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Can CP-violation be observed in heavy-ion collisions?

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CAPTURE OF DARK MATTER BY THE SOLAR SYSTEM

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1. Introduction

Dark matter density in our Galaxy is

$$\rho_g \simeq 4 \cdot 10^{-25}\ \mathrm{g/cm}^3 . \tag{1}$$

Only upper limits on the level of $10^{-19}\ \mathrm{g/cm}^3$ are known for dm density ρ_{SS} in Solar System.

But even these limits are derived under quite strong assumption that distribution of dmp density in SS is spherically-symmetric with respect to the Sun.

Information on ρ_{SS} is important for experiments aimed at detection of dm.

2. Total mass of captured dark matter

SS is immersed in halo of dark matter (dm) and moves together with it around center of Galaxy. In reference frame, comoving with halo, velocities v of dm particles in halo have Maxwell distribution

$$f(v)\,dv = \sqrt{54/\pi}\,(v^2 dv/u^3)\exp\left(-3v^2/2u^2\right) \longrightarrow \sqrt{54/\pi}\,(v^2 dv/u^3);$$

local rms velocity $u \simeq 220$ km/s is large as compared to typical planetary $v \simeq 30$ km/s.

Particle cannot be captured by Sun alone. Interaction with planet is necessary for it, this is three-body problem. Capture is dominated by particles whose orbits are close to parabolic ones with respect to the Sun, and whose perihelia are close to the planet orbit since such trajectories are most sensitive to additional attraction by planet.

Capture is effectively described by restricted three-body problem:
Interaction between heavy bodies (the Sun and a planet) is treated exactly.
As exactly is treated motion of the third, light body (dmp) in gravitational field of two heavy ones.
One neglects back reaction of light particle upon motion of two heavy bodies. This approximation is fully legitimate for our purpose.
Still, even restricted three-body problem remains quite complicated, and requires both subtle analytical treatment and serious numerical calculations.

We resort instead to dimensional and qualitative estimates.

Total mass captured by the Sun (its mass is M) together with a planet with mass m_p, during lifetime $T \simeq 4.5 \cdot 10^9$ years $\simeq 10^{17}$ s of SS, is

$$\Delta m_p = \rho_g T < \sigma v > ;$$

σ is capture cross-section. Product σv is averaged here over distribution $f(v)\,dv = \sqrt{54/\pi}\,(v^2 dv/u^3)$.
We estimate $< \sigma v >$ with dimensional arguments, supplemented by two physical requirements: masses m_p and M of two heavy components of restricted three-body problem should enter result symmetrically, and mass of dmp should not enter result at all in virtue of the equivalence principle.

Then final estimate for captured mass is

$$\Delta m_p \sim \rho_g T \sqrt{54\pi}\;\; k^2\, m_p\, M / u^3\,. \tag{2}$$

here k is the Newton gravitation constant; an extra power of π, inserted into this expression, is perhaps inherent in σ.
Capture gets possible due to dmp interaction both with the planet and the Sun, which results in the product of corresponding coupling constants kM and km_p.

For the Earth it constitutes

$$\mu_E \sim 4 \cdot 10^{18}\, \mathrm{g}\,. \tag{3}$$

3. Capture cross-section

By dimensional reasons, capture cross-section for the Earth is

$$\sigma \sim \pi k^2 m_E M / \tilde{v}^4 , \tag{4}$$

$\tilde{v}$ is some velocity. We assume that:
1) Capture occurs close to the Earth, i.e. at distances $\sim r_E$ from the Sun.
2) Initial velocities of captured dmp's exceed only slightly parabolic one v_{par}, (but to our accuracy, omit 2 in $v_{par}^2 = 2kM/r_E$).
3) Their final velocities are only slightly less than v_{par}.
Just put $\tilde{v}^2 \sim v_E^2 = kM/r_E$ ($v_E = 30$ km/s). Thus,

$$\sigma \sim \pi k^2 m_E M / v_E^4 \tag{5}$$

or

$$\sigma \sim \pi r_E^2 (m_E/M) \,. \tag{6}$$

Corresponding impact parameter is

$$r_{\rm imp} \sim r_E (m_E/M)^{1/2} \ll r_E \,. \tag{7}$$

Quite natural: $r_{\rm imp}$ corresponds to distance
at which attraction to the Earth exceeds attraction to the Sun,
i.e. where

$$km/r^2 > kM/r_E^2 \,, \quad r \ll r_E \,.$$

Up to now we dealt with capture cross-section integrated over directions of dmp velocity $\mathbf{v}$. In fact, cross-section depends essentially on the mutual orientation of $\mathbf{v}$ and $\mathbf{v}_E$. It is maximal when these velocities are parallel and as close as possible by modulus. Besides, the impact parameter $r_{\rm imp} \ll r_E$. Therefore, it is quite natural to identify $\tilde{v}$ in (4) with the relative velocity $\mathbf{v} - \mathbf{v}_E$ of the dmp and the Earth, i.e. to generalize formula (5) as follows:

$$d\sigma \sim \frac{k^2 m_p M}{(\mathbf{v} - \mathbf{v}_E)^4} \frac{1}{4} d\Omega \tag{8}$$

(factor $1/4$ is introduced here for the correspondence with the factor π in (5): $(1/4) \int d\Omega = \pi$).

Thus derived total cross-section is

$$\sigma \sim \frac{1}{4}\int d\Omega\, \frac{k^2\, m_p\, M}{(\mathbf{v} - \mathbf{v}_E)^4} = \frac{\pi\, k^2\, m_p\, M}{(v^2 - v_E^2)^2}\,. \tag{9}$$

Clearly, it is the particles moving initially with the velocities only slightly above the parabolic one $\sqrt{2}\, v_E = 42$ km/s that are captured predominantly, and thus, with $v = \sqrt{2}\, v_E$, cross-sections (5) and (9) practically coincide.

On the other hand, it follows from (9) that
in the vicinity of the Earth the captured particles move with respect to it with velocities close to $(\sqrt{2} - 1)v_p \simeq$ 12 km/s.

4. Space distribution of captured dark matter

Captured dmp's had initial trajectories predominantly close to parabolas focussed at the Sun, and their velocities change slightly as a result of scattering. Therefore, their trajectories become elongate ellipses with large semimajor axes, still focussed at the Sun. The ratio of their maximum $r_{\max}$ and minimum $r_{\min}$ distances from the Sun is

$$r_{\max}/r_{\min} = (1+e)/(1-e)\,,$$

e is eccentricity of trajectory. As result of capture, eccentricity changes from $1+\varepsilon_1$ to $1-\varepsilon_2$, $\varepsilon_{1,2} \ll 1$. Loss of eccentricity is due to gravitational perturbation by the Earth, and thus is proportional to m_E. $r_{\min}$ is close to radius r_E of the Earth orbit. Thus, for dimensional reasons,

$$r_{\max} \sim r_E\,(M/m_E)\,.$$

Analogous estimate for Jupiter complies with numerical calculations by Petrosky.

Semimajor axis $a_{\rm dmp}$ of dmp trajectory is of same order as $r_{\rm max}$.
Time spent by dmp with velocity $\sim v_E$ at distance $\sim r_E$ from the Sun, is close to orbital period of the Earth $\sim$ $T_E = 1$ year.
Dmp orbital period T is related to semimajor axis a as $T \sim a^{3/2}$.
Thus, orbital period of captured dmp is

$$T_{\rm dmp} \sim T_E \left(M/m_E\right)^{3/2}. \qquad (10)$$

For the Earth, $T_{\rm dmp} \sim 10^8$ years. Still, it is much less than $T_{SS} \sim 5 \cdot 10^9$ years. Thus, relative time spent by a dmp at distances $\sim r_E$ from the Earth is $\sim (m_E/M)^{3/2}$. Moreover, with impact parameter $r_{\rm imp} \sim r_E \left(m_E/M\right)^{1/2}$, relative time spent by dmp sufficiently close to the Earth to be captured, is estimated as $(m_E/M)^2$.

With impact parameter $\sim r_E\,(m_E/M)^{1/2}$, volume V, crucial for capture, is centered at the Earth and can be estimated as

$$V \sim (4\pi/3)\, r_E^3\, (m_E/M)^{3/2} \ll (4\pi/3)\, r_E^3. \qquad (11)$$

Combine total captured mass with volume occupied by this mass and with estimate $(m_E/M)^2$ for relative time spent by dmp within impact parameter with respect to the Earth. Thus we arrive at following estimate for dm density, captured by SS, in vicinity of the Earth:

$$\rho_E \sim 5 \cdot 10^{-25}\ \mathrm{g/cm^3}\,. \qquad (12)$$

Same result can be derived otherwise. Captured dm close to the Sun effectively fills in torus of large radius r_E, small radius $r_E\,(m_E/M)^{1/2}$, and volume

$$V_1 \sim r_E^3\,(m_E/M)^2\,. \tag{13}$$

With above ratio of orbital periods $T_{\rm dmp} \sim T_E\,(M/m_E)^{3/2}$, we arrive at the same estimate for dm density, captured by SS, in vicinity of the Earth:

$$\rho_E \sim 5 \cdot 10^{-25}\ \text{g/cm}^3\,. \tag{14}$$

This estimate coincides practically with value $\rho_g \sim 4 \cdot 10^{-25}\ \text{g/cm}^3$ of galactic dm density!

Our results for total mass and density of captured dm should be considered as upper limits only, since we neglected inverse process: ejection of the captured dm from the SS. Characteristic time of inverse process is not exactly clear now. Therefore, it cannot be excluded that it is comparable to, or even larger than, the lifetime $\boldsymbol{T}$ of the SS. Then our estimates are valid.

If this is the case indeed, then

dm around the Earth consists of
two components of comparable densities:
common component with typical velocity $\boldsymbol{u} \sim \mathbf{220}$ km/s,
and one more, with velocity relative to the Earth $\simeq \mathbf{12}$ km/s.