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Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond'

26 - 28 May 2010

Tsallis-Pareto like distributions in hadron-hadron collisions

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Tsallis-Pareto like distributions
in
hadron-hadron collisions

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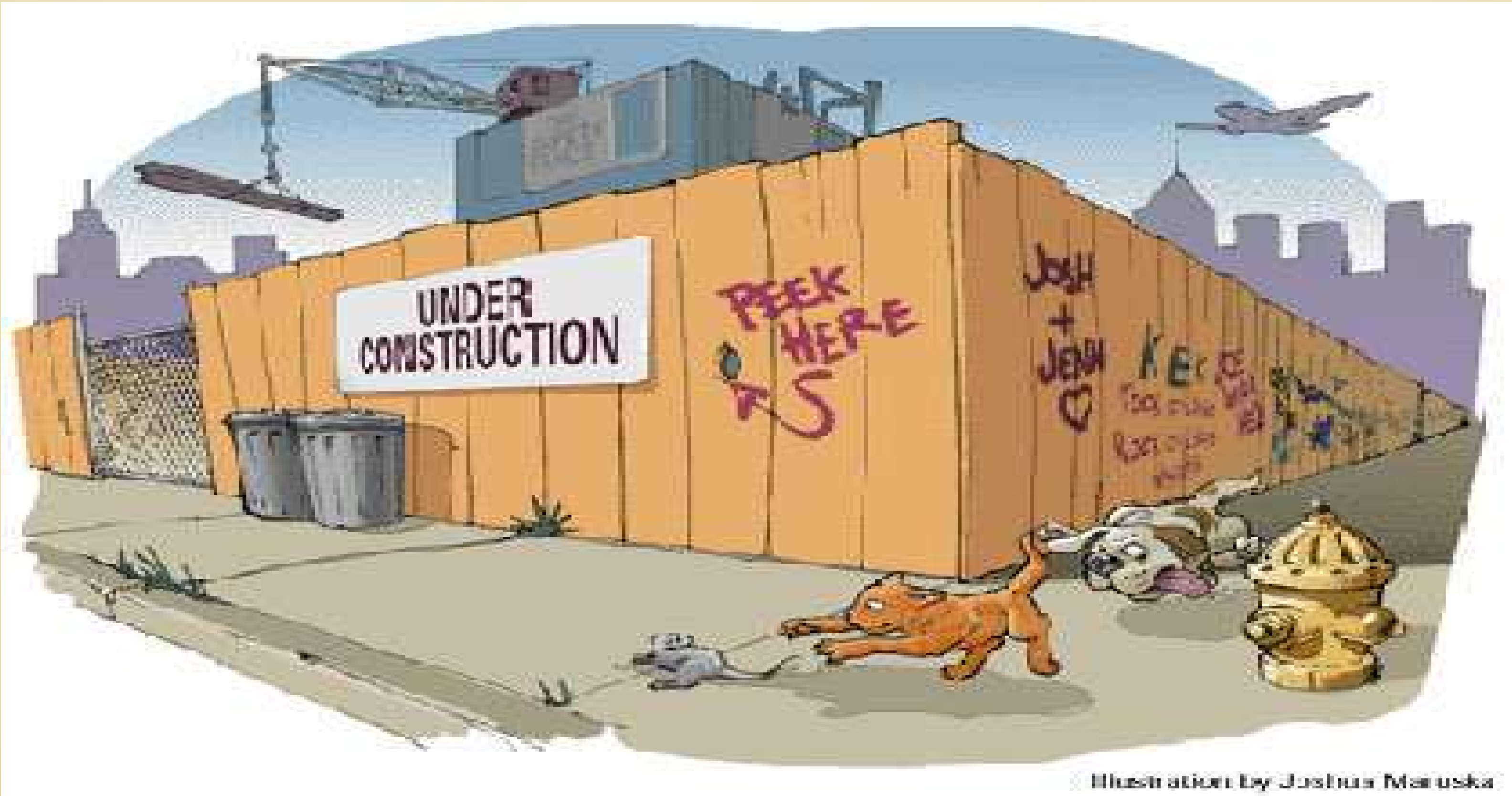
T.S. Bíró, G. Kalmár, K. Ürmössy, P. Ván

Gribov '80 Memorial Workshop, ICTP - Trieste, Italy 26-28 May 2010

Tsallis-Pareto like distributions
in
hadron-hadron collisions



History: Tsallis, Rényi, Pareto...

- Constantino Tsallis

  1943 (Greece) – (Brazil), statistical phyisicist

  Applying Generlised Entropy: $S_q(p) = \frac{1}{q-1}\left(1 - \int (p(x))^q \, dx\right),$

- Alfréd Rényi

  1921 – 1970 (Hungary), mathematician

  Define a Generalised Enrtopy: $H_\alpha(X) = \frac{1}{1-\alpha} \log\left(\sum_{i=1}^{n} p_i^\alpha\right)$

- Vilfredo Federico Damaso Pareto

  1848 (Paris) – 1923 (Geneva), (Italian)

  industrialist, sociologist, economist, philosopher

  Generalised Pareto Distrib.: $f_{(\xi,\mu,\sigma)}(x) = \frac{1}{\sigma}\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{\left(-\frac{1}{\xi}-1\right)}.$

O U T L I N E

- Motivation:

  CMS data, (V)HMPID, measuring FFs

- Mysteries of hadron spectra

  Hadron spectra at high & low p_T

- Notes on fragmentation

  Fragmentation functions, and parton evolution

- On the non-extensive termodynamics

  E.g. in AA (pp) collisons or within a jet, generally non-thermalised → non-extensive thermodynamics

- Simple joint models, tests & outlook

M O T I V A T I O N

- Measuring jet fragmentation

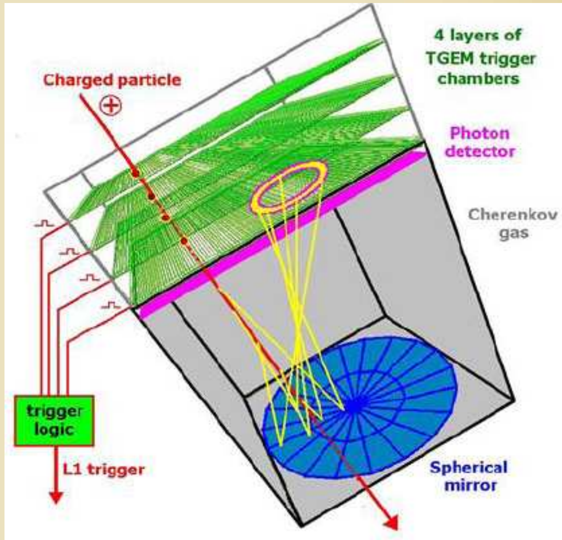
  Available: Jet fragmentation can be measured via jet analysis, data are from: DESY, Tevatron, RHIC, LHC...

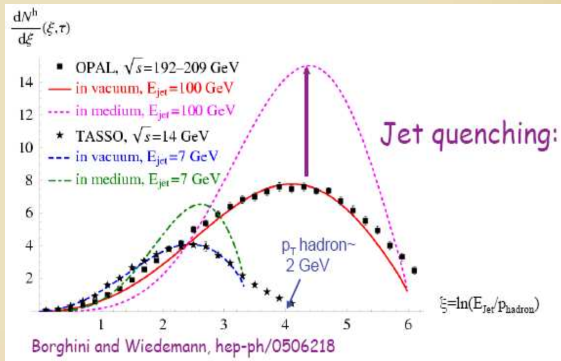
- Future PID measurement in HIC

  Under development: ALICE VHMPID detectors might measure it

- Fragmentation in "matter"

  Aim/dream: Measure differential jet-parameters, identifying final state effects (jet-quenching modifications).





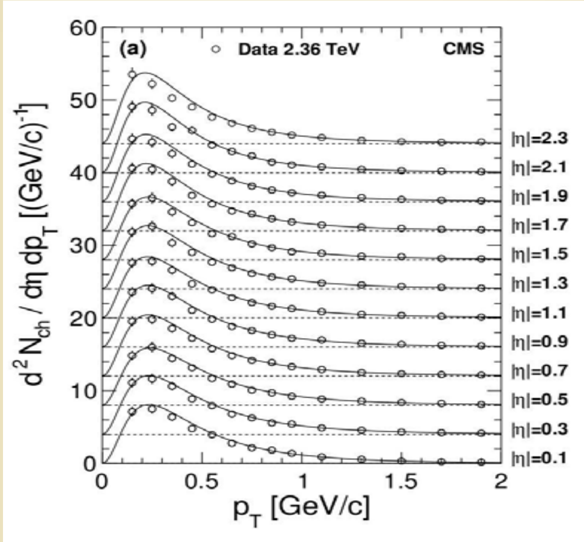
MOTIVATION

- New LHC data (CMS)

  K. Krajczár et al.: JHEP 1002:041(2010)

  fitted Tsallis distribution for p_T spectra:

$$E\frac{d^3N_{\rm ch}}{dp^3} = \frac{1}{2\pi p_T}\frac{E}{p}\frac{d^2N_{\rm ch}}{d\eta dp_T} = C(n,T,m)\frac{dN_{\rm ch}}{dy}\left(1+\frac{E_T}{nT}\right)^{-n}$$



  Parameters:

  0.9 TeV T= 130 MeV, q=1.13

  2.36 TeV T= 140 MeV, q=1.15 $n := (q-1)^{-1}$

- RHIC analysis on AuAu data (y=0)

  Cooper-Frye model: K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

  Parameters: $f(E) = A[1+(q-1)E/T]^{-1/(q-1)}$

  200 GeV T= 51 MeV, q= 1.062 (fit for p_T < 6 GeV/c)

Mysteries of hadron spectra

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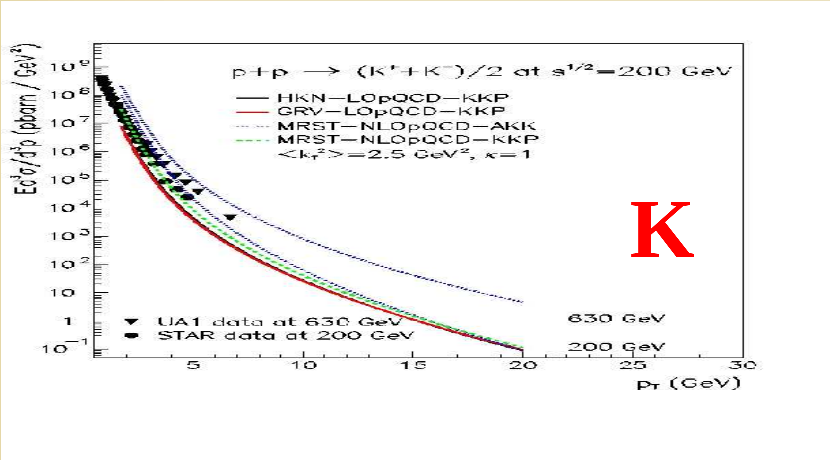
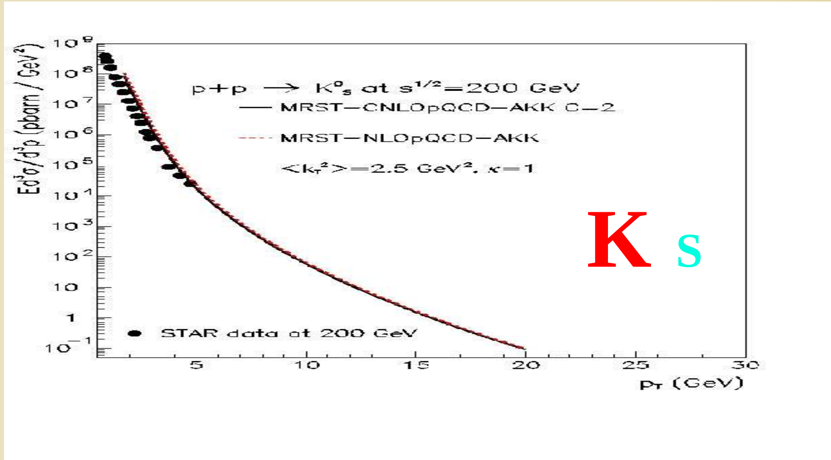
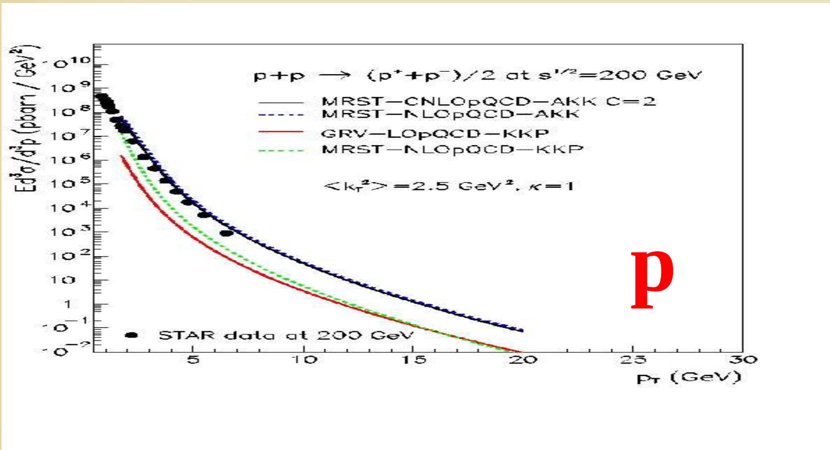
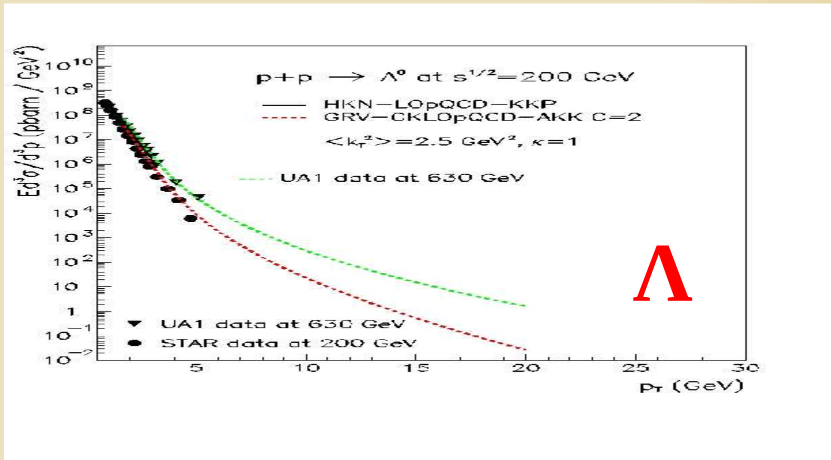
open questions

pQCD with AKK FF and coalescence

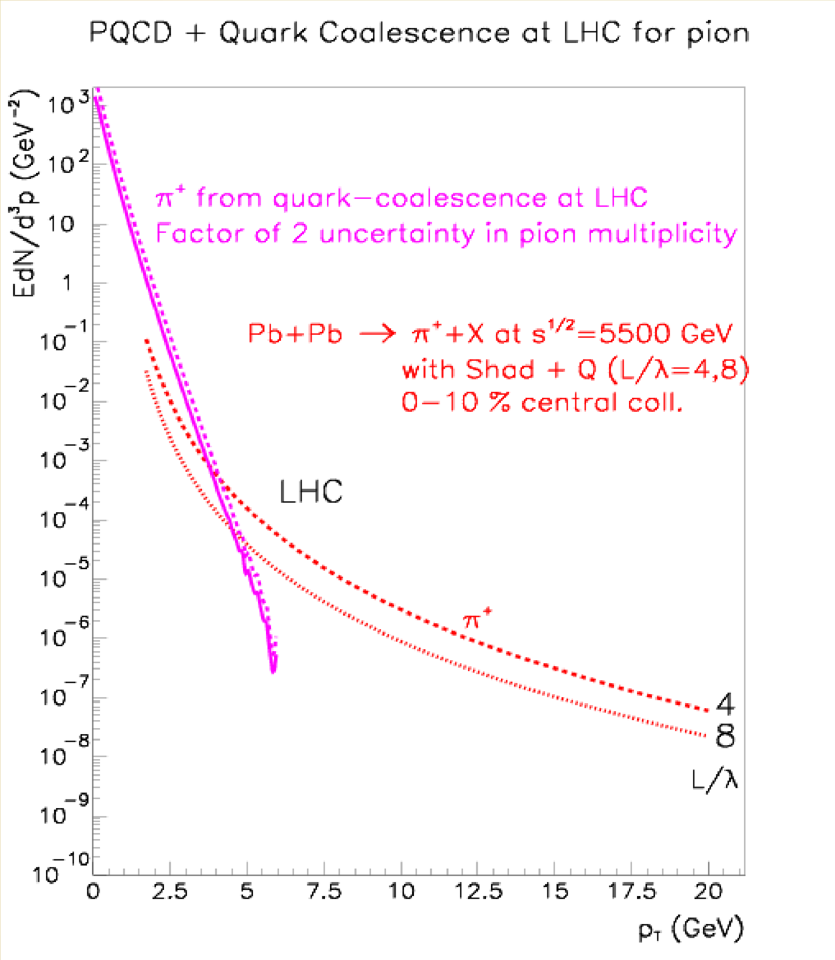


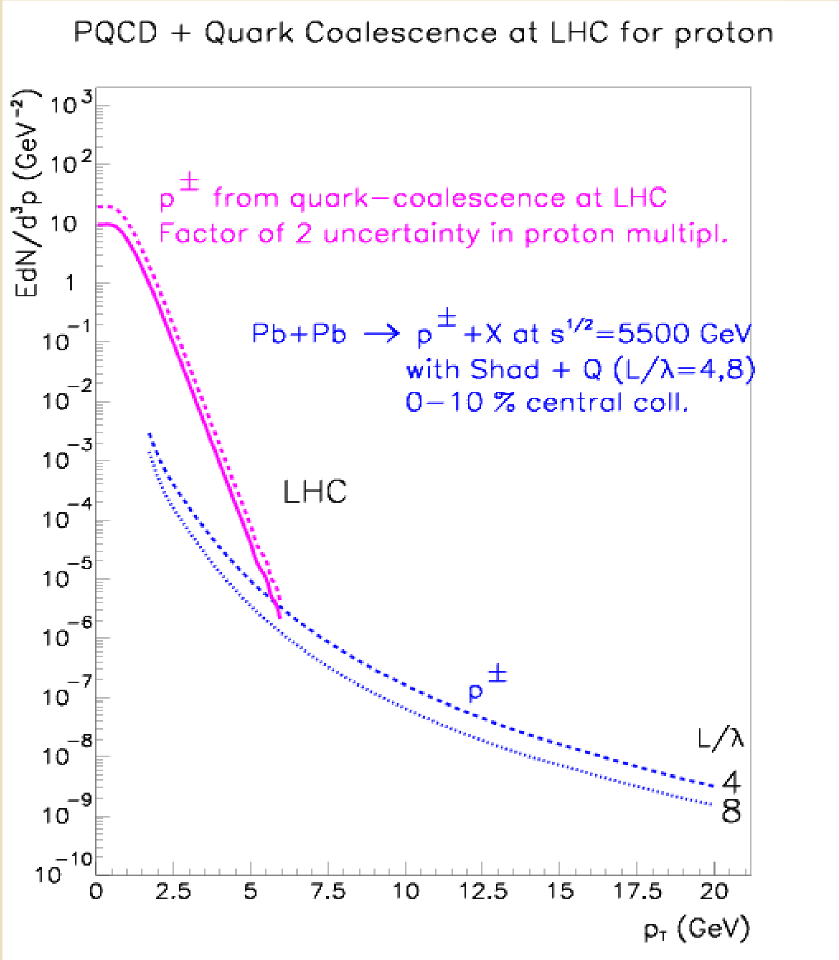




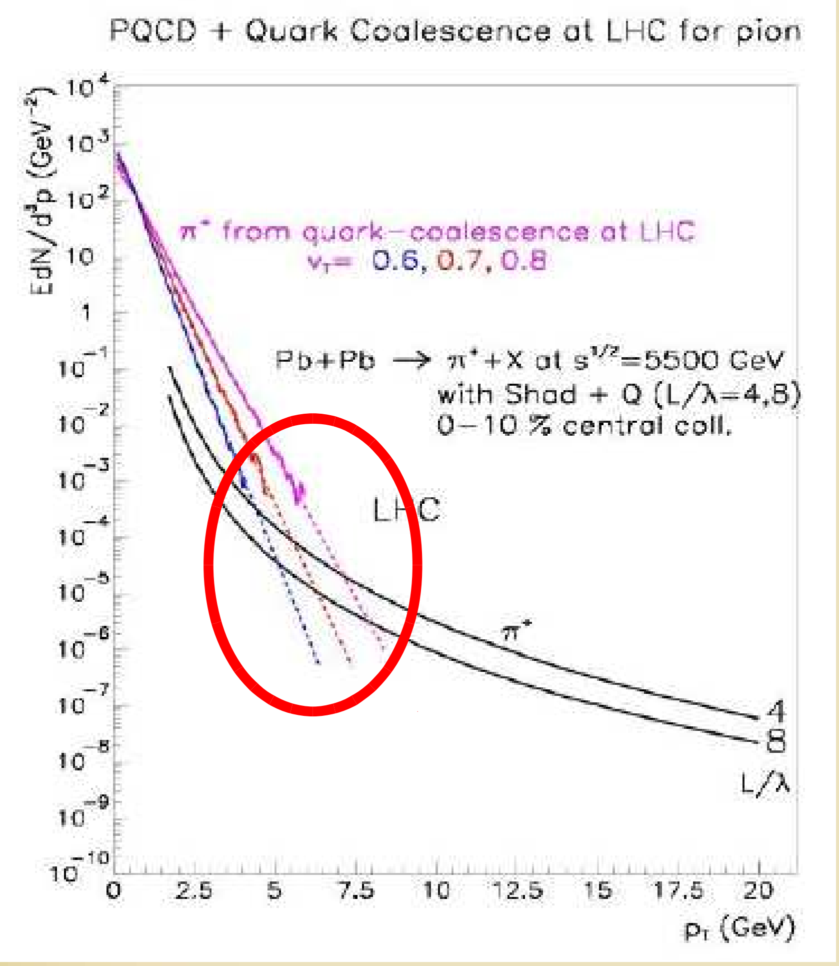


pQCD with AKK FF and coalescence





pQCD with AKK FF and coalescence



- pQCD based parton model:
  - QCD at T → 0 temperature
  - power law distribution
  - strong dependence on FF
  - good for high-p_T hadrons
- Quark-coalescence model
  - Thermal, finite temperature
  - exponential distribution $e^{-m/T}$
  - parton-hadron duality
  - good for high-p_T hadrons

P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

Mysteries of hadron spectra

- No unified physical model

  Different phyisical picture for low and high p_T production

- Non (exactly) exact theory → phenomenology

  (p)QCD + measurements + many parameters

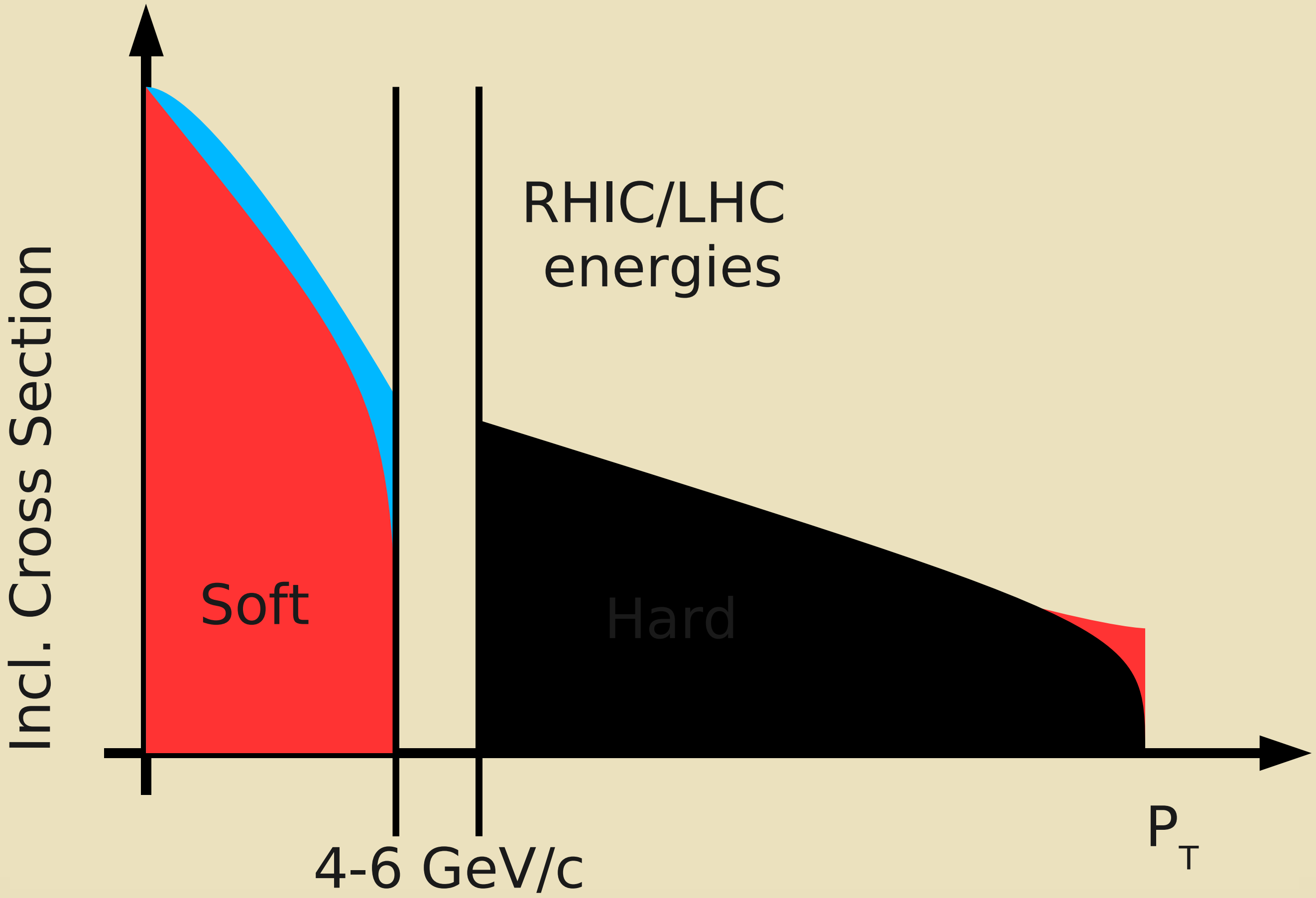
- Connecting theories with conflicting assumptions

  Basic assumptions are in conflict when maching.

⇉ Can we understand the hadronization?

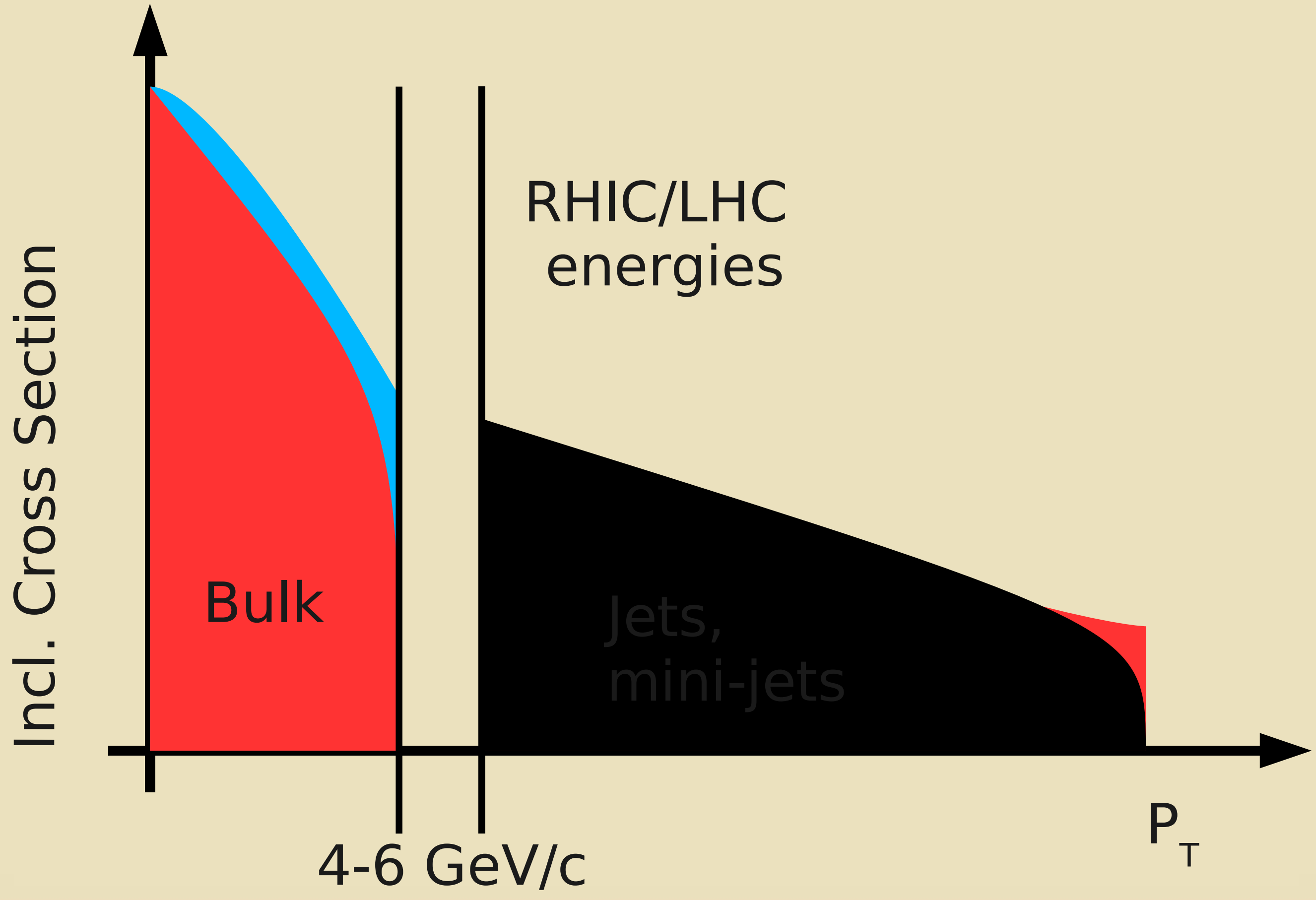
Mysteries of hadron spectra

- 1. interpretation 'soft' & 'hard' processes



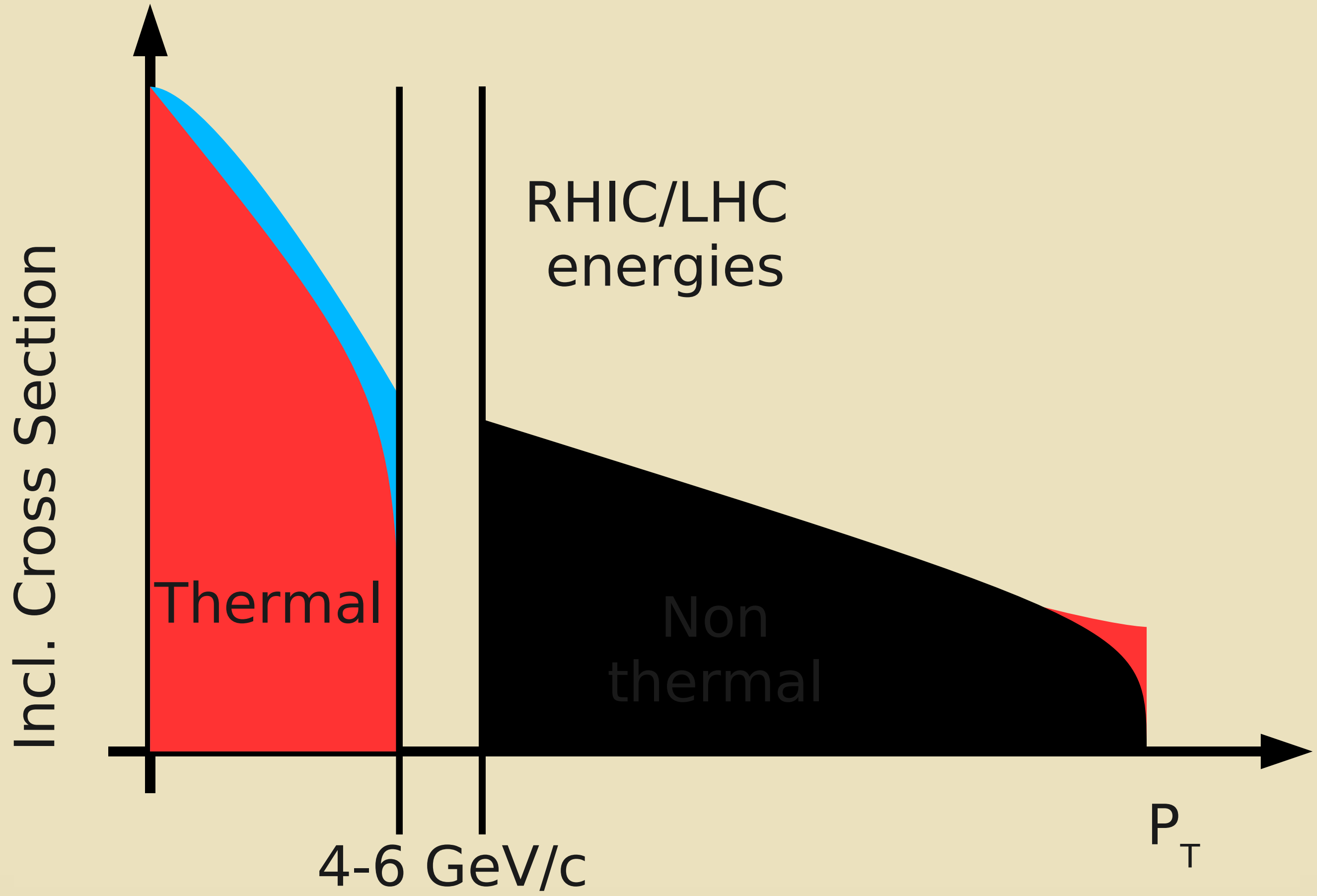
Mysteries of hadron spectra

- 2. interpretation 'bulk' & 'jet-like' processes



Mysteries of hadron spectra

- 3. interpretation 'thermal' & 'non-thermal' models

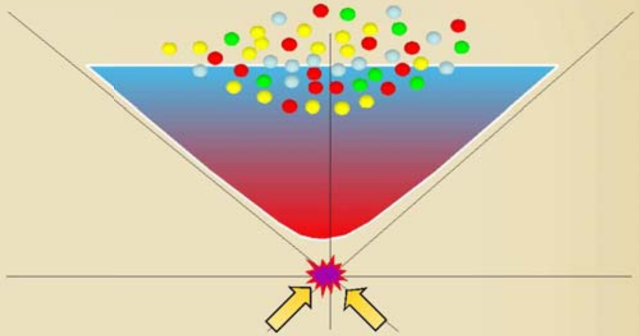


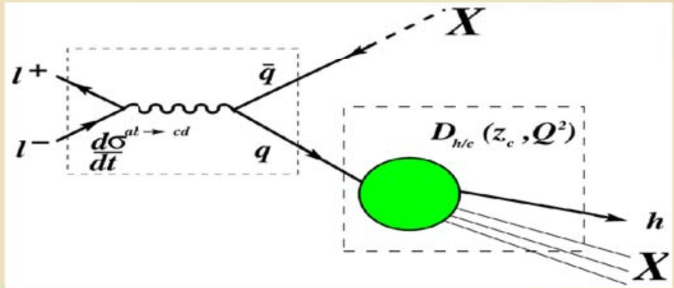
Notes on fragmentation

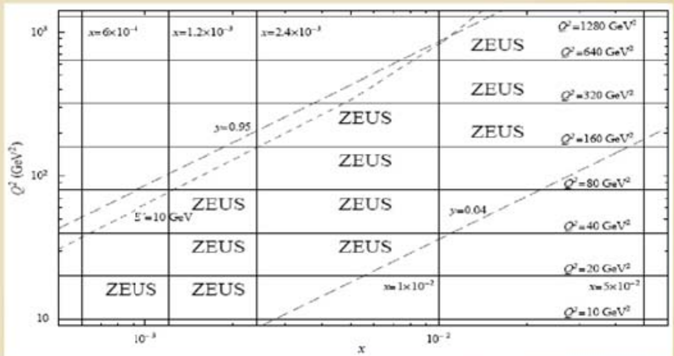
integrated vs. statistical

Hadronization processes & fragmentation

- Hadronization: requires a model, based on local parton-hadron duality (kvantum numbers & momenta connected to a cone around or to the leading particle.)
- Parton/hadron shower evolution comes from statistical processes (step-by-step MC evolution).
- Fragmentation function (FF) carries integrated (phenomenological) information on how parton fragment into hadron.
- Measurement lepton-antilepton annihilation



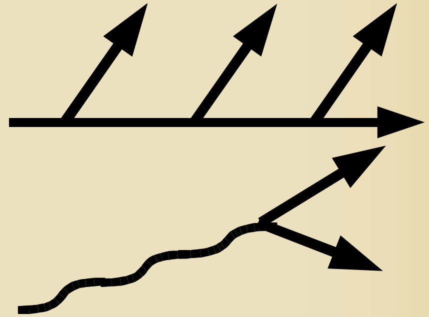




Models for fragmentation

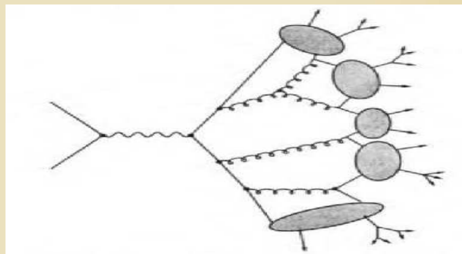
- Independent fragmentation model (Feynman - Field)

  Simplest model for fragmentation by
  Field & Feynman : q & g channels



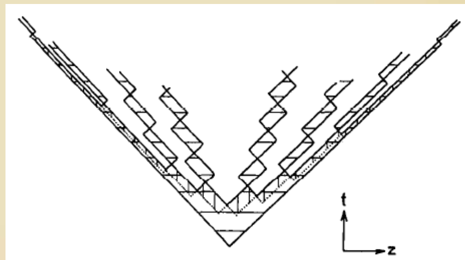
- (Quark) string model

  color strings between $q\bar{q}$, breaking into
  quark-antiquark pair → mesons



- Cluster model (Lund model)

  Lund model: phase-space separation,
  froming clusters: $q\bar{q}$ → M, qqq → B



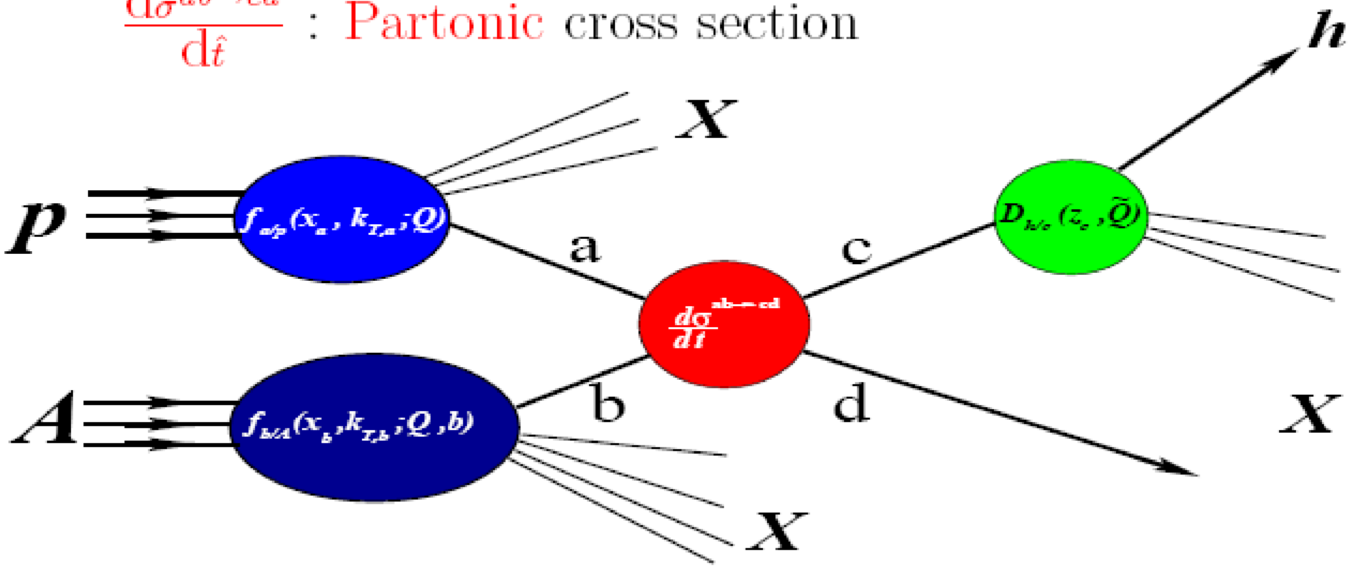
pQCD based parton model

$$E_\pi \frac{\mathrm{d}\sigma_\pi^{pA}}{\mathrm{d}^3 p_\pi} \sim f_{a/p}(x_a, Q^2; k_T) \otimes f_{b/A}(x_b, Q^2; k_T, b) \otimes \frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}} \otimes \frac{D_{\pi/c}(z_c, \widehat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a, Q^2; k_T, b)$: Parton Dist. Function (PDF), at scale Q^2

$D_{\pi/c}(z_c, \widehat{Q}^2)$: Fragmentation Function for π (FF), at scale $\widehat{Q}^2$

$\frac{\mathrm{d}\sigma^{ab\to cd}}{\mathrm{d}\hat{t}}$: Partonic cross section



Fragmentation processes in parton model

In a pQCD based parton model, fragmentation functions (FF) gives how parton (a) fragment into a hadron (h), $D_{h/a}(z,Q^2)$.

DGLAP scale evolution:

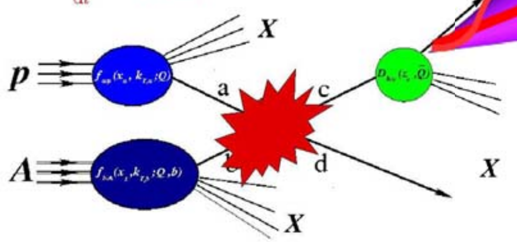
$$\frac{\partial}{\partial \ln Q^2} D_i^h(x,Q^2) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_S}{4\pi} P_{ji}\left(\frac{x}{z},Q^2\right) D_i^h(z,Q^2)$$

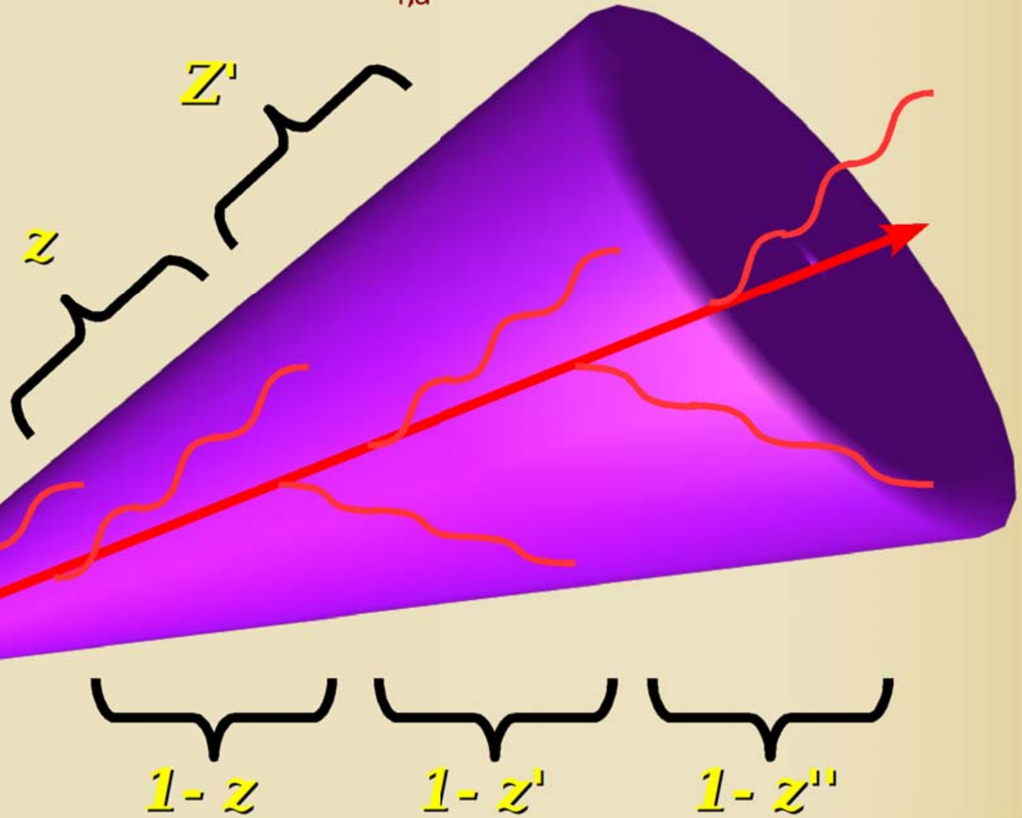
$$E_\pi \frac{d\sigma_\pi^{pA}}{d^3p_\pi} \sim f_{a/p}(x_a,Q^2;k_T) \otimes f_{b/A}(x_b,Q^2;k_T,b) \otimes \frac{d\sigma^{ab\to cd}}{d\hat t} \otimes \frac{D_{\pi/c}(z_c,\hat{Q}^2)}{\pi z_c^2}.$$

$f_{b/A}(x_a,Q^2;k_T,b)$: Parton Dist. Function (PDF), at scale Q^2

$D_{\pi/c}(z_c,\hat{Q}^2)$: Fragmentation Function for π (FF), at scale $\hat{Q}^2$

$\frac{d\sigma^{ab\to cd}}{d\hat t}$: Partonic cross section





On the non-extensive

thermodynamics

Basics of non-extensive thermodynamics

Non-extensive thermodynamics: associative composition rule, (non-additive) T.S. Biró: EPL84, 56003,2008:

$$h(h(x,y),z) = h(x,h(y,z))$$

Then should exist a strict monotonic function, X(x) 'generalised logarithm' (an entropy-like quantity), for which:

$$h(x,y) = X^{-1}\left(X(x) + X(y)\right) \qquad X\left(h(x,y)\right) = X(x) + X(y).$$

Example: (i) Classical thermodynamics:

$$f(E) = e^{-\beta E}/Z \qquad h(x,y) = x + y.$$

(ii) Tsallis – Pareto distribution

$$h(x,y) = x + y + axy \qquad a = q - 1$$

$$f(E) = \frac{1}{Z} e^{-\frac{\beta}{a}\ln(1+aE)} = \frac{1}{Z}(1+aE)^{-\beta/a} \qquad S = \int f \frac{e^{-a\ln(f)} - 1}{a} = \frac{1}{a}\int (f^{1-a} - f).$$

Associative composition

Non-extensive Gibbs, generalised

logarithm: $f(x) = \frac{1}{Z} e^{-\beta X(x)}$

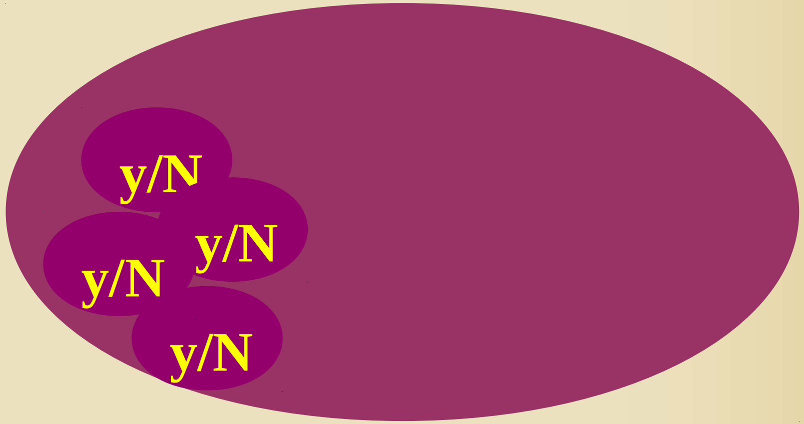
Compositin rule for sub-systems:

$$x_N(y) := \underbrace{h \circ \ldots \circ h}_{N-1} \left(\frac{y}{N}, \ldots, \frac{y}{N} \right)$$

Meanwhile satisfy: $\lim_{N \to \infty} x_N(y) < \infty$

Assimptotically, if $N_1, N_2 \to \infty$:

$$x_{N_1+N_2} = \varphi(x_{N_1}, x_{N_2})$$



recursive equation can be given:

$x_n = h\left(x_{n-1}, \frac{y}{N}\right)$, where $h(x,0) = x$.

$$x_n - x_{n-1} = h\left(x_{n-1}, \frac{y}{N}\right) - h(x_{n-1}, 0).$$

Evolution equation can carry out:

$$\frac{dx}{dt} = \frac{y}{t_f} h_2'(x, 0^+)$$

$$L(x) = \int_0^x \frac{dz}{h_2'(z, 0+)} = y \frac{t}{t_f}.$$

Fragmentation via associative composition

Program:

1) Search and fit Tsallis-Pareto distribution to data.

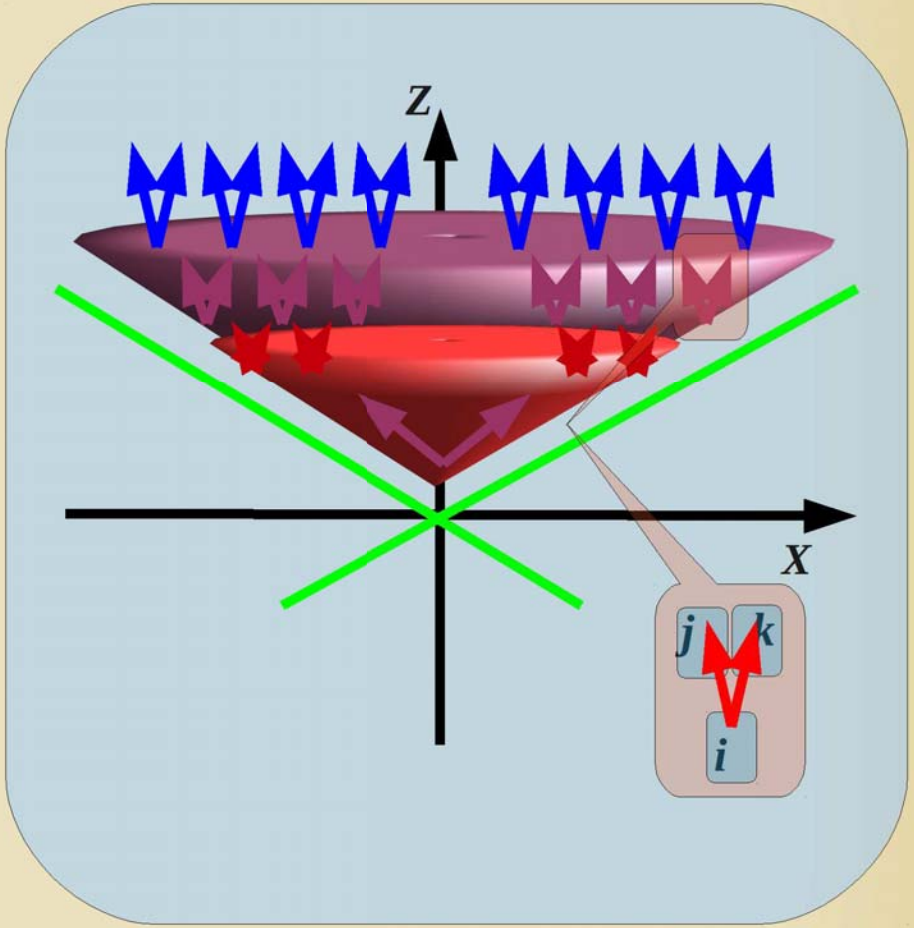
2) Serach for physical meaning of T and q parameters.

3) Components of the sub-systems are e.g. 'splitting functions' P_{qg} , P_{gg}

4) Test: can a DGLAP-like evolution equation be obtained?

$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$

$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$

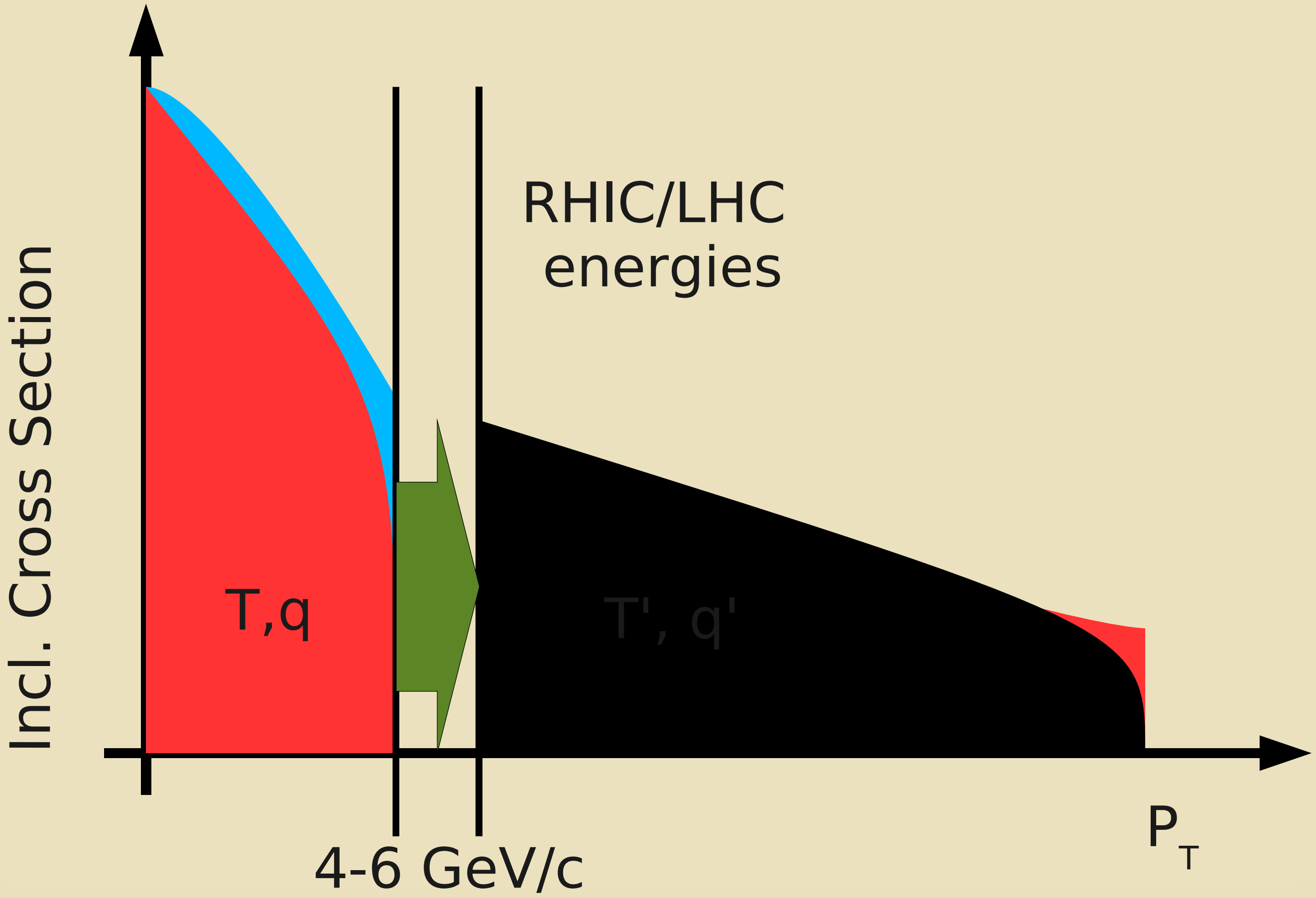


A test in two models

joint ideas…

1^{st} way: to resolve mysteries

- A suggested new way: Tsallis-Pareto (like) distribution



1st joint model: recombination & pQCD in AA

Find a distribution for low & high momentum spectra:

$$e^{-\beta E} \longrightarrow F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

In AA collisions particle energy modified by the flow:

$$E = \gamma(m_t - v_{flow} p_t), \quad m_t = \sqrt{m^2 + p_t^2}$$

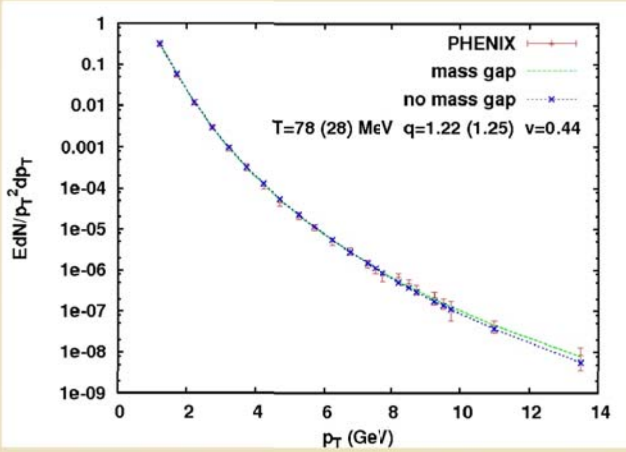
Slope for particle spectra can be fitted:

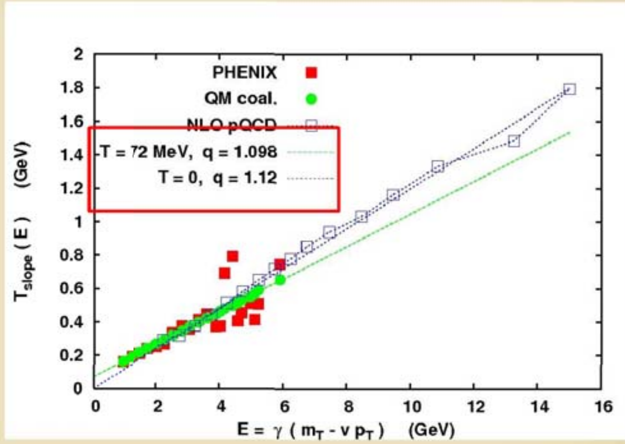
$$T_{slope} = -\frac{\partial E}{\partial ln(F_h(E))} = T(1 + aE) = T + (q-1)E$$

furthermore, if E>>m, then $T \rightarrow T\,[\,(1+v)\,/\,(1-v)\,]^{1/2}$

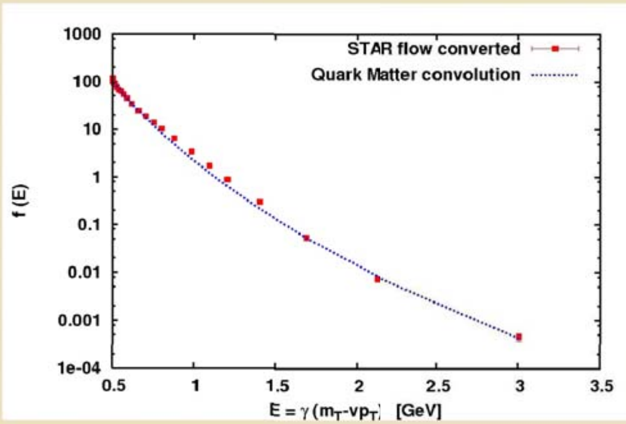
1^{st} joint model: recombination & pQCD in AA

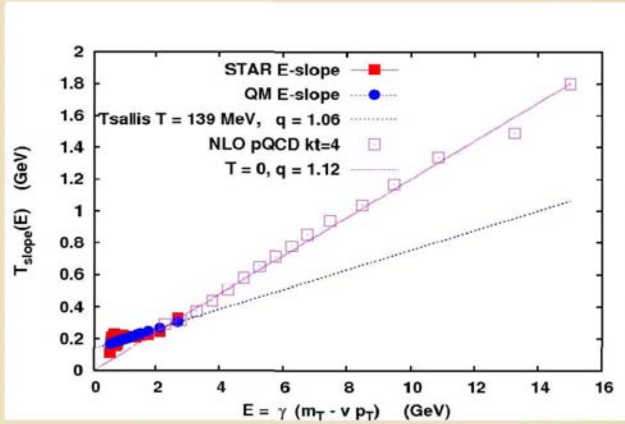
Pion spectrum





Kaon spectrum



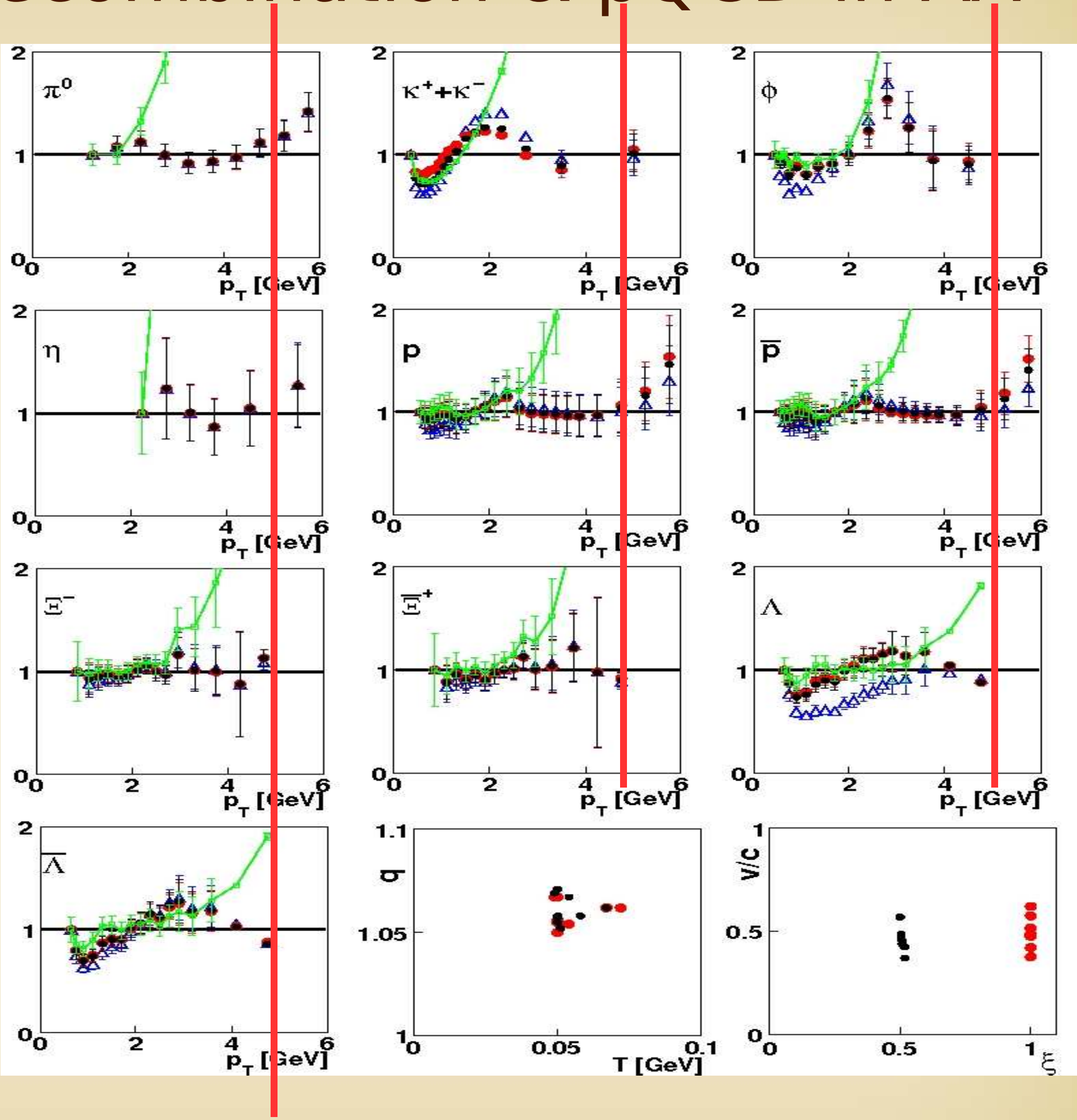


Fitted T & q Tsallis parameters at RHIC energies: (recombination and NLO pQCD+AKK FF, *v=0.5* for kaons): T.S. Bíró, K. Ürmössy, GGB: JPG35, 044012 (2008)

1st joint model: recombination & pQCD in AA

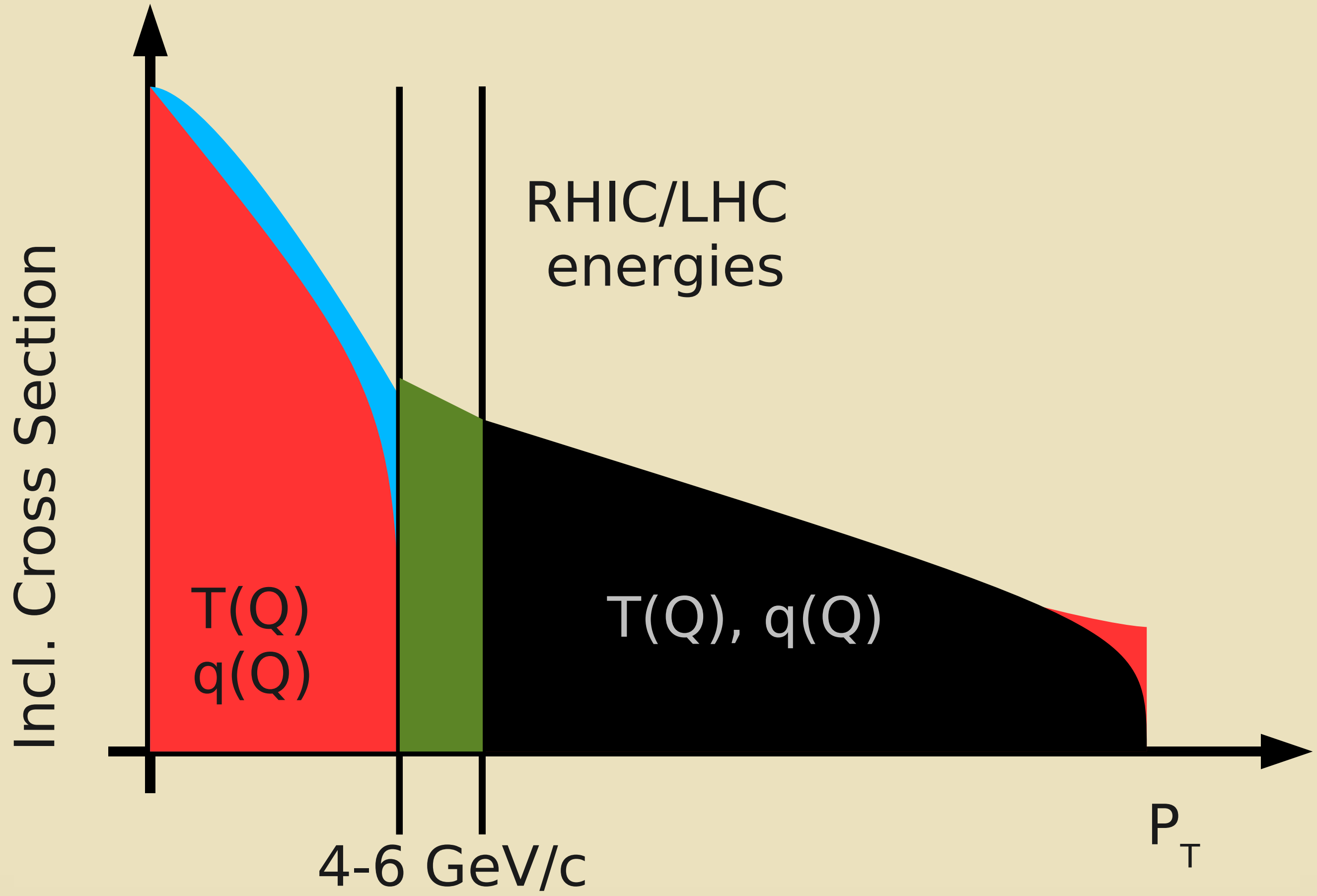
Analysing
Meson & Baryon
specta in
AuAu collisions

- Ratio of theoretical or experimental p_T spectra in y=0, AuAu collisions fitted by Tsallis distribution.
- T=50-70MeV, q=1.06
- Here the fit is only for $p_T < 6$ GeV/c.
- K. Ürmössy, T.S. Bíró: PLB689:14 (2010)



2nd way: to resolve mysteries

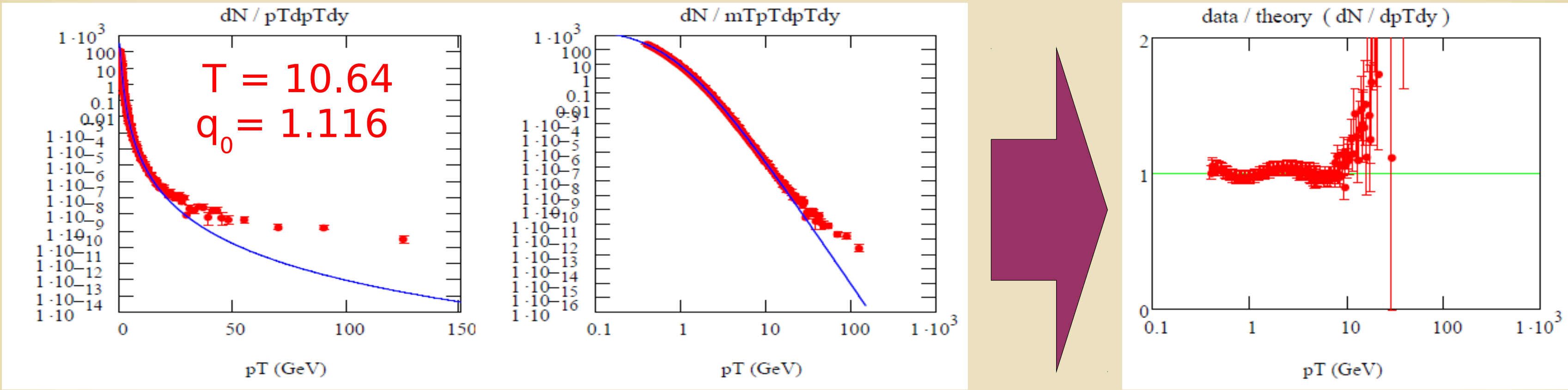
- Suggested interpetation: Tsallis-Pareto + evolution



2nd joint model: Tsallis – Pareto with evolution

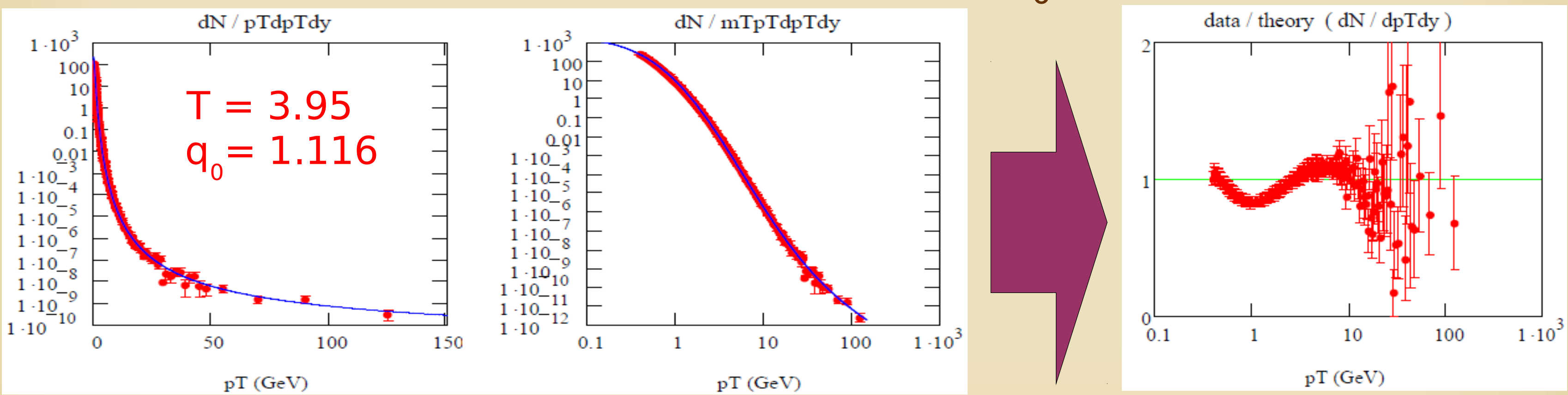
- TEST on CDF pion data in pp @ 1.96 TeV |y|<1

NO evolution



- DGLAP motivated evolution: $n = (q_0-1)^{-1} - 2*\log(\log(Q))$

With evolution



Recall: Mysteries of hadron spectra

- No unified phyisical model?

  Unified distribution for both: low & high p_T regions.

- Non (exactly) exact theory → phenomenology

  (p)QCD + measurements + less parameters

- Connecting theories with conflicting assumptions

  Continuation of one theory to another – at least by non-conflicting parameter level.

⇉ Aim: Can we understand hadron distribution?

B A C K U P

DGLAP-like evolution

Test of the DGLAP like evolution via parameters T, q, & normalisation