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Screening in plasma with charged Bose condensate

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SCREENING IN PLASMA WITH CHARGED BOSE CONDENSATE

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Based on:

A.D. Dolgov, A. Lepidi, G. Piccinelli JCAP 0902 (2009) 027; Phys. Rev D, 80 (2009) 125009, arXiv: 1005.2702 [astro-ph]

Similar results in:

G. Gabadadze, R.A. Rosen Phys. Lett. B 658 (2008) 266; JCAP 0810 (2008) 030; JCAP 1004 (2010) 028.

Textbook formula for screening:

$$U(r) = \frac{Q}{4\pi r} \to \frac{Q\,\exp(-m_D r)}{4\pi r},$$

because the time-time component of the photon propagator acquires "mass" term:

$$k^2 \to k^2 + \Pi_{00}(k) = k^2 + m_D^2\,,$$

where e.g. for relativistic fermions

$$m_D^2 = e^2\left(T^2/3 + \mu^2/\pi^2\right).$$

Strangely untill recently effects on screening from condensate of a charged Bose field were not considered.

Consider electrically neutral plasma with large electric charge density of fermions compensated by charged bosons Bosons condense when their chemical potential reaches maximum value:

$$\mu_B = m_B \, .$$

Equilibrium distribution of condensed bosons:

$$f_B = C\delta^{(3)}(\mathrm{q}) + \frac{1}{\exp\left[(E - m_B)/T\right] \pm 1}$$

annihilates collision integral for an arbitrary constant C.

Calculations:

$$\partial_\nu F^{\mu\nu}(x) = \mathcal{J}^\mu(x)\,,$$

where

$$\mathcal{J}^\mu(x) = -i\,e[(\phi^\dagger(x)\partial^\mu\phi(x)) - (\partial^\mu\phi^\dagger(x))\phi(x)] + 2e^2 A^\mu(x)|\phi(x)|^2$$

Express ϕ through A_μ:

$$\phi(x) = \phi_0(x) + \int d^4y\, G_B(x-y)\mathcal{J}_\phi(y)\,,$$

where

$$\mathcal{J}_\phi(x) = -i\,e[\partial_\mu A^\mu + 2A_\mu\partial^\mu]\phi + e^2 A^\mu A_\mu\phi\,.$$

$$\partial_\nu F^{\mu\nu}(x) = -ie\,\phi_0^\dagger(x)\times$$
$$\partial^\mu\left[\int d^4y\,G_B(x-y)\mathcal{J}_\phi(y)\right]+\ldots$$

where

$$\mathcal{J}_\phi(x) = +i\,e[\partial_\mu A^\mu(x) + 2A_\mu(x)\partial^\mu]\phi_0(x).$$

Averaging Maxwell equation over medium:

$$\langle a^\dagger(\mathrm{q})a(\mathrm{q}')\rangle = f_B(E_q)\delta^{(3)}(\mathrm{q}-\mathrm{q}'),$$
$$\langle a(\mathrm{q})a^\dagger(\mathrm{q}')\rangle = [1+f_B(E_p)]\delta^{(3)}(\mathrm{q}-\mathrm{q}')\,.$$

$$\Pi_{00}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}[f_B(E_B,\mu_B)$$

$$+\bar{f}_B(E_B,\bar{\mu}_B)][1+\frac{E_B^2}{kq}\ln|\frac{2q+k}{2q-k}|] \,.$$

Without condensate one obtaines the usual k-independent Debye screening:

$$\Pi_{00}(0,k) = m_D^2 \,.$$

Corrections to Π_{00} at low k are infrared singular:

$$\frac{\Delta\Pi_{00}}{e^2} = \frac{m_B^2 T}{2k} + \frac{C}{m_B(2\pi)^3}\left(1 + \frac{4m_B^2}{k^2}\right)$$

Both terms in r.h.s. appear only if $\mu = m_B$.

Instead of exponential the screening becomes power law and oscillating, depending upon parameters, m_j:

$$\Pi_{00} = m_0^2 + m_1^3/k + m_2^4/k^2.$$

May this have something to do with confinement? Recent paper: P. Gaete, E. Spalucci, 0902.00905 – confinement in Higgs phase.

Contribution from poles (in some limit):

$$U(r)_{pole} = \frac{Q}{4\pi r} \exp\left(-\sqrt{e/2}\, m_2 r\right) \times \cos\left(\sqrt{e/2}\, m_2 r\right).$$

Osicllating screening is known for fermions, Friedel oscillations. Observed in experiment.

Contribution from the integral along imaginary axis.

If $m_2 \neq 0$, the dominant term is

$$U(r) = -\frac{12Qm_1^3}{\pi^2 e^2 r^6 m_2^8}.$$

If $T \neq 0$, $\mu = m_B$, but the condensate is not yet formed, the asymptotic decrease of the potential becomes:

$$U(r) = -\frac{Q}{\pi^2 e^2 r^4 m_1^3} = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}.$$

Contribution from logarithmic cuts (analogous to Friedel oscillations for fermions).

If the first "pinch" dominates:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2}\frac{e^{-z}}{\ln^2(2\sqrt{2}z)}\sin z\,,$$

where $z = 2r\sqrt{2\pi T m_B}$.

NB: $U_1(r)$ **is inversely proportional to** e^2 **and formally vanishes at** $T \to 0$, **but remains finite if** $\sqrt{T m_B} r \neq 0$.

All pinches are comparable:

$$U(r) \approx -\frac{3Q}{2e^2T^2m_B^3r^6 \ln^3(\sqrt{8m_BTr})}.$$

$U \sim T^{-2}$ **valid if** $r \ll 1/\sqrt{16\pi T m_B}$, **i.e. if** $T = 0.1\text{K}$ **and** $m_B = 1GeV$ **the distance should be bounded from above as** $r \ll 3 \cdot 10^{-8}$ **cm.**

Condensation of vector bosons.
$W^{\pm}$ would condense in the early universe if lepton asymmetry was sufficiently high. Plasma neutrality was maintained by quarks and leptons.
Depending on the sign of the pairwise spin-spin coulings W's would condense either in $S = 0$ (scalar) state or in $S = 2$ (ferromagnetic) state.

Magnetic spin-spin interaction through one photon exchange (similar to Breit equation):

$$U_{em}^{spin}(r) = \frac{e^2\rho^2}{4\pi m_W^2}\Big[\frac{(S_1 \cdot S_2)}{r^3} - 3\,\frac{(S_1 \cdot r)(S_2 \cdot r)}{r^5} - \frac{8\pi}{3}(S_1 \cdot S_2)\delta^{(3)}(r)\Big].$$

Here ρ is the ratio of magnetic moment of W to the standard one.

For S-wave the energy is shifted by the last term only.

Local quartic self-coupling of W:

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W}(S_1 S_2)\delta^{(3)}(r).$$

The net result $U_{em} + U_{4W}$ is negative, so $S = 2$ state is energetically favorable and spontaneous magnetization in the early universe is possible – seeds for large scale magnetic fields.