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International Centre for Theoretical Physics**



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**Gribov-80 Memorial Workshop on Quantum Chromodynamics and
Beyond'**

26 - 28 May 2010

Non-perturbative particle production in strong non-Abelian fields

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Non-perturbative particle production in time-dependent strong non-Abelian fields

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Phys. Rev. D71 (2005) 094010.

Phys. Rev. D78 (2008) 054004.

J. Phys. G36 (2009) 064068.

arXiv: 0909.2323 [hep-th]

Gribov – 80 Workshop, ICTP

27 May 2010, Trieste, Italy

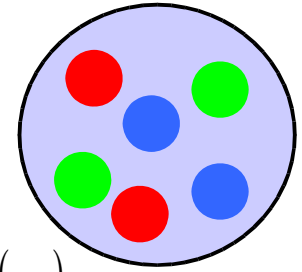
Particle production mechanisms in high energy HI collisions:

I. Dilute parton gas limit as initial condition + parton cascade:

PDF(p,n) + pQCD + Glauber + [Shad; Multisc; Quench; Fluct; ...]

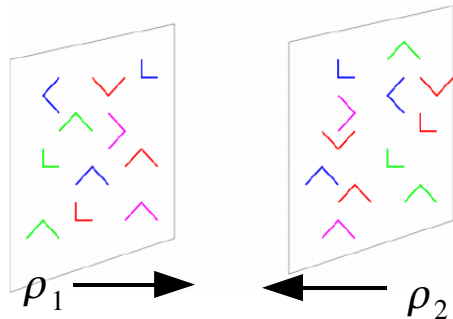
$$E_\pi \frac{d\sigma^{pp}}{d^3 p_\pi} = \int dx_1 \int dx_2 \int dz_c f_{a/p}(x_a, Q^2) f_{b/p}(x_b, Q^2) \frac{d\sigma}{dt} \frac{D_c^\pi(z_c)}{\pi z_c^2}$$

$$E_\pi \frac{d\sigma^{AB}}{d^3 p_\pi} = \int d^2 b d^2 r t_A(\vec{r}) t_B(|\vec{b}-\vec{r}|) E_\pi \frac{d\sigma^{pp}}{d^3 p_\pi} \otimes S(\dots) \otimes M(\dots) \otimes Q(\dots) \otimes F(\dots)$$



Dilute gas

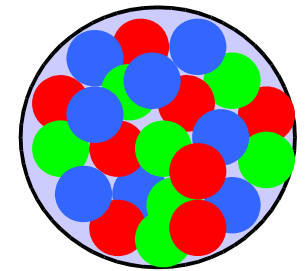
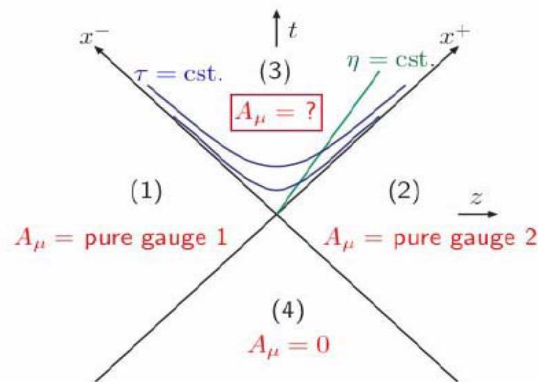
II. Dense gluon matter limit as initial condition + hydro:



CGC initial condition:

$$J^\mu = \delta^{\mu+} \delta(x^-) \rho_1(\mathbf{x}_T) + \delta^{\mu-} \delta(x^+) \rho_2(\mathbf{x}_T)$$

where $-D_i \alpha_{(m)}^i = \rho_{(m)}(\mathbf{x}_\perp)$. **and** α_1, α_2 gluon fields of nuclei

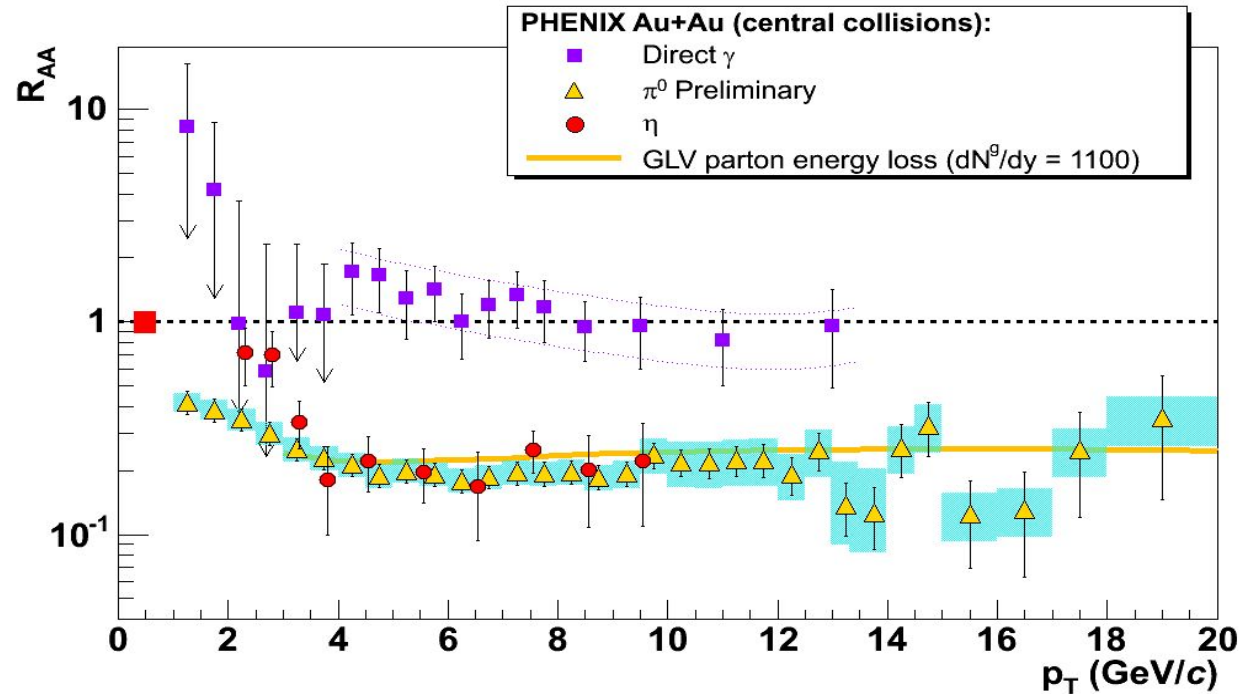


CGC: high density gluons

Successful applications of I and II:

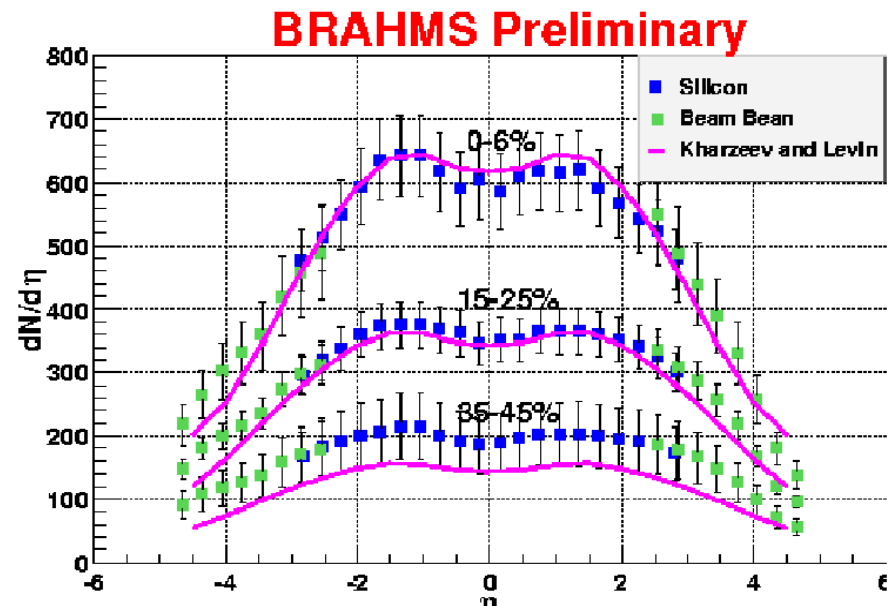
I. pQCD model:

- *hard probes*
- *high- p_T physics*
- *jets*
- *h - h correlations*
- ...



II. CGC model:

- *soft physics*
- *multiplicities*
- *centrality dependence*
- *E_T production*
- *rapidity distributions*
- ...



Problems:

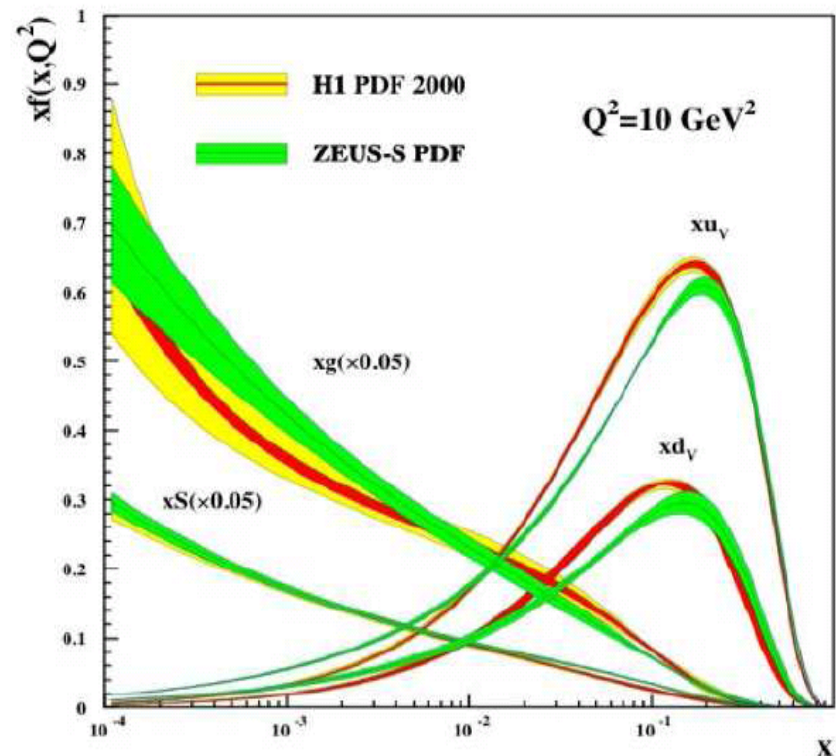
I. pQCD model (Feynman graphs):

- LO, NLO, ... ?*
- factorization (k_T)*
- resummations*
- soft physics*
- heavy quark quenching*
- ...*

II. CGC model (asymptotic):

- hard probes*
- jet physics*
- correlations*
- ...*

Connection between I and II:



Large-x: valence partons

random color charge, $\rho^a(x)$

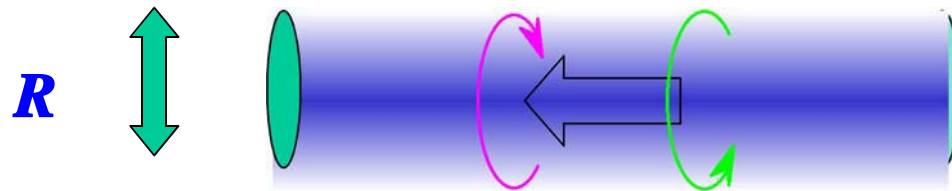
Small-x: radiation field,

created by $\rho^a(x)$

A further model for particle production:

III. Non-perturbative, non-asymptotic color transport: “confined flux tube formation and breaking”

- phenomenological approximations are known (string, rope)**
- phenomenology is applied successfully in string-based codes**
- FRITIOF, PYTHIA, HIJING are using strings**
- URQMD, HIJING-BB is using ropes (melted strings)**
- good agreement with data at different energies**
- ...**



- formal QCD-based equations are known (Heinz, Mrowczynski)**
- YM-field evolution in 3+1 dim, collision (Poschl, Müller)**
- lattice-QCD calculations have been started (Krasnitz, Lappi)**
- ...**

A further model for particle production:

**III. Non-perturbative, non-asymptotic color transport:
“pair-creation in strong fields”**

**--- strong (Abelian) static E field: Schwinger mechanism
probability of pair-creation:**

$$P(p_T) d^2 p_T = -\frac{eE}{4\pi^3} \ln(1 - \exp[-\pi \frac{m^2 + p_T^2}{eE}]) d^2 p_T$$

integrated probability at mass m:

$$P_m = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp[-\pi \frac{nm^2}{eE}]$$

ratio of production rates (e.g. strange to light)

$$\gamma_s = \frac{P(s\bar{s})}{P(q\bar{q})} = \exp[-\pi \frac{m_s^2 - m_q^2}{eE}] \quad eE = 0.9 \text{ GeV/fm}$$

--- strong time dependent SU(N) color fields:

Kinetic Equation for the color Wigner function

A.V. Prozokevich, S.A. Smolyansky, S.V. Ilyin, hep-ph/0301169.

Kinetic equation for fermion pair production:

Wigner function: $W(k_1, k_2, k_3)$

Color decomposition: $W = W^s + W^a t^a$, where $a = 1, 2, \dots, N_c^2 - 1$

Spinor decomposition: $W^{s;a} = a^{s;a} + b_\mu^{s;a} \gamma^\mu + c_{\mu\nu}^{s;a} \sigma^{\mu\nu} + d_\mu^{s;a} \gamma^\mu \gamma^5 + i e^{s;a} \gamma^5$

Color vector field (longit.): $A_\mu^a = (0, -\vec{A}) = (0, 0, 0, A_3^a)$

Kinetic equation for Wigner function:

$$\begin{aligned} \partial_t W + \frac{g}{8} \frac{\partial}{\partial k_i} \left(4 \{ W, F_{0,i} \} + 2 \{ F_{i\nu}, [W, \gamma^0 \gamma^\nu] \} - [F_{i\nu}, \{ W, \gamma^0 \gamma^\nu \}] \right) = \\ = i k_i \{ \gamma^0 \gamma^i, W \} - i m [\gamma^0, W] + i g [A_i, [\gamma^0 \gamma^i, W]]. \end{aligned}$$

for details see V.V. Skokov, PL: PRD71 (2005) 094010 for U(1)

PRD78 (2008) 054004 for SU(2)

in preparation for SU(3)

Distribution function for fermions with mass m :

$$f_f(\vec{k}, t) = \frac{m a^s(\vec{k}, t) + \vec{k} \vec{b}^s(\vec{k}, t)}{\omega(\vec{k})} + \frac{1}{2}$$

Time dependent external field, $E(t)$ and neglected mass, $m=0$:

A, Pulse field (dotted):

$$E_{pulse}(t) = E_0 \left[1 - \tanh^2(t/\delta) \right]$$

B, Constant field (dashed):

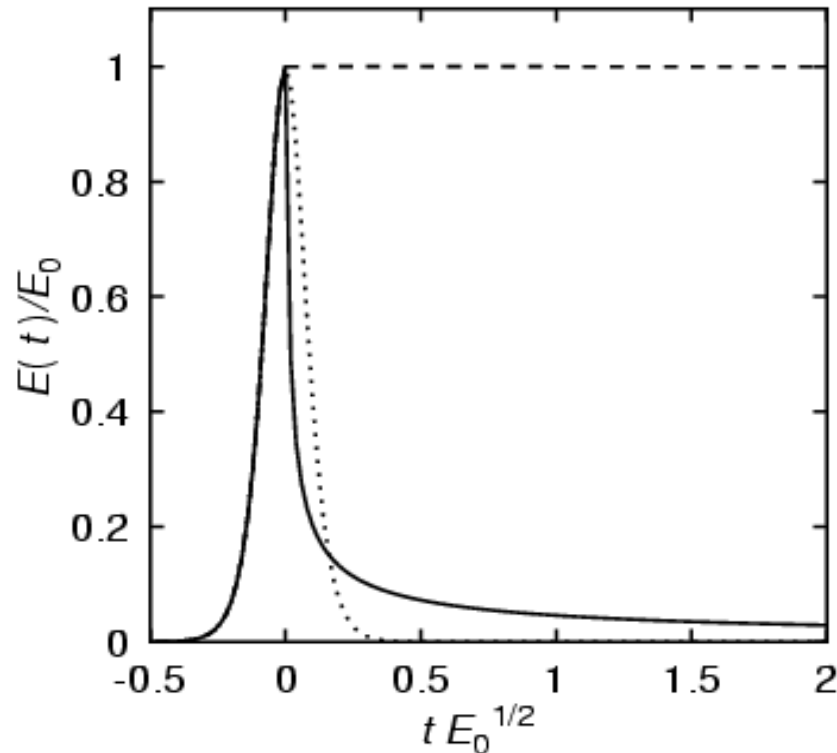
$$E_{const}(t) = E_{pulse}(t) \quad \text{at } t < 0$$

$$E_{const}(t) = E_0 \quad \text{at } t > 0$$

C, Scaled field (solid):

$$E_{scaled}(t) = E_{pulse}(t) \quad \text{at } t < 0$$

$$E_{scaled}(t) = \frac{E_0}{(1+t/t_0)^\kappa} \quad \text{at } t > 0$$



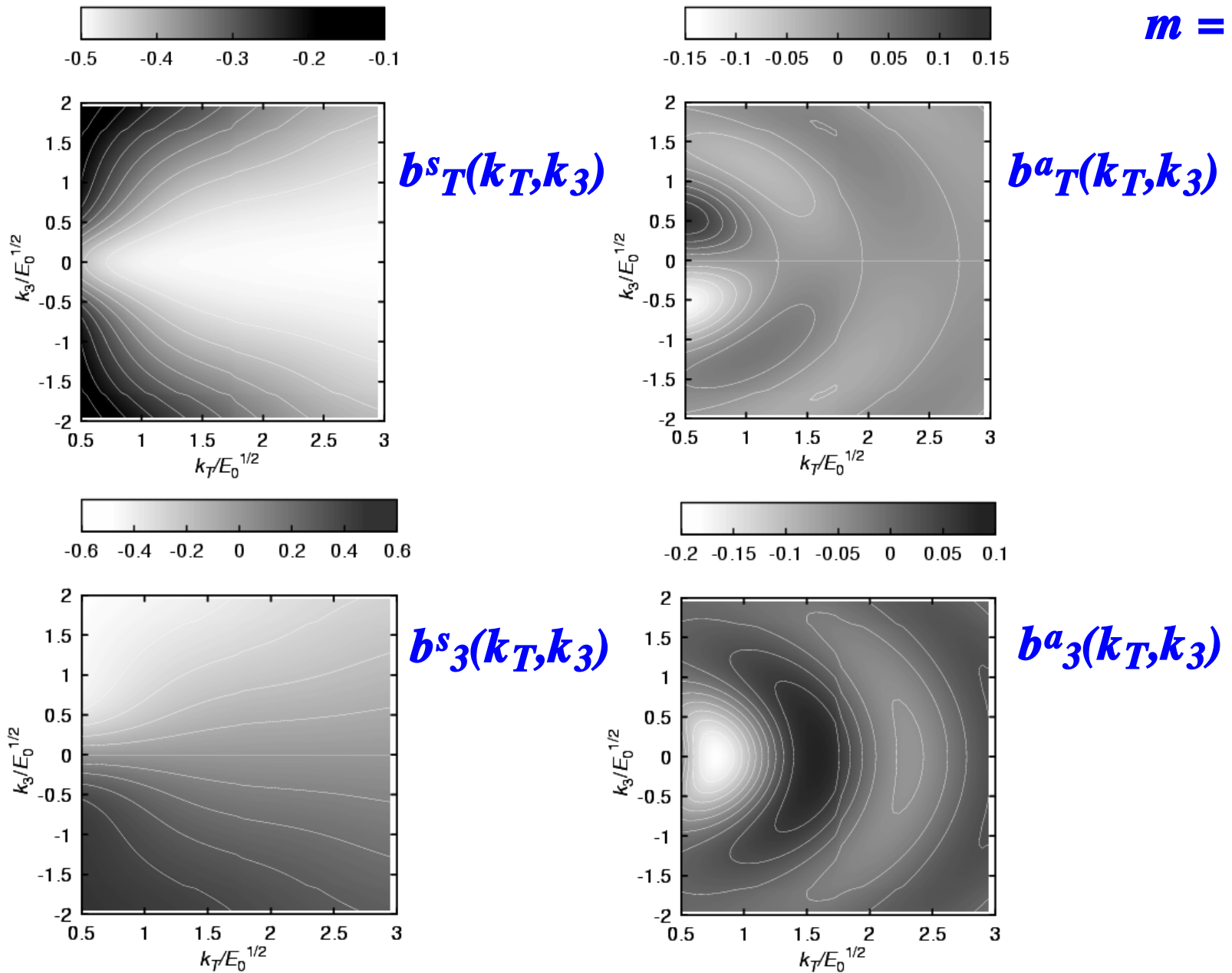
$$\delta = 0.1 / E_0^{1/2} \quad \text{at RHIC energy}$$

$$\kappa = 2/3 \quad \text{for scaled Bjorken expans.}$$

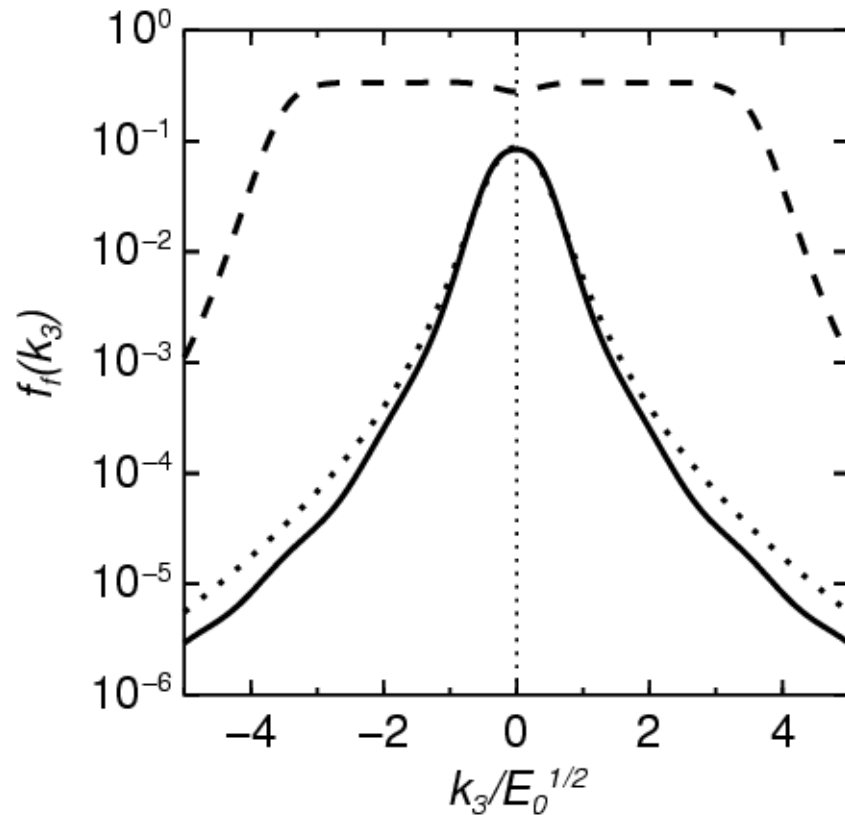
$$\text{with } t_0 = 0.01 / E_0^{1/2}$$

Numerical results (b^i) for the Bjorken expansion at $t=2/\sqrt{E_0}$ in $SU(2)$:

$m = 0$

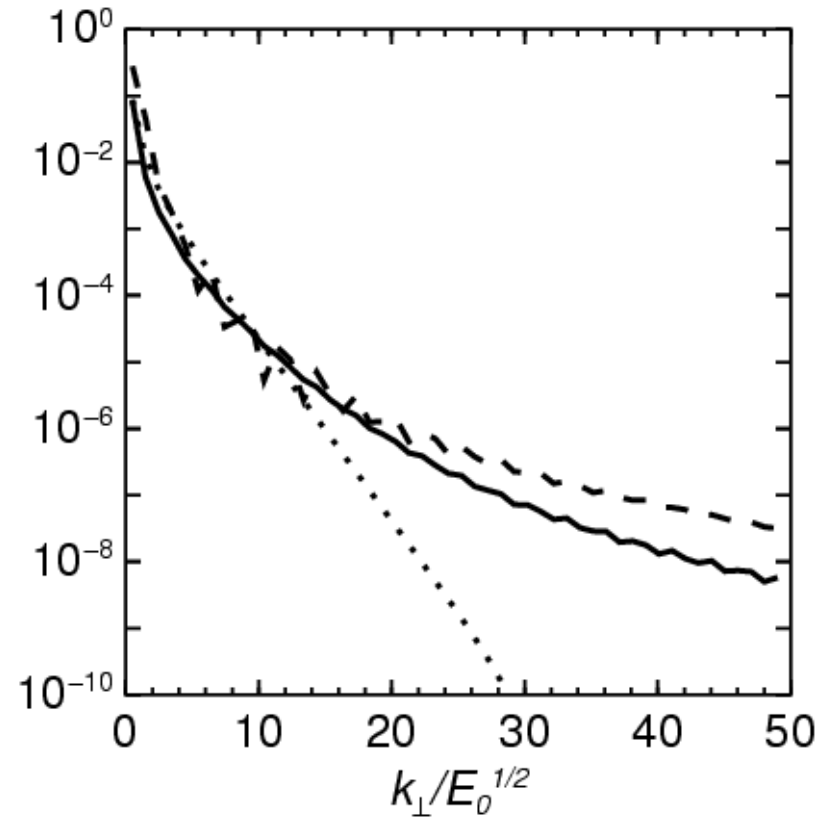


Numerical results for fermion distributions at $t=2/\sqrt{E_0}$ in SU(2):



$f_f(k_3)$: longitudinal mom. distr.

$k_T/\sqrt{E_0} = 0.5$



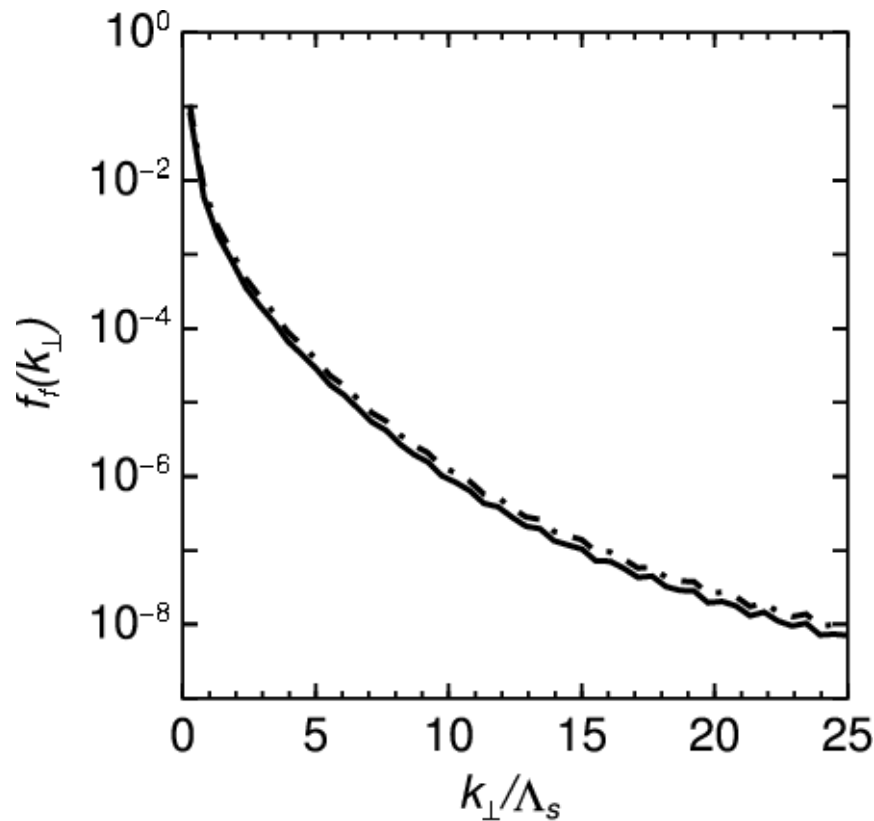
$f_f(k_T)$: transv. mom. distr.

$k_3 = 0$

\Rightarrow exponential (pulse)

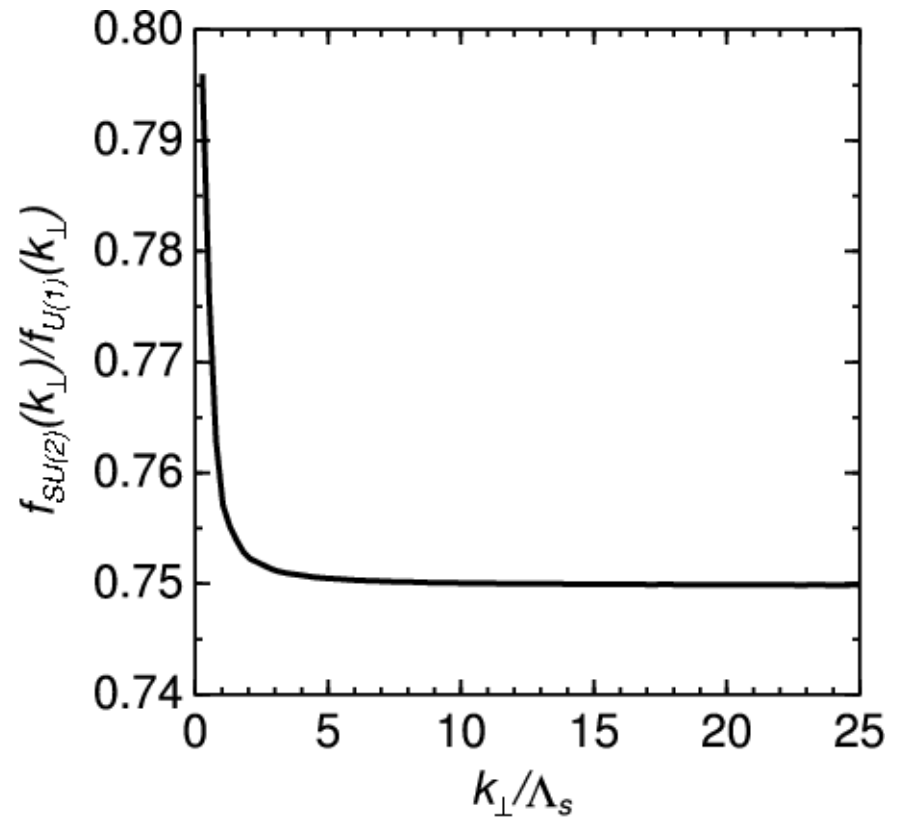
\Rightarrow polinomial (scaled)

Transverse momentum distr: scaling between U(1) and SU(2) at high-pT



f_T(k_T): transv. mom. distr.

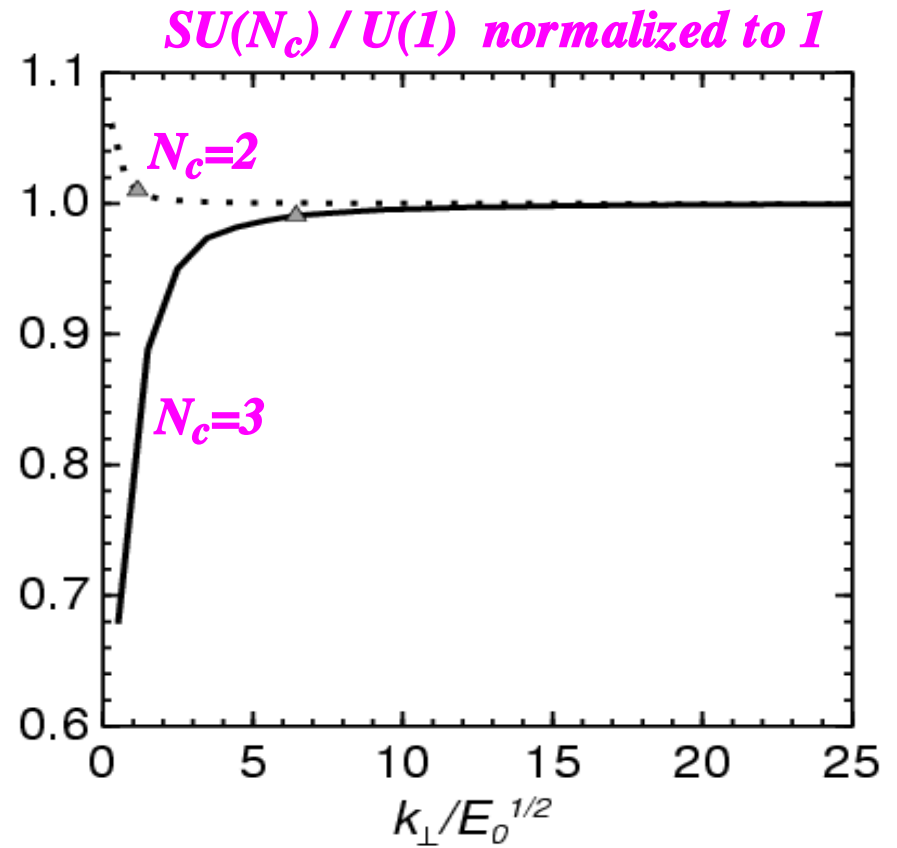
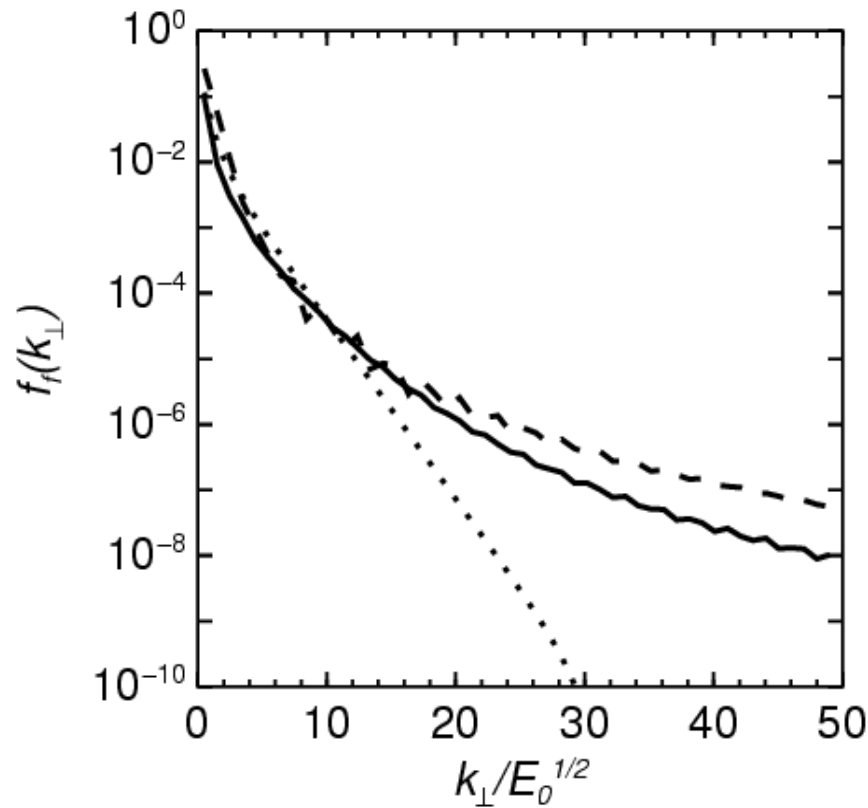
***at $k_T/\sqrt{E_0} = 0.5$
in U(1) and SU(2)
[Bjorken scaled]***



ratio: SU(2) / U(1)

***⇒ 3/4 at $k_T/\Lambda_s > 3$
(scaling in the Kinetic Eq.)***

Transverse momentum distr: scaling in SU(3) at high-pT (m=0)



$f_f(k_T)$: transv. mom. distr.

in SU(3)

3 cases of $E(t)$

[similar to SU(2)]

Ratios (scaled time evol.):

$SU(2) / U(1) \Rightarrow 3/4$

$SU(3) / U(1) \Rightarrow 4/3$

(scaling in the Kinetic Eq.)

Quark-pair production in strong $SU(2)$ field
--- quark mass dependence ---

Mass dependent fermion production in SU(2):

Quark-pair production depends on the mass:

$$\begin{aligned}m(\text{light}) &= 8 \text{ MeV} \\m(\text{strange}) &= 150 \text{ MeV} \\m(\text{charm}) &= 1200 \text{ MeV} \\m(\text{bottom}) &= 4200 \text{ MeV}\end{aligned}$$

Usually 'm' mass behaves as a scale (see electron mass in QED).

But, what about zero mass limit?

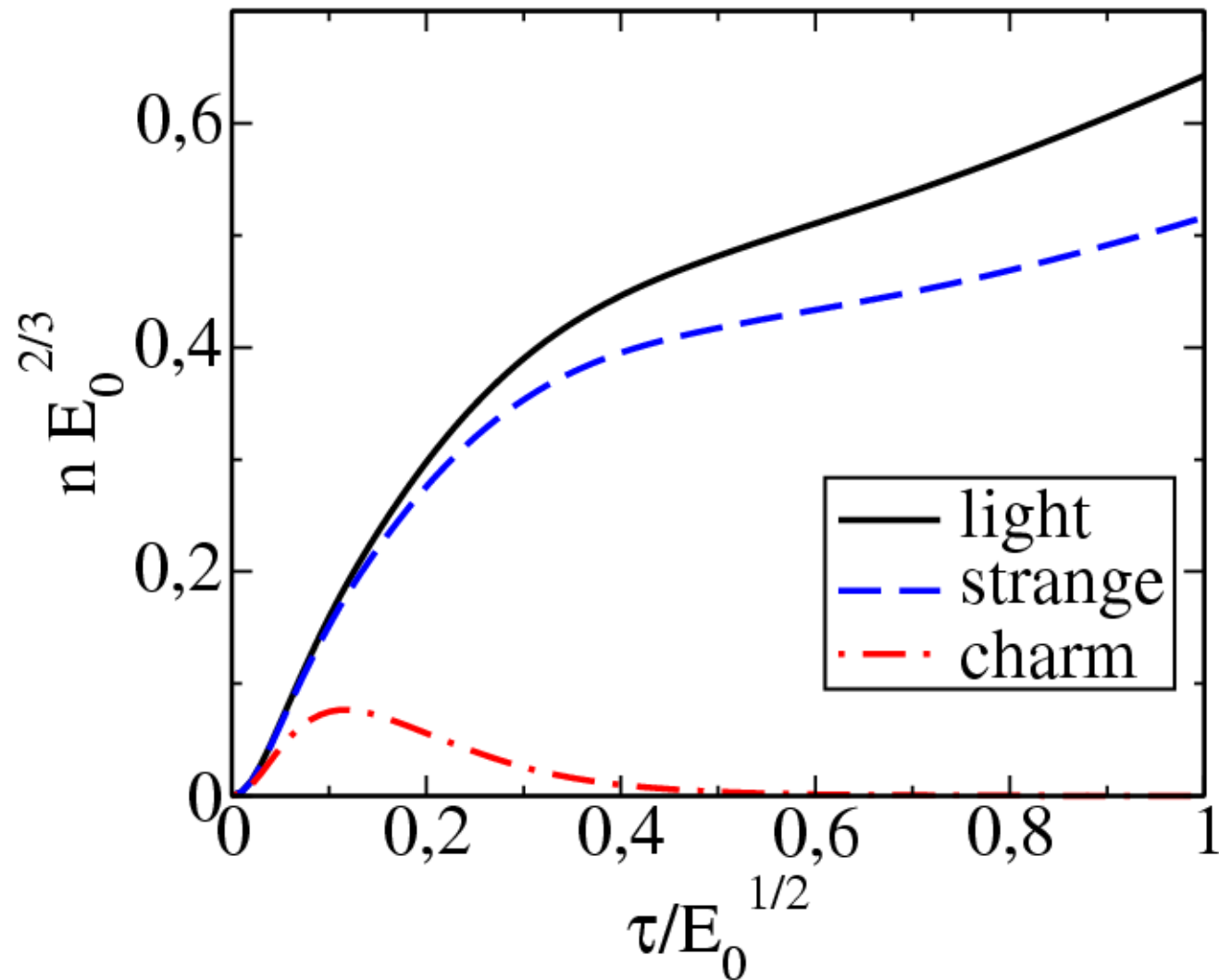
What is the scale in that case?

*Since we have non-zero fermion production,
then some scale must exist.*

The characteristic time of the changes in $E(t)$??

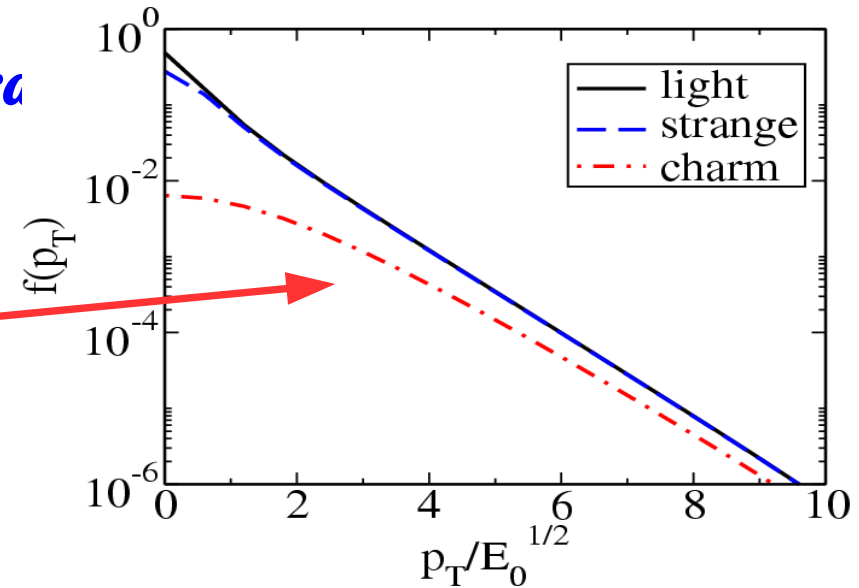
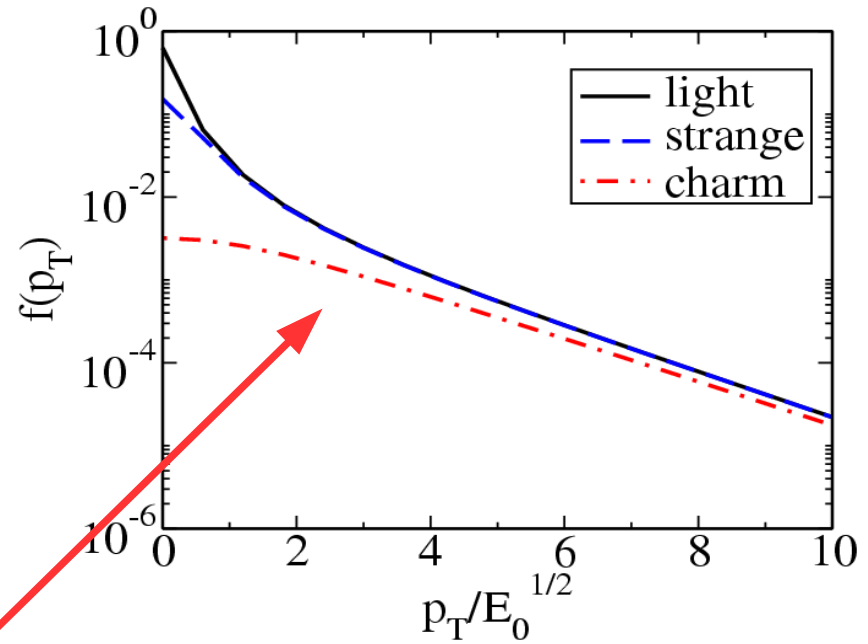
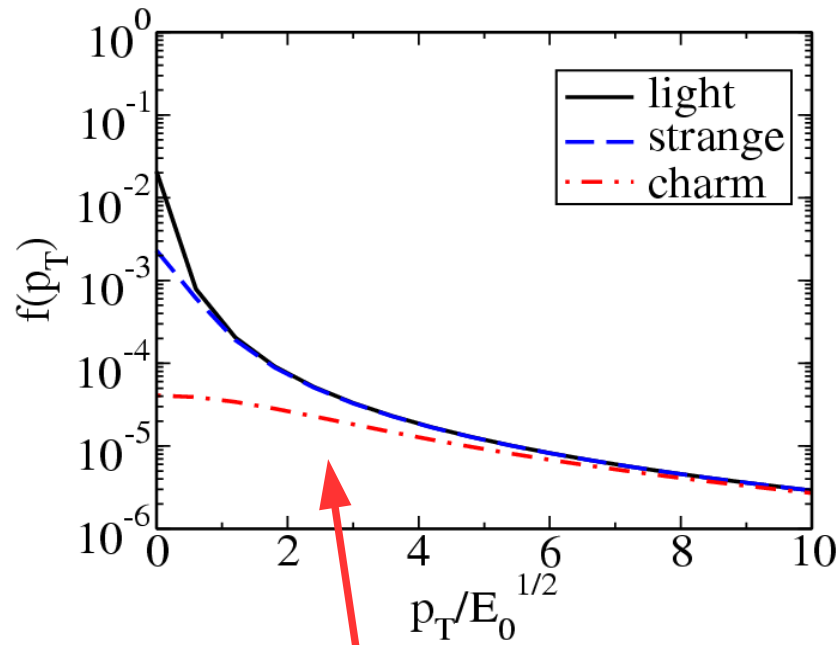
$$\tau \Rightarrow \delta$$

Mass dependent fermion production in SU(2) [pulse-like time dep.]



Fermion number (n) depends on the characteristic time of the pulse width: $\tau = \delta$ in the pulse scenario

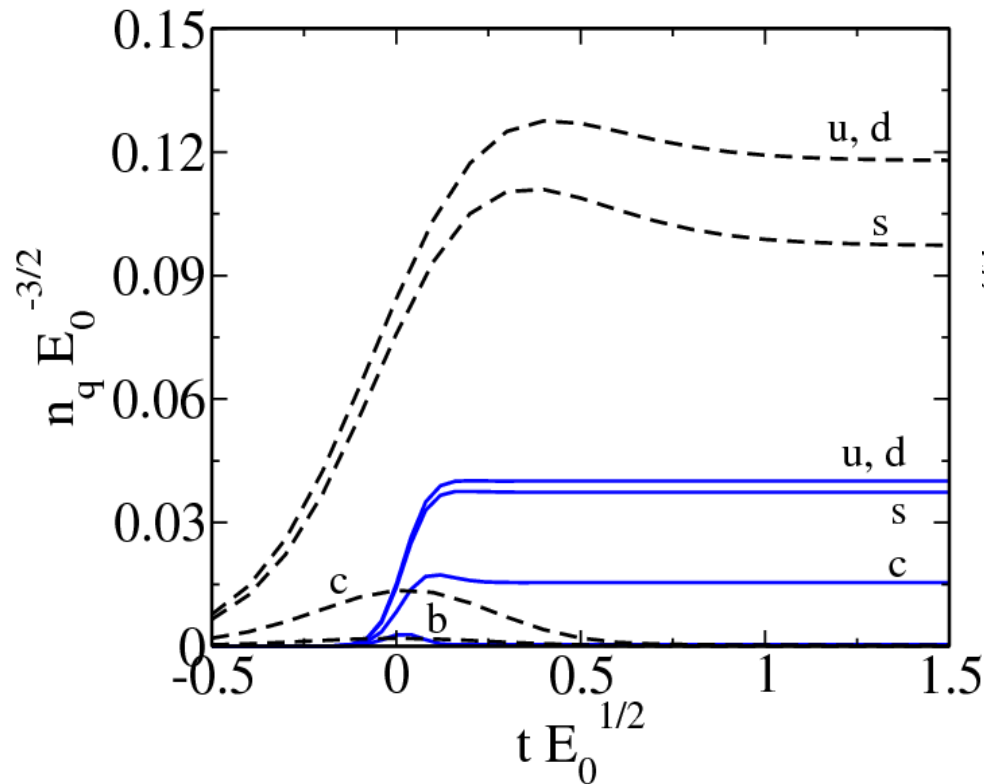
Mass dependent fermion production in SU(2) [pulse-like time dep.]



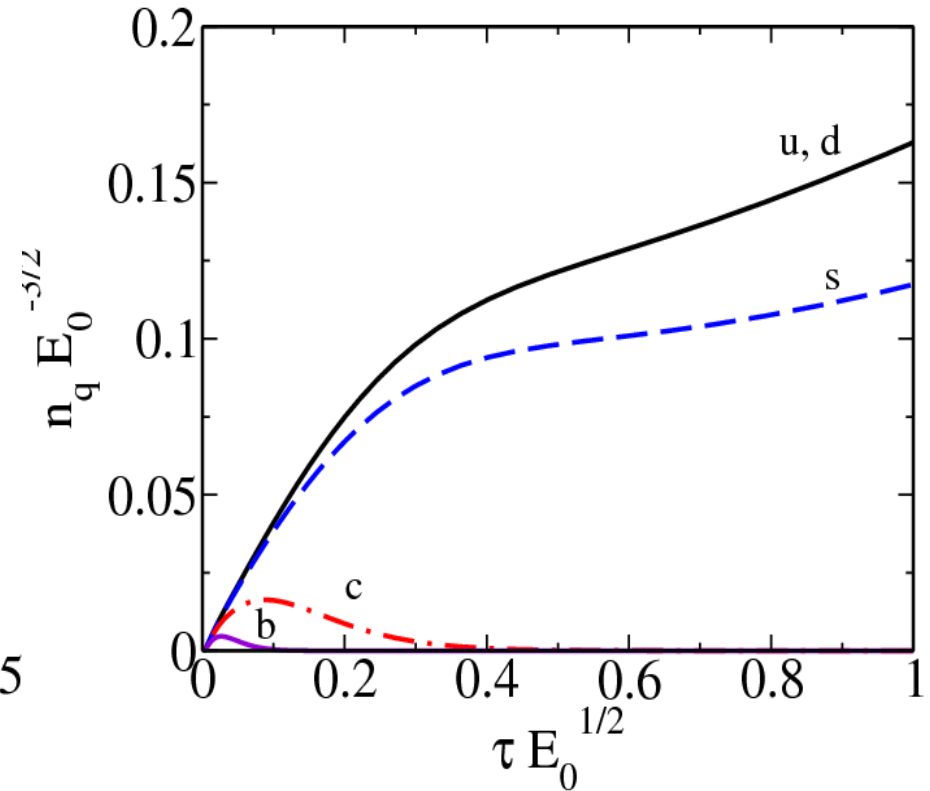
**Transverse momentum spectra
at different pulse width:**

$\tau\sqrt{E_0} = 0.01; 0.1; 0.2$

Mass dependent fermion production in SU(2) [pulse-like time dep.]



t: time in the CM frame



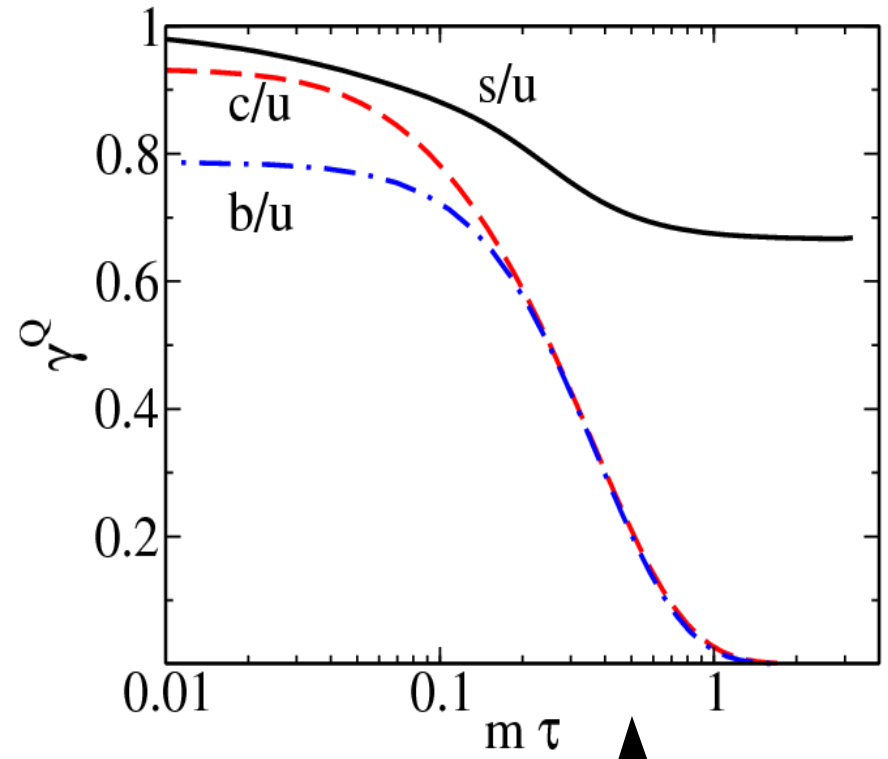
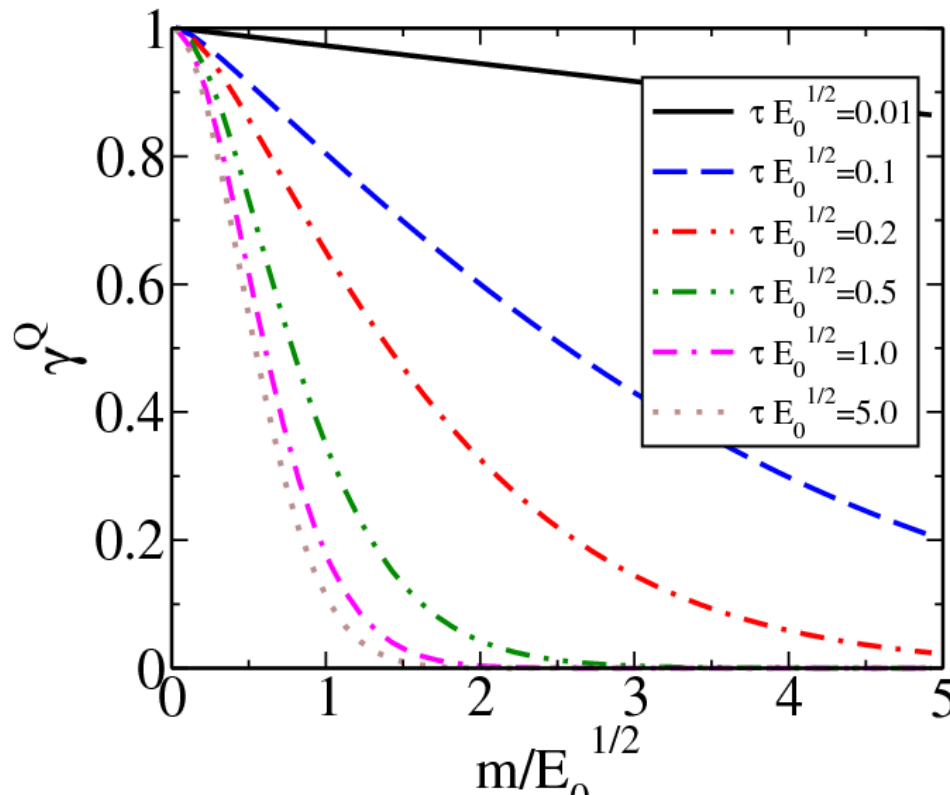
τ : pulse width ($t \rightarrow \infty$)

Full line: $\tau \sqrt{E_0} = 0.1$ ($\tau = 0.05 \text{ fm}$)

Dashed line: $\tau \sqrt{E_0} = 0.5$ ($\tau = 0.25 \text{ fm}$)

$E_0 = 0.68 \text{ GeV/fm}$, $g=2$ $\rightarrow \rightarrow \rightarrow$ $g \cdot E_0 \propto \kappa = 1.17 \text{ GeV/fm}$

Mass dependent fermion production in SU(2) [pulse-like time dep.]



$$\gamma^Q = \lim (t \rightarrow \infty) n_Q(t) / n_u(t)$$

flavour suppression factor

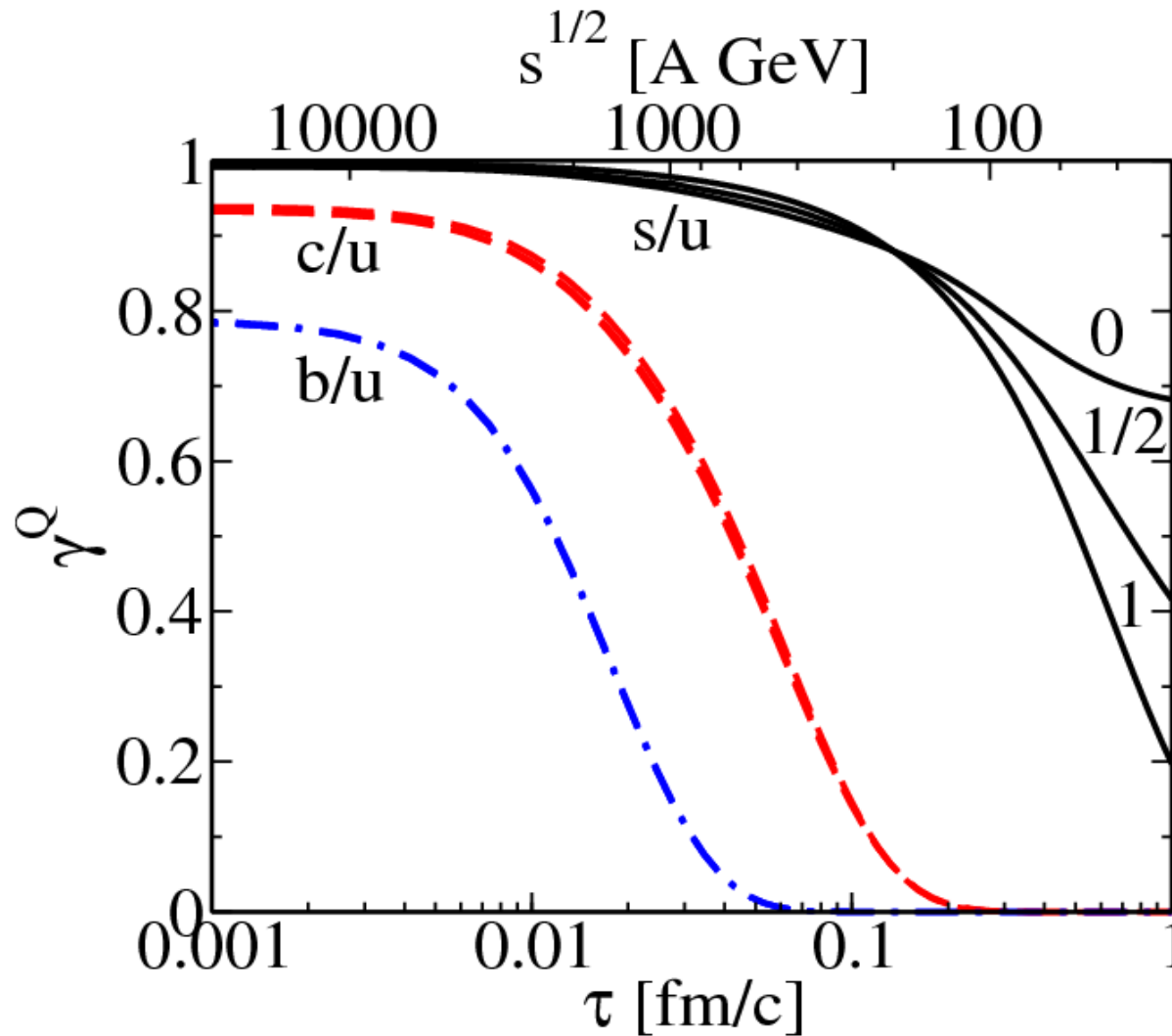
Blue line: $\tau\sqrt{E_0} = 0.1$ ($\tau = 0.05$ fm)

At large τ heavy quarks are suppressed.

Enhanced heavy fermion production at small τ

$$\tau_{\text{eff}} = \delta + m^{-1} \quad [m_{\text{eff}} \Rightarrow \delta^{-1}]$$

Mass dependent fermion production in SU(2) [pulse-like time dep.]



Collisional energy dependence of the quark flavour suppression
+ $E_0(t) = E_0 (\tau_0 / \tau)^\beta$ where $\beta : 0, 1/2, 1$

Mass dependent fermion production in SU(2)

Numerical values for suppression factors :

| | Schwinger | 130 AGeV | 200 AGeV | 1 ATeV | 2 ATeV | 5.5 ATeV |
|-----------------|--------------------------|--------------------------|------------------------|---------------|---------------|-----------------|
| <i>s</i> | 0.74 | 0.84 | 0.88 | 0.96 | 0.98 | 0.99 |
| <i>c</i> | 3 10⁻⁹ | 9 10⁻³ | 0.06 | 0.66 | 0.82 | 0.91 |
| <i>b</i> | ≈ 0 | ≈ 0 | 10⁻⁶ | 0.15 | 0.45 | 0.72 |

Effective string constants and massive fermion suppression in SU(2)

Schwinger formula for static field and static string:

$$\frac{dN}{dt d^3 x} = \frac{\kappa^2}{4\pi^3} \exp(-\pi m^2 / \kappa)$$

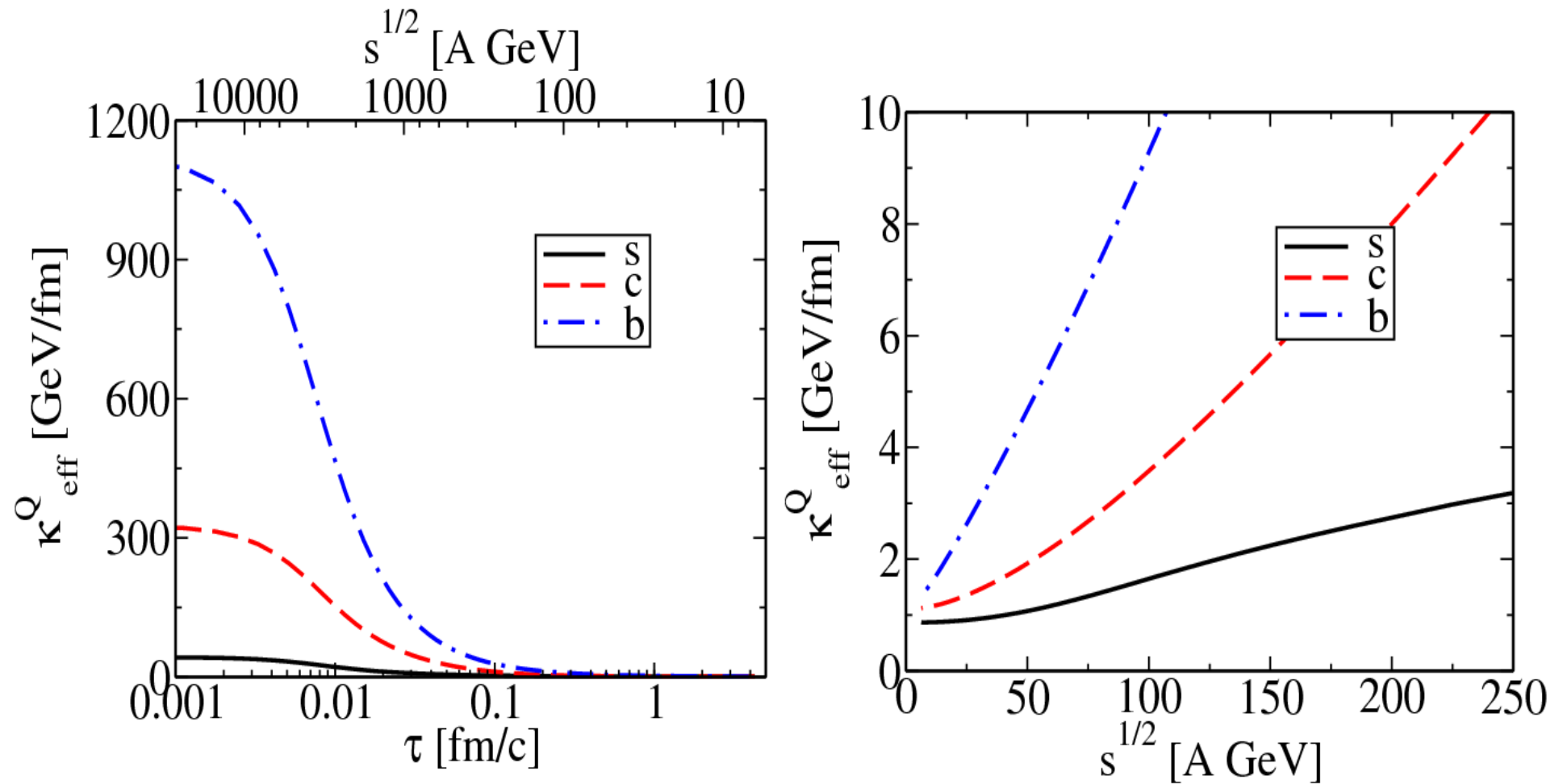
Suppression factor:

$$\gamma^Q = \exp(-\pi (m_Q^2 - m_q^2) / \kappa)$$

**Results of our dynamical calculation can be fit by
an effective string tension, κ_{eff} :**

$$\gamma_{\infty}^Q(\kappa_{eff}^Q) = \gamma^{(Q)}(\tau)$$

Effective string constants and massive fermion suppression in SU(2)



***Pulse width and collisional energy dependence
of the flavour dependent effective string constant
---- too much difference (and what about for light quarks)***

Effective string constants and massive fermion suppression in SU(2)

Solution:

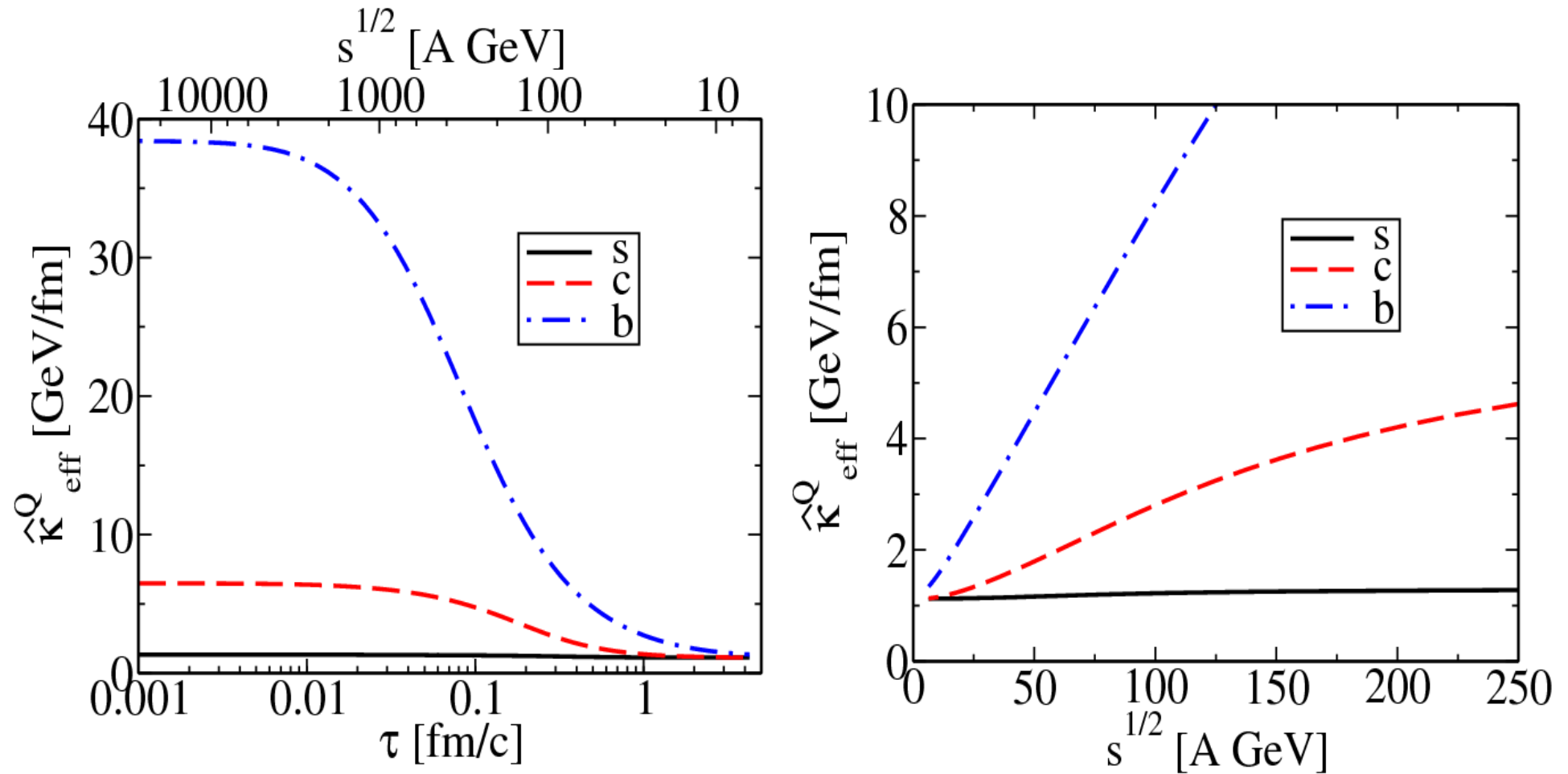
Let us keep a fixed string constant for the light quarks

$$\kappa_{eff}^u = 1.17 \text{ GeV} / \text{fm}$$

and fix flavour specific effective string constant for the heavier quarks (strange, charm, bottom):

$$\gamma_{\infty}^Q = \left(\frac{\kappa_{eff}^Q}{\kappa_{eff}^u} \right)^2 \exp \left(-\pi \frac{m_Q^2}{\kappa_{eff}^Q} + \pi \frac{m_u^2}{\kappa_{eff}^u} \right) = \gamma^Q(\tau)$$

Effective string constants and massive fermion suppression in SU(2)



***Pulse width and collisional energy dependence
of the flavour specific effective string constants***

--> strange string constant is nice, for heavy Q we get large values

Effective string constants and massive fermion suppression in SU(2)

**Numerical values for flavour specific effective string constants
in GeV/fm:**

| | 130 AGeV | 200 AGeV | 1 ATeV | 2 ATeV | 5.5 ATeV |
|-------------------|-----------------|-----------------|---------------|---------------|-----------------|
| <i>u,d</i> | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 |
| <i>s</i> | 1.24 | 1.26 | 1.32 | 1.33 | 1.34 |
| <i>c</i> | 3.32 | 4.2 | 6.1 | 6.3 | 6.5 |
| <i>b</i> | 10.3 | 14.7 | 32 | 36 | 38 |

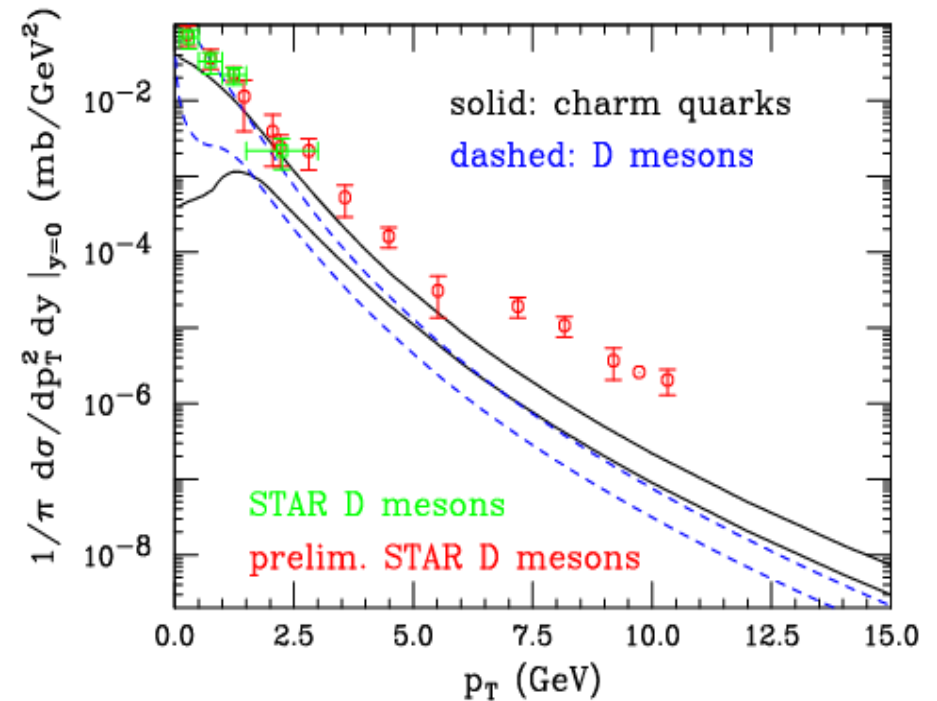
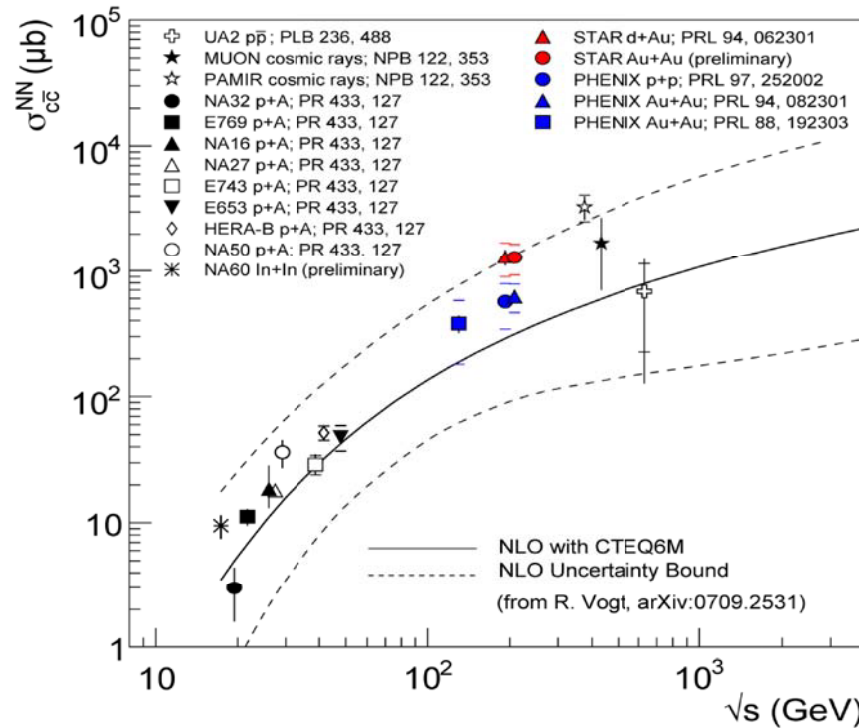
Saturation at higher LHC energies !!!!

Discussion: How large is the primary charm production ?

Do we have room for non-perturbative charm yield ?

**Charm pair production can be (must be ?) calculated in pQCD:
LO, NLO, NLL, FONLL, ...**

Results at RHIC energies



R. Vogt, EPJ ST 155 (2008) 213.

M. Cacciari, ..., PRL95,122001

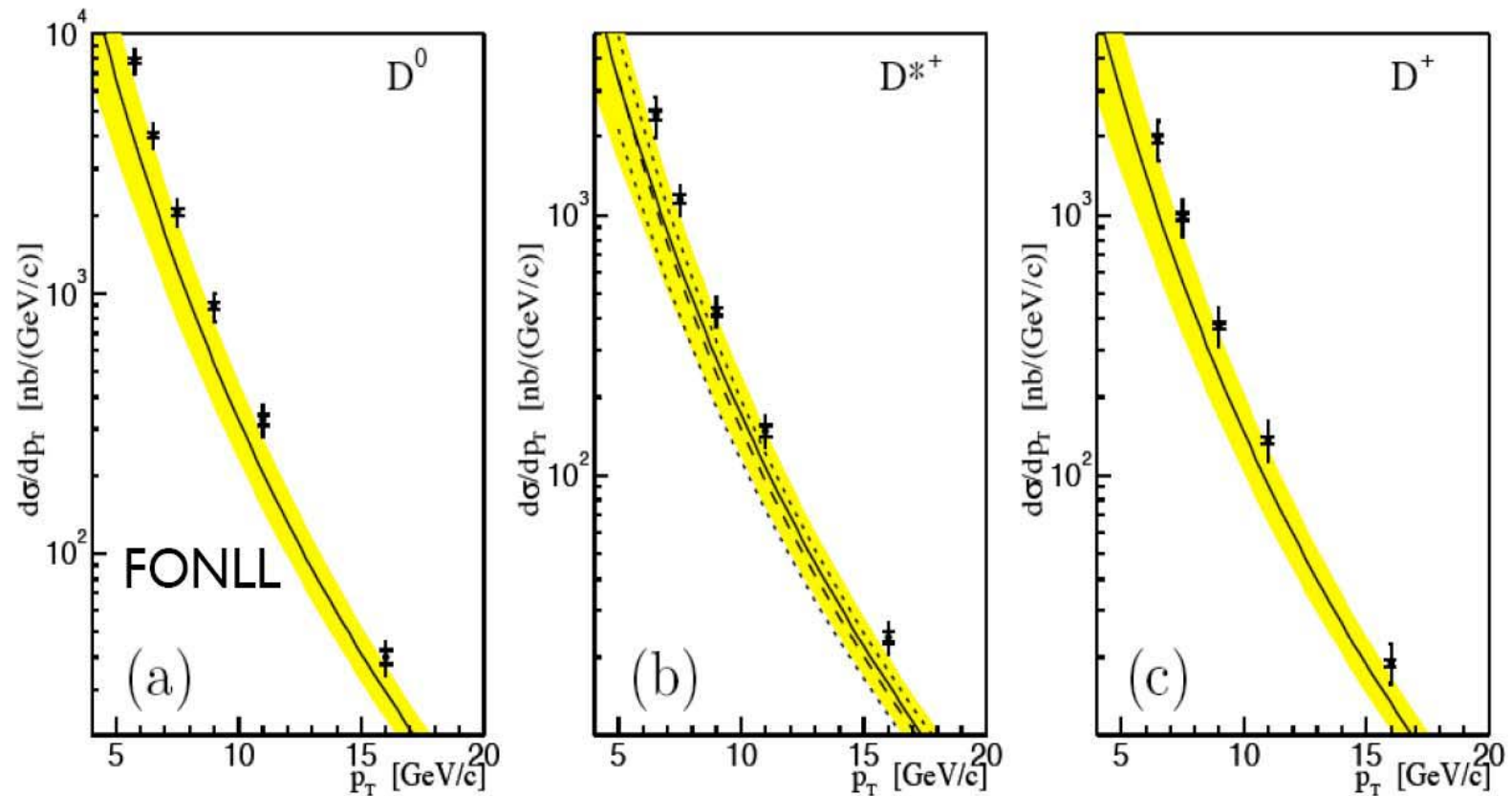
Data are at the upper limit of theory (or beyond) !?? ($m_c = 1.2 \text{ GeV}$)

Discussion: How large is the primary charm production ?

Do we have room for non-perturbative charm yield ?

Charm production at FERMILAB energies ($p\bar{p}$, $\sqrt{s} = 1.96$ TeV)

CDF Run II $c \rightarrow D$ data [PRL 91:241804,2003]

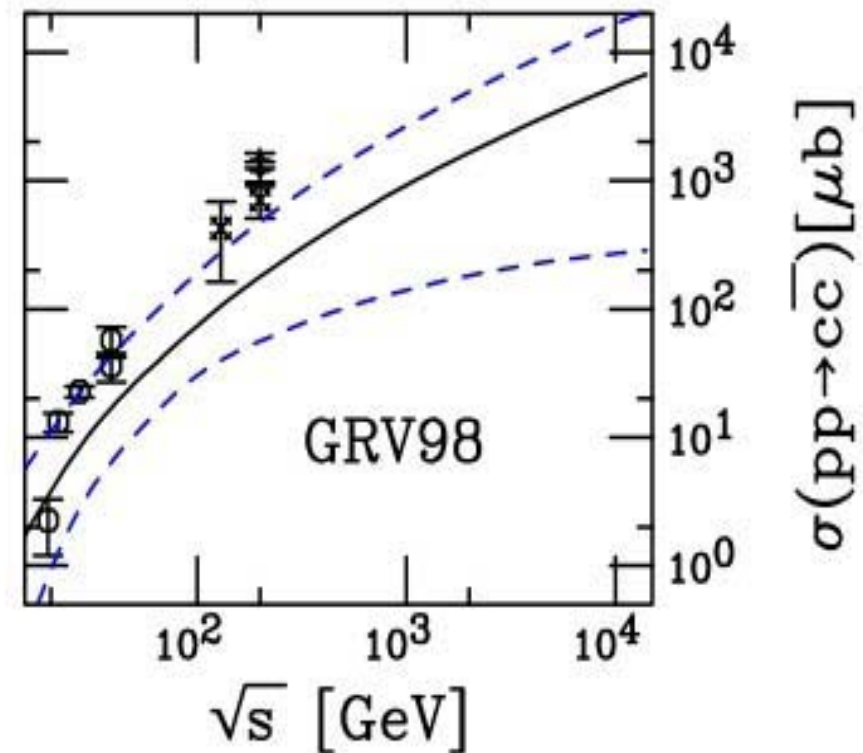
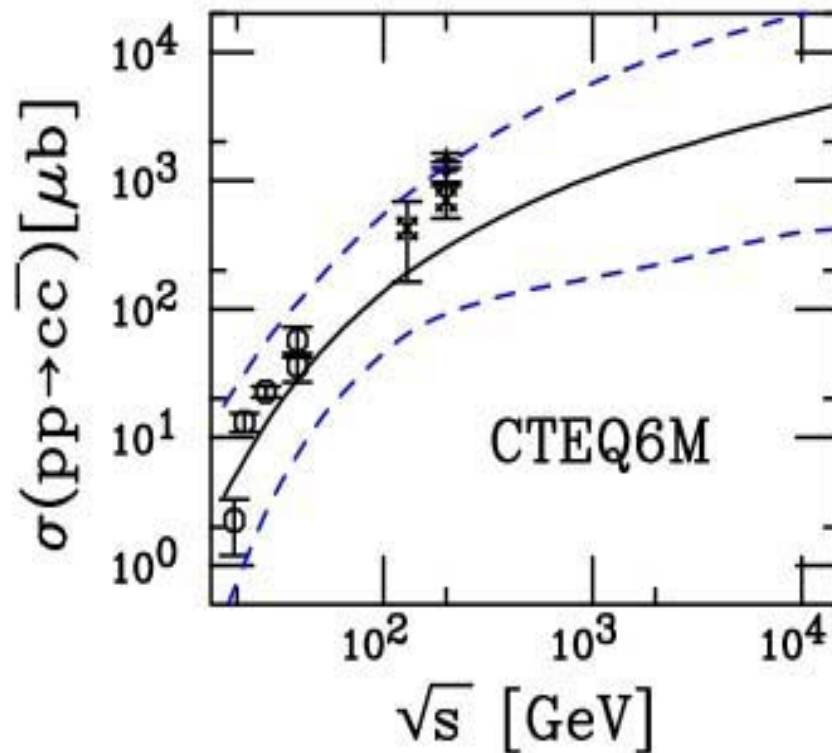


Data are at the upper limit of theory (or beyond) !?? (factor of 2 ?)

Discussion: How large is the primary charm production ?

Do we have room for non-perturbative charm yield ?

Charm production at LHC energies (pp , $\sqrt{s} = 2-14$ TeV)



R. Vogt, Private comm., 2009

Large uncertainties --> more data are needed to fix parameters

There is room for non-perturbative contributions (today).

Conclusions:

- 1. Particle production mechanisms are not fully explored in non-Abelian cases, especially in case of strong fields.***
- 2. If the overlap of heavy ions is very short, and the time scale of the initial phase is also short, then heavy quark production is not suppressed by the heavy mass.***
- 3. Short pulse: the time scale of the initial 'pulse' determines the heavy quark production and not the charm mass.***
- 4. Thus: heavy quark production can carry message about the time scale of the initial overlap at LHC energies.
(strange quark mass is too close to light quark mass)***
- 5. LHC data are extremely interesting, turning point is $\sim 1-2$ TeV.***