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Confinement in Yang-Mills Theories: Elements of a Big Picture

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Gribov-80 Memorial Workshop
on QCD and Beyond

May 26-28, 2010, Triest

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Confinement in Yang-Mills: Elements of a Big Picture

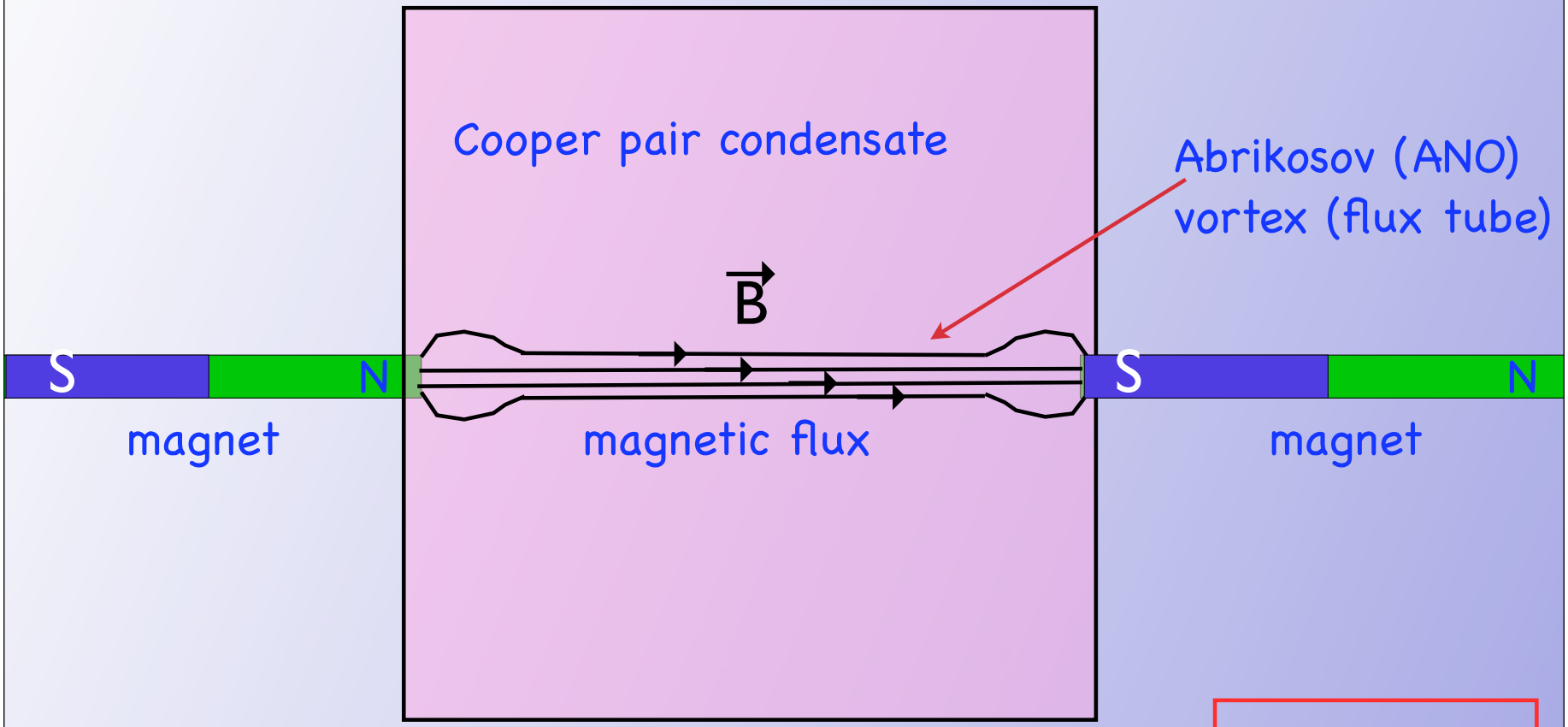
With A. Yung

5. STRONG COUPLING

6. CONFINEMENT

Are we aware of precedents?

☞ Yes, the Meissner effect! 1930s, 1960s



Superconductor of the 2nd kind

Abelian ☚

Prototype model

$$S = \int \mathrm{d}^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}|D_\mu a^a|^2 \right.$$

$$+ \mathrm{Tr}\left(\nabla_\mu \Phi\right)^\dagger \left(\nabla^\mu \Phi\right) + \frac{g_2^2}{2}\left[\mathrm{Tr}\left(\Phi^\dagger T^a \Phi\right)\right]^2 + \frac{g_1^2}{8}\left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right) - N\xi\right]^2$$

$$\left. + \frac{1}{2}\mathrm{Tr}\left|a^a T^a \Phi + \Phi\sqrt{2}M\right|^2 + \frac{i\,\theta}{32\,\pi^2} F^a_{\mu\nu}\tilde{F}^{a\,\mu\nu} \right\},$$

$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} \\ \varphi^{21} & \varphi^{22} \end{pmatrix}$$

$$M = \begin{pmatrix} m & 0 \\ 0 & -m \end{pmatrix}$$

U(2) gauge group, 2 flavors of (scalar) quarks

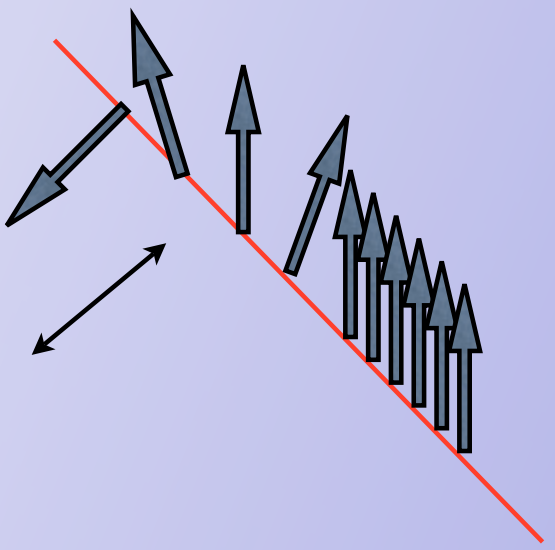
SU(2) Gluons A^a_μ + U(1) photon + gluinos+ photino

$\Phi = \sqrt{\xi} \times \mathrm{I}$

Basic idea:

- **Color-flavor locking in the bulk → Global symmetry G;**
- **G is broken down to H on the given string;**
- **G/H coset; G/H sigma model on the world sheet.**

"Non-Abelian" string is formed if all non-Abelian degrees of freedom participate in dynamics at the scale of string formation



2003: Hanany, Tong
Auzzi et al.
Yung + M.S.

classically gapless excitation

SU(2)/U(1) = CP(1)~O(3) sigma model

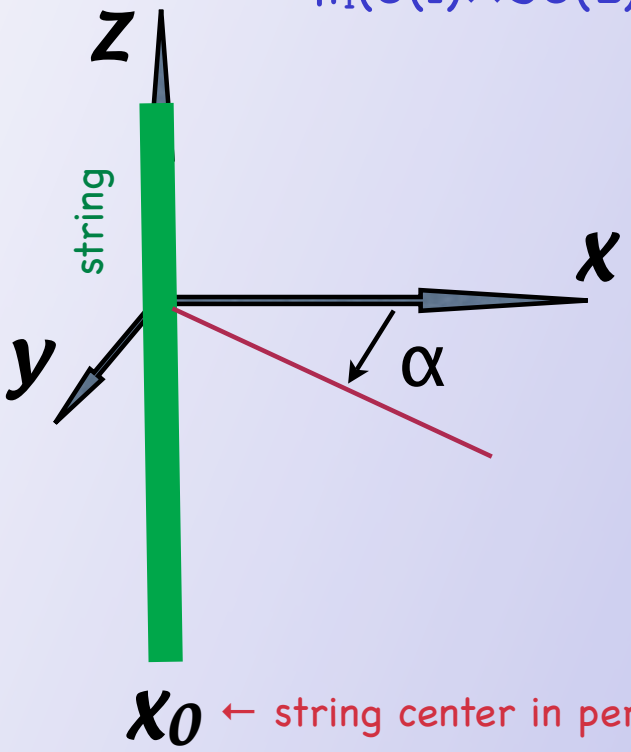
★ ANO strings are there because of U(1)!

★ New strings:

$\pi_1(SU(2)\times U(1)) = Z_2$: **rotate by π around 3-d axis in SU(2)**

→ **-1; another -1 rotate by π in U(1)**

$\pi_1(U(1)\times SU(2))$ nontrivial due to Z_2 center of SU(2)



ANO $\quad \sqrt{\xi}\, e^{i\alpha} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$T=4\pi\xi$

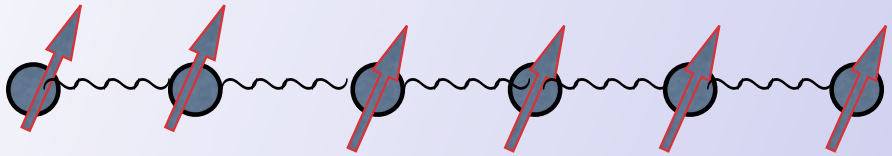
Non-Abelian $\quad \sqrt{\xi} \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}$

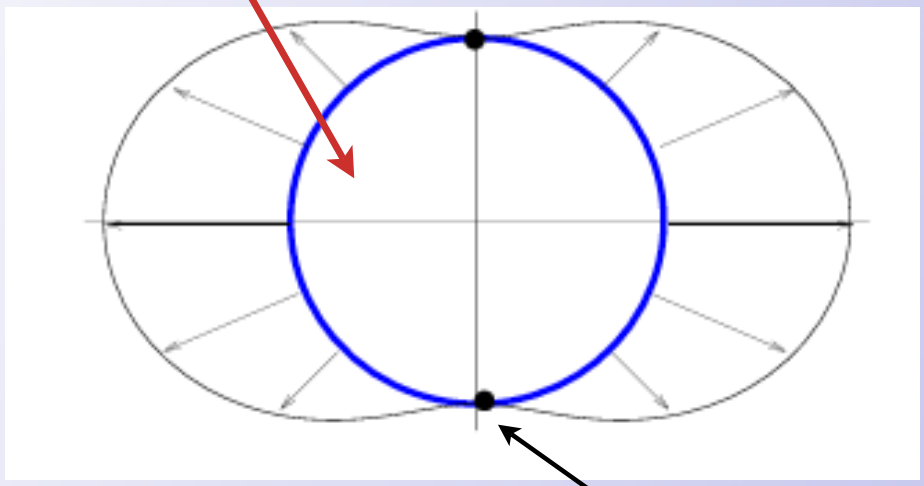
$T_{U(1)} \pm T^3_{SU(2)}$

$T=2\pi\xi$

SU(2)/U(1) ←orientational moduli; O(3) σ model

Worldsheet theory

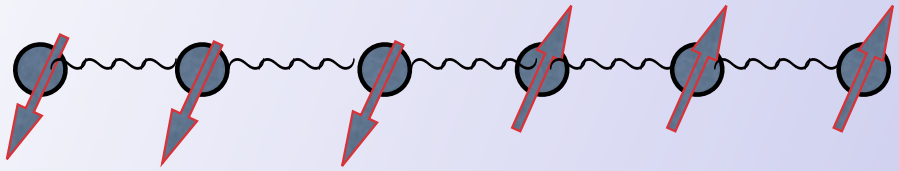


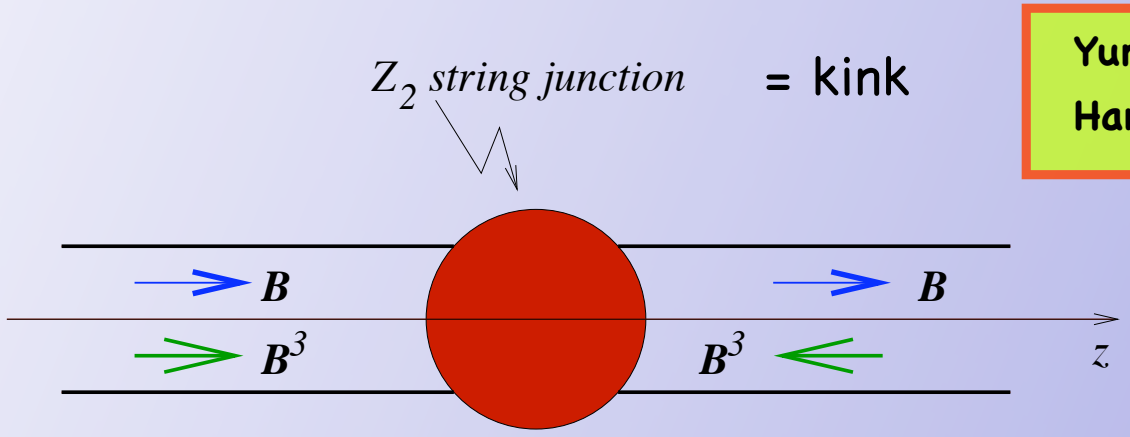


Global SU(2) is gone!
U(1) remains intact

CP(1) model with twisted mass

$$S = \int d^2x \left\{ \frac{2}{g^2} \frac{\partial_\mu \bar{\phi} \partial^\mu \phi - (\Delta m)^2 \bar{\phi}\phi}{(1+\bar{\phi}\phi)^2} + fermions \right\}$$





Yung + M.S.
Hanany, Tong

Evolution in dimensionless parameter m^2/ξ

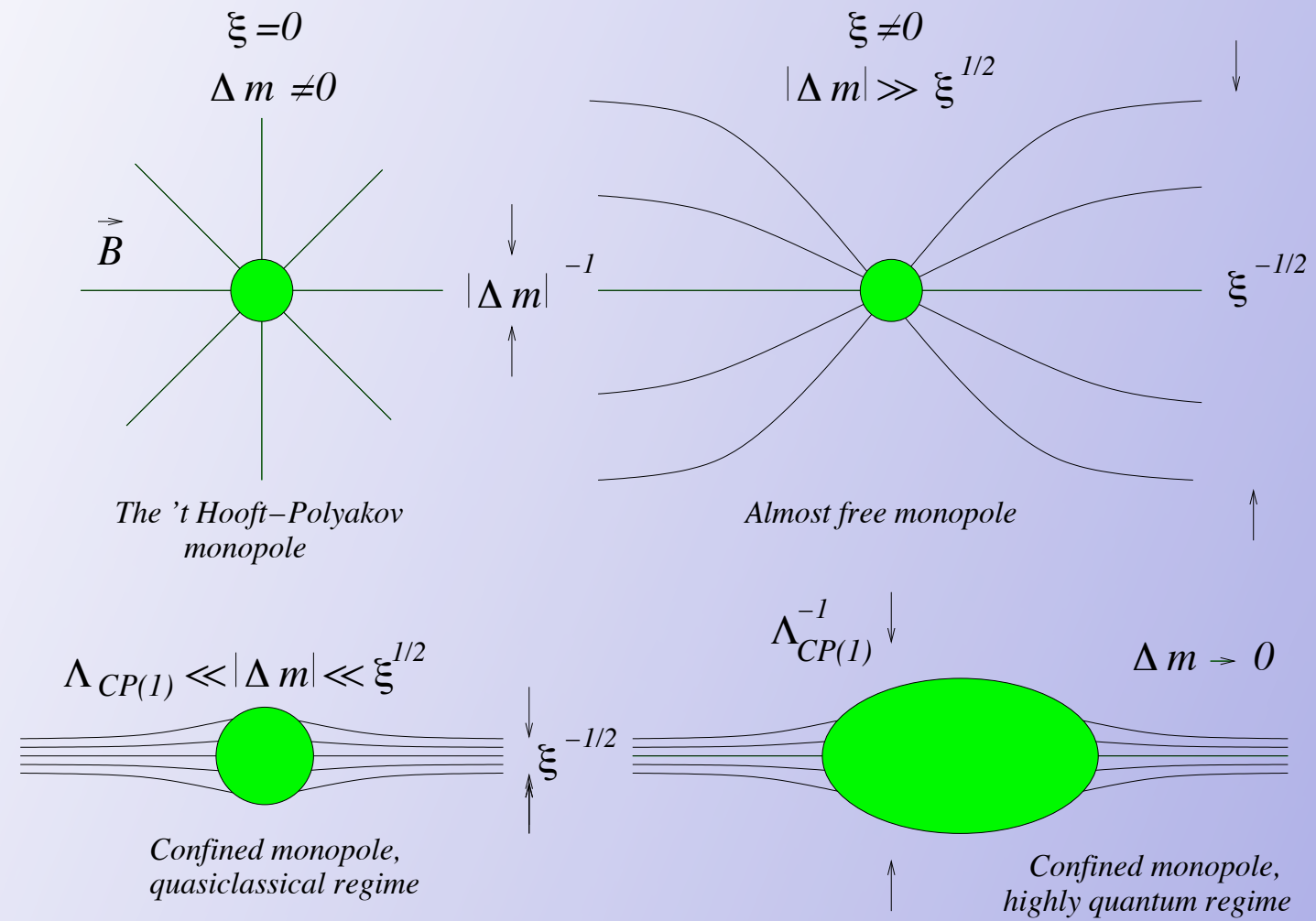
- Kinks are confined in 4D (attached to strings).
- Kinks are confined in 2D:

  ★only kink-antikink in the spectrum★

  if SUSY is unbroken (explained by Witten)

Kink = Confined Monopole

Why?


$\xi = 0$
$\Delta m \neq 0$
$\vec{B}$
$|\Delta m|^{-1}$
$\xi \neq 0$
$|\Delta m| \gg \xi^{1/2}$
$\xi^{-1/2}$
The 't Hooft–Polyakov monopole
Almost free monopole
$\Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2}$
$\xi^{-1/2}$
$\Lambda_{CP(1)}^{-1}$
$\Delta m \to 0$
Confined monopole, quasiclassical regime
Confined monopole, highly quantum regime

Break N = 2 down to N = 1 in the bulk

Tong
Yung + M.S.

Heterotic deformation of CP(N-1)

(2,2) supersymmetry is broken down to (0,2)

$$L_{heterotic} = \zeta_R^\dagger i\partial_L \zeta_R + \left[\gamma \zeta_R R \left(i\partial_L \phi^\dagger\right)\psi_R + H.c.\right] - g_0^2|\gamma|^2 \left(\zeta_R^\dagger \zeta_R\right)\left(R\psi_L^\dagger \psi_L\right)$$

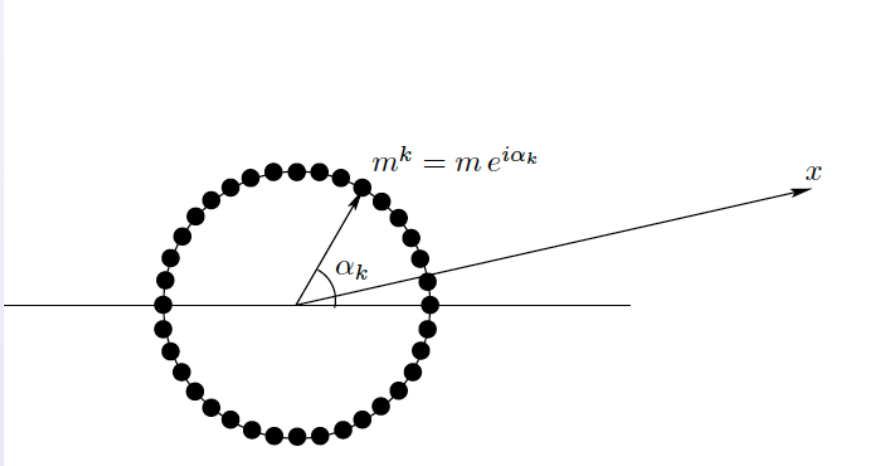
at small γ
ζ_R is Goldstino

$$\mathcal{E}_{vac} = |\gamma|^2 \left|\langle R\psi_R^\dagger \psi_L\rangle\right|^2$$

(0,2) supersymmetry is spontaneously broken!

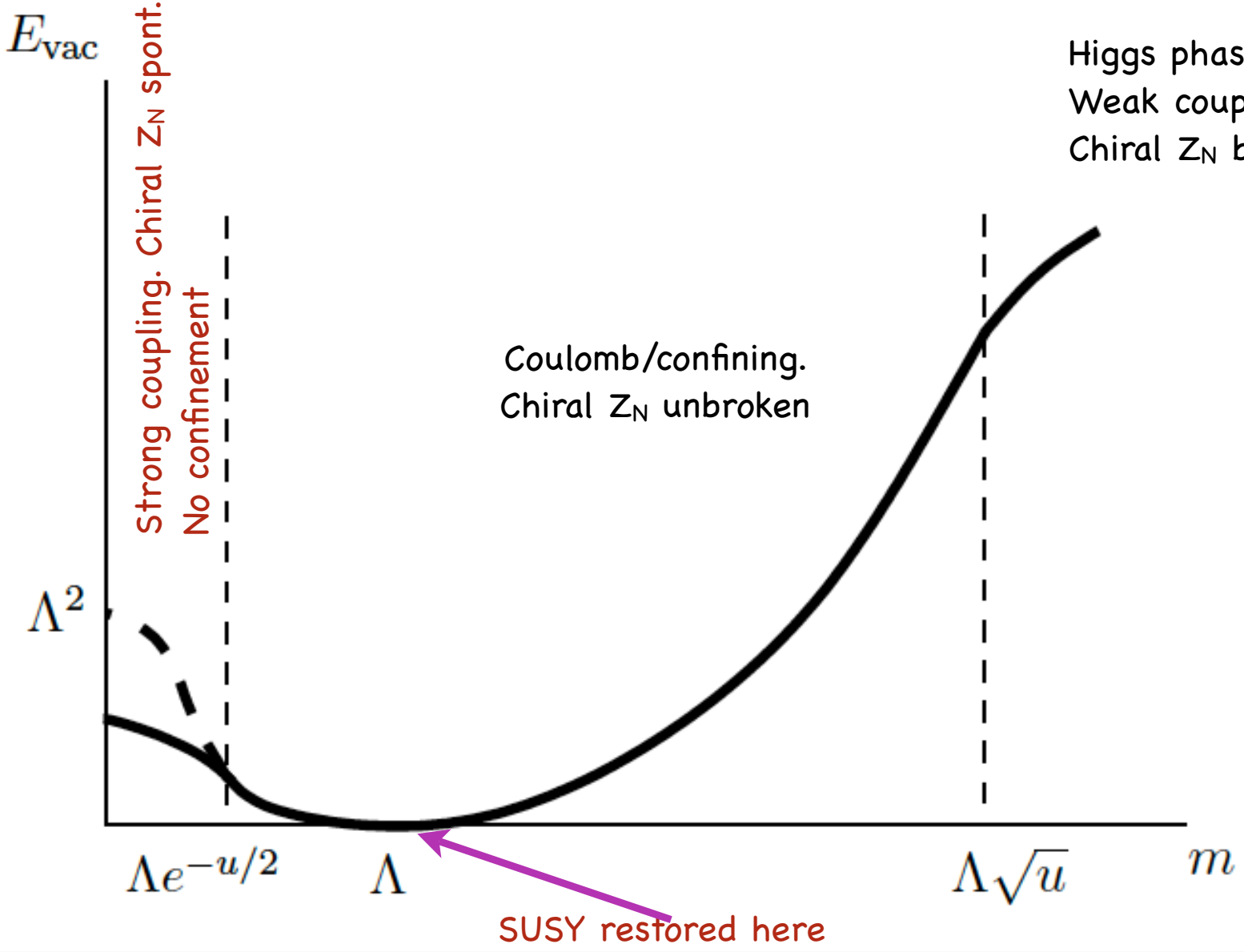
At large N heterotic CP(N-1) is solvable (a là Witten) and presents a treasure trove of various phases

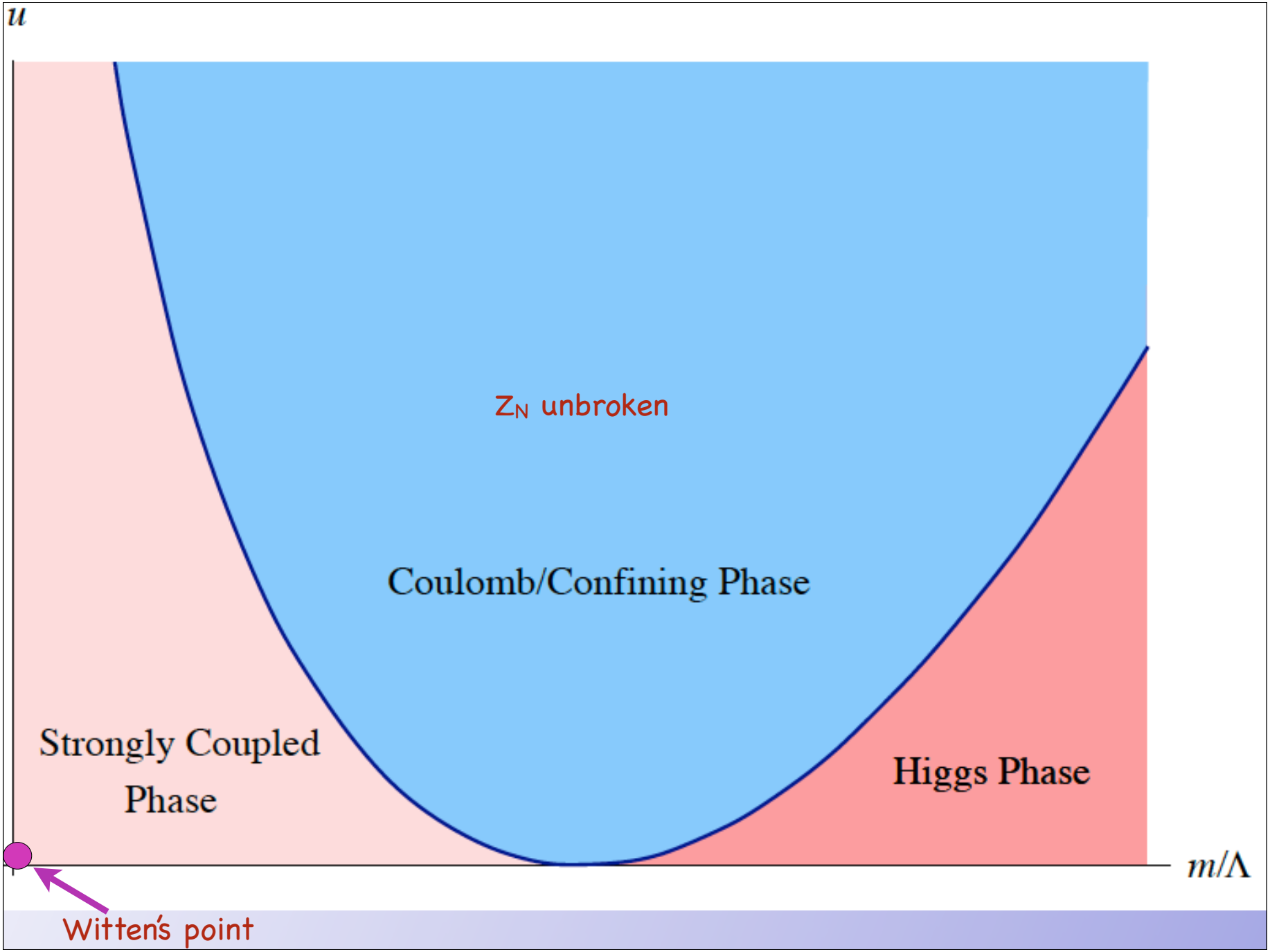
We have two parameters, γ and m, and a nontrivial phase diagram



With this choice of mass parameters we have Z_N symmetry, and phases with broken/unbroken Z_N. SUSY is spontaneously broken

$\gamma >> 1 \quad (u >> 1)$

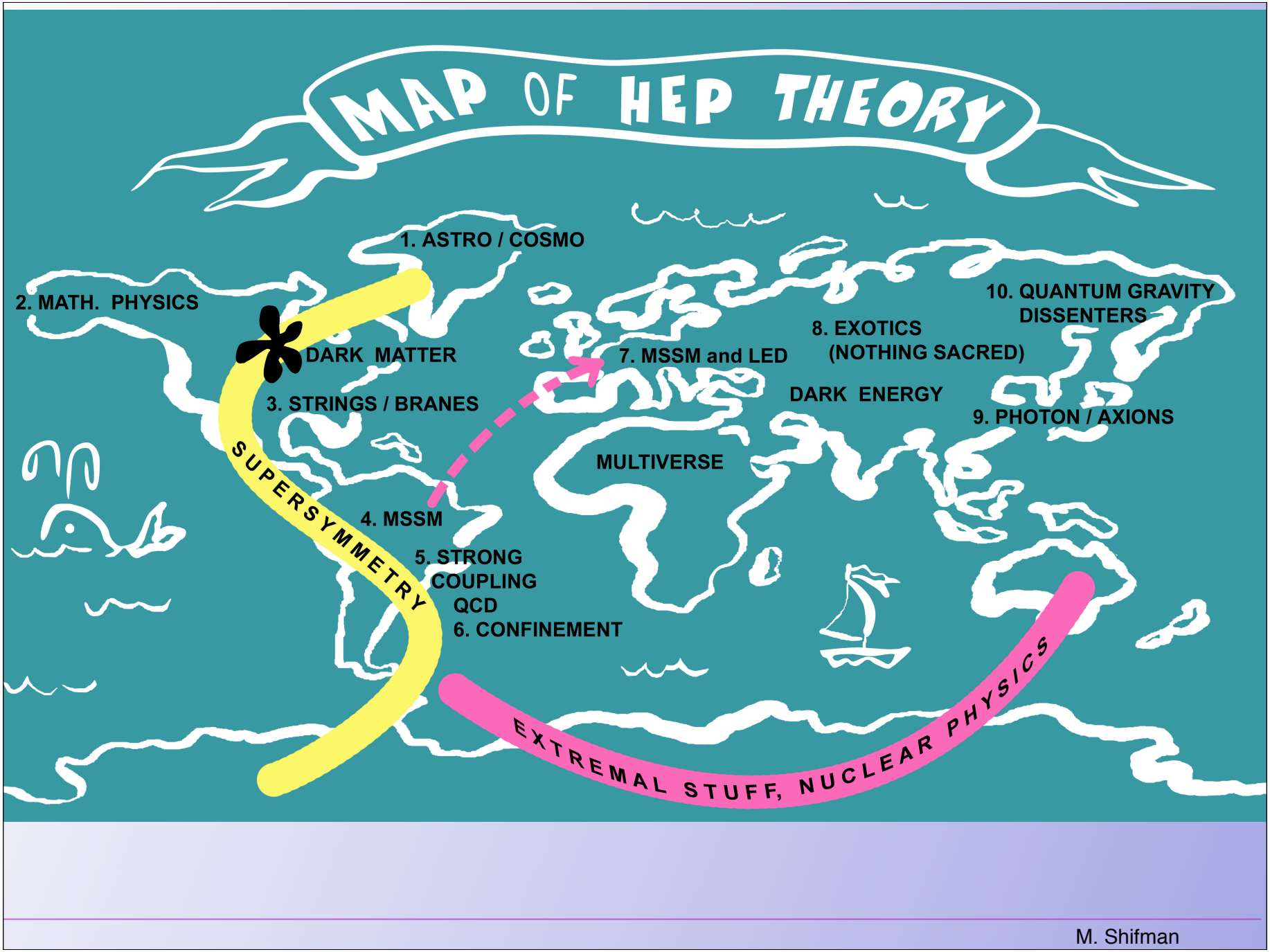



u
Z_N unbroken
Coulomb/Confining Phase
Strongly Coupled Phase
Higgs Phase
m/Λ
Witten's point

All phase transitions are of the second kind!

Conclusions

In this paper we presented the large-N solution of the two-dimensional heterotic $\mathcal{N} = (0, 2)$ CP$(N-1)$ model. Our studies were motivated by the fact that this model emerges on the world sheet of non-Abelian strings supported in a class of four-dimensional $\mathcal{N} = 1$ Yang–Mills theories. The non-trivial dynamics which we observed – with three distinct phases, confinement and no confinement, and two phase transitions – must somehow reflect dynamics of appropriate four-dimensional theories. If so, we open a window to a multitude of unexplored dynamical scenarios in $\mathcal{N} = 1$ theories. But this is a topic for a separate investigation.


MAP OF HEP THEORY
1. ASTRO / COSMO
2. MATH. PHYSICS
DARK MATTER
3. STRINGS / BRANES
SUPERSYMMETRY
4. MSSM
5. STRONG
COUPLING
QCD
6. CONFINEMENT
7. MSSM and LED
MULTIVERSE
8. EXOTICS
(NOTHING SACRED)
DARK ENERGY
9. PHOTON / AXIONS
10. QUANTUM GRAVITY
DISSENTERS
EXTREMAL STUFF, NUCLEAR PHYSICS

M. Shifman