The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

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2146-23

Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond'

26 - 28 May 2010

Photon Polarization Asymmetry as an Insight into Light Meson and Muon Decays

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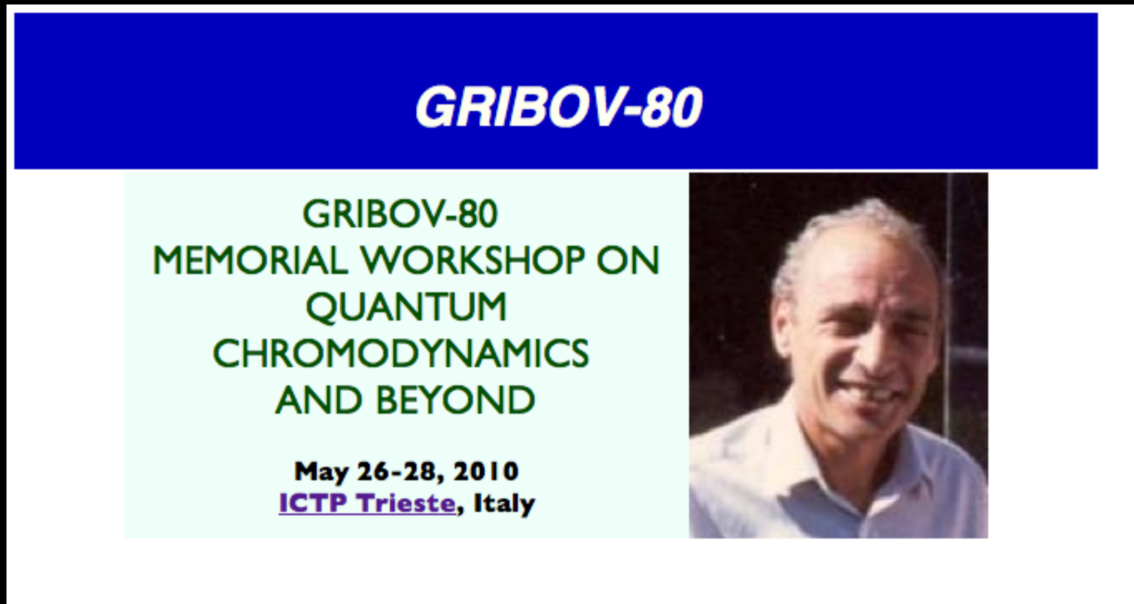
Photon Polarization Asymmetry as an Insight into Light Mesons (and Muon) Radiative Decays



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27-05-2010

E. Gabrielli, L.T.
Nucl.Phys.B792:48-88,2008

We systematically compute and discuss meson and muon polarized radiative decays. Doubly differential distributions in terms of momenta and helicities of the final lepton and photon are explicitly computed. The undergoing dynamics giving rise to lepton and photon polarizations is examined and analyzed in the soft and hard region of momenta. The particular configurations made by right-handed leptons with accompanying photons are investigated and interpreted as a manifestation of the axial anomaly. The photon polarization asymmetry is evaluated. Finiteness of polarized amplitudes against infrared and collinear singularities is shown to take place with mechanisms distinguishing between right handed and left handed final leptons. We propose a possible test using photon polarization to clarify a recently observed discrepancy in radiative meson decays.

- Polarized decays of Mesons and of the Muon. More detailed information with respect to unpolarized
- Anomalous helicity flip interpretation
- R- and L-handed lepton distributions
- Photon polarization asymmetry
- sources of precision tests of the SM

Precision tests of the Standard Model in radiative decays of mesons and of the muon are well known

Polarized decays contain additional information

Pion

L. Trentadue and M. Verbeni, Phys. Lett. **B 478** (2000) 137; Nucl. Phys. **B 583** (2000) 307, and references therein;

Muon

M. Fischer, S. Groote, J.G. Koerner and M.C. Mauser, Phys. Rev. **D 67** (2003) 113008, and references therein.

V.S. Schulz and L.M. Sehgal, Phys. Lett. **B 594** (2004) 153.

Axial Anomaly interpretation of helicity flip

A.D. Dolgov and V.I. Zakharov, Nucl. Phys. **B 27** (1971) 525.

B. Falk and L. M. Sehgal, Phys. Lett. **B 325** (1994) 509;
L. M. Sehgal, Phys. Lett. **B 569** (2003) 25.

Finiteness and separate cancellation of
collinear and infra-red singularities in
Right handed and Left-handed polarized final leptons
amplitudes

L. Trentadue and M. Verbeni, Phys. Lett. **B 478** (2000) 137; Nucl. Phys. **B 583** (2000) 307, and references therein;

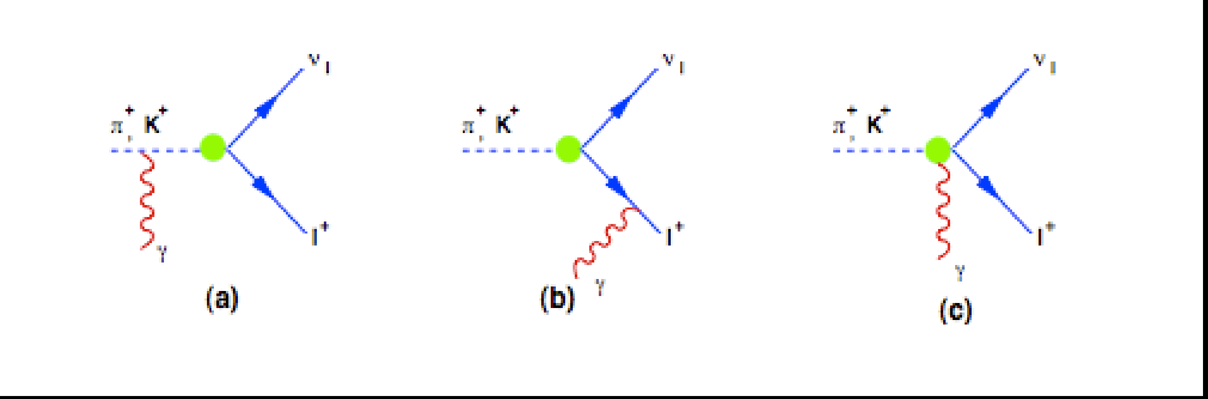
We calculate also photon polarization - not considered before ***

$$M^+(p) \to \nu_l(p_\nu)\; l^+(p_l, \lambda_l)\; \gamma(k, \lambda_\gamma)\,,$$

$$\mathrm{M}^{(\lambda_l,\lambda_\gamma)} = \mathrm{M}_{IB}^{(\lambda_l,\lambda_\gamma)} + \mathrm{M}_{SD}^{(\lambda_l,\lambda_\gamma)}$$

$$\mathrm{M}_{IB}^{(\lambda_l,\lambda_\gamma)} = \frac{ieG_F}{\sqrt{2}} m_l f_M V_{uq}\; \epsilon_\mu^\star(k,\lambda_\gamma) \left[\bar{u}(p_\nu)\left(\frac{p^\mu}{(p\cdot k)} - \frac{\not{k}\gamma^\mu + 2p_l^\mu}{2\,(p_l\cdot k)}\right)(1+\gamma_5)\, v(p_l,\lambda_l)\right]$$

$$\begin{aligned}\mathrm{M}_{SD}^{(\lambda_l,\lambda_\gamma)} \;=\;& -\frac{ieG_F}{\sqrt{2}} V_{uq}\; \epsilon_\mu^\star(k,\lambda_\gamma) \left\{(p\cdot k)\,\frac{A}{m_M}\left(-\eta^{\mu\nu} + \frac{p^\mu k^\nu}{(p\cdot k)}\right) + i\epsilon^{\mu\nu\alpha\beta}\frac{V}{m_M} k_\alpha p_\beta\right\} \\ & \times\; \left[\bar{u}(p_\nu)\gamma_\nu\,(1-\gamma_5)\, v(p_l,\lambda_l)\right],\end{aligned}$$



$$x \equiv \frac{2p\cdot k}{m_M^2}, \quad y \equiv \frac{2p\cdot p_l}{m_M^2}, \quad z \equiv \frac{2p_l\cdot k}{m_M^2} = y - 1 + x - r_l$$

$$\begin{aligned}
\mathrm{M}_{IB}^{(\lambda_l,\lambda_\gamma)} &= eG_F m_l f_M V_{uq}\, \frac{2}{z}\left\{\delta_{\lambda_l,-1}\left(\delta_{\lambda_\gamma,-1}\ \hat{E}_\gamma + \hat{E}_\nu\right) R_+ \sin\theta \right.\\
&+ \left.\delta_{\lambda_l,+1}\ \delta_{\lambda_\gamma,-1}\ \hat{E}_\gamma R_- \,(1-\cos\theta)\right\} e^{i\lambda_\gamma\varphi}
\end{aligned}$$

$$\begin{aligned}
\mathrm{M}_{SD^\pm}^{(\lambda_l,\lambda_\gamma)} &= eG_F m_M^2 V_{uq}\, \frac{(V\pm A)}{2}\, \delta_{\lambda_\gamma,\pm1}\ x\left\{\mp\delta_{\lambda_l,-1}\ R_- \sin\theta \right.\\
&\pm \left.\delta_{\lambda_l,+1} R_+ \,(\cos\theta \pm 1)\right\}\ e^{i\lambda_\gamma\varphi},
\end{aligned}$$

$\lambda_\gamma = -1$ left-handed (L) and $\lambda_\gamma = 1$ right-handed

θ is the angle between photon and lepton

$$\begin{aligned}
\hat{E}_\gamma &= \frac{x}{2\sqrt{1-x}}, \quad \hat{E}_\nu = \frac{1-x-r_l}{2\sqrt{1-x}}, \quad \hat{E}_l = \frac{1-x+r_l}{2\sqrt{1-x}},\\
\cos\theta &= \frac{(x-2)(1-x+r_l)+2y(1-x)}{x(1-r_l-x)}
\end{aligned}$$

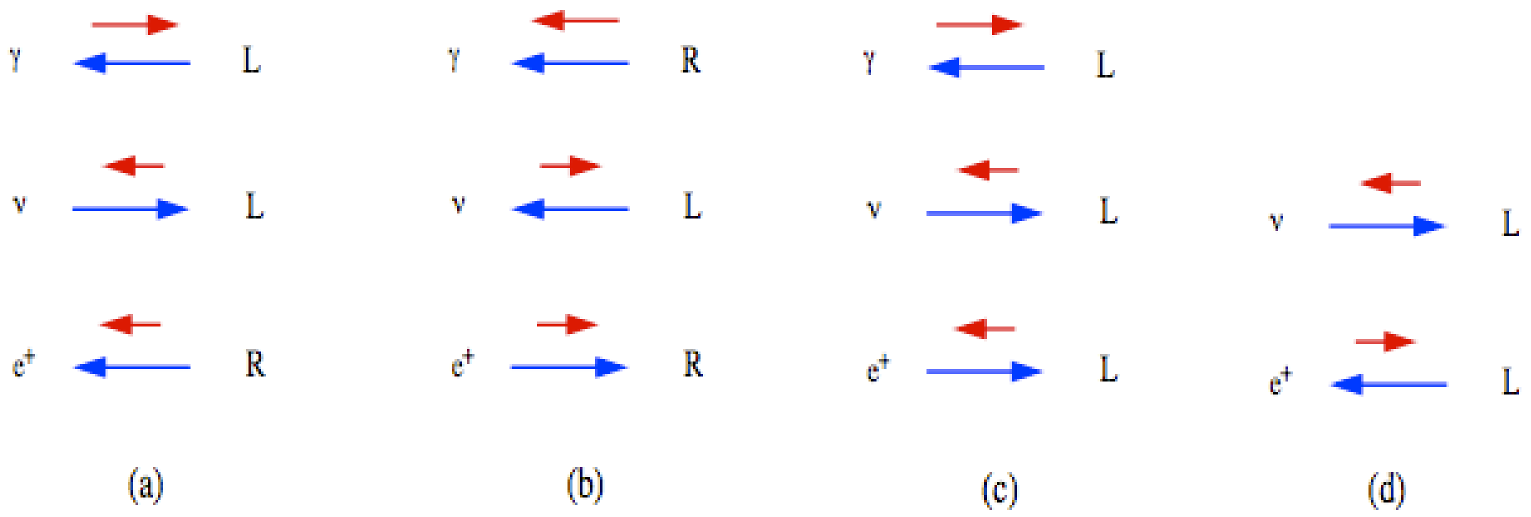
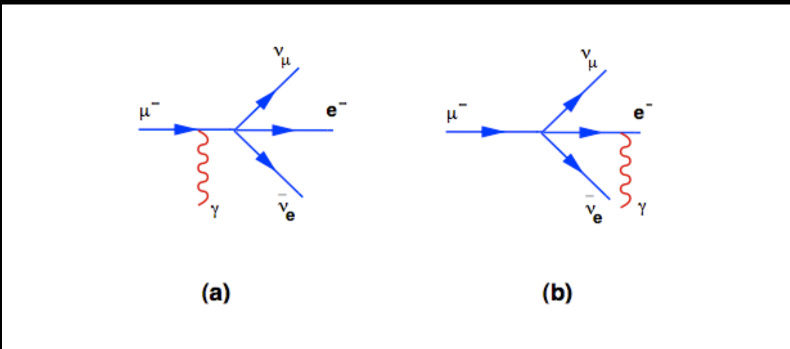


Figure 2: Allowed helicity (in red) configurations of γ, ν and e^+ for $\pi^+ \to e^+\nu_e\gamma$ decay in π^+ rest frame, figures (a), (b), (c), when all momenta (in blue) are aligned on the same axis. Direction of photon momentum is fixed by convention. Figure (d) corresponds to the non radiative decay $\pi^+ \to e^+\nu_e$. Analogous spin configurations hold for the corresponding K^+ decays as well.

$$\mu^-(p) \to \nu_\mu(q_1)\,\bar{\nu}_e(q_2)\,e^-(p_e)\,\gamma(k)$$



$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(\lambda_\gamma\,,\,\lambda_e)}}{dx\,dy\,dz} = -\frac{\alpha}{2\pi}\left\{\frac{M_\alpha^{(\lambda_\gamma,\lambda_e)\dagger}M_\beta^{(\lambda_\gamma,\lambda_e)}\,N^{\alpha\beta}}{4\,m_\mu^2}\right\}.$$

$$x = \frac{2E_\gamma}{m_\mu}, \qquad y = \frac{2E_e}{m_\mu}, \qquad z = \frac{x}{2}\,(y - \mathrm{A}_e\,\cos\theta)\,,$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(\lambda_\gamma\,,\,\lambda_e)}}{dx\,dy} = \frac{\alpha}{24\pi}\,\frac{1}{\mathrm{A}_e\,x}\left[G_0 + \lambda_\gamma\,\bar{G}_0 + \lambda_e\,(G_1 + \lambda_\gamma\,\bar{G}_1)\right],$$

$$\begin{aligned}
\lim_{r\to 0}\frac{1}{\Gamma_0}\frac{d\Gamma^{(R,L)}}{dx\,dy} &= \frac{\alpha}{3\pi}\,\frac{y^2}{x}\Big\{-3\,(x-3)\,\left(x^2-2\right) + (12 + x\,(9 + x\,(2\,x-5)))\,y \\
&+ (2\,x + 2\,y - 3)\,(3\,\log(r) - 6\,\log(y))\Big\} \\
\lim_{r\to 0}\frac{1}{\Gamma_0}\frac{d\Gamma^{(R,R)}}{dx\,dy} &= 0 \\
\lim_{r\to 0}\frac{1}{\Gamma_0}\frac{d\Gamma^{(L,L)}}{dx\,dy} &= \frac{\alpha}{2\pi}\,\frac{1}{x}\Big\{4\,x^3\,\left(1 + \log(r) + y\right) + 2\,x\,(y-1)\,y\,\Big(12 + 6\,\log(r) + y \\
&+ 2\,(2 + \log(r))\,y^2\,(2\,y - 3) + x^2\,\Big(6\,\log(r)\,(2\,y-1) \\
&+ y\,(16 + 5\,y) - 6\Big) - 4\,(x+y)^2\,(2\,x + 2\,y - 3)\,\log(y)\Big\} \\
\lim_{r\to 0}\frac{1}{\Gamma_0}\frac{d\Gamma^{(L,R)}}{dx\,dy} &= \frac{\alpha}{\pi}\,x\,(3 - 2\,x - 2\,y)\,,
\end{aligned}$$

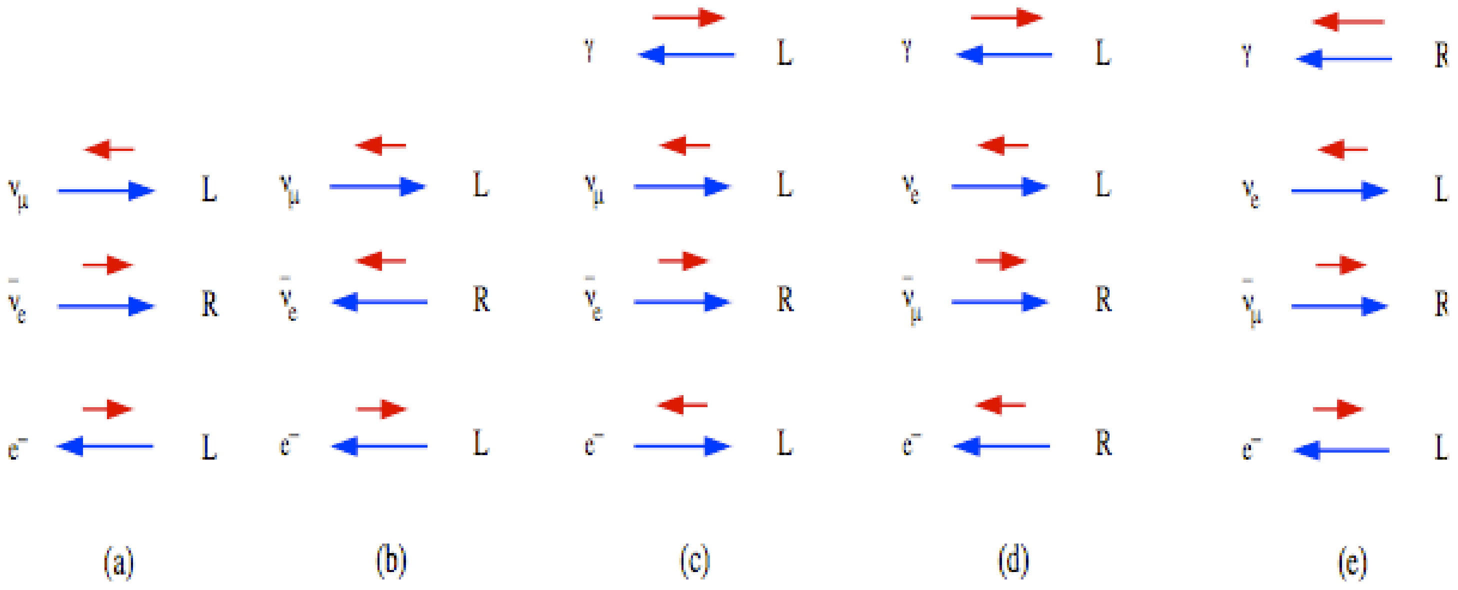


Figure 10: Some helicity (in red) configurations of γ, ν_μ $\bar{\nu}_e$ and e^- for $\mu^- \to \nu_\nu \bar{\nu}_e e^- \gamma$ decay in μ^- rest frame, figures (c)-(e), when all momenta (in blue) are aligned on the same axis. Figures (a) and (b) correspond to the non-radiative decay $\mu^- \to \nu_\nu \bar{\nu}_e$. Direction of electron momentum in (a)-(b), as well as photon momentum in (c)-(e) diagrams, is fixed by convention.

$$
\begin{aligned}
\lim_{r\to 0} \frac{1}{\Gamma_0}\frac{d\Gamma^{(R,L)}}{dx\,dy} &= \frac{\alpha}{3\pi}\,\frac{y^2}{x}\Big\{ -3\,(x-3)\,\big(x^2-2\big) + (12 + x\,(9 + x\,(2\,x-5)))\,y \\
&+ (2\,x + 2\,y - 3)\,(3\,\log(r) - 6\,\log(y))\Big\} \\
\lim_{r\to 0} \frac{1}{\Gamma_0}\frac{d\Gamma^{(R,R)}}{dx\,dy} &= 0 \\
\lim_{r\to 0} \frac{1}{\Gamma_0}\frac{d\Gamma^{(L,L)}}{dx\,dy} &= \frac{\alpha}{2\pi}\,\frac{1}{x}\Big\{4\,x^3\,\big(1 + \log(r) + y\big) + 2\,x\,(y-1)\,y\,\big(12 + 6\,\log(r) + y\big) \\
&+ 2\,(2 + \log(r))\,y^2\,(2\,y-3) + x^2\,\Big(6\,\log(r)\,(2\,y-1) \\
&+ y\,(16 + 5\,y) - 6\Big) - 4\,(x+y)^2\,(2\,x + 2\,y - 3)\,\log(y)\Big\} \\
\lim_{r\to 0} \frac{1}{\Gamma_0}\frac{d\Gamma^{(L,R)}}{dx\,dy} &= \frac{\alpha}{\pi}\,x\,(3 - 2\,x - 2\,y)\;,
\end{aligned}
$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(R,L)}}{dy} = \frac{\alpha}{\pi} y^2 \left\{ \left[\log(x_0) - \log(1-y) \right] (3-2\,y)\,(2+\log(r)-2\,\log(y)) + \frac{1}{18}(1-y)\Big(57+36\,\log(r)+28\,y+y^2+4\,y^3-72\,\log(y)\Big)\right\}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(R,R)}}{dy} = 0$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(L,L)}}{dy} = \frac{\alpha}{\pi} \left\{ \left[\log(x_0) - \log(1-y) \right] y^2 \, (3-2\,y)\,(2+\log(r)-2\,\log(y)) - \frac{1}{12}(y-1)^2 \, \Big(10+96\,y+5\,y^2+2\,\log(r)\,(5+22\,y) - 4\,(5+22\,y)\,\log(y)\Big)\right\}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma^{(L,R)}}{dy} = \frac{\alpha}{6\pi}\,(1-y)^2\,(5-2y)\,.$$

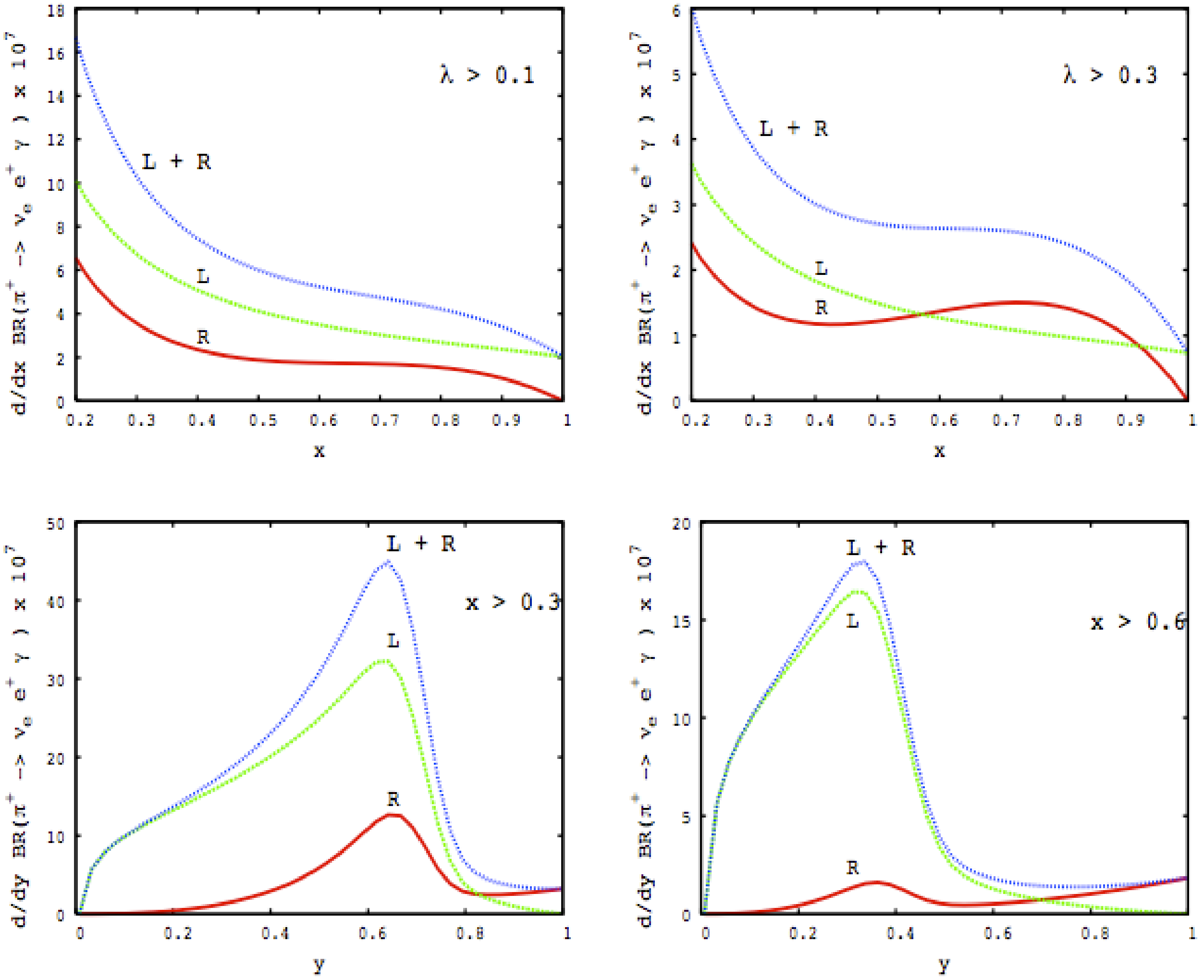


Figure 4: The photon energy spectrum $\frac{dBR_\gamma}{dx}$ versus x (top plots) and electron energy spectrum $\frac{dBR_\gamma}{dy}$ versus y (bottom plots), for pion decay $\pi^+ \to \nu_e e^+ \gamma$. The labels L and R attached to the curves indicate pure left-handed and right-handed photon polarizations contributions respectively, while $L + R$ correspond to the sum. Kinematical cuts $\lambda > 0.1$ (top-left), $\lambda > 0.3$ (top-right) and $x > 0.3$ (bottom-left), $x > 0.6$ (bottom-right) are applied respectively.

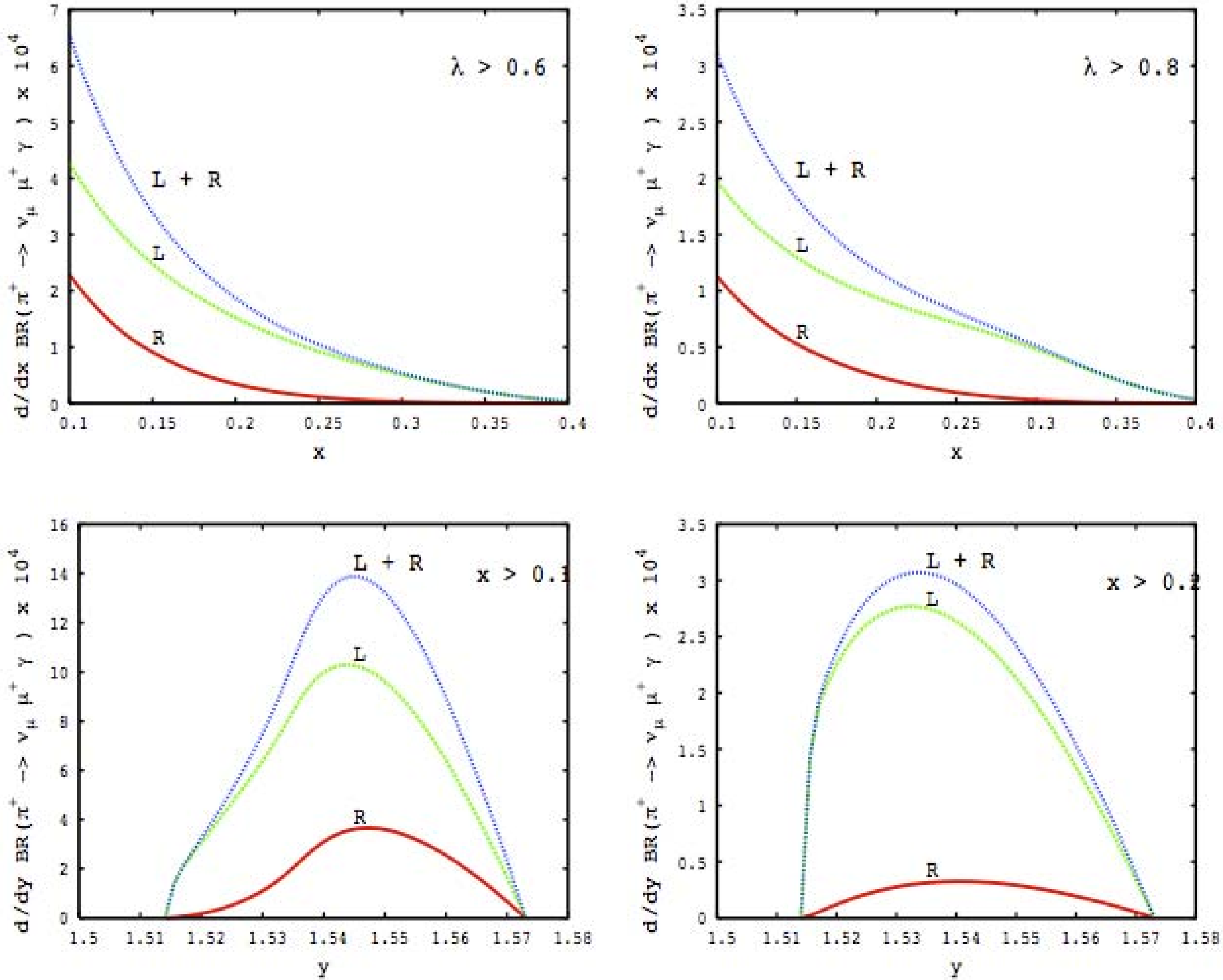


Figure 5: As in Fig. 4, but for pion decay $\pi^+ \to \nu_\mu \mu^+ \gamma$, and with kinematical cuts $\lambda > 0.6$ (top-left), $\lambda > 0.8$ (top-right) and $x > 0.1$ (bottom-left), $x > 0.2$ (bottom-right).

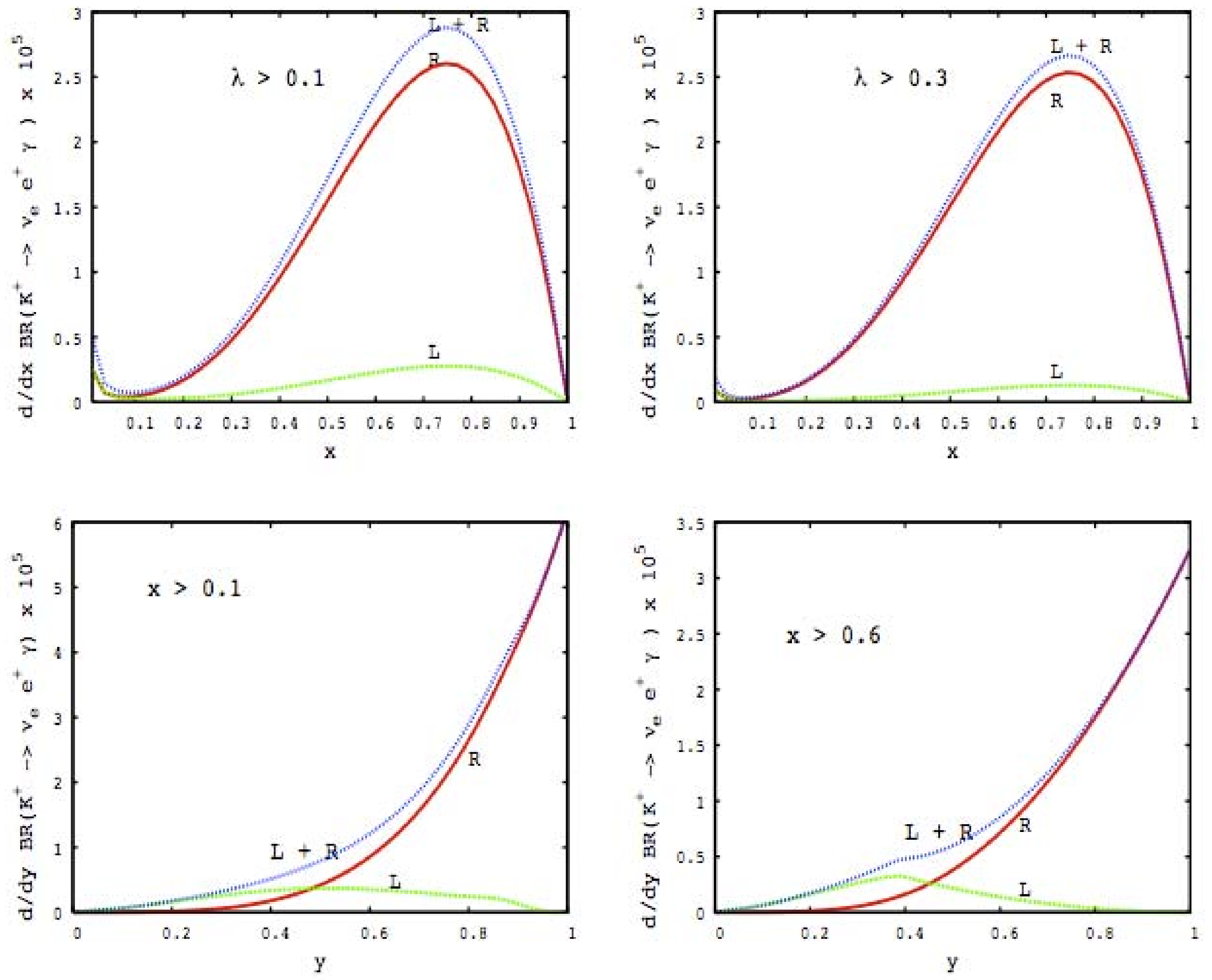


Figure 7: As in Fig. 4, but for kaon decay in $K^+ \to \nu_e e^+ \gamma$. Curves correspond to kinematical cuts as reported in the figures.

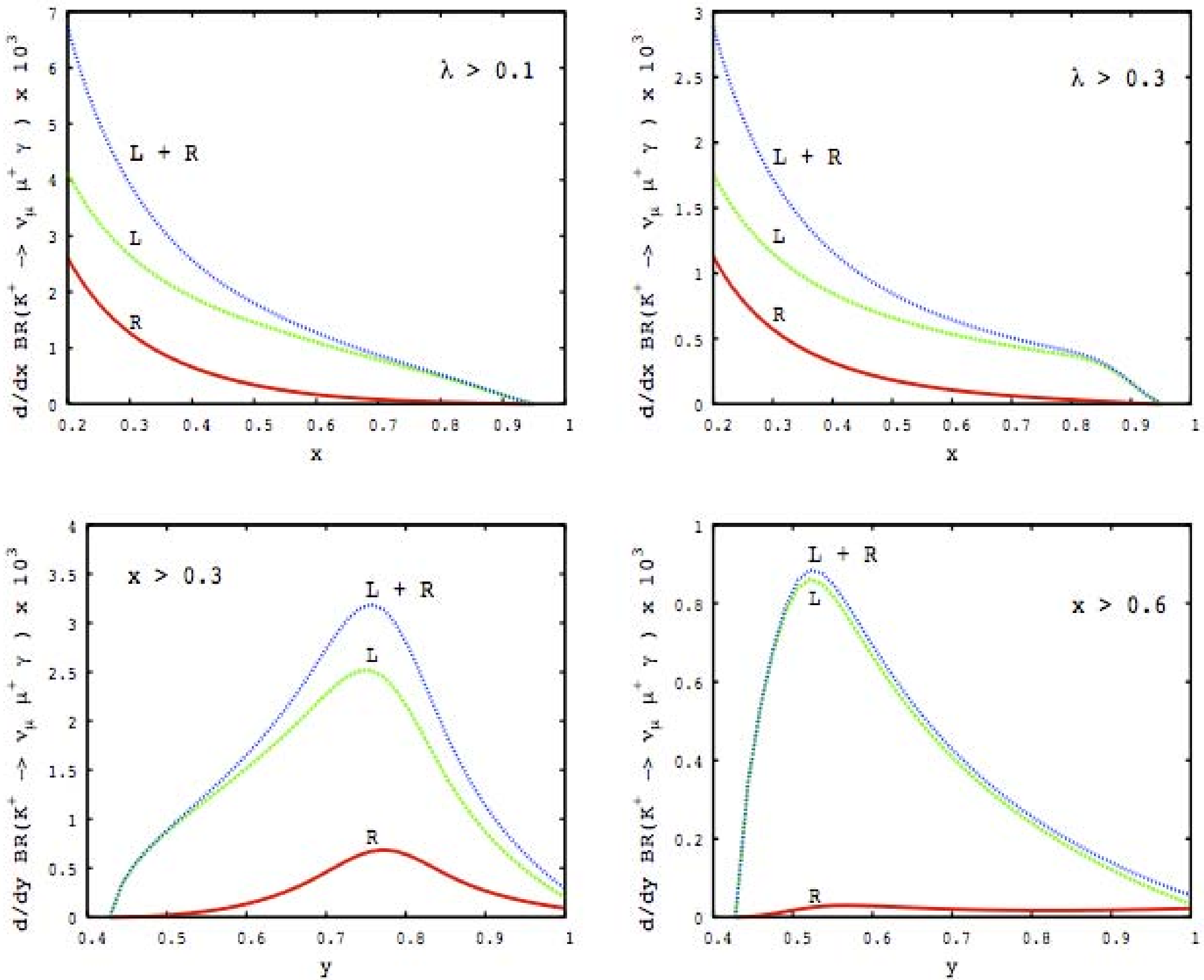


Figure 8: As in Fig. 7, but for kaon decay in $K^+ \to \nu_\mu \mu^+ \gamma$. Curves correspond to kinematical cuts as indicated in the figures.

Photon polarization asymmetry

$$\frac{\mathrm{dA}_\gamma}{\mathrm{d}\xi} \equiv \frac{\mathrm{d}_\xi(\mathrm{BR_L}) - \mathrm{d}_\xi(\mathrm{BR_R})}{\mathrm{d}_\xi(\mathrm{BR_L}) + \mathrm{d}_\xi(\mathrm{BR_R})}$$

$$\mathrm{d}_\xi(\mathrm{BR_{L,R}}) \equiv \frac{\mathrm{d\,BR_{L,R}}}{\mathrm{d}\xi}$$

$$\xi = \{x, y\}$$

is an infrared finite quantity

$$\lim_{x\to 0} \{\rho_\mathrm{L}(x,y) - \rho_\mathrm{R}(x,y)\} \;\to\; \mathcal{O}(x)$$

$$\lim_{x\to 0} \{\rho_\mathrm{L}(x,y) + \rho_\mathrm{R}(x,y)\} \to \log(x)$$

- **very sensitive to hadronic form factors V and A**

provides a direct measure of parity violation

Photon electron and muon spectra

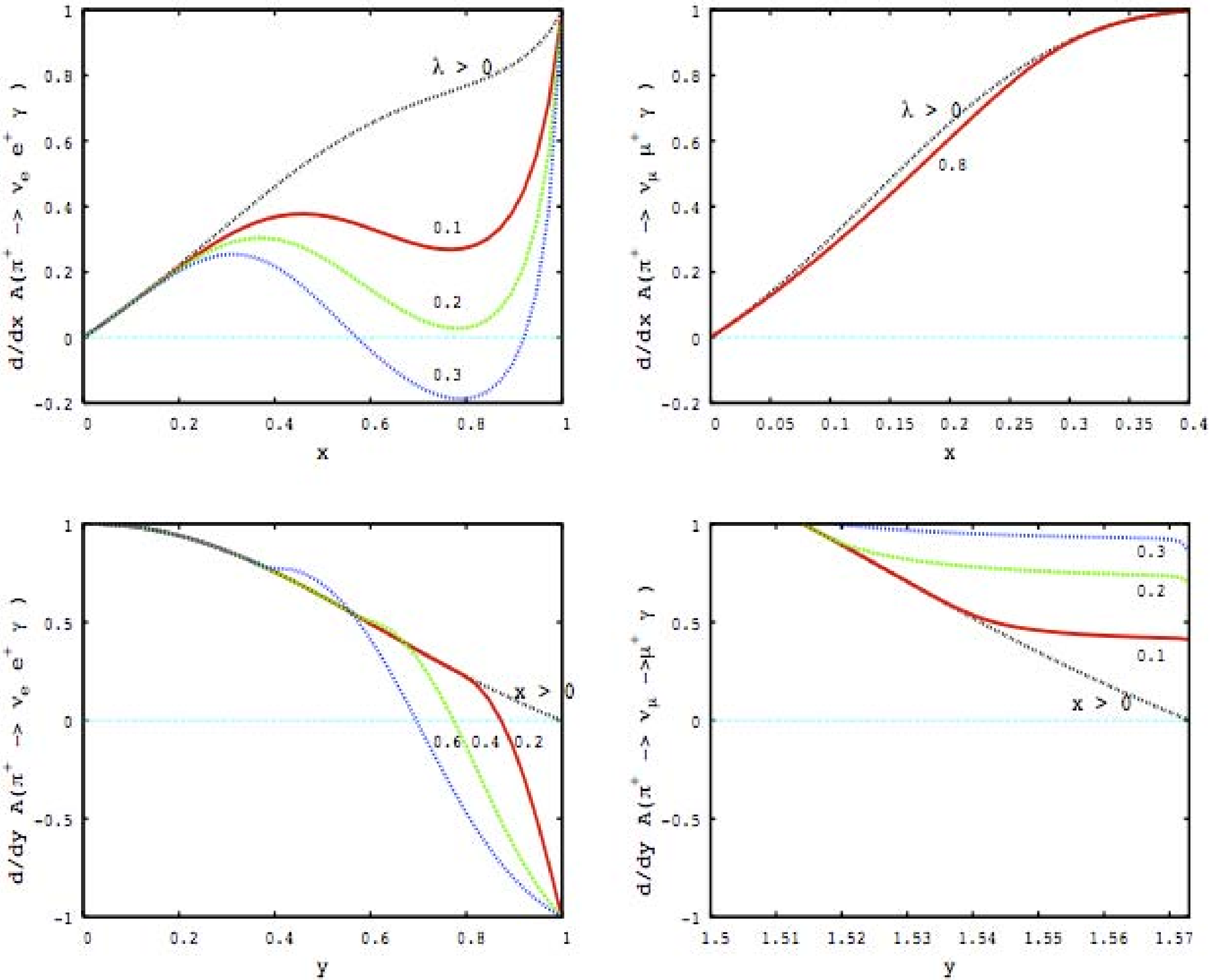
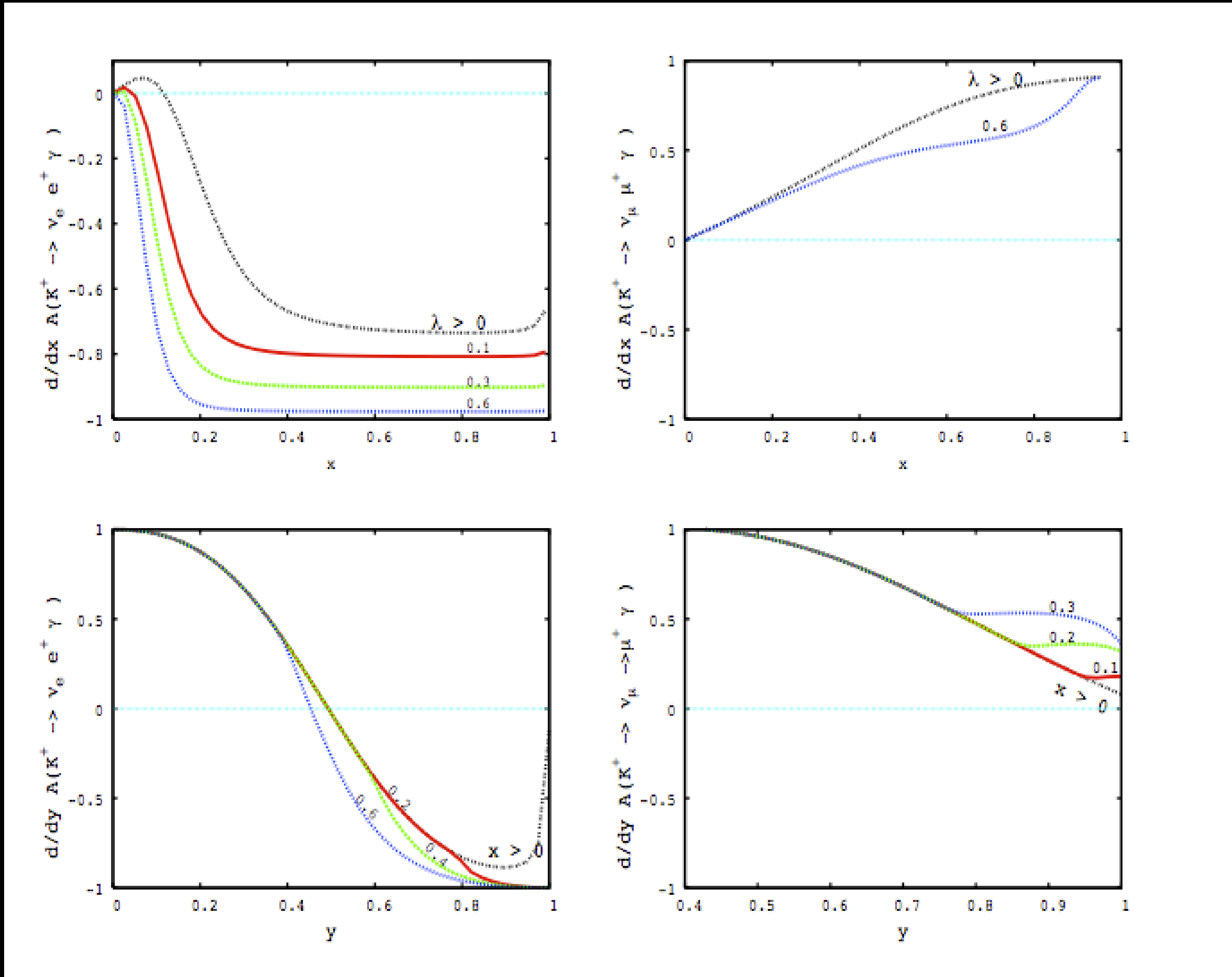


Figure 6: The differential asymmetry $\frac{dA_\gamma}{dx}$ versus x (top) and $\frac{dA_\gamma}{dy}$ versus y (bottom), with kinematical cuts $\lambda > 0$, 0.1, 0.2, 0.3 (top-left), $\lambda > 0$, 0.8 (top-right) and $y > 0$, 0.2, 0.4, 0.6 (bottom-left), $y > 0, 0.10.2, 0.3$ (bottom-right) respectively.



d/dx A(K+ -> νe e+ γ)
λ > 0
0.1
0.3
0.6
x
d/dx A(K+ -> νμ μ+ γ)
λ > 0
0.6
x
d/dy A(K+ -> νe e+ γ)
0.2
0.6
0.4
x > 0
y
d/dy A(K+ -> νμ ->μ+ γ)
0.3
0.2
0.1
x > 0
y

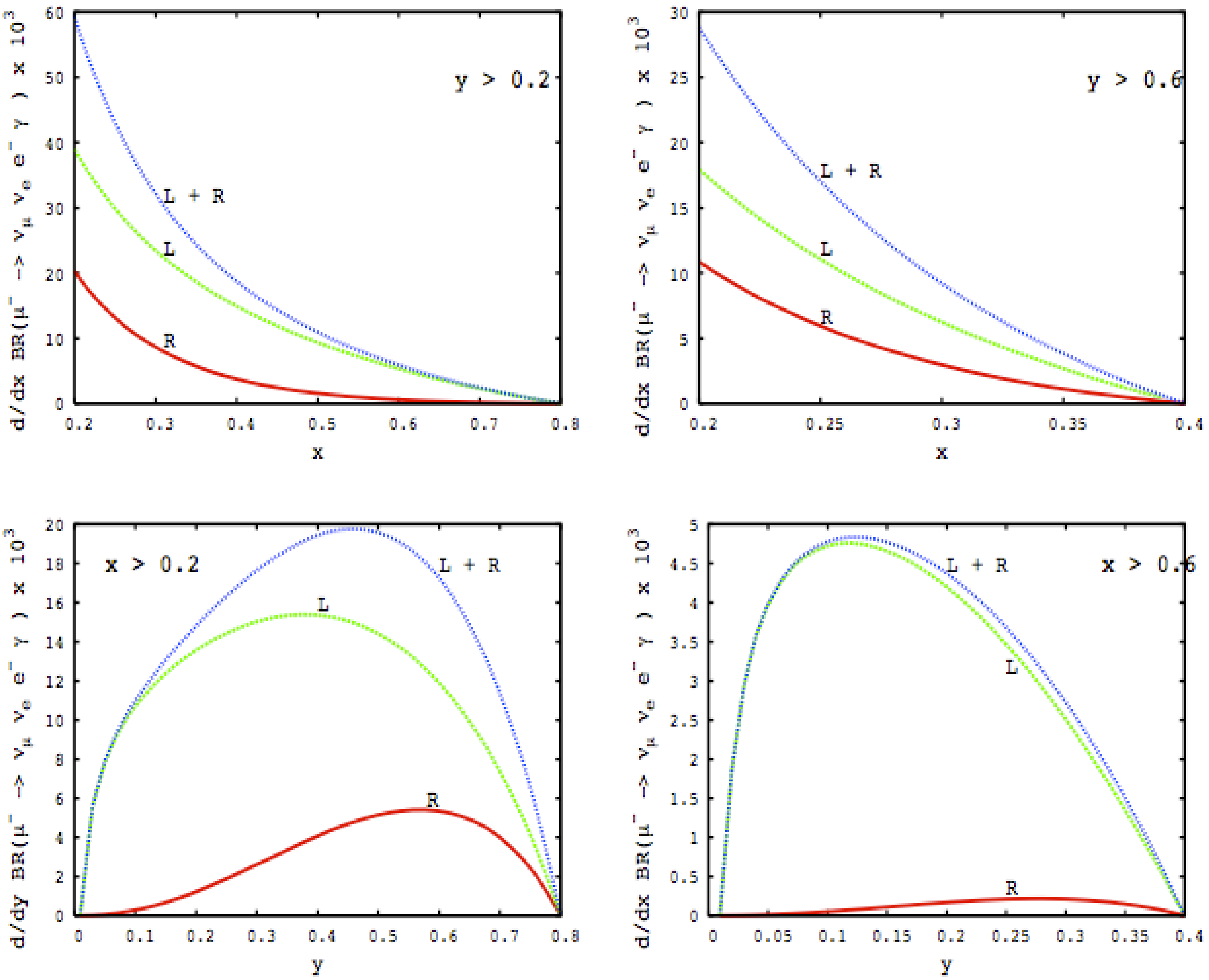


Figure 11: The photon energy (E_γ) spectrum $\frac{\mathrm{dBR}_\gamma}{\mathrm{dx}}$ versus $x = 2\,E_\gamma/m_\mu$ (top) and electron energy (E_γ) spectrum $\frac{\mathrm{dBR}_\gamma}{\mathrm{dy}}$ versus $y = 2\,E_e/m_\mu$ (bottom) for muon decay $\mu^+ \to \bar{\nu}_\mu \nu_e e^+$ with left-handed (L) and right-handed (R) photon polarizations. and for kinematical cuts $y > 0.2$ (top-left), $y > 0.6$ (top-right) and $x > 0.2$ (bottom-left), $x > 0.6$ (bottom-right) respectively.

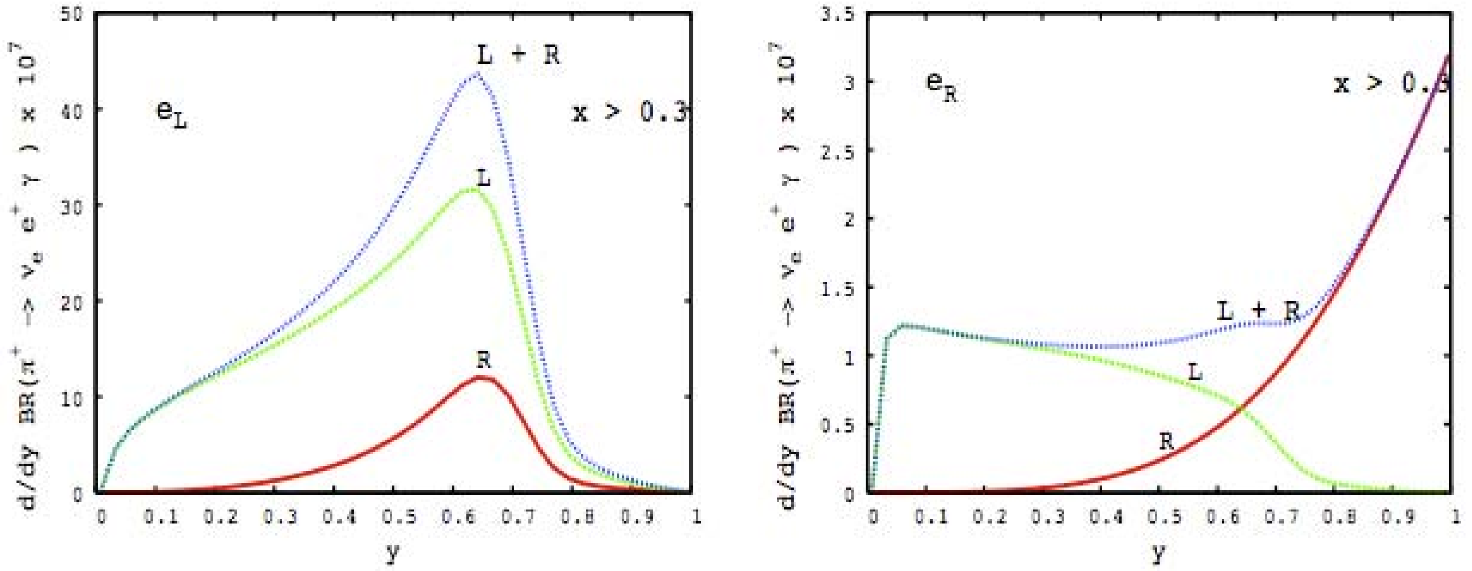


Figure 13: The electron energy spectrum $\frac{\mathrm{dBR}}{\mathrm{dy}}$ versus y for $\pi^+ \to \nu_e e^+ \gamma$, for left-handed (left plot) and right-handed electron polarizations (right plot), with photon energy cut $x > 0.3$. As in previous figures, the labels L and R labels attached to the curves indicate pure left-handed and right-handed photon polarizations contributions respectively, while $L + R$ correspond to the sum.

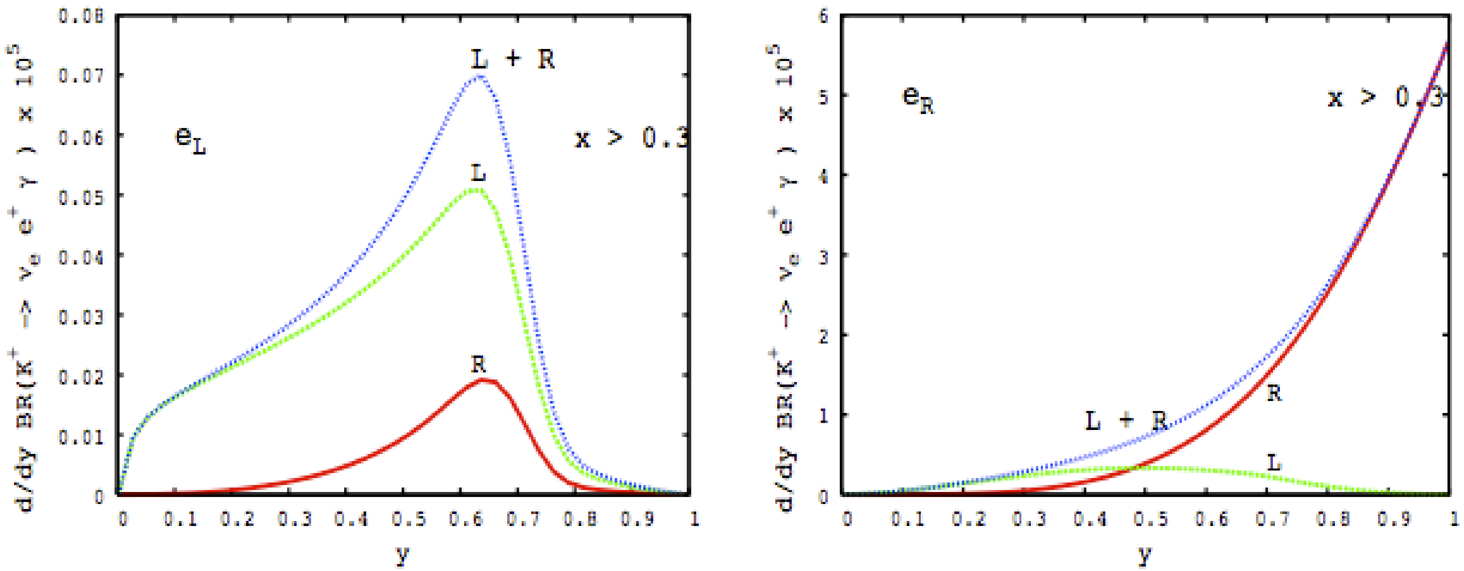


Figure 14: As in Fig.13, but for $K^+ \to \nu_e e^+ \gamma$.

The anomalous helicity flip contribution

- naive prediction of massless QED is that helicity flip contribution vanishes at any order in perturbation theory
- Lee and Nauenberg ('64) pointed out that there will be a non-zero helicity-flip contribution from collinear photon emission surviving the limit m_e ----> 0
- this phenomenon happens also in meson and muon radiative decays (T. and Verbeni ('03), Fischer et al. ('02), Sehgal and Schulz ('03))

- in the radiative scattering of electrons from Coulomb field the probability of helicity flip in the massless limit does not vanish

$$\frac{d\sigma}{d\theta^2} \sim \frac{\left(\frac{m_e}{E_e}\right)^2}{\left(\left(\frac{m_e}{E_e}\right)^2 + \theta^2\right)^2}$$

- in massless electron theory, positron will be emitted left-handed in IB contribution to meson decay and analogously for the electron in muon decay

- finite right-handed positron/electron contribution in the massless limit

- axial anomaly can be traced to the existence of an anomalous helicity-flip contribution to the absorbitive part of VVA triangle diagram in massless QED (Dolgov-Zakharov ('71))

Cancellation of mass singularities

$$\lim_{r_l \to 0} \lim_{y \to 1} \frac{1}{\Gamma_0} \frac{d\Gamma_{IB}^{(L,R)}}{dy} = \frac{\alpha}{2\pi}\Big(1-y\Big) = 0\,.$$

by summing over the polarizations one obtains the usual results

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = D(y,r)[1+\frac{\alpha}{\pi}R_l(y)].$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = \delta(1-y) + \frac{\alpha}{\pi}(L-1)P^{(1)}(y) + \frac{\alpha}{\pi}R_l(y)].$$

$$R_l(y) = 1 - y - \frac{1}{2}(1-y)\log(1-y) + \frac{1+y^2}{1-y}\log\, y.$$

$$\frac{\Gamma}{\Gamma_0} = 1 + \frac{\alpha}{\pi}[\frac{15}{8} - \frac{\pi^2}{3}].$$

Conclusions

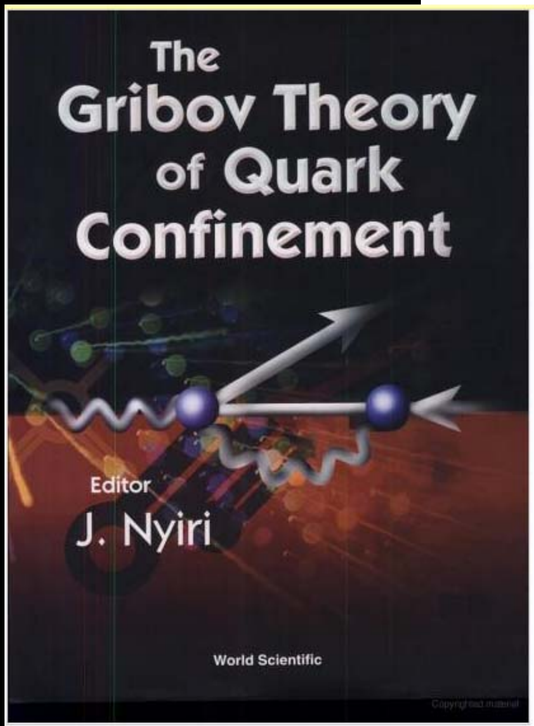
- we have analyzed polarized distributions in radiative meson and muon decays and the definition of photon-polarization-asymmetry has been introduced allowing a new approach to investigate interaction dynamics via a finite and universal quantity directly connected to parity violation.
- In the pion case the production of hard photons in association with soft positron are mainly left-handed polarized

$E_\gamma \gtrsim 25$ MeV and $E_{e^+} \gtrsim 120$ MeV

- on the contrary in Kaon case when energy cuts are imposed both positron and photon are mainly right-handed polarized
- regarding the meson decay in muon channel, photon is mainly left-handed polarized
- Mechanisms of cancellations of mass singularities have been analyzed for polarized processes
- right-handed lepton contributions are finite while left-handed ones are only finite for inclusive processes, i.e. virtual + radiative photons.
- Looking forward to see these properties in the experiments

Budapest 1984

preprint
KFKI-1981-66



V. N. Gribov

Anomalies, as a manifestation of the high momentum collective motion in the vacuum

It is shown that the source of any anomalies in the theory is the collective motion of the vacuum particles with arbitrarily large moments.

1. General description of the phenomenon

This paper has a mainly pedagogical character. Still, I feel that it is not absolutely meaningless to rediscuss in another language one of the most beautiful and non-trivial phenomena in the modern field theory. What is the mystery of the phenomenon we call anomalies? We have a theory in which the energy-momentum tensor and for example the axial charge are conserved on the classical level. We also have the quantum perturbation expansion in the theory which preserves conservations of energy-momentum of the particles and their helicities in any order. But, when we start to calculate the energy-momentum tensor and the axial current explicitly, for example in a given external field, we find out, that these quantities are not conserved even in second order in the external field.

This discovery [1] becomes very important, when we are outside the region where perturbation theory is valid [2]. In order to understand what is going on, let us consider the simplest case — free vacuum of massless fermions in external electromagnetic field. Even more, let us imagine this vacuum, as a gas of classical massless particles, moving independently in external field.

Volume 194, number 1 PHYSICS LETTERS B 30 July 1987

ANOMALIES AND A POSSIBLE SOLUTION OF PROBLEMS OF ZERO-CHARGE AND INFRA-RED INSTABILITY

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Received 30 April 1987

We discuss a possible solution of the problems of zero-charge in QED and of infra-red instability in QCD. The solution is based on the introduction of extra currents which compensate the Pauli–Villars subtraction in the amplitudes of photon–photon or gluon–gluon scattering.

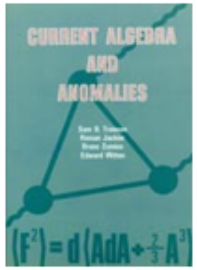
1. Introduction. One of the main hypotheses of modern field theory is the hypothesis that the theory can be defined by a lagrangian with an ultra-violet cut-off λ. Practically, this means that when calculating any loops we can neglect all contributions of particles of momenta larger than $\sqrt{\lambda}$. The only manifestation of momenta beyond $\sqrt{\lambda}$ is that the coupling constant must depend on λ. This hypothesis is commonly accepted as a very natural one. Theories which do not satisfy this condition, such as those which contain an axial anomaly, are considered as not self-consistent. But even in a usual theory like QED or QCD, the existence of anomalies in the axial current and the energy–momentum tensor shows that fluctuations with frequencies higher than $\sqrt{\lambda}$ can be essential. In this paper we shall show that

theory, to some degrees of freedom known a priori. It is not a current of real particles of the theory, because all the particles have a momentum less than $\sqrt{\lambda}$. It could be a current of some condensate, but condensates do not show up in perturbation theory.

The existence of this anomalous part of the current means that the bare vacuum is unstable and that inclusion of interactions will lead to the creation of some condensates and, maybe, of some new physical states. If we knew these condensates and states, we could in principle formulate the theory in terms of new fields in the usual lagrangian way and write the current down as a gauge invariant form. In perturbation theory we have no choice at the moment, but to look at the development of the vacuum state as a function of time in the process of switching on the

CURRENT ALGEBRA AND ANOMALIES

by **S Treiman**, **R Jackiw**, **B Zumino**, & **E Witten**



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V. N. Gribov, *Anomalies as a Manifestation of the High Momentum Collective Motion in the Vacuum* 74

Preprint KFKI-1981-66 (1981); Proc. of the Workshop and Conference on