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New exact solutions of perfect fluid hydrodynamics: theory and applications in high-energy experiments

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Outline

- Heavy-ion physics: studying the phase structure of QCD matter
- Collective phenomena: experimental signs of phase structure
- Hydrodynamical modelling: exact vs. numerical
- Solutions to the hydrodynamical equations
  - Nonrelativistic solutions
  - Review of well-known solutions
  - Recent results in relativistic perfect fluid hydrodynamics
- Phenomenological applications

Motivation

Goal of heavy-ion physics: Understand the phase structure of the strongly interacting matter (hard task)

Experimentally: the matter created in heavy-ion collisions is an almost perfect fluid

- Hydrodynamics is the only way which dynamically connects the initial state, the final state, and the equation of state (EoS)
- To explore the EoS is perhaps the final aim: but we can get to it only by knowing properly the space-time picture of the collisions
- Lots of models failed to describe the observations correctly

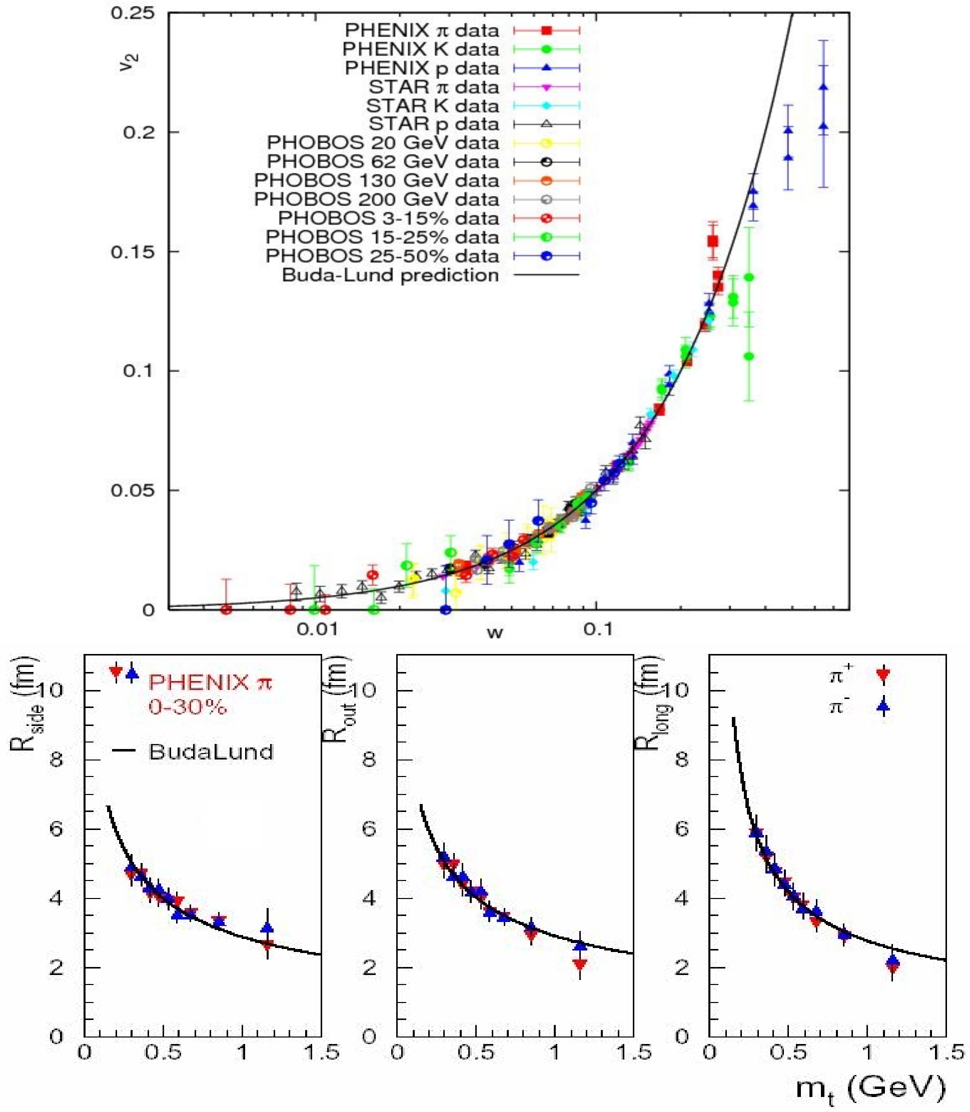
Hydrodynamical modelling can tell about the dynamics of the matter, thus finally on the equation of state

Perfect fluid hydrodynamics

Perfectness of the fluid: important question

- In the relativistic case, even the equations are not well-defined for viscous fluids
- In nonrelativistic case, equations are easier to solve
- Surprisingly, they lead to a good description of the observables
- Elliptic flow: scaling behaviour predicted

Hydrodynamics naturally leads to scaling



Relativistic hydrodynamics

Only assumption: local thermal equilibrium & local energy-momentum conservation (no internal scale -> scaling laws)

Stress-energy-momentum tensor: $T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$

- Equations follow from that its four-divergence vanishes $\partial_\nu T^{\mu\nu} = 0$

Equations for perfect fluid:

- Euler equation: $w u^\nu \partial_\nu u^\mu = \left(g^{\mu\rho} - u^\mu u^\rho\right) \partial_\rho p$
- Energy conservation: $w \partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon$
- Entropy conservation: $\partial_\mu \left(\sigma u^\mu\right) = 0.$

Hydrodynamical modelling

Different types of hydrodynamical models in heavy ion physics:

- Hydrodynamics inspired parametrization: describes just the final state (at freeze-out), time evolution not considered
- Numerical models: solve the equations numerically
- Parametric solutions: exact solutions with fit parameters
- Models based on exact solutions: either use a particular solution directly, or a parametrization that simplifies to known exact solutions

A hydrodynamical model is a combination of

initial conditions + dynamical equations + freeze-out condition

- Common: calculate observables from thermal distributions

Historical solutions

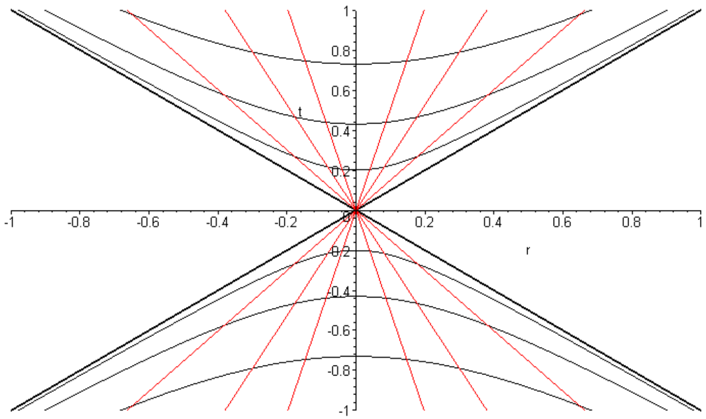
Landau-Khalatnikov solution: implicit, exact, accelerating 1+1D

- Used in the description of elementary particle reactions (p+p in cosmic rays); hydrodynamics really "begins" in p+p (not in $e^+ + e^-$)
- Initially: a finite slab of matter, final observables: approximately Gaussian rapidity distribution; formulas are complicated

L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953)

Hwa-Bjorken solution: exact explicit 1+1D, boost-invariant

- Final rapidity distribution is constant), but important for estimating initial energy density



R. C. Hwa, Phys. Rev. D 10, 2260 (1974), J. D. Bjorken, Phys. Rev. D 27, 140 (1983)

Nonrelativistic solutions

In the NR limit, there are many 1+3D realistic exact solutions:

- First exact Hubble-like solution in a broad class (with finite density):

J. Bondorf, S. Garpman, J. Zimányi, Nucl. Phys. A 296 320 B 37 483 (1978)

- Directional Hubble velocity field, general temperature profile:

T. Csörgő, Acta Phys. Polon. B 37 483 (2006)

- Directional Hubble velocity field, Gaussian density profile, for arbitrary temperature-dependent speed of sound:

T. Csörgő, S. V. Akkelin, Y. Hama, B. Lukács, Y. M. Sinyukov, PRC 67 034904 (2003)

To my best knowledge, this is the only exact solution where arbitrary EoS (e.g. lattice QCD results) can be directly utilized (at μ_B=0)

- Buda-Lund hydrodynamical model: based on these solutions, describes scaling of elliptic flow, RHIC HBT puzzle, etc ...

Relativistic solutions

Homogeneous expansion:

- An example: spherical expansion & ellipsoidally symmetric pressure:

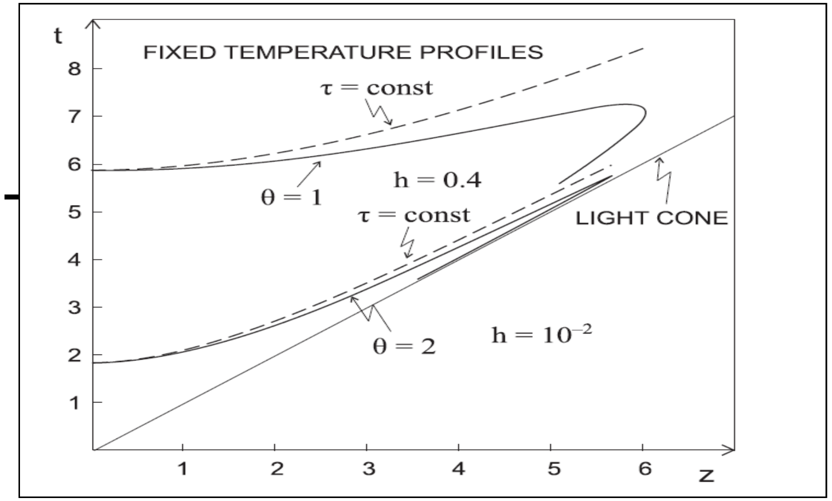
$$u^\mu = \frac{x^\mu}{\tau} \qquad s = \frac{r_x^2}{\dot{X}_0^2 t^2} + \frac{r_y^2}{\dot{Y}_0^2 t^2} + \frac{r_z^2}{\dot{Z}_0^2 t^2}$$

T. Csörgő, L. P. Csernai, Y. Hama, T. Kodama, Heavy Ion Phys. A 21, 73 (2004)

- Many other solutions ...

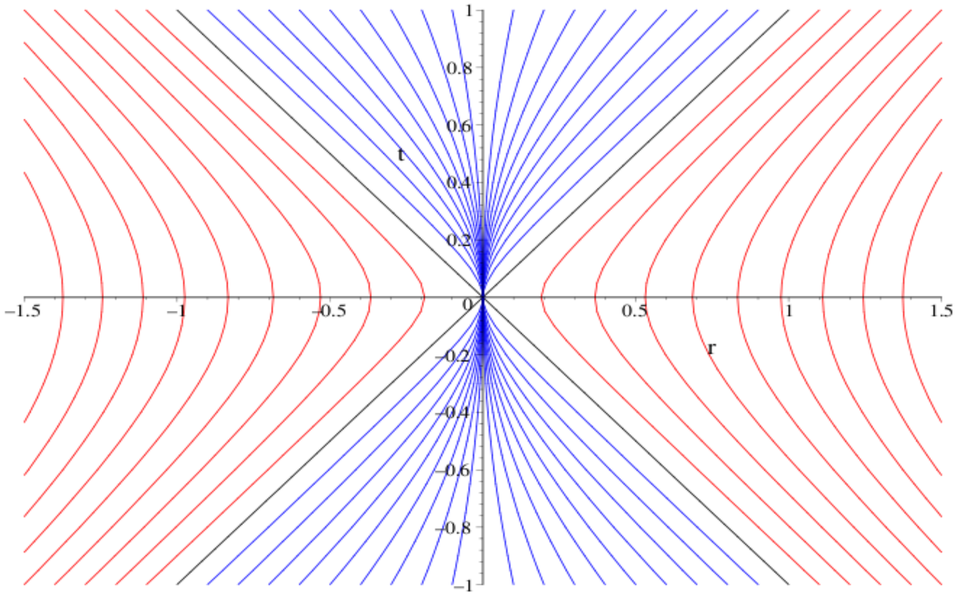
1+1D, interpolation between Landau-Khalatnikov & Hwa-Bjorken:

- valid for arbitrary (constant) speed of sound



A. Bialas, R. Janik, R. Peschanski, Phys. Rev. C 76 054901 (2007)

Relativistic solutions



- First accelerating explicit solution: constant acceleration in rest frame
- Framework for Unruh-effect
- An extension:

$$v = \frac{2tr}{t^2 + r^2} \;\; \text{->} \;\; v = \tanh \lambda\eta,$$

For a very stiff EoS (κ=1), the equations become a wave-equation for a potential function: exact solution for any initial condition

M. Nagy, T. Csörgő, M. Csanád, Phys. Rev. C 77 024908 (2008)

(see also: M. Borshch, V. Zhdanov, SIGMA 3, 16 (2007))

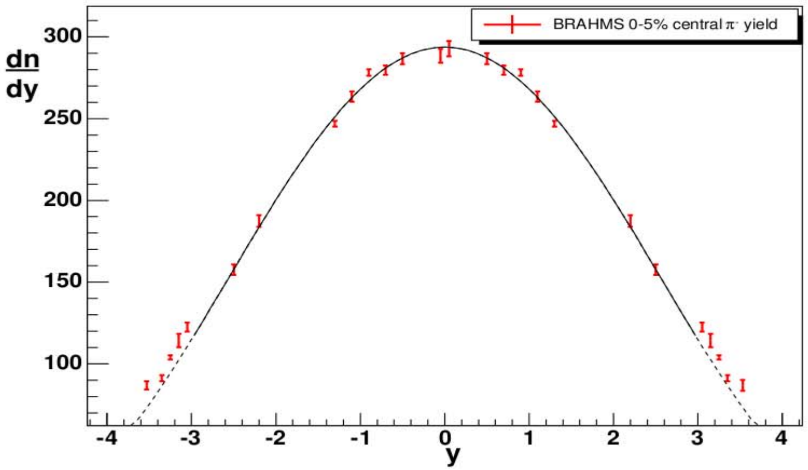
Applications

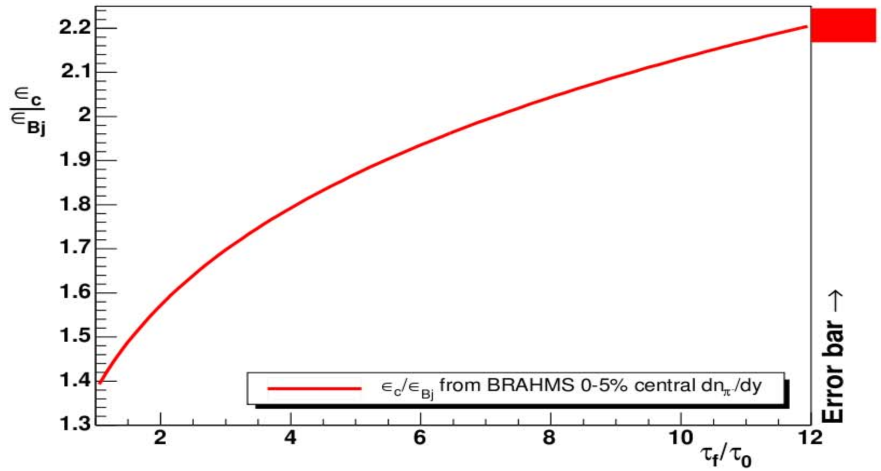
Rapidity distribution: finite

- Acceleration parameter fitted to RHIC measurements (BRAHMS)

  - energy density: a ~**120% increase** compared to the Bjorken-estimate, and with more realistic EoS, conjectured **further ~50%:** Bjorken estimate: 5 GeV/fm^3, improved estimate is ~15 GeV/fm^3,

  - Critical value: 1 GeV/fm^3, ~170MeV, qualitative agreement with the temperature measurement at PHENIX (~300 MeV based on thermal photons) since $\varepsilon \sim T^4$





New relativistic solutions

A special problem investigated:

- Can the collisionless evolution of matter particles maintain local thermal equilibrium (contrary to intuition)?
- In the nonrelativistic case: there are such expanding solutions
- Relativistic case: new results

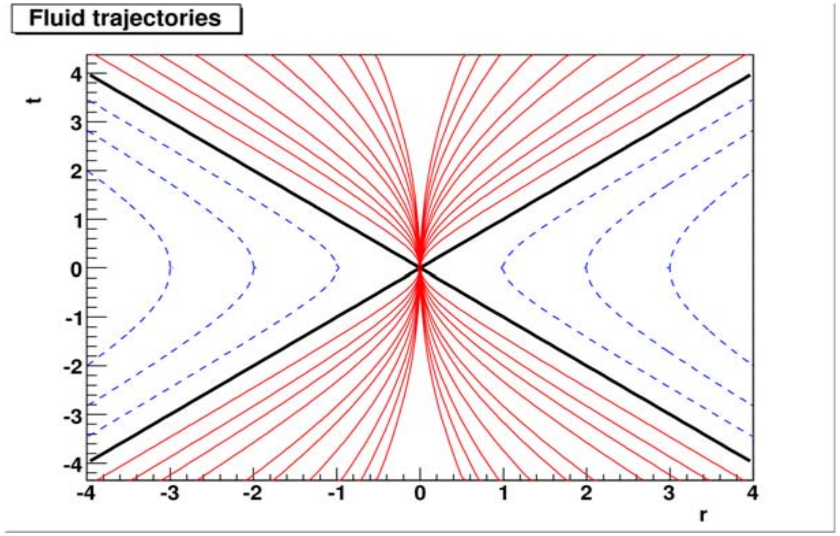
Generalizing these solutions:

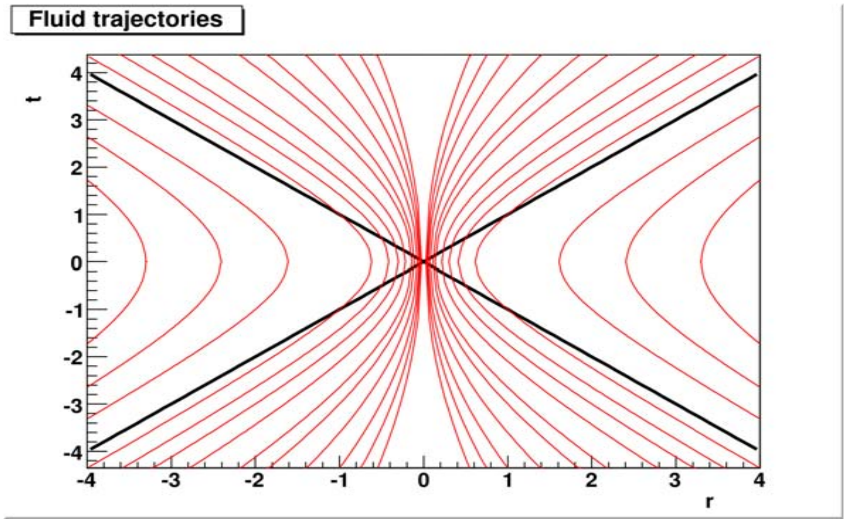
- Interesting solutions on their own: regular, spatially finite pressure
- Rotating and coasting, realistic 3D solutions (for the first time)

M. I. Nagy, arXiv:0909.4285

New relativistic solutions

Example of the new solutions:





Already known (μ=0) New, regular (finite μ)

Other generalizations:

Rotating relativistic solutions: maybe useful for studying non-central collisions (e.g. local parity violation ...)

Summary

Exact (parametric) hydrodynamical solutions:

- Useful in heavy-ion physics: follow the time evolution (constraints on initial conditions provided we know the final state), or if initial state is modelled, constrains EoS from data

- It is natural to develop the models with exact solutions (challenging)

- Now: there are many exact solutions, some of them are realistic in EoS (works for any speed of sound), realistic in geometry (3D expansion, ellipsoidal symmetry) or realistic in flow profile: asymptotics (Hubble), or acceleration

Future work needed:

- New exact results are essential steps in understanding the dynamics of heavy-ion reactions, leading to understanding the properties of the QCD matter experimentally

Thank you for your attention!