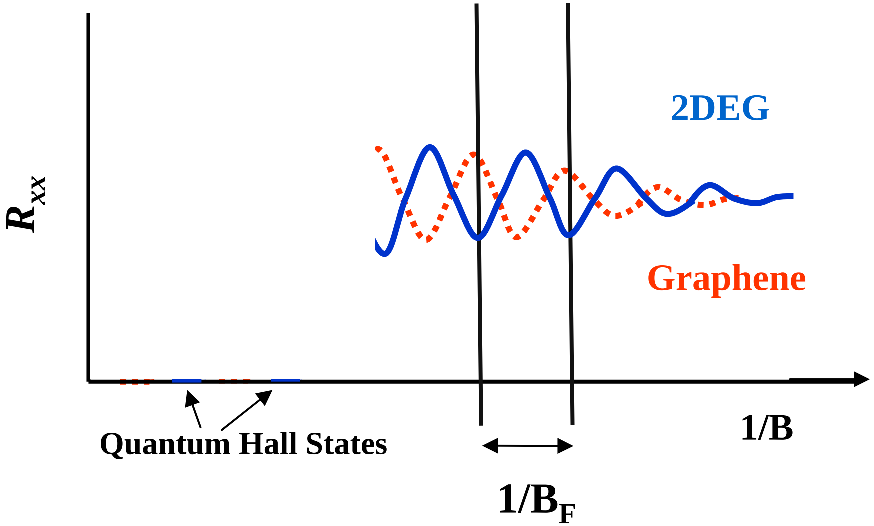

**Magnetic flux in cyclotron orbit**

$$\Phi_B = \Phi_o\, B_F/B$$

$$\Phi_o = h/e$$

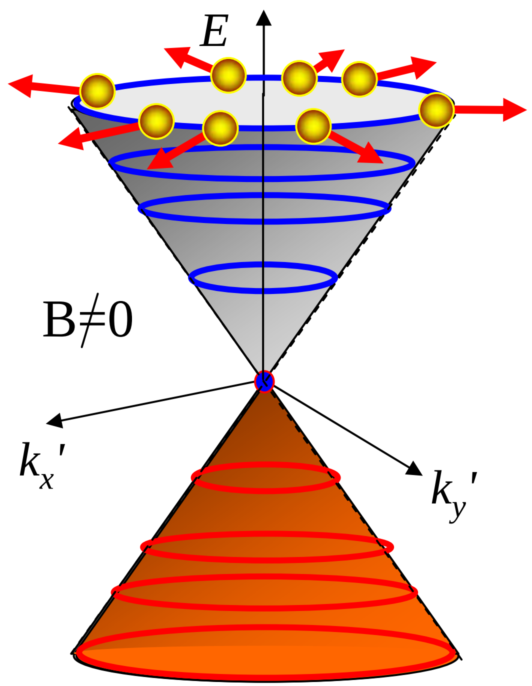
$$B_F = \Phi_o\, k_F^{\,2}/4\pi$$

$$\psi \sim \psi_o \,\exp[2\pi(\Phi_B/\Phi_o)]$$




**Graphene:** $H_{eff} = \hbar v_F\, \vec{\sigma} \cdot \vec{k}'_{\perp}$

Pseudo Spin

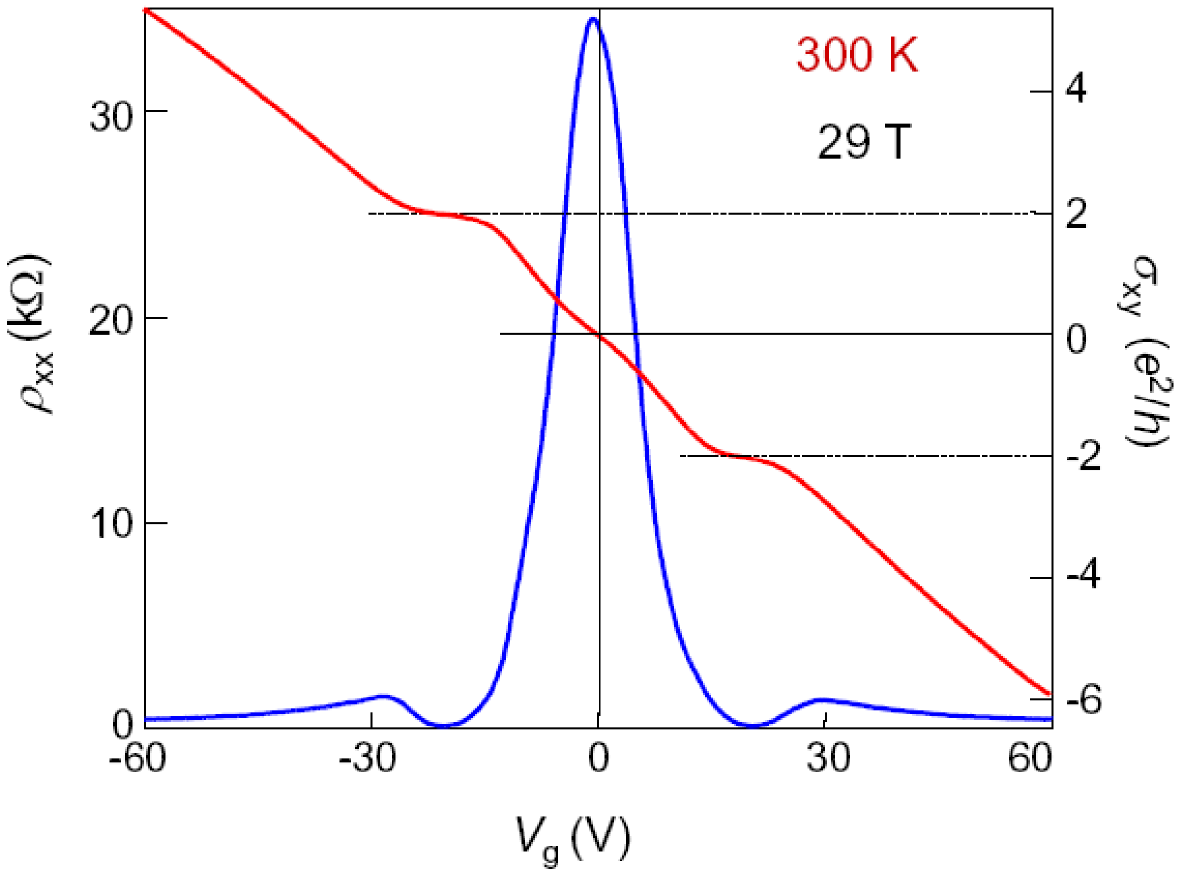



$S = \hbar/2$

$\theta = 2\pi$

$$\psi \sim \psi_o \,\exp[2\pi(\Phi_B/\Phi_o) - \underbrace{S\theta/\hbar}_{\pi}]$$

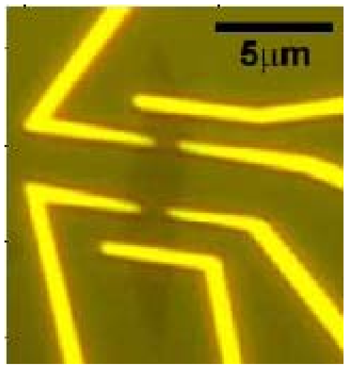
# Room Temperature Quantum Hall Effect




$$E_n = \pm\sqrt{2e\hbar v_F^2 |n| B}$$

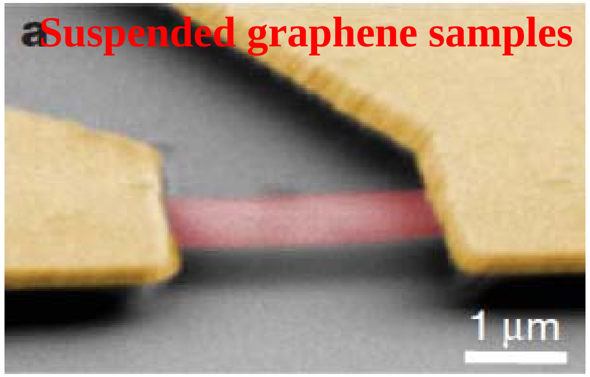
$E_1$ ~ 100 meV @ 5 T

**Typical sample on $SiO_2$**
**mobility: ~ 15,000 $cm^2$/Vsec**

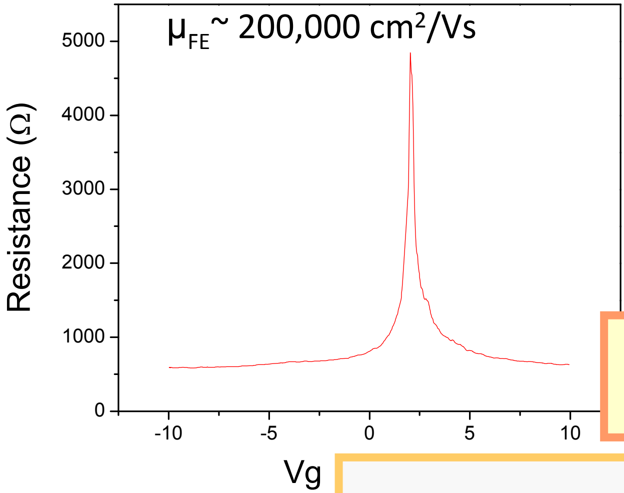



Novoselov, Jiang, Kim and Geim et al. *Science* (2007)

# Quantum Hall Effect in Suspended Graphene



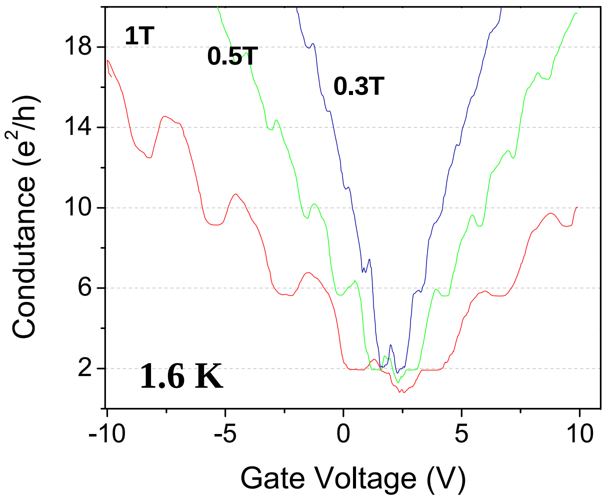

Bolotin et al., (2008); Du et al., (2008)



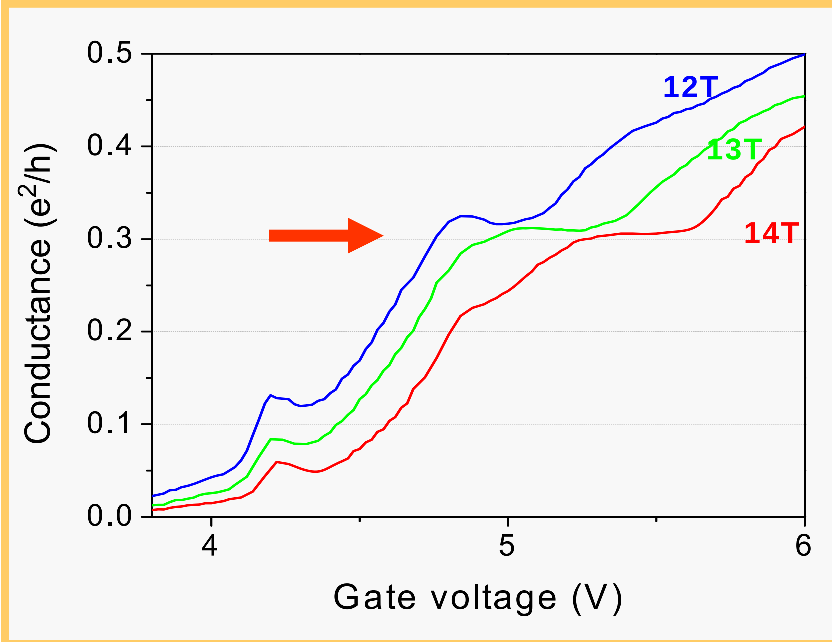

- Cleaning: current annealing
  Bachtold et al. (2007)
- Mechanical stability
  J. Lau et al. (2009)

**ν=1/3 FQHE is observed!!**

**Low Field QHE**

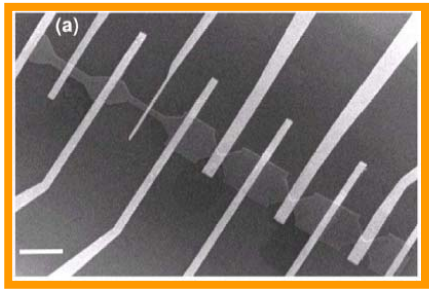



**Increasing B**

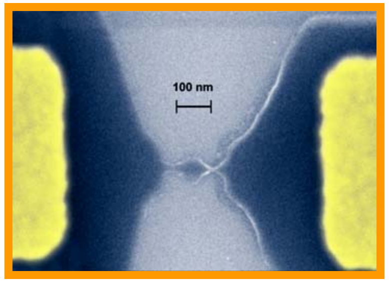


# Creation of Energy Gap in Graphene

• Confinement of Dirac Particles: Nanoribbons, Quantum Dot



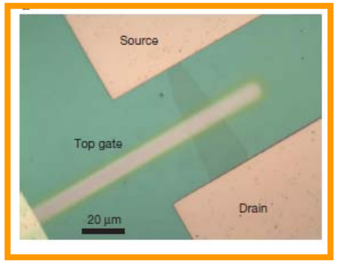

(Columbia, IBM, …)




(Manchester, ETH, …)

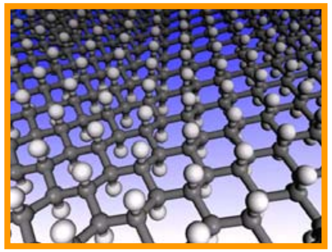
• Breaking Symmetry: Biased Bilayer Graphene

(Manchester, DELFT, Berkeley, Columbia, …)

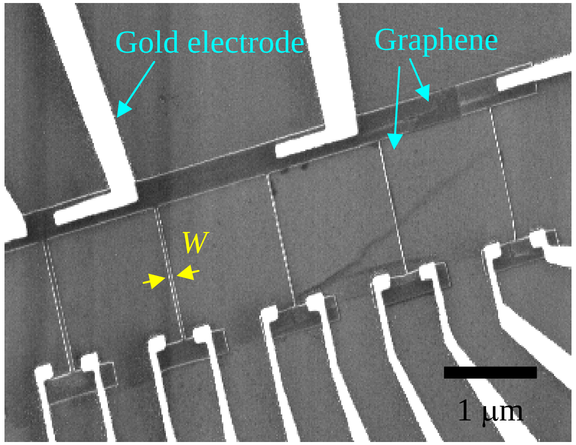



• Chemical Treatment: Graphane, Graphene Oxide

(Manchester, Rutgers, …)

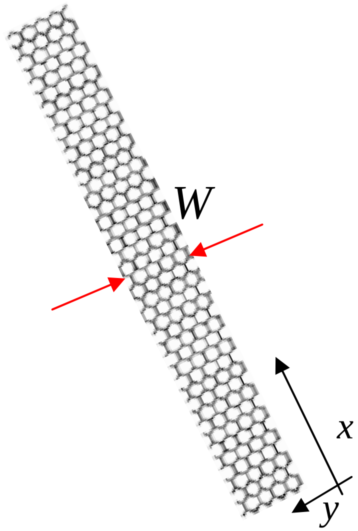


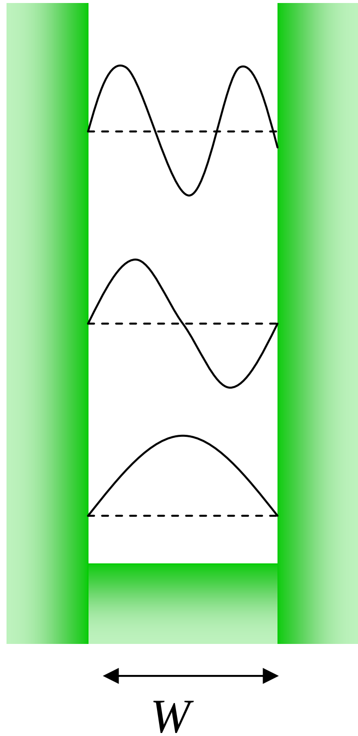
# Graphene Nanoribbons: Confined Dirac Particles




10 nm < $W$ < 100 nm

**Dirac Particle Confinement**




$$k_y = \frac{3 \cdot \pi}{W}$$

$$k_y = \frac{2 \cdot \pi}{W}$$

$$k_y = \frac{1 \cdot \pi}{W}$$

$$\Delta k_y = \frac{\pi}{W}$$

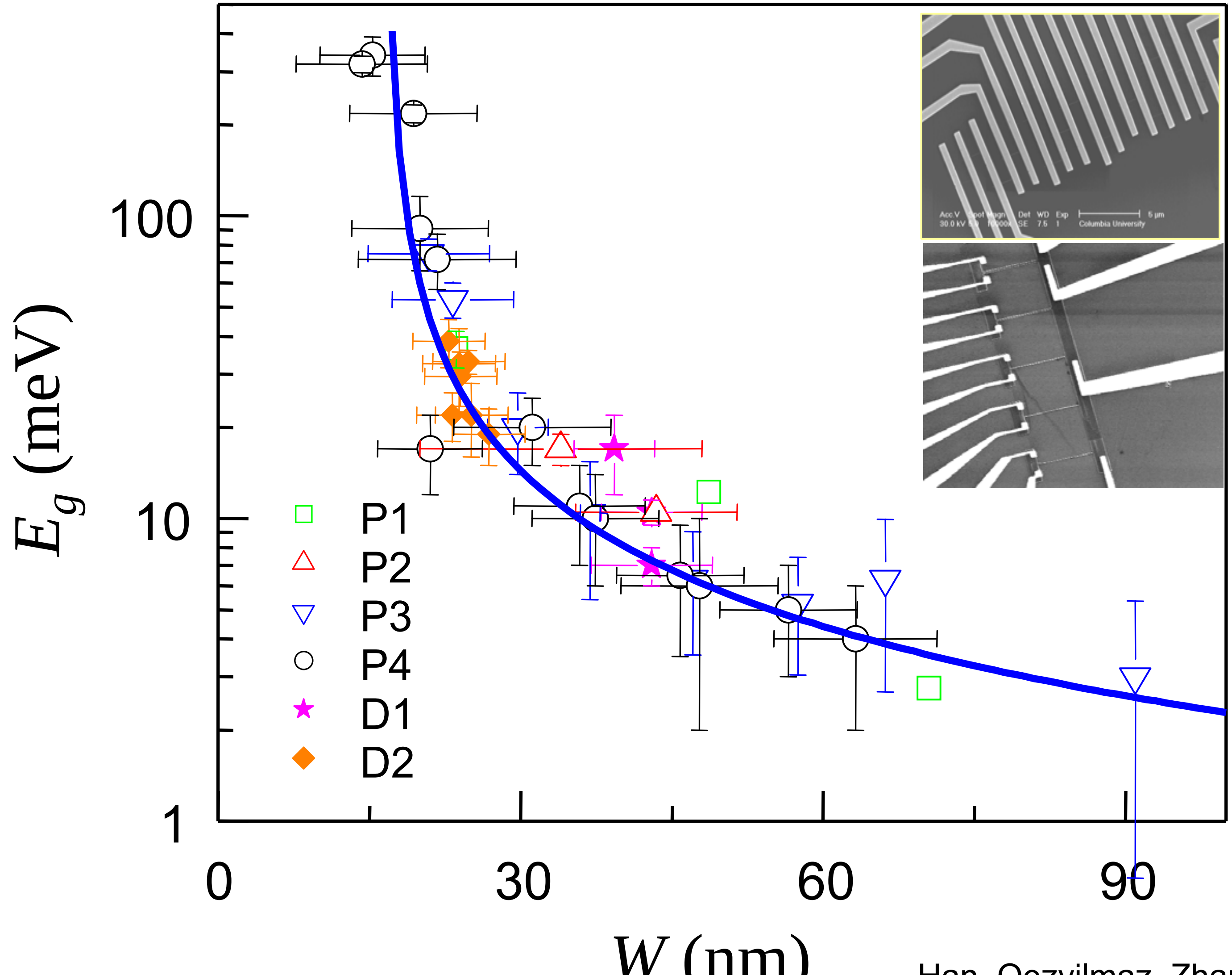
$$E = \pm \hbar v_F \sqrt{k_x^{\,2} + (\pi n / W)^2}$$

$$E_{gap} \sim \hbar v_F \Delta k \sim h v_F / W$$

## Graphene nanoribbon theory partial list

K. Nakada, M. Fujita, G. Dresselhaus, M. S. Dresselhaus, Phys. Rev. B **54**, 17954 (1996).
K. Wakabayashi, M. Fujita, H. Ajiki, M. Sigrist, Phys. Rev. B **59**, 8271 (1999).
Y. Miyamoto, K. Nakada, M. Fujita, Phys. Rev. B **59**, 9858 (1999).
M. Ezawa, Phys. Rev. B **73**, 045432 (2006).
N. M. R. Peres, A. H. Castro Neto, and F. Guinea, Phys. Rev. B **73**, 195411 (2006)
L. Brey and H. A. Fertig, Phys. Rev. B **73**, 235411 (2006).
Y. Ouyang, Y. Yoon, J. K. Fodor, and J. Guo, Appl. Phys. Lett. **89**, 203107 (2006).
Y.-W. Son, M. L. Cohen, S. G. Louie, Nature **444**, 347 (2006)
Y.-W. Son, M. L. Cohen, S. G. Louie, Phys. Rev. Lett. **97**, 216803 (2006).
V. Barone, O. Hod, G. E. Scuseria, Nano Lett **6** 2748 (2006).
D. A. Areshkin, D. Gunlycke, C. T. White, Nano Lett. **7**, 204 (2007).
• • •

# Scaling of Energy Gaps in Graphene Nanoribbons



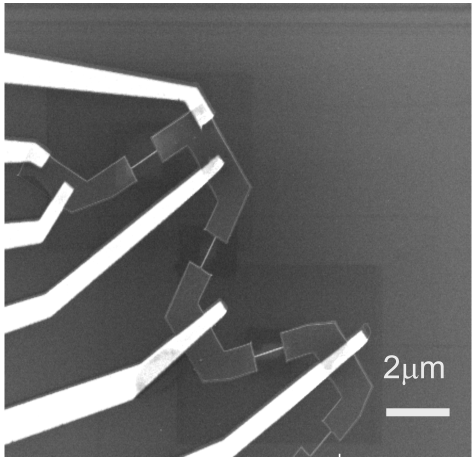

$E_g = E_0/(W-W_0)$

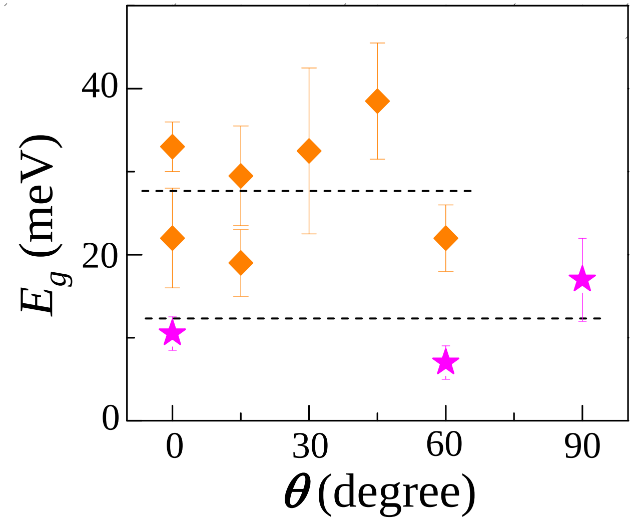
Han, Oezyilmaz, Zhang and Kim PRL (2007)
Chen, Lin, Rooks, and Avouris, Physica E. (2007)

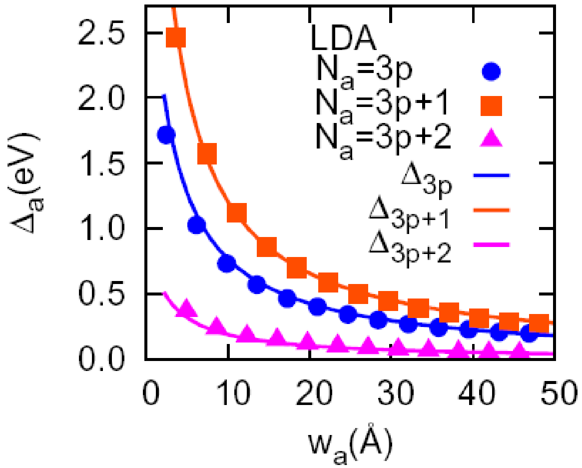
# Graphene Nanoribbons Edge Effect

## Crystallographic Directional Dependence

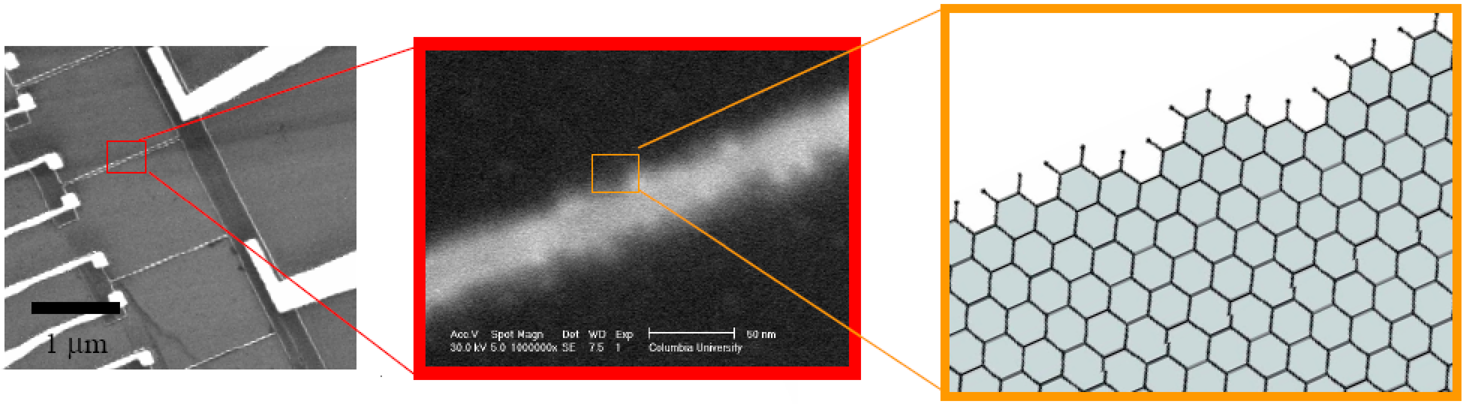
Son, et al, PRL. 97, 216803 (2006)



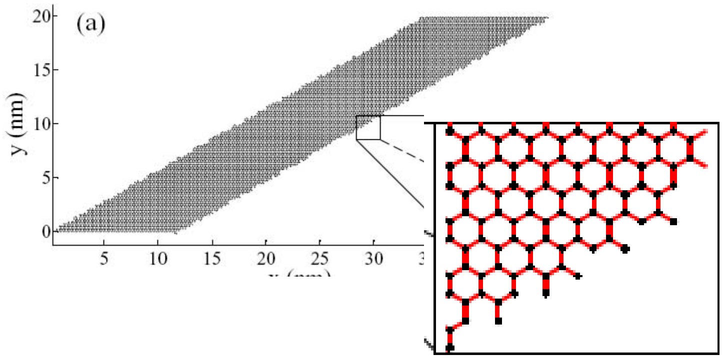




## Rough Graphene Edge Structures



# Localization of Edge Disordered Graphene Nanoribbons

Querlioz et al., Appl. Phys. Lett. **92**, 042108 (2008)




See also:

Gunlycke et al, Appl. Phys. Lett. **90** (14), 142104 (2007).
Areshkin et al, Nano Lett. **7** (1), 204 (2007)
Lherbier et al, PRL 100 036803 (2008)
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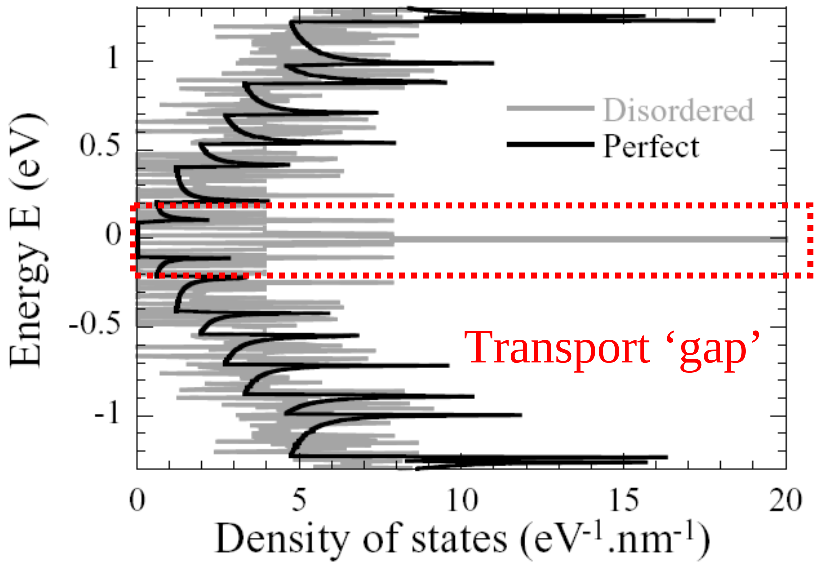
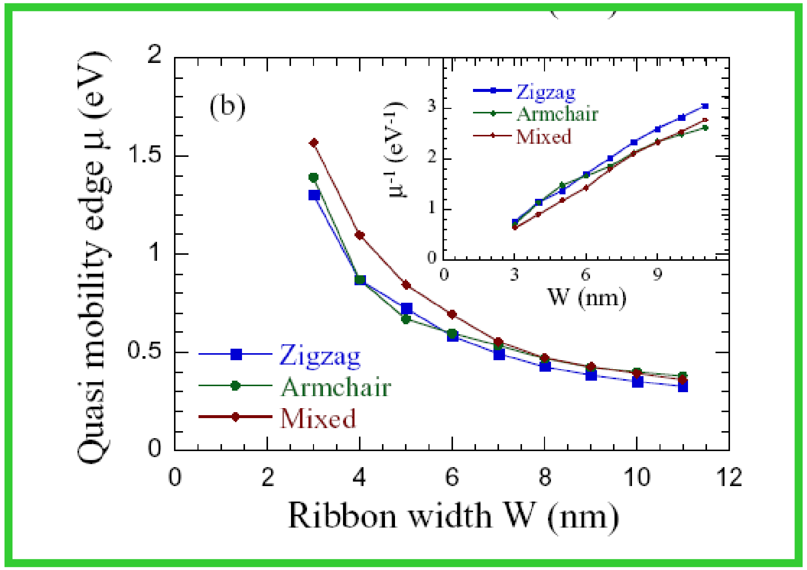
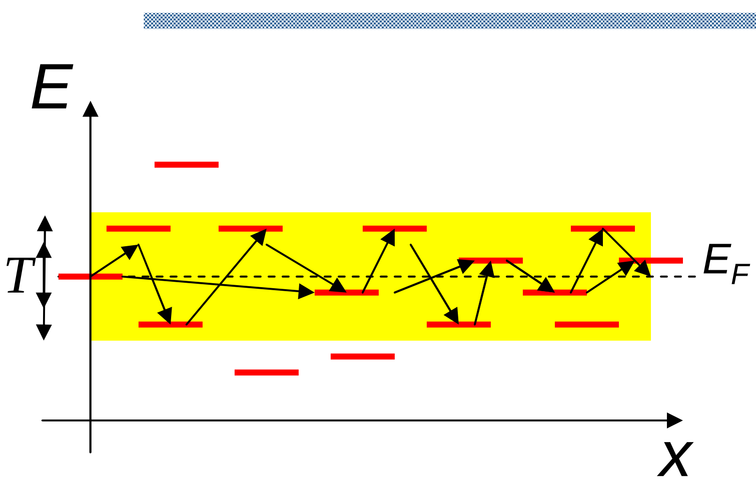


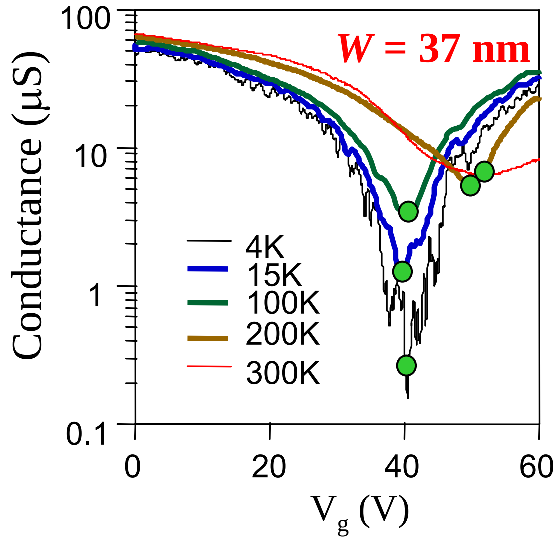



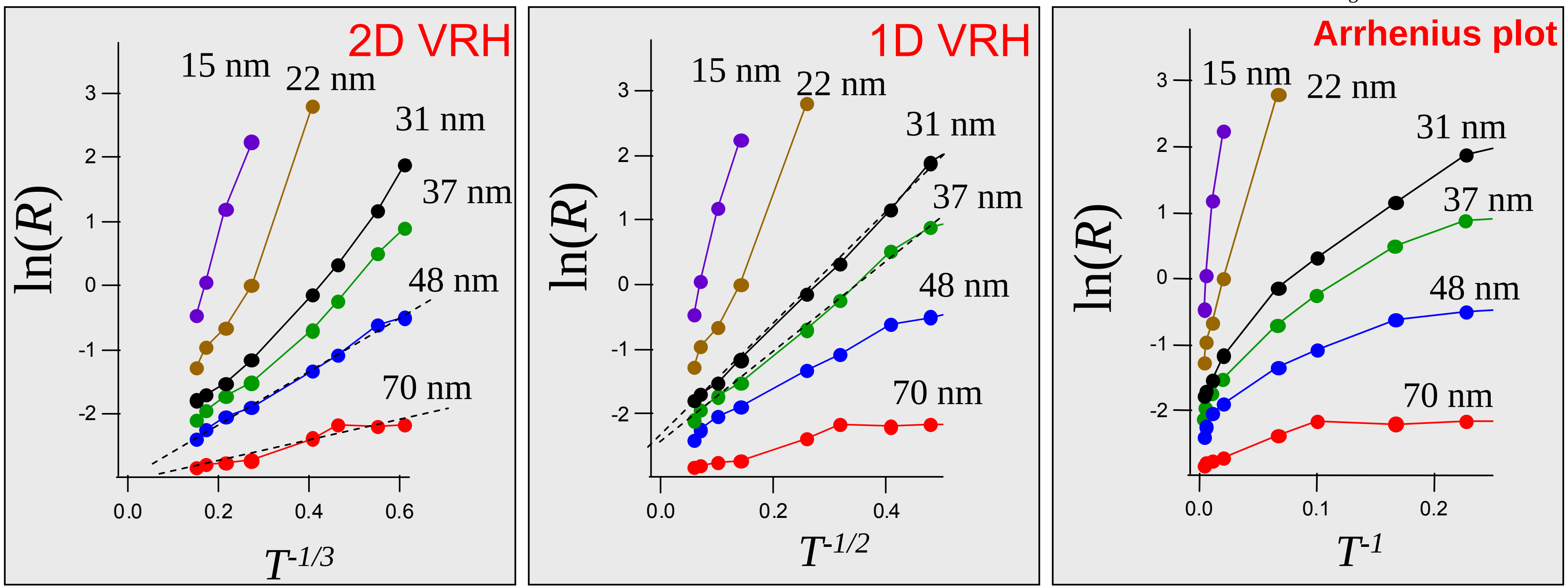
# Variable Range Hopping in Graphene Nanoribbons




$$G_{max} = G_0 \exp\left[-\left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}\right]$$

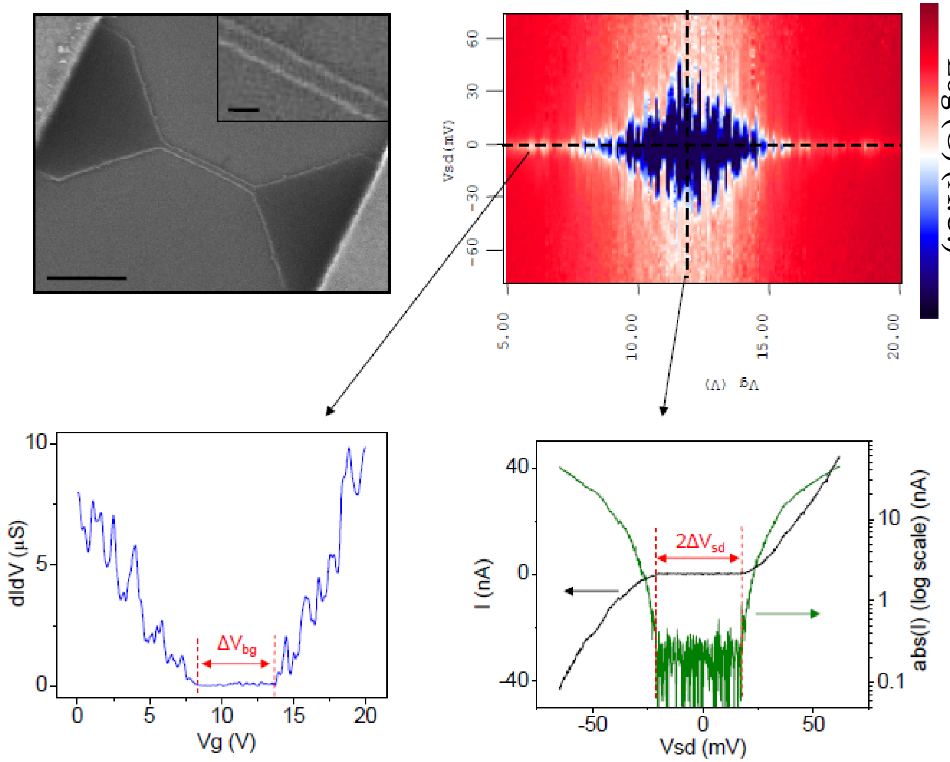
*d*: dimensionality



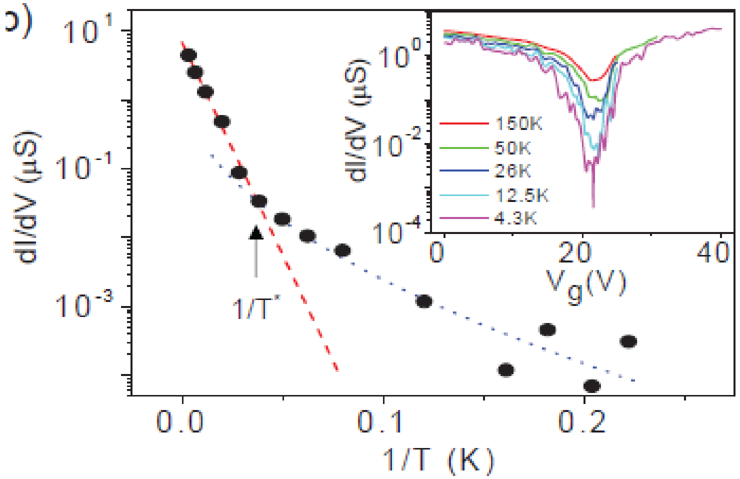



# Nature of Transport Gap in Graphene Nanoribbons

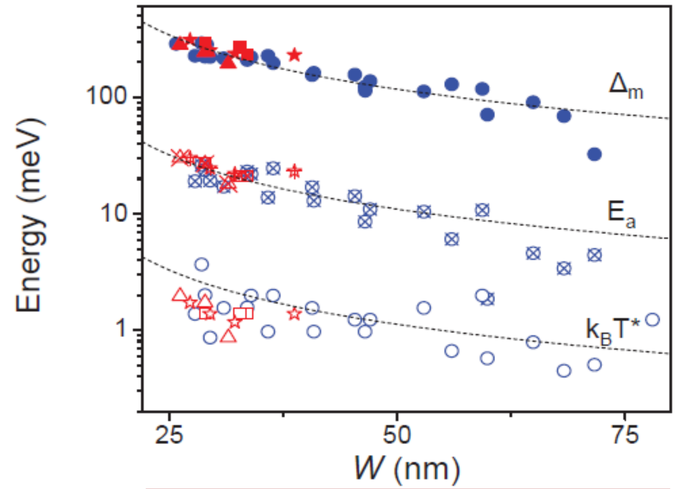
## Transport Characterization: nanoribbons




## Temperature Dependent Transprot




## Energy Gap Scaling in Nanoribbons




Han *et al.*, Phys. Rev. Lett. (2010)

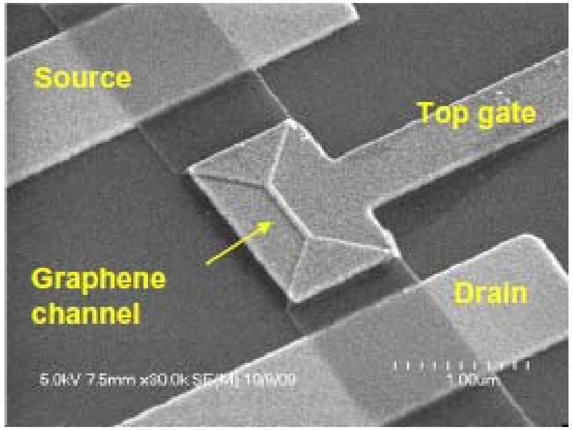
Graphene nanoribbons exhibits three different energy scales for energy gap formation due to the quantum confinement and edge disorders:

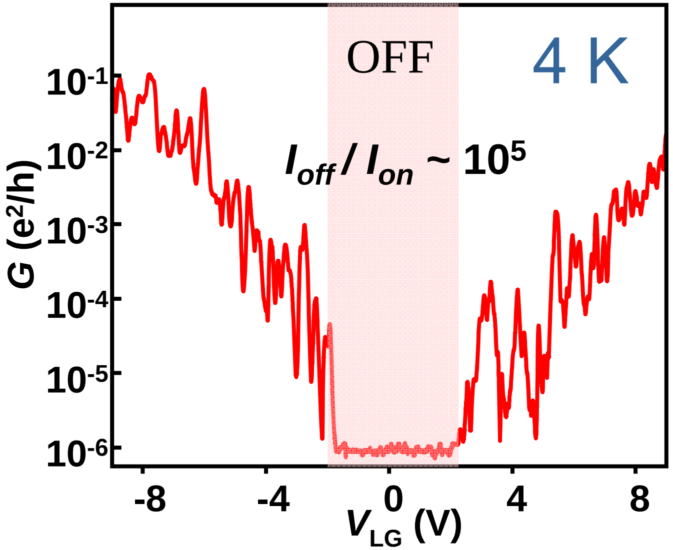
$\Delta_m$: **mobility edges induced by edge disorder**

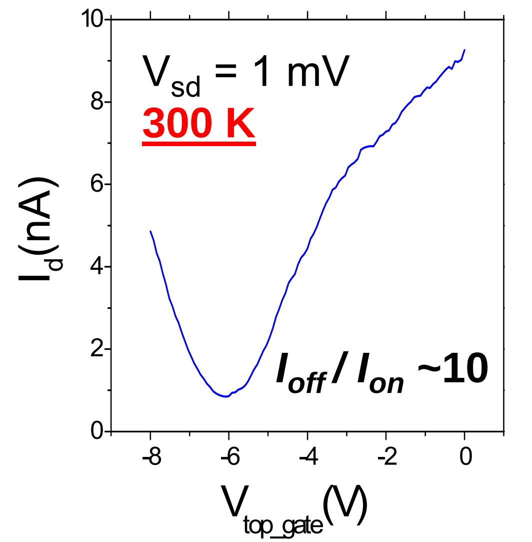
***$E_a$***: **Activation energy from the quantum confinement origin**

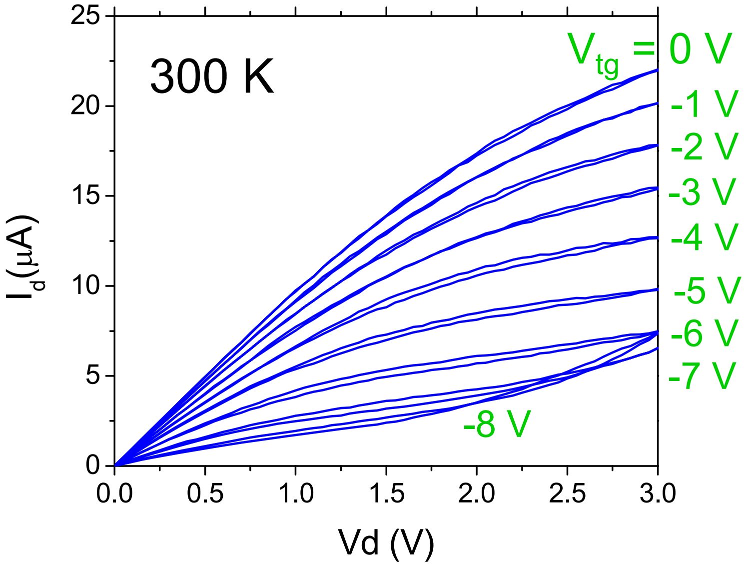
***$k_BT^*$***: **Hoping length to the localized states in gapped regime**

# Top Gated Graphene Nanoribbon FET




**Good**

- **High saturation velocity**

  $v_{Fermi}$ = 1x10⁸ cm/s

  Silicon: 1x10⁷ cm/s

  GaAs: 0.7x10⁷ cm/s

- **Operation current density > 1 mA/μm**

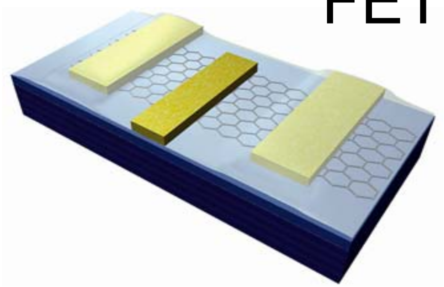
**Bad**

- Mobility: ~ 500-1000 $cm^2$/Vsec
- On-off ratio: ~ 10-30 @RT

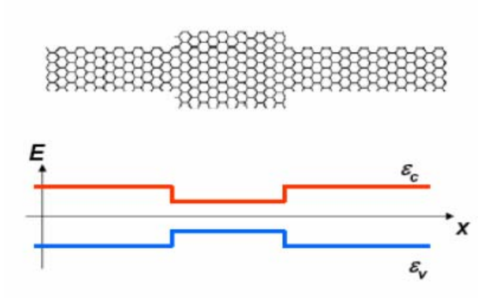
# Graphene Electronics

## Conventional Devices

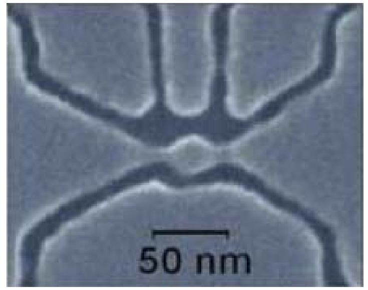
FET



Band gap engineered
Graphene nanoribbons

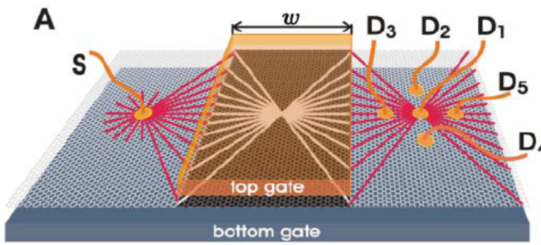



Graphene quantum dot



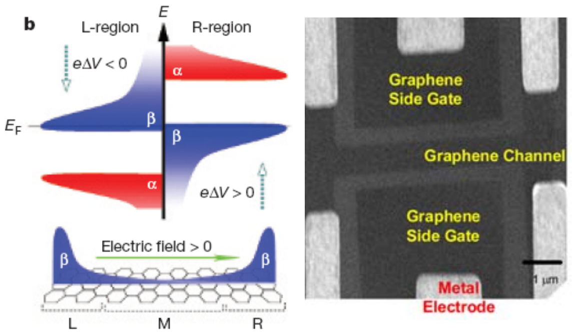

(Manchester group)

## Nonconventional Devices



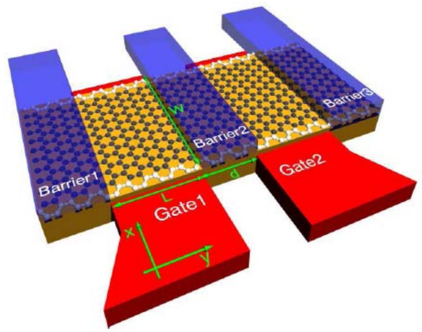

**Graphene Veselago lense**

Cheianov *et al. Science* (07)




**Graphene Spintronics**

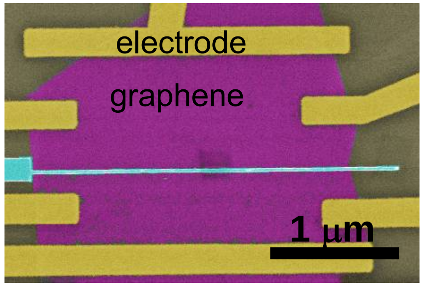
Son *et al. Nature* (07)




**Graphene psedospintronics**

Trauzettel *et al. Nature Phys.* (07)

# Transport Ballistic Graphene Heterojunction



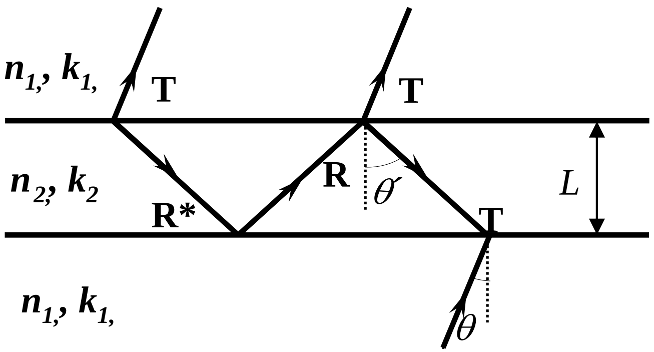

**PN junction resistance**

$$R = (\pi/2)(h/e^2)\sqrt{\hbar v \,/\, e|F_{pn}|}$$

Cheianov and Fal'ko (2006)

See also Shavchenko et al and Goldhaber-Gordon's recent PRL

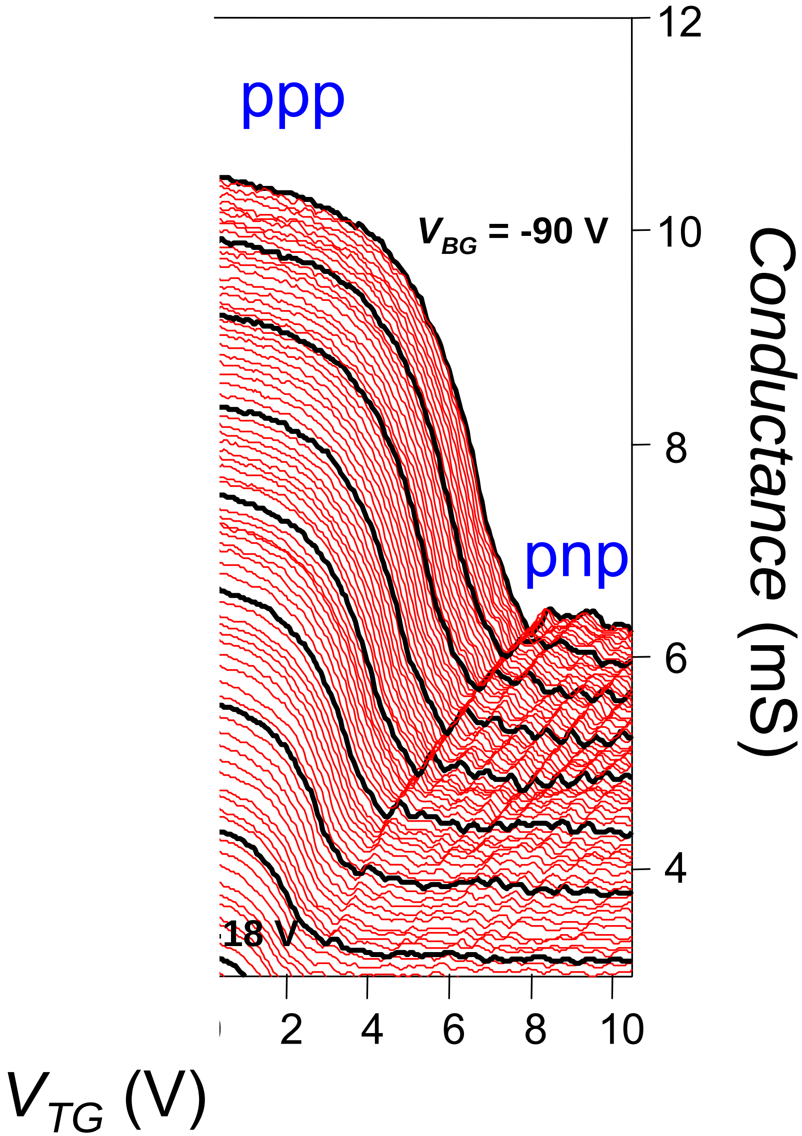
**Conductance Oscillation: Fabry-Perot**




$k_1/k_2 = \sin\theta' / \sin\theta$

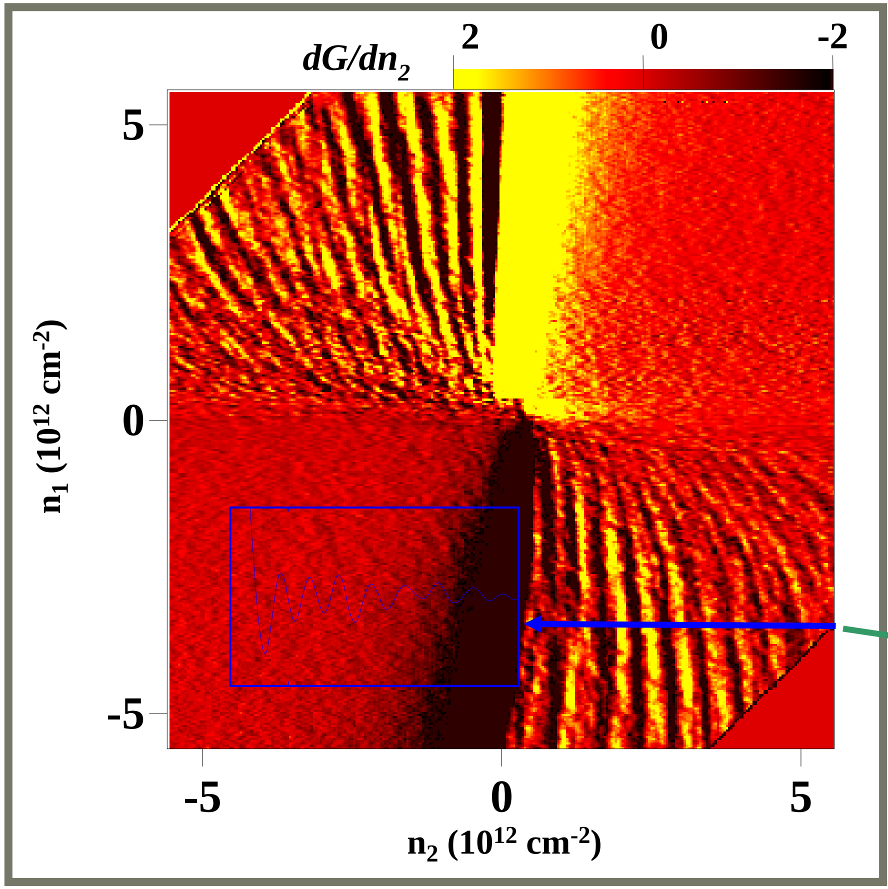
$\Delta\phi = 2L/\cos\theta$

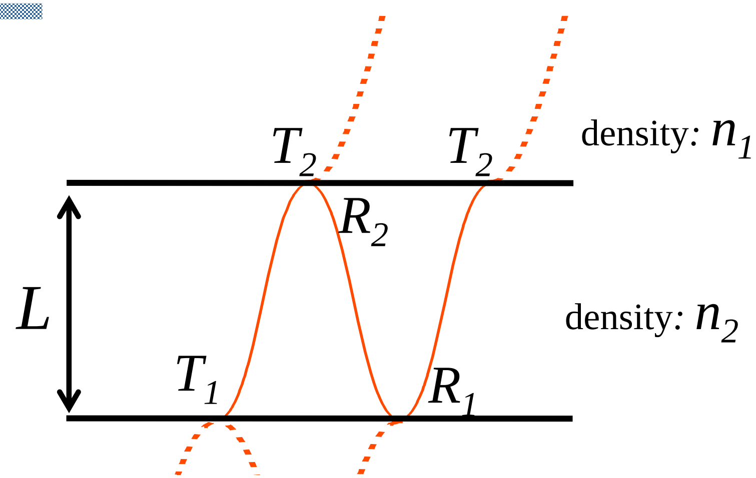
Young and Kim (2008)

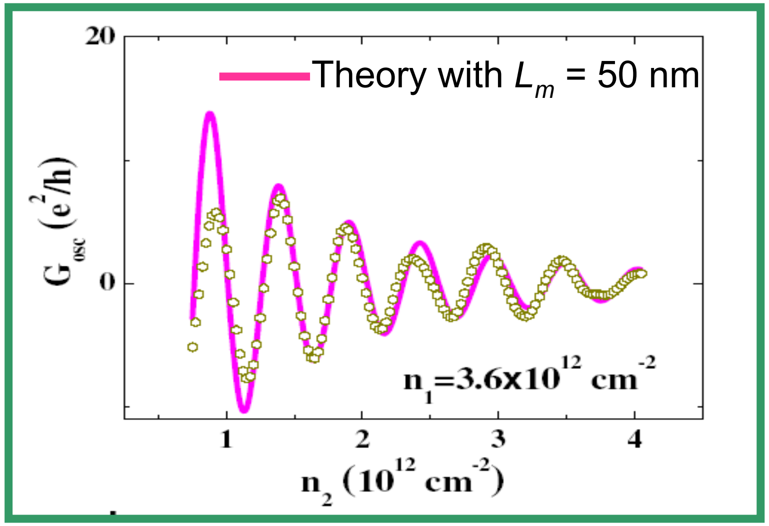


# Fabry-Perot Oscillations in Ballistic Graphene Heterojunction

Derivative Map

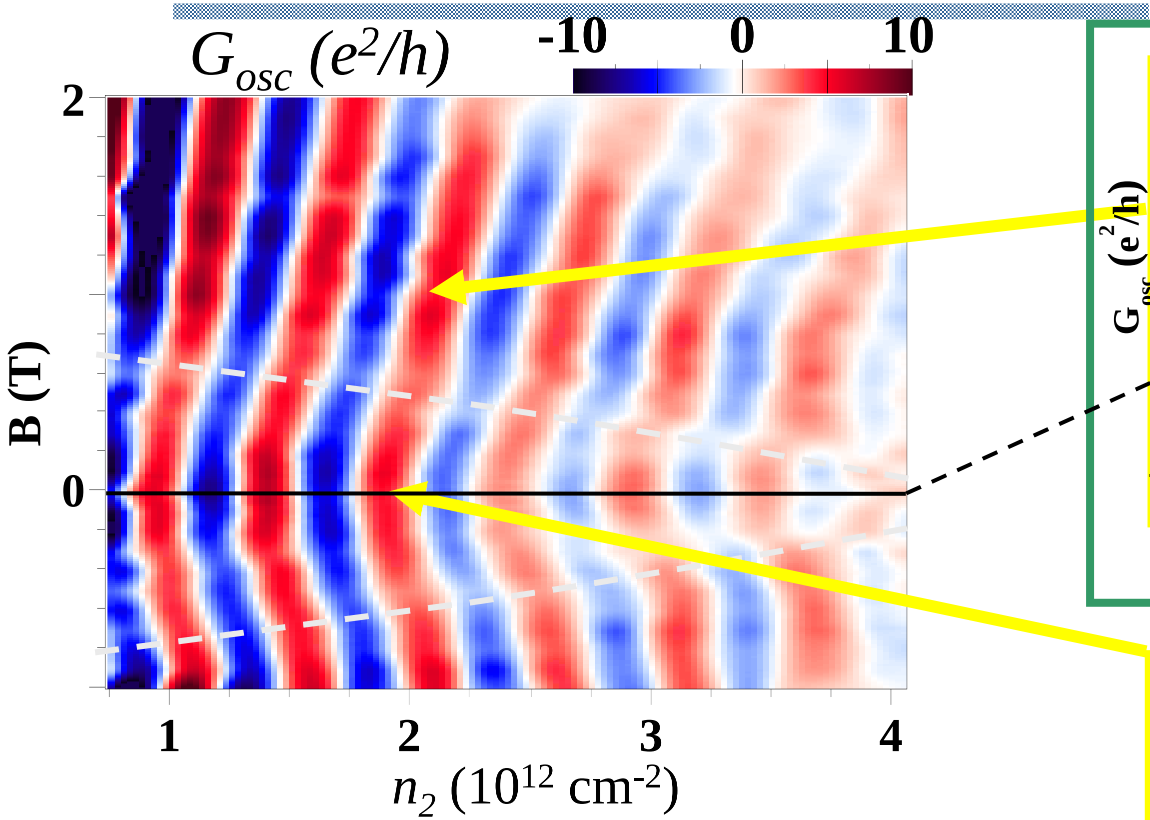


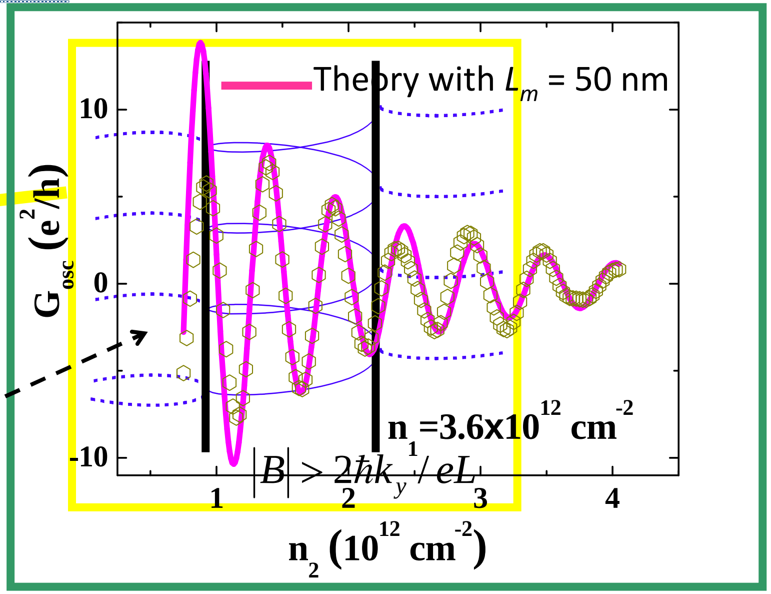



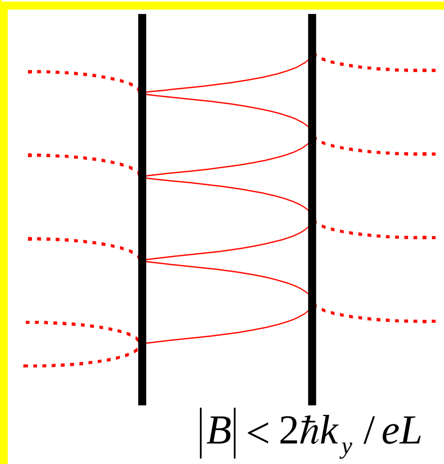

$$G_{osc} \sim \frac{8e^2}{h} \sum_{k_y} |T_1|^2 \, |T_2|^2 \, |R_1| \, |R_2| \cos\left(\theta_{WKB}\right) e^{-4L/L_m}$$

# Pseudo Spin Control with Magnetic Field




Young and Kim, Nat. Phys. (2009)




With increasing Magnetic field:

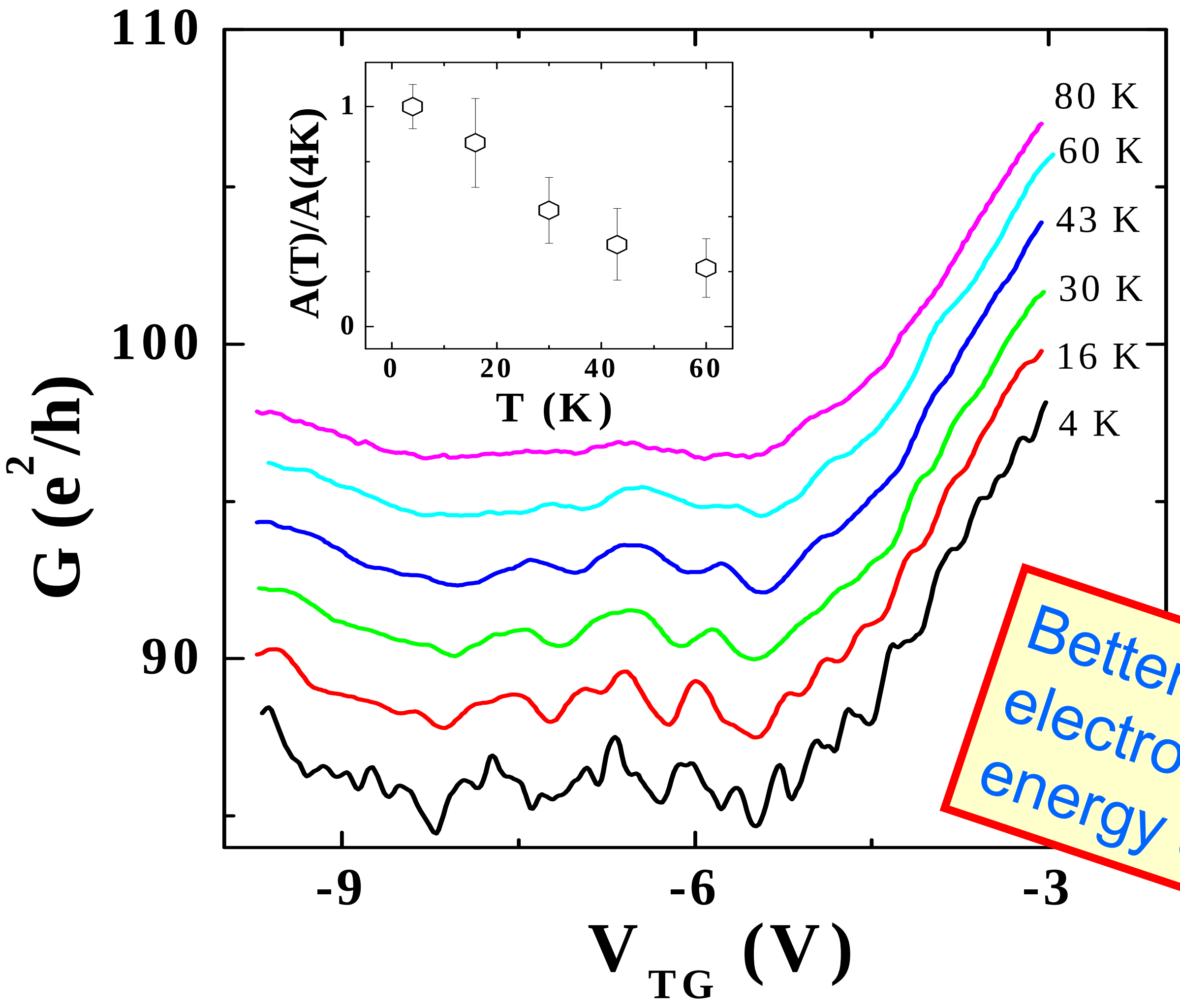
* Oscillations phase shift

* Abrupt phase at a certain magnetic field

Berry phase associated with pseudo spin

Levitov et al. (2008)

# Temperature Dependence



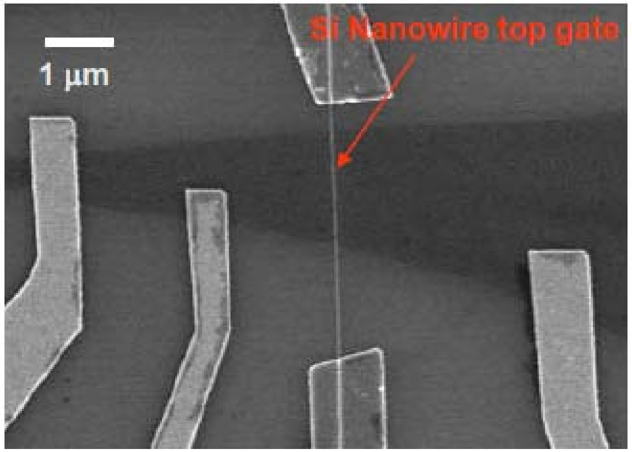

- FP resonance to ~80K
- Energy scale

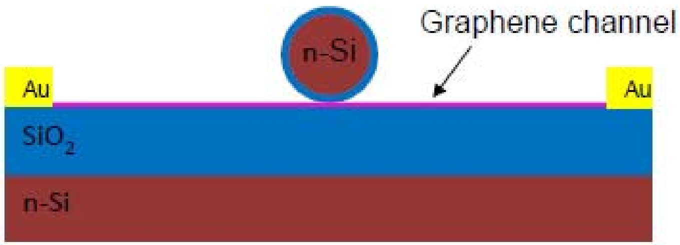
$$T \approx \frac{\hbar v_F}{L} \approx 100\text{K}$$

Better mobility and better electrostatic will make the energy scale larger!

# Nanowire Top Gate: Ultimate Electrostati

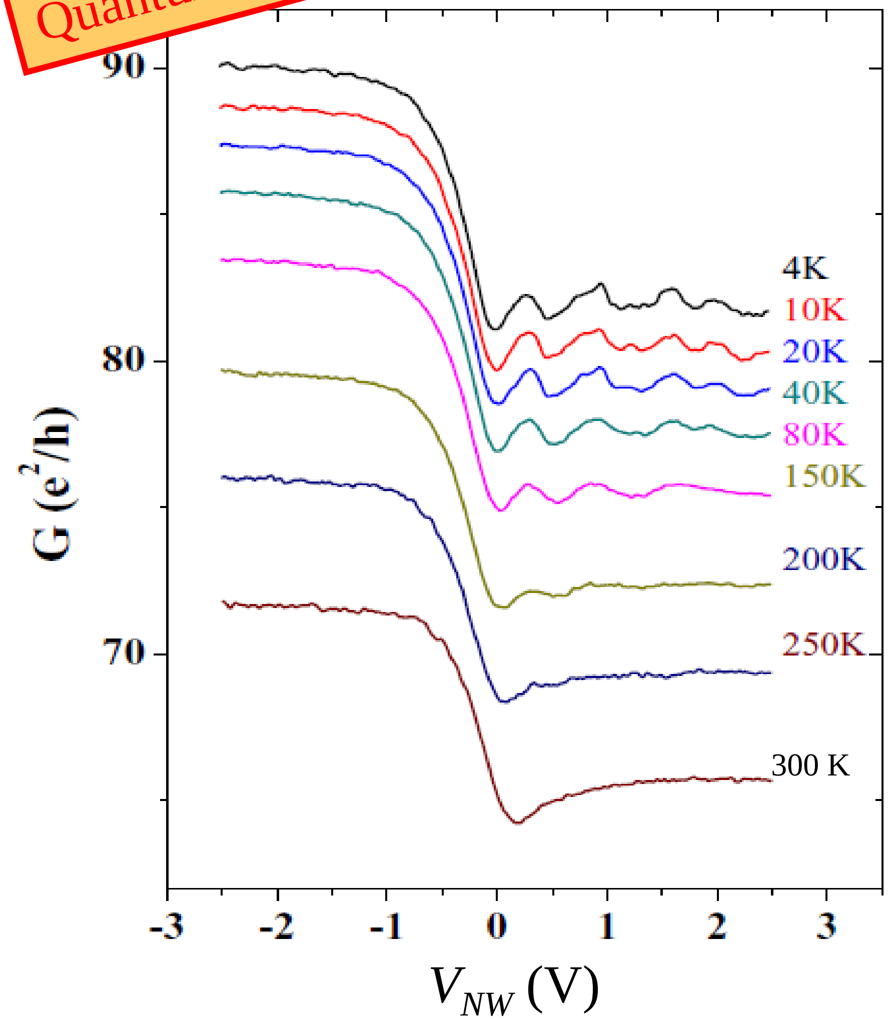
Quantum Oscillations at elevated temperatures



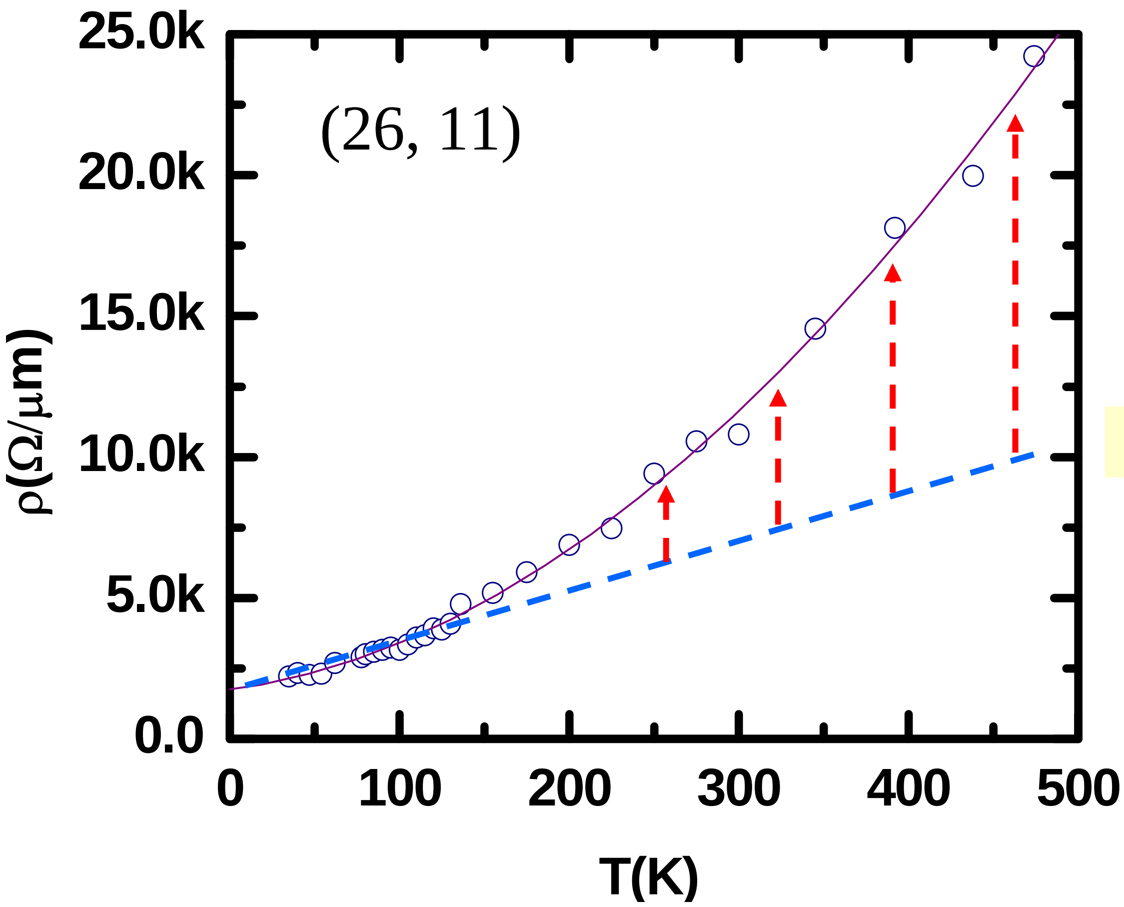

Si Nanowire: diameter ~ 25 nm

Natural $SiO_2$ dielectric ~ 2 nm




Temperature Dependence

# Nanotube on $SiO_2$ substrate



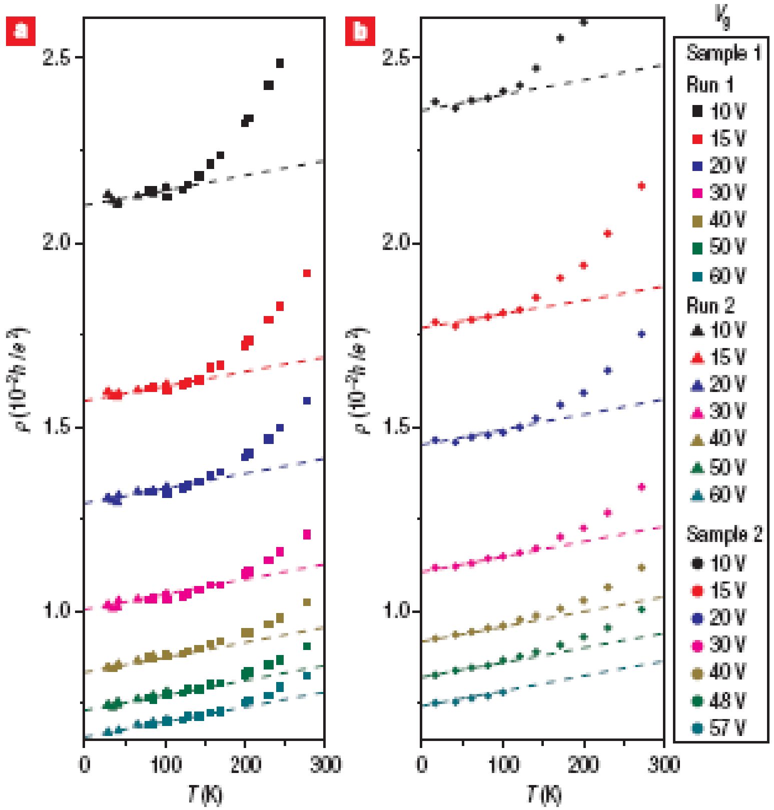

Super linear behaviors for T > 150 K

**Scattering due to acoustic phonons**

$$\frac{1}{\tau_{ac}} = 2\frac{2\pi}{\hbar}\Xi^2\left(\frac{k_B T}{2\rho v_s^2}\right)\frac{1}{h v_F}$$

Enhanced scattering activated T > 150 K

# Optical phonons in substrates

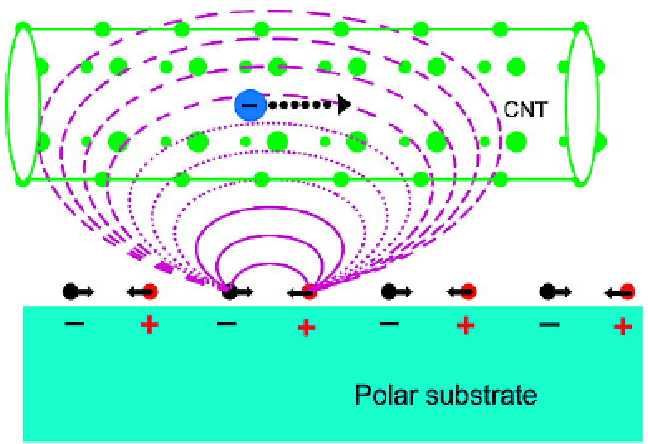



**The Effects of Substrate Phonon Mode Scattering on Transport in Carbon Nanotubes**

Vasili Perebeinos, Slava V. Rotkin, Alexey G. Petrov, and Phaedon Avouris

*Nano Lett.*, **2009**, 9 (1), 312-316 • DOI: 10.1021/nl8030086 • Publication Date (Web): 04 December 2008






**Transport in graphene**
**Chen et al. Nature Phys (2008)**

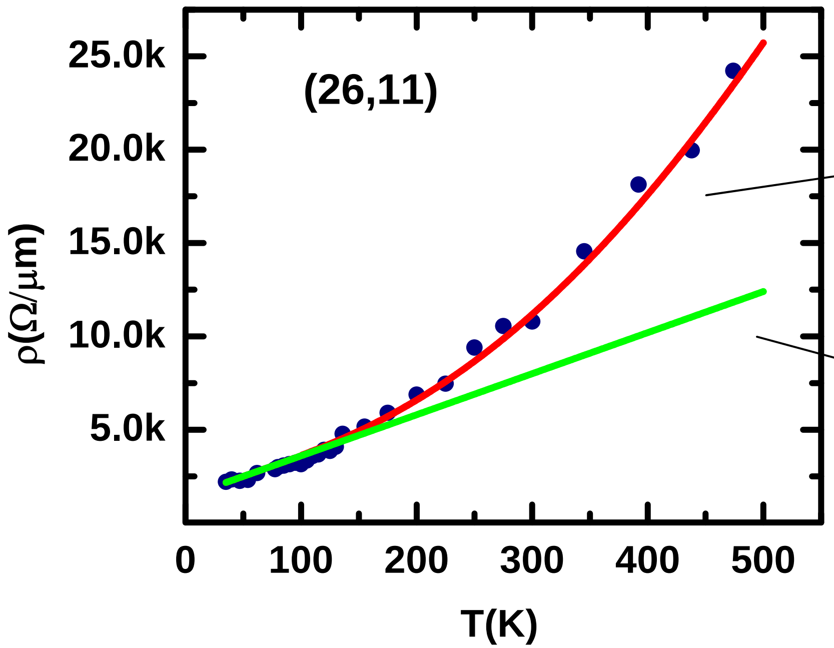
Optical phonons $SiO_2$ substrate is much softer ! (~ 30-50 meV)

# Substrate Phonon Scattering Analysis

Perebeinos *et al.* Nano Lett. **9**, 312 (2009)

$$\rho_{Total} = \rho_{static} + \alpha_{acoustic} T + \alpha_{SPP} (n_B(\hbar w_2) + \beta n_B(\hbar w_{4,5}))$$

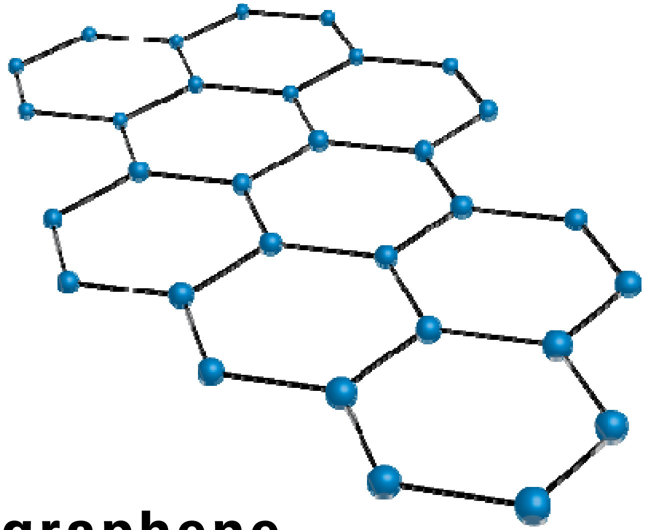
$$F_v^{\,2} = \frac{\hbar w_{SO,v}}{2\pi}\left(\frac{1}{\varepsilon_\infty + 1} - \frac{1}{\varepsilon_0 + 1}\right) \qquad \beta = \frac{(F_4^{\,2} + F_5^{\,2})}{F_2^{\,2}} \qquad n_B(\hbar w_v) = \frac{1}{(e^{\hbar w_v / k_B T} - 1)}$$



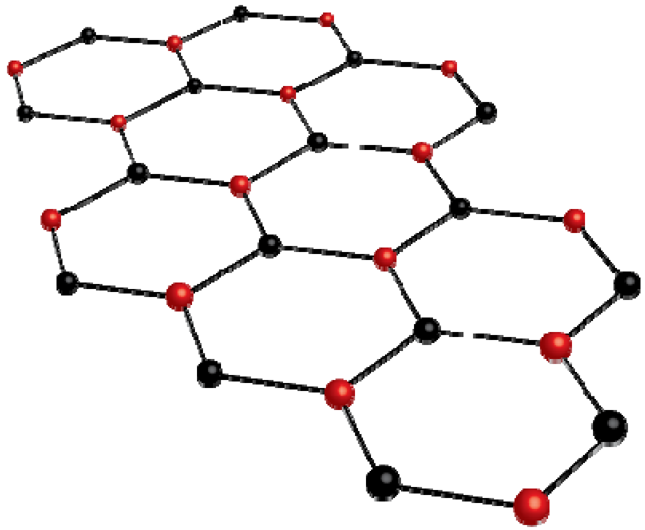

$$\rho_{Total} = \rho_{static} + \alpha_{acoustic} T + \alpha_{SPP} (n_B(\hbar w_2) + \beta n_B(\hbar w_{4,5}))$$

$$\rho_{Total} = \rho_{static} + \alpha_{acoustic} T$$

# hexa-Boron Nitride: Ideal Dielectric



**graphene**

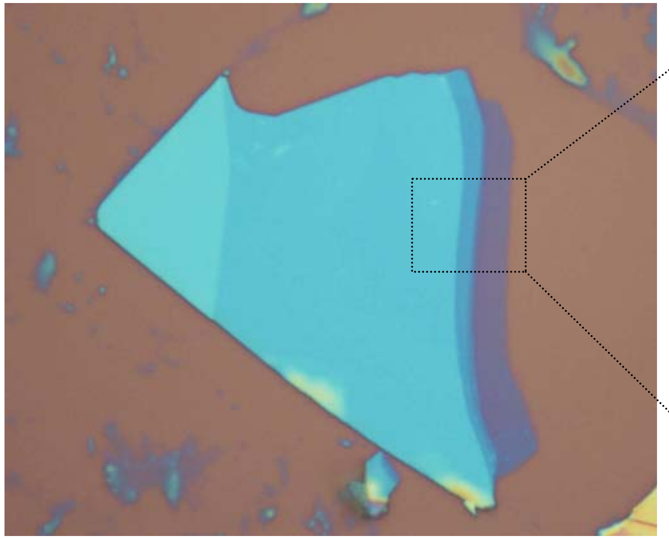


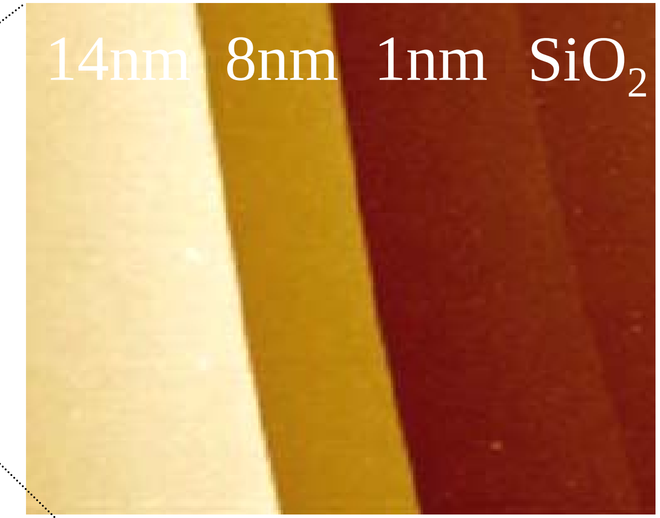
**Boron Nitride**

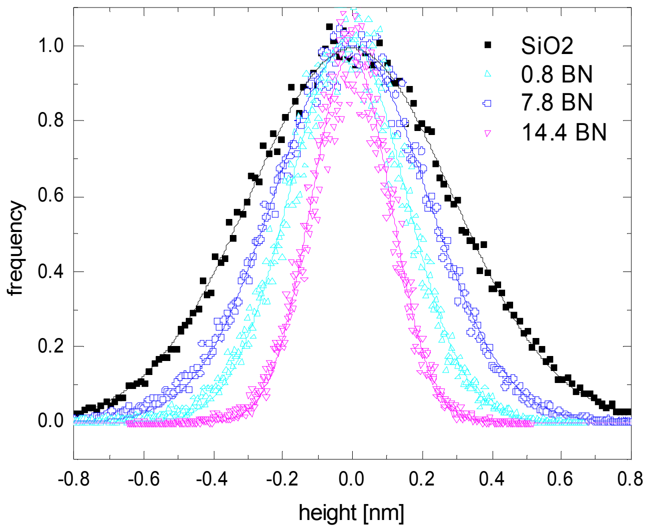
Comparison of h-BN and $SiO_2$

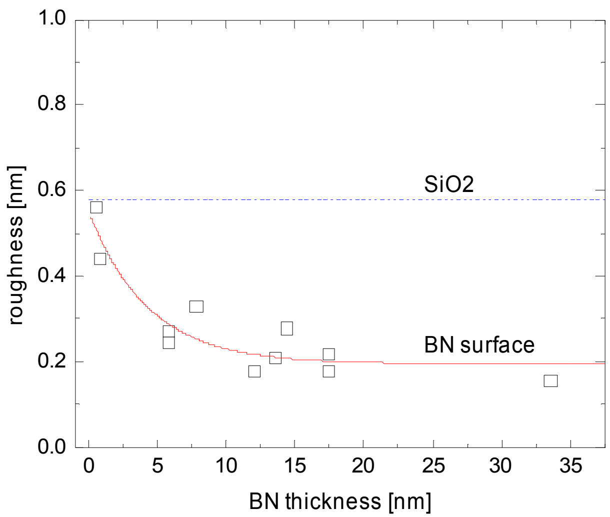
| | Band Gap | Dielectric Constant | Optical Phonon Energy | Structure |
|---|---|---|---|---|
| BN | 3.6 - 7.1 eV | ~4 | >150 meV | Layered crystal |
| SiO2 | 8.9 eV | 3.9 | 59 meV | Amorphous |

# Mechanical Exfoliation of BN



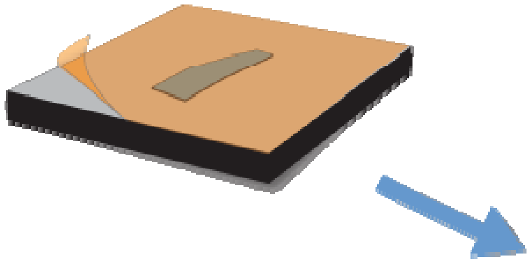


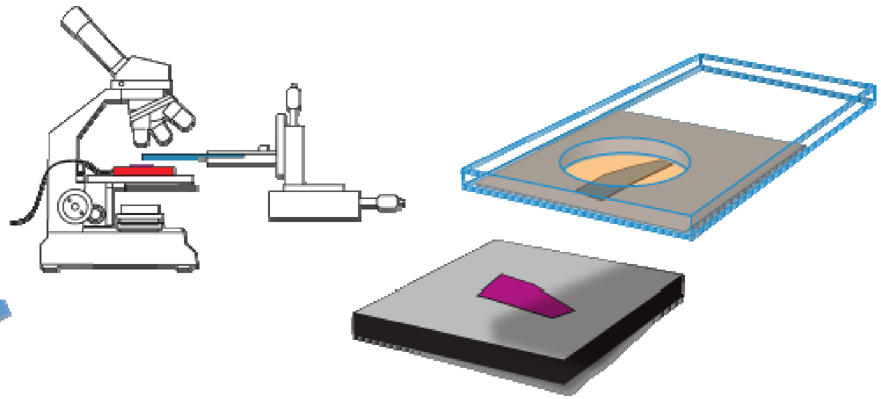




- Mechanically cleavable
- Atomically flat

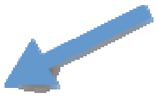
# Precision Transfer Technique

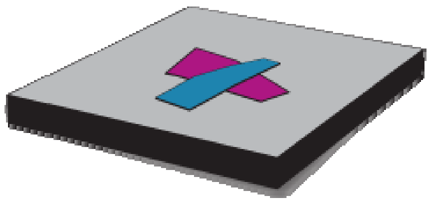


1. spin coat substrate with PMMA and scratch onto top surface



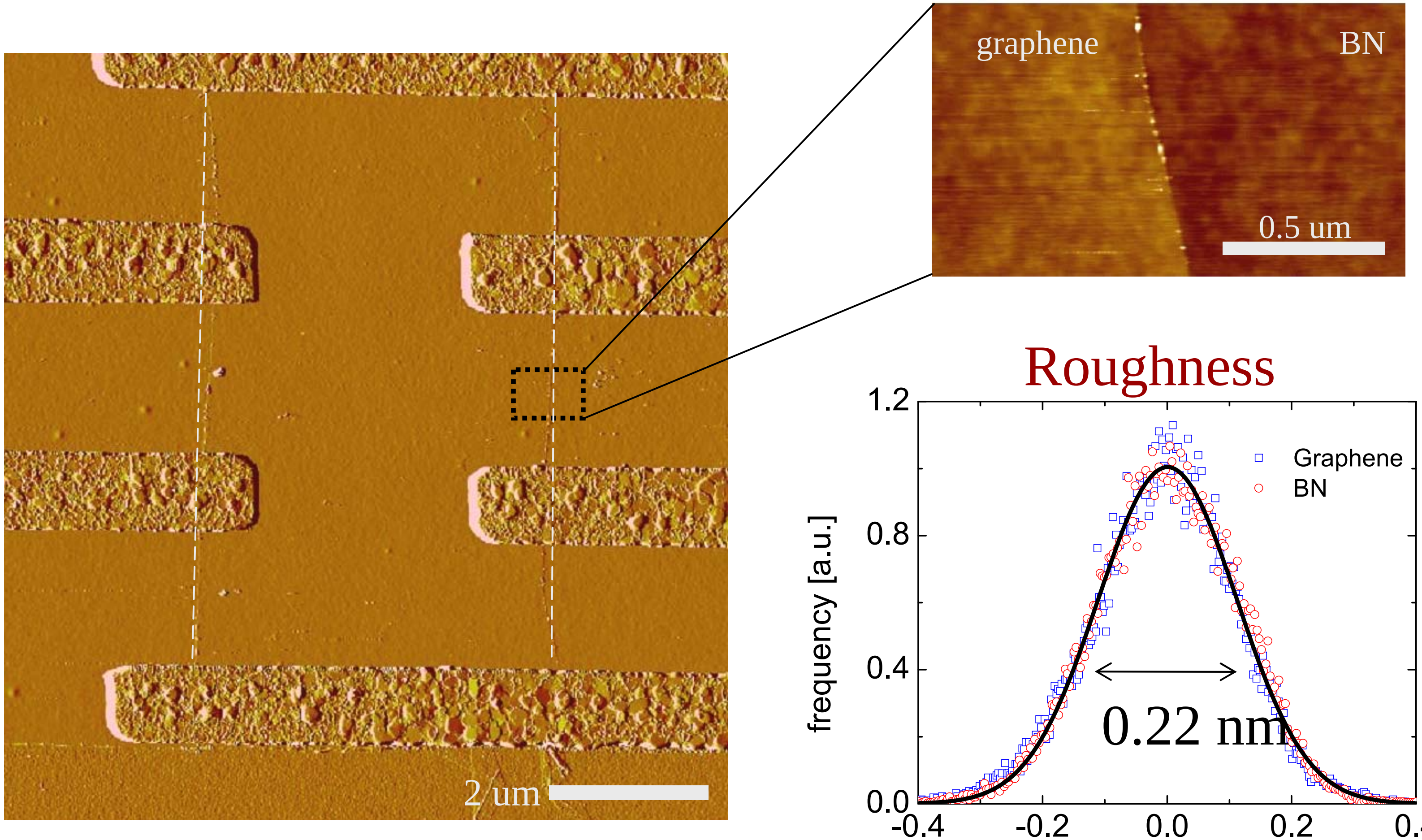
2. Lift off PMMA; graphene comes with it!



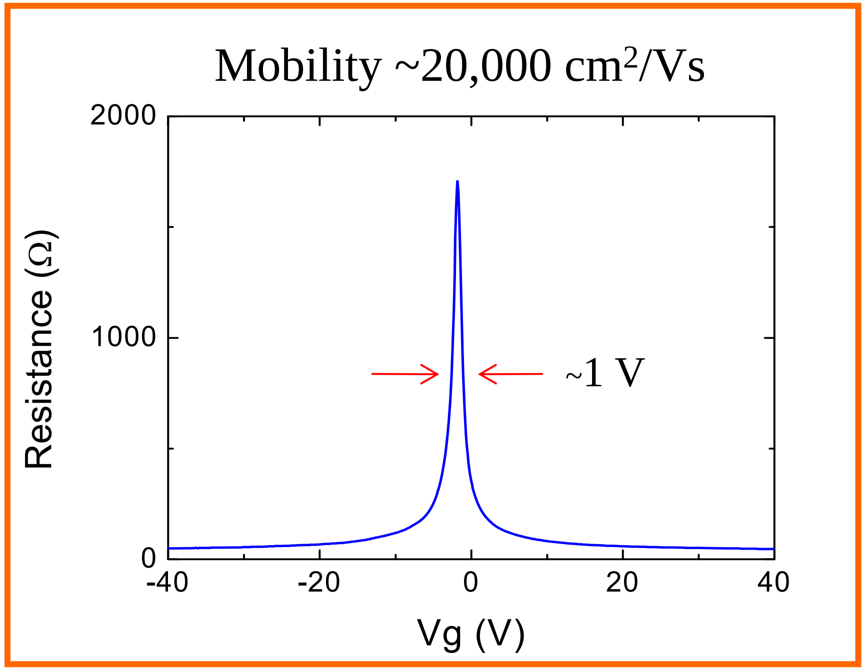


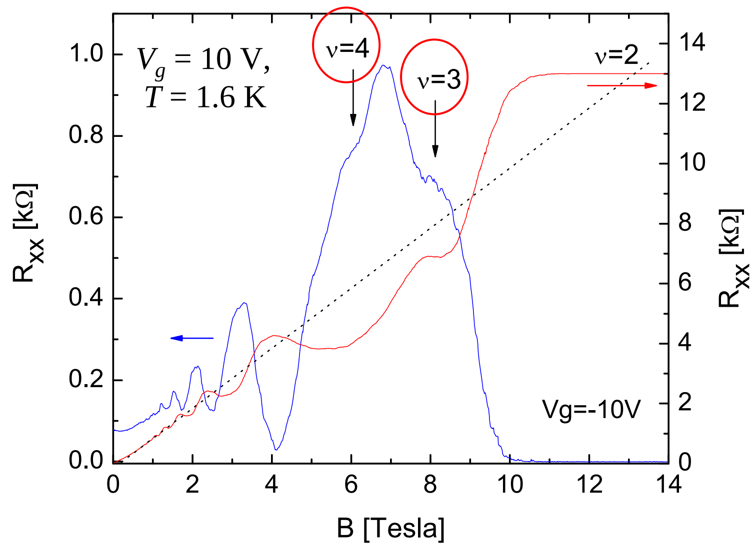
3. Align graphene over target using a micro-manipulator. PMMA is brought into contact with target and annealed.

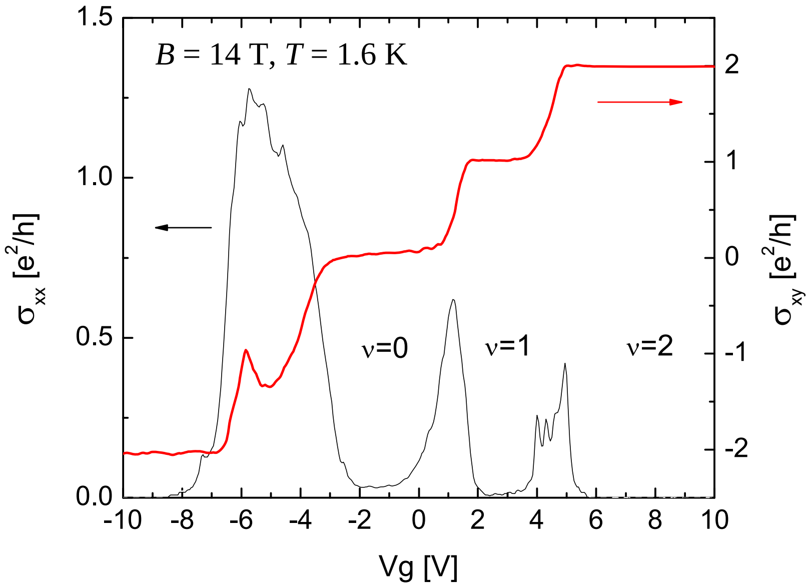
# Graphene on h-BN is Flat!



# Transport in Graphene/h-BN

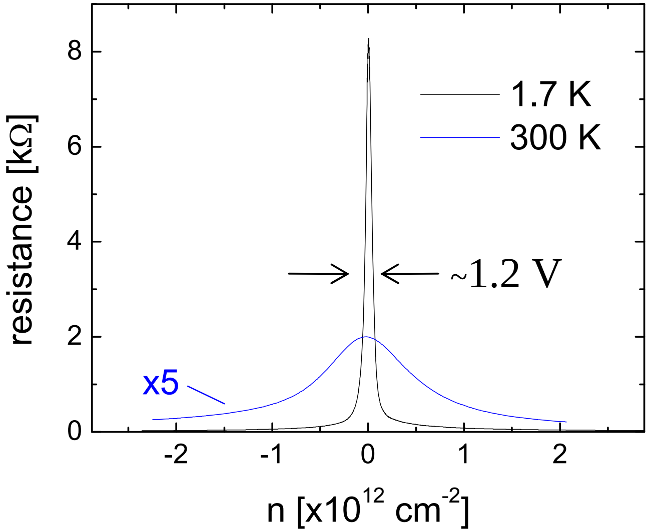


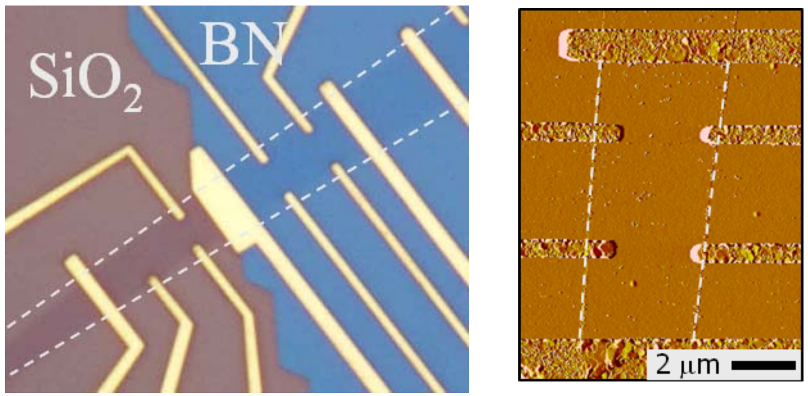



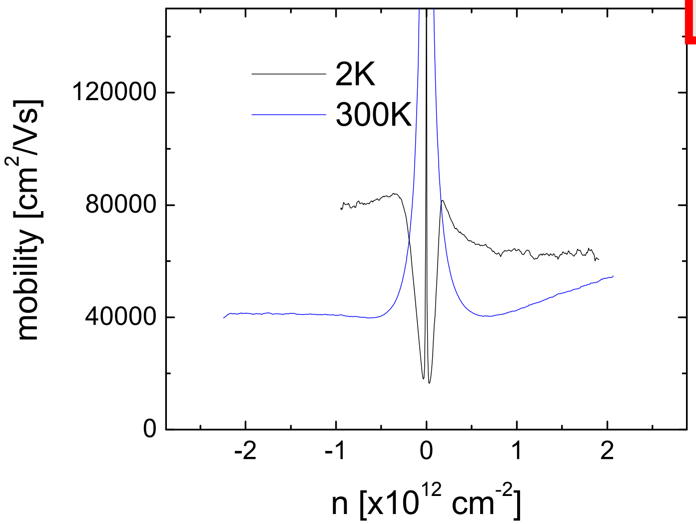

- Enhanced mobility on BN versus $SiO_2$ for the same flake
- Very narrow DP peak: reduced inhomogeneity.
- Reduced chemical reactivity (no appreciable doping by H/Ar annealing)

# Bilayer graphene on Boron Nitride



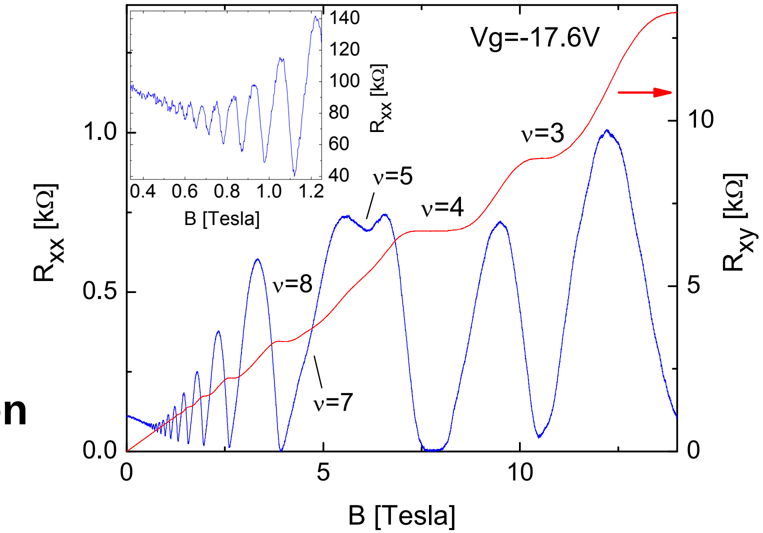




Bilayer graphene transferred over BN




**RT mobility > 40,000 $cm^2$/sec**

**SdH Oscillation**

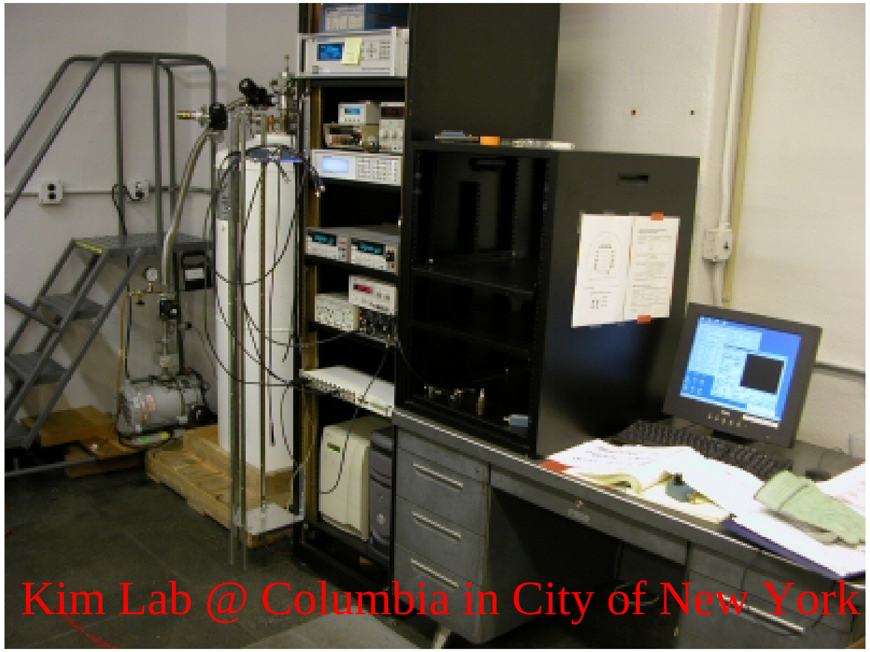


# Conclusions

Relativistic QM: High Energy Physics

CERN Electro-Positron Collider

Quasi Relativistic QM: Low Energy Physics



Kim Lab @ Columbia in City of New York

*Dirac Equation:* $\tilde{H} = c^{*}\,\vec{\sigma}\cdot\vec{p}$

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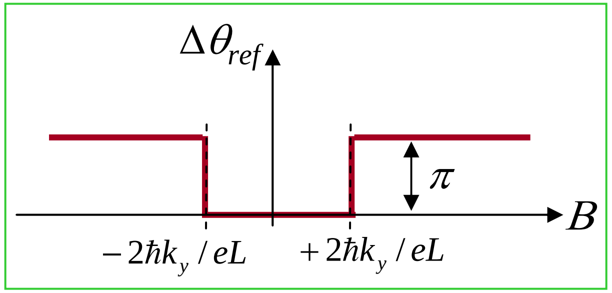
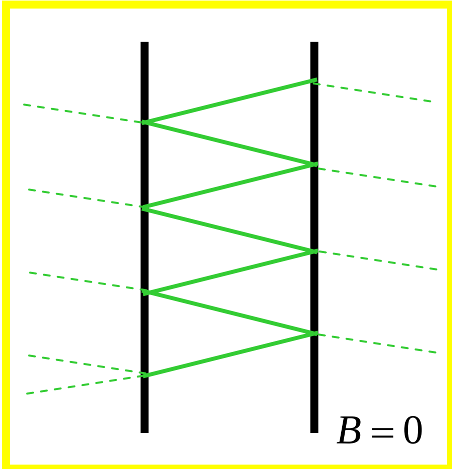
# FP resonances and Berry Phase




$$\theta_{WKB} = \Re \int_{-L/2}^{L/2} \sqrt{\pi |n(x)| - \left(k_y - \frac{e}{\hbar} B x\right)^2}$$

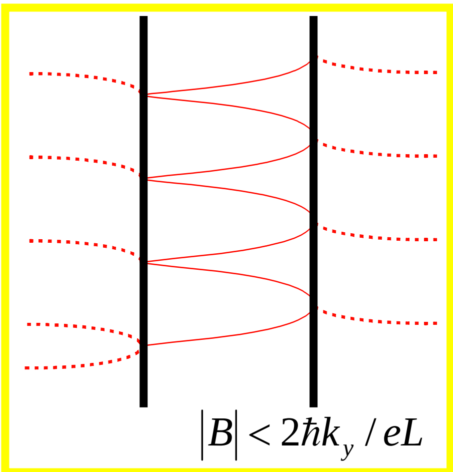
$$\Delta\theta_{ref} = \pi\left[\theta(k_y + \frac{eBL}{2\hbar}) - \theta(k_y - \frac{eBL}{2\hbar})\right]$$

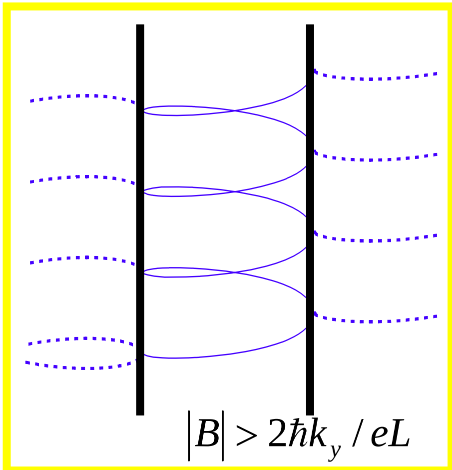
Real Space trajectories



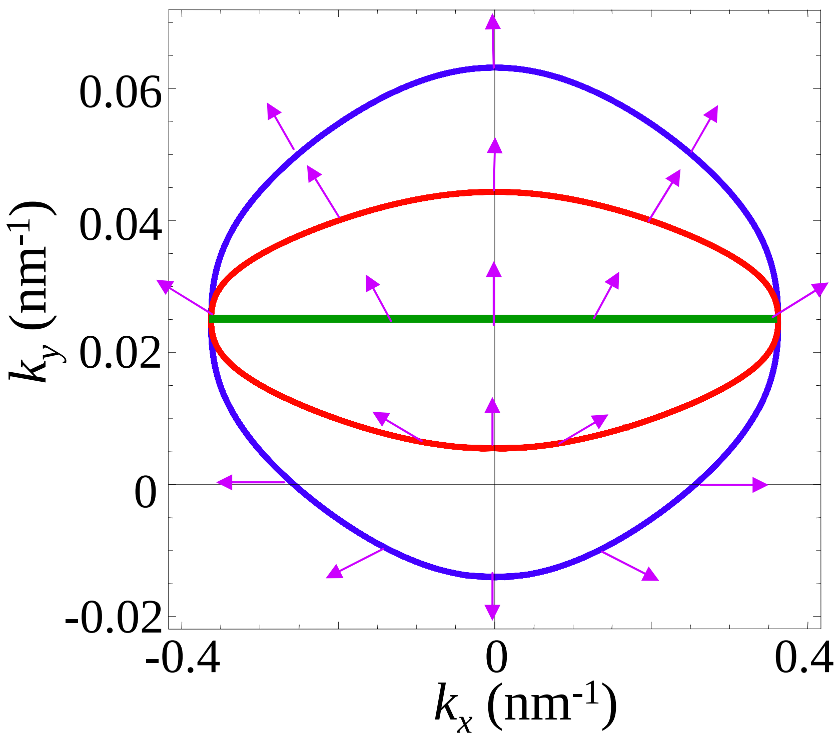

**Critical field**

$$B_c = 2\hbar k_y / eL$$



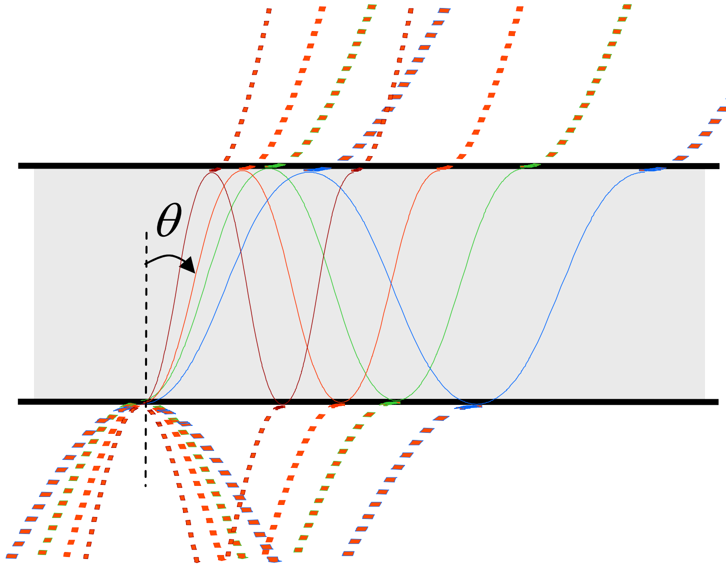

**Momentum Space Trajectories**



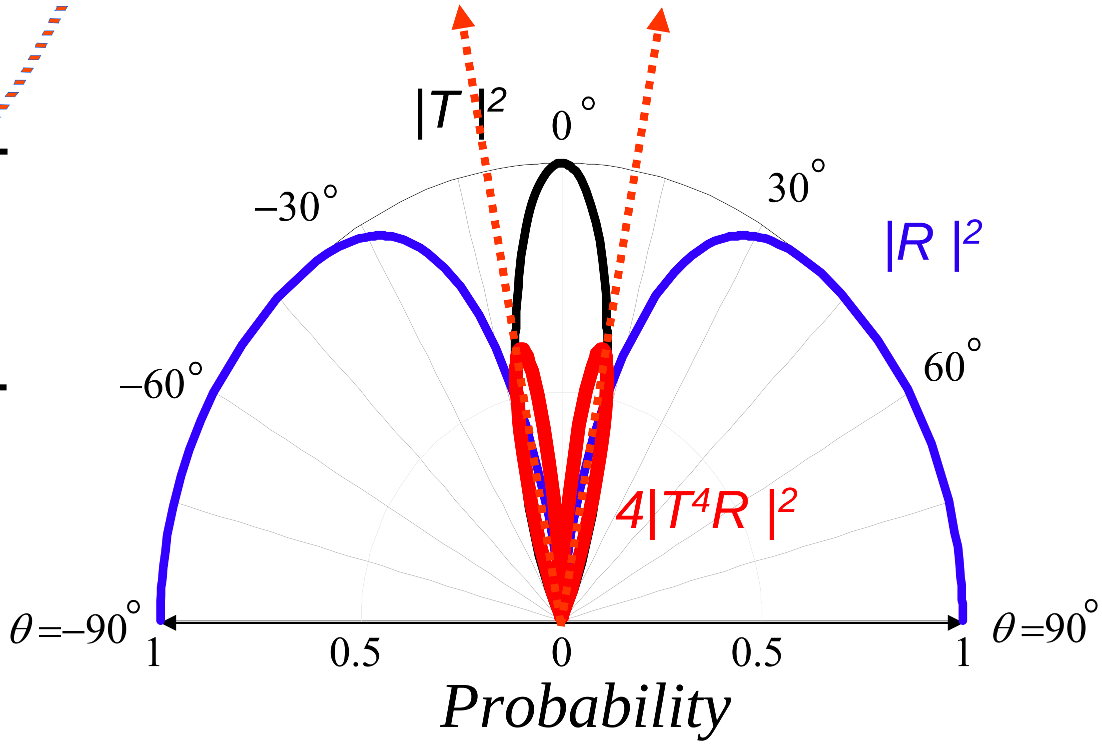
# Wave Function Collimation in Diffusive Transport

$$G_{osc} \sim \frac{8e^2}{h} \sum_{k_y} |T_1|^2 \, |T_2|^2 \, |R_1| \, |R_2| \cos\left(\theta_{WKB}\right)$$

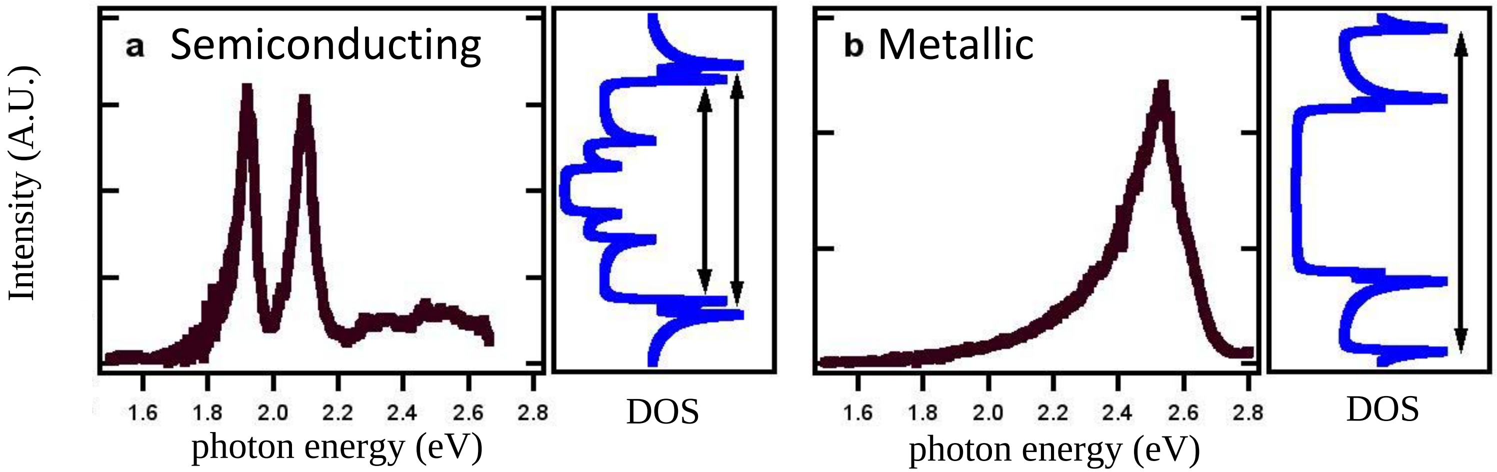
**Diffusive transport outside of heterojunction regime**



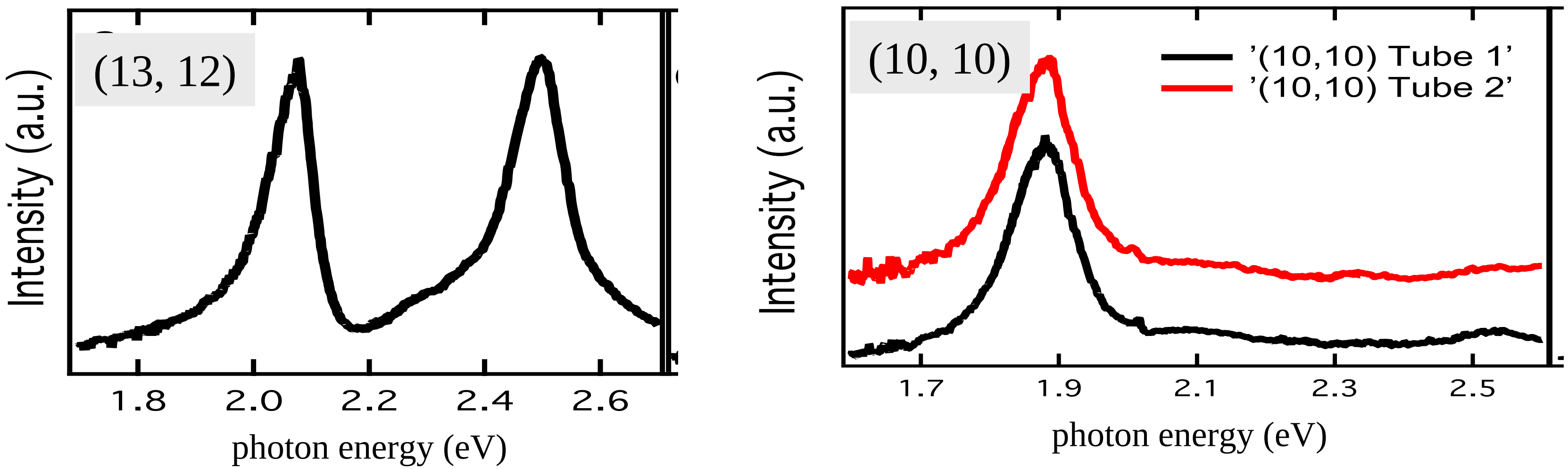

**Collimated beam direction**



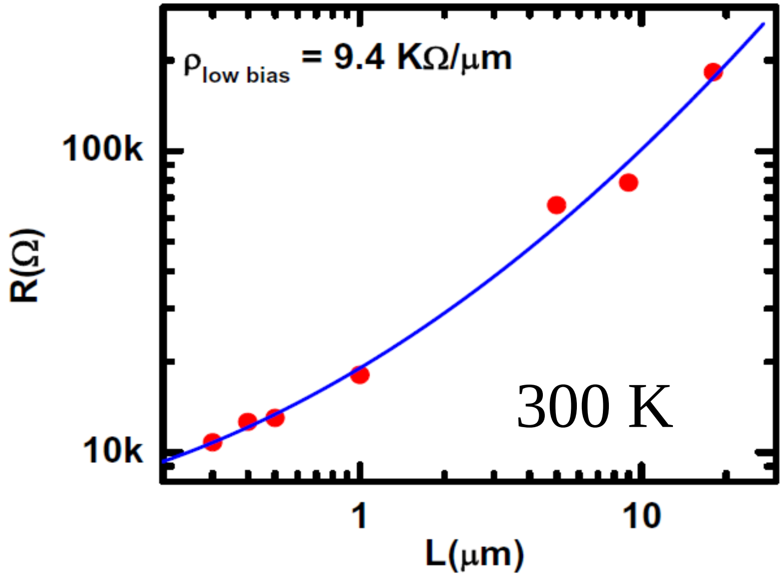
# Rayleigh Light Scattering of Individual Tubes



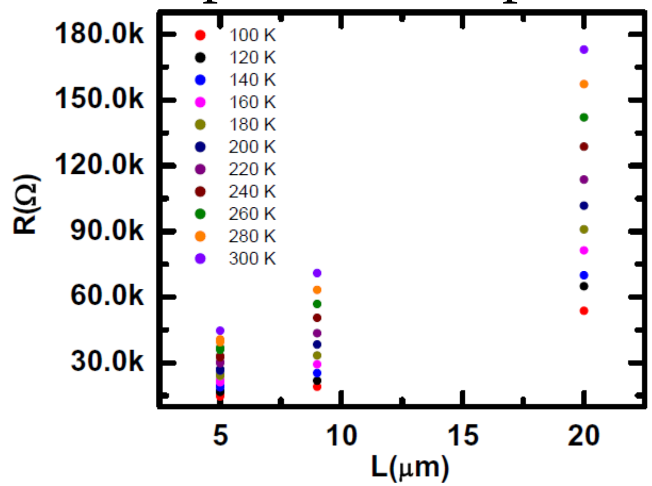

Structure can be analyzed from the Rayleigh Scattering Spectrum



# Resistivity of (26, 11) Nanotubes

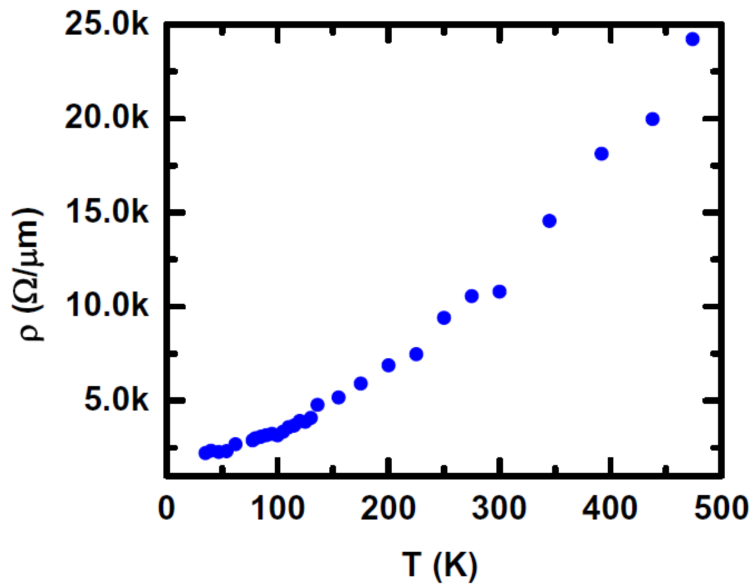



Temperature Dependence




1-dimensional resistance scaling

$$R(L) = \frac{h}{Ne^2} + \rho_{1d} L$$




Measurement of resistivity of known chirality in 4-500 K