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Gribov Reggeon calculus in QCD, SUSY and gravity

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Gribov Reggeon calculus
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1 Regge ansatz for amplitudes

Regge kinematics

$$s = 4E^2 >> -t = \vec{q}^2$$

t-channel partial wave expansion

$$A^p(s,t) = s \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \left((-s)^\omega - p s^\omega\right) f^p_\omega(t)$$

Regge pole hypothesis

$$f^p_\omega(t) = \frac{\gamma^2(t)}{\omega - \omega_p(t)}$$

Linear Regge trajectories

$$\omega_p(t) = \Delta - \alpha' \vec{q}^2$$

2 Mandelstam cuts

Regge asymptotics

$$A^p_{Regge}(s,t) = \xi_p(t)\, s^{1+\omega_p(t)}\, \gamma^2(t)\,,\;\; \xi_p(t) = e^{-i\pi\omega_p(t)} - p$$

Mandelstam cut contribution

$$A^p_{Mand}(s,t) = \xi_p\, s \int \frac{d^2k}{(2\pi)^2}\Phi^2(k,q-k) s^{\omega_{p_1}(-k^2)} s^{\omega_{p_2}(-(q-k)^2)}$$

Gribov signature conservation rule

$$p = p_1\, p_2$$

Total cross sections and Pomeron

$$\sigma = \frac{1}{s}\, \Im A(s.0)\,,\;\; A(s,t) \approx is\, s^{\Delta_P - \alpha'_P q^2}\,,\;\; \Delta_P << 1$$

3 Gribov Pomeron calculus

Multi-particle unitarity conditions in t-channel

$$\Im_t f_\omega(t) \sim \sum_n \int d\Omega_n |f_\omega^{(n)}|^2$$

Separation of particle clusters in their rapidities

$$0 < y_1 < y_2 < ... < y_k < \ln s\,,\;\; 1 << y_k - y_{k-1} << \ln s$$

Non-relativistic reggeon propagators

$$G_0 = \frac{1}{E + \Delta - \frac{k^2}{2m}}\,,\;\; E = -\omega\,,\;\; \alpha' = \frac{1}{2m}$$

Gribov effective Pomeron action

$$S = \int dy\, d^2\rho \left(\phi^*(\partial_y - \Delta)\phi + \frac{1}{2m}\,|\partial_\mu \phi|^2) + \lambda \phi^* \phi^2 + ...\right)$$

4 Gluon reggeization in QCD

QCD Born amplitude

$$M_{AB}^{A'B'}(s,t)|_{Born} = g\,T^c_{A'A}\,\delta_{\lambda_{A'}\lambda_A}\,\frac{2s}{t}\,g\,T^c_{B'B}\,\delta_{\lambda_{B'}\lambda_B}$$

Leading logarithmic approximation

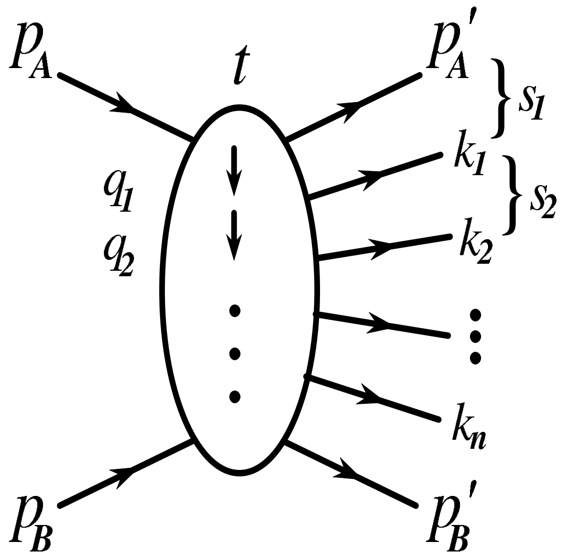
$$M_{AB}^{A'B'}(s,t) = M_{AB}^{A'B'}(s,t)|_{Born}\,s^{\omega(t)},$$

$$\alpha_s \ln s \sim 1\,,\;\; \alpha_s = \frac{g^2}{4\pi} \ll 1$$

Gluon Regge trajectory in LLA

$$\omega(-|q|^2) = -\frac{\alpha_s N_c}{4\pi^2}\int d^2k\,\frac{|q|^2}{|k|^2|q-k|^2} \approx -\frac{\alpha_s N_c}{2\pi}\,\ln\frac{|q^2|}{\lambda^2}$$

5 Amplitudes in multi-Regge kinematics



$$M_{2\to 2+n}^{BFKL} \sim \frac{s_1^{\omega_1}}{|q_1|^2}\, gT^{d_1}_{c_2 c_1} C(q_2,q_1)\,\frac{s_2^{\omega_2}}{|q_2|^2} ... gT^{d_n}_{c_{n+1}c_n}\, C(q_{n+1},q_n)\,\frac{s_{n+1}^{\omega_{n+1}}}{|q_{n+1}|^2}\,,$$

$$\omega_r = -\frac{\alpha_s N_c}{2\pi}\left(\ln\frac{|q_r^2|}{\mu^2} - \frac{1}{\epsilon}\right)\,,\ C(q_2,q_1) = \frac{q_2\, q_1^*}{q_2^* - q_1^*}\,,\ \sigma_t = \sum_n \int d\Gamma_n\, |M_{2\to 2+n}|^2$$

6 Analyticity, unitarity and bootstrap

Steinmann relations for overlapping channels

$$\Delta_{s_r}\Delta_{s_{r+1}}M_{2\to 2+n}=0$$

Dispersion representation for $M_{2\to 3}$ in the Regge ansatz

$$M_{2\to 3}=c_1(-s)^{j(t_2)}(-s_1)^{j(t_1)-j(t_2)}+c_2(-s)^{j(t_1)}(-s_2)^{j(t_2)-j(t_1)}$$

Dispersion representation for $M_{2\to 4}$ in the Regge ansatz

$$M_{2\to 4}=d_1(-s)^{j_3}(-s_{012})^{j_2-j_3}(-s_1)^{j_1-j_2}+d_2(-s)^{j_1}(-s_{123})^{j_2-j_1}(-s_3)^{j_3-j_2}$$
$$+d_3(-s)^{j_3}(-s_{012})^{j_1-j_3}(-s_2)^{j_2-j_1}+d_4(-s)^{j_1}(-s_{123})^{j_3-j_1}(-s_2)^{j_2-j_3}$$
$$+d_5(-s)^{j_2}(-s_1)^{j_1-j_2}(-s_3)^{j_3-j_2}\,,\;j_r=j(t_r)$$

Bootstrap relation in LLA (BFKL (1975-1978))

$$\pi\omega(t_1)\,M_{2\to 2+n}=\sum_r \Im_{s_{0r}}M_{2\to 2+n}=\sum_t M_{2\to 2+t}M_{2+t\to 2+n}$$

7 BFKL equation (1975)

Balitsky-Fadin-Kuraev-Lipatov equation

$$E\,\Psi(\vec{\rho}_1,\vec{\rho}_2) = H_{12}\,\Psi(\vec{\rho}_1,\vec{\rho}_2)\,,\ \sigma_t \sim s^{\Delta}\,,\ \Delta = -\frac{\alpha_s N_c}{2\pi}\,E_0$$

BFKL Hamiltonian

$$H_{12} = \frac{1}{p_1 p_2^*}(\ln|\rho_{12}|^2)p_1 p_2^* + \frac{1}{p_1^* p_2}(\ln|\rho_{12}|^2)p_1^* p_2 + \ln\,|p_1 p_2|^2 - 4\psi(1)\,,$$

$$\rho_{12} = \rho_1 - \rho_2\,,\ \rho_r = x_r + iy_r\,,\ \Delta = 4\alpha N_c\,\ln 2\,/\pi$$

Möbius invariance and eigenvalues (L. (1986))

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d}\,,\ m = \gamma + n/2\,,\ \widetilde{m} = \gamma - n/2\,,\ \gamma = 1/2 + i\nu\,,$$

$$E = \psi(m) + \psi(1-m) + \psi(\widetilde{m}) + \psi(1-\widetilde{m}) - 4\psi(1)$$

8 Effective action approach

Locality in the rapidity space

$$y = \frac{1}{2} \ln \frac{\epsilon_k + |k|}{\epsilon_k - |k|}\,, \;\; |y - y_0| < \eta\,, \;\; \eta << \ln\, s$$

Gluon and Reggeized gluon fields

$$v_\mu(x) = -iT^a v_\mu^a(x)\,, \;\; A_\pm(x) = -iT^a A_\pm^a(x)\,, \, \delta A_\pm(x) = 0$$

Effective action for reggeized gluons (L., 1995)

$$S = \int d^4x \left(L_{QCD} + Tr(V_+ \partial_\mu^2 A_- + V_- \partial_\mu^2 A_+) \right)\,,$$

$$V_+ = -\frac{1}{g} \partial_+ \, P \exp \left(-\frac{g}{2} \int_{-\infty}^{x^+} v_+(x') d(x')^+ \right) = v_+ - g v_+ \frac{1}{\partial_+} v_+ + ...$$

9 Pomeron in $N = 4$ SUSY

BFKL kernel in two loops (F., L. and C.,C. (1998))

$$\omega = 4\,\hat{a}\;\chi(n,\gamma) + 4\;\hat{a}^2\,\Delta(n,\gamma)\,,\;\;\hat{a} = g^2 N_c/(16\pi^2)\,,$$

Hermitian separability in $N = 4$ SUSY (K.,L. (2000))

$$\Delta(n,\gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2\hat{a}/\omega}\,,\;M = \gamma + \frac{|n|}{2}\,,$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2)\,,\;\beta'(z) = \frac{1}{4}\left[\Psi'\Big(\frac{z+1}{2}\Big) - \Psi'\Big(\frac{z}{2}\Big)\right]$$

Maximal transcendentality (K.,L. (2002))

$$\phi(M) = 3\zeta(3) + \Psi^{''}(M) - 2\Phi(M) + 2\beta^{'}(M)\Big(\Psi(1) - \Psi(M)\Big),$$

$$\Phi(M) = \sum_{k=0}^{\infty}\frac{(-1)^k}{k+M}\left(\Psi'(k+1)\;-\;\frac{\Psi(k+1) - \Psi(1)}{k+M}\right)$$

10 Pomeron and reggeized graviton

BFKL Pomeron in a diffusion approximation

$$j = 2 - \Delta - D\,\nu^2\,,\ \gamma = 1 + \frac{j-2}{2} + i\nu$$

Constraint from the energy-momentum conservation

$$\gamma = (j-2)\left(\frac{1}{2} - \frac{1/\Delta}{1+\sqrt{1+(j-2)/\Delta}}\right)$$

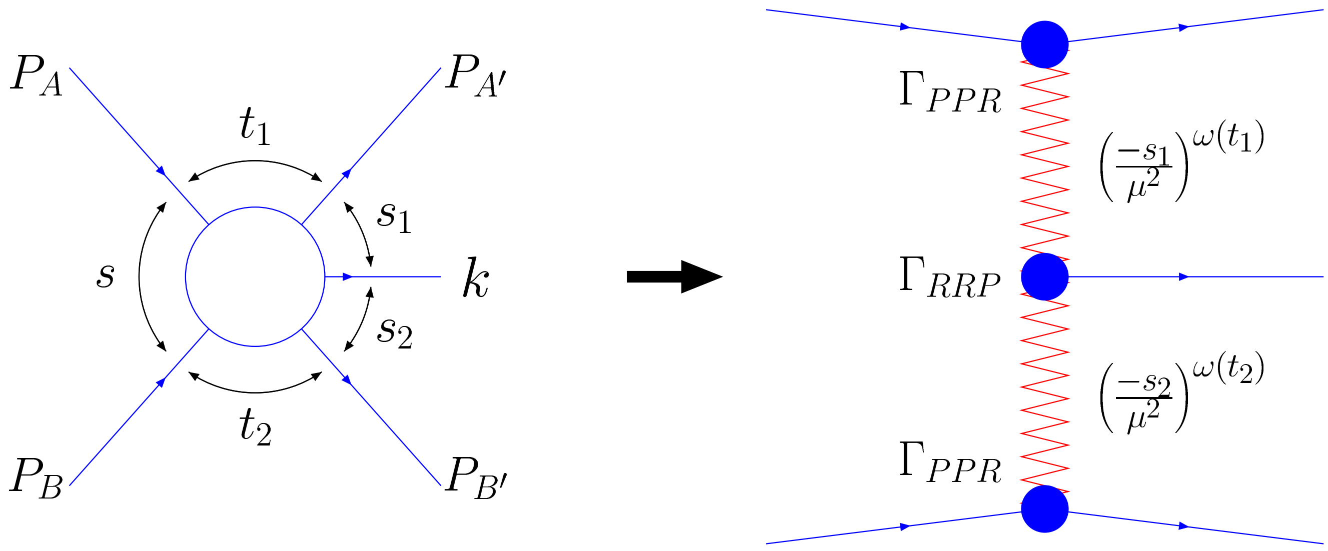
AdS/CFT relation for the graviton Regge trajectry

$$j = 2 + \frac{\alpha'}{2}\,t\,,\ t = E^2/R^2\,,\ \alpha' = \frac{R^2}{2}\,\Delta$$

Large coupling asymptotics for Δ (KLOV, BPST)

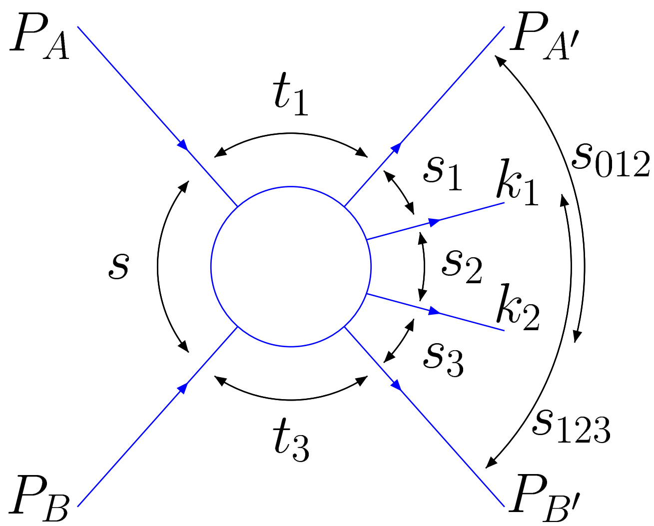
$$\gamma = -\sqrt{2\pi(j-2)}\,\hat{a}^{1/4}\,,\ j = 2 - \Delta\,,\ \Delta = \frac{1}{2\pi}\,\hat{a}^{-1/2}$$

11 BDS production amplitude (BLS)



$$\ln \Gamma_{21} = -\frac{1}{2}\left(\omega(t_1)+\omega(t_2) - \int_0^a \frac{da'}{a'}\left(\frac{\gamma_K(a')}{4\epsilon}+\beta(a')\right)\right)\ln\frac{-k_\perp^2}{\mu^2} -$$
$$\frac{\gamma_K(a)}{16}\left(\ln^2\frac{-k_\perp^2}{\mu^2} - \ln^2\frac{-t_1}{-t_2} - \zeta_2\right) - \frac{1}{2}\int_0^a \frac{da'}{a'}\ln\frac{a}{a'}\left(\frac{\gamma_K(a')}{4\epsilon^2}+\frac{\beta(a')}{\epsilon}+\delta(a')\right)$$

12 Regge factorization violation (BLS)



$$M_{2\to 4}^{BDS}|_{s,s_2>0;\, s_1,s_3<0} = C\,\Gamma_1 \left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)} \Gamma_{21} \left(\frac{-s_2}{\mu^2}\right)^{\omega(t_2)} \Gamma_{32} \left(\frac{-s_3}{\mu^2}\right)^{\omega(t_3)} \Gamma_3\,,$$

$$C = \exp\left[\frac{\gamma_K(a)}{4}\, i\pi \left(\ln\frac{t_1 t_2}{(\vec{k}_1+\vec{k}_2)^2\mu^2} - \frac{1}{\epsilon}\right)\right]$$

13 Mandelstam cuts in j_2-plane

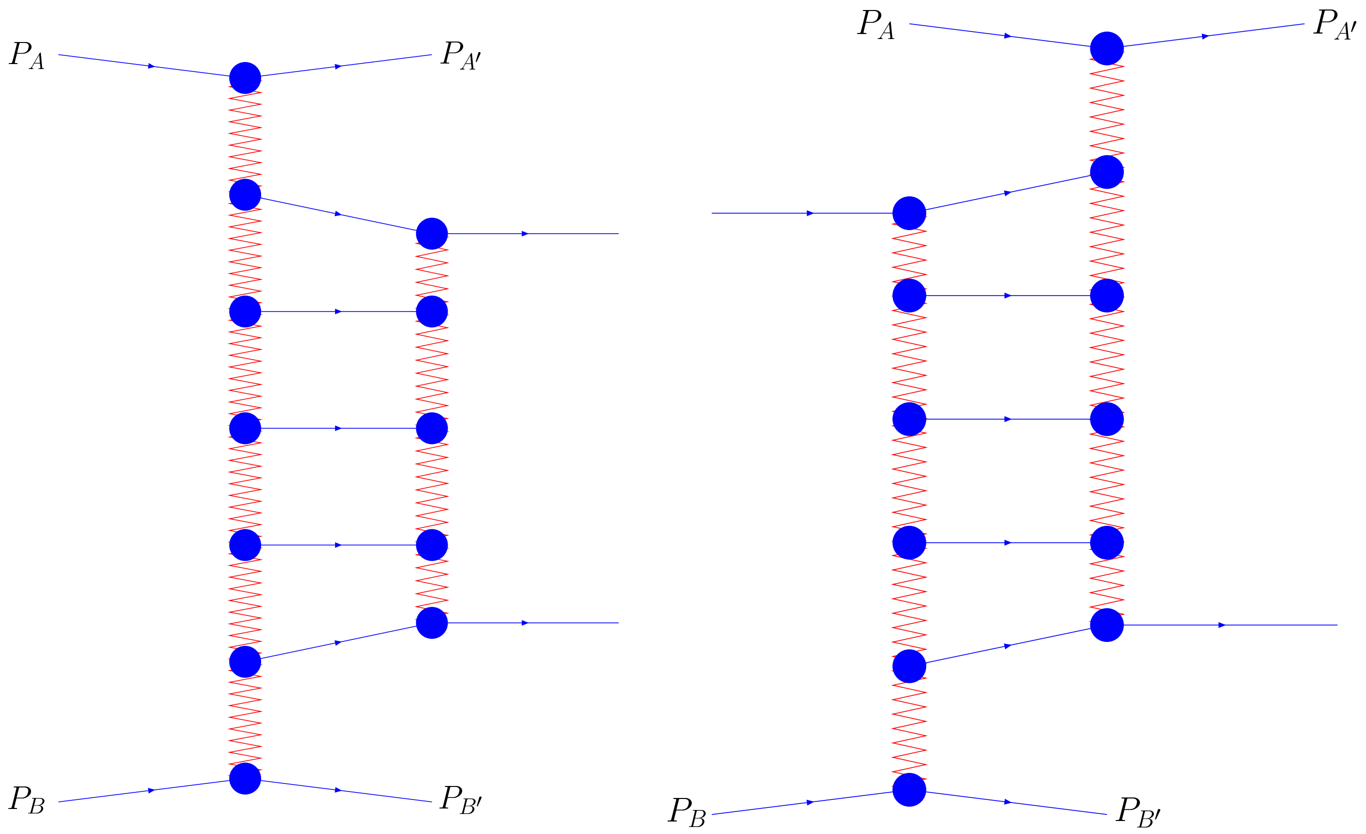


Figure 1: BFKL ladders in $M_{2\to4}$ and $M_{3\to3}$

14 Correction to the BDS result (BLS)

Factorization of infrared divergencies in LLA

$$M^{2\to 4}_{|_{s,s_2>0;s_1,s_3<0}} = (1+i\Delta_{2\to 4})\; M^{BDS}_{2\to 4}\,,$$

Analytic result in LLA in the region $a\ln s_2 \sim 1$

$$\Delta_{2\to 4} = \frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n\int_{-\infty}^{\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}\,(V^*)^{i\nu-\frac{n}{2}}\,V^{i\nu+\frac{n}{2}}\left(s_2^{\delta(\nu,n)}-1\right)$$

Duality transformation (L.L. (1999))

$$V=\frac{q_3k_1}{k_2q_1}\to\frac{z_{03}z_{0'1}}{z_{0'3}z_{01}}$$

Functions of 4-dimensional anharmonic ratios

$$i\Delta_{2\to 4} = \frac{a^2}{4}Li_2(\chi)\ln\frac{\chi t_2 s_{13}}{s_3t_1}\ln\frac{\chi t_2 s_{02}}{t_3 s_1}+...\,,\;\; \chi = 1-\frac{ss_2}{s_{012}s_{123}}$$

15 Multi-gluon states in octet channels

Holomorphic hamiltonian for n-gluon composite states

$$h = \ln(Z_1^2 \partial_1) - 2\psi(1) + \ln \partial_{n-1} + \sum_{k=1}^{n-2} h_{k,k+1}\,,\;\; p_k = Z_{k-1,k}\,,\;\; Z_0 = 0\,,\;\; Z_n = \infty$$

Pair hamiltonian of the spin chain

$$h_{1,2} = \ln(Z_{12}^2 \partial_1) + \ln(Z_{12}^2 \partial_2) - 2\ln Z_{12} - 2\psi(1)$$

Monodromy matrix (L. (1993))

$$t(u) = L_1(u) L_2(u) ... L_{n-1}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

Integrals of motion and Baxter equation for integrable spin chain

$$[D(u), h] = 0\,,\;\; D(u)Q(u) = (u-i)^{n-1} Q(u-i)$$

16 Effective action for gravity

Locality in the rapidity space

$$y = \frac{1}{2}\ln\frac{\epsilon_k + |k|}{\epsilon_k - |k|}\,,\ \ |y - y_0| < \eta\,,\ \ \eta << \ln\, s$$

Metric tensor and reggeized graviton fields

$$d^2S = \sum_{\mu\nu} g_{\mu\nu}\, dx^\mu\, dx^\nu\,,\ \delta A^{++}(x) = \delta A^{--}(x) = 0$$

Effective action for reggeized gravitons

$$S = \int d^4x \left(-\frac{\sqrt{-g}}{2\kappa}\, R + G_{++}A^{++} + G_{--}A^{--}\right)$$

Effective metric tensor

$$G_{++} = g_{++} + (\partial_\sigma g_{++})\frac{1}{4\partial_+^2}\partial_\sigma g_{++} - (\partial_\sigma g_{++})\frac{1}{\partial_+}g_{+\sigma} - g_{+\sigma}^2 + \ldots$$

17 Discussion

1. Locality in the rapidity space
2. Steinmann relations and bootstrap.
3. Remarkable properties of NLLA in $N = 4$ SUSY.
4. Breakdown of the BDS ansatz.
5. Mandelstam cuts in planar amplitudes at $n \geq 6$.
6. Integrability of planar scattering amplitudes
7. Effective action for the reggeized gravitons