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**2146-18**

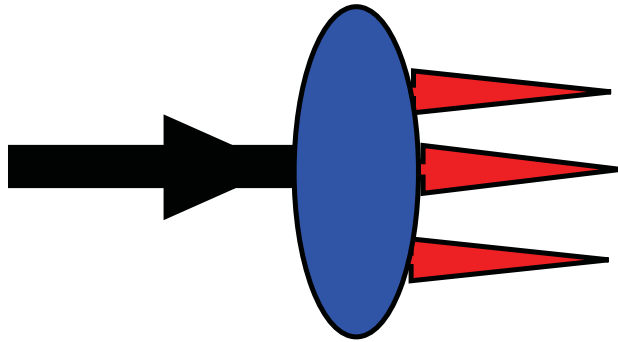
**Gribov-80 Memorial Workshop on Quantum Chromodynamics and  
Beyond'**

*26 - 28 May 2010*

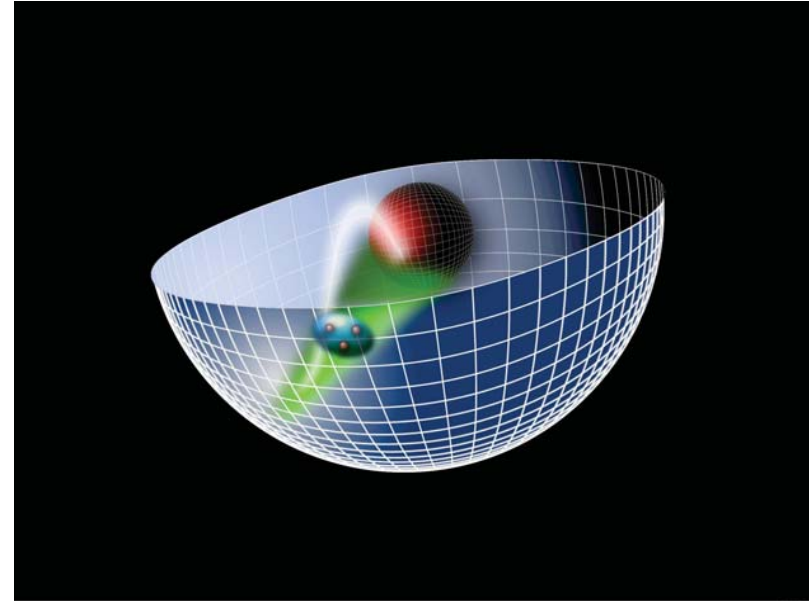
**New insights into hadron physics from AdS/QCD and light-front holography**

Stanley Brodsky  
*Stanford University*  
*U.S.A.*

# *New Insights into Hadron Physics from AdS/QCD and Light-Front Holography*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



**Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond**



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*May 27, 2010*



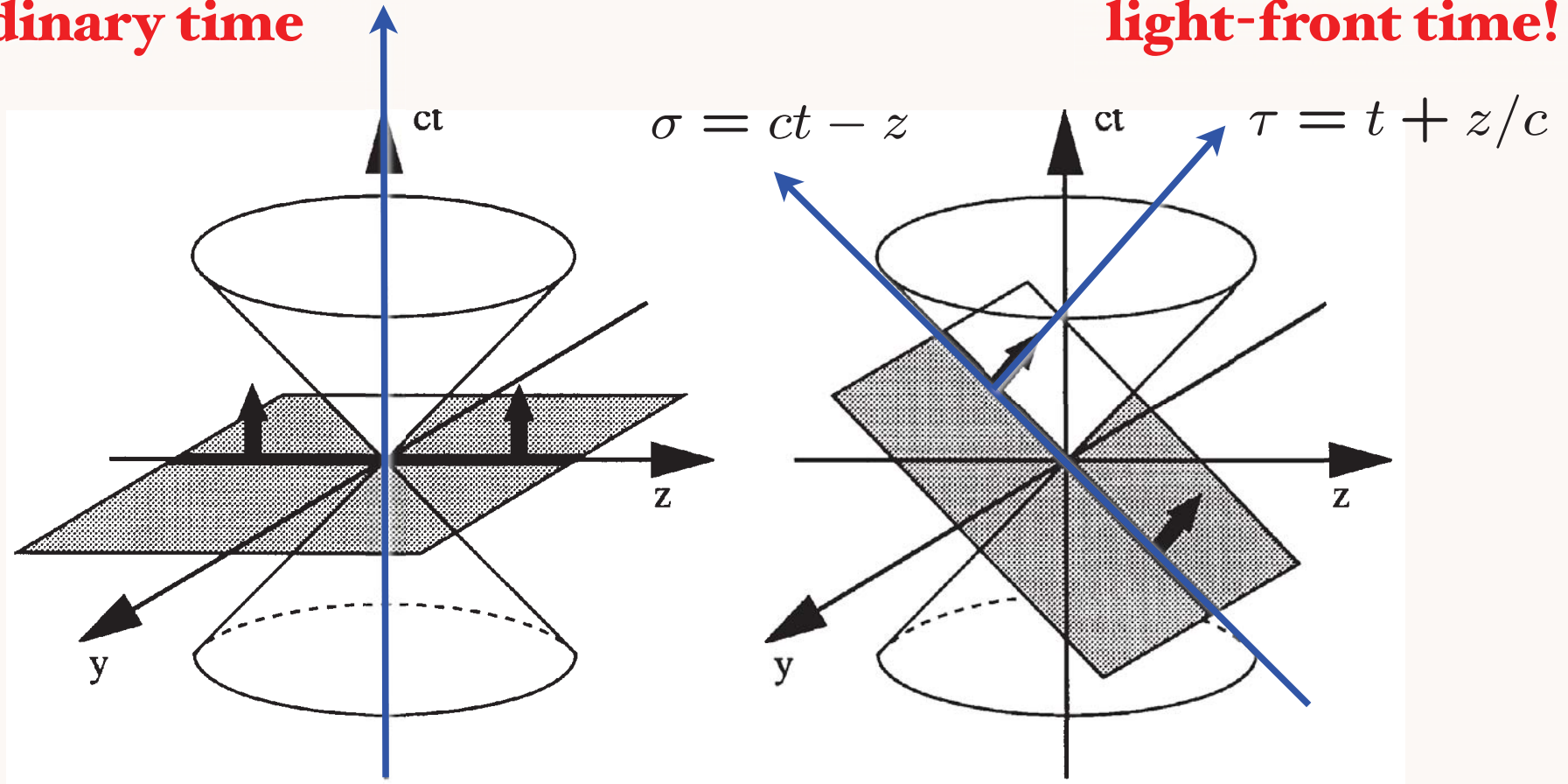
**Stan Brodsky**  **& CP<sup>3</sup> -Origins**

**SLAC**  
NATIONAL ACCELERATOR LABORATORY

# Dirac's Amazing Idea: The Front Form

**Evolve in  
ordinary time**

**Evolve in  
light-front time!**



**Instant Form**

**Front Form**

*Each element of  
flash photograph  
illuminated  
at same Light Front  
time*

$$\tau = t + z/c$$

*Evolve in LF time*

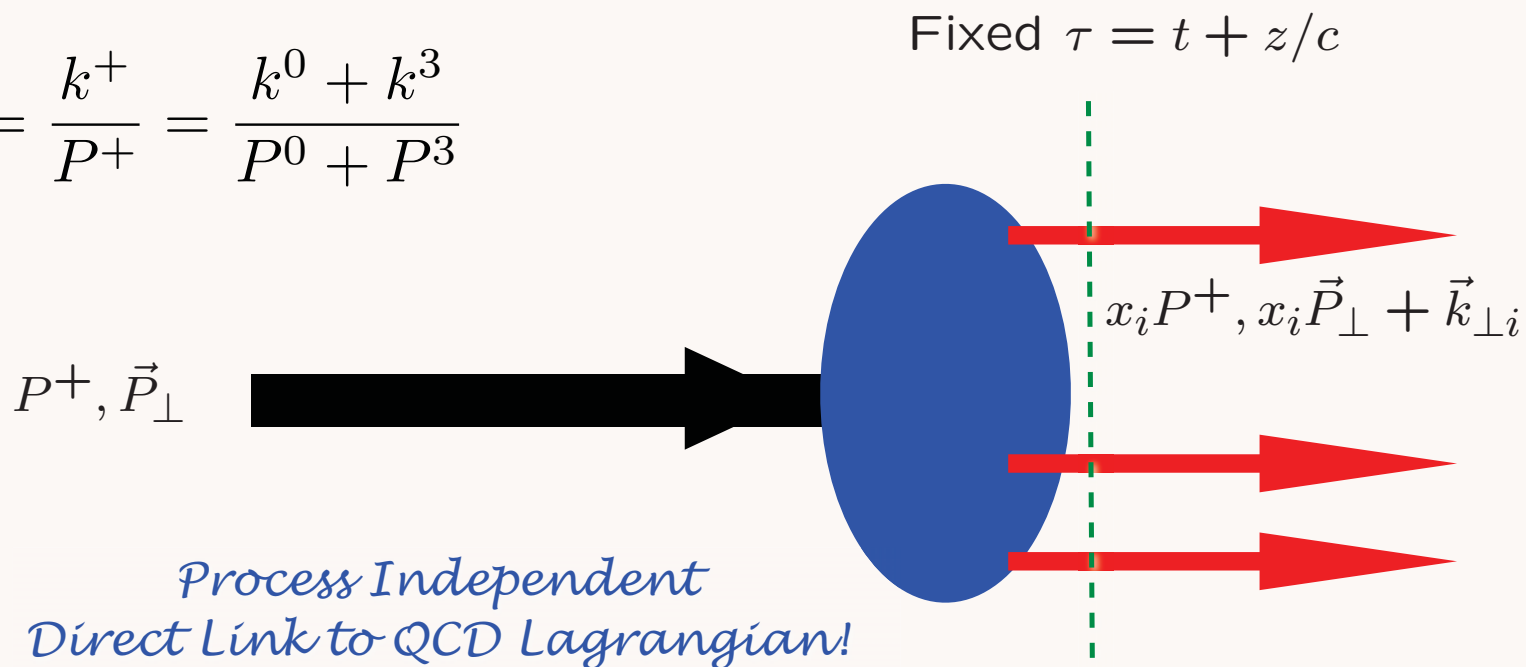
$$P^- = i \frac{d}{d\tau}$$

***Causal, Trivial Vacuum***



# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

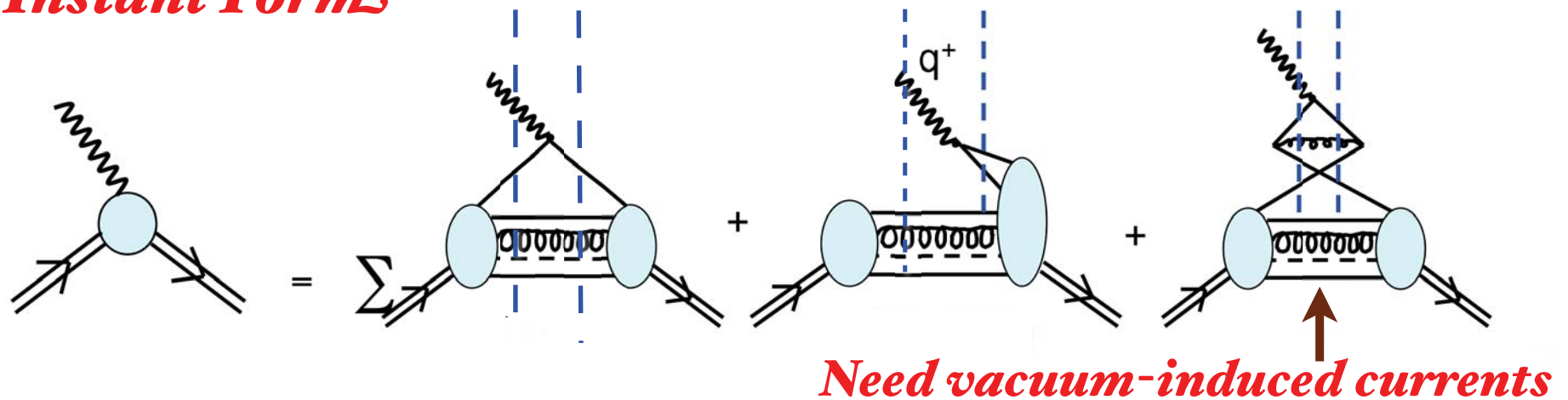
$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

*Invariant under boosts! Independent of  $P^\mu$*

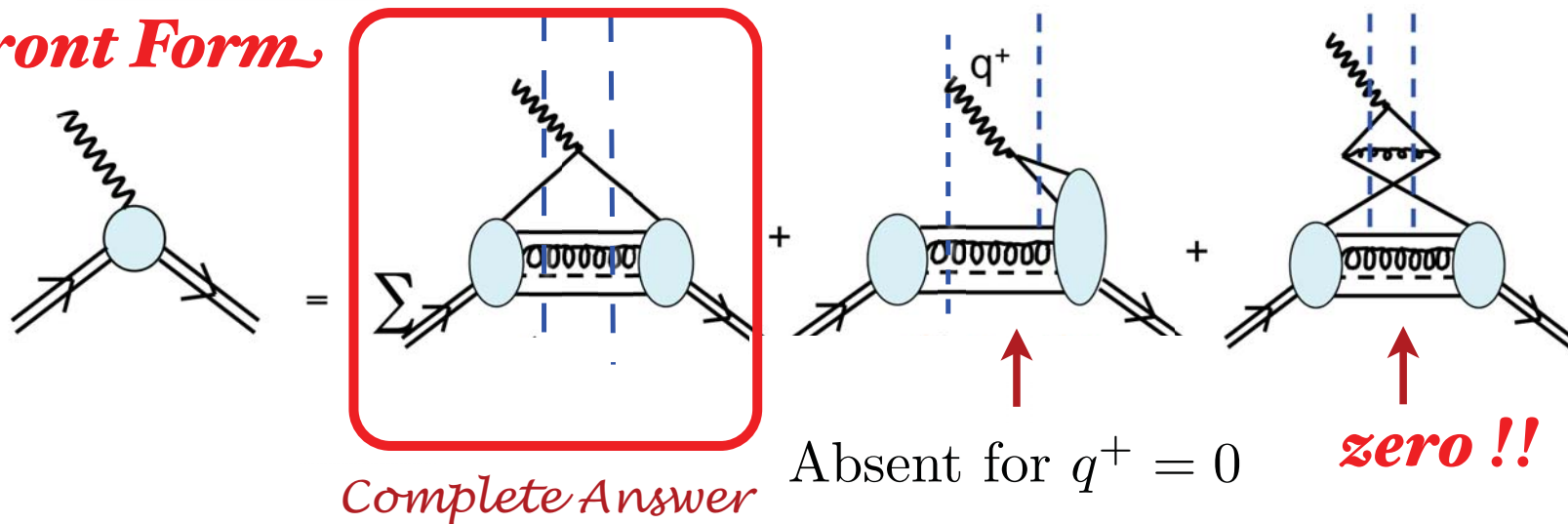
## Calculation of Form Factors in Equal-Time Theory

### Instant Form



## Calculation of Form Factors in Light-Front Theory

### Front Form

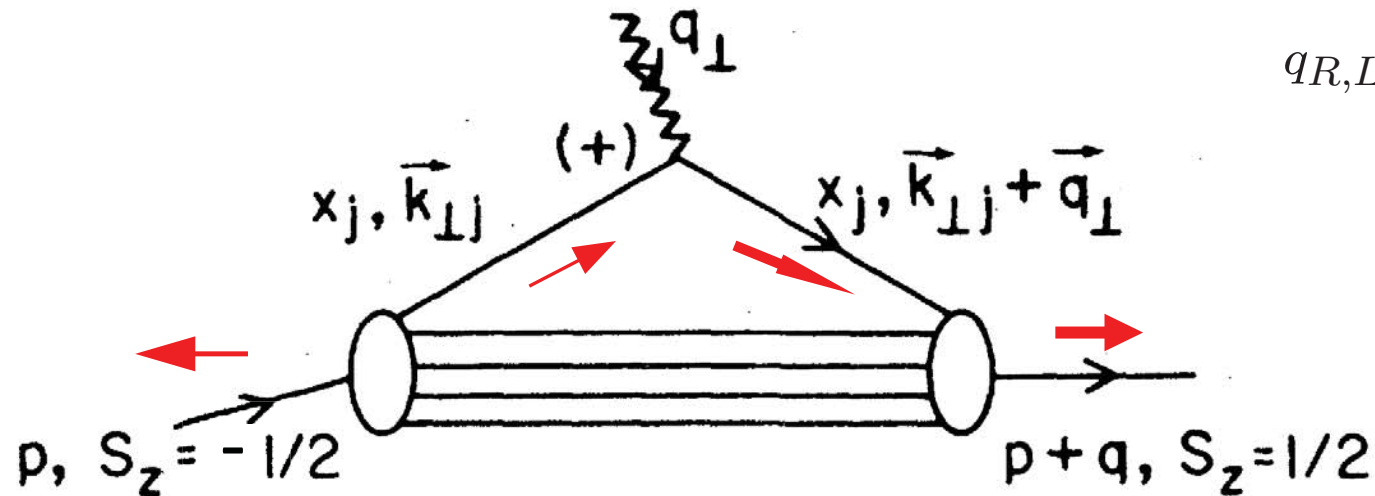


$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$

$$\left[ -\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

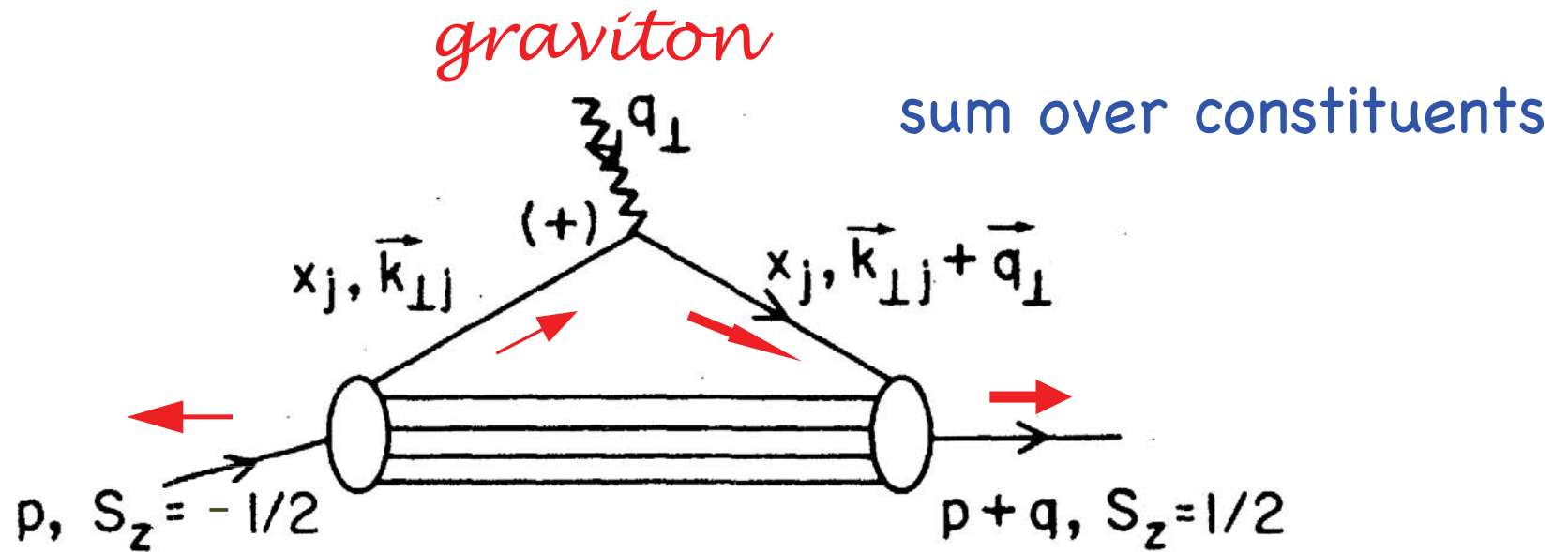


Must have  $\Delta \ell_z = \pm 1$  to have nonzero  $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->  
Nonzero orbital quark angular momentum*

# Anomalous gravitomagnetic moment $B(0)$

**Terayev, Okun, et al:**  $B(0)$  Must vanish because of Equivalence Theorem



**Hwang, Schmidt, sjb;  
Holstein et al**

$$B(0) = 0$$

*Each Fock State*



# Angular Momentum on the Light-Front

*Jaffe definition*  
*LC gauge*

$$J^z = \sum_{i=1}^n s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

**Conserved**  
**LF Fock state by Fock State**

**Gluon orbital angular momentum defined in physical lc gauge**

$$l_j^z = -i \left( k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right) \quad \text{n-1 orbital angular momenta}$$

*Orbital Angular Momentum is a property of LFWFS*

$$|p, S_z\rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i\rangle$$

*sum over states with  $n=3, 4, \dots$  constituents*

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum  $P^\mu$ .

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

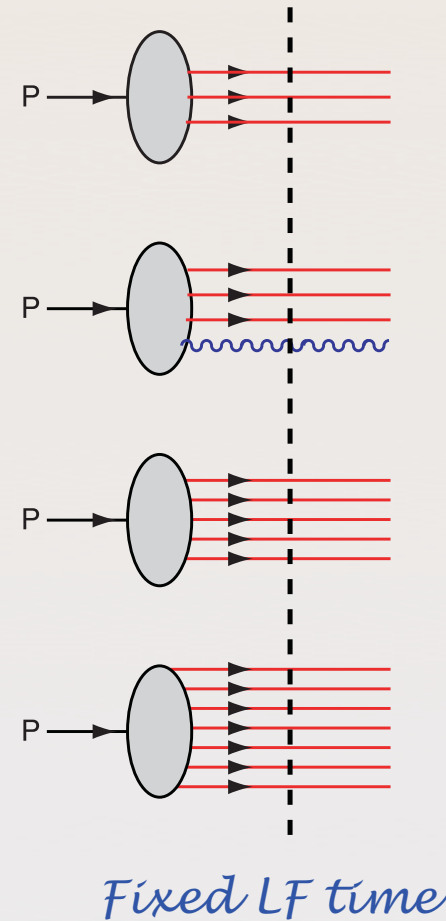
are boost invariant.

$$\sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$

*Intrinsic heavy quarks*  
 **$c(x), b(x)$  at high  $x$ !**

$$\bar{s}(x) \neq s(x)$$

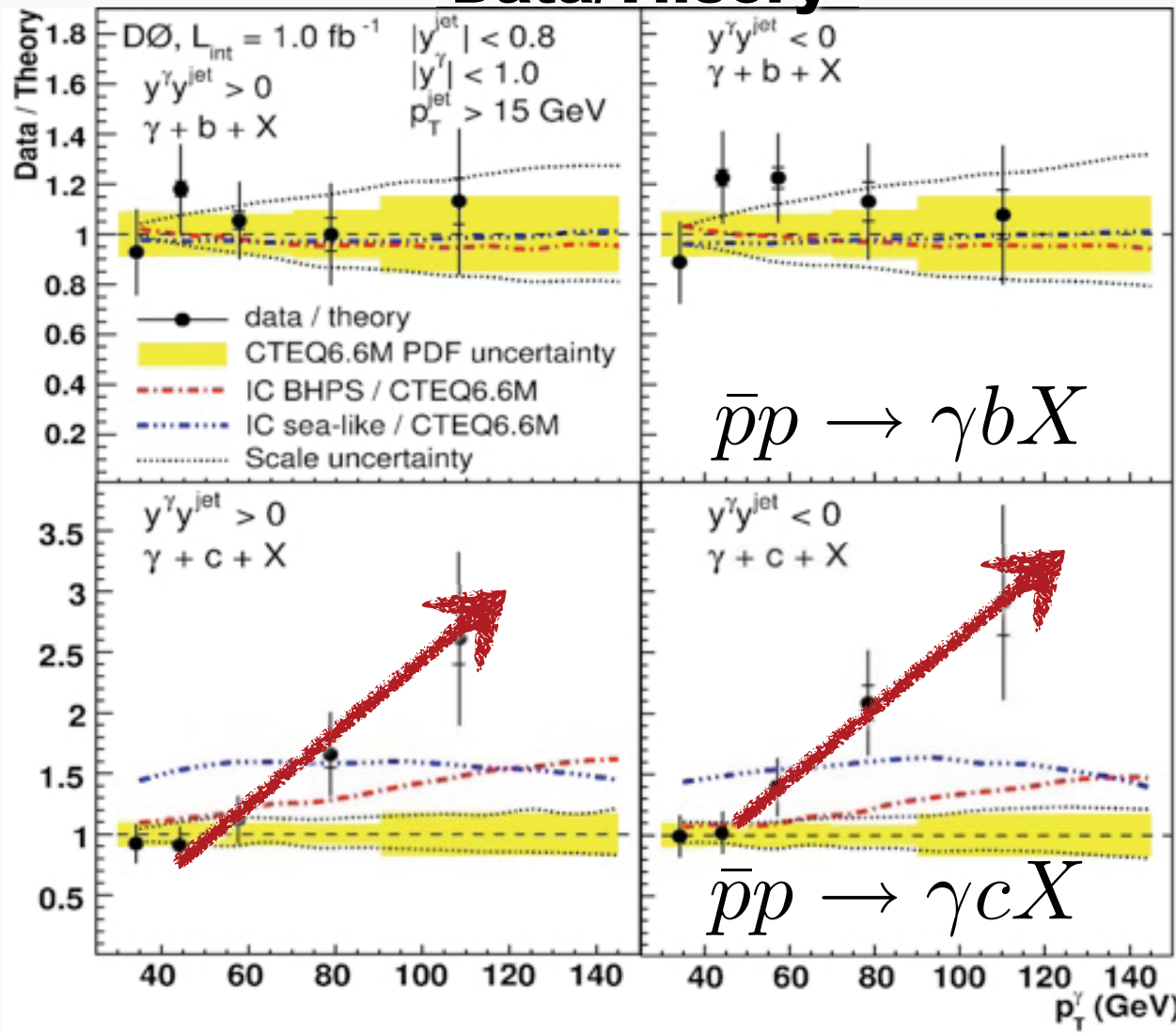
$$\bar{u}(x) \neq \bar{d}(x)$$



Mueller: gluon Fock states  $\rightarrow$  BFKL Pomeron *Hidden Color*

Measurement of  $\gamma + b + X$  and  $\gamma + c + X$  Production Cross Sections  
in  $p\bar{p}$  Collisions at  $\sqrt{s} = 1.96$  TeV

**Data/Theory**



$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

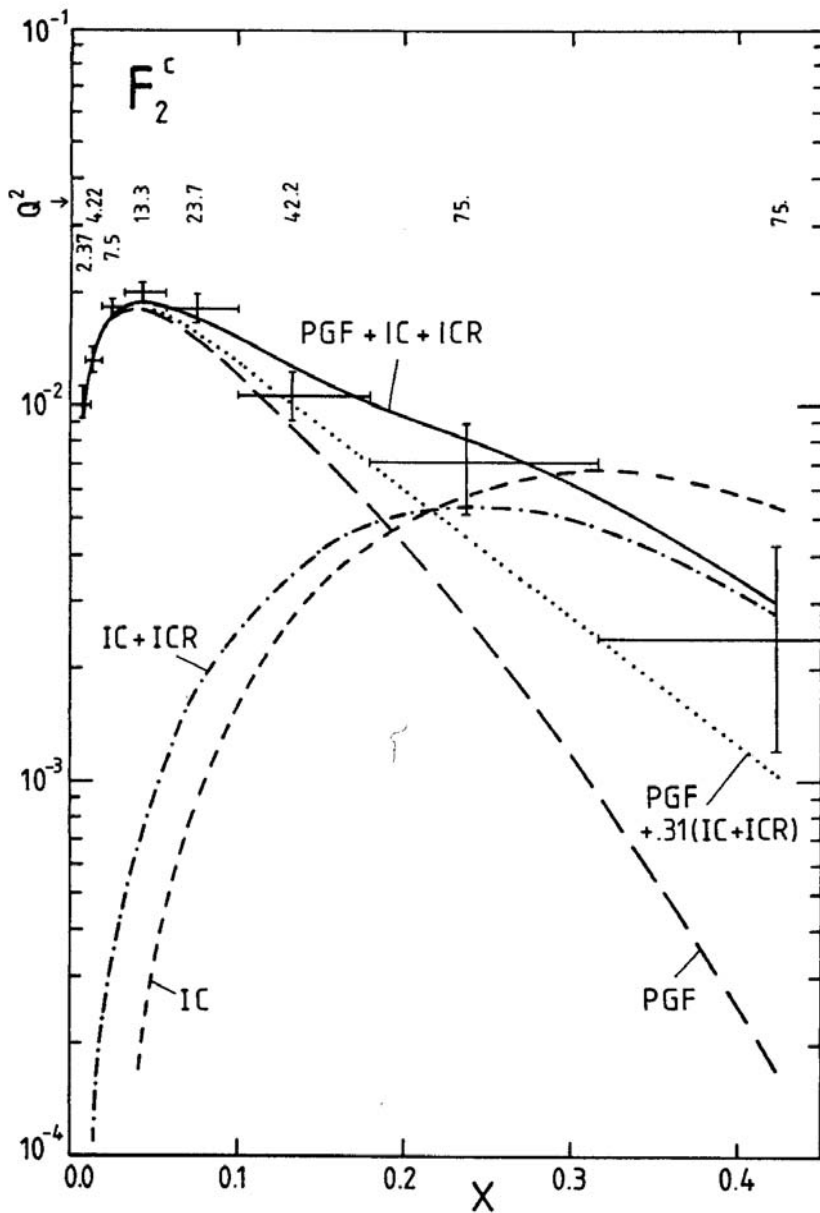
**Ratio  
insensitive to  
gluon PDF,  
scales**

**Signal for  
significant IC  
at  $x > 0.1$**

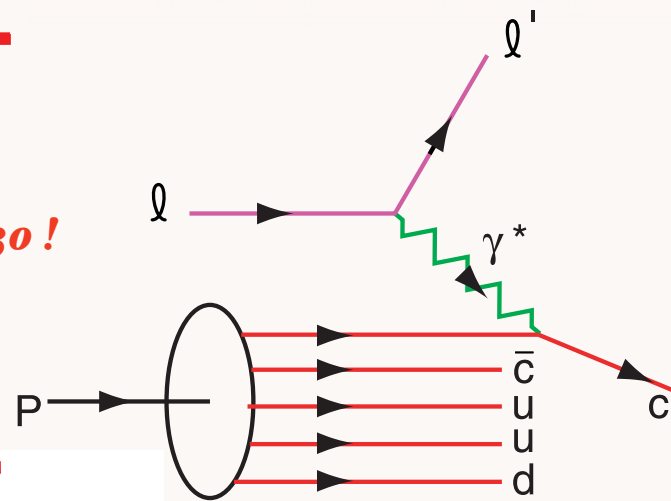
# Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV  $\mu^+$  - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm  
*Never been checked!*

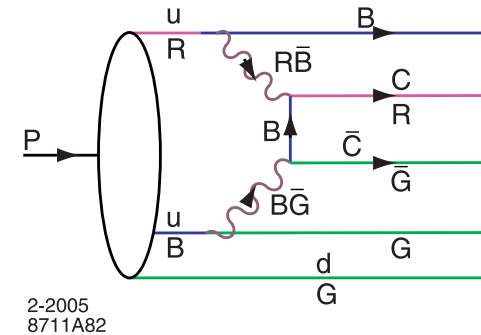


*factor of 30!*



***DGLAP / Photon-Gluon Fusion: factor of 30 too small***

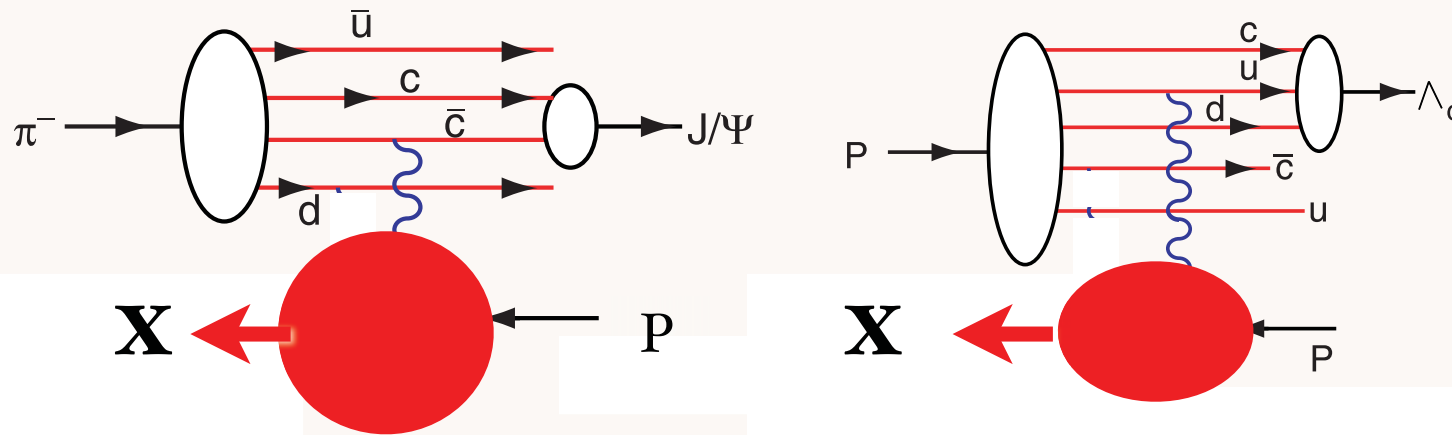
# Intrinsic Heavy-Quark Fock States



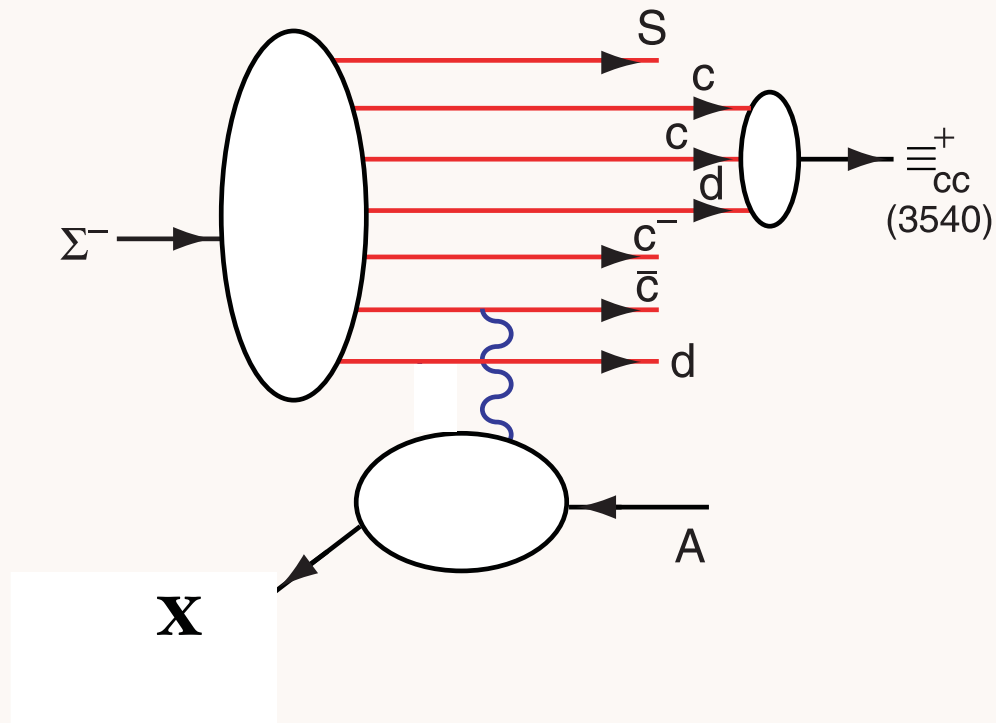
- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability  $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$   $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$   $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

- EMC data:  $c(x, Q^2) > 30 \times \text{DGLAP}$   
 $Q^2 = 75 \text{ GeV}^2, x = 0.42$
- High  $x_F$   $pp \rightarrow J/\psi X$
- High  $x_F$   $pp \rightarrow J/\psi J/\psi X$
- High  $x_F$   $pp \rightarrow \Lambda_c X$            ISR
- High  $x_F$   $pp \rightarrow \Lambda_b X$            ISR
- High  $x_F$   $pp \rightarrow \Xi(ccd) X$  (SELEX)

# Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$



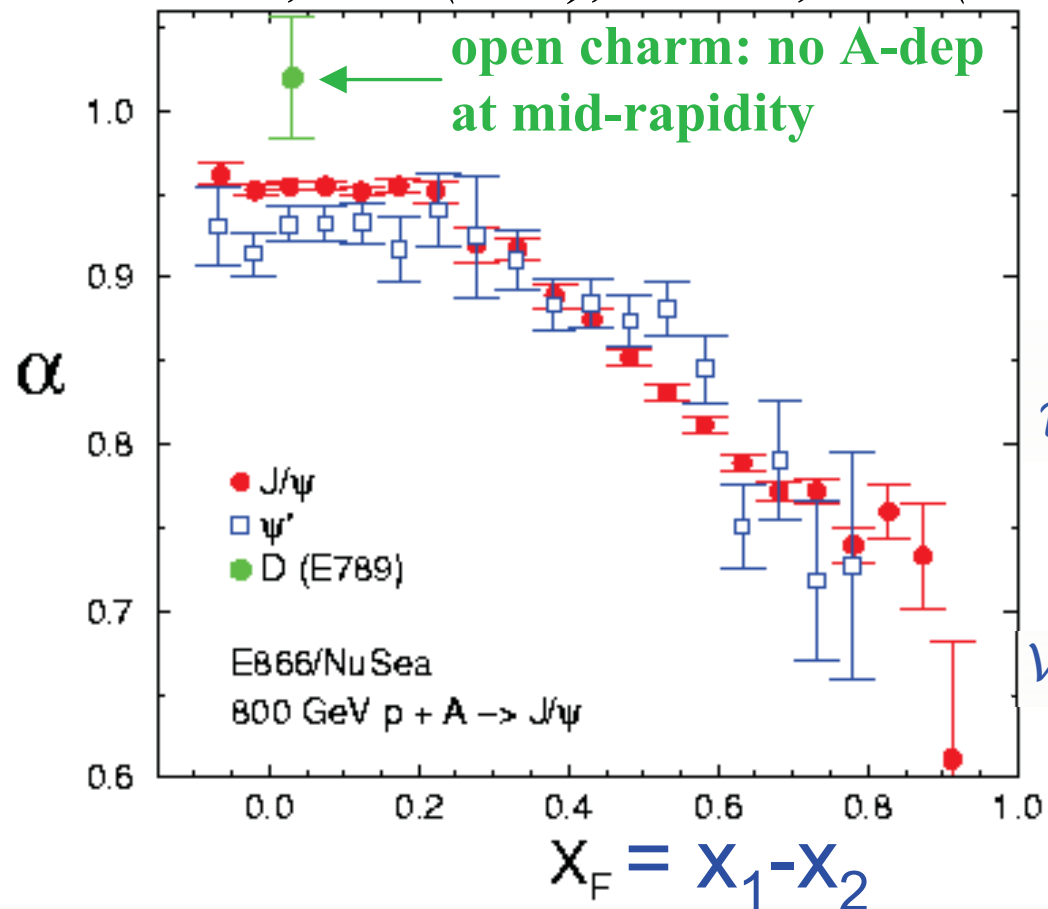
## *Production of a Double-Charm Baryon*

**SELEX high  $x_F$**        $\langle x_F \rangle = 0.33$



800 GeV p-A (FNAL)  $\sigma_A = \sigma_p * A^\alpha$   
*PRL 84, 3256 (2000); PRL 72, 2542 (1994)*

$$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$$



*Remarkably Strong Nuclear Dependence for Fast Charmonium*

*Violation of PQCD Factorization!*

Violation of factorization in charm hadroproduction.

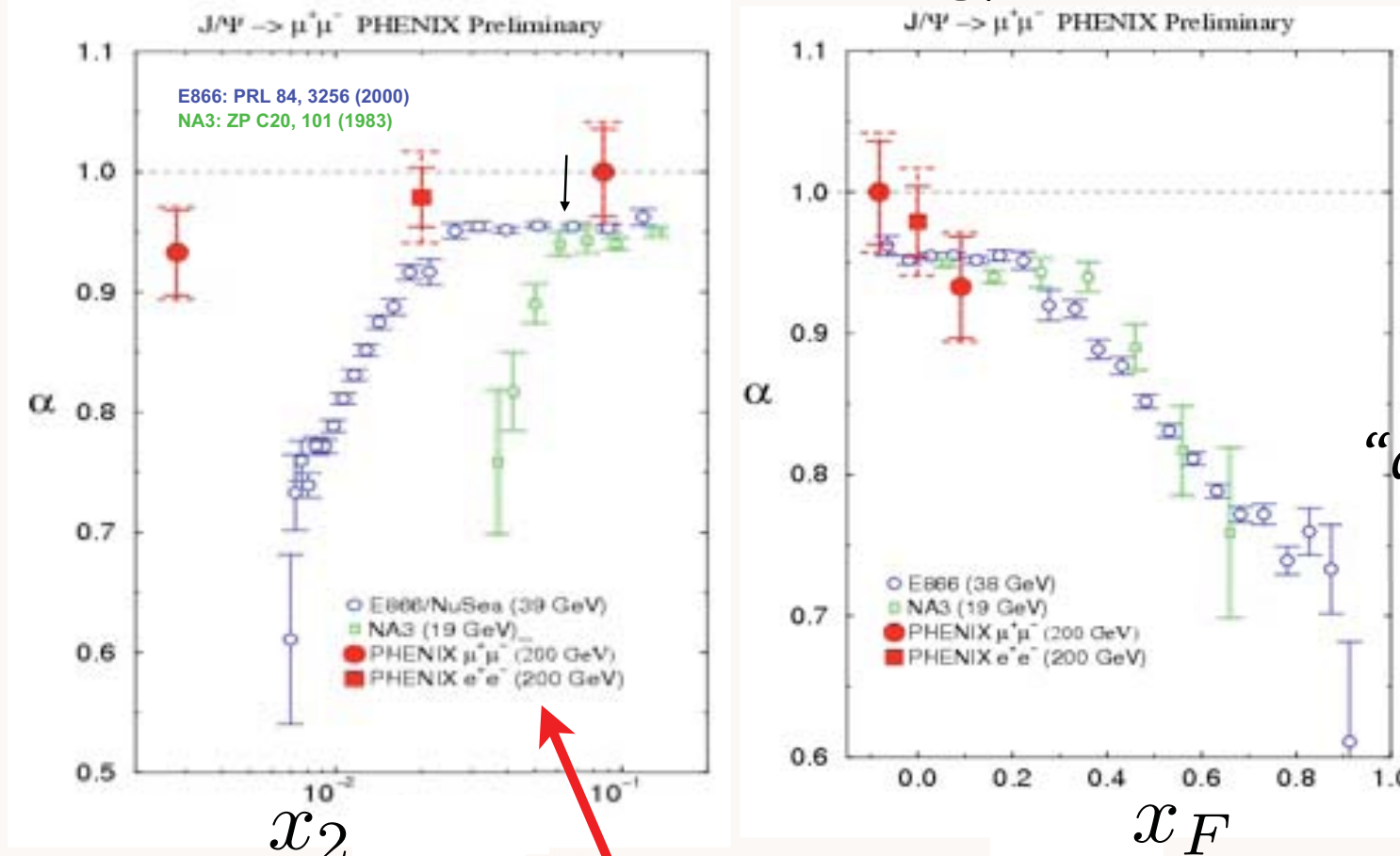
[P. Hoyer](#), [M. Vanttinen \(Helsinki U.\)](#), [U. Sukhatme \(Illinois U., Chicago\)](#). HU-TFT-90-14, May 1990. 7pp.

Published in Phys.Lett.B246:217-220,1990

# J/ψ nuclear dependence vrs rapidity, $x_{Au}$ , $x_F$

M.Leitch

## PHENIX compared to lower energy measurements



Huge  
"absorption"  
effect

Klein, Vogt, PRL 91:142301, 2003  
Kopeliovich, NP A696:669, 2001

Violates PQCD  
factorization!

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

Trieste ICTP  
May 27, 2010

AdS/QCD and Hadronic Physics

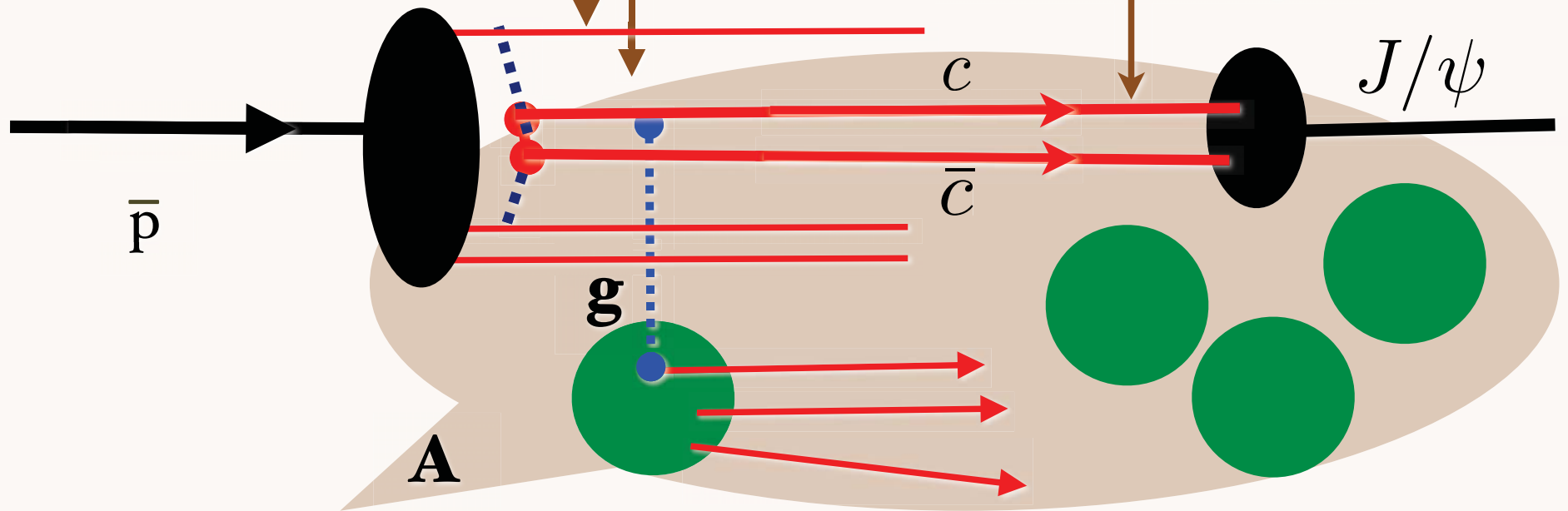
Stan Brodsky  
SLAC-CP3

*Color-Opaque IC Fock state  
interacts on nuclear front surface*

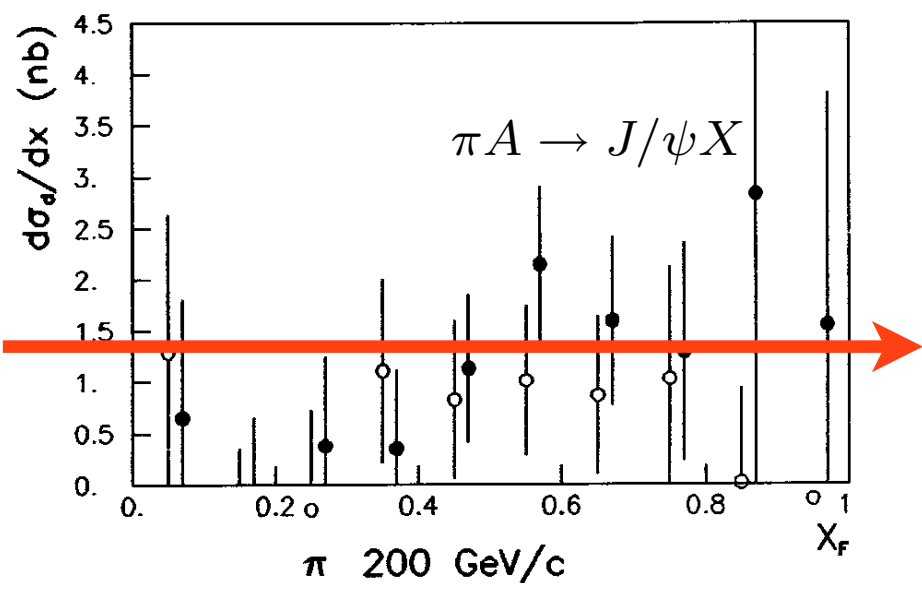
*Scattering on front-face nucleon produces color-singlet  $c\bar{c}$  pair*

*Octet-Octet IC Fock State*

*No absorption of  
small color-singlet*

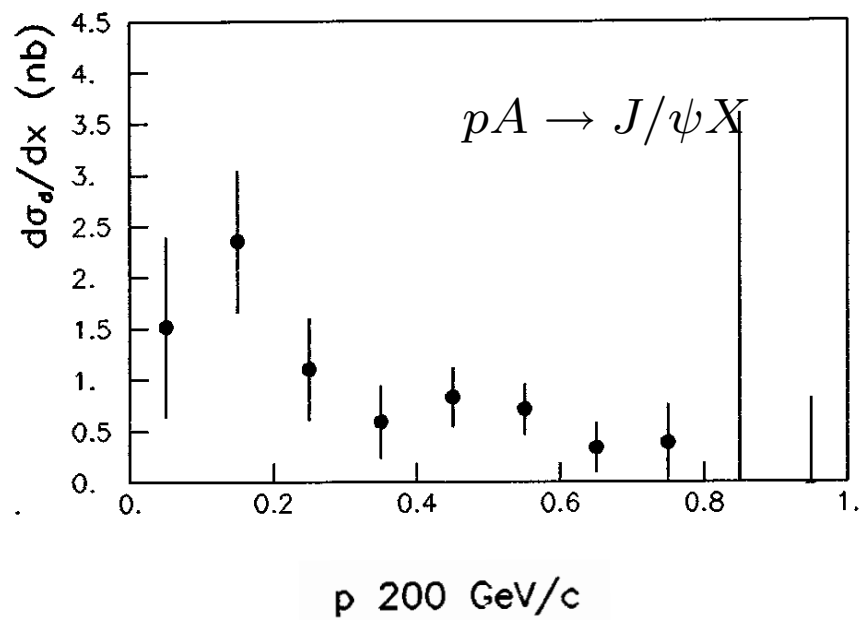


$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$



$A^{2/3}$  component

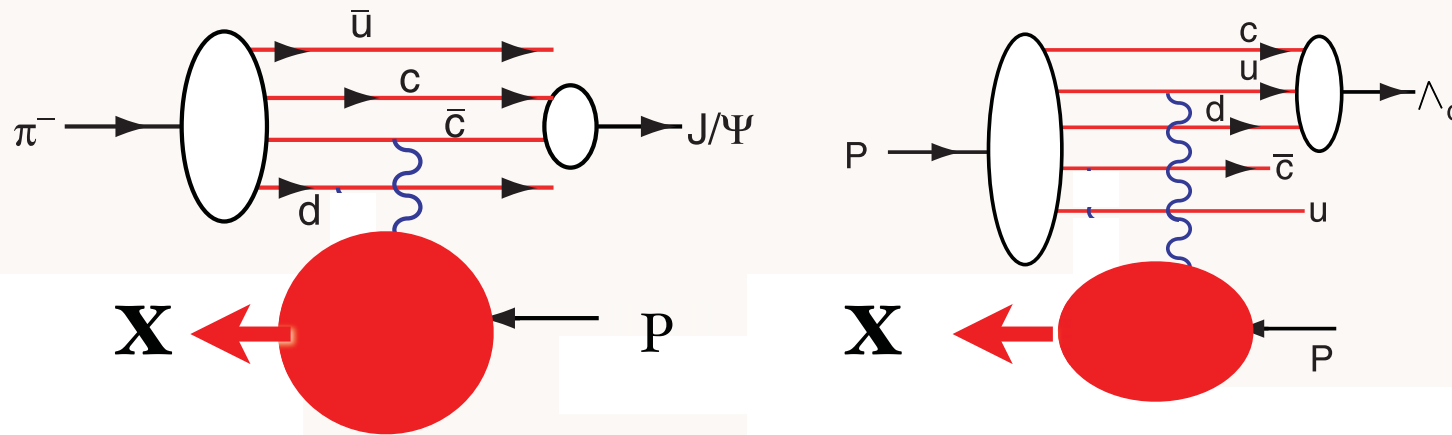
J. Badier et al, NA3



$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

**Excess beyond conventional PQCD subprocesses**

# Leading Hadron Production from Intrinsic Charm



Coalescence of Comoving Charm and Valence Quarks  
Produce  $J/\psi$ ,  $\Lambda_c$  and other Charm Hadrons at High  $x_F$

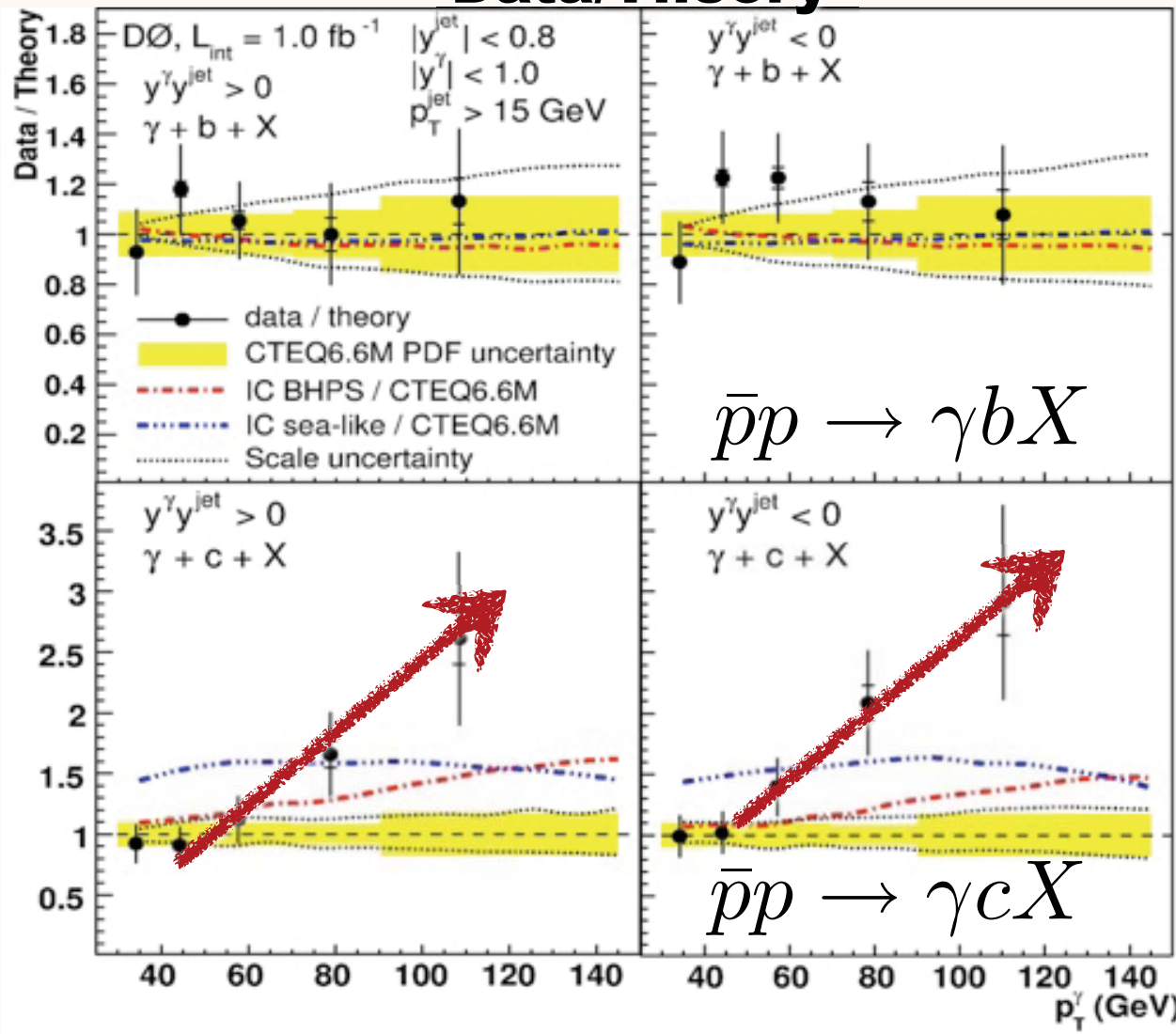
- IC Explains Anomalous  $\alpha(x_F)$  not  $\alpha(x_2)$  dependence of  $pA \rightarrow J/\psi X$   
(Mueller, Gunion, Tang, SJB)
- Color Octet IC Explains  $A^{2/3}$  behavior at high  $x_F$  (NA3, Fermilab) *Color Opacity*  
(Kopeliovitch, Schmidt, Soffer, SJB)
- IC Explains  $J/\psi \rightarrow \rho\pi$  puzzle  
(Karliner, SJB)
- IC leads to new effects in  $B$  decay  
(Gardner, SJB)

**Higgs production at  $x_F = 0.8$  !**

Goldhaber, Kopeliovich,  
Schmidt, Soffer, sjb

Measurement of  $\gamma + b + X$  and  $\gamma + c + X$  Production Cross Sections  
in  $p\bar{p}$  Collisions at  $\sqrt{s} = 1.96$  TeV

**Data/Theory**

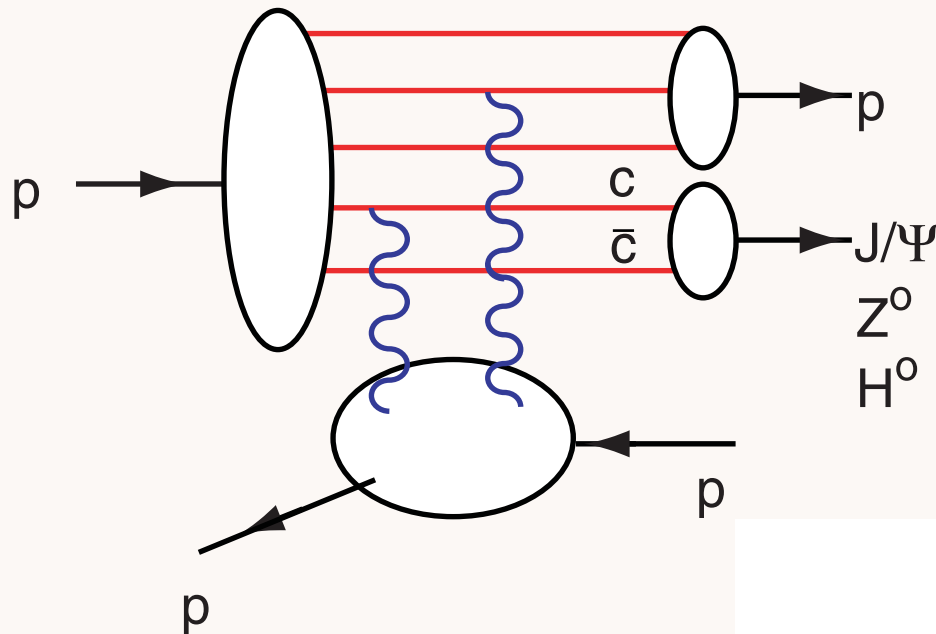


$$\frac{\Delta\sigma(\bar{p}p \rightarrow \gamma c X)}{\Delta\sigma(\bar{p}p \rightarrow \gamma b X)}$$

**Ratio  
insensitive to  
gluon PDF,  
scales**

**Signal for  
significant IC  
at  $x > 0.1$**

# Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$p p \rightarrow J/\psi p p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

*Exclusive Diffractive  
High- $x_F$  Higgs Production!*

Kopeliovich, Schmidt,  
Soffer, sjb

Intrinsic  $c\bar{c}$  pair formed in color octet  $8_C$  in proton wavefunction Large Color Dipole

Collision produces color-singlet  $J/\psi$  through color exchange

**RHIC Experiment**



# Hadron Distribution Amplitudes

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

$\sum_i x_i = 1$

*Lepage, sjb*  
 $k_\perp^2 < Q^2$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE,

- Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

*Lepage, sjb*  
*Efremov, Radyushkin*  
*Sachrajda, Frishman Lepage, sjb*  
*Braun, Gardi*

# Light-Front formalism links dynamics to spectroscopy

Physical gauge:  $A^+ = 0$

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

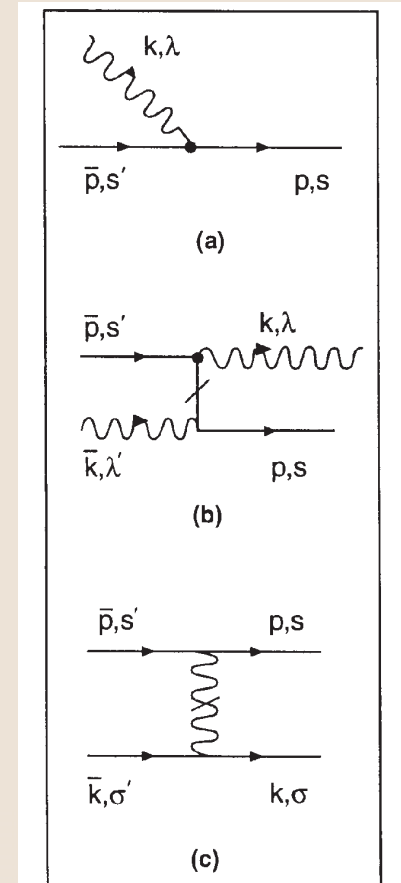
Heisenberg Matrix  
Formulation

$$H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

$H_{LF}^{int}$ : Matrix in Fock Space

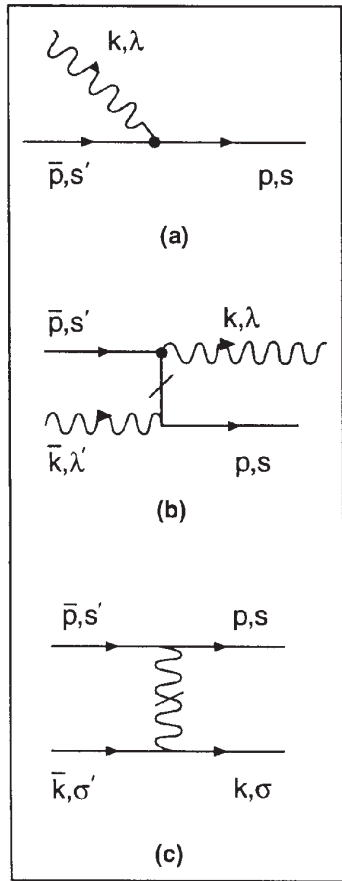
$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions



$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

## Heisenberg Matrix Formulation



| n  | Sector  | 1           | 2  | 3             | 4                       | 5   | 6              | 7                         | 8                                   | 9    | 10              | 11                         | 12                                    | 13  |
|----|---|-------------|----|---------------|-------------------------|-----|----------------|---------------------------|-------------------------------------|------|-----------------|----------------------------|---------------------------------------|---|
|    |   | q $\bar{q}$ | gg | q $\bar{q}$ g | q $\bar{q}$ q $\bar{q}$ | ggg | q $\bar{q}$ gg | q $\bar{q}$ q $\bar{q}$ g | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ | gggg | q $\bar{q}$ ggg | q $\bar{q}$ q $\bar{q}$ gg | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ |
| 1  | q $\bar{q}$                                     |             |    |               |                         | .   |                | .                         | .                                   | .    | .               | .                          | .                                     | .   |
| 2  | gg  |             |    |               | .                       |     |                | .                         | .                                   |      | .               | .                          | .                                     | .   |
| 3  | q $\bar{q}$ g                                   |             |    |               |                         |     |                |                           | .                                   | .    |                 | .                          | .                                     | .   |
| 4  | q $\bar{q}$ q $\bar{q}$                         |             | .  |               |                         | .   |                |                           | .                                   | .    | .               |                            | .                                     | .   |
| 5  | ggg   | .           |    |               | .                       |     |                | .                         | .                                   |      |                 | .                          | .                                     | .   |
| 6  | q $\bar{q}$ gg                                  |             |    |               |                         |     |                |                           | .                                   |      |                 |                            | .                                     | .   |
| 7  | q $\bar{q}$ q $\bar{q}$ g                       | .           | .  |               |                         | .   |                |                           |                                     | .    |                 |                            |                                       | .   |
| 8  | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$             | .           | .  | .             |                         | .   | .              |                           |                                     | .    |                 |                            |                                       | .   |
| 9  | gggg  | .           |    | .             | .                       |     |                | .                         | .                                   |      |                 | .                          | .                                     | .   |
| 10 | q $\bar{q}$ ggg                                 | .           | .  |               | .                       |     |                |                           | .                                   |      |                 |                            | .                                     | .   |
| 11 | q $\bar{q}$ q $\bar{q}$ gg                      | .           | .  | .             |                         | .   |                |                           |                                     | .    |                 |                            |                                       | .   |
| 12 | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g           | .           | .  | .             | .                       | .   | .              |                           |                                     | .    |                 |                            |                                       |   |
| 13 | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ | .           | .  | .             | .                       | .   | .              | .                         |                                     | .    | .               | .                          |                                       |   |

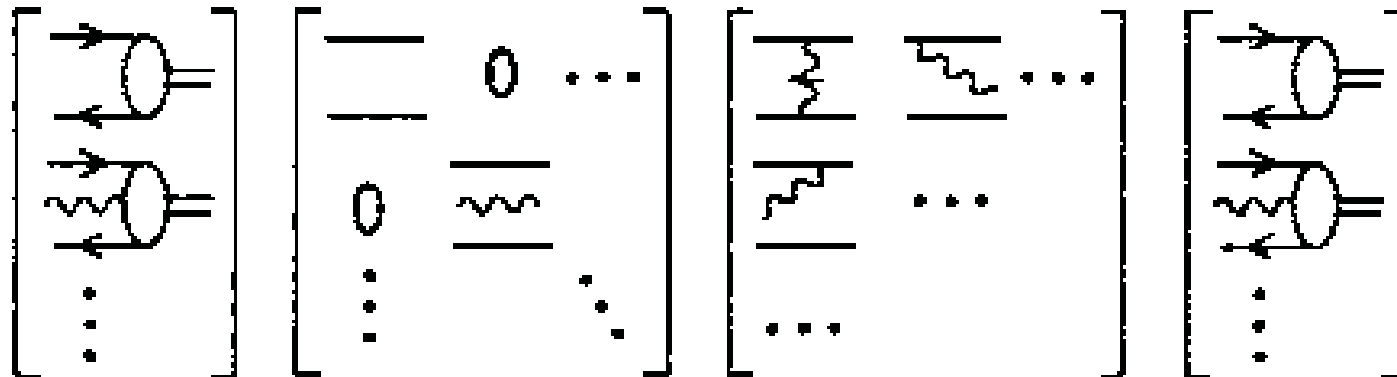
Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

*DLCQ: Frame-independent, No fermion doubling; Minkowski Space*

DLCQ: Periodic BC in  $x^-$ . Discrete  $k^+$ ; frame-independent truncation

# LIGHT-FRONT SCHRÖDINGER EQUATION

$$\left( M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{q}g \rangle & \cdots \\ \langle q\bar{q}g | V | q\bar{q} \rangle & \langle q\bar{q}g | V | q\bar{q}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$



$$A^+ = 0$$

G.P. Lepage, sjb

# *Goal: an analytic first approximation to QCD*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

**de Teramond, Deur, Shrock, Roberts, Tandy**

# Light-Front Wavefunctions

Dirac's Front Form: Fixed  $\tau = t + z/c$

$$\Psi(x, k_{\perp}) \quad x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of  $P^{\mu}$

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

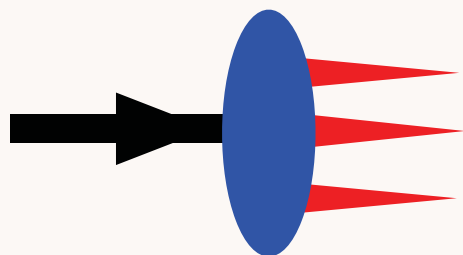
*Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space*

# Light-Front Holography and Non-Perturbative QCD

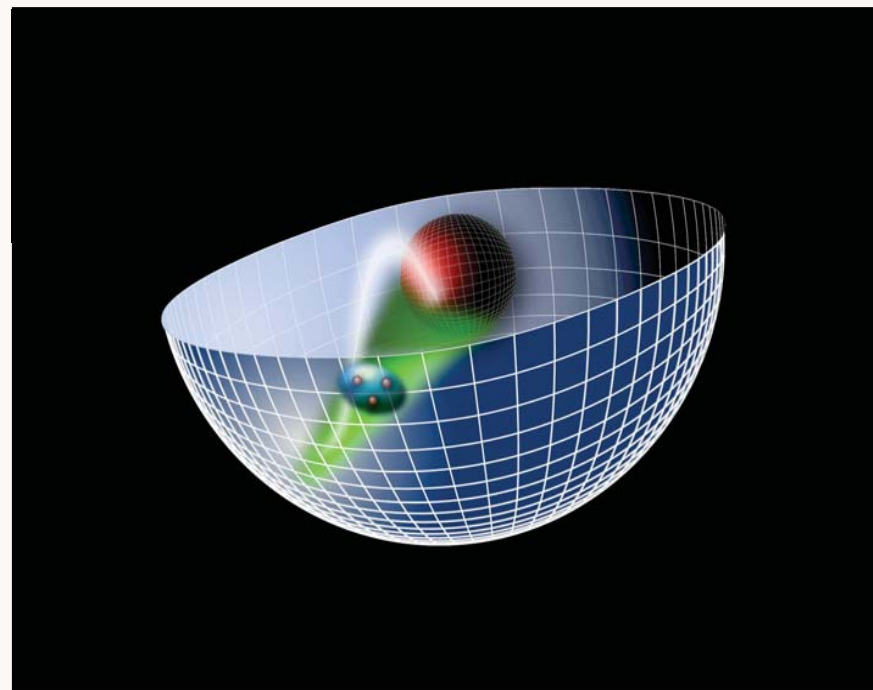
**Goal:**

**Use AdS/QCD duality to construct  
a first approximation to QCD**

Hadron Spectrum  
Light-Front Wavefunctions,  
Running coupling in IR



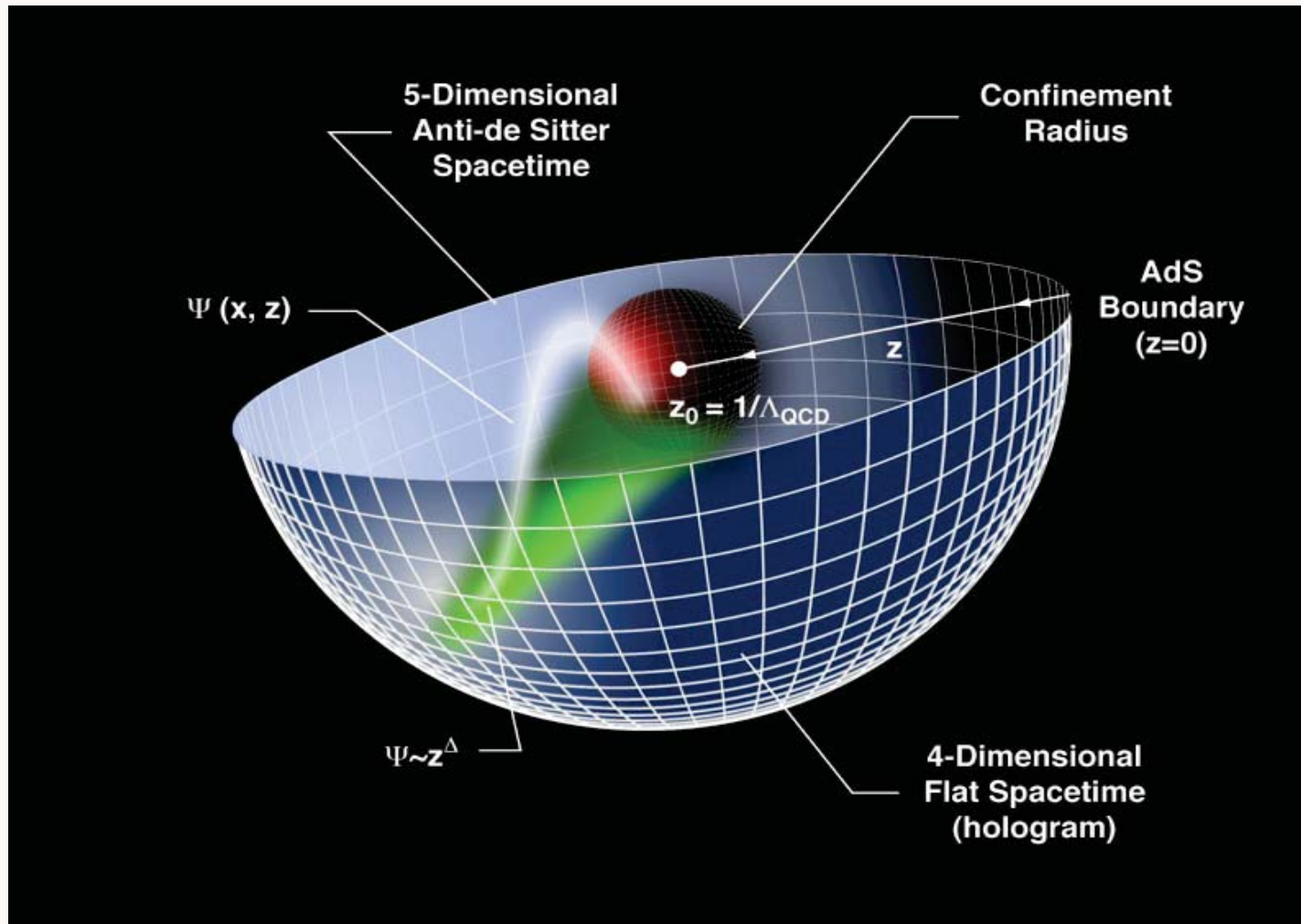
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



**in collaboration with  
Guy de Teramond and Alexandre Deur**

**Central problem for strongly-coupled gauge theories**

# Applications of AdS/CFT to QCD



*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

**in collaboration with Guy de Teramond**

Trieste ICTP  
May 27, 2010

AdS/QCD and Hadronic Physics

31

Stan Brodsky  
SLAC-CP3



*Conformal Theories are invariant under the Poincare and conformal transformations with*

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$

*the generators of  $SO(4,2)$*

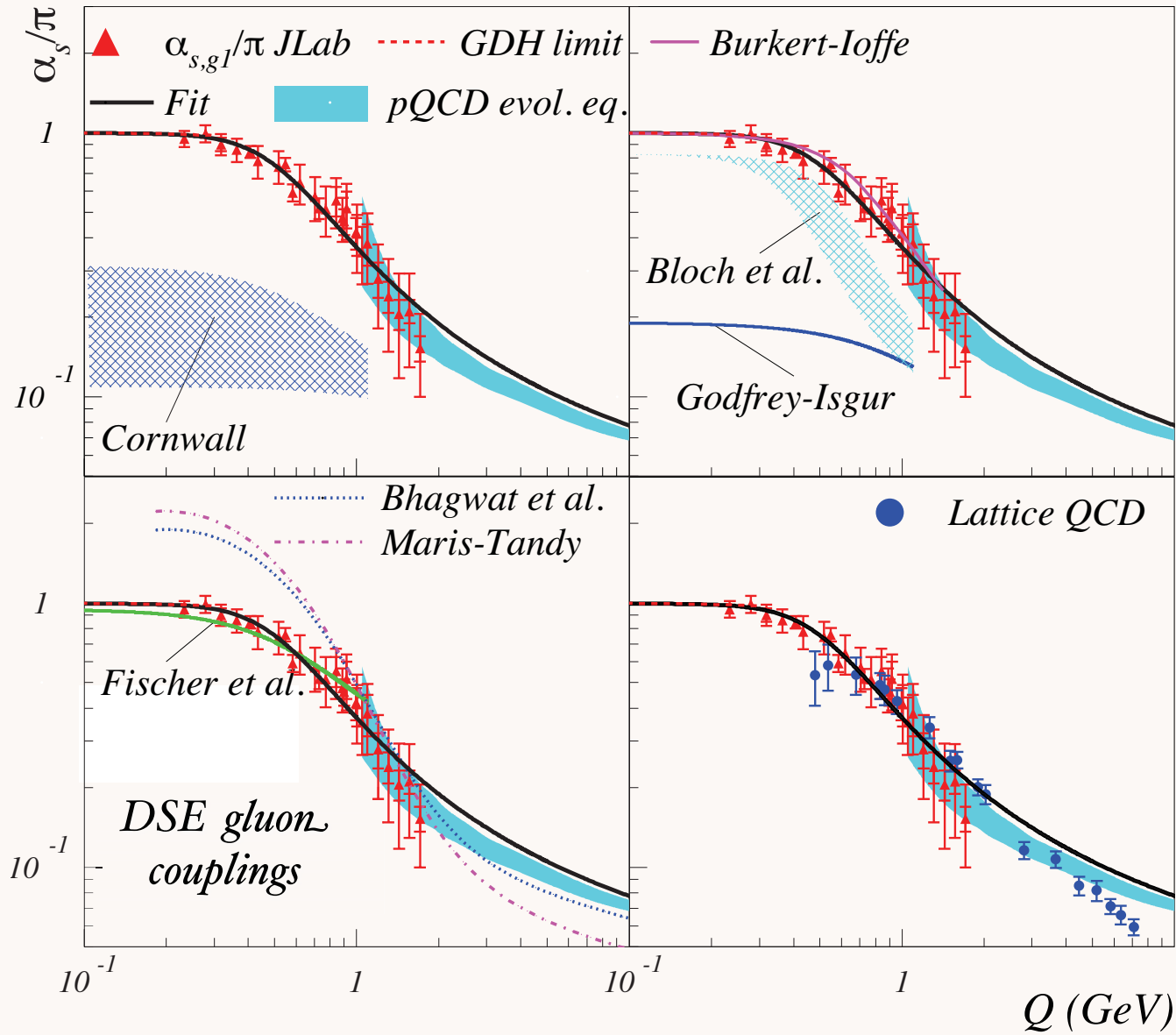
**$SO(4,2)$  has a mathematical representation on  $AdS_5$**

# *AdS/CFT*: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map  $AdS_5 \times S^5$  to conformal  $N=4$  SUSY

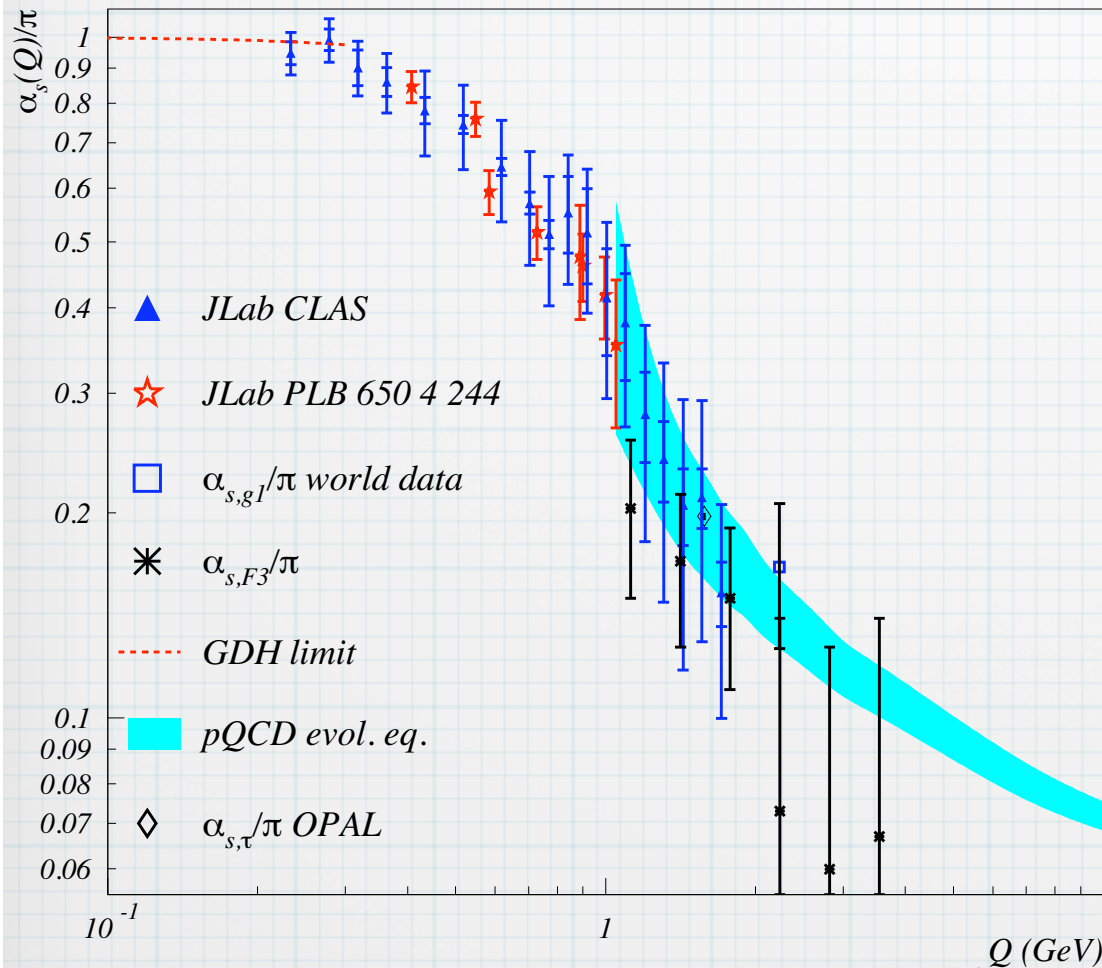
- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window**:  $\alpha_s(Q^2) \simeq \text{const}$  at small  $Q^2$
- Use mathematical mapping of the conformal group  $SO(4,2)$  to  $AdS_5$  space



# Nearly conformal QCD?

Define  $\alpha_s$  from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left( g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s g_1}{\pi} \right)$$



$gl$  = spin dependent structure function

Recent JLab data from EQ1(2008), CLAS, and Hall A

$\alpha_s$  runs only modestly at small  $Q^2$

Gribov

Fig. from 08034119, Duer et al.

# Maximal Wavelength of Confined Fields

$$(x - y)^2 < \Lambda_{QCD}^{-2}$$

- **Colored fields confined to finite domain**
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

**Shrock, sjb**

*Quark and Gluon vacuum polarization insertions  
decouple: IR fixed Point*

J. D. Bjorken,  
SLAC-PUB 1053  
Cargese Lectures 1989

***A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).***

**Trieste ICTP  
May 27, 2010**


**AdS/QCD and Hadronic Physics**

**Stan Brodsky  
SLAC-CP3**

## Scale Transformations

- Isomorphism of  $SO(4, 2)$  of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure* 

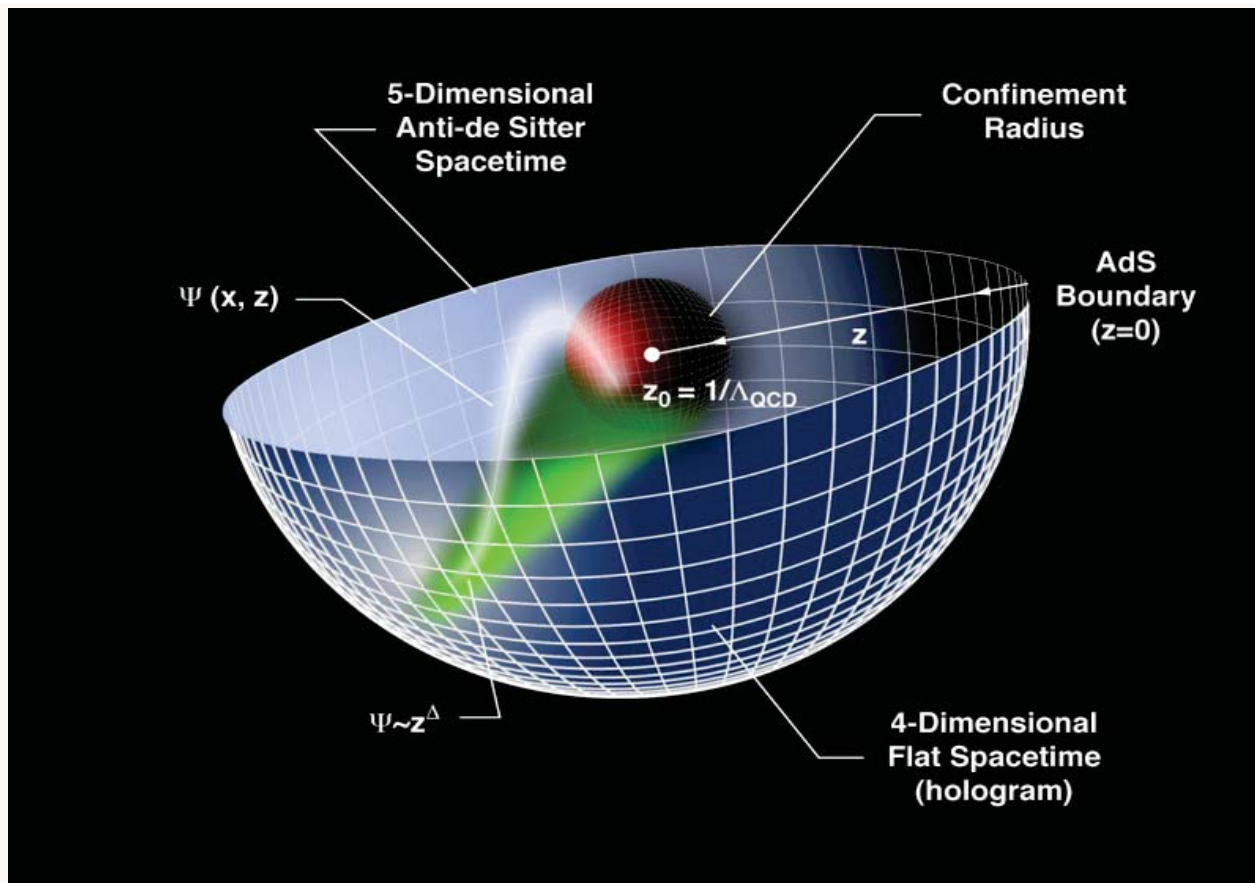
$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate  $z$ .

- AdS mode in  $z$  is the extension of the hadron wf into the fifth dimension.
- Different values of  $z$  correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$ : invariant separation between quarks

- The AdS boundary at  $z \rightarrow 0$  correspond to the  $Q \rightarrow \infty$ , UV zero separation limit.



8-2007  
8685A14

- Truncated AdS/CFT (Hard-Wall) model: cut-off at  $z_0 = 1/\Lambda_{\text{QCD}}$  breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) [Polchinski and Strassler \(2001\)](#).
- Smooth cutoff: introduction of a background dilaton field  $\varphi(z)$  – usual linear Regge dependence can be obtained (Soft-Wall Model) [Karch, Katz, Son and Stephanov \(2006\)](#).

# Bosonic Solutions: Hard Wall Model

- Conformal metric:  $ds^2 = g_{\ell m} dx^\ell dx^m$ .  $x^\ell = (x^\mu, z)$ ,  $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$ .
- Action for massive scalar modes on  $\text{AdS}_{d+1}$ :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along  $x^\mu$ -coordinates,  $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$ ,  $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution:  $\Phi(z) \rightarrow z^\Delta$  as  $z \rightarrow 0$ ,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$



$$\text{Let } \Phi(z) = z^{3/2} \phi(z)$$

*AdS Schrodinger Equation for bound state  
of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

**L: light-front orbital angular  
momentum**

*Derived from variation of Action in AdS<sub>5</sub>*

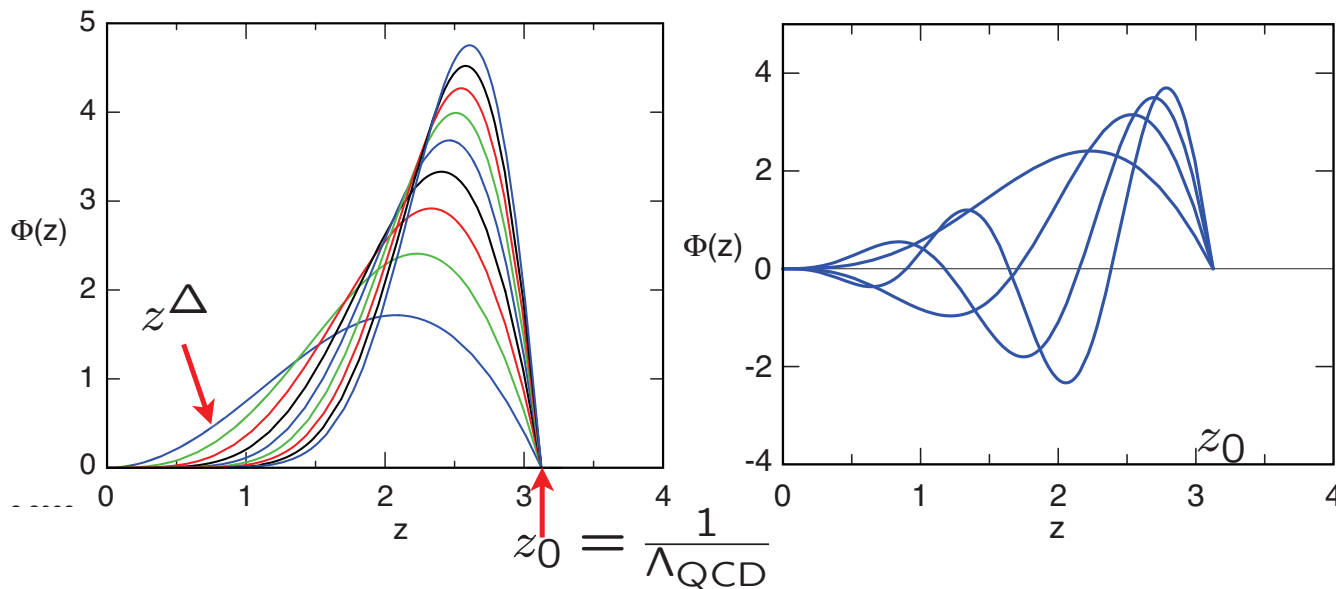
*Hard wall model: truncated space*

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

*Match fall-off at small  $z$  to conformal twist-dimension  
at short distances*

*twist*

- Pseudoscalar mesons:  $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1 \dots \ell_m\}} \psi$  ( $\Phi_\mu = 0$  gauge).  $\Delta = 2 + L$
- 4- $d$  mass spectrum from boundary conditions on the normalizable string modes at  $z = z_0$ ,  $\Phi(x, z_0) = 0$ , given by the zeros of Bessel functions  $\beta_{\alpha,k}$ :  $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes  $\Phi(z)$



$S = 0$  Meson orbital and radial AdS modes for  $\Lambda_{QCD} = 0.32$  GeV.

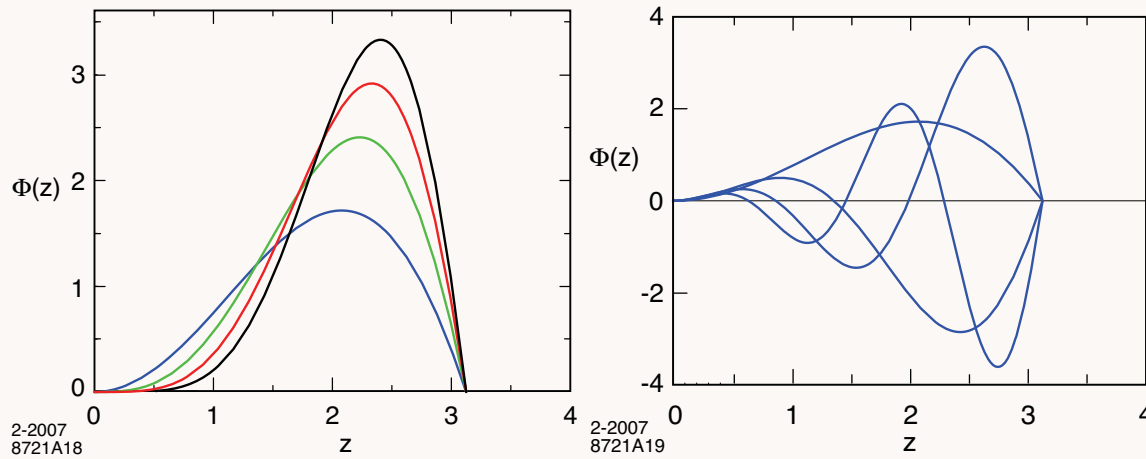


Fig: Orbital and radial AdS modes in the hard wall model for  $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$ .

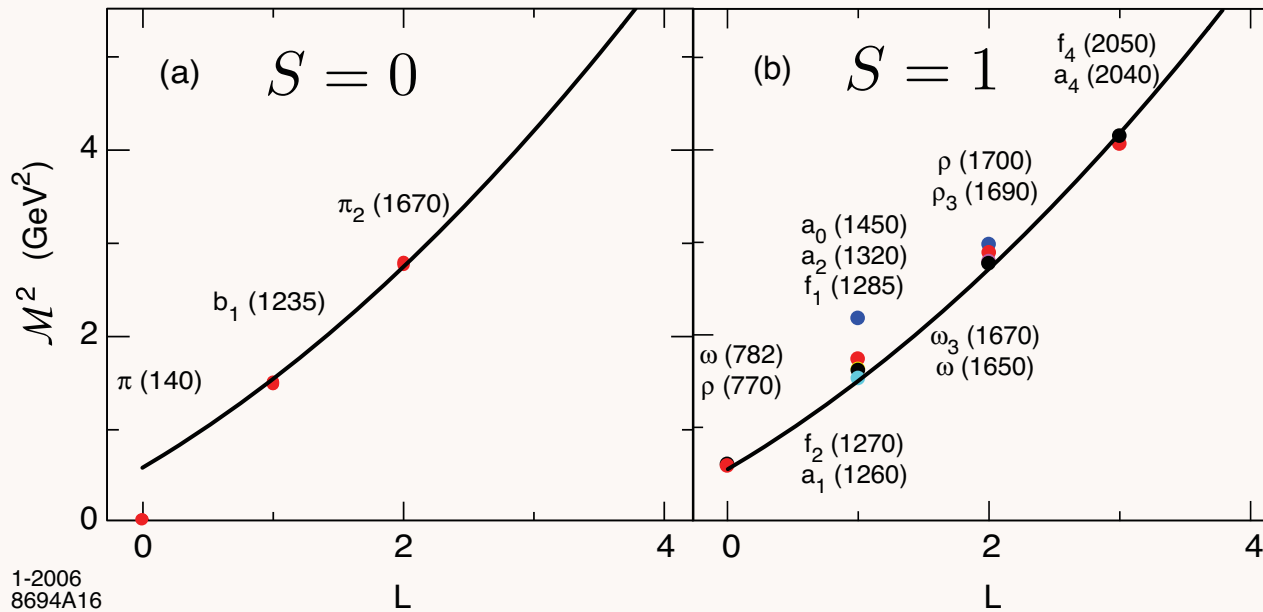


Fig: Light meson and vector meson orbital spectrum  $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$

# Soft-Wall Model

$$S = \int d^4x dz \sqrt{g} e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2$$

**Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field**

Karch, Katz, Son and Stephanov (2006)]

- Equation of motion for scalar field  $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with  $(\mu R)^2 \geq -4$ .

- LH holography requires 'plus dilaton'  $\varphi = +\kappa^2 z^2$ . Lowest possible state  $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

*Massless pion*

*AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action  
Dilaton-Modified AdS<sub>5</sub>*

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

Quark separation increases with  $L$

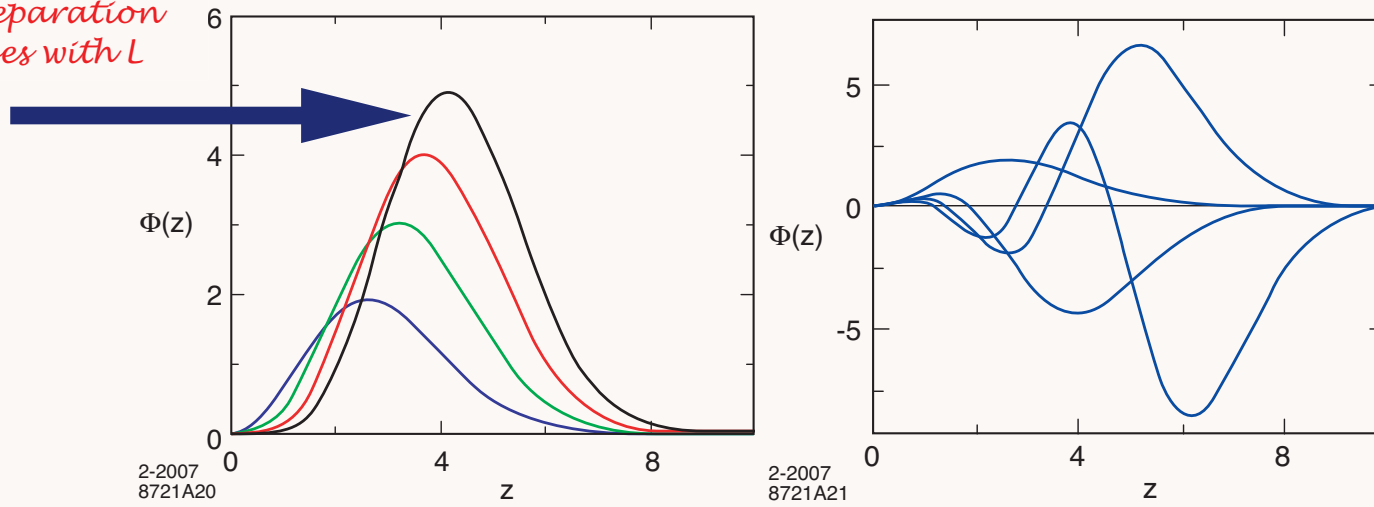


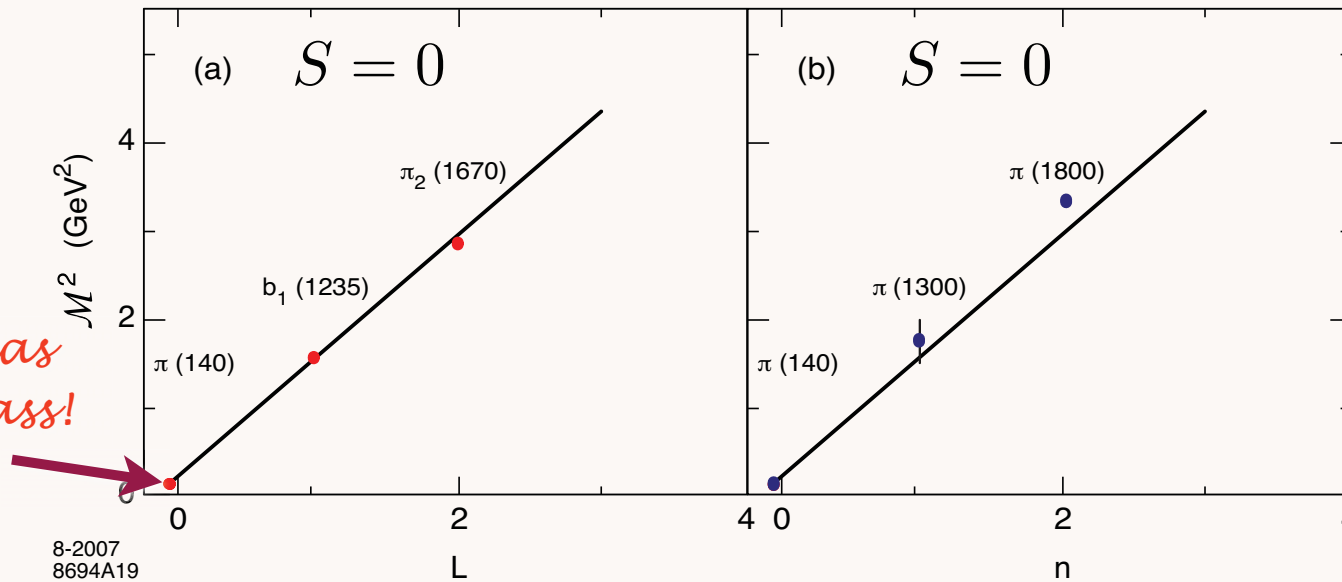
Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

*Soft Wall Model*

**Pion mass automatically zero!**

$$m_q = 0$$

*Pion has zero mass!*



Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

# Higher-Spin Hadrons

- Obtain spin- $J$  mode  $\Phi_{\mu_1 \dots \mu_J}$  with all indices along 3+1 coordinates from  $\Phi$  by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for  $\Phi$

$$\left[ z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution  $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

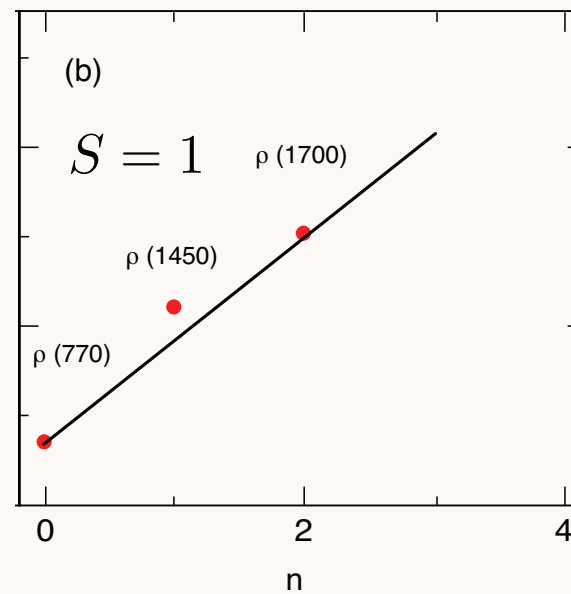
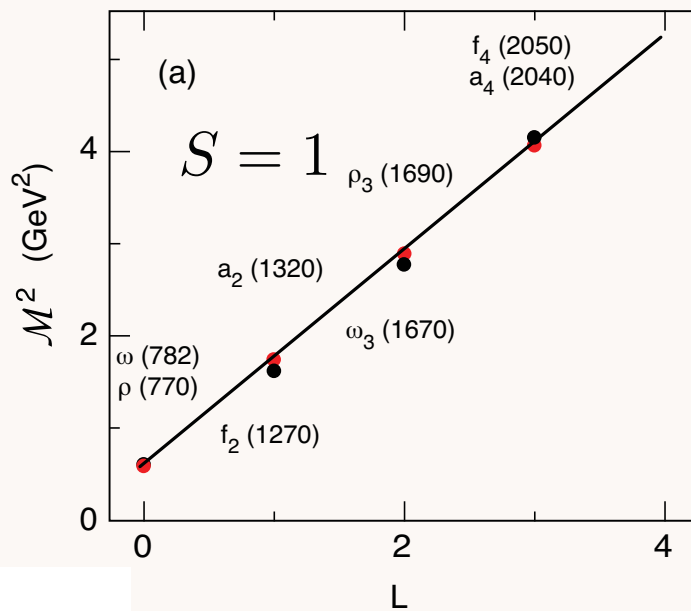
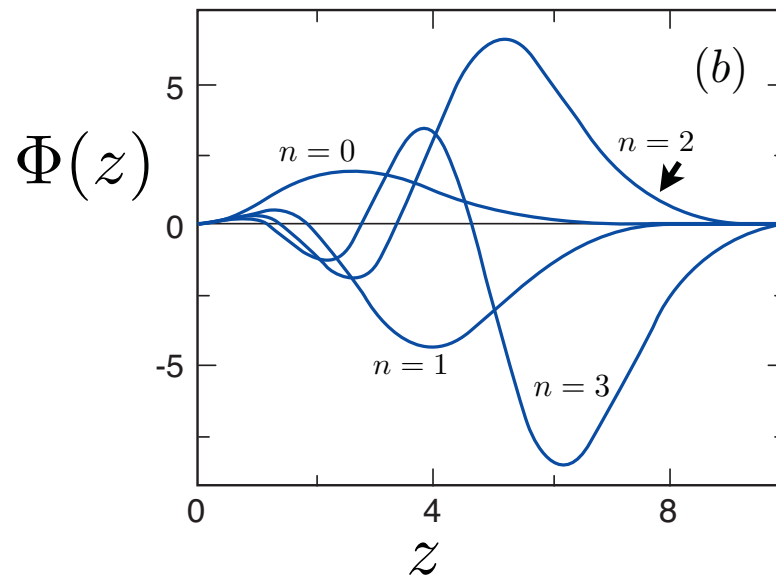
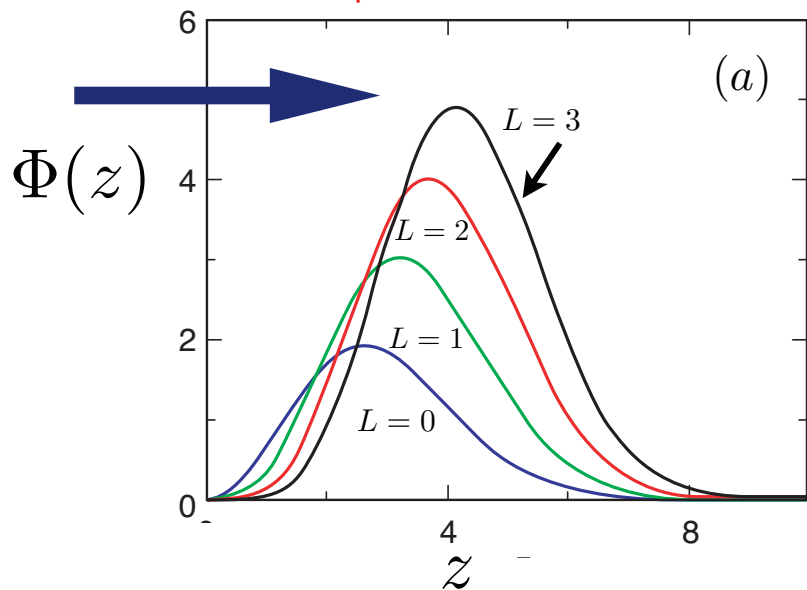
we find the LF wave equation

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$

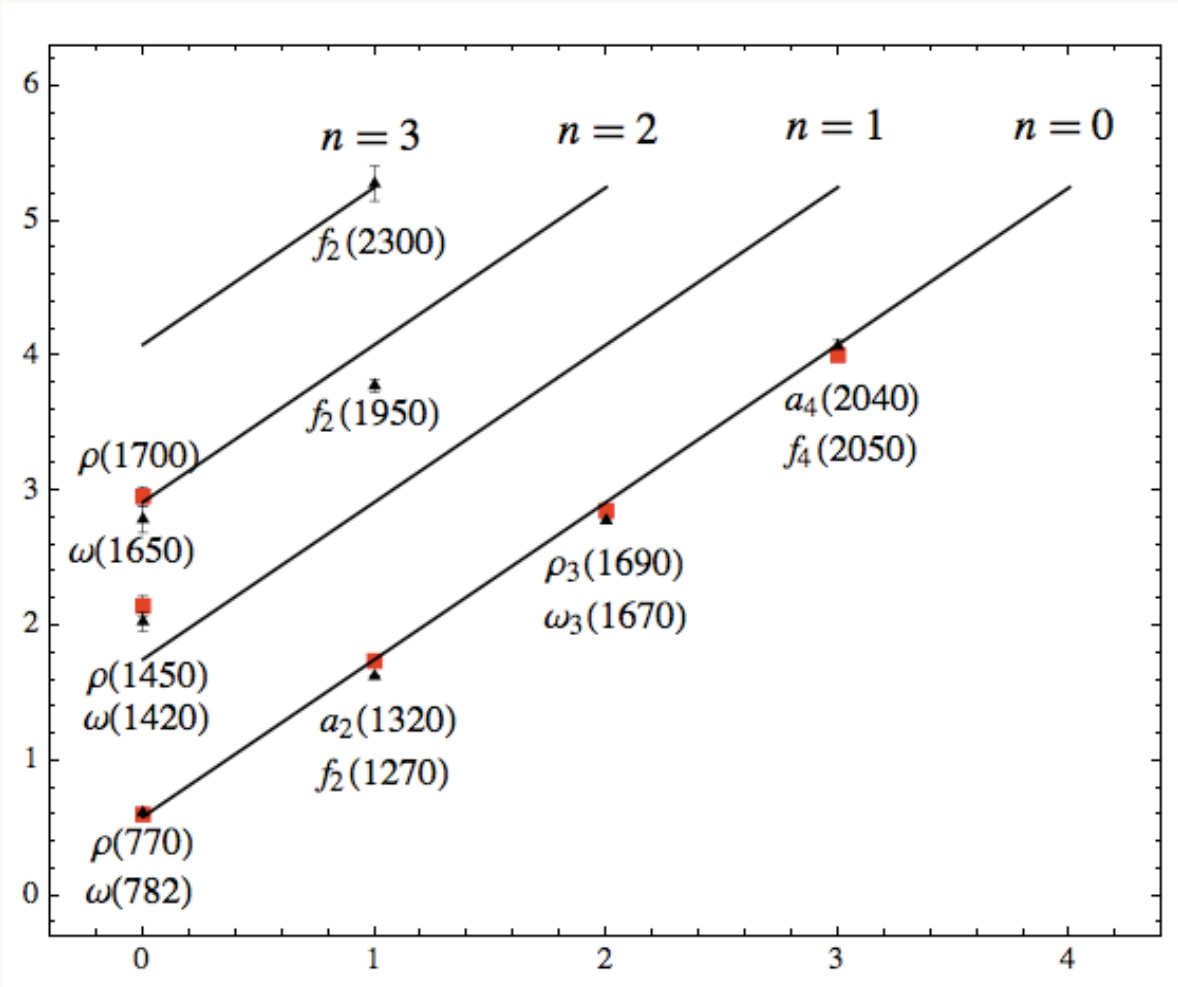


with  $(\mu R)^2 = -(2 - J)^2 + L^2$

Quark separation increases with  $L$





$1^{--}$  $2^{++}$  $3^{--}$  $4^{++}$  $J^{PC}$  $\mathcal{M}^2$  $L$ 

Parent and daughter Regge trajectories for the  $I = 1$   $\rho$ -meson family (red)  
and the  $I = 0$   $\omega$ -meson family (black) for  $\kappa = 0.54$  GeV

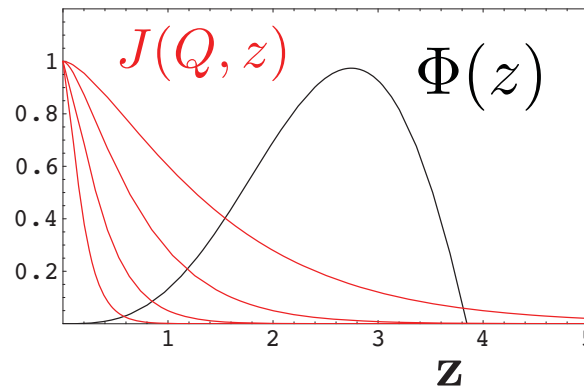
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High  $Q^2$   
from  
small  $z \sim 1/Q$



**Polchinski, Strassler  
de Teramond, sjb**

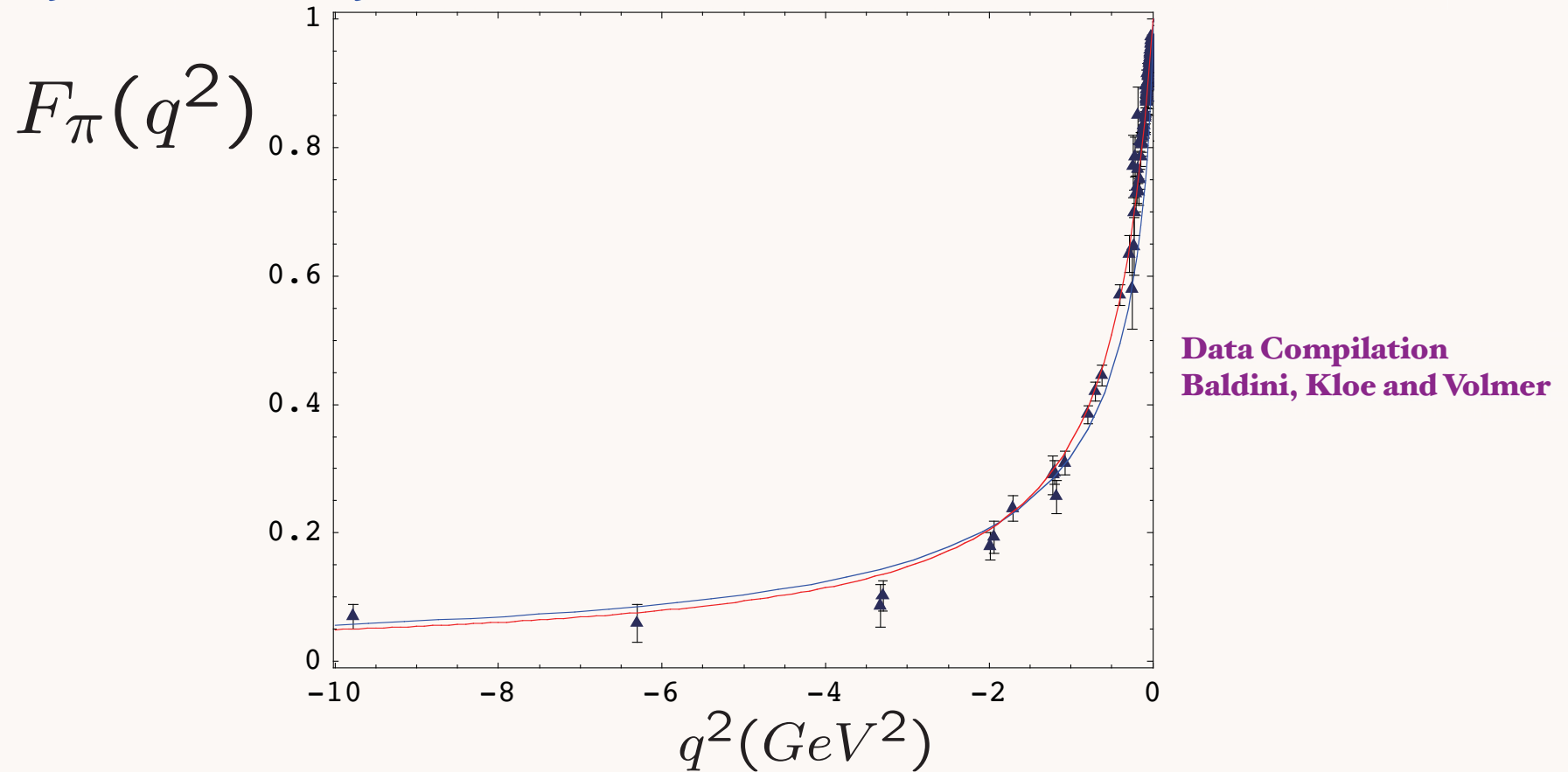
Consider a specific AdS mode  $\Phi^{(n)}$  dual to an  $n$  partonic Fock state  $|n\rangle$ . At small  $z$ ,  $\Phi$  scales as  $\Phi^{(n)} \sim z^{\Delta_n}$ . Thus:

$$F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1},$$

**Dimensional Quark Counting Rules:  
General result from  
AdS/CFT and Conformal Invariance**

where  $\tau = \Delta_n - \sigma_n$ ,  $\sigma_n = \sum_{i=1}^n \sigma_i$ . The twist is equal to the number of partons,  $\tau = n$ .

# Spacelike pion form factor from AdS/CFT



- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant.*

de Teramond, sjb  
See also: Radyushkin

# Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space  $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ( $b = |\vec{b}_\perp|$ ):

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

**Soper**

## Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with  $\tilde{\rho}(x, \zeta)$  QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary  $Q$  using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for  $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$  !

# Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where  $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for  $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary  $Q$

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

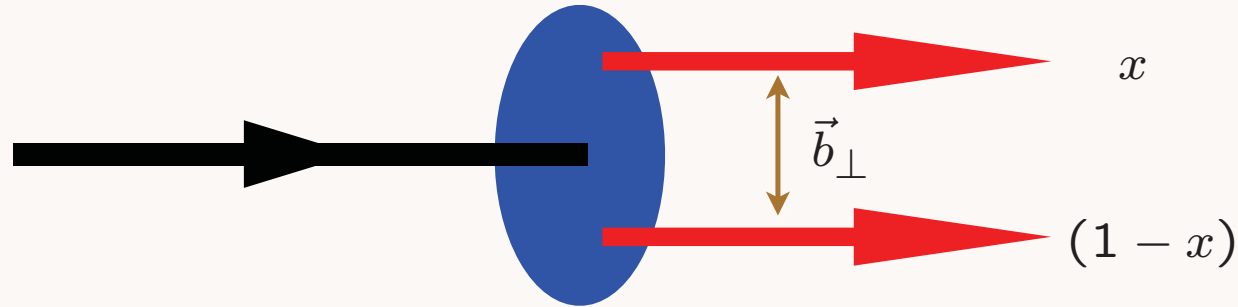
*Identical to LF Holography obtained from electromagnetic current*

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

*Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

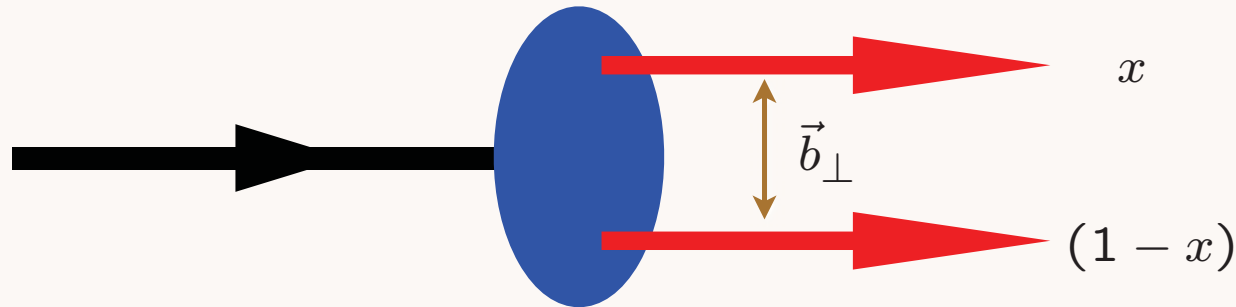
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(z) = \kappa^4 z^2 + 2\kappa^2(L + S - 1)$$

*soft wall  
confining potential*

**G. de Teramond, sjb**

**Trieste ICTP**  
May 27, 2010

**AdS/QCD and Hadronic Physics**

55

**Stan Brodsky**  
**SLAC-CP3**



- Propagation of external current inside AdS space described by the AdS wave equation

$$[z^2 \partial_z^2 - z(1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where  $U(a, b, c)$  is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background  $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

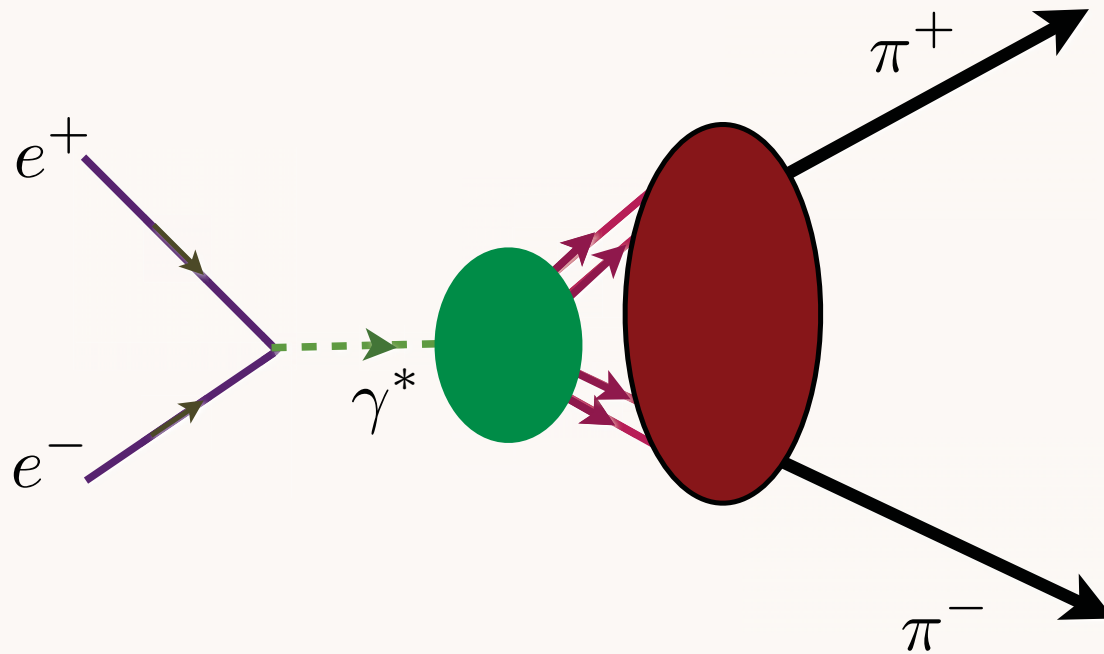
- For large  $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Soft Wall  
Model*

*Dressed soft-wall current bring in higher Fock states and more vector meson poles*



# Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

...

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \quad N,$$

Positive Dilaton Background  $\exp(+\kappa^2 z^2)$        $\mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right)$

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \quad Q^2 \rightarrow \infty$$

*Constituent Counting*

AdS/CFT now extensive field---apologies for all omitted references  
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: “fancy”  
Start from string theory, develop QCD analogs  
on lower dimensional branes

**Sakai & Sugimoto**

“Bottom-up”  
Anticipate what 5D Lagrangian must be (guess),  
directly involving desired  $\rho$ ,  $\pi$ ,  $a_1$ , ... fields and  
connect to matching QCD structures

**Erlich et al.  
Da Rold & Pomarol**

EM form factors in “bottom-up” approach

**Brodsky & de Teramond  
Radyushkin & Grigoryan**

Gravitational form factors in bottom-up approach

**Zainul Abidin & me**

Soft-wall

**Karch, Katz, Son, and Stephanov  
Batell, Gherghetta, and Sword**

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp)$$



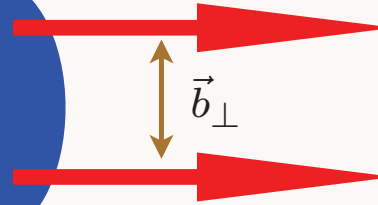
$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$



$$z$$

$$\psi(x, \vec{b}_\perp)$$



$$(1-x)$$

$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta)$$

*Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

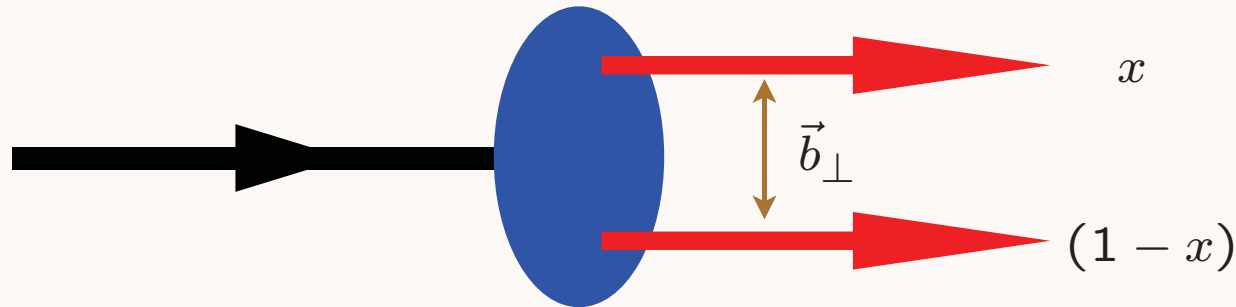
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Relativistic LF radial equation!

Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

G. de Teramond, sjb

*soft wall  
confining potential:*

# Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\begin{aligned} \mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left( -\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions.} \end{aligned}$$

**Change  
variables**

$$(\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned} \mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \end{aligned}$$

$$H_{QED}$$

*QED atoms: positronium and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

*Coupled Fock states*

$$\left[-\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$\left[-\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell)\right] \psi(r) = E \psi(r)$$

*Spherical Basis*  $r, \theta, \phi$

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

**Bohr Spectrum**

*Semiclassical first approximation to QED*



$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

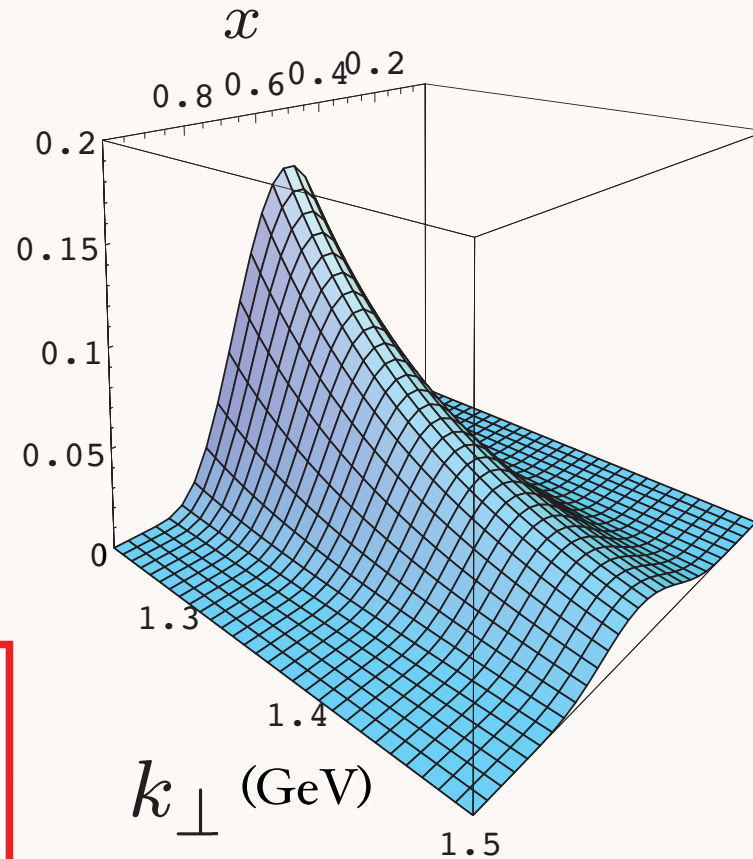
Semiclassical first approximation to QCD

Confining AdS/QCD potential

# Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$$\psi_M(x, k_{\perp}^2)$$



**“Soft Wall”  
model**

$$\kappa = 0.375 \text{ GeV}$$

massless quarks

**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

*Connection of Confinement to TMDs*

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# Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

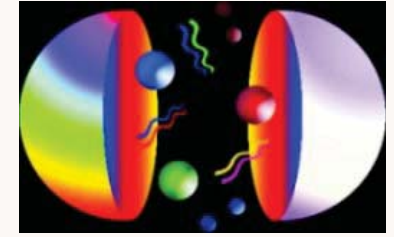
$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

- Baryons Spectrum in "bottom-up" holographic QCD  
GdT and Brodsky: hep-th/0409074, hep-th/0501022.



From Nick Evans

## Baryons in AdS/CFT

- Action for massive fermionic modes on AdS<sub>5</sub>:

$$S[\bar{\Psi}, \Psi] = \int d^4x dz \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion:  $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution ( $\mu R = \nu + 1/2$ )

$$\Psi(z) = C z^{5/2} [J_\nu(z\mathcal{M})u_+ + J_{\nu+1}(z\mathcal{M})u_-]$$

- Hadronic mass spectrum determined from IR boundary conditions  $\psi_\pm(z = 1/\Lambda_{\text{QCD}}) = 0$

$$\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}$$

with scale independent mass ratio

- Obtain spin- $J$  mode  $\Phi_{\mu_1 \dots \mu_{J-1/2}}$ ,  $J > \frac{1}{2}$ , with all indices along 3+1 from  $\Psi$  by shifting dimensions

# Baryons

## Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin- $\frac{1}{2}$  modes  $\psi(\zeta)$  and spin- $\frac{3}{2}$  modes  $\psi_\mu(\zeta)$  are **two-component spinor** solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where  $H_{LF} = \alpha\Pi$  and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint  $\Pi_L^\dagger(\zeta)$  satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$

- Note: in the Weyl representation ( $i\alpha = \gamma_5\beta$ )

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension  $3 + L$  ( $\nu = L + 1$ )

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_-].$$

Baryonic modes propagating in AdS space have two components: orbital  $L$  and  $L + 1$ .

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,$$

given by

$$\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},$$

with a scale independent mass ratio.

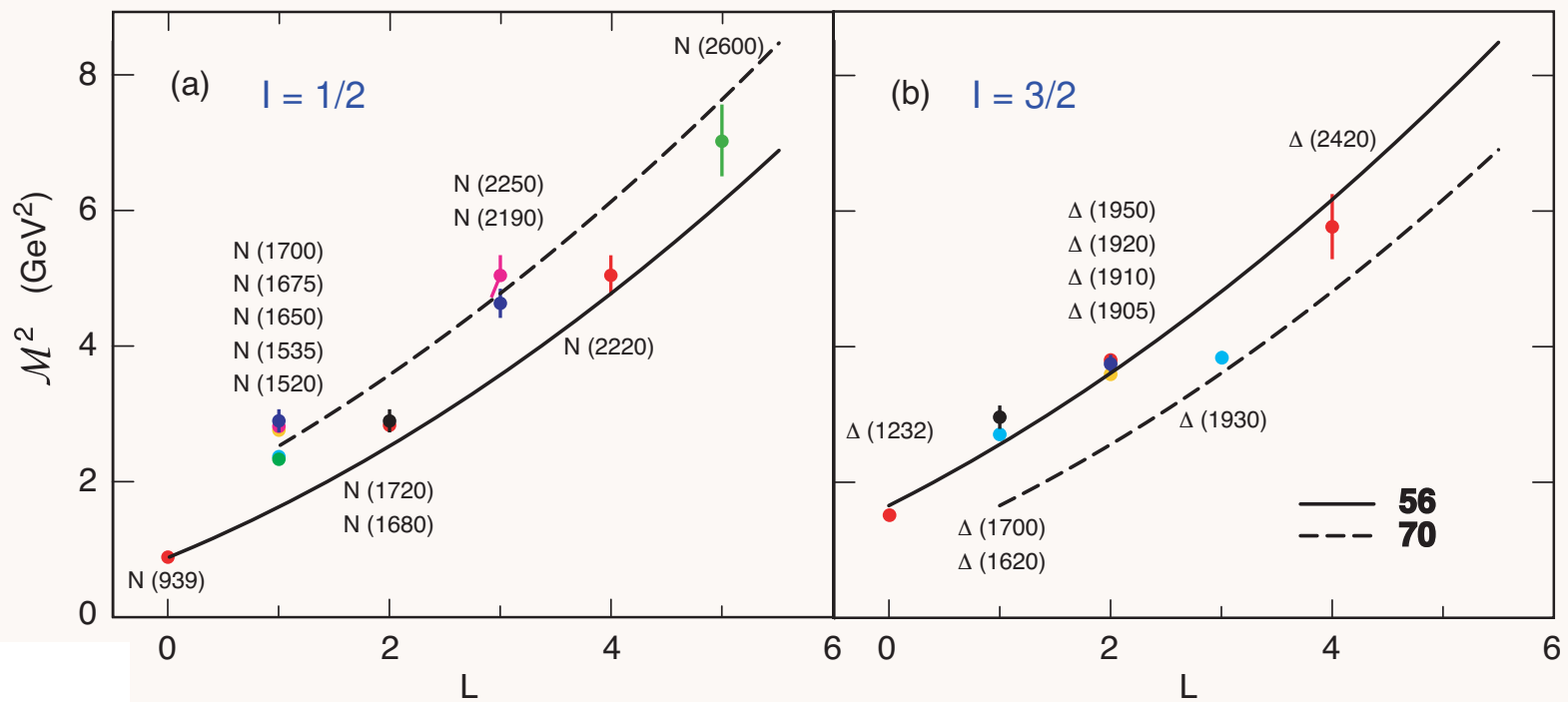


Fig: Light baryon orbital spectrum for  $\Lambda_{QCD} = 0.25$  GeV in the HW model. The  $56$  trajectory corresponds to  $L$  even  $P = +$  states, and the  $70$  to  $L$  odd  $P = -$  states.

## Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator  $\Pi$

$$\nu = L + 1$$

$$\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint  $\Pi^\dagger$ , with commutation relations

$$\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$



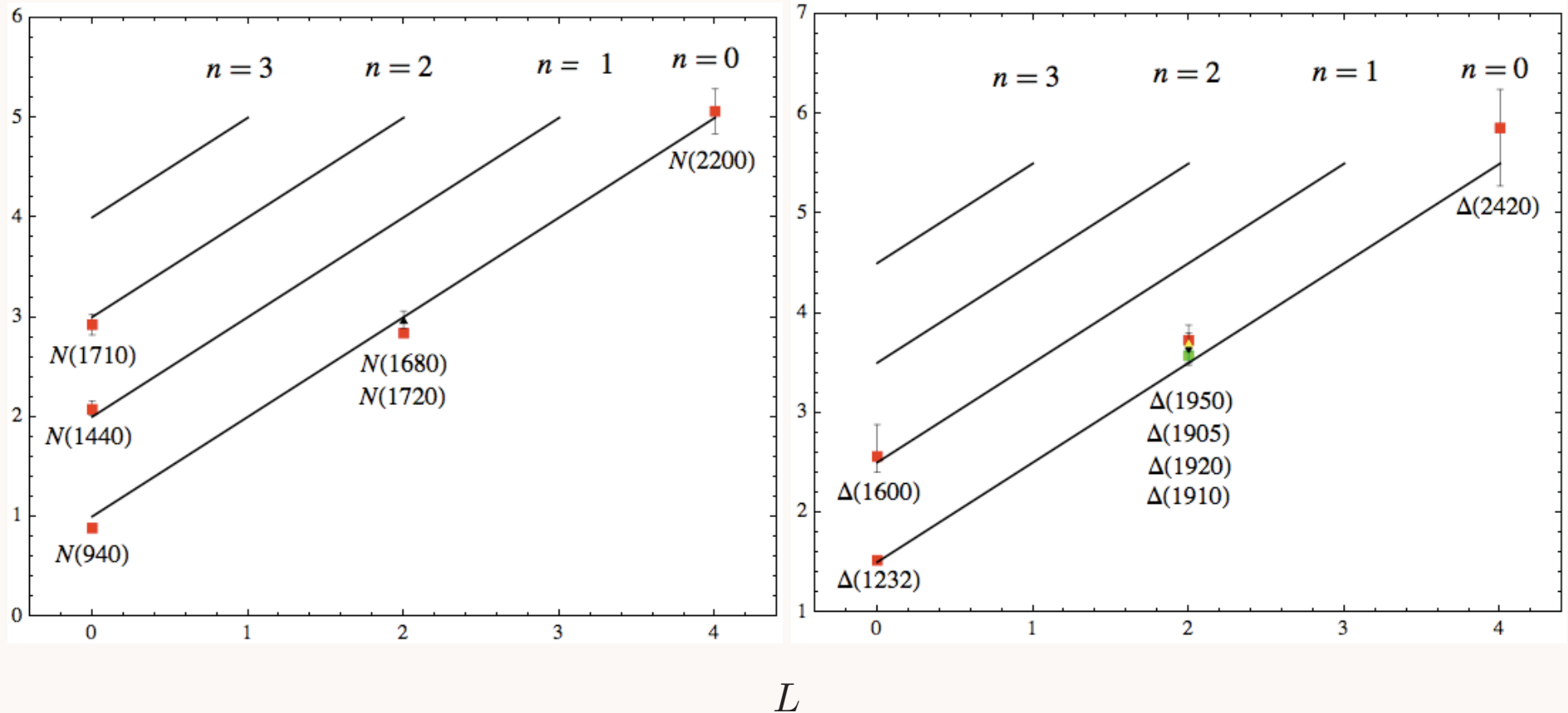
- $\Delta$  spectrum identical to Forkel and Klempt, Phys. Lett. B 679, 77 (2009)

$$4\kappa^2 \text{ for } \Delta n = 1$$

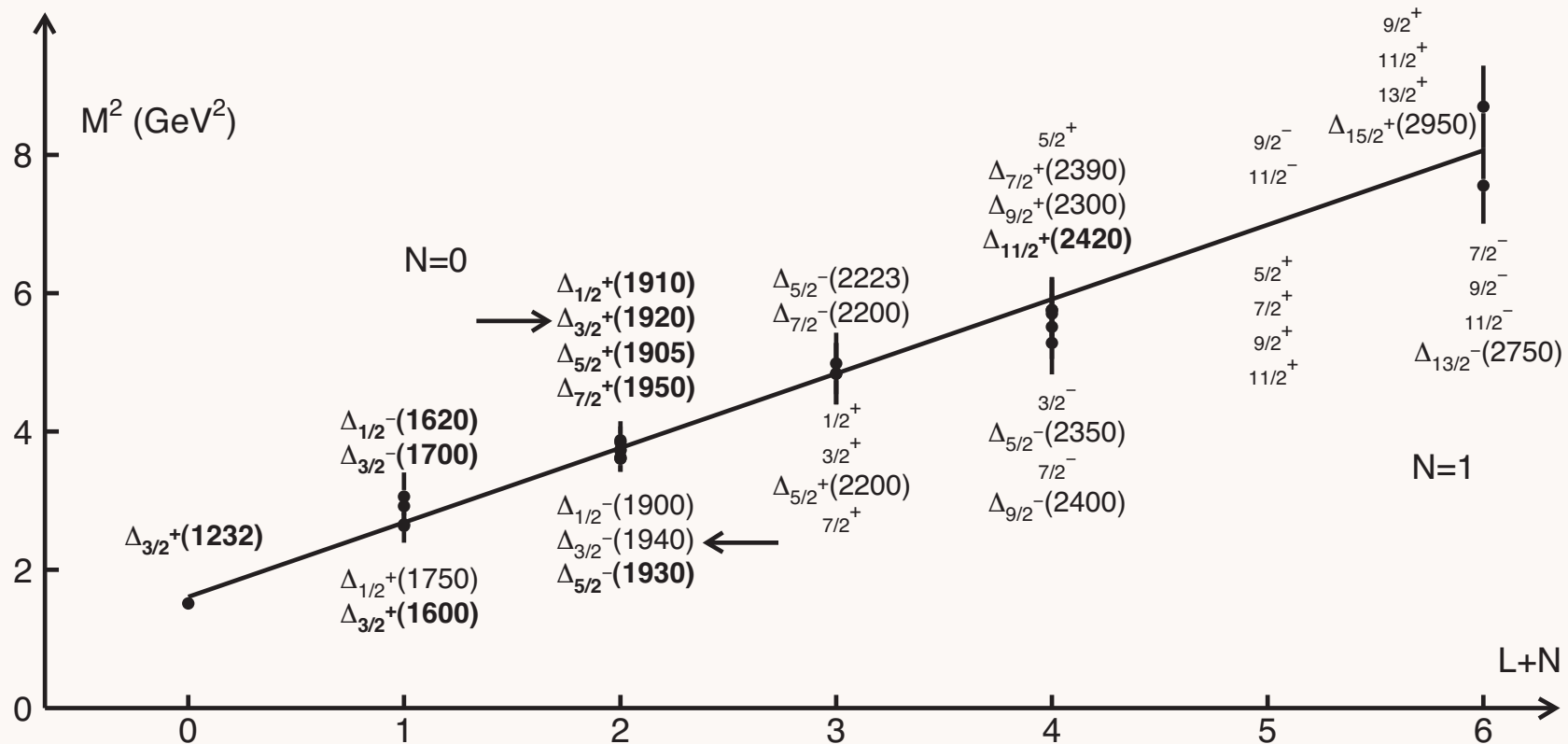
$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$

$$\mathcal{M}^2$$



Parent and daughter **56** Regge trajectories for the  $N$  and  $\Delta$  baryon families for  $\kappa = 0.5$  GeV



E. Klempt *et al.*:  $\Delta^*$  resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.  
 H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2,$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory.

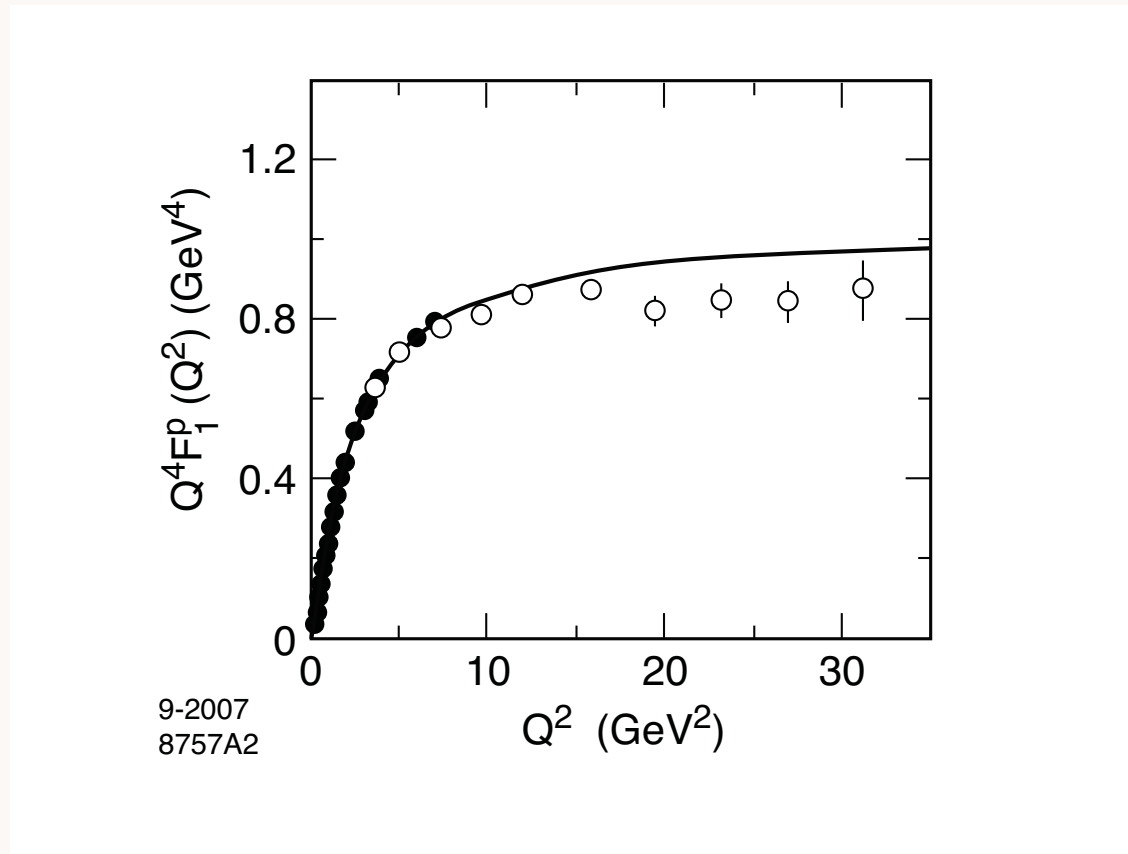
- Choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\psi_+(\zeta)$  and  $\psi_-(\zeta)$  correspond to nucleons with  $J^z = +1/2$  and  $-1/2$ .
- For  $SU(6)$  spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2,$$

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) [|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2],$$

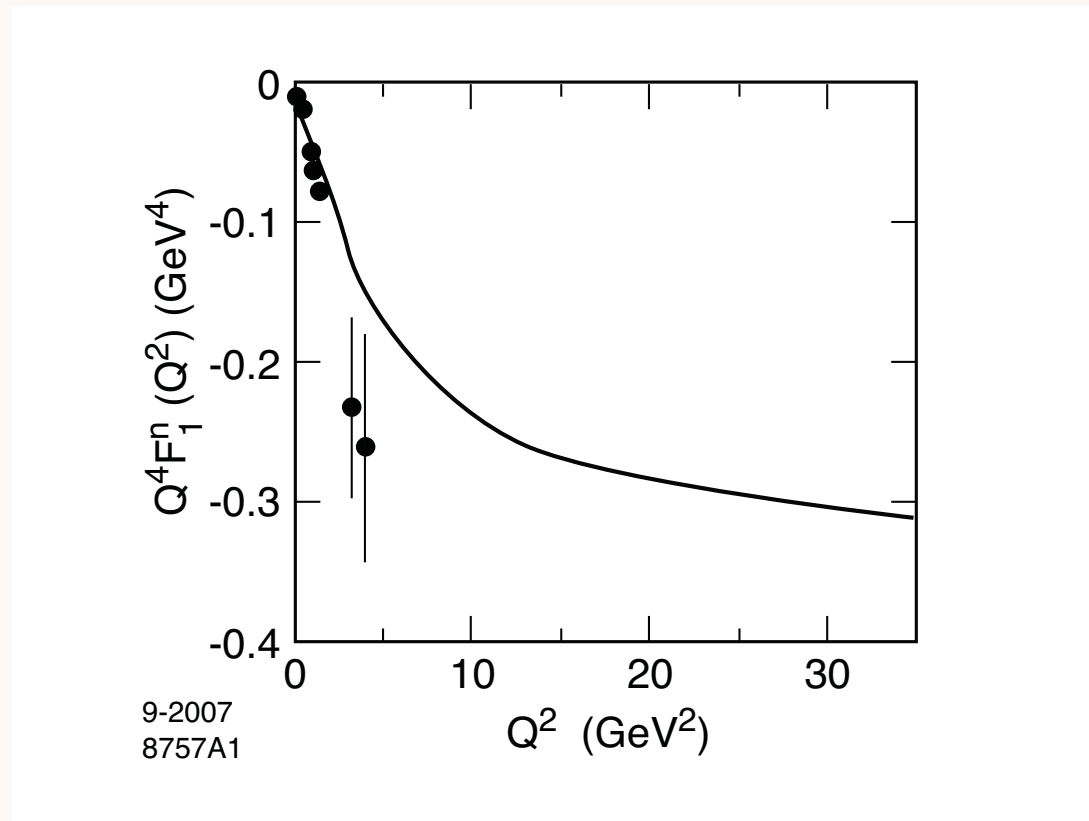
where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ .

- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  Proton  $\tau = 3$



SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large  $Q^2$ :  $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$  Neutron  $\tau = 3$

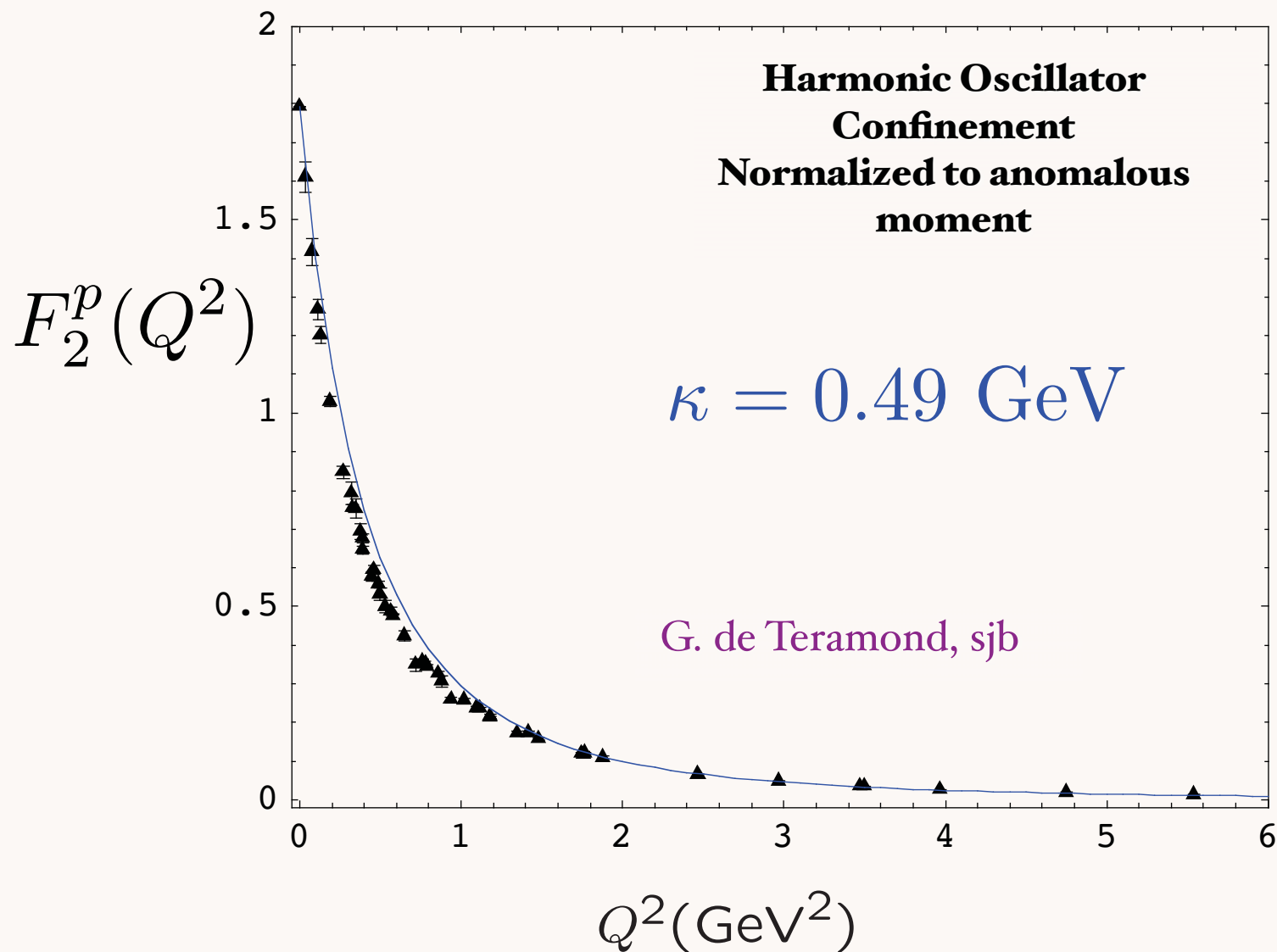


SW model predictions for  $\kappa = 0.424$  GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

# Spacelike Pauli Form Factor

Preliminary

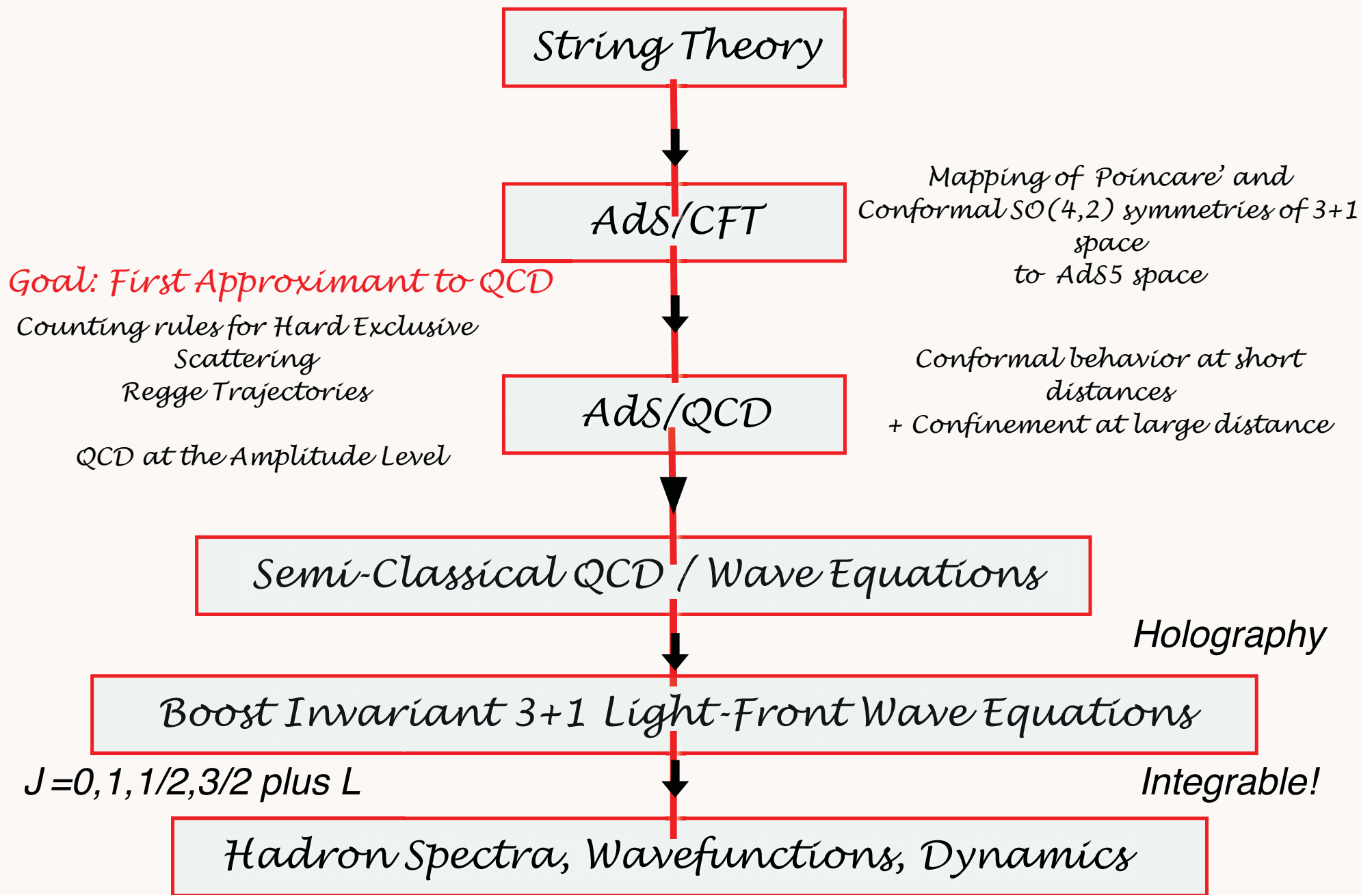
From overlap of  $L = 1$  and  $L = 0$  LFWFs



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# Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS<sub>5</sub> space in dilaton background  $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling  $g_5(z)$  incorporates the non-conformal dynamics of confinement

- YM coupling  $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$  is the five dim coupling up to a factor:  $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale  $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

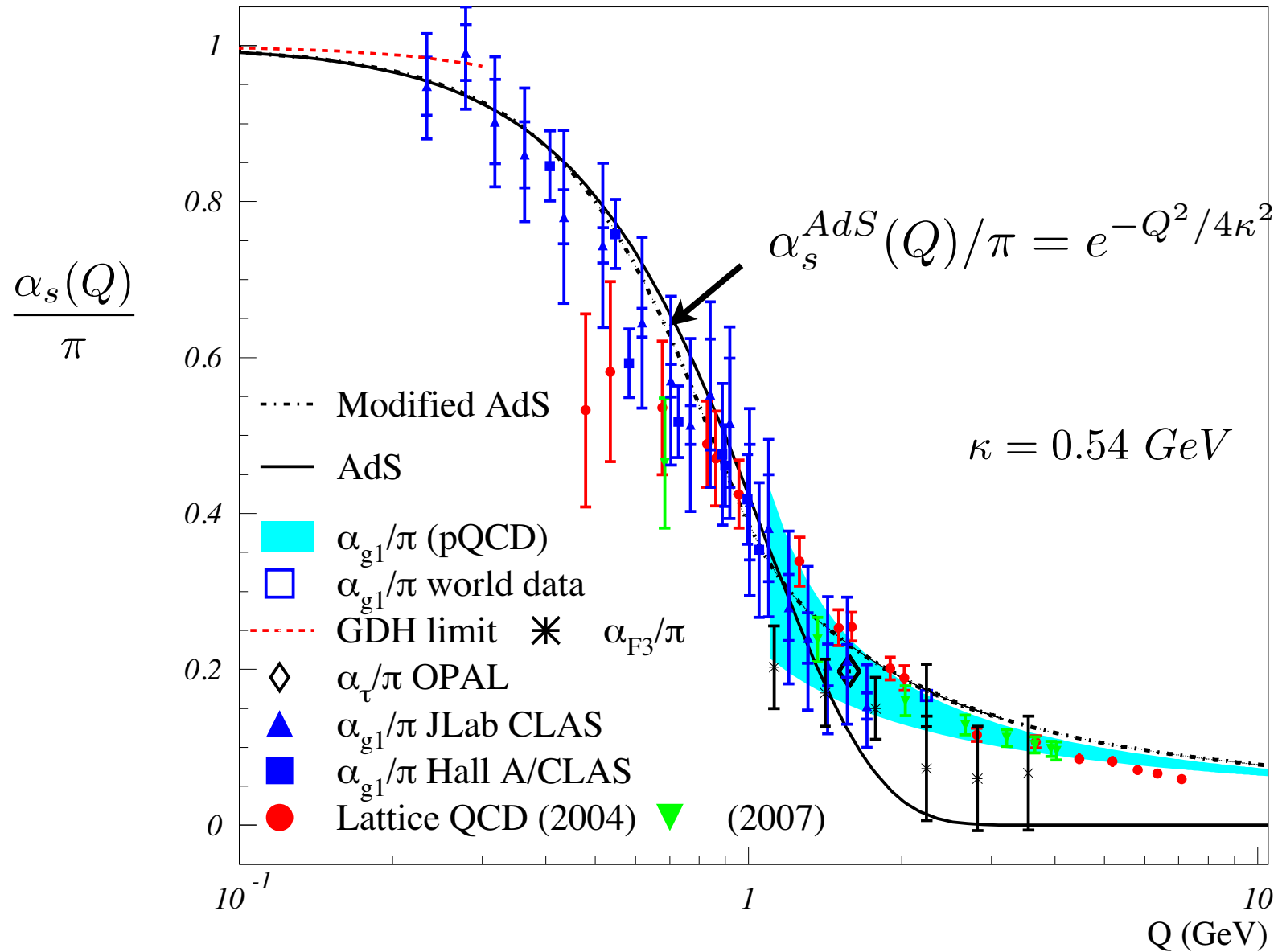
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}.$$

where the coupling  $\alpha_s^{AdS}$  incorporates the non-conformal dynamics of confinement



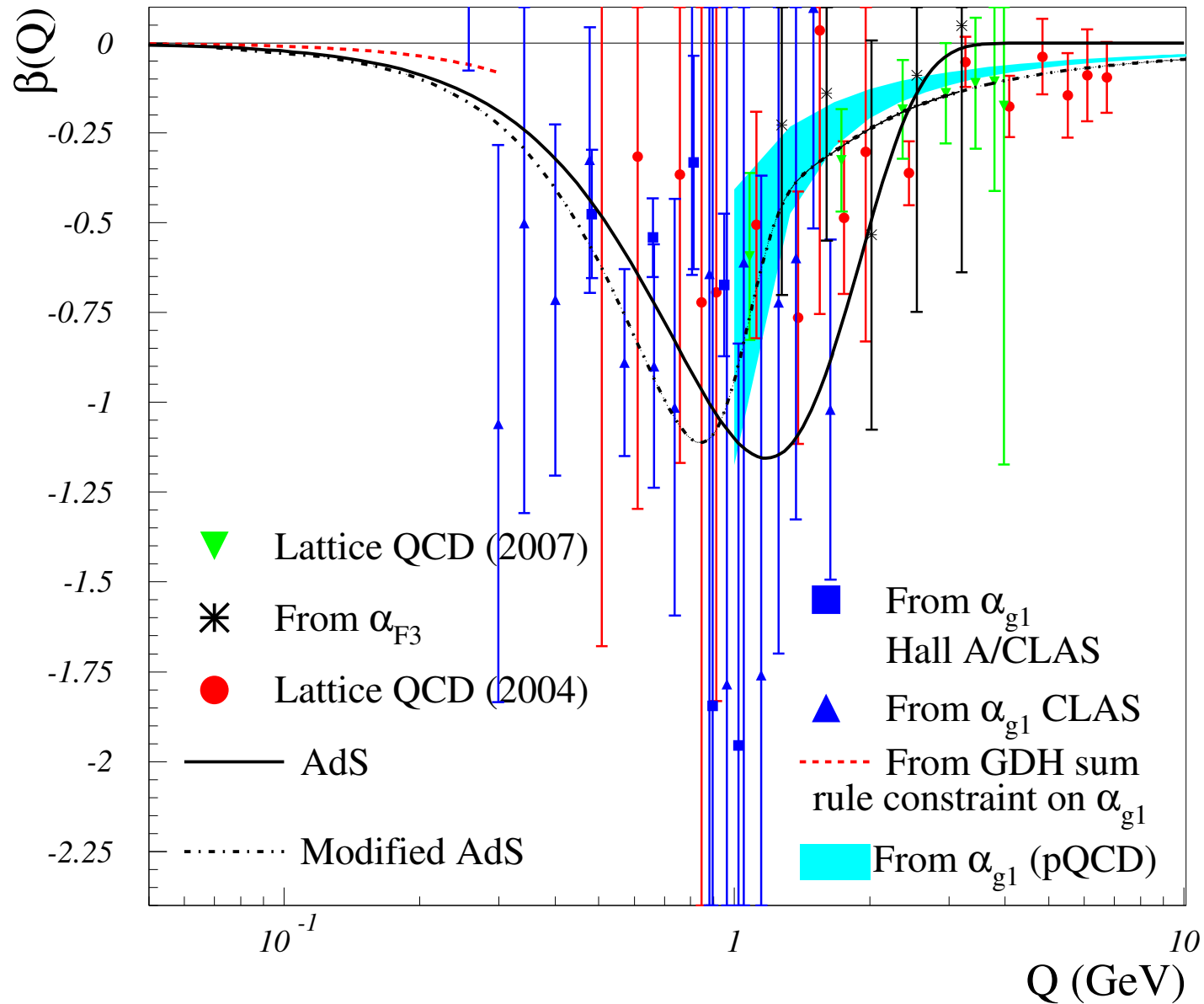
# Running Coupling from Light-Front Holography and AdS/QCD

**Analytic, defined at all scales, IR Fixed Point**



Deur, de Teramond, sjb

$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



Deur, de Teramond, sjb

# *Features of Soft-Wall AdS/QCD*

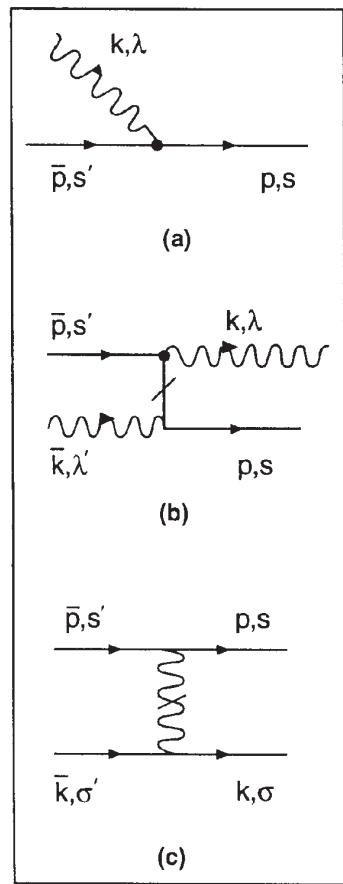
- **Single-variable frame-independent radial Schrodinger equation**
- **Massless pion ( $m_q = 0$ )**
- **Regge Trajectories: universal slope in  $n$  and  $L$**
- **Valid for all integer  $J$  &  $S$ . Spectrum is independent of  $S$**
- **Dimensional Counting Rules for Hard Exclusive Processes**
- **Phenomenology: Space-like and Time-like Form Factors**
- **LF Holography: LFWFs; broad distribution amplitude**
- **No large  $N_c$  limit**
- **Add quark masses to LF kinetic energy**
- **Systematically improvable -- diagonalize  $H_{LF}$  on AdS basis**

*Use AdS/CFT orthonormal Light Front Wavefunctions  
as a basis for diagonalizing the QCD LF Hamiltonian*

- Good initial approximation
- Better than plane wave basis **Pauli, Hornbostel,  
Hiller, McCartor, sjb**
- DLCQ discretization -- highly successful I+I
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- Hamiltonian light-front field theory within an AdS/QCD basis.  
**J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,**  
**G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, sjb**

Light-Front QCD  
Heisenberg Equation

$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$



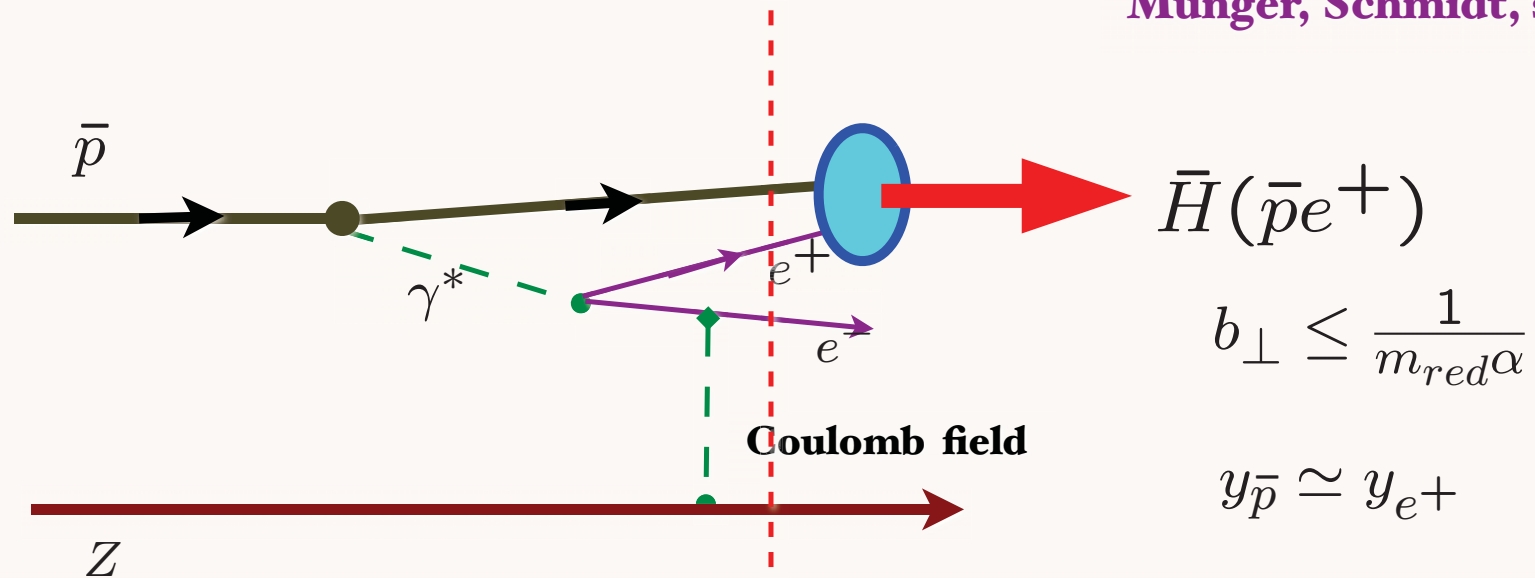
| n  | Sector  | 1<br>q $\bar{q}$ | 2<br>gg | 3<br>q $\bar{q}$ g | 4<br>q $\bar{q}$ q $\bar{q}$ | 5<br>gg g | 6<br>q $\bar{q}$ gg | 7<br>q $\bar{q}$ q $\bar{q}$ g | 8<br>q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ | 9<br>gg gg | 10<br>q $\bar{q}$ gg g | 11<br>q $\bar{q}$ q $\bar{q}$ gg | 12<br>q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g | 13<br>q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ |
|----|---|------------------|---------|--------------------|------------------------------|-----------|---------------------|--------------------------------|--|------------|------------------------|----------------------------------|---|---|
| 1  | q $\bar{q}$                                     |                  |         |                    |                              | .         |                     | .                              | .  | .          | .                      | .                                | .   | .   |
| 2  | gg  |                  |         |                    | .                            |           |                     | .                              | .  |            | .                      | .                                | .   | .   |
| 3  | q $\bar{q}$ g                                   |                  |         |                    |                              |           |                     |                                | .  | .          |                        | .                                | .   | .   |
| 4  | q $\bar{q}$ q $\bar{q}$                         |                  | .       |                    |                              | .         |                     |                                |  | .          | .                      |                                  | .   | .   |
| 5  | gg g  | .                |         |                    | .                            |           |                     | .                              | .  |            |                        | .                                | .   | .   |
| 6  | q $\bar{q}$ gg                                  |                  |         |                    |                              |           |                     |                                | .  |            |                        |                                  | .   | .   |
| 7  | q $\bar{q}$ q $\bar{q}$ g                       | .                | .       |                    |                              | .         |                     |                                |  | .          |                        |                                  |   | .   |
| 8  | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$             | .                | .       | .                  |                              | .         | .                   |                                |  | .          | .                      |                                  |   |   |
| 9  | gg gg   | .                |         | .                  | .                            |           |                     | .                              | .  |            |                        | .                                | .   | .   |
| 10 | q $\bar{q}$ gg g                                | .                | .       |                    | .                            |           |                     |                                | .  |            |                        |                                  | .   | .   |
| 11 | q $\bar{q}$ q $\bar{q}$ gg                      | .                | .       | .                  |                              | .         |                     |                                |  | .          |                        |                                  |   | .   |
| 12 | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g           | .                | .       | .                  | .                            | .         | .                   |                                |  | .          | .                      |                                  |   |   |
| 13 | q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ | .                | .       | .                  | .                            | .         | .                   | .                              |  | .          | .                      | .                                |   |   |

Use AdS/QCD basis functions

# Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

Munger, Schmidt, sjb



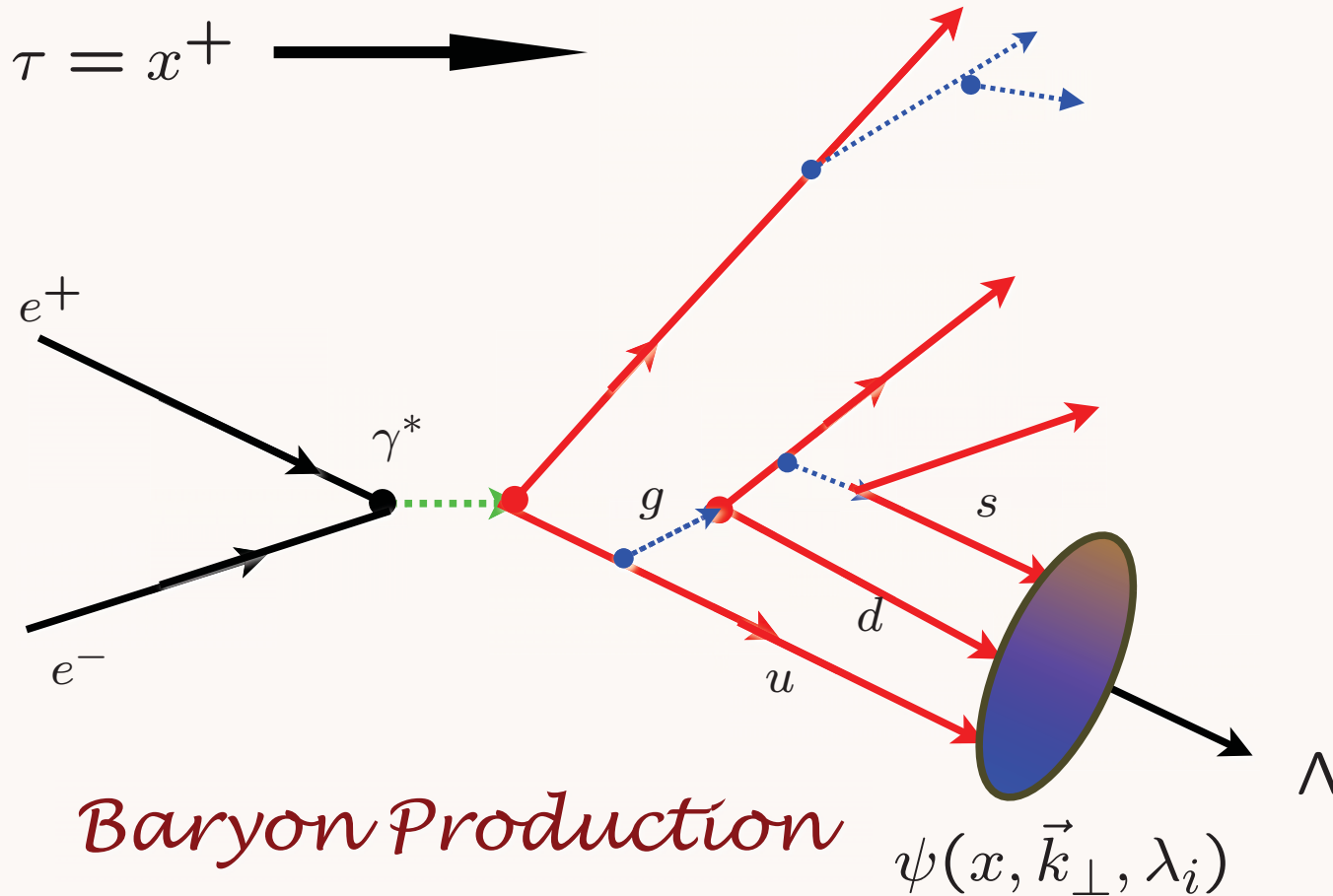
**Coalescence of off-shell co-moving positron and antiproton**

*Wavefunction maximal at small impact separation and equal rapidity*

*“Hadronization” at the Amplitude Level*



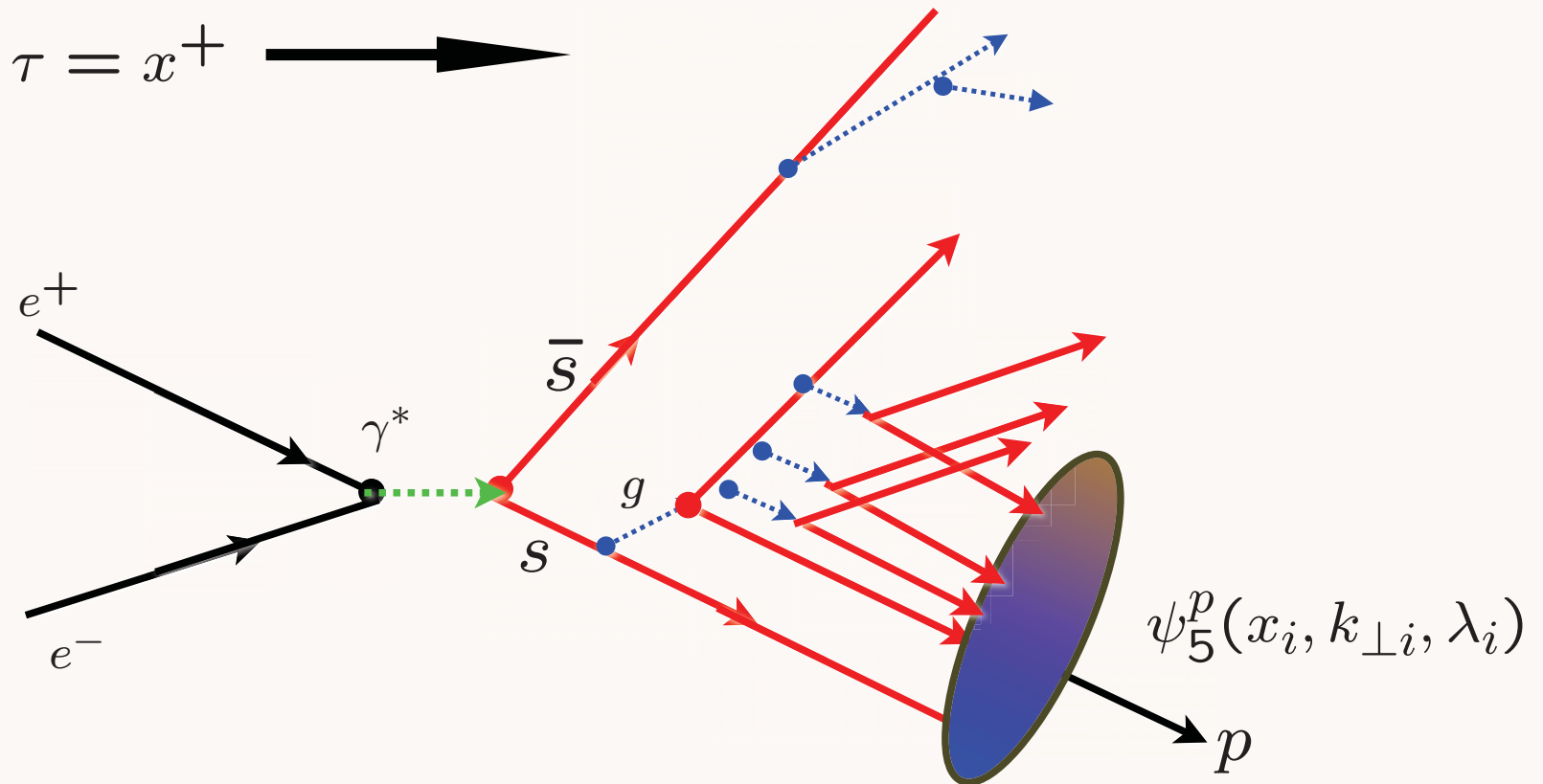
# Hadronization at the Amplitude Level



Construct helicity amplitude using Light-Front  
Perturbation theory; coalesce quarks via LFWFs



# Hadronization at the Amplitude Level



**Higher Fock State Coalescence**  $|uuds\bar{s}\rangle$

**Asymmetric Hadronization!**  $D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z)$

B-Q Ma, sjb

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# Features of LF T-Matrix Formalism

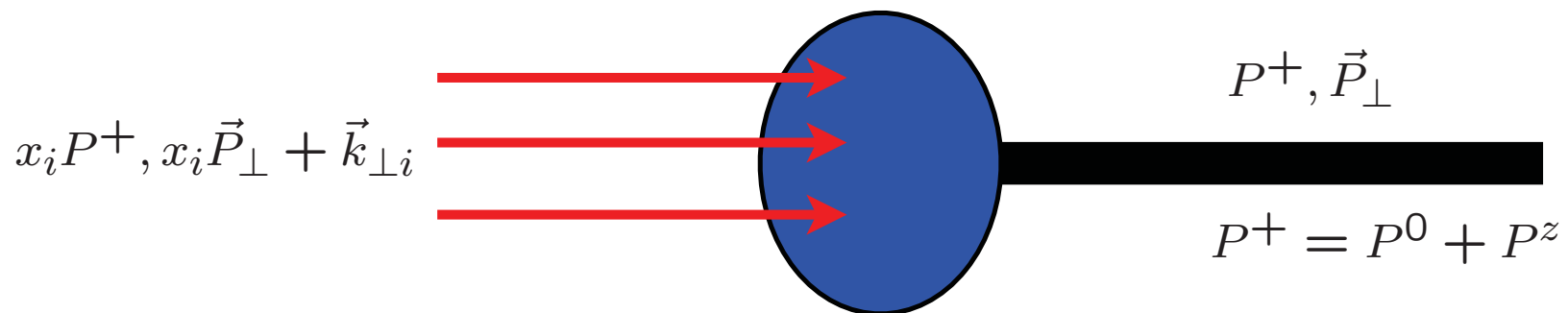
## “Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if

$$\mathcal{M}_n^2 = \sum_{i=1}^n \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2$$

- The coalescence probability amplitude is the LF wavefunction  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

- No IR divergences: Maximal gluon and quark wavelength from confinement



*Baryon can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

**p**

$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

**Bjorken**  
**Blankenbecler, Gunion, sjb**  
**Berger, sjb**  
**Hoyer, et al: Semi-Exclusive**

**Sickles; sjb**

*Small color-singlet*  
*Color Transparent*  
*Minimal same-side energy*

**u**

**d**

**u**

**gg**

**gg**

$$\mathbf{n}_{\text{active}} = 6$$

$$qq \rightarrow B\bar{q}$$

$$\mathbf{n}_{\text{eff}} = 2\mathbf{n}_{\text{active}} - 4$$

$$\mathbf{n}_{\text{eff}} = 8$$

**d**

*Collision can produce 3  
collinear quarks*

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**AdS/QCD and Hadronic Physics**

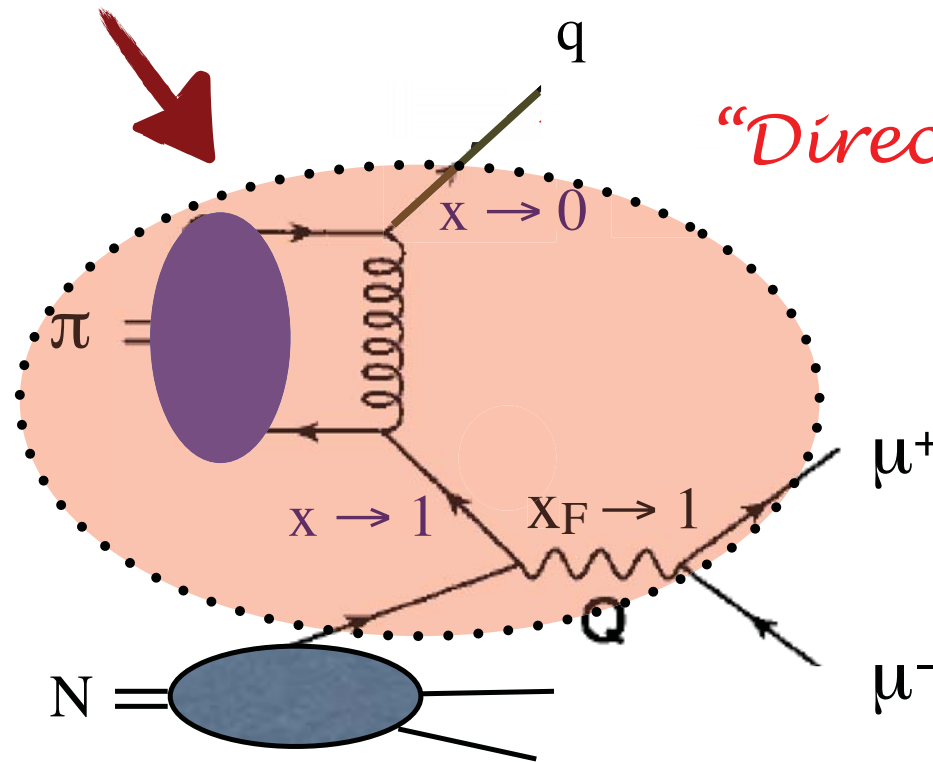
**Stan Brodsky**  
**SLAC-CP3**

$$\pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F$$

In the limit where  $(1-x_F)Q^2$  is fixed as  $Q^2 \rightarrow \infty$

*Light-Front Wavefunctions from AdS/CFT*

Entire pion wf contributes to hard process



*“Direct” subprocess*

Virtual photon is longitudinally polarized

Berger, sjb  
Khoze, Brandenburg, Muller, sjb

Hoyer Vanttinen

$$\pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV}/c$$

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin 2\theta \cos\phi + \omega \sin^2\theta \cos 2\phi.$$

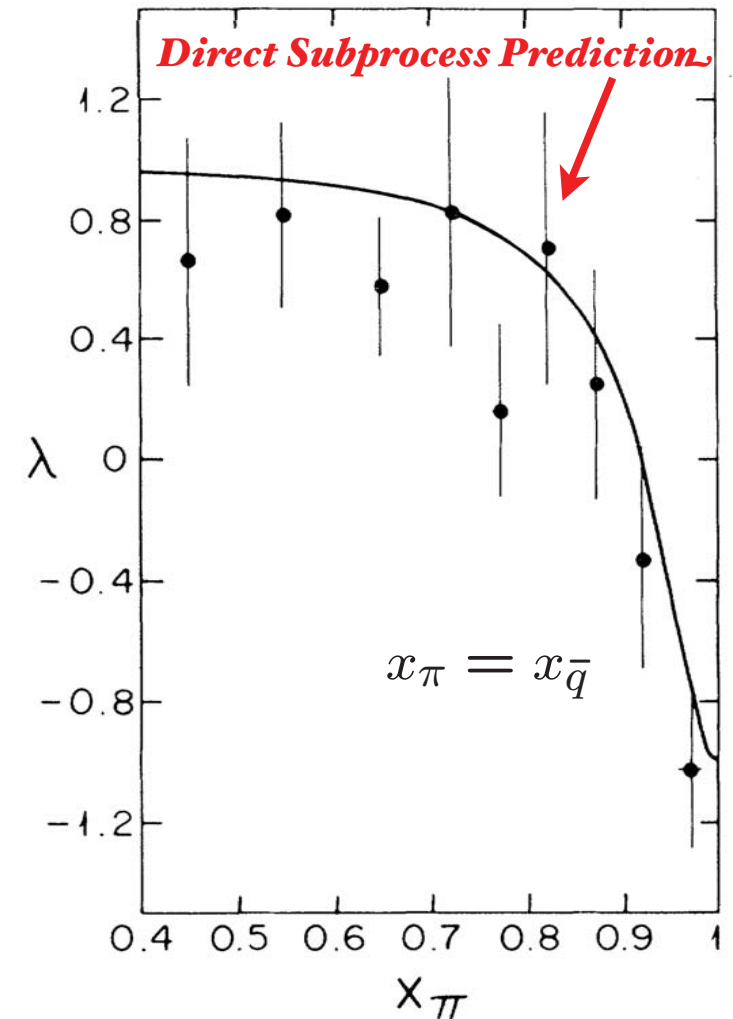
$$\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[ (1-x_\pi)^2 (1 + \cos^2\theta) + \frac{4}{9} \frac{\langle k_T^2 \rangle}{M^2} \sin^2\theta \right]$$

$$\langle k_T^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2$$

$$Q^2 = M^2$$

*Dramatic change in  
angular distribution at  
large  $x_F$*

**Example of a higher-twist  
direct subprocess**



Chicago-Princeton  
Collaboration

Phys.Rev.Lett.55:2649,1985

*Crucial Test of Leading-Twist QCD:  
Scaling at fixed  $x_T$*

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E \frac{d\sigma}{d^3p}(pN \rightarrow \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

***Parton model:  $n_{eff} = 4$***

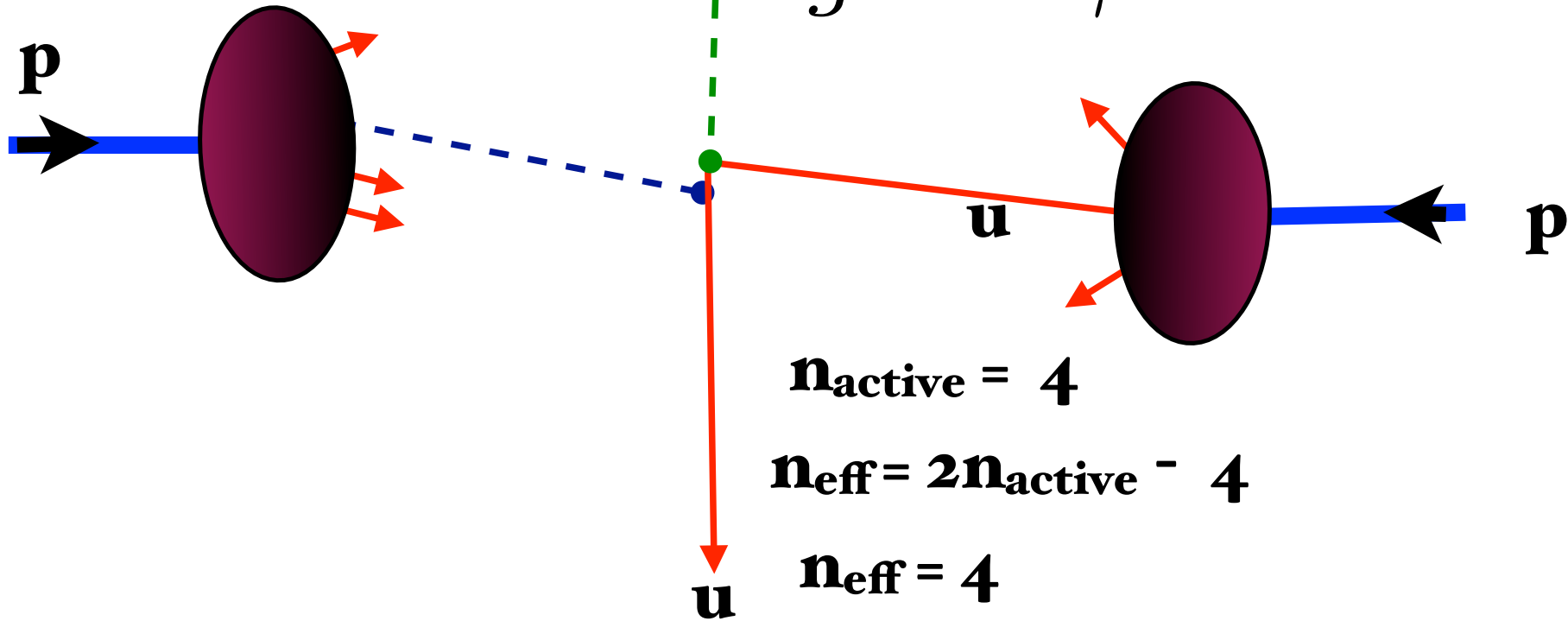
***As fundamental as Bjorken scaling in DIS***

**Conformal scaling:  $n_{eff} = 2 n_{active} - 4$**

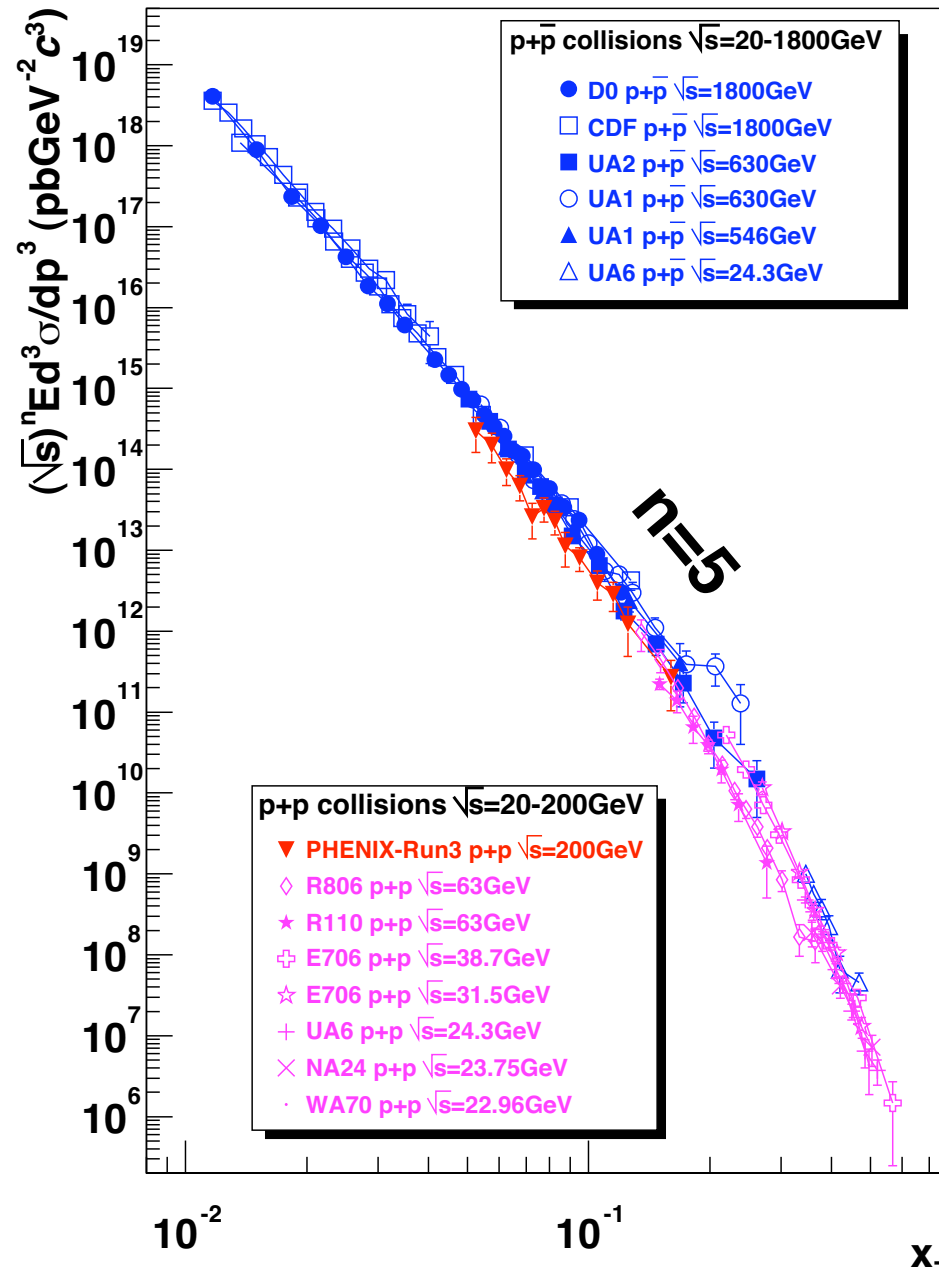
$$pp \rightarrow \gamma X$$

$$E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \rightarrow \gamma u$$



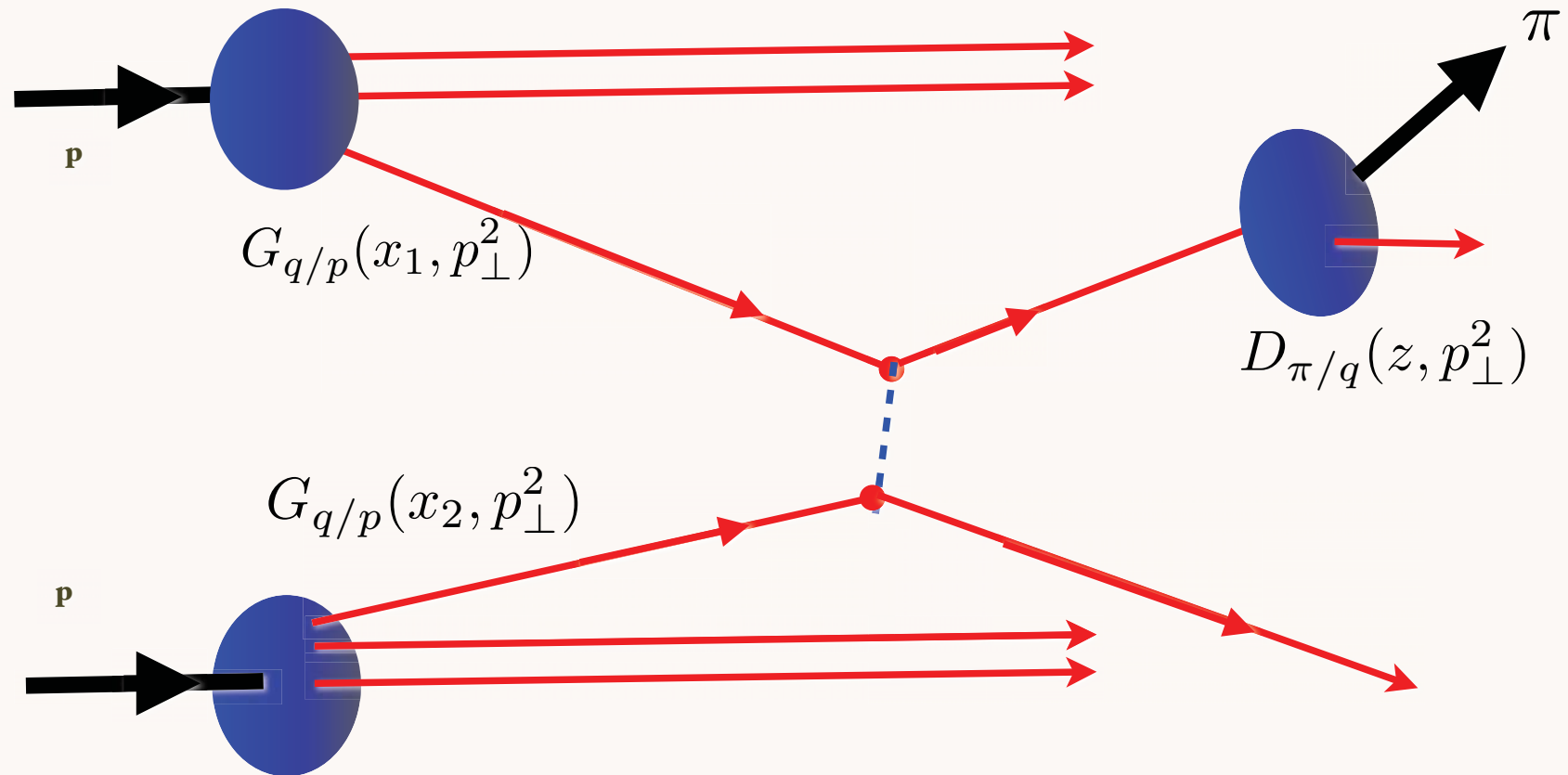
$$\sqrt{s}^n E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) \text{ at fixed } x_T$$



**$x_T$ -scaling of  
direct photon  
production is  
consistent with  
PQCD**



# Leading-Twist Contribution to Hadron Production

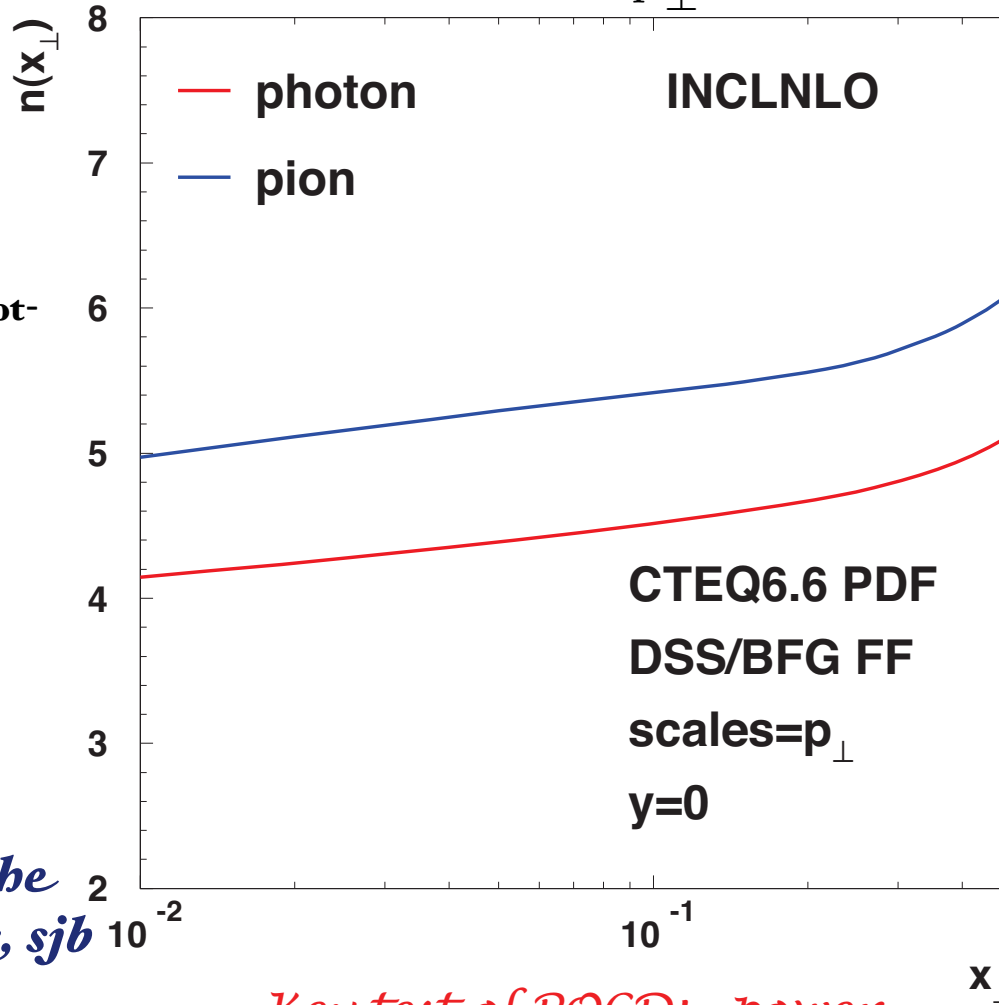


*Parton model and Conformal Scaling:*

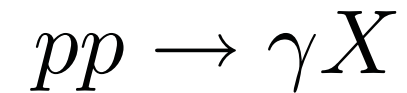
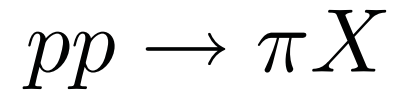
$$\frac{d\sigma}{d^3 p / E} = \alpha_s^2 \frac{F(x_{\perp}, y)}{p_{\perp}^4}$$

QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_{\perp}, y)}{p_{\perp} n(x_{\perp})}$$



DSS  
(De Florian-Sassot-Stratmann)



Arleo, Aurenche  
Hwang, Sickles, sjb

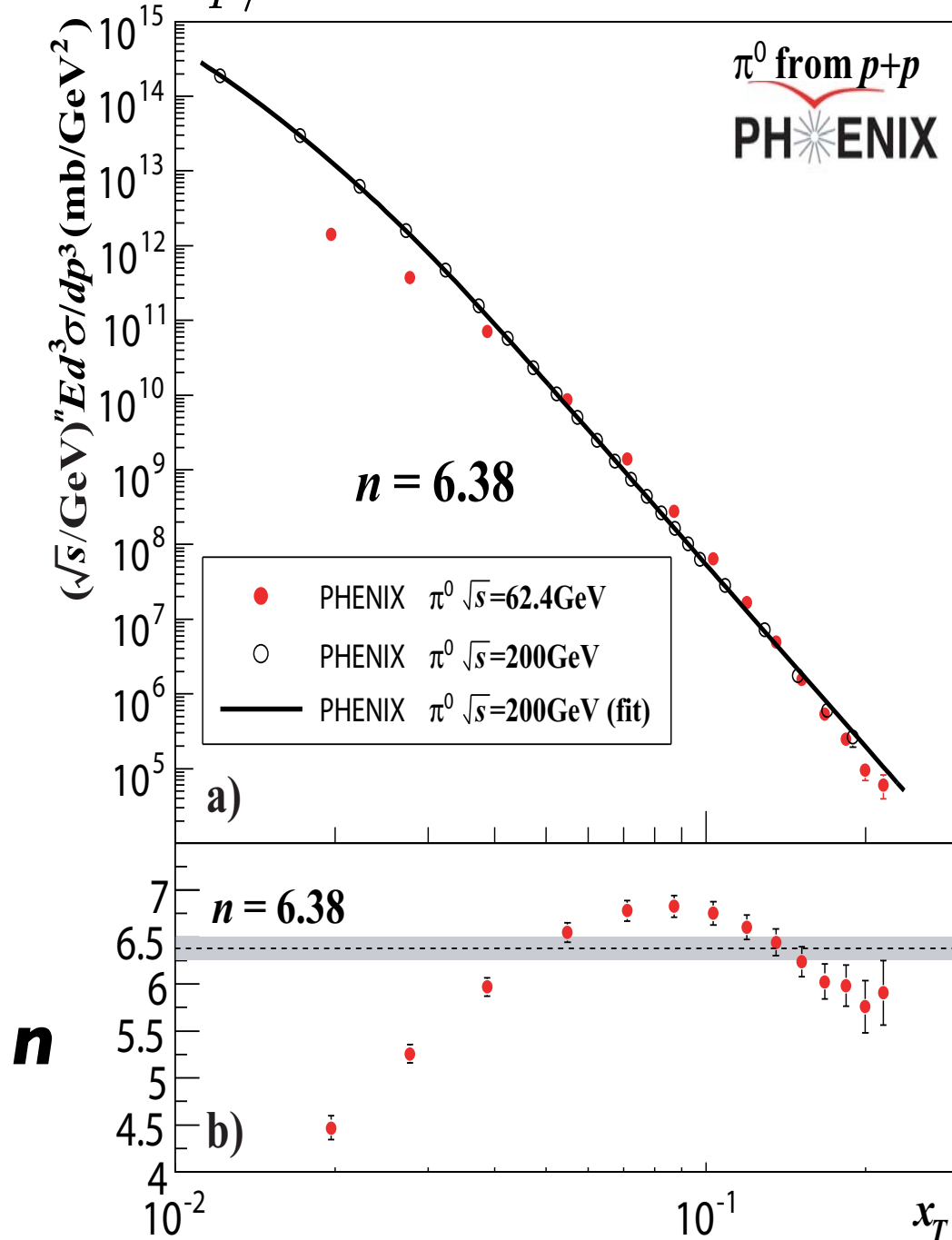
$$5 < p_{\perp} < 20 \text{ GeV}$$

$$70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV}$$

Pirner, Raufeisen, sjb

Key test of PQCD: power-law fall-off at fixed  $x_{\perp}$

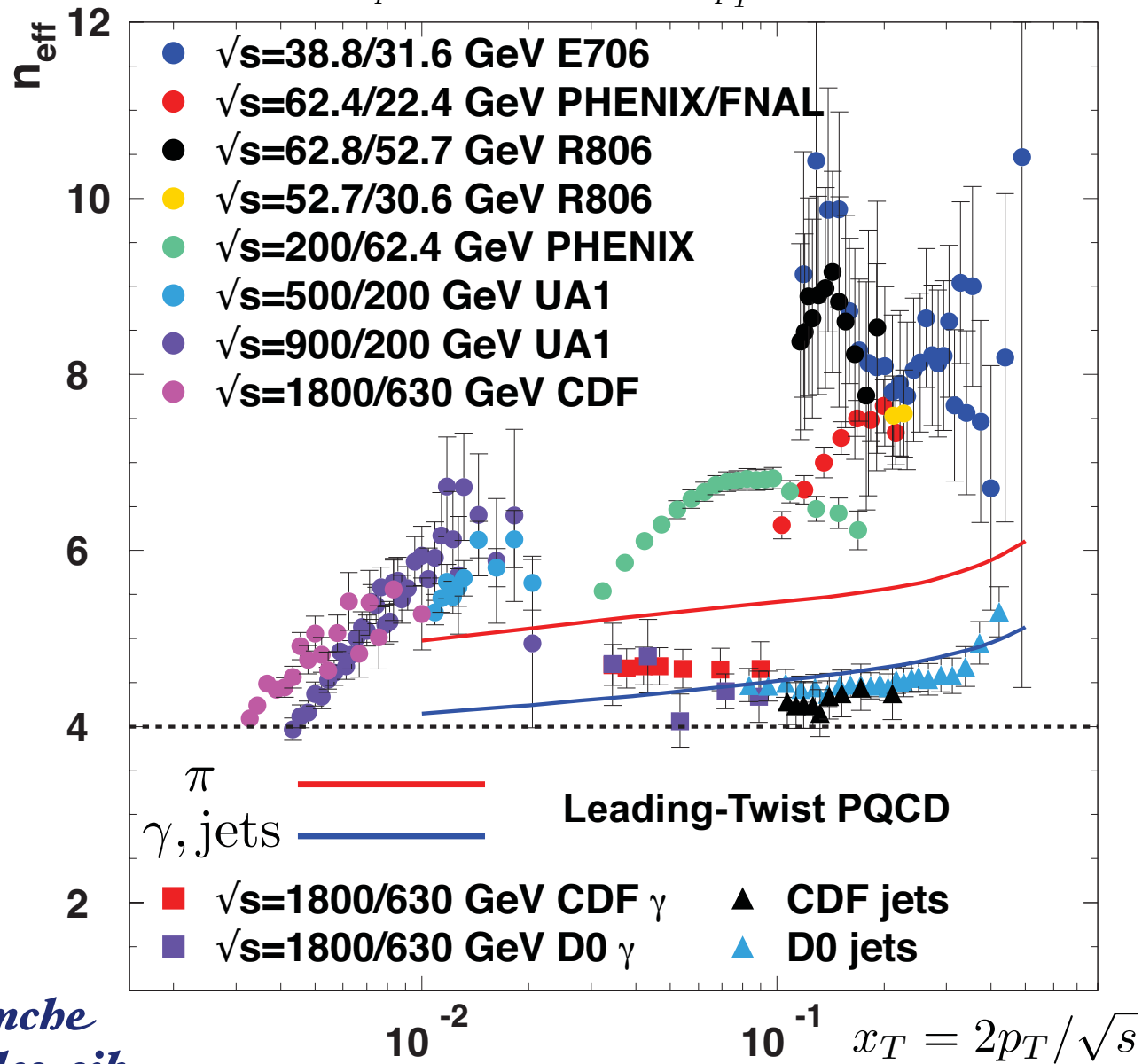
$$[\sqrt{s}]^n \frac{d\sigma}{d^3p/E} (pp \rightarrow \pi^0 X) \text{ at fixed } x_T = \frac{2p_T}{\sqrt{s}}$$



M. J.  
Tannenbaum

PHENIX  
62.4 and 200  
GeV data

$$E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\text{eff}}}}$$



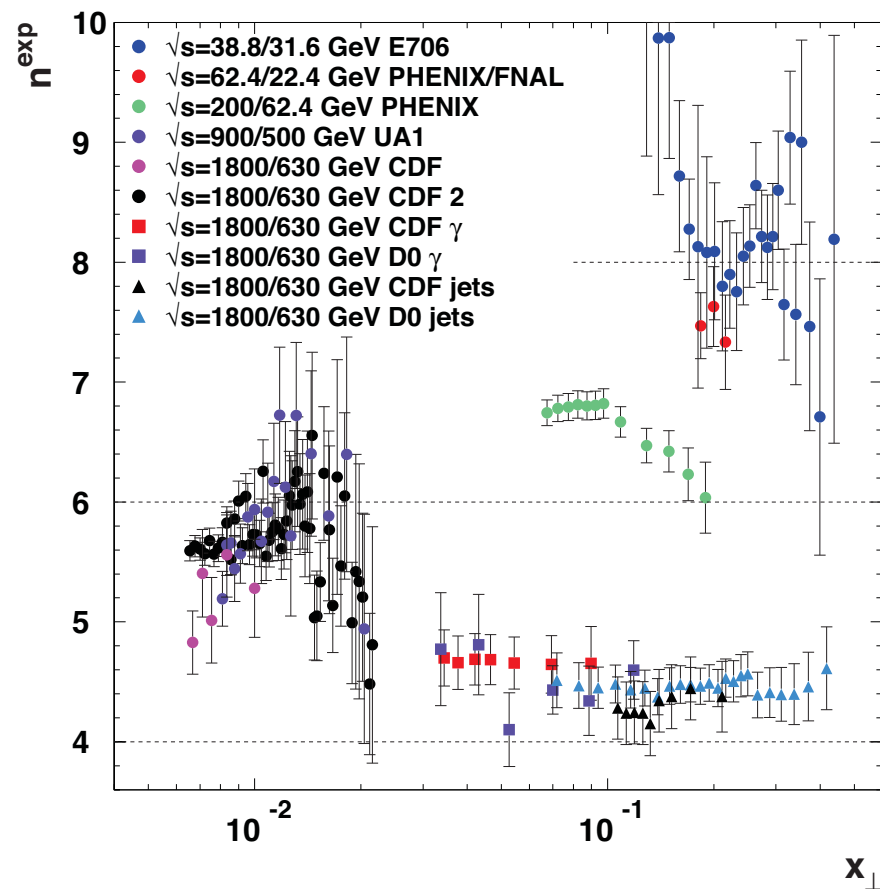
*Arleo, Aurenche  
Hwang, Sickles, sjb*

Trieste ICTP  
May 27, 2010

AdS/QCD and Hadronic Physics

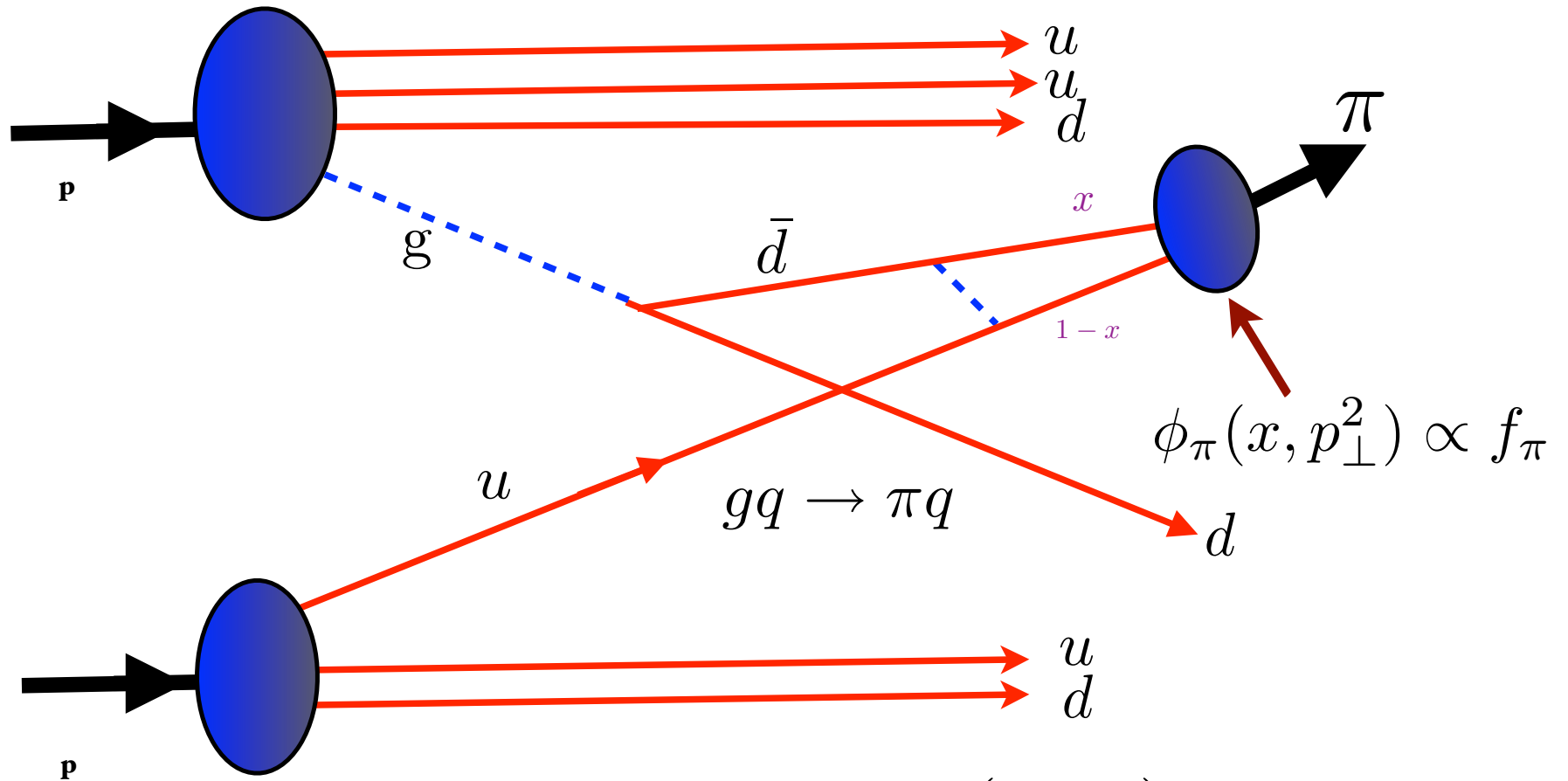
99

Stan Brodsky  
SLAC-CP3



- Significant increase of the hadron  $n^{\text{exp}}$  with  $x_{\perp}$ 
  - $n^{\text{exp}} \simeq 8$  at large  $x_{\perp}$
- Huge contrast with photons and jets !
  - $n^{\text{exp}}$  constant and slight above 4 at all  $x_{\perp}$

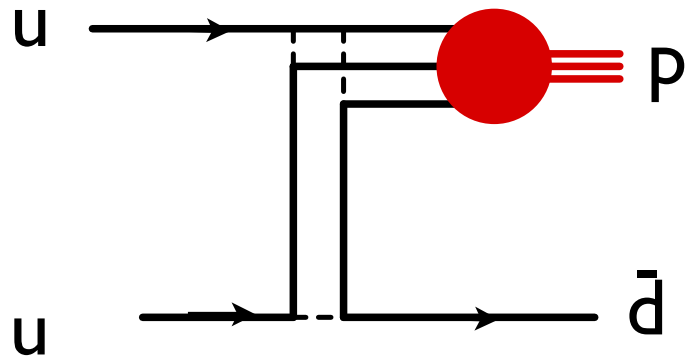
# Direct Contribution to Hadron Production



$$\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_\perp, y)}{p_\perp^6}$$

No Fragmentation Function

# Direct Proton Production



$$n_{\text{active}} = 6$$

$$E \frac{d\sigma}{d^3p}(p p \rightarrow p X) \sim \frac{F(x_{\perp}, \vartheta^{\text{cm}})}{p_{\perp}^8}$$

**Explains “Baryon anomaly” at RHIC**

Sickles, sjb

*Baryon can be made directly within hard subprocess*

**Coalescence  
within hard  
subprocess**

**Bjorken**  
**Blankenbecler, Gunion, sjb**  
**Berger, sjb**  
**Hoyer, et al: Semi-Exclusive**

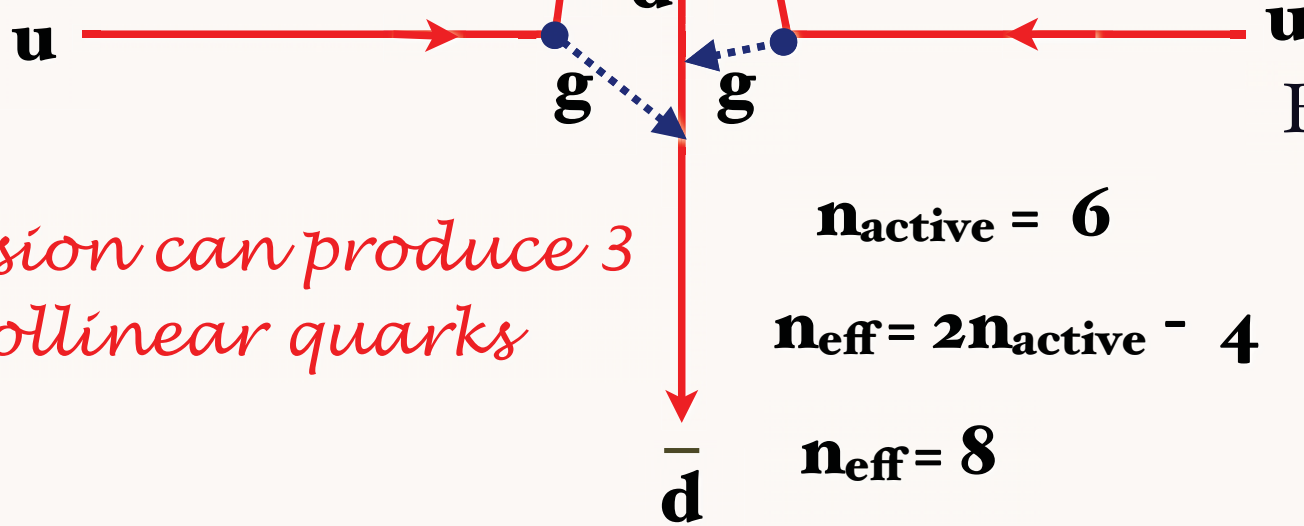
**p**

$$uu \rightarrow p\bar{d}$$

$$\phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2$$

**Sickles; sjb**

*Small color-singlet*  
*Color Transparent*  
*Minimal same-side energy*



Baryon anomaly

$$qq \rightarrow B\bar{q}$$

$$n_{\text{active}} = 6$$

$$n_{\text{eff}} = 2n_{\text{active}} - 4$$

$$n_{\text{eff}} = 8$$

*Collision can produce 3  
collinear quarks*



# Chiral Symmetry Breaking in AdS/QCD

Erlich et  
al.

- **Chiral symmetry breaking effect in AdS/QCD depends on weighted  $z^2$  distribution, not constant condensate**

$$\delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \phi^2(z) z^2$$

- **$z^2$  weighting consistent with higher Fock states at periphery of hadron wavefunction**
- **AdS/QCD: confined condensate**
- **Suggests “In-Hadron” Condensates**

de Teramond, Shrock, sjb

In presence of quark masses the Holographic LF wave equation is ( $\zeta = z$ )

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2} \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (1)$$

and thus

$$\boxed{\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.} \quad (2)$$

The parameter  $a$  is determined by the Weisberger term

$$a = \frac{2}{\sqrt{x}}.$$

Thus

$$\boxed{X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,} \quad (3)$$

and

$$\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi} \psi \rangle \langle z^2 \rangle + \langle \bar{\psi} \psi \rangle^2 \langle z^4 \rangle, \quad (4)$$

where we have used the sum over fractional longitudinal momentum  $\sum_i x_i = 1$ .

*Mass shift from dynamics inside hadronic boundary*

## Chiral magnetism (or magnetohydrochironics)

Aharon Casher and Leonard Susskind  
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel  
(Received 20 March 1973)

### I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.<sup>1</sup> Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.<sup>2</sup> A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.<sup>3</sup>

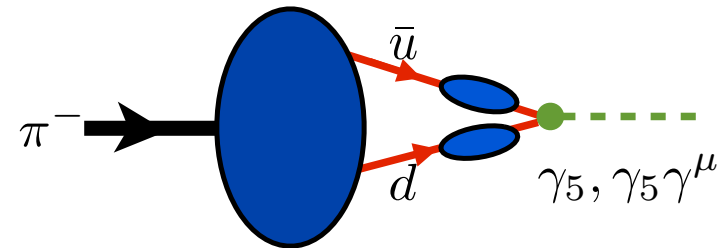
*Light-Front  
Formalism*

# Bethe-Salpeter Analysis

$$f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \gamma^\mu \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

Maris,  
Roberts, Tandy

$f_H$  Meson Decay Constant  
 $T_H$  flavor projection operator,  
 $Z_2(\Lambda), Z_4(\Lambda)$  renormalization constants  
 $\mathcal{S}(p)$  dressed quark propagator  
 $\Gamma_H(q; P) = F.T. \langle H | \psi(x_a) \bar{\psi}(x_b) | 0 \rangle$   
 Bethe-Salpeter bound-state vertex amplitude.



$$i\rho_\zeta^H \equiv \frac{-\langle q\bar{q} \rangle_\zeta^H}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \mathcal{S}(\frac{1}{2}P + q) \Gamma_H(q; P) \mathcal{S}(\frac{1}{2}P - q)]$$

*In-Hadron Condensate!*

$$f_H m_H^2 = -\rho_\zeta^H \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q$$

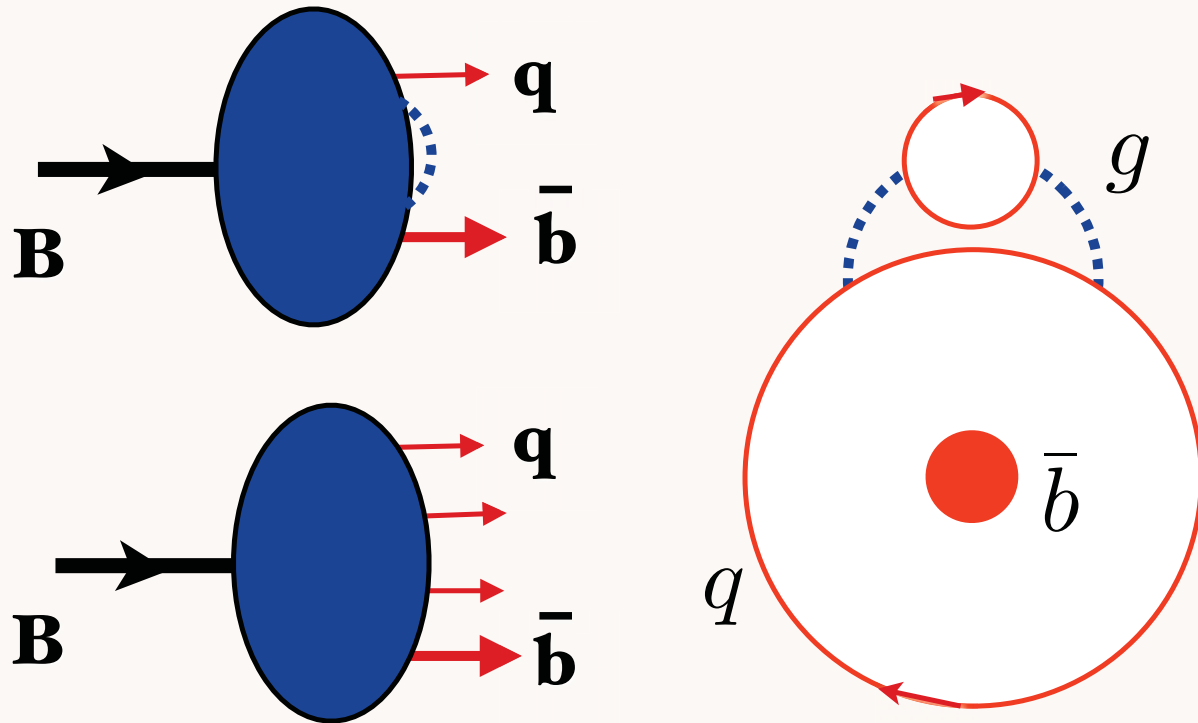
$$m_\pi^2 \propto (m_q + m_{\bar{q}}) / f_\pi \quad \text{GMOR}$$

# Simple physical argument for "in-hadron" condensate

Roberts, Shrock, Tandy, sjb

**Gribov pairs**

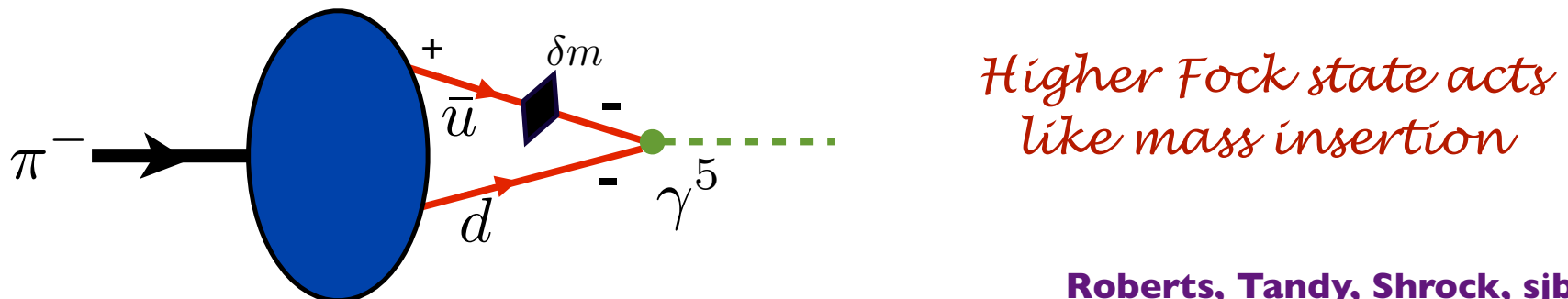
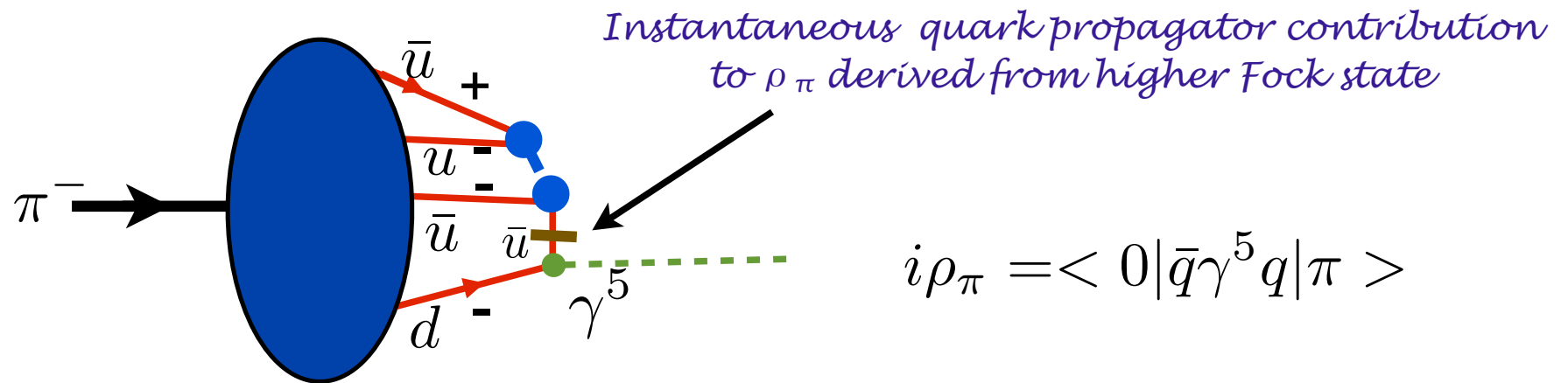
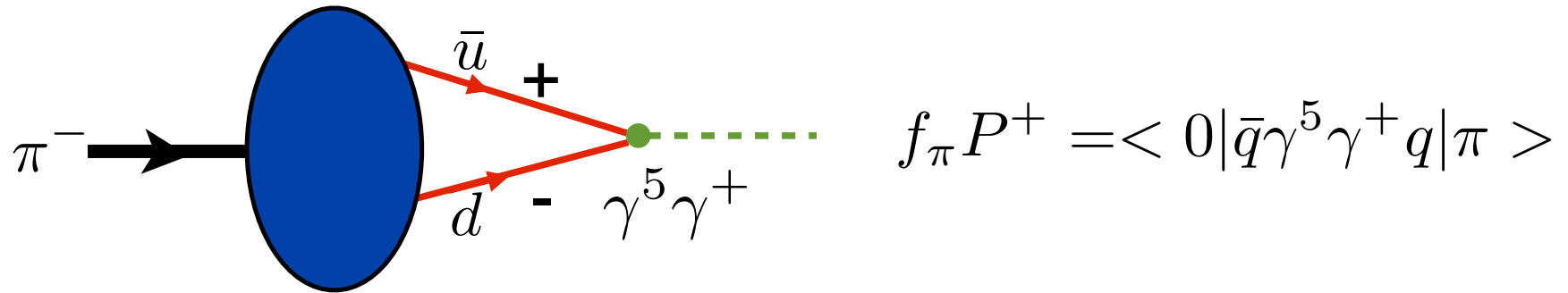
*B-Meson*



*Use Dyson-Schwinger Equation for bound-state quark propagator:  
find confined condensate*

$$\langle B | \bar{q}q | B \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle$$

# Higher Light-Front Fock State of Pion Simulates DCSB



Roberts, Tandy, Shrock, sjb

# Essence of the vacuum quark condensate

Stanley J. Brodsky,<sup>1,2</sup> Craig D. Roberts,<sup>3,4</sup> Robert Shrock,<sup>5</sup> and Peter C. Tandy<sup>6</sup>

<sup>1</sup>*SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309*

<sup>2</sup>*Centre for Particle Physics Phenomenology: CP<sup>3</sup>-Origins,  
University of Southern Denmark, Odense 5230 M, Denmark*

<sup>3</sup>*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

<sup>4</sup>*Department of Physics, Peking University, Beijing 100871, China*

<sup>5</sup>*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794*

<sup>6</sup>*Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA*

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

**ArXiv: 10005.4610**

# *Quark and Gluon condensates reside within hadrons, not vacuum*

**Casher and Susskind    Maris, Roberts, Tandy    Shrock and sjb**

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant --  
Eliminates 45 orders of magnitude conflict**

**R. Shrock, sjb  
PNAS  
ArXiv:0905.1151**



*“One of the gravest puzzles of  
theoretical physics”*

**DARK ENERGY AND  
THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA  
Kavil Institute for Theoretical Physics, University of California,  
Santa Barbara, CA 93106, USA  
zee@kitp.ucsb.edu*

$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$

$$\Omega_{\Lambda} = 0.76(\text{expt})$$

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$

*QCD Problem Solved if Quark and Gluon condensates reside  
within hadrons, not vacuum!*

**R. Shrock, sjb**

arXiv:0905.1151 [hep- th], Proc. Nat'l. Acad. Sci., (in press);  
“Condensates in Quantum Chromodynamics and the Cosmological Constant.”

*Quark and Gluon condensates reside within  
hadrons, not LF vacuum*

**Maris, Roberts,  
Tandy**

**Casher  
Susskind**

- **Bound-State Dyson-Schwinger Equations**
- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**
- **Finite size phase transition - infinite # Fock constituents**
- **AdS/QCD Description -- CSB is in-hadron Effect**
- **Analogous to finite-size superconductor!**
- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**
- **Implications for cosmological constant**

**Shrock, sjb**

*“Confined QCD Condensates”*

# Determinations of the vacuum Gluon Condensate

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle [\text{GeV}^4]$$

$-0.005 \pm 0.003$  from  $\tau$  decay.

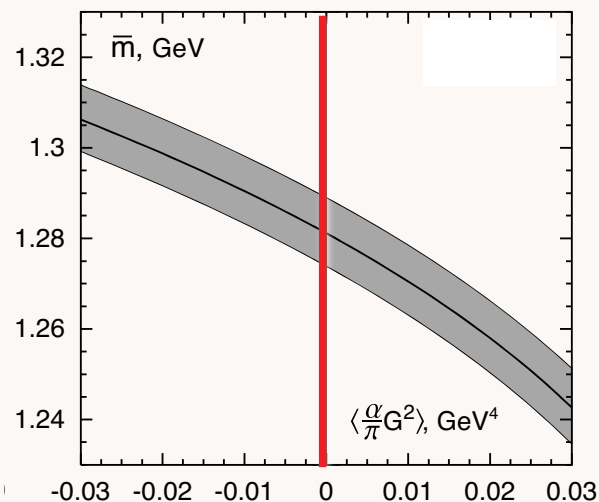
Davier et al.

$+0.006 \pm 0.012$  from  $\tau$  decay.

Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$  from charmonium sum rules

Ioffe, Zyablyuk



*Consistent with zero  
vacuum condensate*

# Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS<sub>5</sub>**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite  $N_c = 3$ : Baryons built on 3 quarks -- Large  $N_c$  limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

# *An analytic first approximation to QCD*

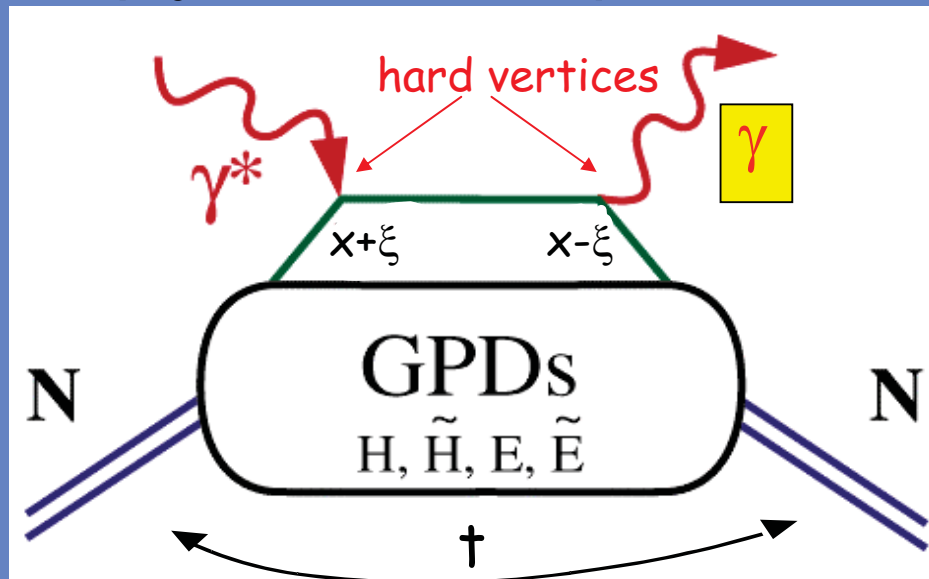
## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable  $\zeta$  conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter  $\kappa$**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

# GPDs & Deeply Virtual Exclusive Processes

## - New Insight into Nucleon Structure

### Deeply Virtual Compton Scattering (DVCS)



$x$  - quark momentum fraction

$\xi$  - longitudinal momentum transfer

$\sqrt{-t}$  - Fourier conjugate to transverse impact parameter

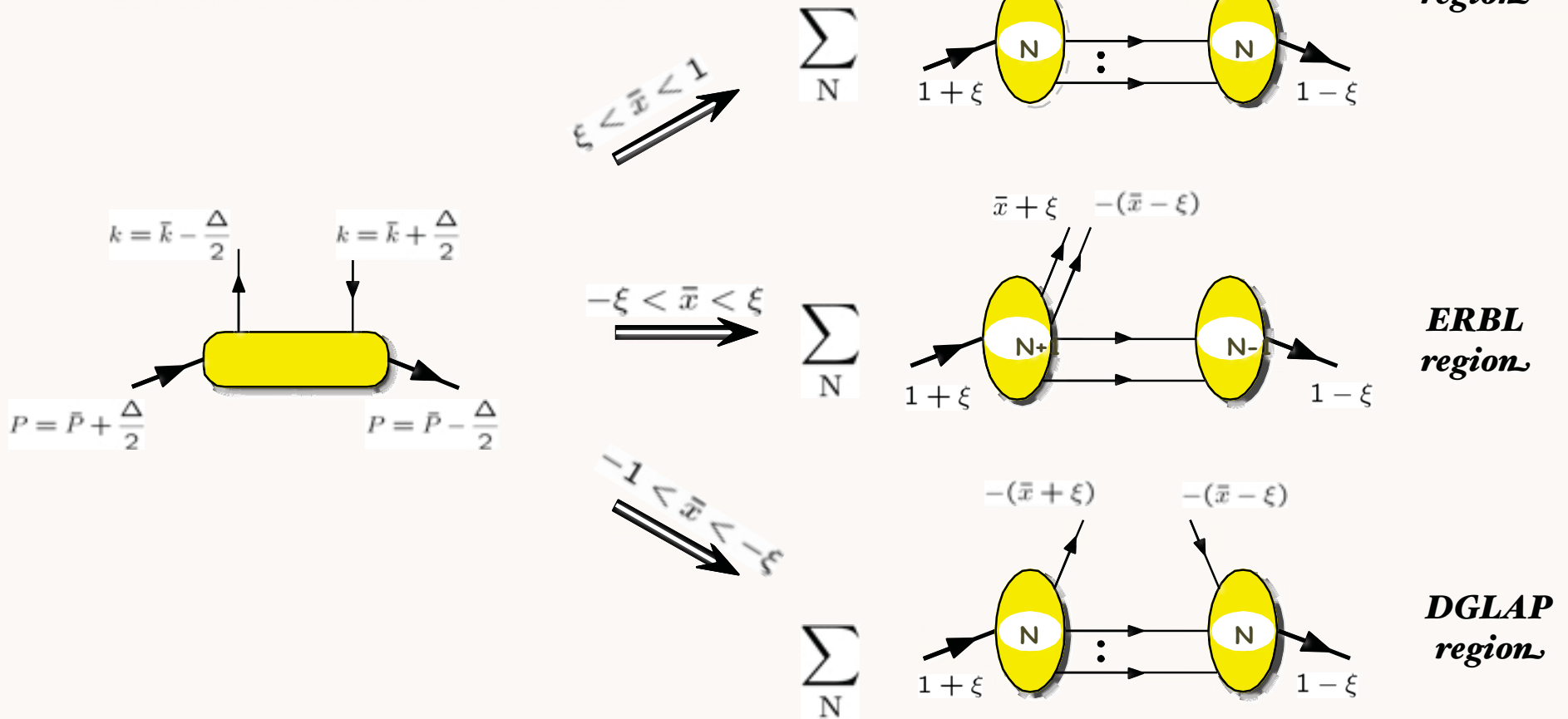
$H(x, \xi, t), E(x, \xi, t), \dots$  "Generalized Parton Distributions"

# Light-Front Wave Function Overlap Representation

## DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll





# Example of LFWF representation of GPDs ( $n+1 \Rightarrow n-1$ )

Diehl, Hwang, sjb

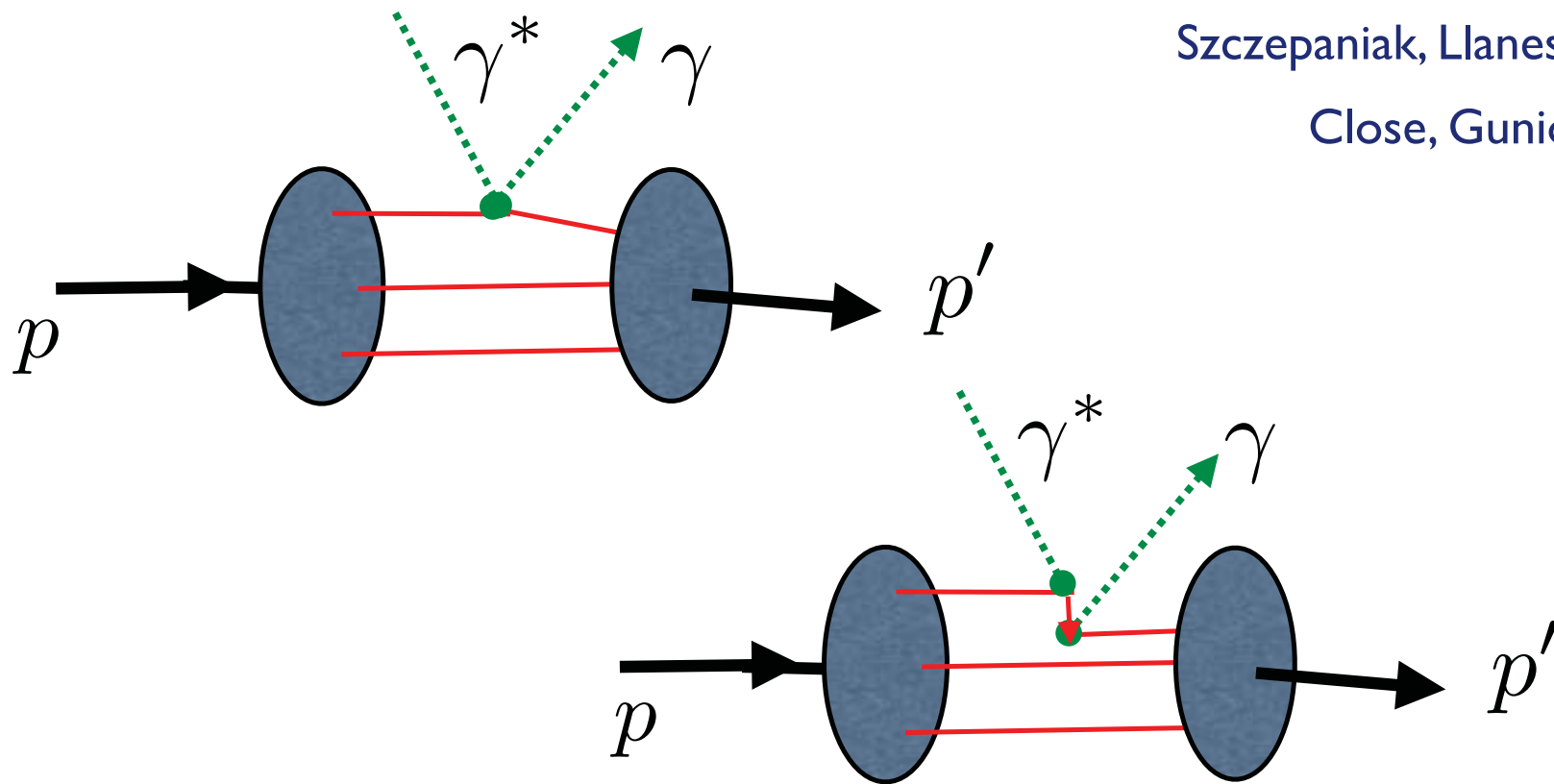
$$\begin{aligned}
 & \frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\Delta^2}{2M} E_{(n+1 \rightarrow n-1)}(x, \zeta, t) \\
 &= (\sqrt{1-\zeta})^{3-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n+1} \frac{dx_i d^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^{n+1} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n+1} \vec{k}_{\perp j}\right) \\
 & \quad \times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_{\perp}) \\
 & \quad \times \delta(x - x_1) \psi_{(n-1)}^{\uparrow*}(x'_i, \vec{k}'_{\perp i}, \lambda_i) \psi_{(n+1)}^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},
 \end{aligned}$$

where  $i = 2, \dots, n$  label the  $n - 1$  spectator partons which appear in the final-state hadron wavefunction with

$$x'_i = \frac{x_i}{1 - \zeta}, \quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1 - \zeta} \vec{\Delta}_{\perp}.$$

# $J=0$ Fixed Pole Contribution to DVCS

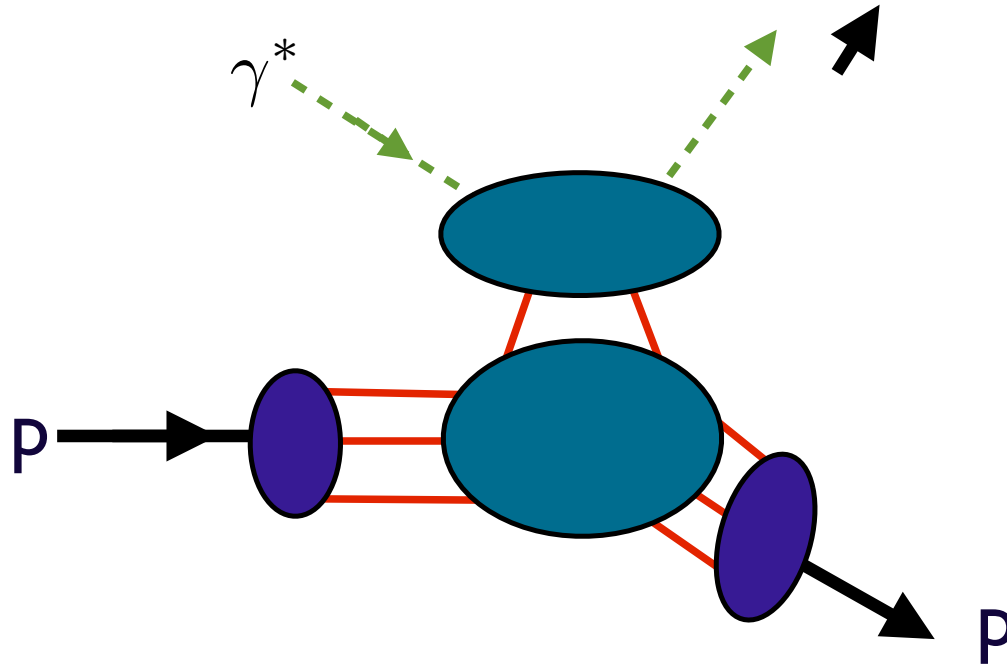
- $J=0$  fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator



Real amplitude, independent of  $Q^2$  at fixed  $t$

# Deeply Virtual Compton Scattering

$$\gamma^* p \rightarrow \gamma p$$



*Hard Reggeon Domain*

$$s \gg -t, Q^2 \gg \Lambda_{QCD}^2$$

$$T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^\alpha(t) \beta_R(t)$$

$$\alpha_R(t) \rightarrow 0$$

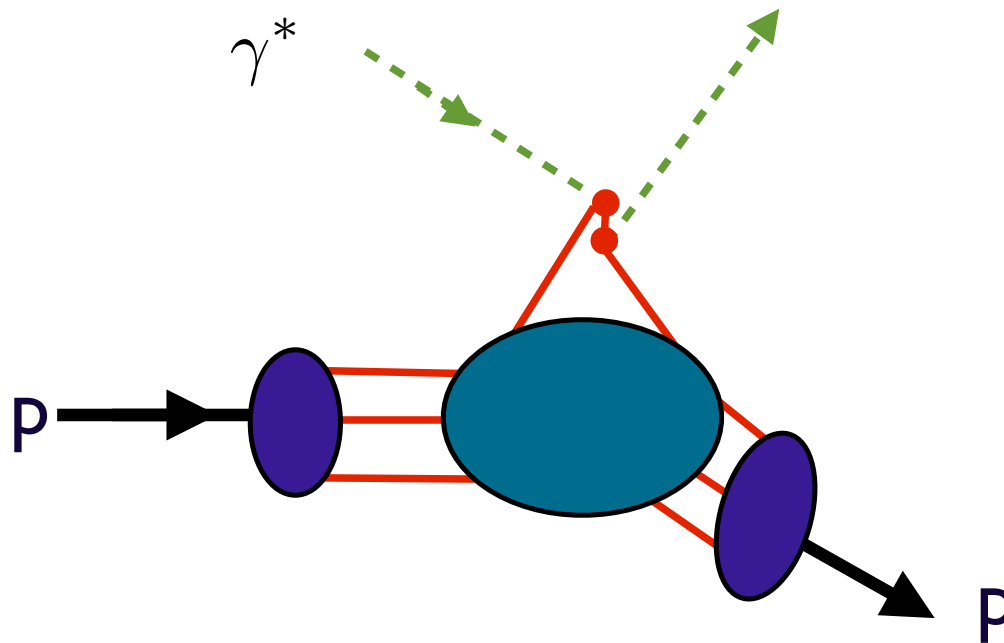
*Reflects elementary coupling of two photons to quarks*

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

# Deeply Virtual Compton Scattering

$$\gamma^* p \rightarrow \gamma p$$



*Seagull interaction  
(instantaneous quark  
exchange or Z-graph)*

$$s \gg -t, Q^2 \gg \Lambda_{QCD}^2$$

*Hard Reggeon  
Domain*

$$T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^\alpha(t) \beta_R(t)$$

$$\alpha_R(t) \rightarrow 0$$

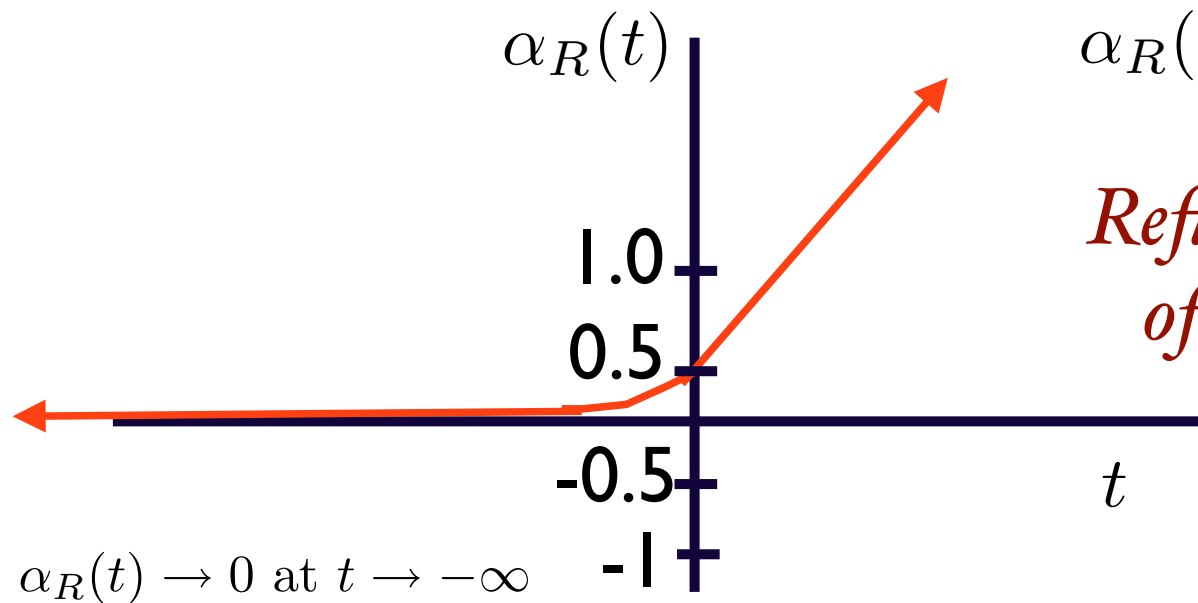
*Reflects elementary coupling of two photons to quarks*

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

# Regge domain

$$T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha_R(t)} \beta_R(t) \quad s \gg -t, Q^2$$



$$\alpha_R(t) \rightarrow 0 \text{ at } t \rightarrow -\infty$$

J=0 fixed pole

*Reflects elementary couplin  
of two photons to quarks*

$$\beta_R(t) \sim \frac{1}{t^2}$$

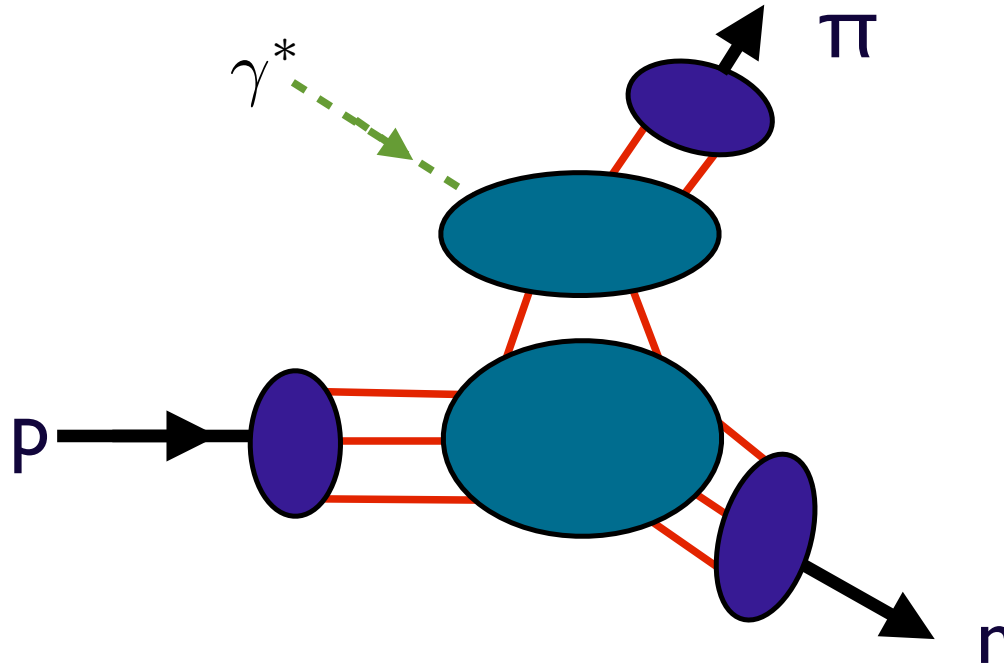
$$\alpha_R(t) \rightarrow 0 \text{ at } t \rightarrow -\infty$$

$$\frac{d\sigma}{dt}(\gamma^* p \rightarrow \gamma p) \rightarrow \frac{1}{s^2} \beta_R^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s}$$

*Fundamental test of QCD*

# Exclusive Electroproduction

$$ep \rightarrow e' \pi^+ n$$



*Hard Reggeon Domain*

$$s \gg -t, Q^2 \gg \Lambda_{QCD}^2$$

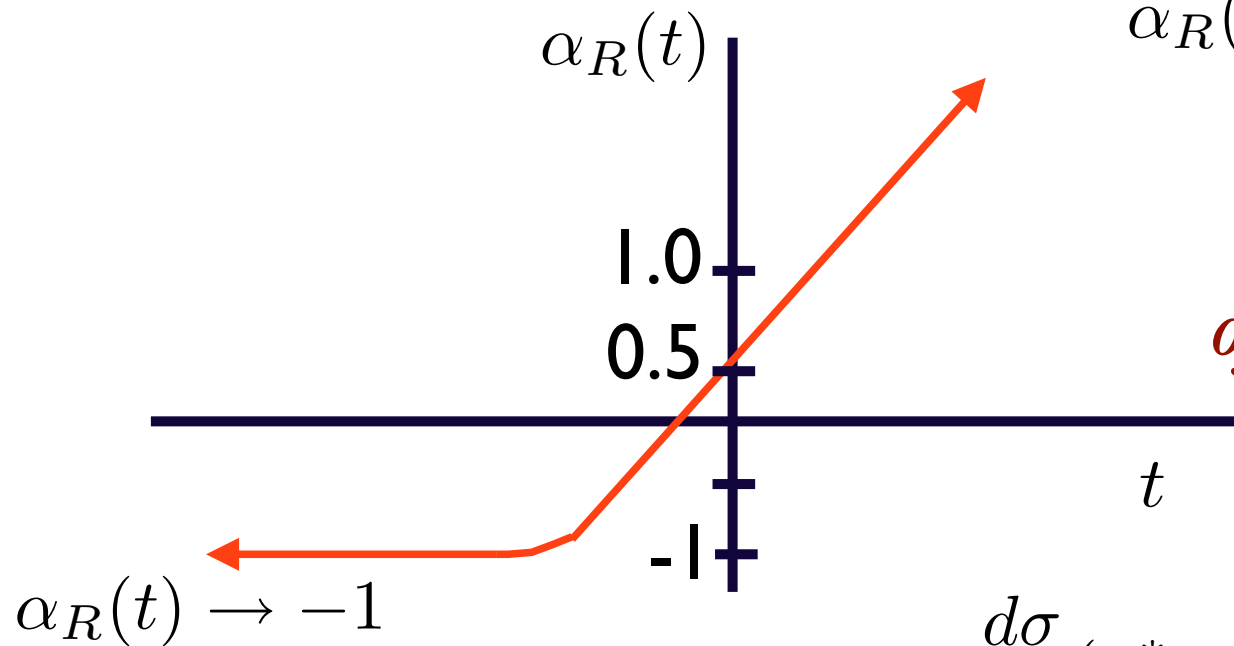
$$T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^\alpha(t) \beta_R(t)$$

$$\alpha_R(t) \rightarrow -1 \quad \text{Reflects elementary exchange of quarks in } t\text{-channel}$$

$$\beta_R(t) \sim \frac{1}{t^2} \quad \frac{d\sigma}{dt} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

# Regge domain

$$T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha_R(t)} \beta_R(t) \quad s \gg -t, Q^2$$



$$\alpha_R(t) \rightarrow -1 \text{ at } t \rightarrow -\infty$$

*Reflects elementary  
exchange  
of quarks in t-channel*

$$\beta_R(t) \sim \frac{1}{t^2}$$

$$\frac{d\sigma}{dt}(\gamma^* p \rightarrow \pi^+ n) \rightarrow \frac{1}{s^3} \beta_R^2(t)$$

$$\frac{d\sigma}{dt} \sim \frac{1}{s^3} \frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s}$$

*Fundamental test of QCD*

# *J=0 Fixed pole in real and virtual Compton scattering*

Damashek, Gilmar  
Close, Gunion, sjb  
Llanes-Estrada,  
Szczeponiak, sjb

Effective two-photon contact term

Seagull for scalar quarks

Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of  $Q^2$  at fixed  $t$

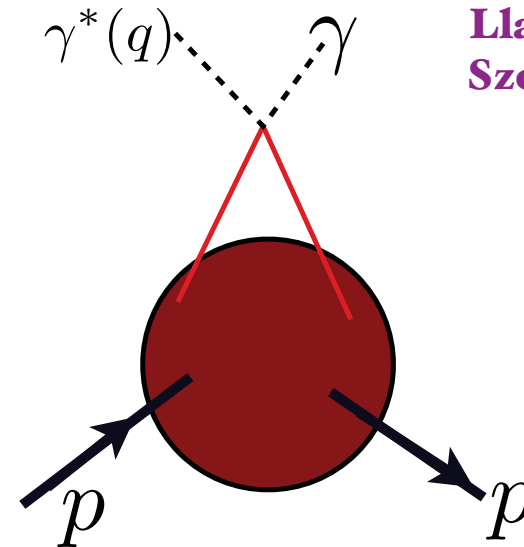
$\langle I/x \rangle$  Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

No ambiguity in D-term

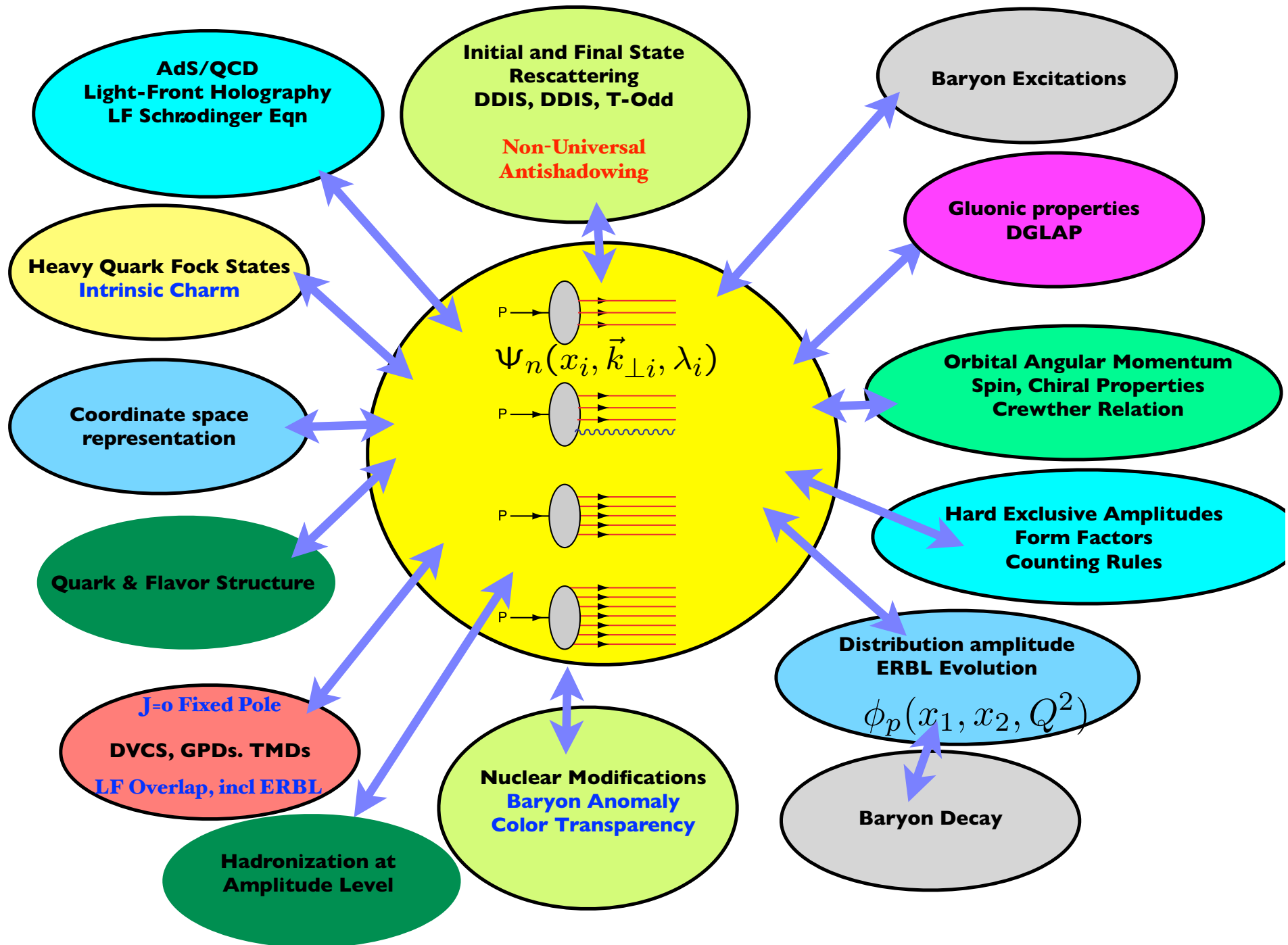
$Q^2$ -independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt} (\gamma^* p \rightarrow \gamma p) = F^2(t)$$





# QCD and the LF Hadron Wavefunctions



*Single-spin asymmetries*

**Leading Twist  
Sivers Effect**

Hwang,  
Schmidt, sjb

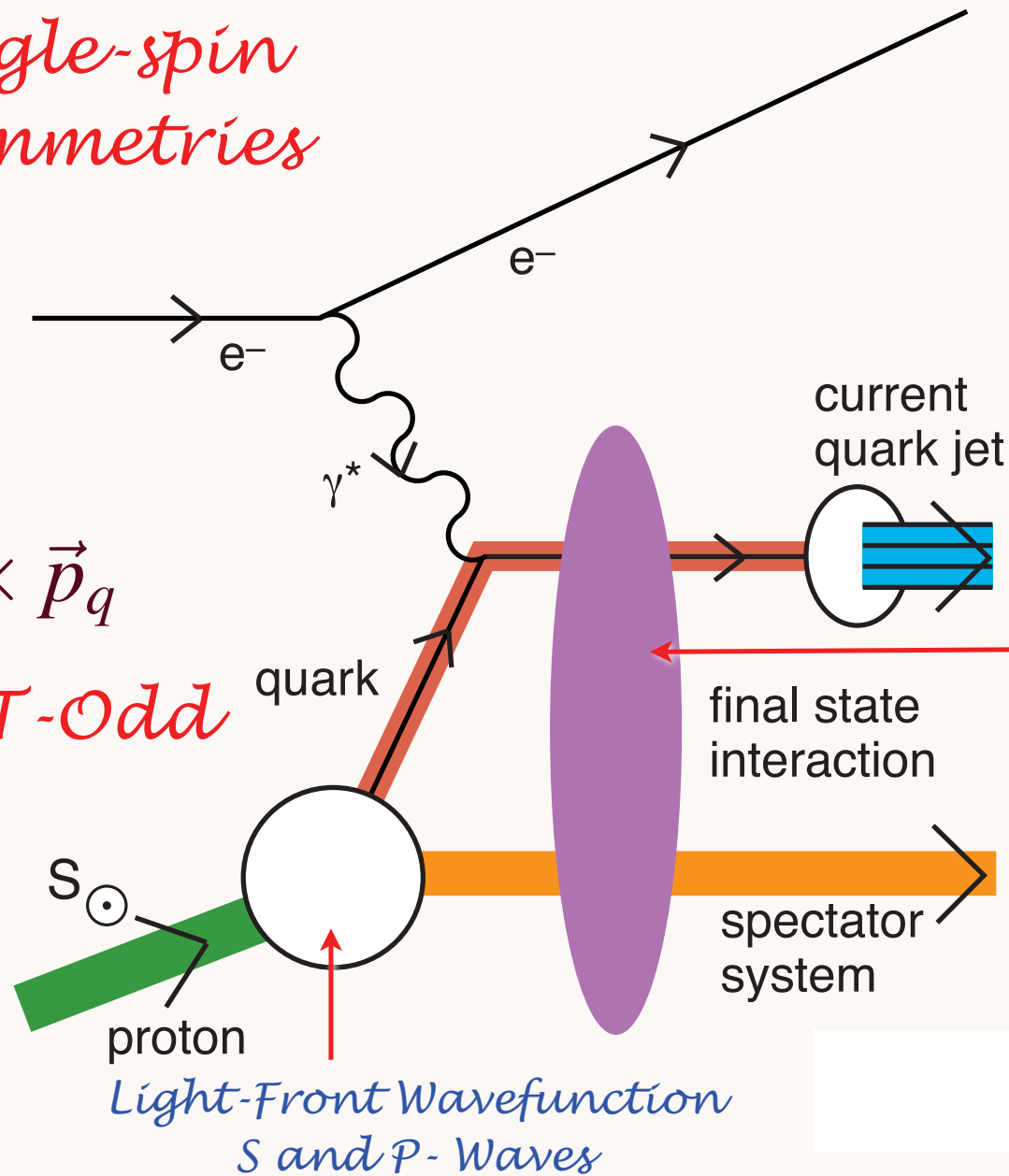
Collins, Burkardt  
Ji, Yuan

*QCD S- and P-  
Coulomb Phases  
--Wilson Line*

*Leading-Twist  
Rescattering  
Violates pQCD  
Factorization!*

$$i \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo-T-Odd*

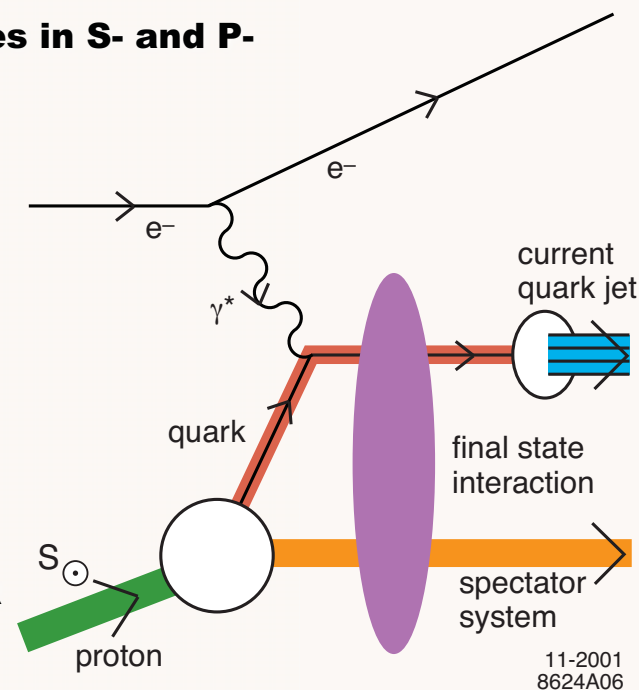


# Final-State Interactions Produce Pseudo-T-Odd (Sivers Effect)

Hwang, Schmidt, sjb  
Collins

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;**
- **Wilson line effect -- gauge independent**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**

$$\mathbf{i} \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$



Pasquini, Xiao, Yuan, sjb  
Mulders, Boer Qiu, Sterman

Trieste ICTP  
May 27, 2010

AdS/QCD and Hadronic Physics

130

Stan Brodsky  
SLAC-CP3

# Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero anomalous gavitomagnetic moment)

$$\vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

Hwang, Schmidt.  
sjb; Burkardt

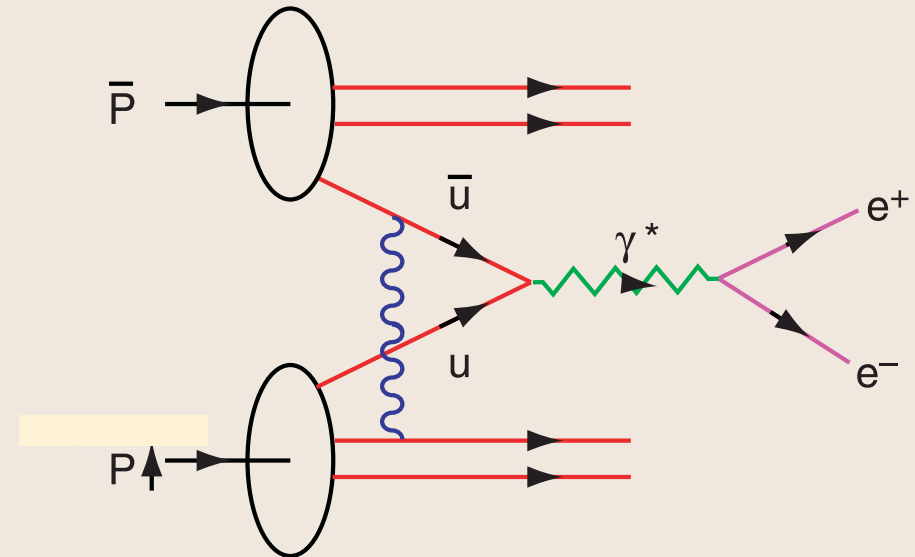
# Key QCD Experiment

Collins;  
Hwang,  
Schmidt, sjb

Measure single-spin asymmetry  $A_N$   
in Drell-Yan reactions

Leading-twist Bjorken-scaling  $A_N$   
from  $S, P$ -wave  
initial-state gluonic interactions

Predict:  $A_N(DY) = -A_N(DIS)$   
Opposite in sign!

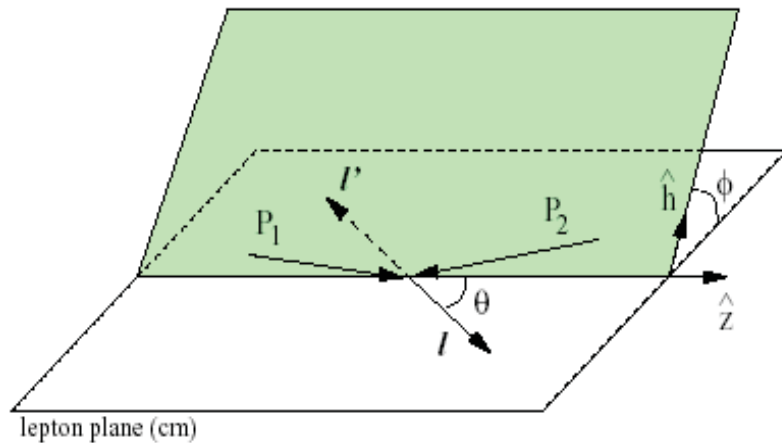


$$\bar{p} p_{\uparrow} \rightarrow l^{+} l^{-} X$$

$$\vec{S} \cdot \vec{q} \times \vec{p} \text{ correlation}$$

# Drell-Yan angular distribution

## *Unpolarized DY*



- Experimentally, a violation of the Lam-Tung sum rule is observed by sizeable  $\cos 2\Phi$  moments
- Several model explanations
  - higher twist
  - spin correlation due to non-trivial QCD vacuum
  - Non-zero Boer Mulders function

Lam – Tung SR :  $1 - \lambda = 2\nu$

NLO pQCD :  $\lambda \approx 1 \quad \mu \approx 0 \quad \nu \approx 0$

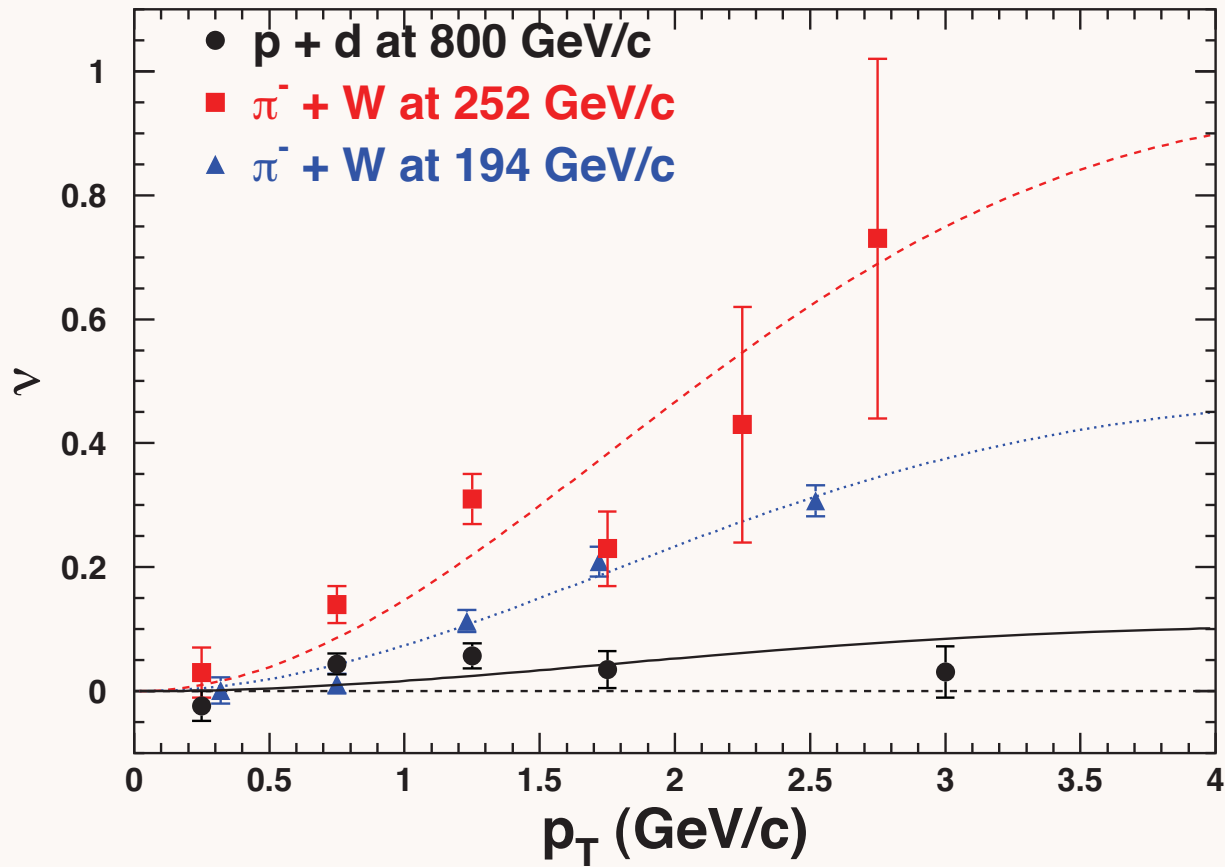
$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

Experiment:  $\nu \simeq 0.6$

**B. Seitz**

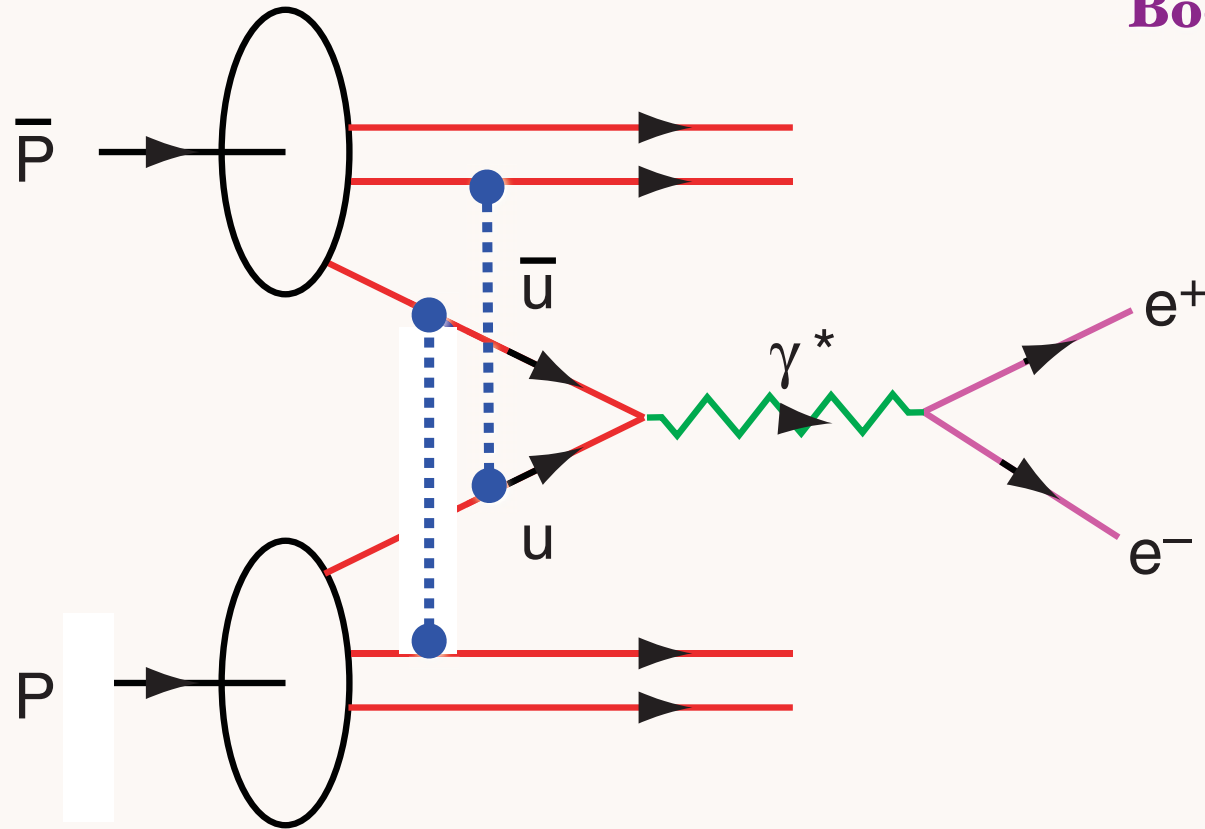
# Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)



Huge Effect in  
 $\pi W \rightarrow \mu^+ \mu^- X$   
 Negligible Effect  
 $pd \rightarrow \mu^+ \mu^- X$

Parameter  $\nu$  vs.  $p_T$  in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and  $M_C = 2.4 \text{ GeV}/c^2$  are also shown.



**$DY \cos 2\phi$  correlation at leading twist from double ISI**

*Product of Boer - Mulders Functions*

$$h_1^\perp(x_1, \mathbf{p}_\perp^2) \times \bar{h}_1^\perp(x_2, \mathbf{k}_\perp^2)$$



$$f_1 = \text{[Diagram: Yellow circle with a light blue dot in the center.]}$$

*Unpolarized  
Distribution*

$$g_{1L} = \text{[Diagram: Yellow circle with a light blue dot on the left and a right-pointing arrow.]} - \text{[Diagram: Yellow circle with a light blue dot on the right and a left-pointing arrow.]}$$

*Bj Sum Rule*

$$h_{1T} = \text{[Diagram: Yellow circle with a light blue dot at the bottom and an upward-pointing arrow.]} - \text{[Diagram: Yellow circle with a light blue dot at the top and a downward-pointing arrow.]}$$

*Transversity*

$$f_{1T}^\perp = \text{[Diagram: Yellow circle with a light blue dot in the center and an upward-pointing arrow.]} - \text{[Diagram: Yellow circle with a light blue dot in the center and a downward-pointing arrow.]}$$

*Sivers Function*

$$h_1^\perp = \text{[Diagram: Yellow circle with a light blue dot at the top and a downward-pointing arrow.]} - \text{[Diagram: Yellow circle with a light blue dot at the bottom and an upward-pointing arrow.]}$$

*Boer-Mulders  
Function*

*T-Odd:  
Require ISI or FSI*

# Double Initial-State Interactions

generate anomalous  $\cos 2\phi$

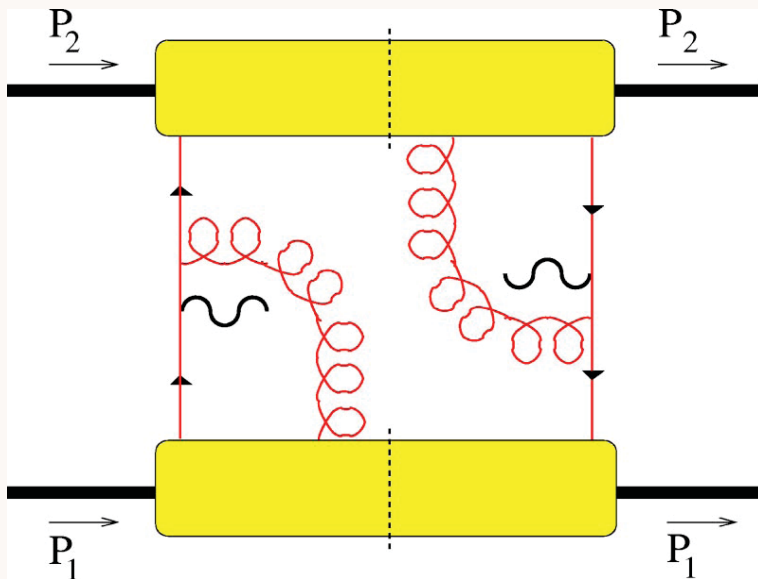
Boer, Hwang, sjb

## Drell-Yan planar correlations

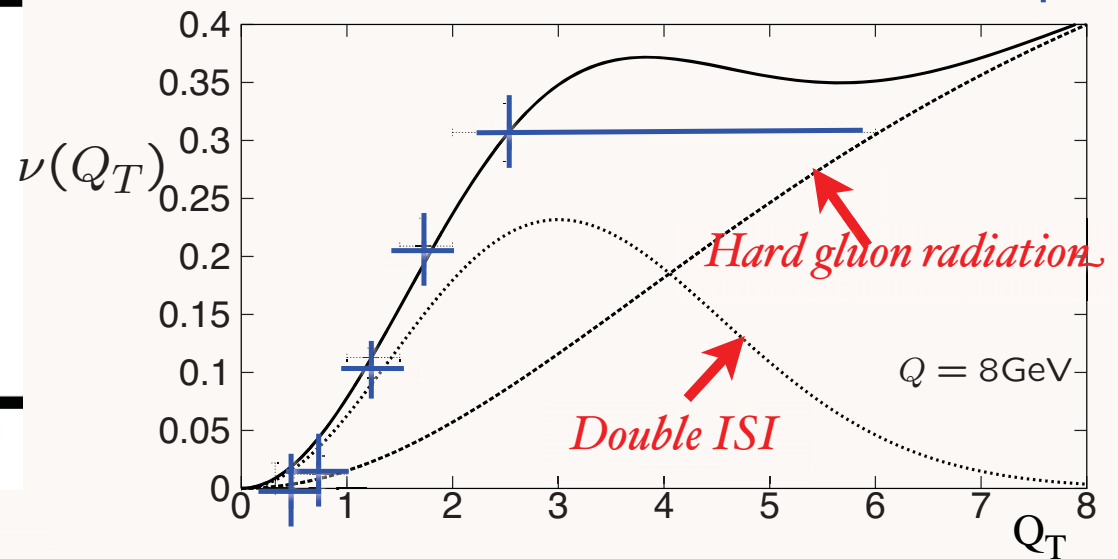
$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

PQCD Factorization (Lam Tung):  $1 - \lambda - 2\nu = 0$

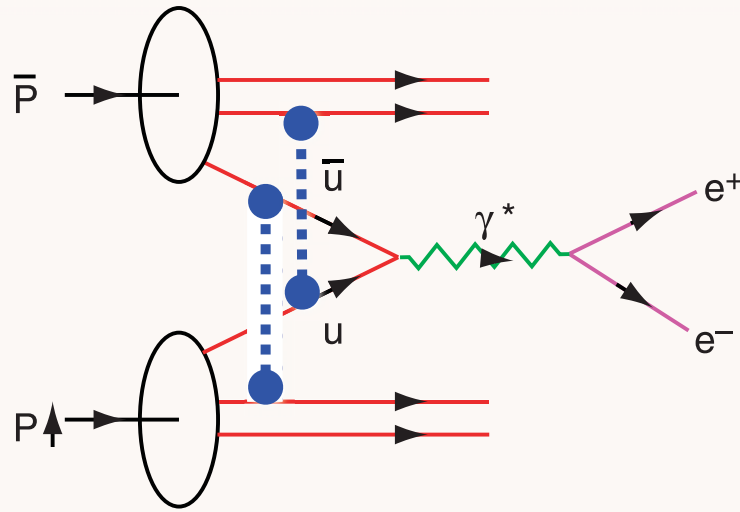
$$\frac{\nu}{2} \propto h_1^\perp(\pi) h_1^\perp(N)$$



**Violates Lam-Tung relation!**



# DY cos 2φ correlation at leading twist from double ISI



*Product of Boer -  
Mulders  
Functions*

$$h_1^\perp(x_1, \mathbf{p}_\perp^2) \times \bar{h}_1^\perp(x_2, \mathbf{k}_\perp^2)$$

$$F \equiv \mathcal{F}[(2\hat{\mathbf{h}} \cdot \mathbf{p}_\perp \hat{\mathbf{h}} \cdot \mathbf{k}_\perp - \mathbf{p}_\perp \cdot \mathbf{k}_\perp) h_1^\perp \bar{h}_1^\perp]$$

$$= \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_\perp) (2\hat{\mathbf{h}} \cdot \mathbf{p}_\perp \hat{\mathbf{h}} \cdot \mathbf{k}_\perp - \mathbf{p}_\perp \cdot \mathbf{k}_\perp) \times h_1^\perp(\Delta, \mathbf{p}_\perp^2) \bar{h}_1^\perp(\bar{\Delta}, \mathbf{k}_\perp^2),$$

$$\nu = \frac{2}{M_1 M_2} \frac{\sum_{a, \bar{a}} e_a^2 F_a}{\sum_{a, \bar{a}} e_a^2 G_a}.$$

$$G \equiv \mathcal{F}[f_1 \bar{f}_1]$$

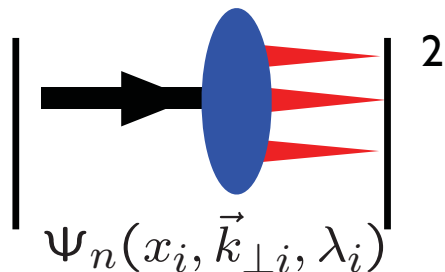
$$= \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_\perp) f_1(\Delta, \mathbf{p}_\perp^2) \bar{f}_1(\bar{\Delta}, \mathbf{k}_\perp^2),$$

(40)

**Boer, Hwang, sjb**

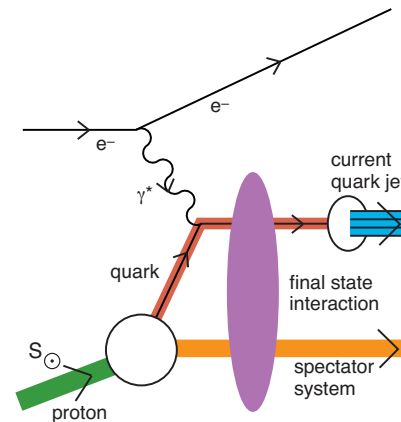
# Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and  $J^z$
- DGLAP Evolution; mod. at large  $x$
- No Diffractive DIS



# Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS



**Hwang,  
Schmidt, sjb,  
Mulders, Boer  
Qiu, Sterman  
Collins, Qiu  
Pasquini, Xiao,  
Yuan, sjb**

# *Applications of Nonperturbative Running Coupling from AdS/QCD*

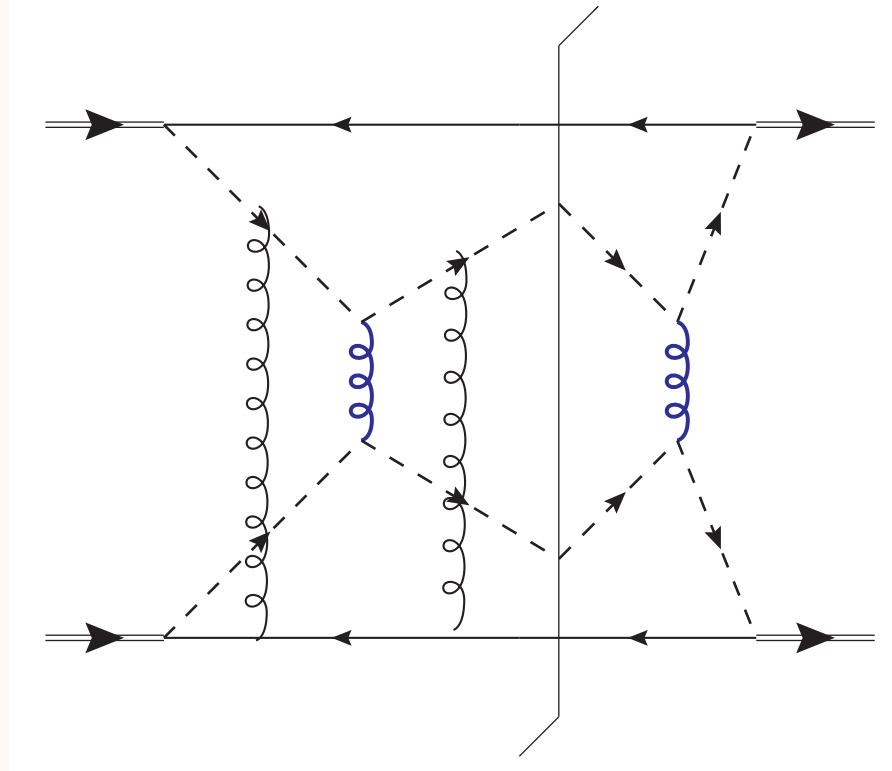
- **Sivers Effect in SIDIS, Drell-Yan**
- **Double Boer-Mulders Effect in DY**
- **Diffraction DIS**
- **Heavy Quark Production at Threshold**

*All involve gluon exchange at small  
momentum transfer*

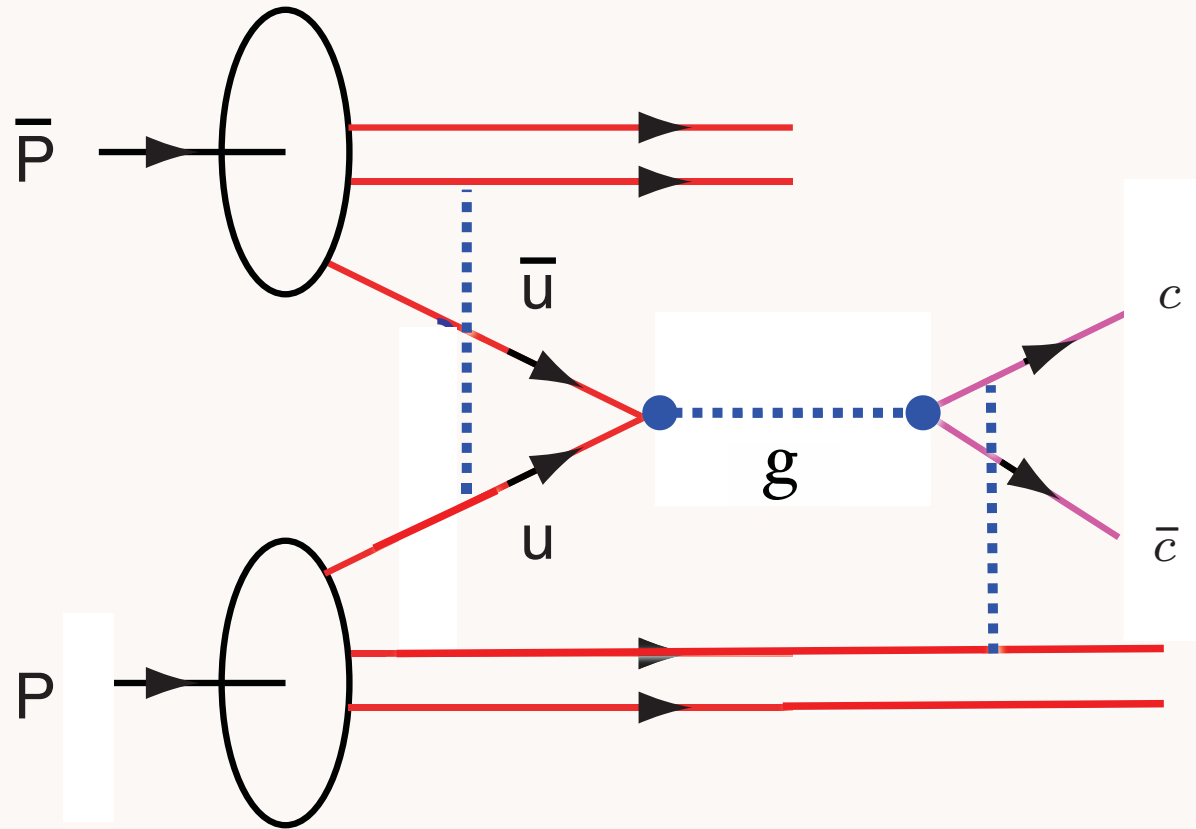
# Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins, [Jian-Wei Qiu](#) . ANL-HEP-PR-07-25, May 2007.

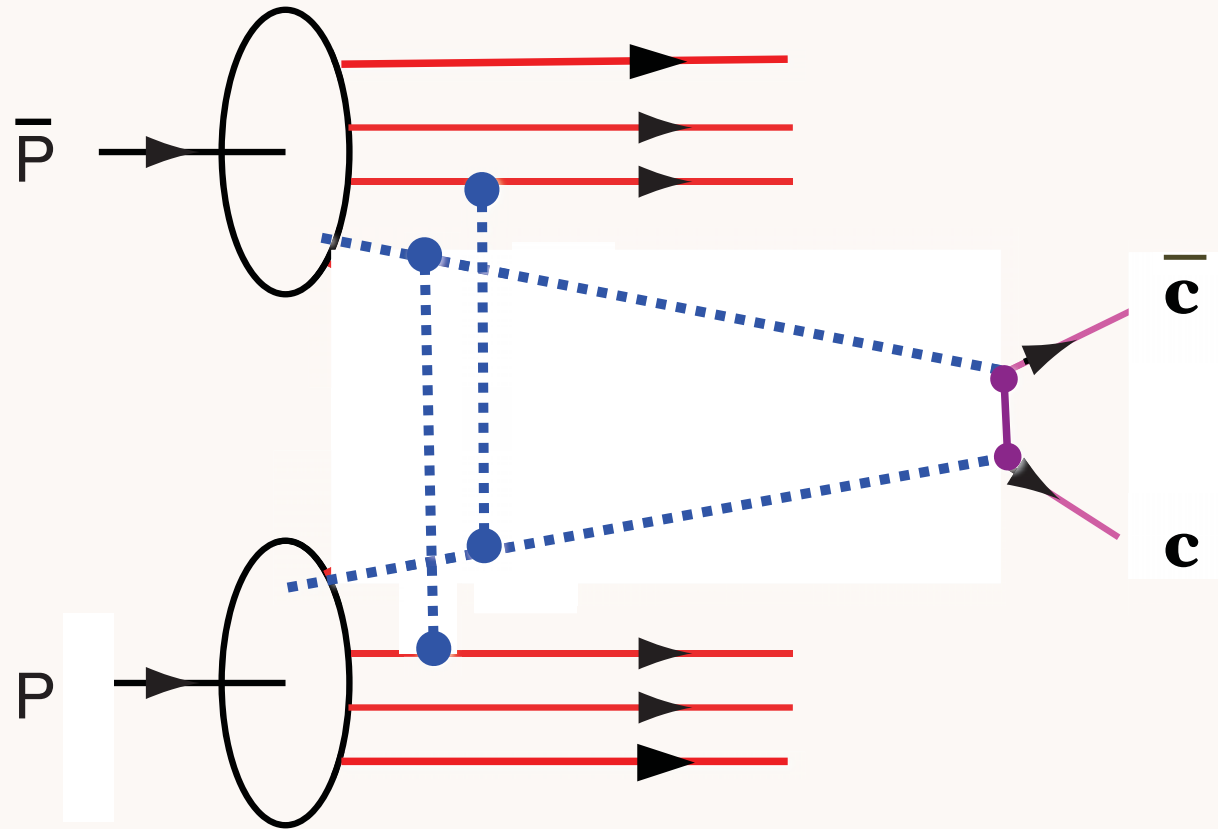
e-Print: [arXiv:0705.2141](#) [hep-ph]



The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

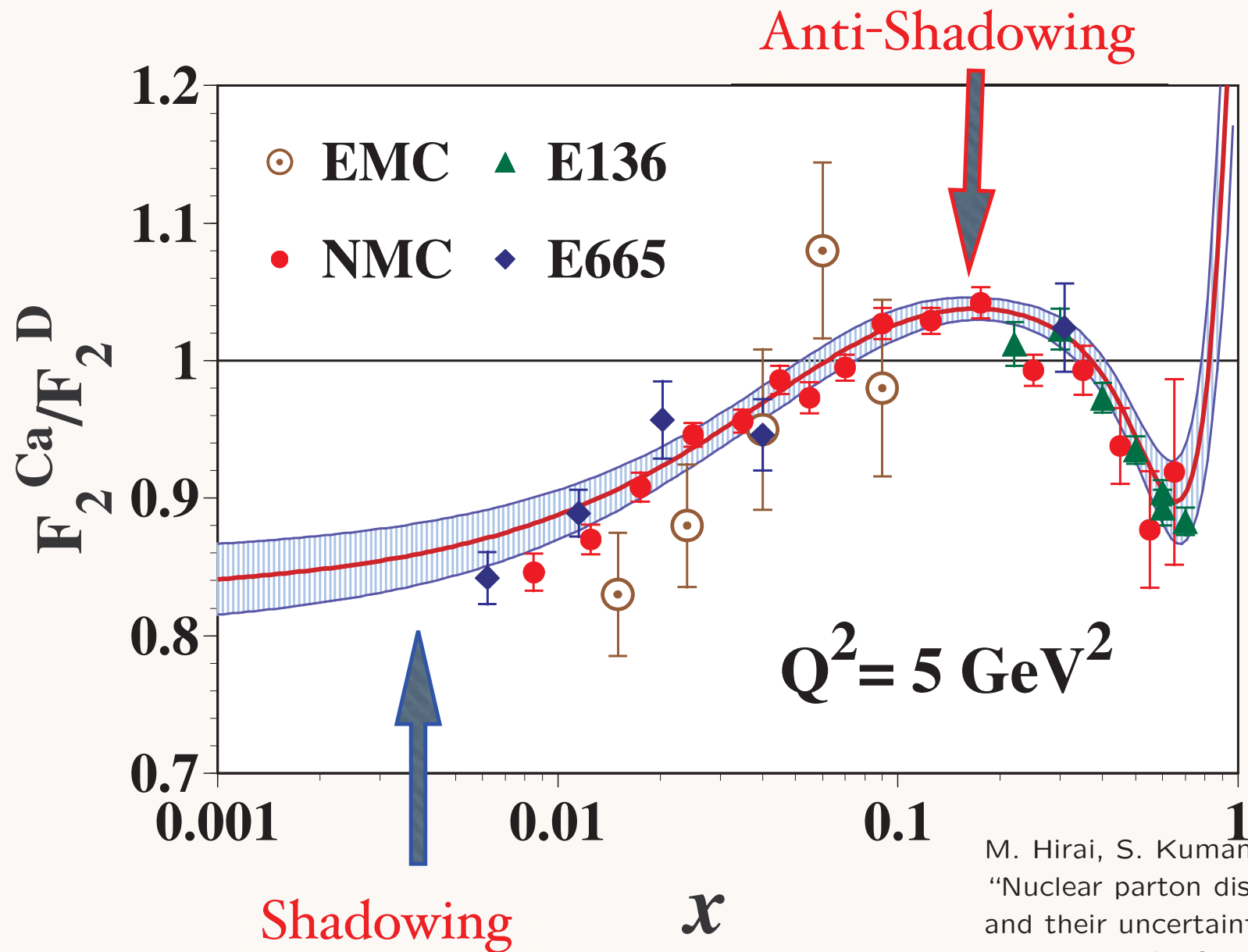


*Problem for factorization when both ISI and FSI occur*



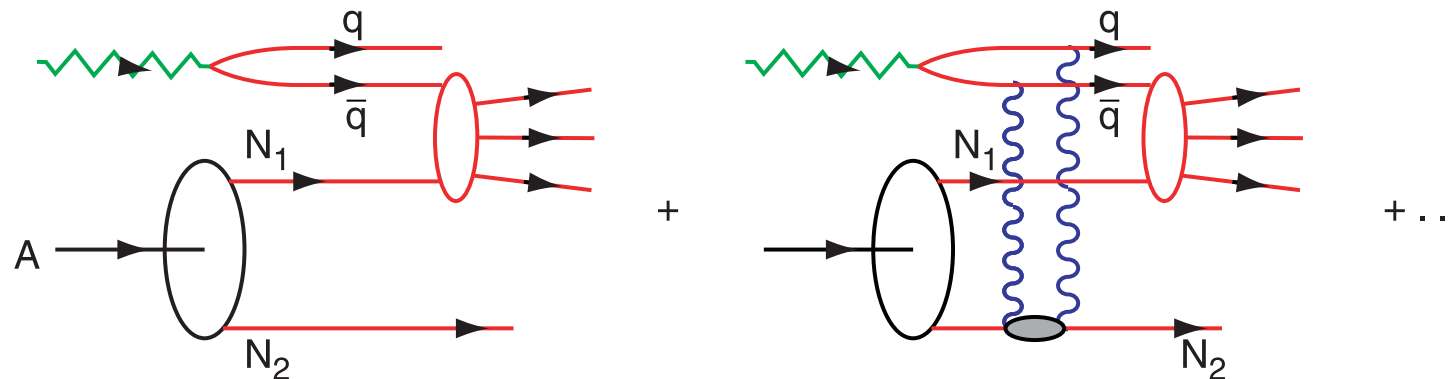
$\cos 2\phi$  correlation for quarkonium production at leading twist from double ISI  
**Enhanced by gluon color charge**





M. Hirai, S. Kumano and T. H. Nagai,  
 "Nuclear parton distribution functions  
 and their uncertainties,"  
 Phys. Rev. C **70**, 044905 (2004)  
 [arXiv:hep-ph/0404093].

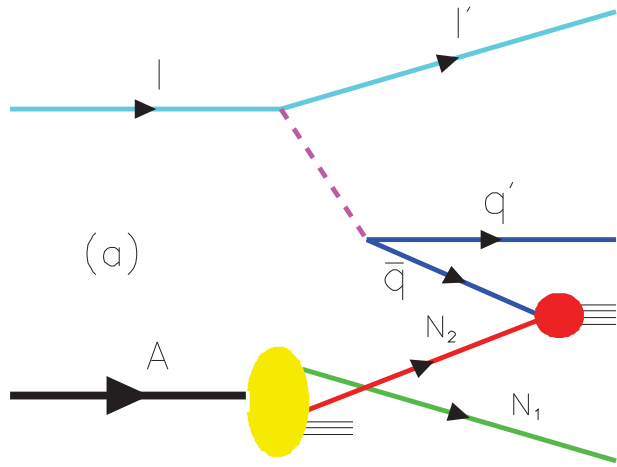
# Nuclear Shadowing in QCD



*Shadowing depends on understanding leading twist-diffraction in DIS*

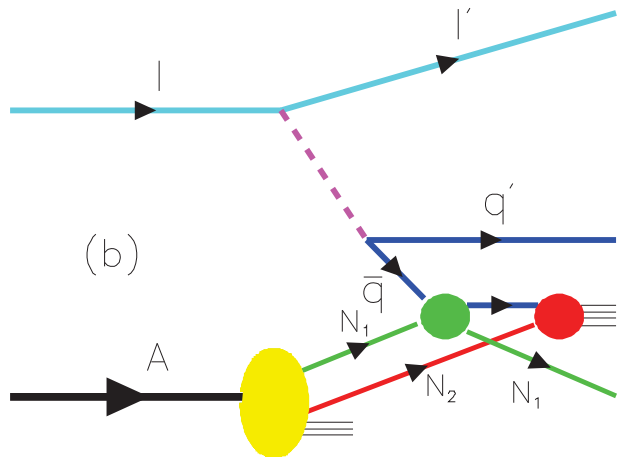
**Nuclear Shadowing not included in nuclear LFWF !**

**Dynamical effect due to virtual photon interacting in nucleus**



The one-step and two-step processes in DIS on a nucleus.

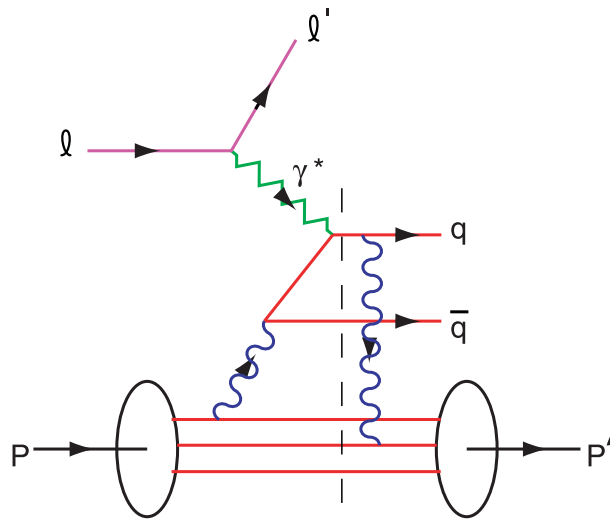
Coherence at small Bjorken  $x_B$  :  
 $1/Mx_B = 2\nu/Q^2 \geq L_A$ .



If the scattering on nucleon  $N_1$  is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the  $\bar{q}$  flux reaching  $N_2$ .

→ Shadowing of the DIS nuclear structure functions.

## Observed HERA DDIS produces nuclear shadowing



*Shadowing depends on leading-twist DDIS*

***Integration over on-shell domain produces phase  $i$***

***Need Imaginary Phase to Generate Pomeron***

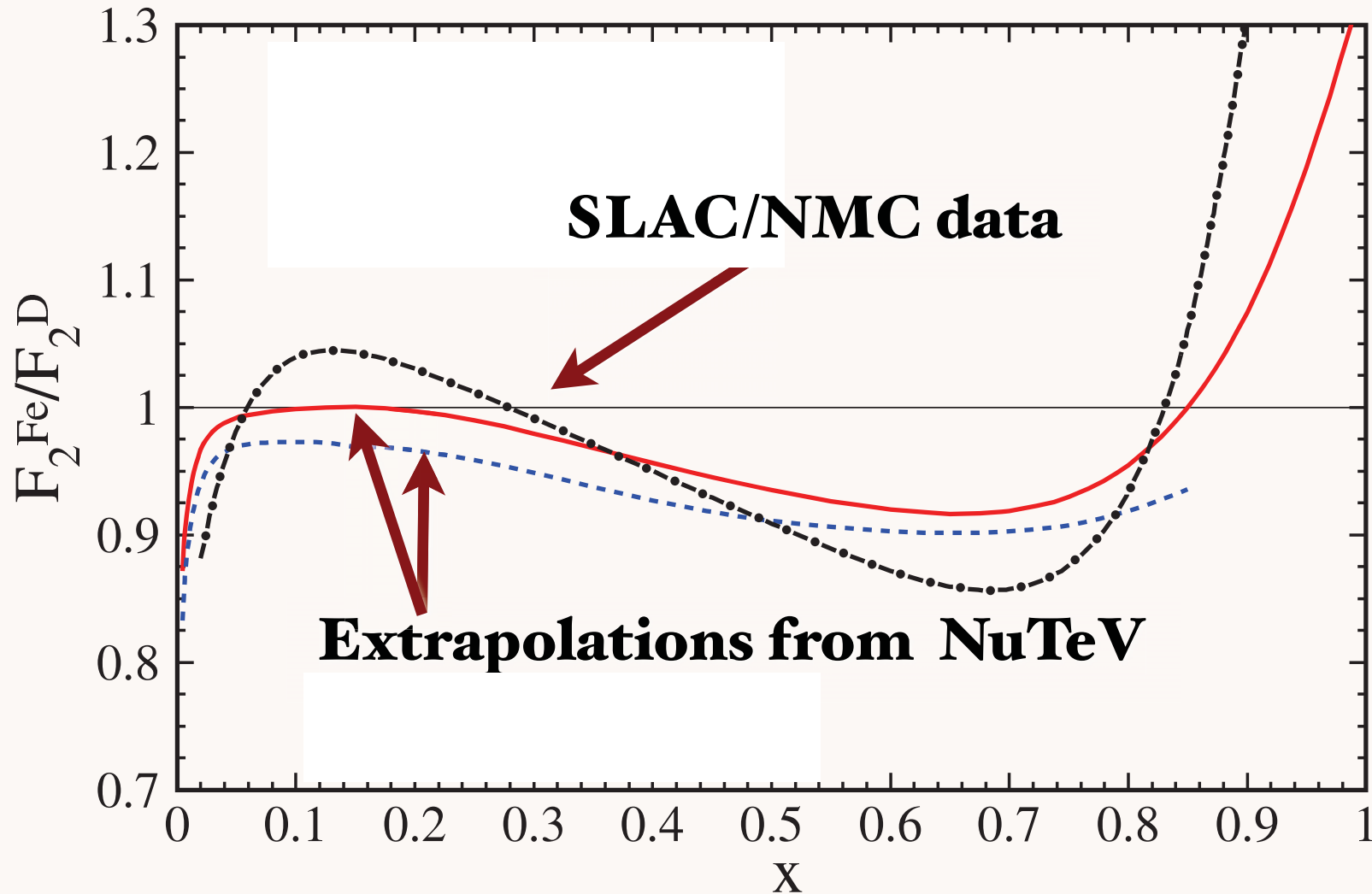
***Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry***

*Physics of FSI not in Wavefunction of Target*

*Antishadowing (Reggeon exchange) is not universal!*

**Schmidt, Yang, sjb**

$$Q^2 = 5 \text{ GeV}^2$$



*Scheinbein, Yu, Keppel, Morfin, Olness, Owens*

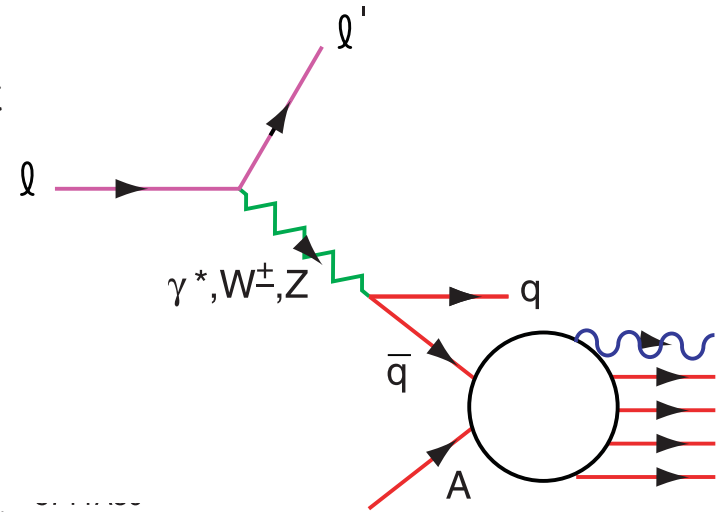
# Origin of Regge Behavior of Deep Inelastic Structure Functions

Antiquark interacts with target nucleus at energy  $\hat{s} \propto \frac{1}{x_{bj}}$

Regge contribution:  $\sigma_{\bar{q}N} \sim \hat{s}^{\alpha_R - 1}$

Nonsinglet Kuti-Weisskoff  $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$  at small  $x_{bj}$ .

Shadowing of  $\sigma_{\bar{q}M}$  produces shadowing of nuclear structure function.



Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Schmidt, Yang, Lu, sjb

# Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1 - i) \times i = \frac{1}{\sqrt{2}}(i + 1)$$

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of  $\gamma^*$ ,  $Z^0$ ,  $W^\pm$

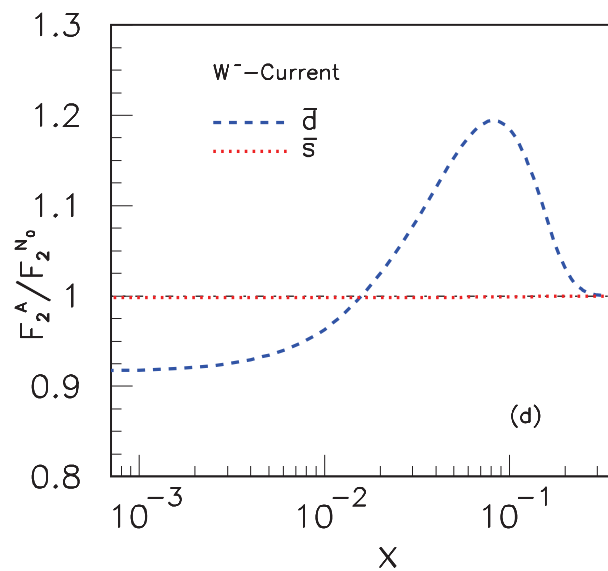
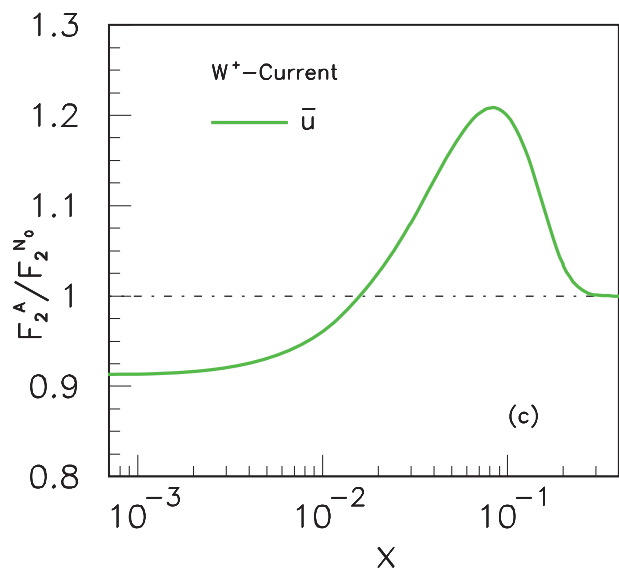
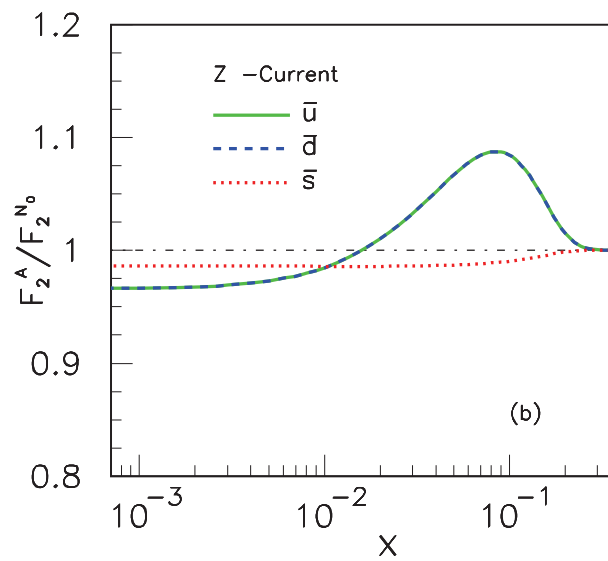
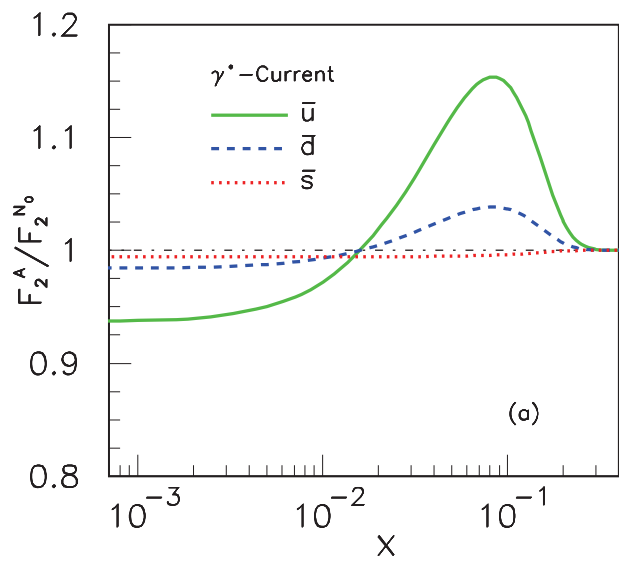
*Critical test: Tagged Drell-Yan*

# Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: **Destructive Interference** of Two-Step and One-Step Processes  
*Pomeron Exchange*
- Antishadowing: **Constructive Interference** of Two-Step and One-Step Processes!  
*Reggeon and Odderon Exchange*
- Antishadowing is Not Universal!  
Electromagnetic and weak currents:  
different nuclear effects !  
**Potentially significant for NuTeV Anomaly}**

Jian-Jun Yang  
Ivan Schmidt  
Hung Jung Lu  
sjb





Schmidt, Yang; sjb

**Modifies  
NuTeV  
extraction of**

$$\sin^2 \theta_W$$

*Nuclear Antishadowing not universal!*

- **Color Confinement: Maximum Wavelength of Quark and Gluons**
- **Conformal symmetry of QCD coupling in IR**
- **Conformal Template (BLM, CSR, BFKL scale)**
- **Motivation for AdS/QCD**
- **QCD Condensates inside of hadronic LFWFs**
- **Technicolor: confined condensates inside of technihadrons -- alternative to Higgs**
- **Simple physical solution to cosmological constant conflict with Standard Model**

**Roberts, Shrock, Tandy, and sjb**