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Beyond'**

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**Theory of leading twist nuclear shadowing and predictions for small-x pdf's and hard
diffractive phenomena**

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The background of the slide is a piece of marbled paper with a complex, swirling pattern of colors including shades of brown, orange, red, and grey. The pattern resembles natural stone or wood grain.

Theory of leading twist nuclear shadowing and predictions for small x pdf's and hard diffractive phenomena

Mark Strikman
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Based on the studies with Frankfurt & Guzey

Gribov - 80 , May 27, 2010

OUTLINE

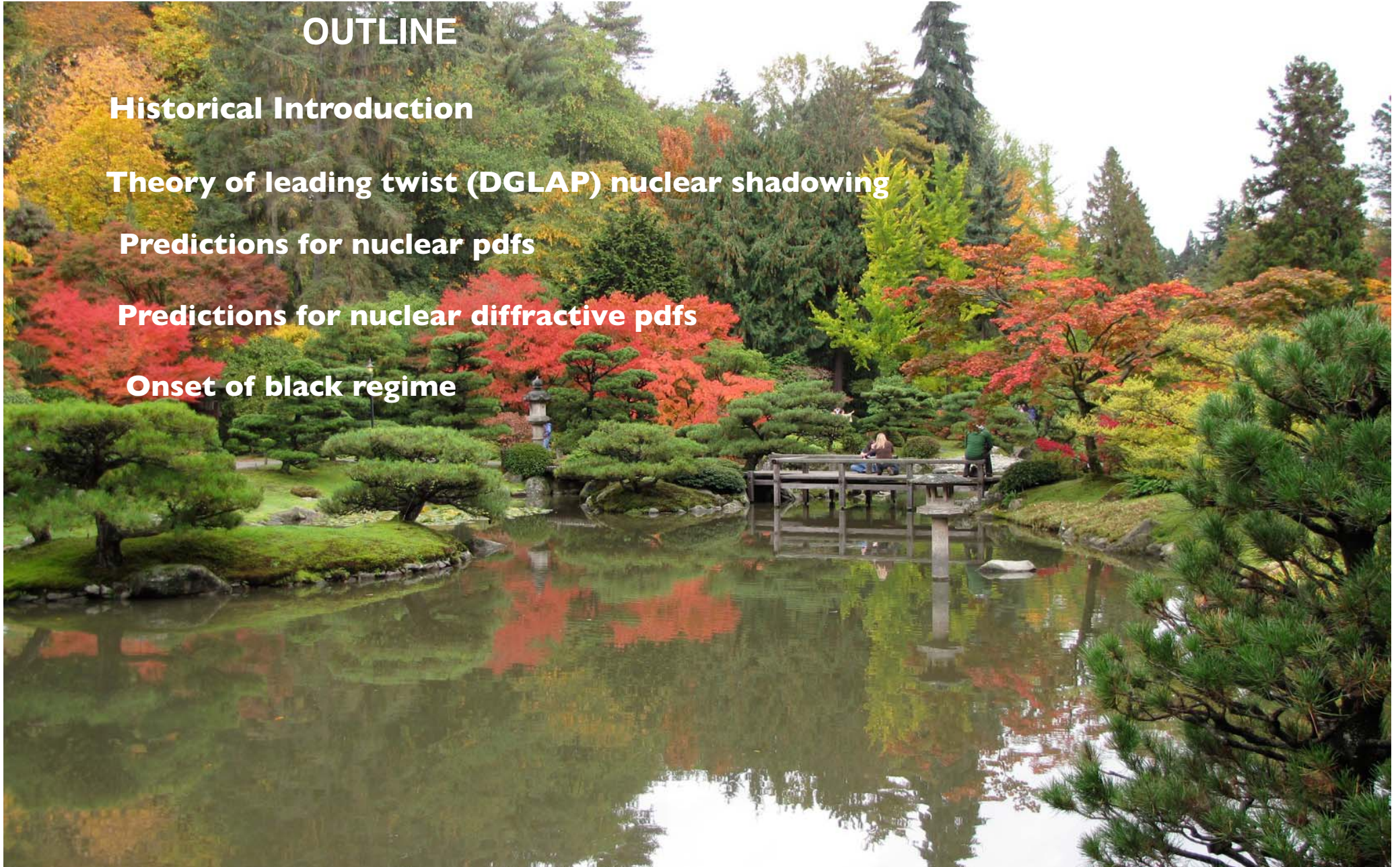
Historical Introduction

Theory of leading twist (DGLAP) nuclear shadowing

Predictions for nuclear pdfs

Predictions for nuclear diffractive pdfs

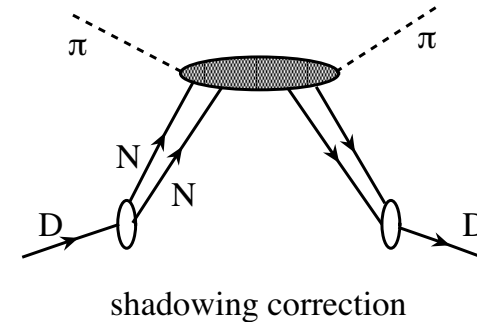
Onset of black regime



During 68 - 70 Gribov made several key observations relevant for description of nuclear shadowing in DIS

✓ Nuclear shadowing in high energy hadron - nucleus scattering

Though the diagrams consider by Glauber in QM treatment of hA scattering are exactly zero at $E_h \gg m_h$ (AFS diagrams), the answer for double scattering



is expressed through the diffractive cross section (elastic + inelastic) at $t \sim 0$. For triple,... rescattering the answer is related to the low t diffraction but cannot be obtained in a model independent way

Natural explanation in the Gribov space-time picture of high energy scattering

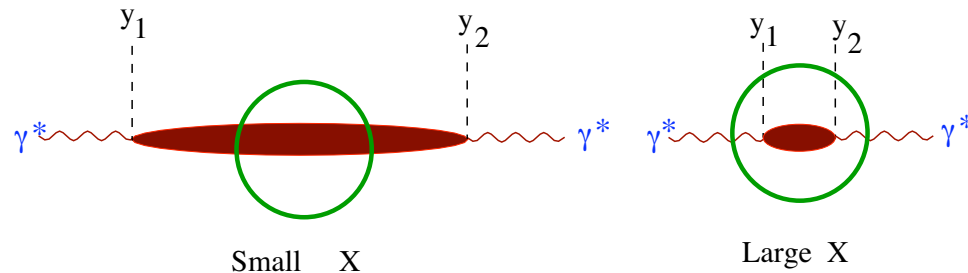
✓ Large longitudinal distance dominate the small x DIS

Gribov, Ioffe, Pomerenchuk 65, Ioffe 68, Gribov 69

Follows from the analysis of the representation of the forward Compton scattering amplitude expressed as a Fourier transform of the matrix element of the commutator of two electromagnetic (weak) current operators:

$$Im A_{\mu\nu}^{\gamma^* N}(q^2, 2qp) \frac{1}{\pi} \int \exp^{iq(y_2 - y_1)} \langle p | [j_\mu(y_2), j_\nu(y_1)] | p \rangle d^4(y_2 - y_1)$$

y_1 and y_2 are the points where γ^* is absorbed and emitted.

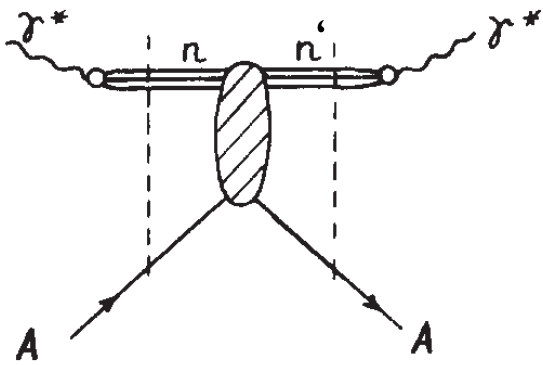


In the nucleus rest frame for z component of $y \equiv z \sim \frac{1}{2m_N x}$

Scaling violation for small $x \Rightarrow z = \lambda_s / 2m_N x$, with $\lambda_s \ll 1$ at large Q^2

Kovchegov & MS, Blok & Frankfurt

- ✓ Nuclear shadowing in the limit of $q_0 \rightarrow \infty$, fixed Q^2 :
relation to the photon polarization operator & Gribov paradox



For large q_0 & $A \gg 1$ all hadronic configurations $|n\rangle$ in γ^* interact with **black disk strength** $\sigma_{\text{tot}} = 2\pi R_A^2$

⇒ only diagonal transitions $n=n'$ survive

$$F_{2A}(\gamma^* A) \propto \frac{Q^2}{Q_0^2} \ln(2R_A m_N / x) 2\pi R_A^2$$

Gross violation of Bjorken scaling - Gribov paradox (Bj)

- ⇒ Parton model solution - aligned jet model - Bj
- ⇒ QCD aligned jet model - color screening and color transparency - F & S



Onset of the Gribov regime is likely in QCD *though at much smaller x*

QCD aligned jet model predicted correct magnitude of shadowing, diffraction at HERA, as well the slow energy dependence of diffraction in DIS.

★ Knowledge of nuclear pdfs is critical for interpretation of AA at the LHC and forward physics at RHIC. However there is no data at sufficiently large Q at small x .

The Gribov theory of nuclear shadowing in which relates shadowing in $\gamma^* A$ and diffraction in the elementary process: $\gamma^* + N \rightarrow X + N$.

Before HERA one had to model ep diffraction to calculate shadowing for $\sigma_{\gamma^* A}$ (FS88-89, Kwiecinski89, Brodsky & Liu 90, Nikolaev & Zakharov 91). More recently several groups (Capella et al) used the HERA diffractive data as input to obtain a reasonable description of the NMC data (however this analysis made several simplifying assumptions). Also the diffractive data were used to describe shadowing in γA scattering without free parameters.

Does not allow to calculate gluon pdfs and hence quark pdfs

Leading twist (LT) nuclear shadowing and diffraction

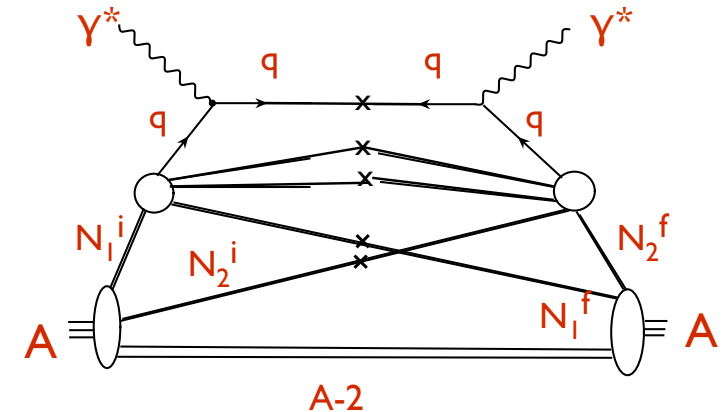
Usually one starts from an impulse approximation for the scattering of a hard probe (γ^*, W) off a nucleus. In the parton language - QCD factorization. Can we trust impulse approximation in the hadronic basis for the nucleus wave function?

Consider interference between γ^* ("Higgs") scattering off two different nucleons

Introduce light cone fraction α for nucleon Free nucleon $\alpha=1$, $\alpha_f \leq 1-x$

For nucleus to have significant overlap of $|\text{in}\rangle$ and $\langle\text{out}|$ states

$$\alpha_{N_1^f} \leq \alpha_{N_1^i} - x \sim 1, \quad \alpha_{N_2^i} \leq \alpha_{N_2^f} - x \sim 1$$



\Rightarrow Interference is very small for $x > 0.1$ and impossible for $x > 0.3$.

\Rightarrow Large interference for $x < 0.01$ due to the final states where small light cone fraction is transferred to the nucleon \equiv diffraction. It results in the leading twist shadowing as well as higher twist shadowing.

How big is HT shadowing is an open question. Issue of duality.

$$\Delta^{\text{diff}} F_{2A}(x, Q^2) = \frac{1}{16\pi} \int d\Delta d^2q_{\perp} G(x, Q^2, \Delta, q_{\perp}) F_A(\Delta, q_{\perp})$$

elementary diffractive blok

where F_A is two nucleon form factor of the nucleus which in the nonrelativistic approximation is

$$F_A(\Delta, q_{\perp}) = \int d^3k_i \psi_A(k_i) \cdot \psi_A(k_1 + \vec{q}, k_2 - \vec{q}, k_3, \dots)$$

Δ - longitudinal momentum transfer LC fraction

We obtained the same expression in the nucleus rest frame as well -FS89

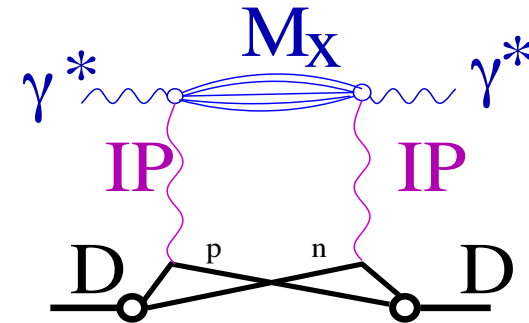
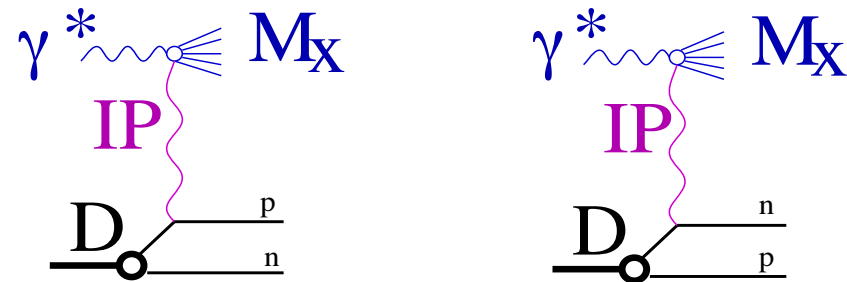
Key element of the logic - nucleus is a system of color singlet clusters - nucleons which are weakly deformed in nuclei - checked by success of the Gribov-Glauber theory of soft hA interactions - $\sigma_{\text{tot}}(hA)$ to few %

Connection between nuclear shadowing and diffraction - nuclear rest frame

Qualitatively, the connection is due to a possibility of small t to the nucleon at small x : $-t_{min} = x^2 m_N^2 (1 + M_{dif}^2 / Q^2)^2$

If $\sqrt{t} \leq$ “average momentum of nucleon in the nucleus” \rightarrow large shadowing / interference

Deuteron example - amplitudes of diffractive scattering off proton and off neutron interfere



Double scattering diagram for the $\gamma^* D$ scattering

$$\frac{d \sigma_{\gamma^*+D \rightarrow M_X + (pn)}}{dt dM_X^2} = \frac{d \sigma_{\gamma^*+N \rightarrow M_X + (pn)}}{dt dM_X^2} (2+2F_D(4t))$$

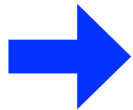
$F_D(t)$ is the deuteron form factor.

For $t=0$ - 100% constructive interference - (pn) system is D . Coherence dies out at large t .

Integrate over t, M_X \rightarrow positive correction to the impulse approximation. Coincides with the Gribov shadowing correction to the total cross section (up to small corrections due to the real part of the amplitude).

However the sign is opposite !!!

Explanation is unitarity - Abramovskii, Gribov, Kancheli cutting rules (AGK) - with some technical differences due to scattering off nuclei - Bertocci & Treliani



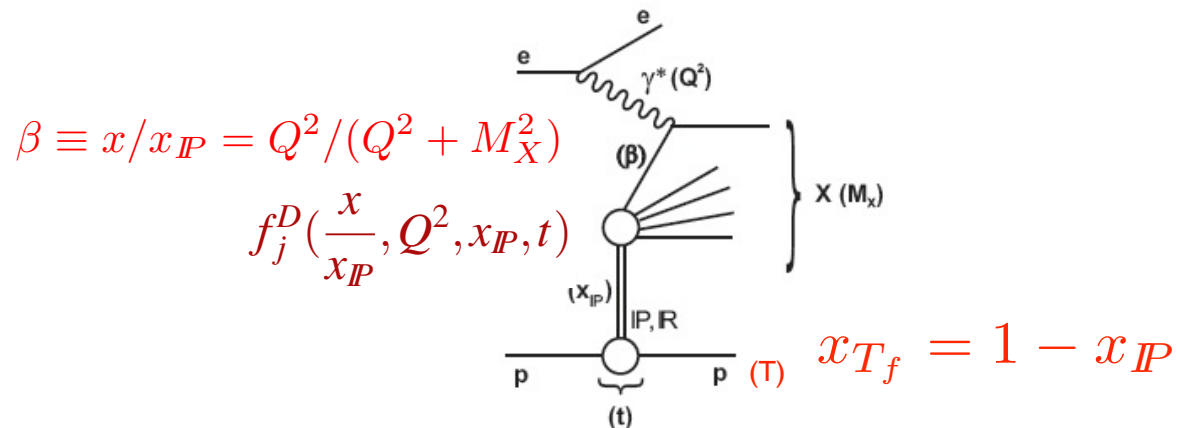
Using AGK cutting rules we re-derived original Gribov result for nuclear shadowing extending it to include the real part effects. This approach does not require separation of diffraction into leading twist and higher twist contributions.

Summary - **Diffractive phenomena - inclusive diffraction and measurement of diffractive pdf's**

Collins factorization theorem: consider hard processes like

$$\gamma^* + T \rightarrow X + T(T'), \quad \gamma^* + T \rightarrow jet_1 + jet_2 + X + T(T')$$

one can define fracture (Trentadue & Veneziano) parton distributions



Theorem:

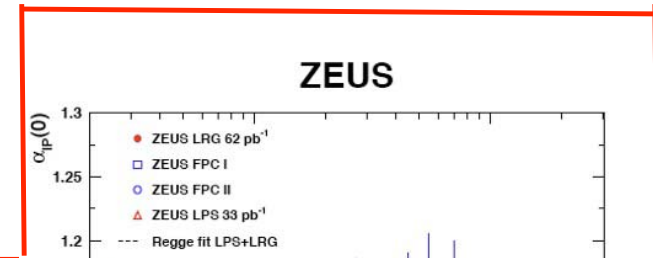
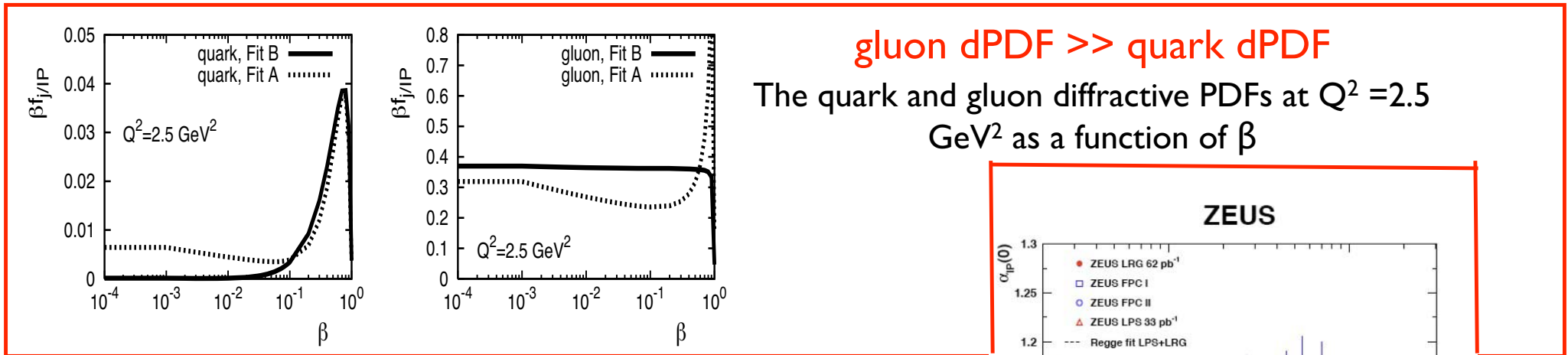
For fixed x_P, t universal fracture pdf + the evolution is the same as for normal pdf's

*Theorem is violated in dipole model of γ^*N diffraction in several ways*

HERA: Good consistency between H1 and ZEUS three sets of measurements

- ☞ Measurements of $F_2^{D(4)}$
- ☞ Measurements of dijet production
- ☞ Diffractive charm production

DGLAP describes totality of the data well **several crosschecks** - Collins factorization theorem valid for discussed Q^2, x range



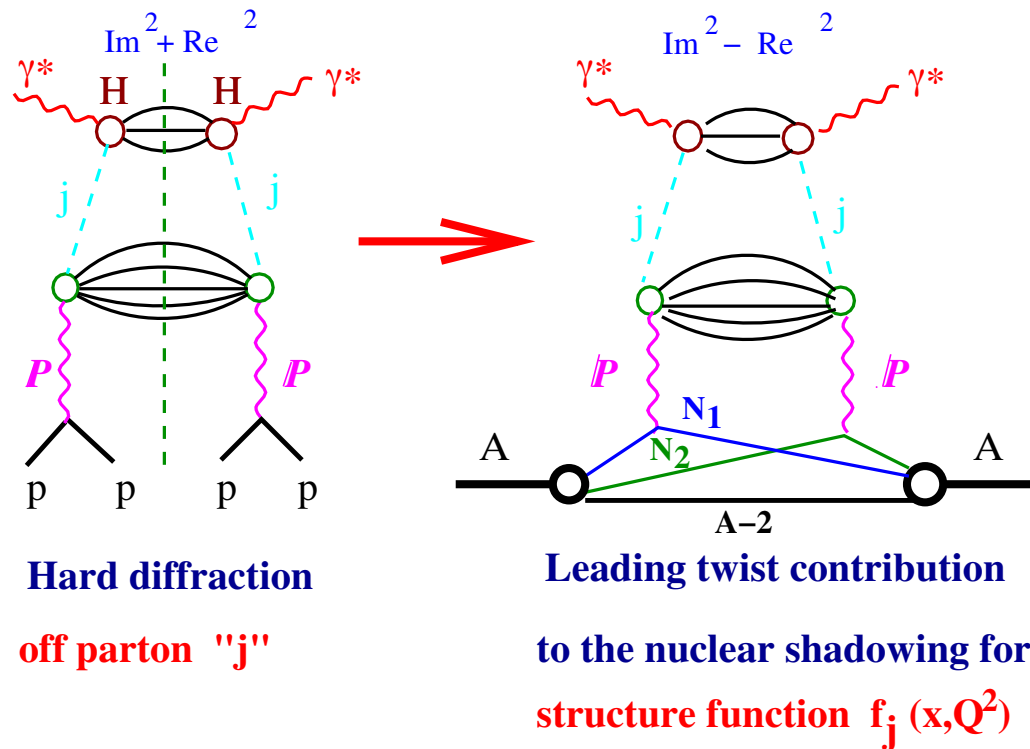
Current fits to soft hadron - hadron interactions find $\alpha_{IP}(0) = 1.09 - 1.10$

☞ Diffraction at HERA is mostly due to the interaction of hadron size components of γ^* not small dipoles

Theoretical expectations for shadowing in the LT limit

Combining Gribov theory of shadowing and pQCD factorization theorem for diffraction in DIS allows to calculate LT shadowing for *all parton densities* (FS98) (instead of calculating F_{2A} only)

Theorem: In the low thickness limit the leading twist nuclear shadowing is unambiguously expressed through the nucleon diffractive parton densities $f_j^D(\frac{x}{x_P}, Q^2, x_P, t)$:



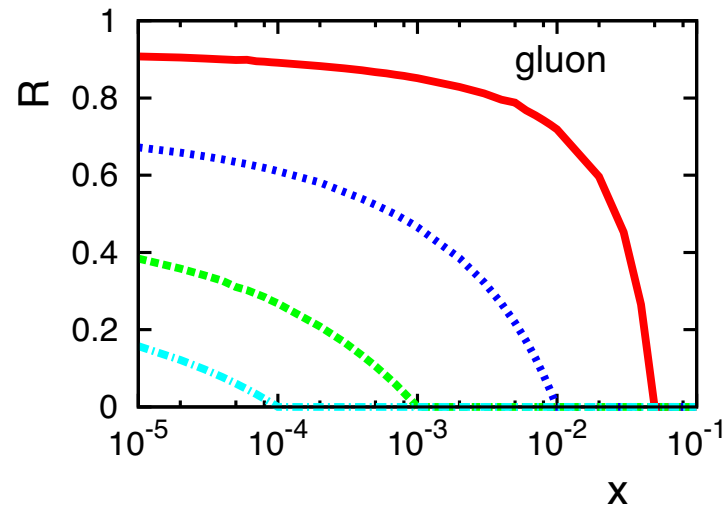
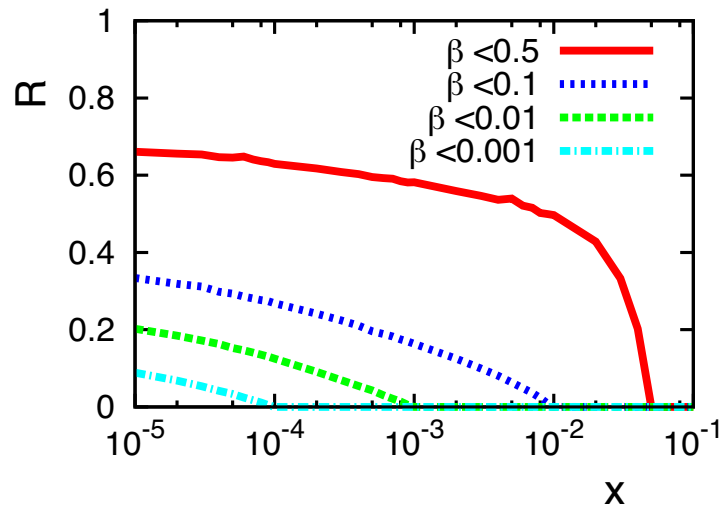
Theorem: in the low thickness limit (or for $x > 0.005$)

$$f_{j/A}(x, Q^2)/A = f_{j/N}(x, Q^2) - \frac{1}{2+2\eta^2} \int d^2b \int_{-\infty}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \int_x^{x_0} dx_{\mathbb{P}} \cdot f_{j/N}^D(\beta, Q^2, x_{\mathbb{P}}, t) \Big|_{k_t^2=0} \rho_A(b, z_1) \rho_A(b, z_2) \text{Re}[(1 - i\eta)^2 \exp(ix_{\mathbb{P}} m_N(z_1 - z_2))],$$

where $f_{j/A}(x, Q^2), f_{j/N}(x, Q^2)$ are nucleus(nucleon) pdf's,

$\eta = \text{Re}A^{diff} / \text{Im}A^{diff} \approx 0.174$, $\rho_A(r)$ nuclear matter density.

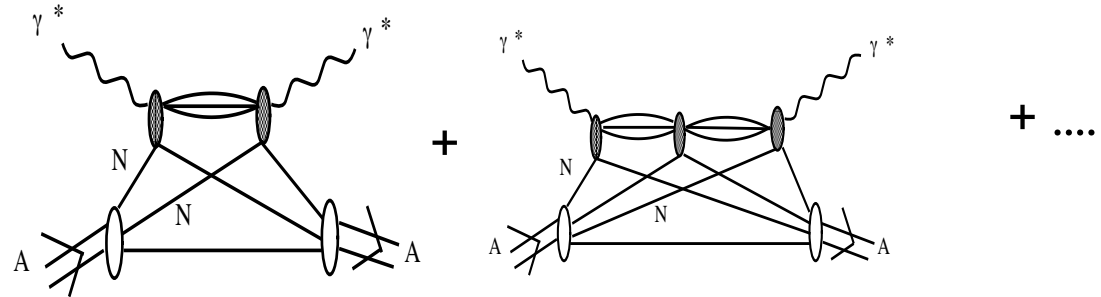
$x_0(\text{quarks}) \sim 0.1$, $x_0(\text{gluons}) \sim 0.03$



Contributions of different $\beta = Q^2/(Q^2 + M^2)$ to shadowing. $M^2 \sim Q^2$ dominate in a wide x range

Confer: Balitski - Kovchegov eq. - dominant contribution $\ln(1/\beta) > 3 \div 4$; $\alpha_P = 1.3 \div 1.5$

Including higher order terms



Color fluctuation approximation: Amplitude to interact with j nucleons $\sim \sigma^j$

$$\begin{aligned}
 x f_{j/A}(x, Q^2) &= \frac{x f_{j/N}(x, Q^2)}{\langle \sigma \rangle_j} 2 \Re e \int d^2 b \left\langle \left(1 - e^{-\frac{A}{2}(1-i\eta)\sigma T_A(b)} \right) \right\rangle_j \\
 &= A x f_{j/N}(x, Q^2) - x f_{j/N}(x, Q^2) \frac{A^2 \langle \sigma^2 \rangle_j}{4 \langle \sigma \rangle_j} \Re e (1 - i\eta)^2 \int d^2 b T_A^2(b) \\
 &\quad - x f_{j/N}(x, Q^2) 2 \Re e \int d^2 b \frac{\sum_{k=3}^{\infty} \left(-\frac{A}{2}(1-i\eta) T_A(b) \right)^k \langle \sigma^k \rangle_j}{k! \langle \sigma \rangle_j},
 \end{aligned}$$

does not depend on f_j

$\langle \dots \rangle_j$ integral over σ with weight $P_j(\sigma)$ - probability for the probe to be in configuration which interacts with cross section σ ;
 $\langle \sigma^k \rangle_j = \int_0^\infty d\sigma P_j(\sigma) \sigma^k$

For intermediate x one needs also to keep finite coherence length factor $e^{i(z_1 - xz_2)m_N x \mathbb{P}}$

Fluctuations with small σ are significant only for $\langle \sigma \rangle$, $\langle \sigma^2 \rangle$
 $\langle \sigma^k \rangle$ for $k > 2$ dominated by soft fluctuations. $\alpha_{IP}(0)=1.1$ - proof that soft
dynamics dominates already for $\langle \sigma^2 \rangle$

$\langle \sigma^k \rangle / \langle \sigma^2 \rangle$ can be modeled based on soft physics - effects of dispersion in
this case known to be small (we did a numerical study in our case).

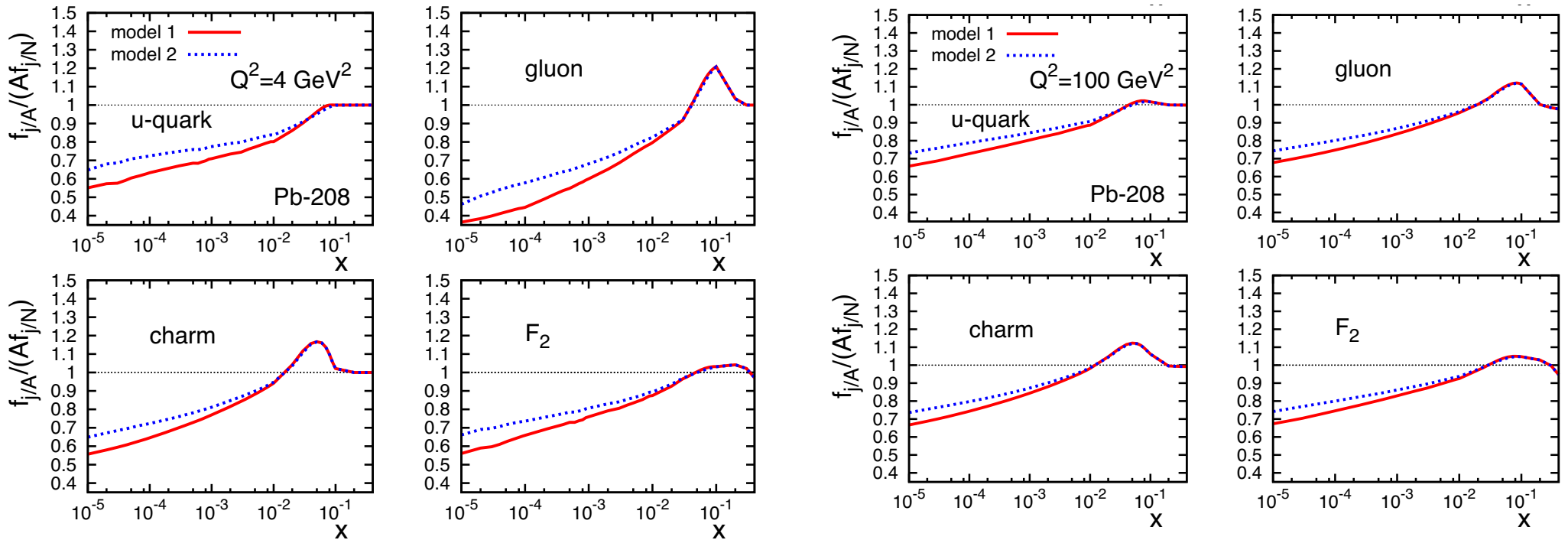
Fluctuation approximation for Q_0^2 :

$$\begin{aligned}
 x f_{j/A}(x, Q^2) &= A x f_{j/N}(x, Q^2) \\
 &- x f_{j/N}(x, Q^2) 8\pi A(A-1) \Re e \frac{(1-i\eta)^2}{1+\eta^2} B_{\text{diff}} \int_x^{0.1} dx_{\mathbb{P}} \beta f_j^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) \\
 &\times \int d^2b \int_{-\infty}^{\infty} dz_1 \int_{z_1}^{\infty} dz_2 \rho_A(\vec{b}, z_1) \rho_A(\vec{b}, z_2) e^{i(z_1-z_2)x_{\mathbb{P}}m_N} e^{-\frac{A}{2}(1-i\eta)\sigma_{\text{soft}}(x, Q^2)} \int_{z_1}^{z_2} dz' \rho_A(\vec{b}, z')
 \end{aligned}$$

where $\sigma_{\text{soft}}(x, Q_0^2) \equiv \langle \sigma^3 \rangle_j / \langle \sigma^2 \rangle_j$ is the only parameter (weakly dependent on x)
which can be estimated only semiquantitatively.

Numerical studies impose antishadowing to satisfy the sum rules for baryon charge and momentum (FS + Liuti 90) - sensitivity to model of fluctuations is weak. At the moment uncertainty from HERA measurements is comparable.

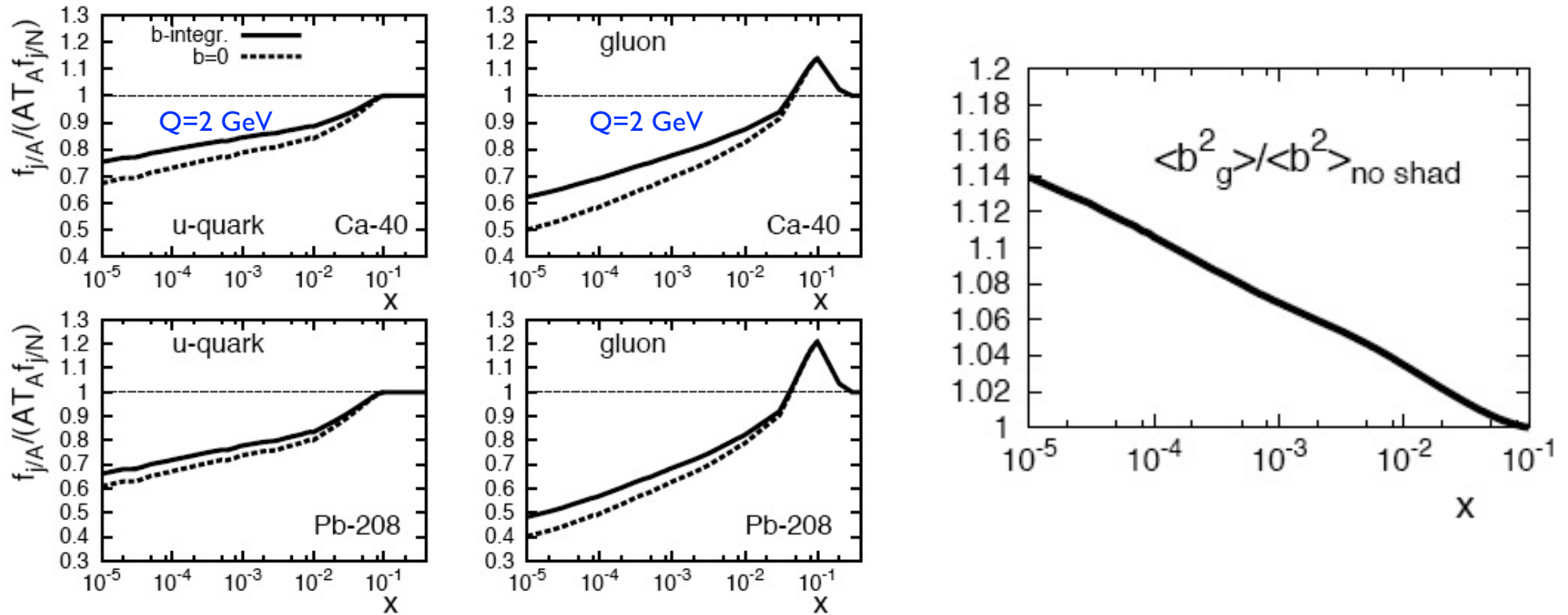
NLO pdfs - as diffractive pdfs are NLO



Note that nuclear shadowing increases with decrease of x - no flattening like in naive ad hoc parametrizations.

Nuclear diagonal generalized parton distributions.

Shadowing strongly depends on the impact parameter, b , - one can formally introduce nuclear diagonal generalized parton distributions. In LT theory - one just needs to remove integral over b .



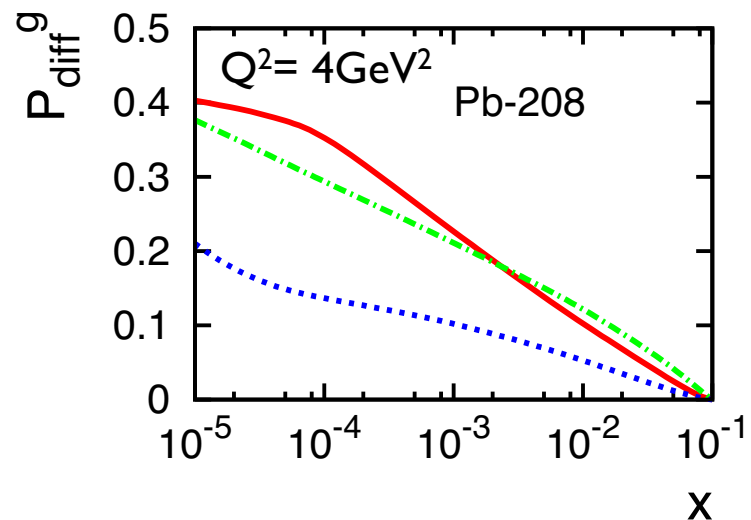
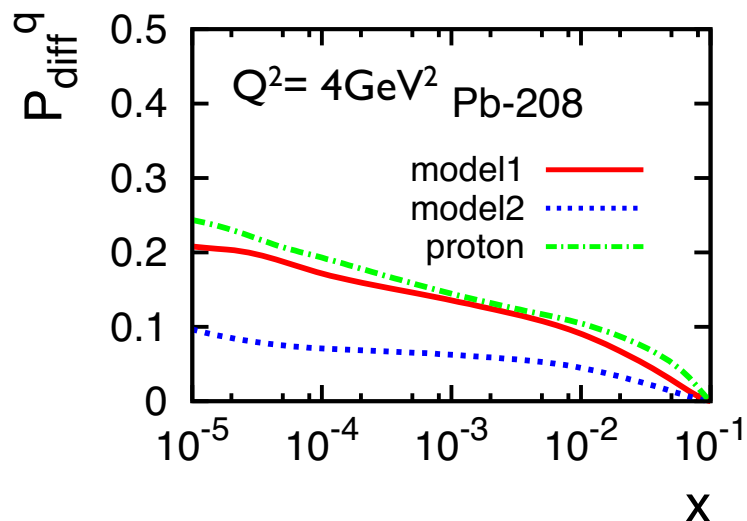
b -dependence can be studied experimentally by comparing the rapidity dependence of the hard processes in peripheral and central collisions

Nuclear Diffractive parton densities

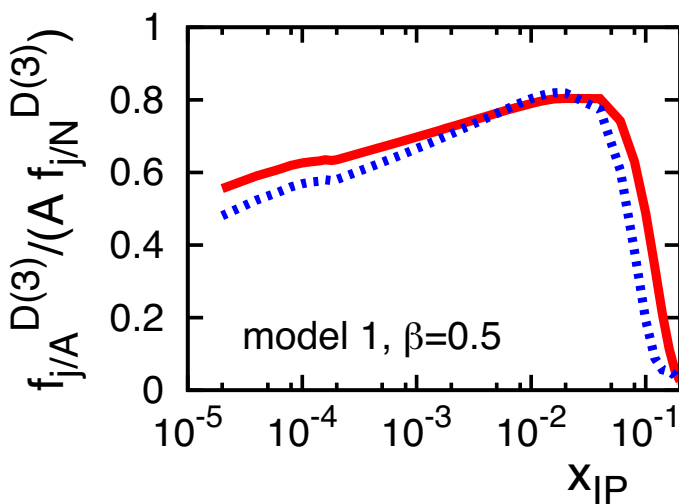
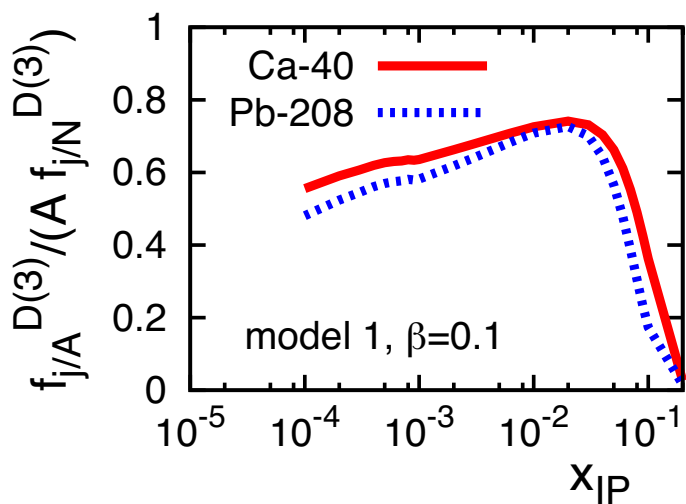
Hard diffraction off nuclei: test of understanding of dynamics, importance of fluctuations, proximity to black disk limit, practical applications for ultraperipheral pA collisions

Nuclear diffractive pdfs were calculated by Guzey et al 03 in the same approximations as LT nuclear pdf's (quasieikonal) and recently in the fluctuation approximation (no model necessary for double rescattering). Difference between QE and fluctuation is the same as in inclusive case

$$f_{j/A}^{D(3)}(x, Q_0^2, x_{\mathbb{P}}) = \frac{A^2}{4} 16\pi f_{j/N}^{D(4)}(x, Q_0^2, x_{\mathbb{P}}, t_{\min}) \\ \times \int d^2b \left| \int_{-\infty}^{\infty} dz \exp\{i x_{\mathbb{P}} m_N z\} \exp\left\{ \sigma_{\text{soft}}^j \frac{A}{2} (1 - i\eta) \int_z^{\infty} dz' \rho_A(b, z') \right\} \rho_A(b, z) \right|^2.$$

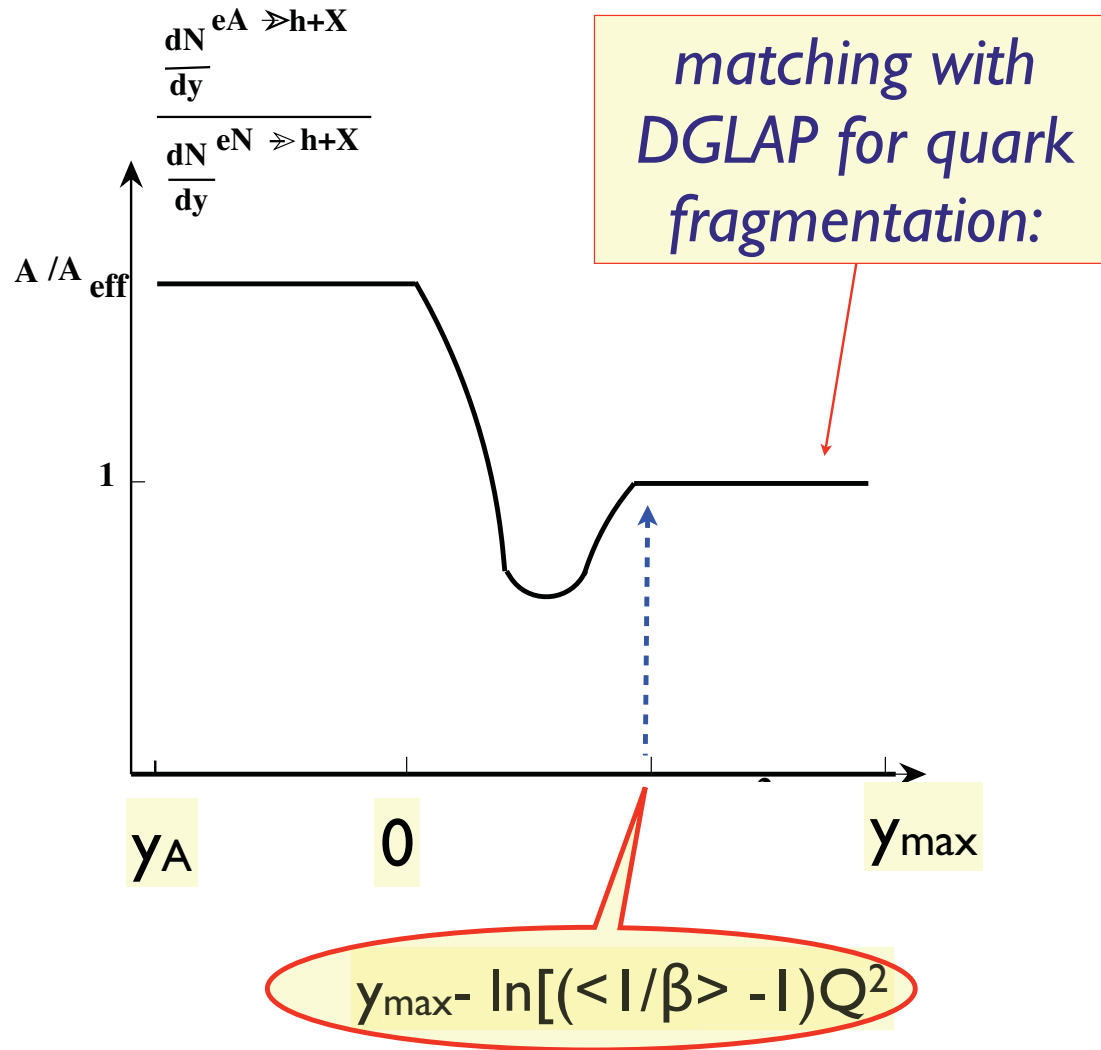


Much larger sensitivity to higher order effects - color fluctuations - large diffraction up to very large Q - will be possible to check soon in ultraperipheral collisions at the LHC



x_{IP}, β dependences are also calculated

Final nondiffractive states - many predictions similar to those for hA scattering
 -but with large color fluctuations effects.



Where DGLAP approximation breaks & non-linear(black disk?) regime suggest in the preQCD logic by Gribov sets in.

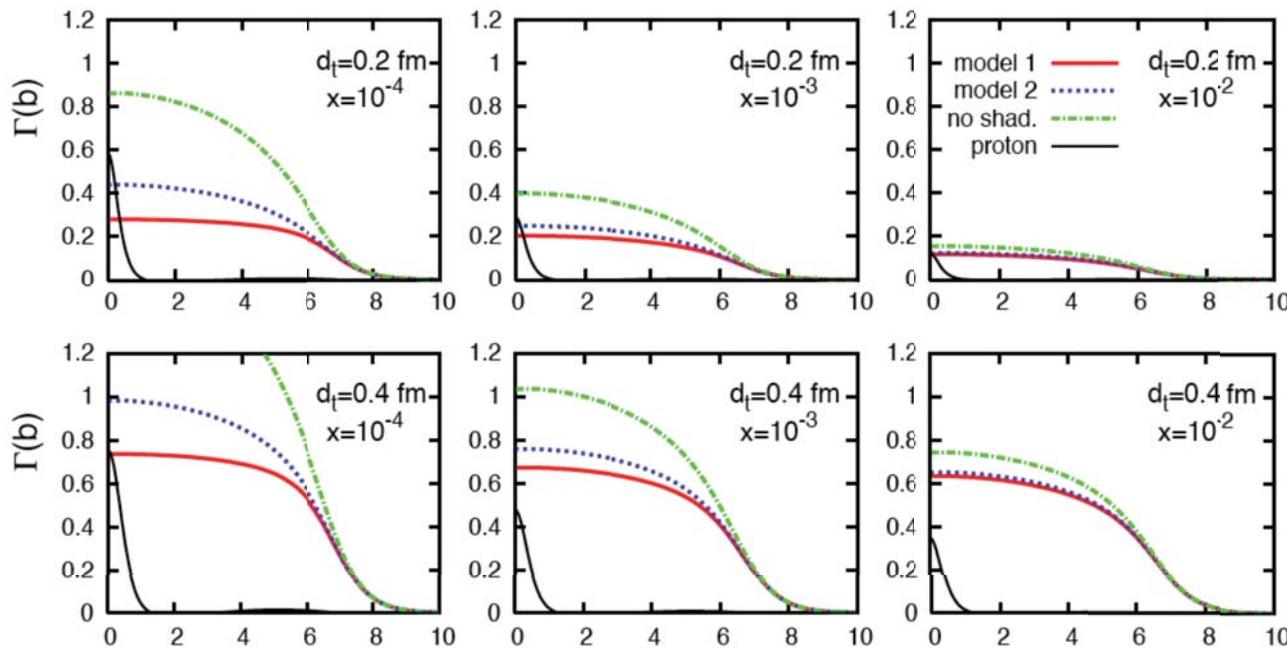
Impact factor $\Gamma(b)$ for quark - antiquark dipole p and dipole -Pb scattering

Update of Rogers et al 03

$$p_t \approx \frac{\pi}{2d}$$

$$p_t \approx 1.5 \text{ GeV}/c$$

$$p_t \approx 0.75 \text{ GeV}/c$$

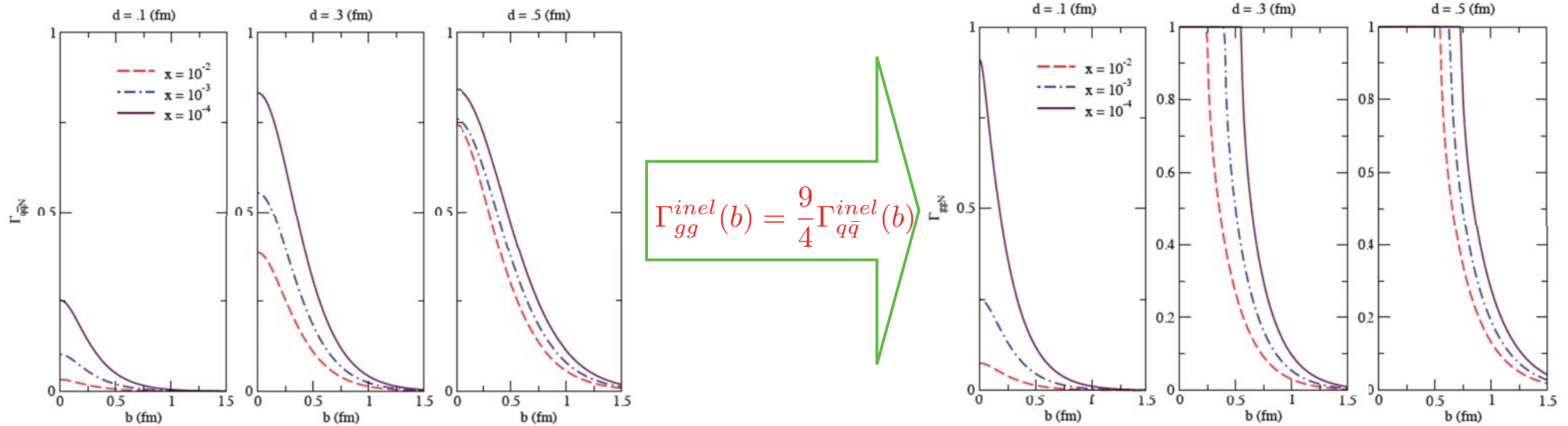


Probability of inelastic interaction is

$$P_{in} = |1 - \Gamma(b)|^2 \rightarrow P_{in} = 3/4$$

for $\Gamma(b) = 1/2$

Glue densities in nuclei and proton at $b=0$ are rather similar. Difference at $\langle b \rangle$ is $\sim 30\%$ larger



At HERA in quark channel range of b where interaction is close to BDR is small except for $Q^2 \sim 1 \text{ GeV}^2$.

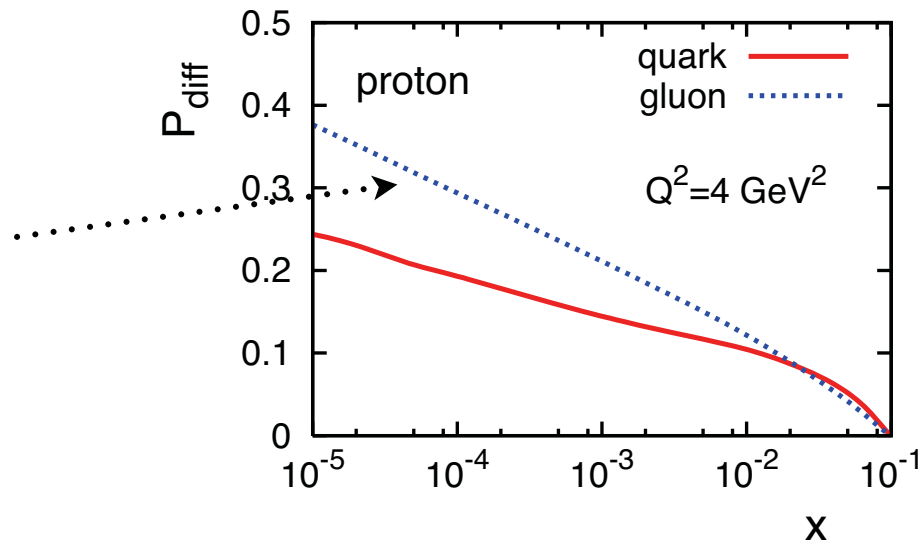
For gluons BDR range is much larger $Q^2 \sim 4 \text{ GeV}^2$ for $x=10^{-4}$?

Test - diffraction in ep DIS at HERA

What is probability that if the parton j is removed, there will be a diffractive final state?

$$P_{\text{diff}}^j = \frac{\int_x^{0.1} dx_{\mathbb{P}} \beta f_j^{D(3)}(\beta, Q^2, x_{\mathbb{P}})}{x f_j(x, Q^2)}$$

gg -N interaction seems close to BDR for $Q^2 \sim 4 \text{ GeV}^2, x \leq 10^{-4}$



The probability of hard diffraction on the nucleon, $P_{j \text{ diff}}$ as a function of x for u quarks and gluons.

Post selection effect in BDR - effective fractional energy losses for $p_t \leq p_t^{(BDR)}$

The simplest case example: Inclusive production of leading hadrons in DIS for $Q < 2p_t^{(BDR)}$

Guzey, McDermott, Frankfurt, MS 2000

Allows to explain dominance of peripheral collisions in DPb in production of leading pions at RHIC as well forward central correlations. Provides further support of onset of BDR for $x \sim 10^{-4}$ in nuclei for interaction with quarks.

Frankfurt, MS 2008

Contradicts expectations of CGC inspired $2 \rightarrow 1$ mechanism

Conclusions

- ☺ Significant LT shadowing effects are present up to large virtualities for $x < 0.01$ and can be calculated with small theoretical uncertainty.
- ☺ For small enough x and in a wide range of virtualities gluon shadowing remains larger than the quark shadowing.
- ☺ Simultaneous measurements of inclusive hard diffraction off nucleon at small t , nuclear shadowing and diffraction will provide stringent tests of the theory and allow to understand interplay of soft and hard dynamics and in particular establish the transition from DGLAP region to black disk regime at $x \sim 10^{-4}$