The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
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International Atomic Energy Agency

**2146-25**

# Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond'

*26 - 28 May 2010*

## The chiral magnetic effect

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# Gribov-80 Workshop, ICTP, May 26-28, 2010

## The Chiral Magnetic Effect

D. Kharzeev

BNL

V.N. Gribov '81:
"Anomalies, as a manifestation of the high momentum collective motion in the vacuum"

**ANOMALIES AND A POSSIBLE SOLUTION OF PROBLEMS OF ZERO-CHARGE AND INFRA-RED INSTABILITY**

V.N. GRIBOV
*Landau Institute, 117334 Moscow, USSR*
*and CERN, CH-1211 Geneva 23, Switzerland*

Can this collective motion be made visible in QCDxQED? yes

can it be observed in heavy ion collisions?
quite likely

# Based on:

DK, hep-ph/0406125 (PLB)

DK, A. Zhitnitsky, arXiv: 0706.1026 (NPA)

DK, L. McLerran, H. Warringa, arXiv:0711.0950 (NPA)

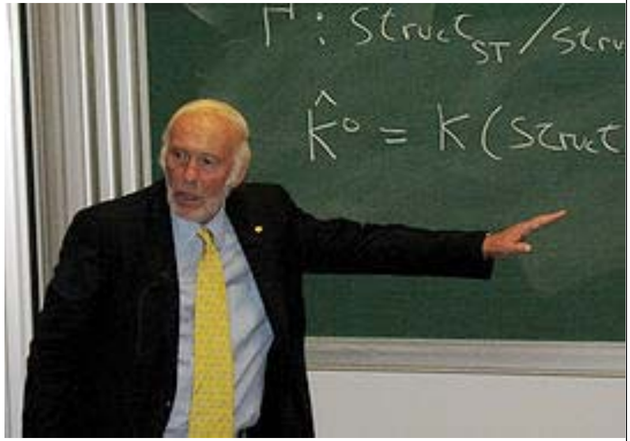
K. Fukushima, DK, H. Warringa, arXiv: 0808.3382 (PRD);
0912.2961 (NPA); 1002.2495 (PRL)

DK, H. Warringa, arXiv: 0907.5007 (PRD)

DK, arXiv: 0911.3715 (Ann. Phys.)

G. Basar, G. Dunne, DK, arXiv: 1003.3464 (PRL)

# Topology and Chern-Simons forms



### 6. Applications to 3-manifolds

In this section $M$ will denote a compact, oriented, Riemannian 3-manifold, and $F(M) \xrightarrow{\pi} M$ will denote its $SO(3)$ oriented frame bundle equipped with the Riemannian connection $\theta$ and curvature tensor $\Omega$. For $A$, $B$ skew symmetric matrices, the specific formula for $P_1$ shows $P_1(A \otimes B) = -(1/8\pi^2) \operatorname{tr} AB$. Calculating from (3.5) shows

$$(6.1) \qquad TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} .$$

## What does it mean for a gauge theory?

# Chern-Simons theory

CHARACTERISTIC FORMS

(6.1) $$TP_1(\theta) = \frac{1}{4\pi^2}\{\theta_{12} \wedge \theta_{13} \wedge \theta_{23} + \theta_{12} \wedge \Omega_{12} + \theta_{13} \wedge \Omega_{13} + \theta_{23} \wedge \Omega_{23}\} .$$

## What does it mean for a gauge theory?

| Geometry | ⟷ | Physics |
|---|---|---|
| Riemannian connection | ⟷ | Gauge field |
| Curvature tensor | ⟷ | Field strength tensor |

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

Abelian non-Abelian

# Chern-Simons theory

$$S_{CS} = \frac{k}{8\pi}\int_M d^3x\ \epsilon^{ijk}\left(A_i F_{jk} + \frac{2}{3}A_i[A_j, A_k]\right)$$

## Remarkable novel properties:

- gauge invariant, up to a boundary term
- topological - does not depend on the metric, knows only about the topology of space-time **M**
- when added to Maxwell action, induces a mass for the gauge boson - different from the Higgs mechanism!
- **breaks Parity invariance**

# Chern-Simons theory and the vacuum of Quantum Chromodynamics

Equation:

$$D^\mu F^a_{\mu\nu} = 0$$

Belavin, Polyakov, Tyupkin, Schwartz;
tunneling events:
't Hooft; Gribov;....

Solution:

$$A^a_\mu(x) = \frac{2\eta_{a\mu\nu}x_\nu}{x^2+\rho^2},$$

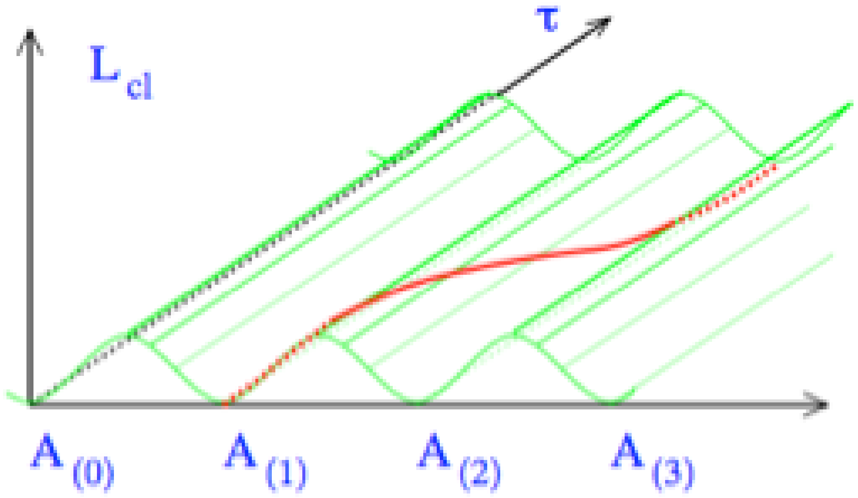
Coupling of space-time and color:

Integer $\quad Q = \int d\sigma_\mu K_\mu$

$$\eta_{a\mu\nu} = \begin{cases} \epsilon_{a\mu\nu} & \mu,\nu = 1,2,3, \\ \delta_{a\mu} & \nu = 4, \\ -\delta_{a\nu} & \mu = 4. \end{cases}$$

$$K_\mu = \frac{1}{16\pi^2}\epsilon_{\mu\alpha\beta\gamma}\left(A^a_\alpha \partial_\beta A^a_\gamma + \frac{1}{3}f^{abc}A^a_\alpha A^b_\beta A^c_\gamma\right)$$ Chern-Simons current

# QCD vacuum as a Bloch crystal




"θ - vacuum"

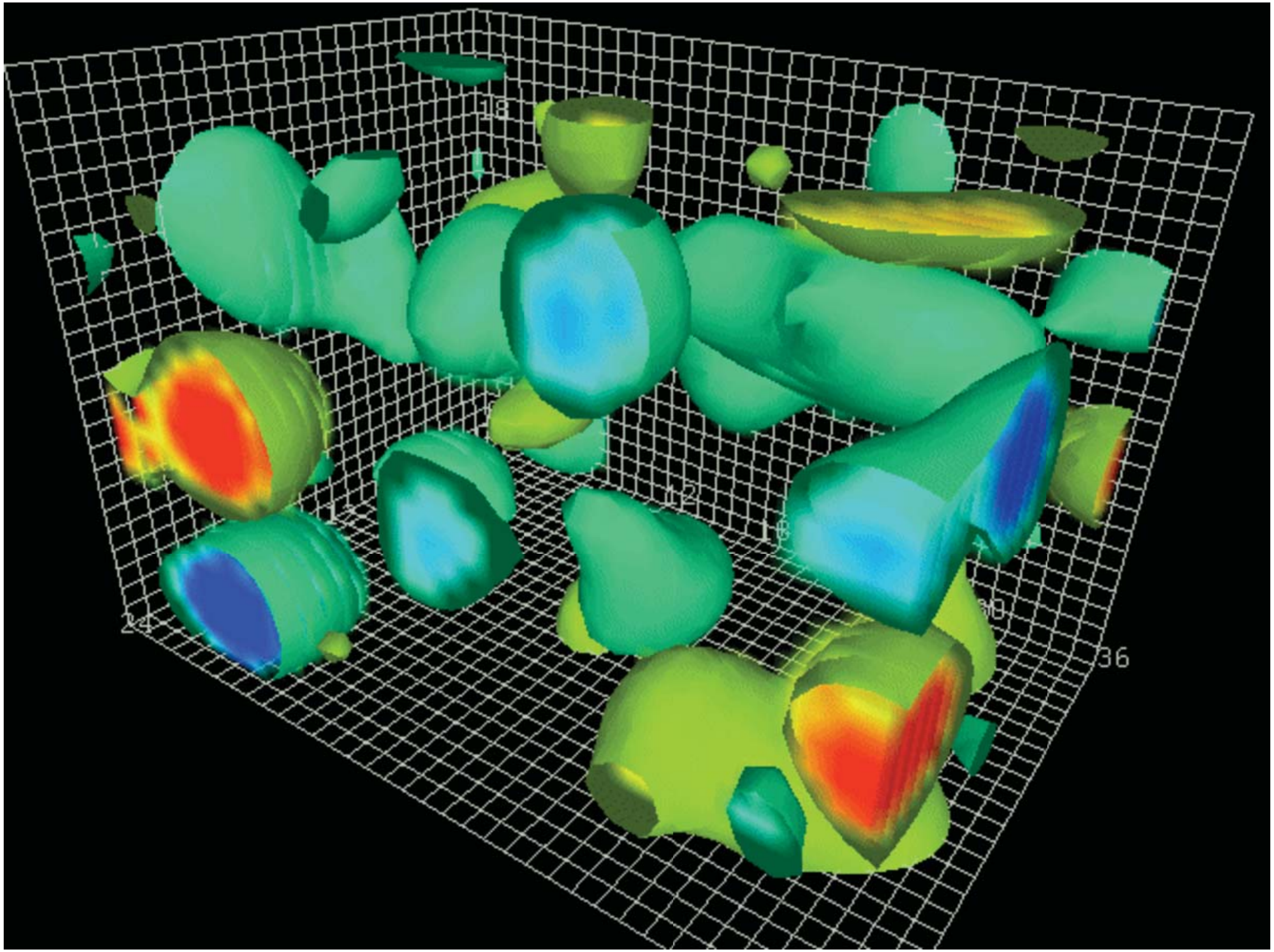
$$|\theta\rangle = \sum_q e^{i\theta q}|q\rangle$$

$$\langle \mathcal{O} \rangle = \sum_q f(q) \int_q D[\psi]D[\bar{\psi}]D[A] \exp(iS_{QCD})\mathcal{O}(\psi, \bar{\psi}, A)$$

$$f(q_1 + q_2) = f(q_1)f(q_2) \longrightarrow f(q) = \exp(i\theta q)$$

"quasi-momentum" "coordinate"

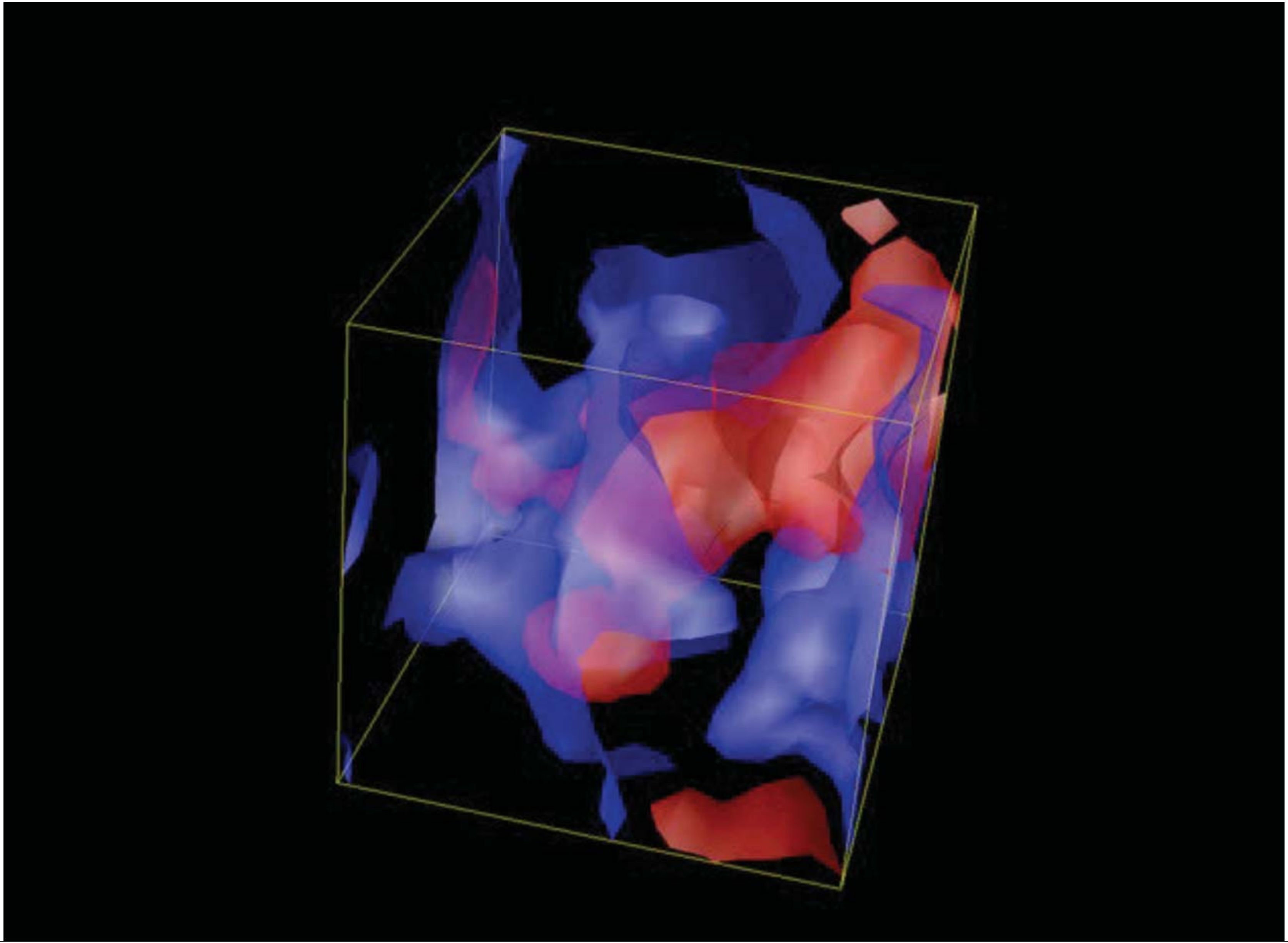
# Topological number fluctuations in QCD vacuum




D. Leinweber

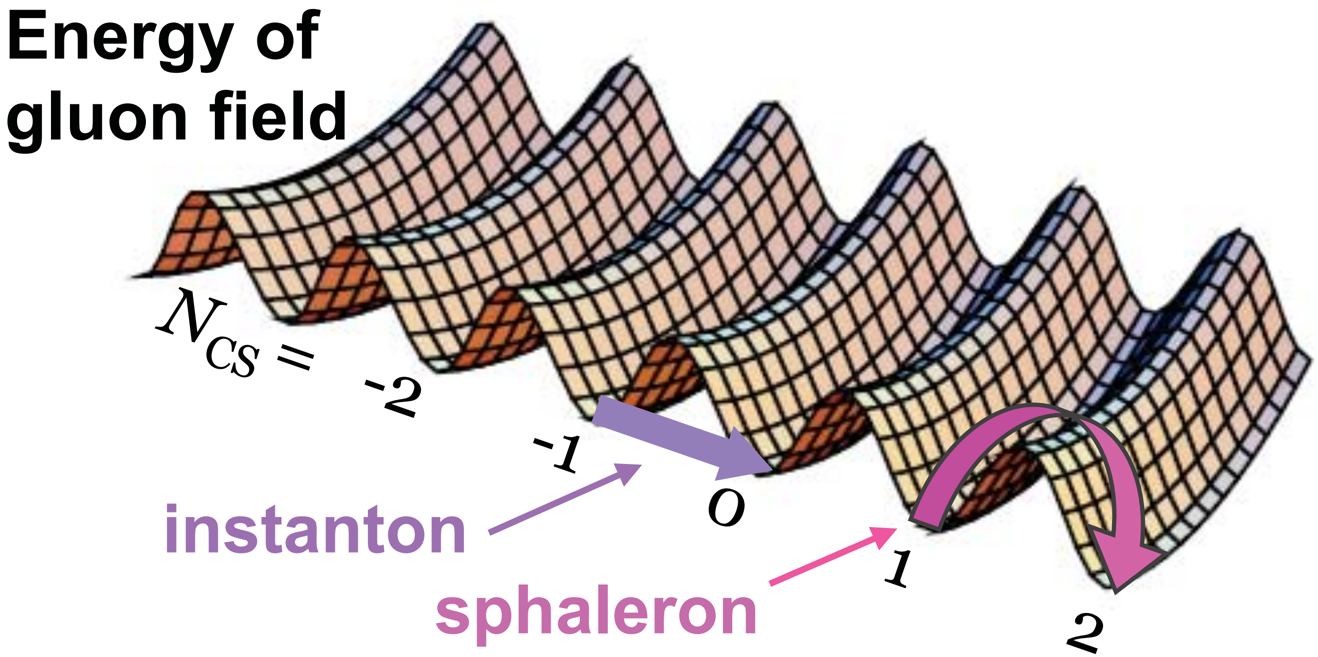
# Topological number fluctuations in QCD vacuum

ITEP Lattice Group



# Sphaleron transitions at finite energy or temperature

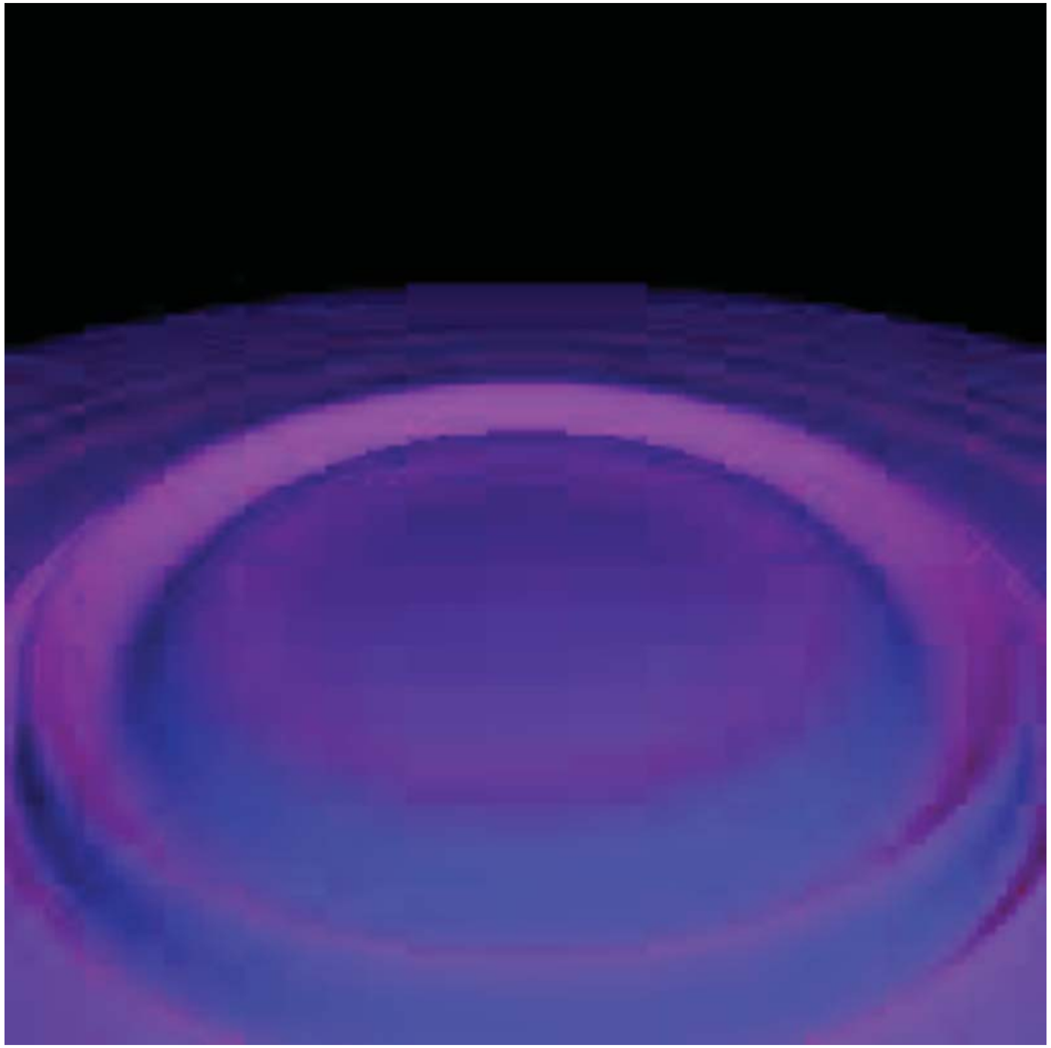
$$\Gamma = \frac{1}{2} \lim_{t\to\infty} \lim_{V\to\infty} \int_0^t \langle (q(x)q(0) + q(0)q(x)) \rangle d^4x$$




Sphalerons:
random walk of
topological charge at finite T:

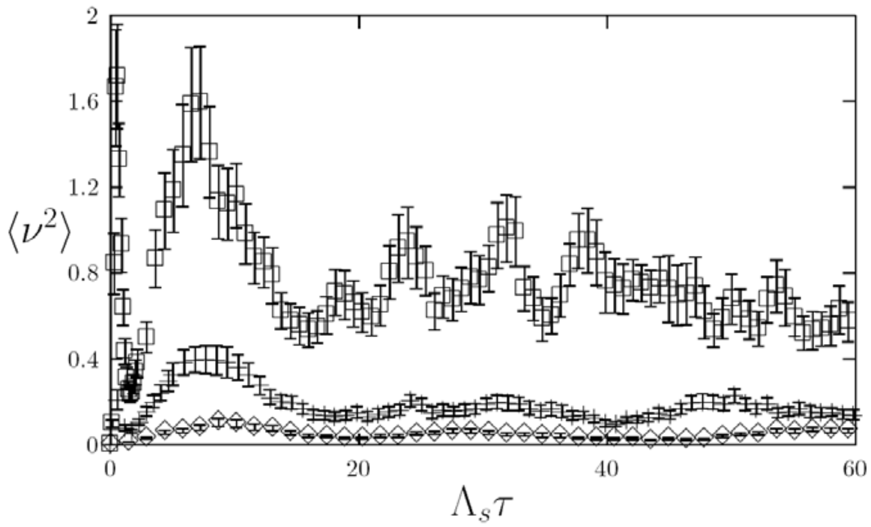
$$\langle Q^2 \rangle = 2\Gamma V t, \quad t \to \infty$$

# Sphaleron transitions at finite energy or temperature

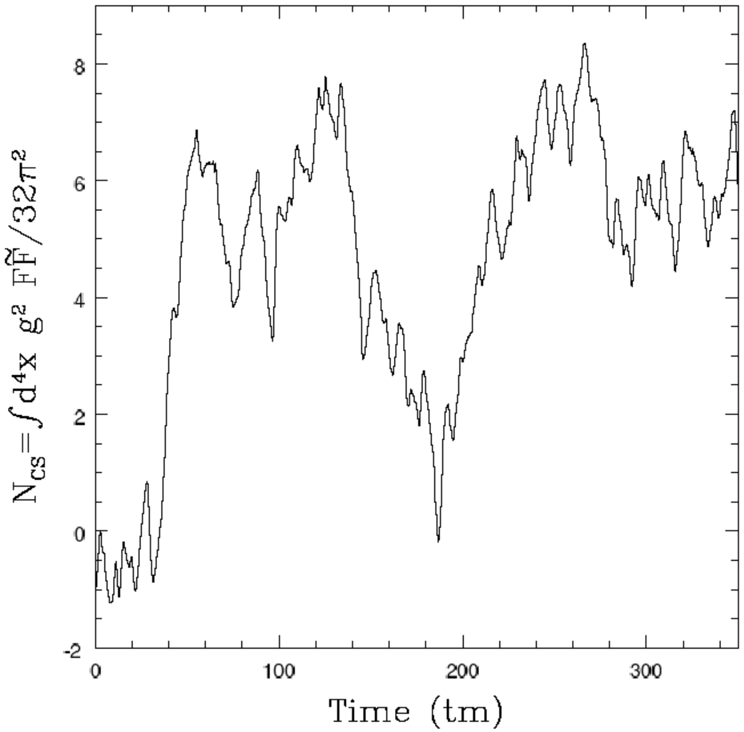


C. Rebbi, http://scv.bu.edu/visualization/gallery

# Diffusion of Chern-Simons number in QCD: real time lattice simulations

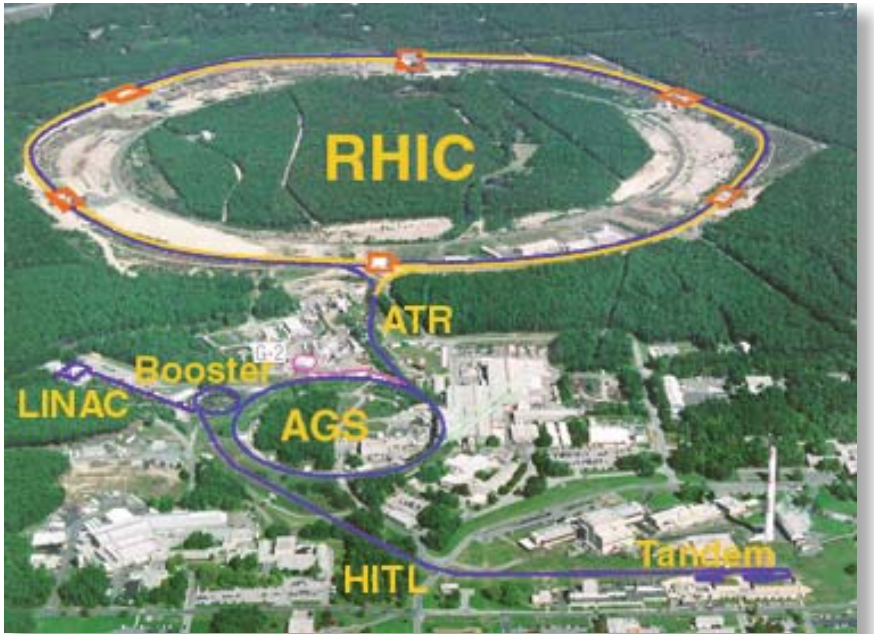


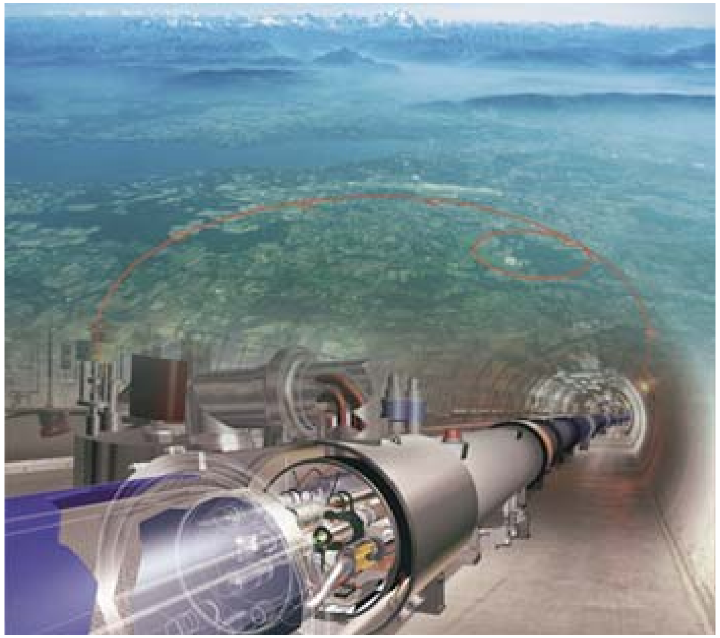

DK, A.Krasnitz and R.Venugopalan,
Phys.Lett.B545:298-306,2002




P.Arnold and G.Moore,
Phys.Rev.D73:025006,2006

# Experimental test of Chern-Simons dynamics in hot QCD: Heavy ion collisions



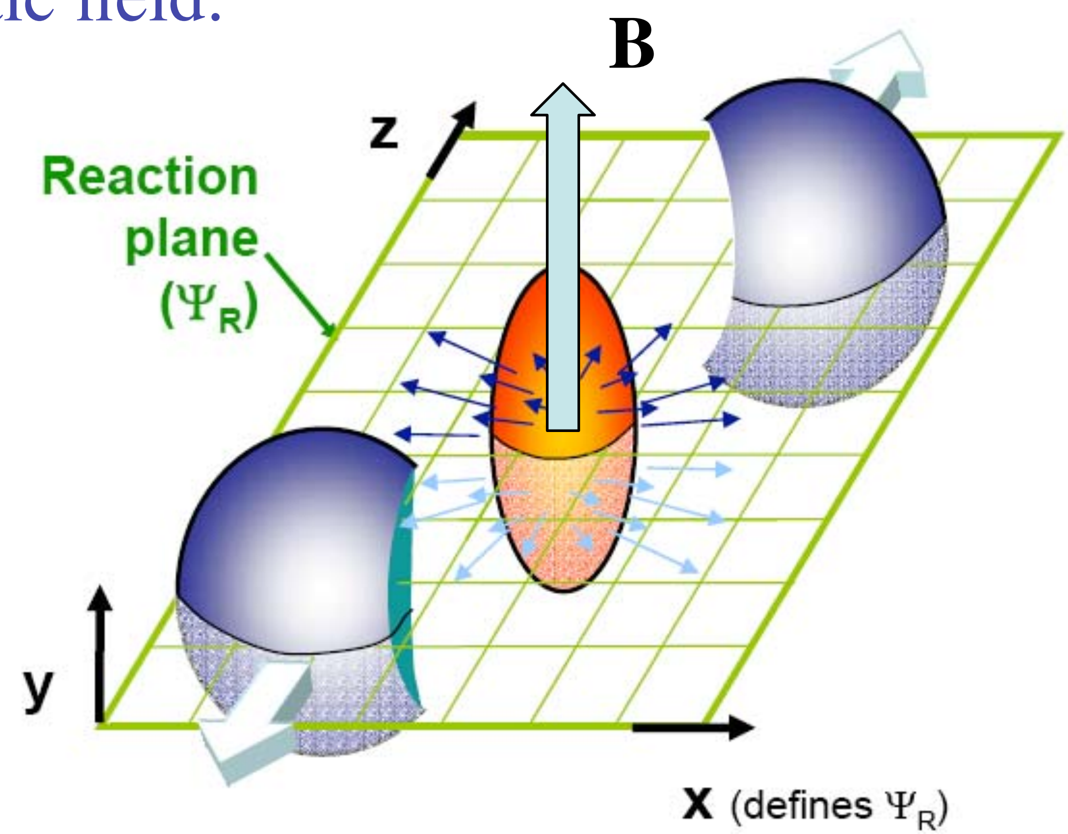



**LHC**

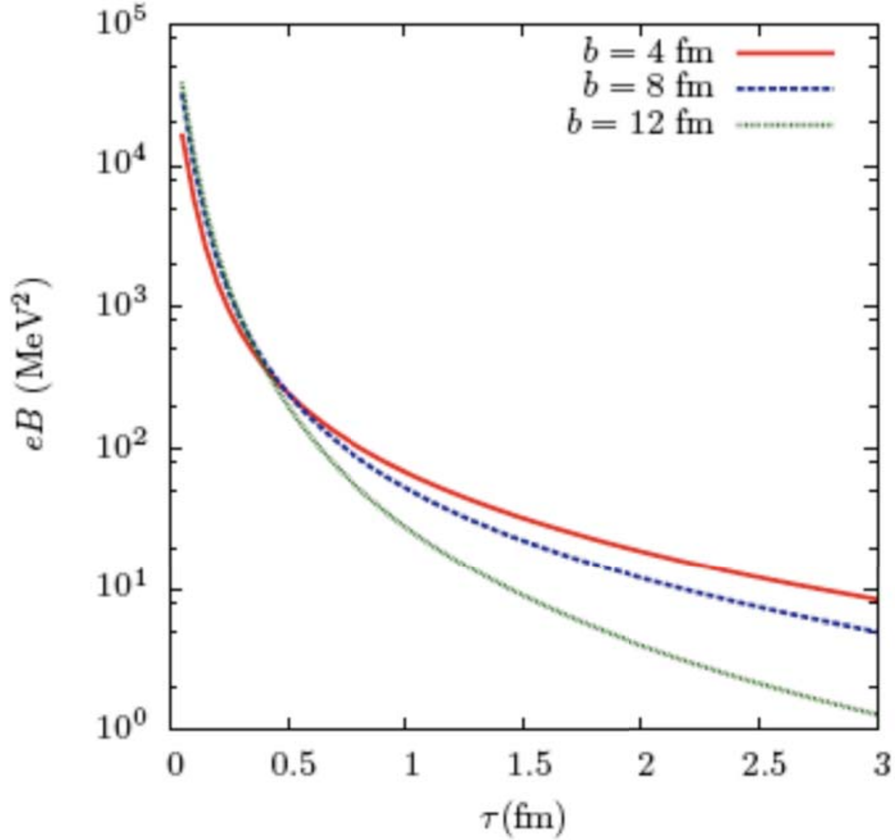
**NICA,**
**JINR**

# Is there a way to observe topological charge fluctuations in experiment?

Relativistic ions create a strong magnetic field:



# Heavy ion collisions as a source of the strongest magnetic fields available in the Laboratory

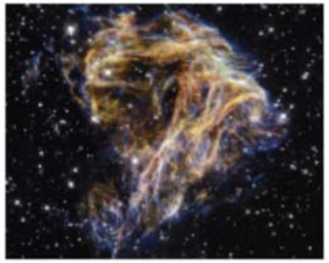



DK, McLerran, Warringa, Nucl Phys A803(2008)227

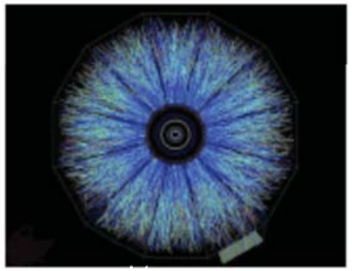
Fig. A.2. Magnetic field at the center of a gold-gold collision, for different impact parameters. Here the center of mass energy is 200 GeV per nucleon pair ($Y_0 = 5.4$).

# Comparison of magnetic fields

| | |
|---|---|
| The Earths magnetic field | 0.6 Gauss |
| A common, hand-held magnet | 100 Gauss |
| The strongest steady magnetic fields achieved so far in the laboratory | $4.5 \times 10^5$ Gauss |
| The strongest man-made fields ever achieved, if only briefly | $10^7$ Gauss |
| Typical surface, polar magnetic fields of radio pulsars | $10^{13}$ Gauss |
| Surface field of Magnetars | $10^{15}$ Gauss |



http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$e\,B(\tau=0.2\,\mathrm{fm}) \;=\; 10^3 \sim 10^4\ \mathrm{MeV}^2 \sim 10^{17}\,\mathrm{Gauss}$$

# From QCD back to electrodynamics: Maxwell-Chern-Simons (axion) theory

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - A_\mu J^\mu + \frac{c}{4}\, P_\mu J^\mu_{CS}$$

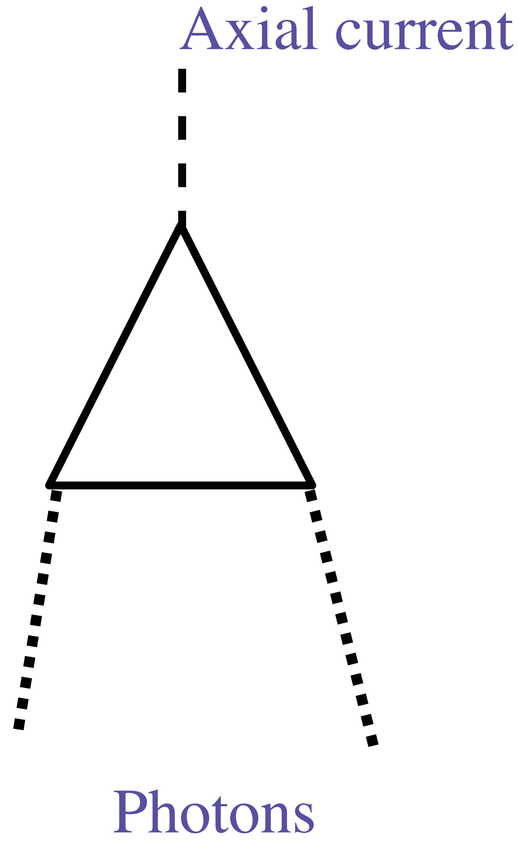
$$J^\mu_{CS} = \epsilon^{\mu\nu\rho\sigma}A_\nu F_{\rho\sigma} \qquad P_\mu = \partial_\mu\theta = (\dot\theta, \vec{P})$$

$$\vec\nabla \times \vec B - \frac{\partial \vec E}{\partial t} = \vec J + \underline{c\left(\dot\theta \vec B - \vec P \times \vec E\right)},$$

$$\vec\nabla \cdot \vec E = \rho + \underline{c\vec P \cdot \vec B},$$

$$\vec\nabla \times \vec E + \frac{\partial \vec B}{\partial t} = 0,$$

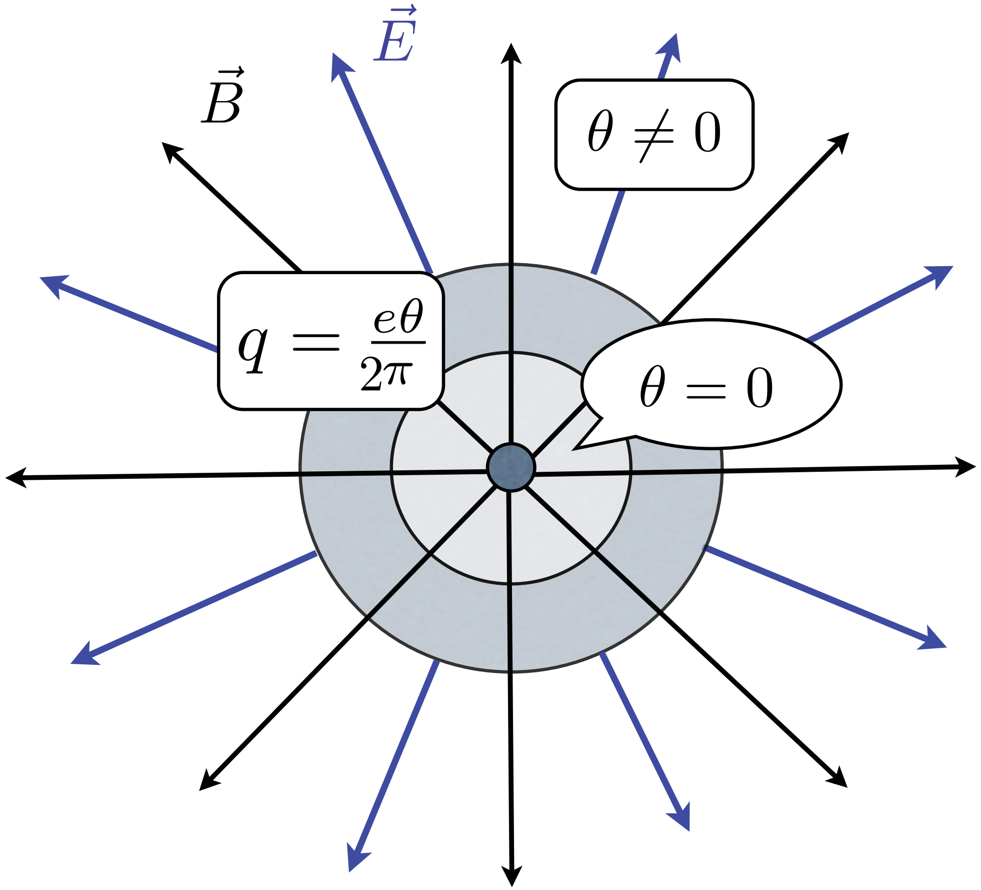
$$\vec\nabla \cdot \vec B = 0,$$



# Magnetic monopole at finite $\theta$ : the Witten effect

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}$$

$$\vec{P} \equiv \vec{\nabla}\theta$$




E. Witten;

F. Wilczek

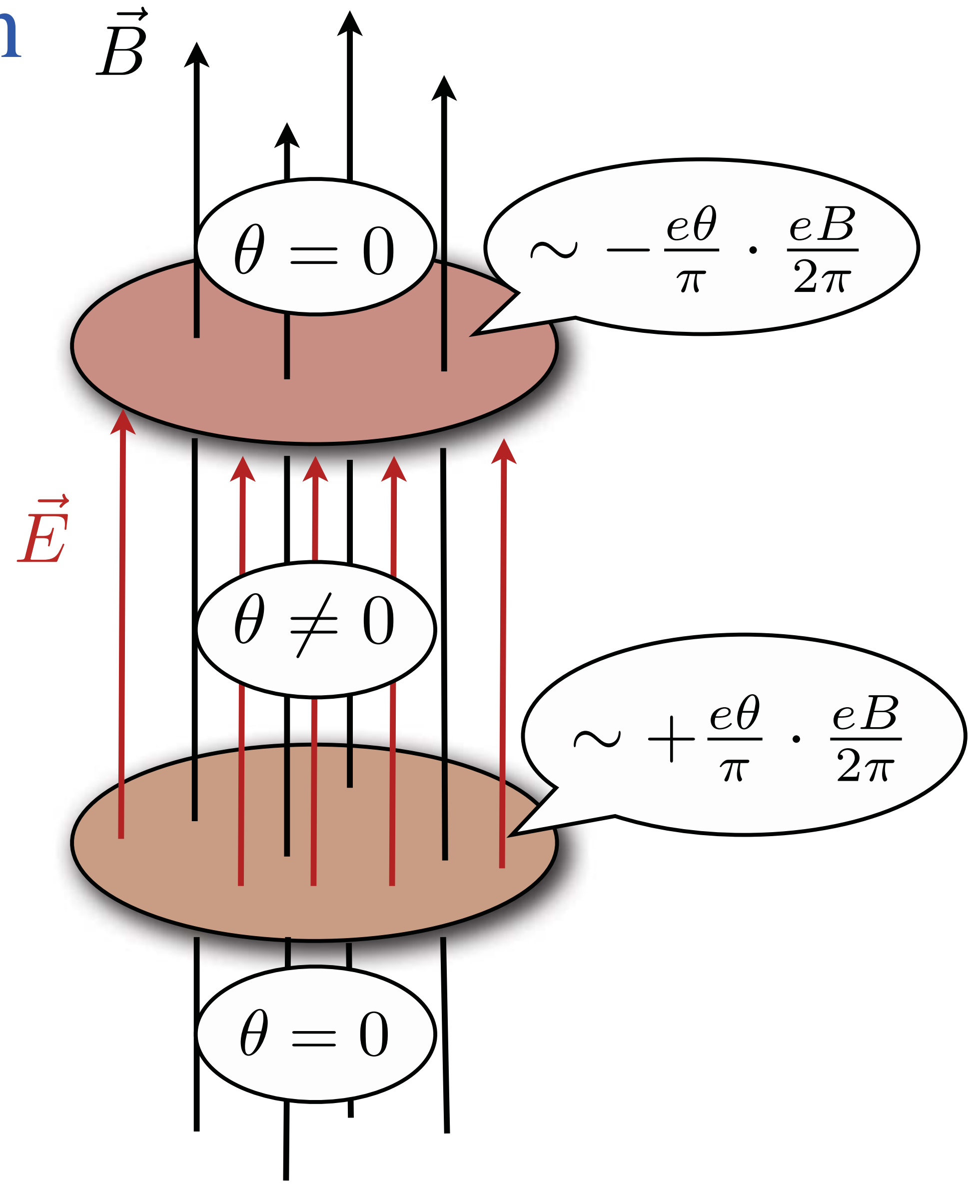
Induced electric charge: $q = c\,\theta\,g = \dfrac{e^2}{2\pi^2}\,\theta\,g = \dfrac{e}{2\pi^2}\,\theta\,(eg) = e\,\dfrac{\theta}{2\pi}$

# The Chiral Magnetic Effect I:
# Charge separation

$$\vec{\nabla} \cdot \vec{E} = \rho + c\vec{P} \cdot \vec{B}$$

$$\vec{P} \equiv \vec{\nabla}\theta$$

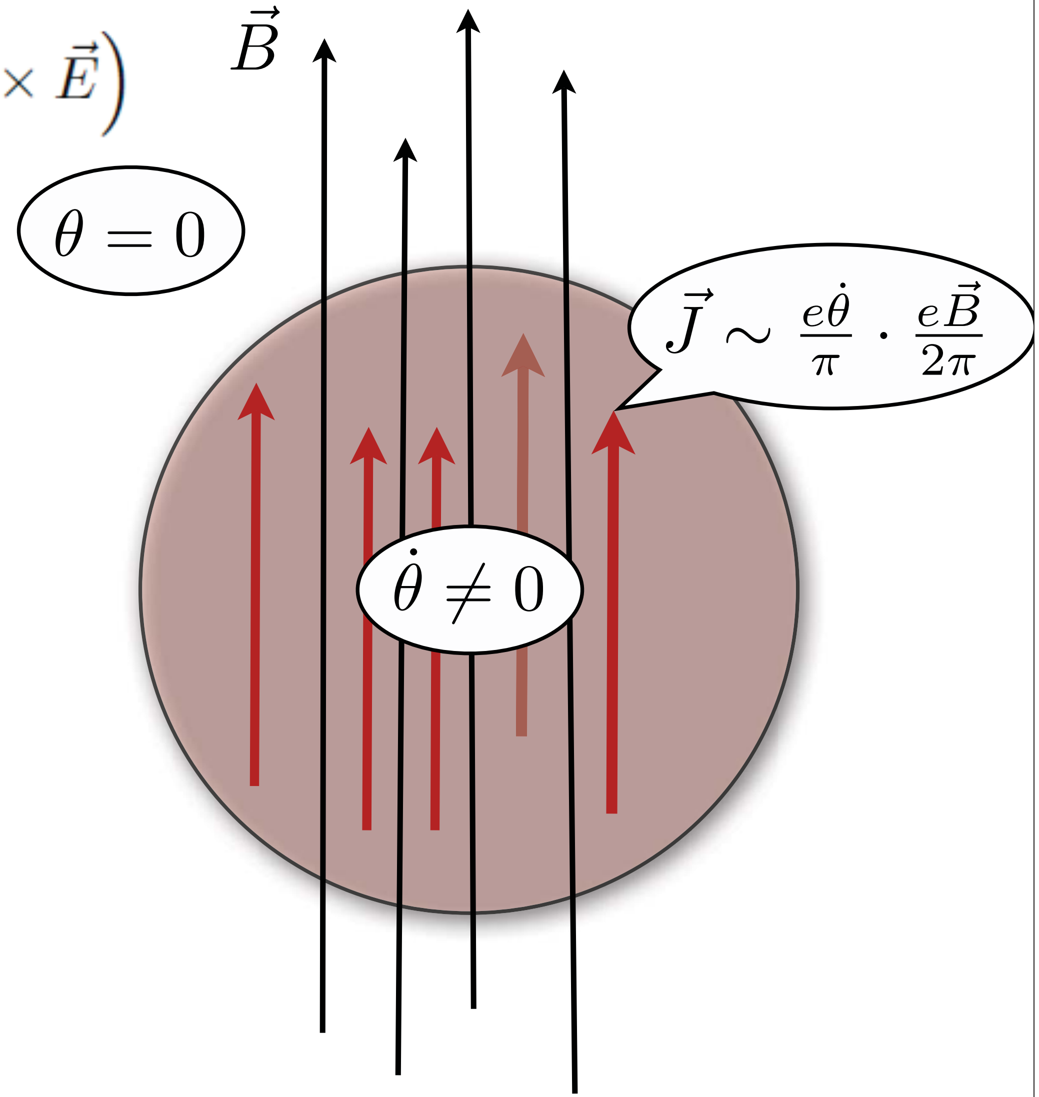
$$d_e = \sum_f q_f^2 \left( e\,\frac{\theta}{\pi} \right) \left( \frac{eB \cdot S}{2\pi} \right) L$$




DK '04;
DK, A. Zhitnitsky '06;
DK arXiv:0911.3715; Annals of Physics (2010)

# The chiral magnetic effect II: chiral induction

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J} + c\left(\dot{\theta}\vec{B} - \vec{P} \times \vec{E}\right)$$




$$\vec{J} = -\frac{e^2}{2\pi^2}\dot{\theta}\vec{B}$$

DK, L. McLerran, H. Warringa '07;
K. Fukushima, DK, H. Warringa '08;
DK, H.Warringa arXiv:0907.5007

# Computing the induced current

Fukushima, DK, Warringa, '08

Chiral chemical potential is formally equivalent to a background chiral gauge field: $\mu_5 = A_5^0$

In this background, vector e.m. current is not conserved:

$$\partial_\mu J^\mu = \frac{e^2}{16\pi^2} \left( F_L^{\mu\nu} \tilde{F}_{L,\mu\nu} - F_R^{\mu\nu} \tilde{F}_{R,\mu\nu} \right)$$

Compute the current through

$$J^\mu = \frac{\partial \log Z[A_\mu, A_\mu^5]}{\partial A_\mu(x)}$$

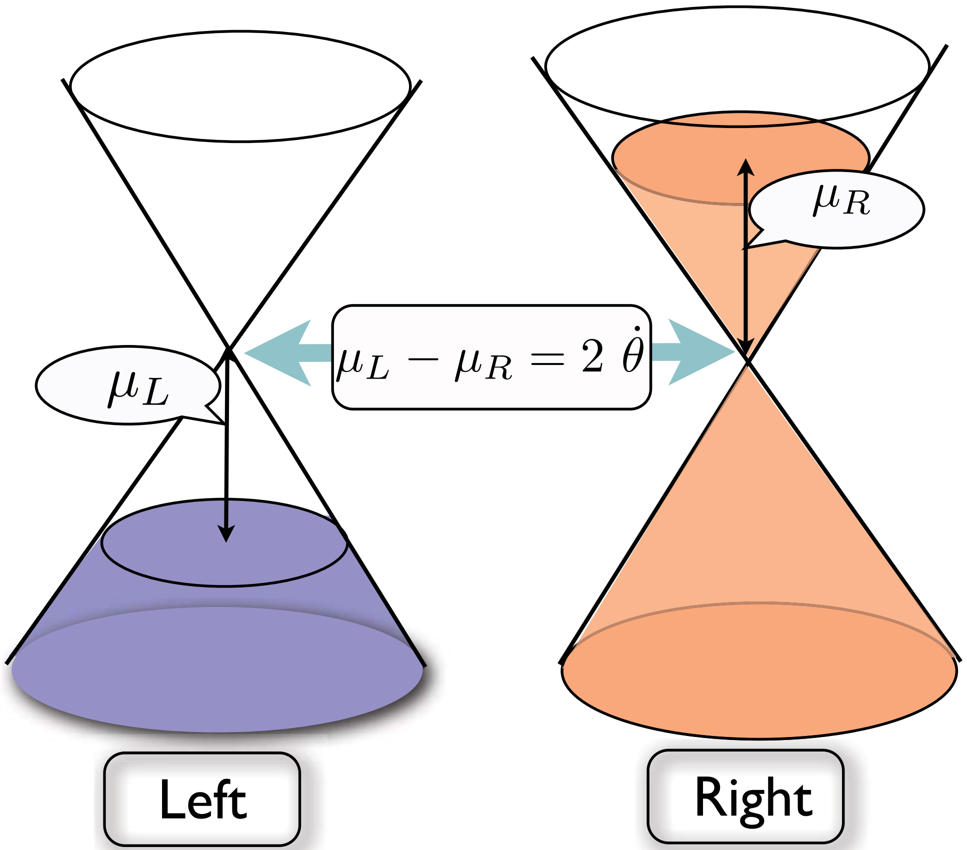
The result:

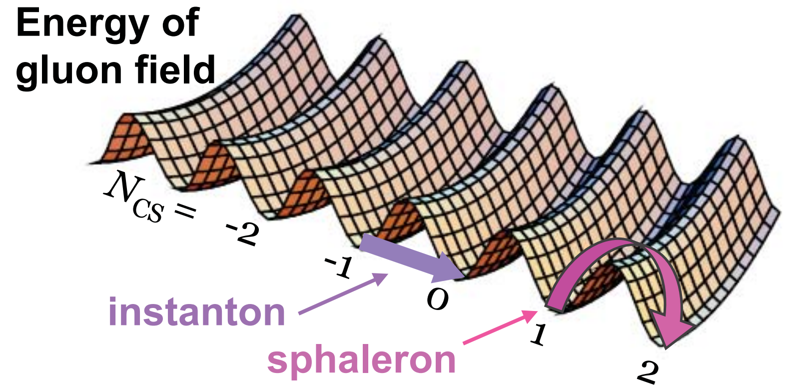
$$\vec{J} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

also: A.Alexeev, V.Cheianov, J.Froelich, '98; M.Giovannini, M.Shaposhnikov, '97

Coefficient is fixed by the axial anomaly, no corrections

# What powers the CME current?



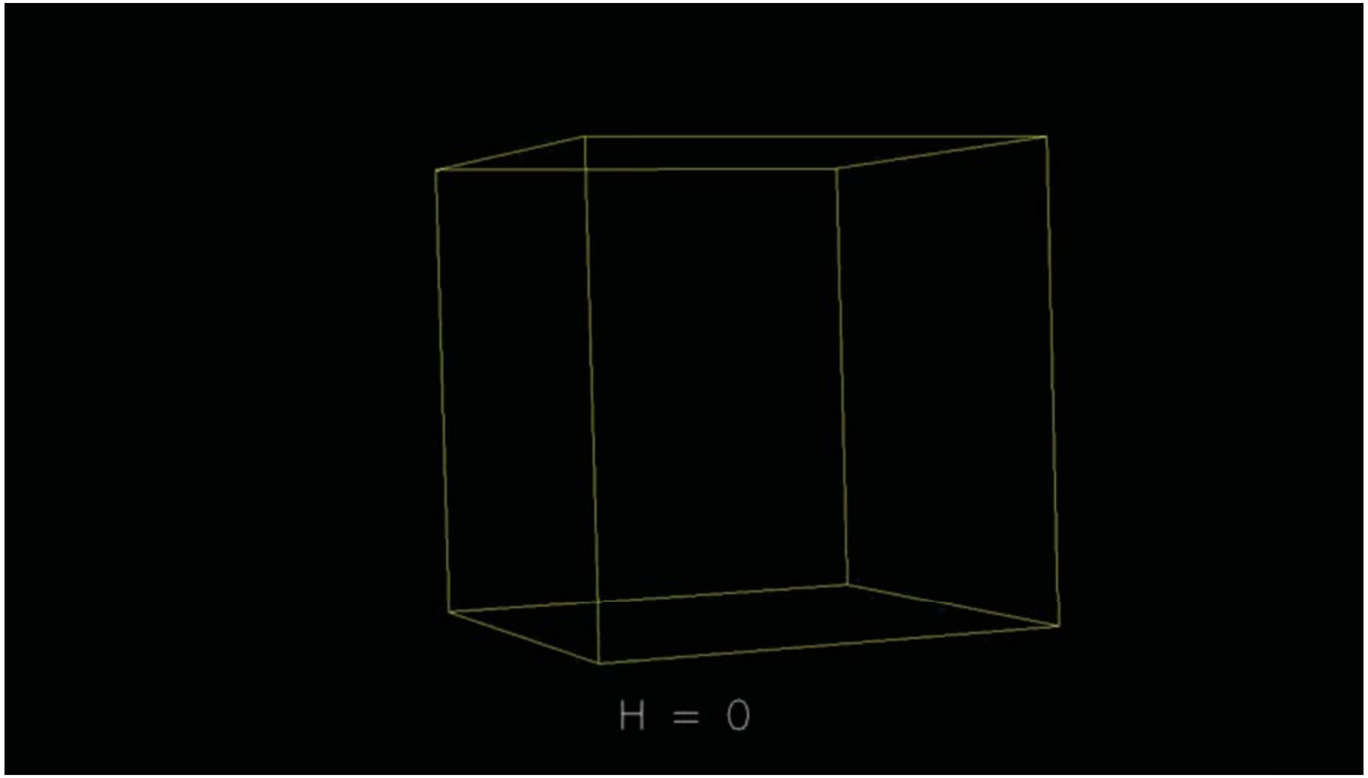




$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$P = \int d^3x\ \vec{J}\cdot\vec{E} = -\dot{\theta}\ \frac{e^2}{2\pi^2}\int d^3x\ \vec{E}\cdot\vec{B} = -\dot{\theta}\ \dot{Q}_5$$

## "Numerical evidence for chiral magnetic effect in lattice gauge theory",

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov, ArXiv 0907.0494; PRD'09




Red - positive charge

Blue - negative charge

SU(2) quenched, Q = 3; Electric charge density (H) - Electric charge density (H=0)

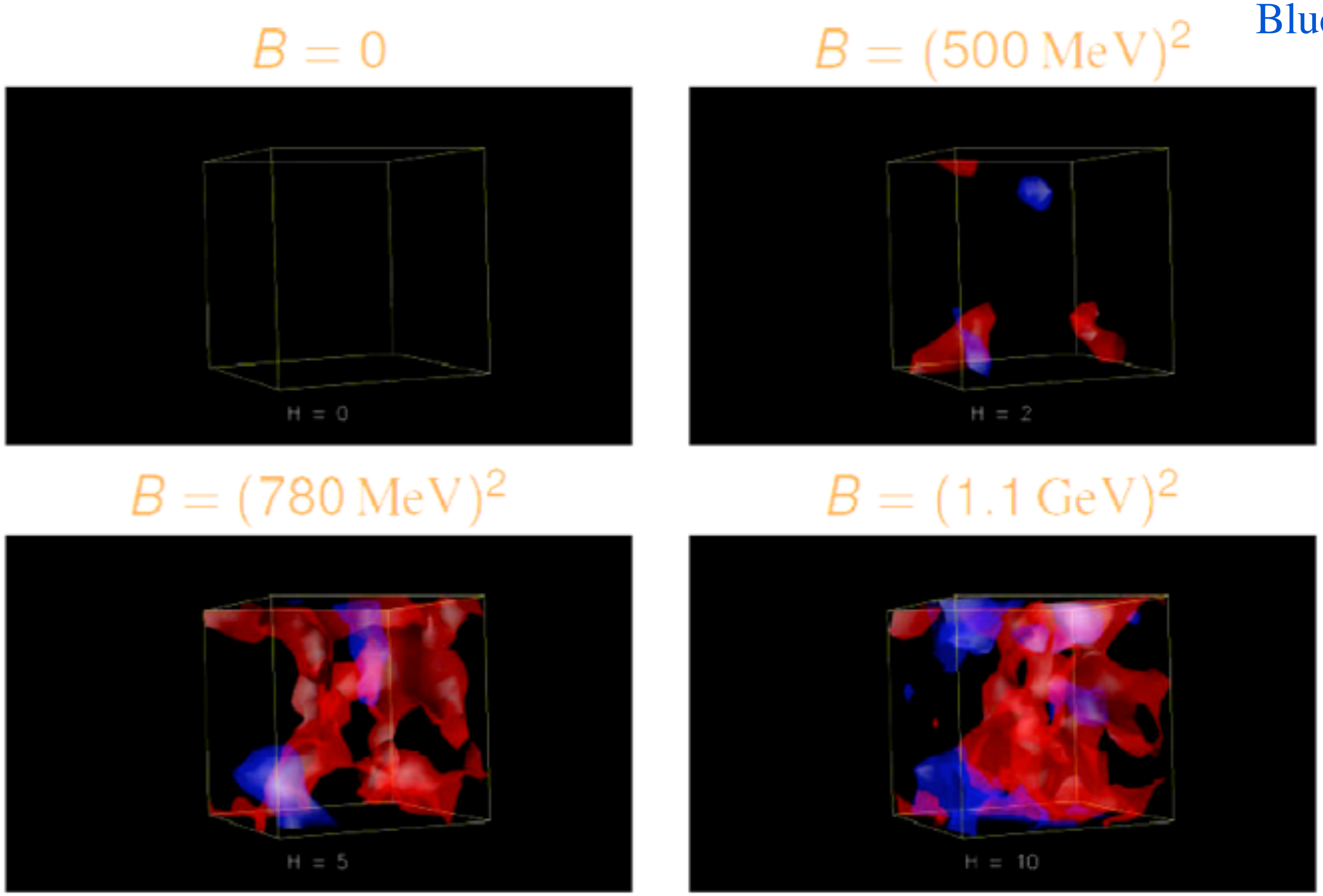
## “Numerical evidence for chiral magnetic effect in lattice gauge theory”,

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov, ArXiv 0907.0494; PRD’09

### Density of the electric charge vs. magnetic field, 3D time slices

Red - positive charge

Blue - negative charge

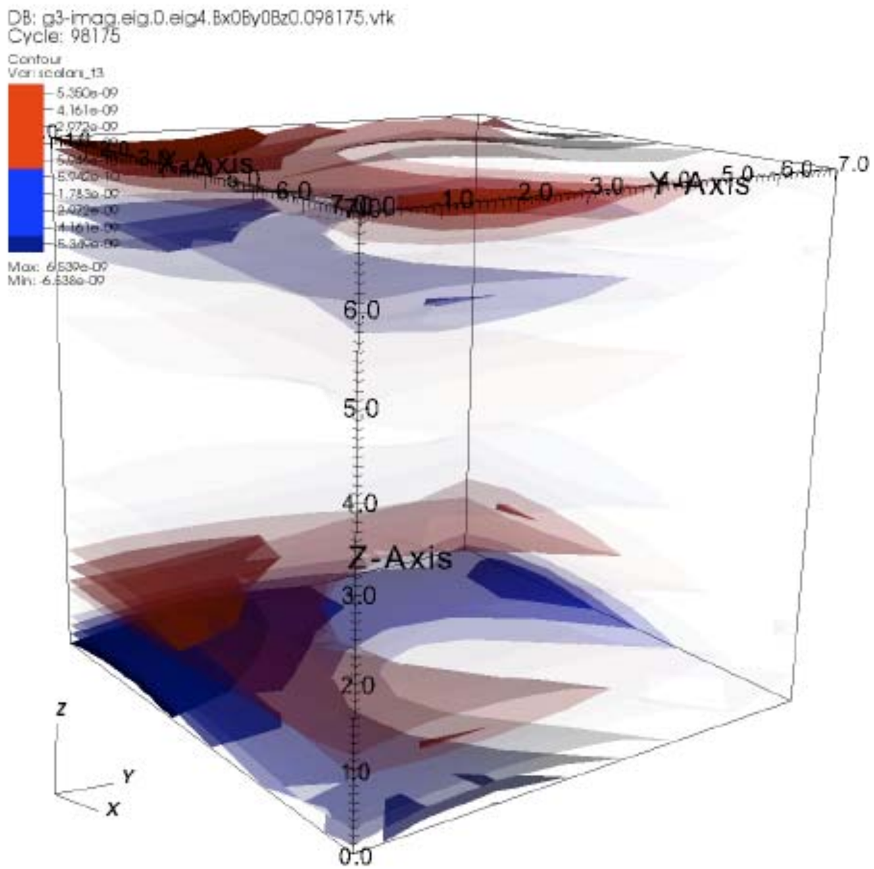


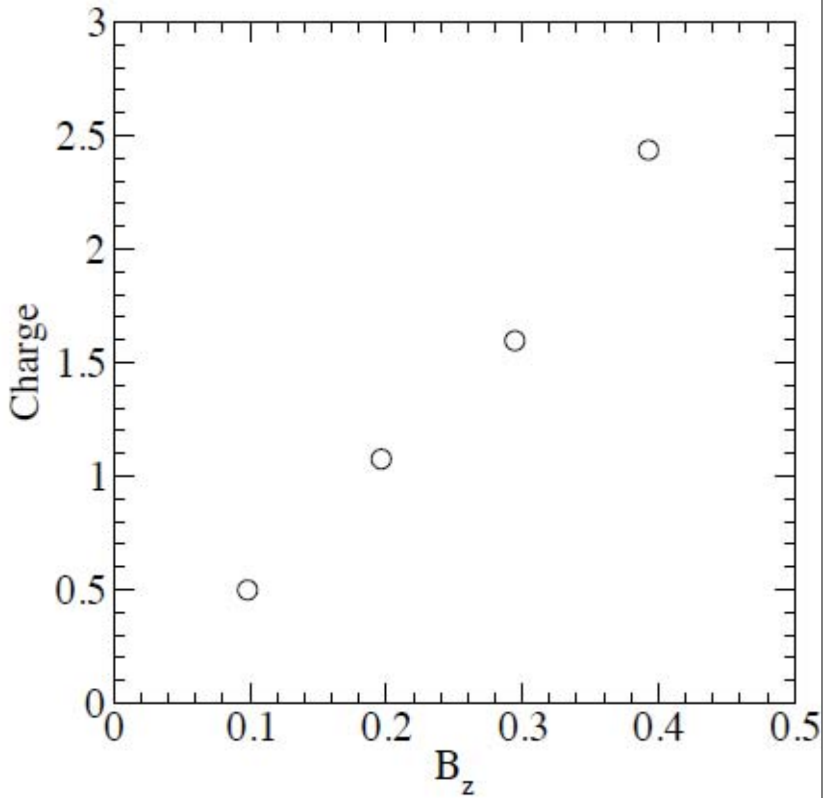

note:
B has to be measured in units of the pion mass^2 !

## “Chiral magnetic effect in 2+1 flavor QCD+QED”,

M. Abramczyk, T. Blum, G. Petropoulos, R. Zhou, ArXiv 0911.1348;
Columbia-Bielefeld-RIKEN-BNL

Red - positive charge
Blue - negative charge



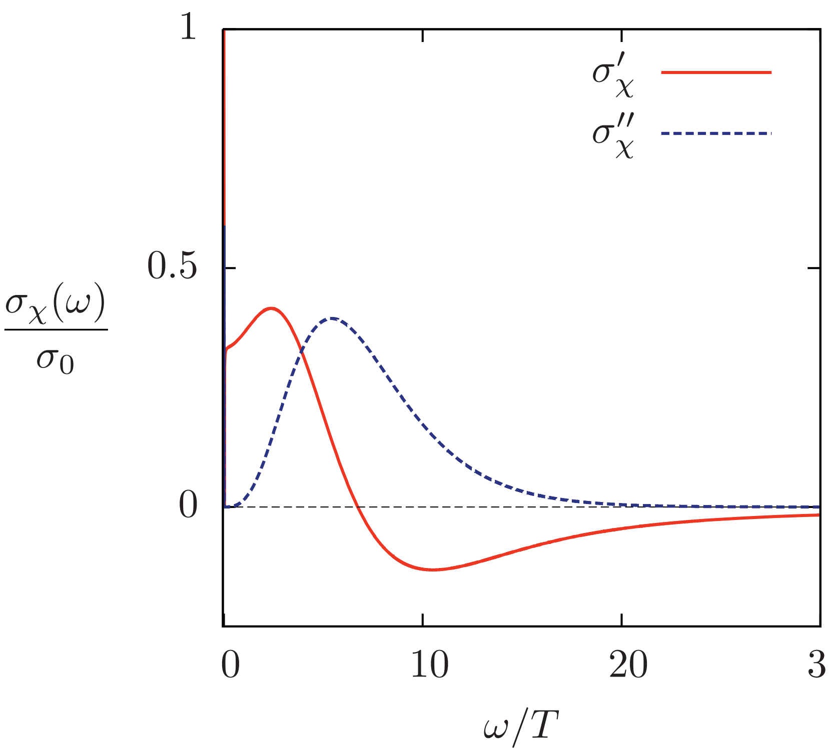



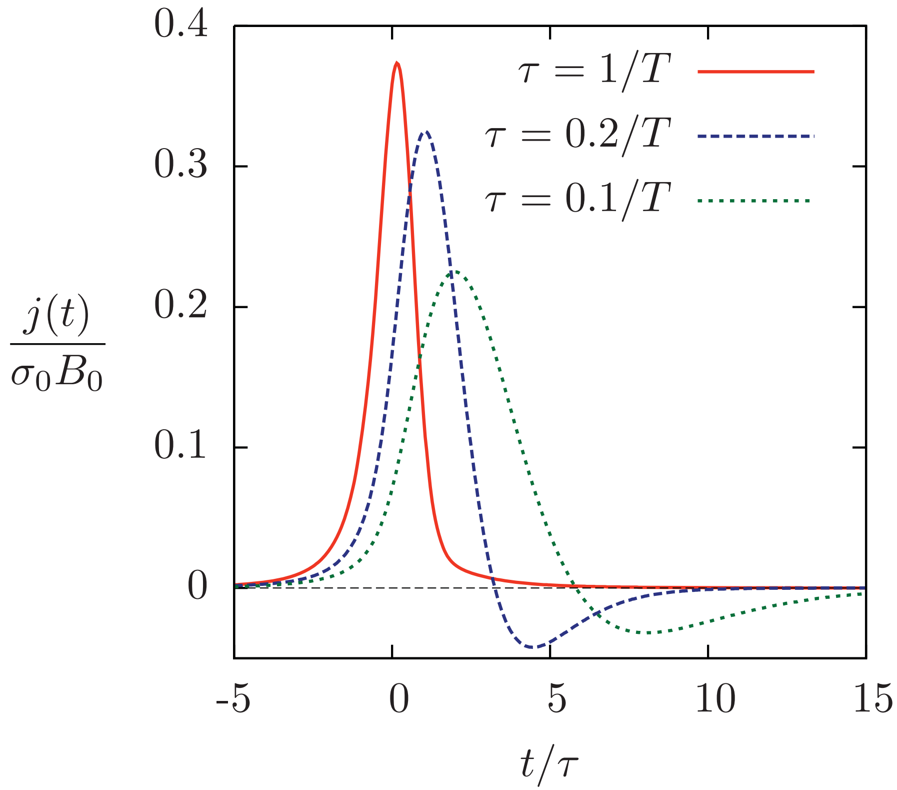

2+1 flavor Domain Wall Fermions, fixed topological sectors, 16^3 x 8 lattice

# Chiral magnetic conductivity

$$\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$$

$$\sigma_\chi(\omega = 0, \boldsymbol{p} = 0) \equiv \sigma_0 = \frac{e^2}{2\pi^2}\mu_5$$



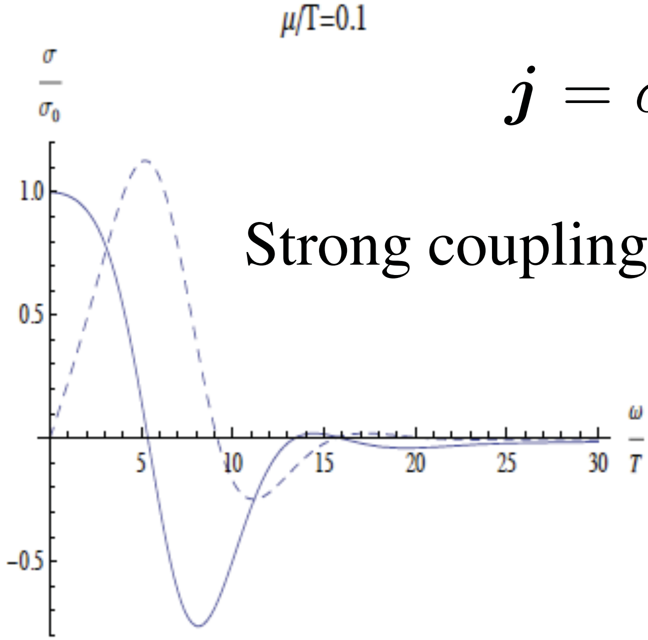




D.K., H. Warringa, Phys Rev D80 (2009) 034028

# Holographic chiral magnetic effect: the strong coupling regime (AdS/CFT)

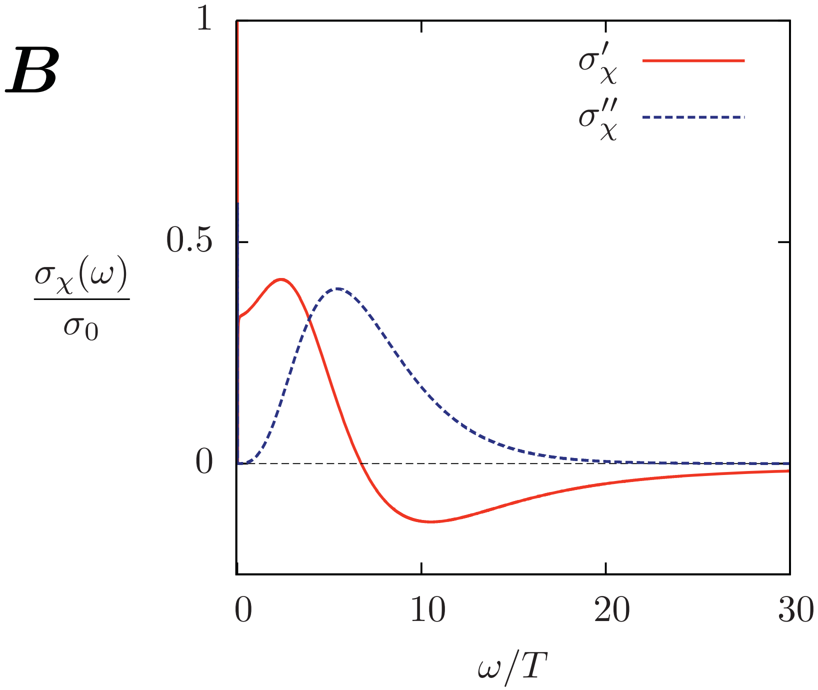
$$\boldsymbol{j} = \sigma_\chi \boldsymbol{B}$$

Strong coupling




H.-U. Yee, arXiv:0908.4189, JHEP 0911:085, 2009

Weak coupling

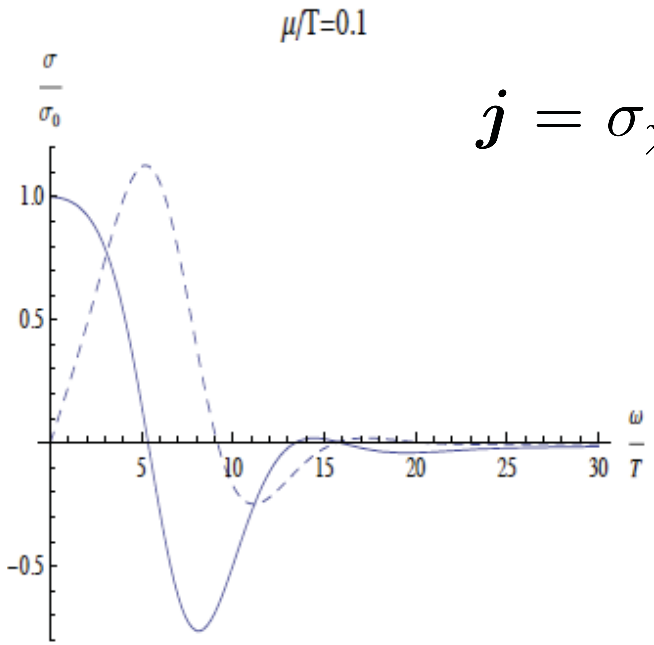



D.K., H. Warringa Phys Rev D80 (2009) 034028

A. Rebhan et al, JHEP 0905, 084 (2009), G.Lifshytz, M.Lippert, arXiv:0904.4772; E. D' Hoker and P. Krauss, arXiv:0911.4518; ...

also: Chiral separation, D. Son and P. Surowka, '09

# Holographic CME: is the current renormalized at strong coupling?




$$j = \sigma_\chi B$$

H.-U. Yee, arXiv:0908.4189, JHEP 0911:085, 2009

H.-U.Yee: No

A. Rebhan et al: Yes (to zero)

**Resolved very recently:**
V.Rubakov, arXiv:1005.1888;
A. Gynther, K. Landsteiner, F. Pena-Benitez, A. Rebhan, arXiv:1005.2587

**CME current is the same at strong and weak coupling**

What carries the current at strong coupling?

# CME in the chirally broken phase

G. Basar, G. Dunne, DK, arXiv: 1003.3464

"Chiral spiral" in (1+1) theories: V. Schoen, M. Thies, hep-th/0008175

Gross-Neveu:

$$\mathcal{L} = \bar{q}\, \mathrm{i}\gamma^\mu \partial_\mu q + \frac{1}{2} g^2 \left[ (\bar{q}q)^2 - \lambda \left( \bar{q}\gamma^5 q \right)^2 \right] - m_0 \bar{q} q$$

't Hooft:

$$\mathcal{L} = \bar{q}\, \mathrm{i} \not{D} q - \frac{1}{2} \mathrm{tr} F_{\mu\nu} F^{\mu\nu} \; , \qquad \not{D} = \gamma^\mu (\partial_\mu + \mathrm{i} g A_\mu)$$

because of constraints on Dirac matrices in 1+1, explicit form e.g.

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \gamma^1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

there is an intricate connection between the vector (baryon) and chiral currents

$$j_V^0 = j_A^1 \; , \qquad j_V^1 = j_A^0$$

Baryon density - chiral current;
chiral density - vector current

# Chiral magnetic spiral

G. Basar, G. Dunne, DK, arXiv: 1003.3464

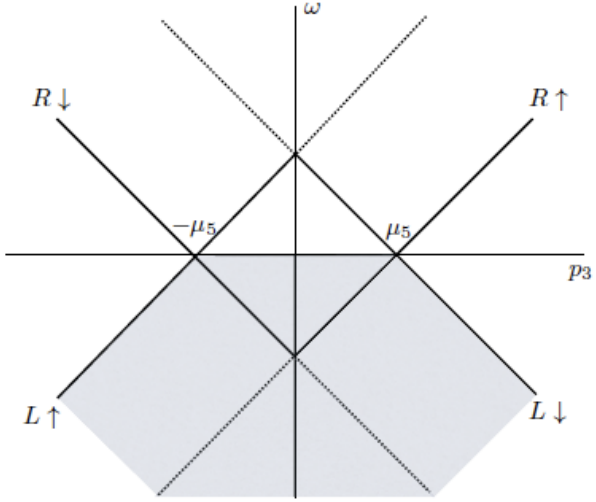
Plane waves describing the pairing fermions acquire a phase difference due to the chemical potential - the spiral nature of condensates.

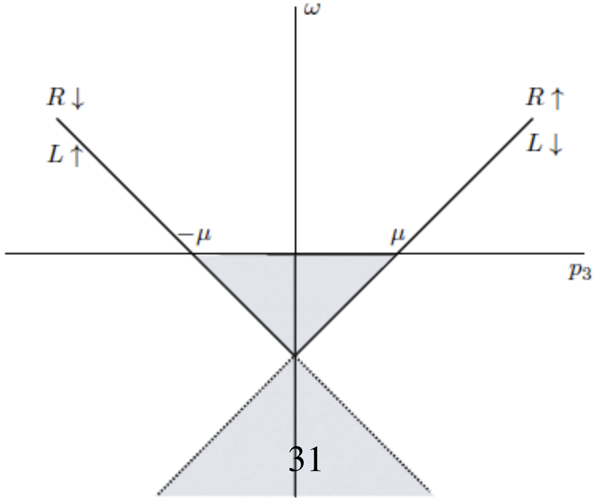
Gapless collective spiral excitation that carries a vector current (at finite chirality) or a chiral current (at finite baryon density).

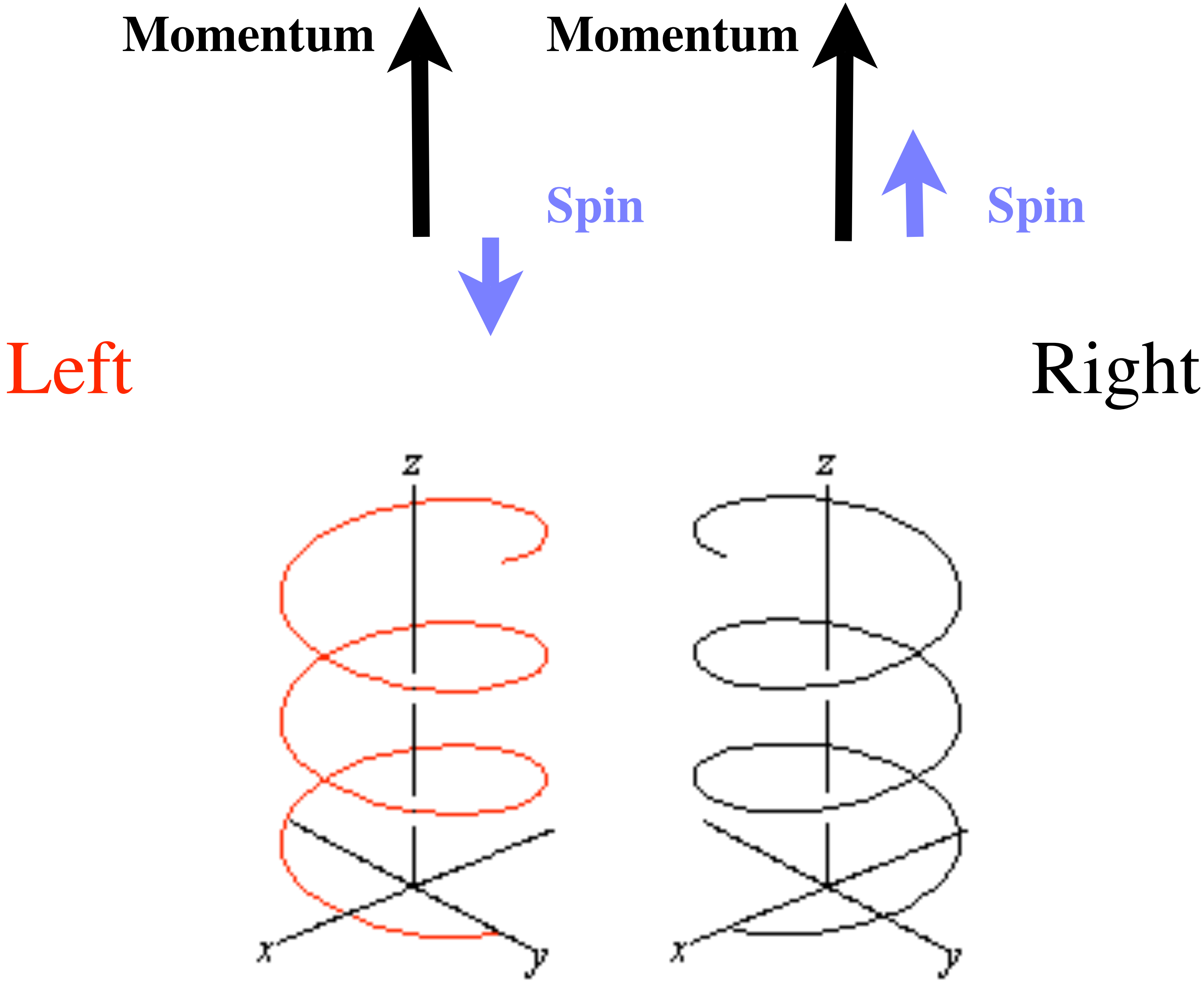
$$\langle J^3 \rangle = \frac{eB}{2\pi}\frac{e\mu_5}{\pi} \qquad \langle J_5^3 \rangle = \frac{eB}{2\pi}\frac{e\mu}{\pi}$$

4 =2x(1+1)

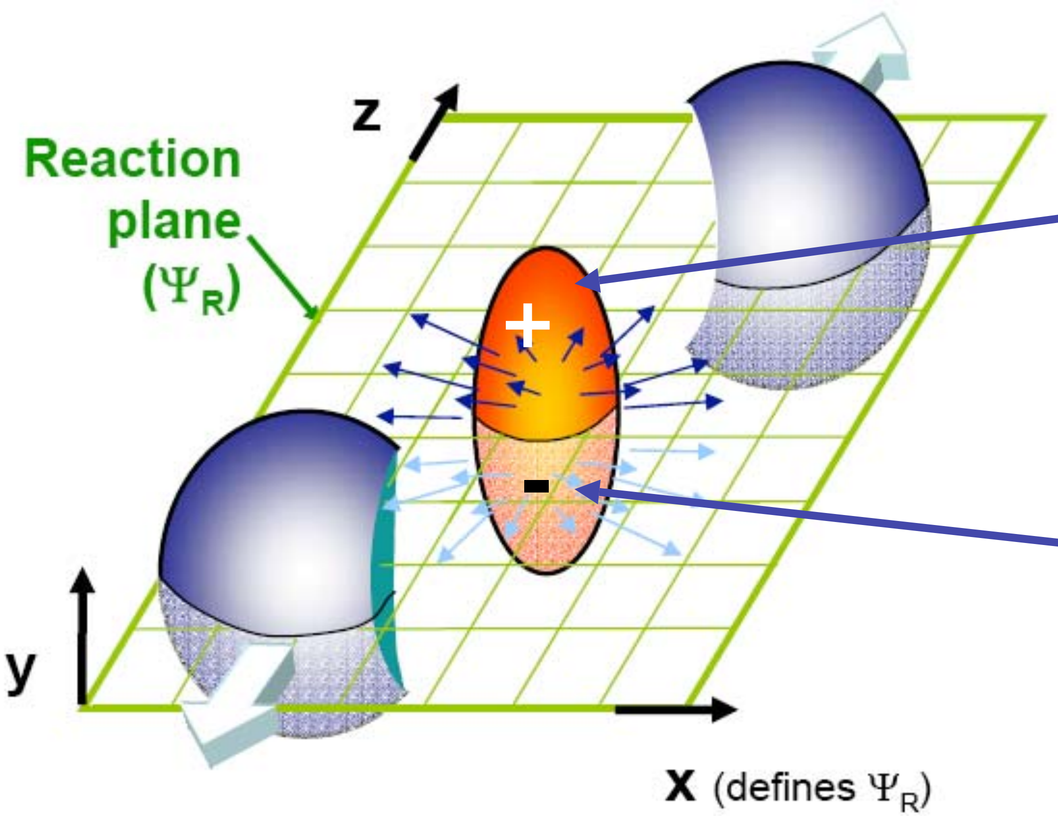
$$\begin{aligned}
\langle J^1 \rangle &= C^2 \cos(2\mu_5\, z - \phi_R) - D^2 \cos(2\mu_5\, z + \phi_L) \\
\langle J^2 \rangle &= -C^2 \sin(2\mu_5\, z - \phi_R) + D^2 \sin(2\mu_5\, z + \phi_L) \\
\langle J_5^1 \rangle &= C^2 \cos(2\mu_5\, z - \phi_R) + D^2 \cos(2\mu_5\, z + \phi_L) \\
\langle J_5^2 \rangle &= -C^2 \sin(2\mu_5\, z - \phi_R) - D^2 \sin(2\mu_5\, z + \phi_L)
\end{aligned}$$




Momentum
Momentum
Spin
Spin
Left
Right
z
z
x
y
x
y

# Charge asymmetry w.r.t. reaction plane as a signature of strong P violation



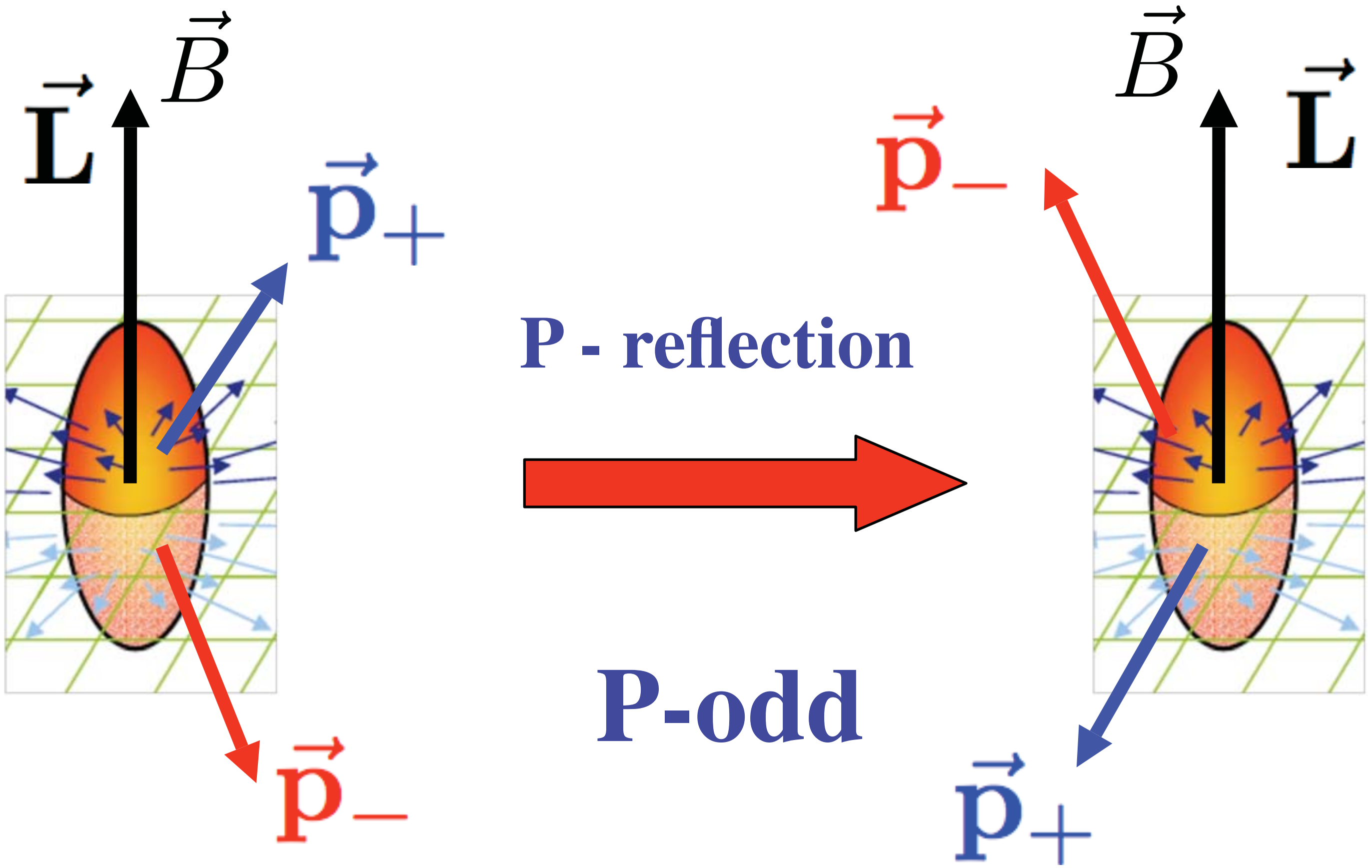

excess of positive charge

excess of negative charge

Electric dipole moment of QCD matter!

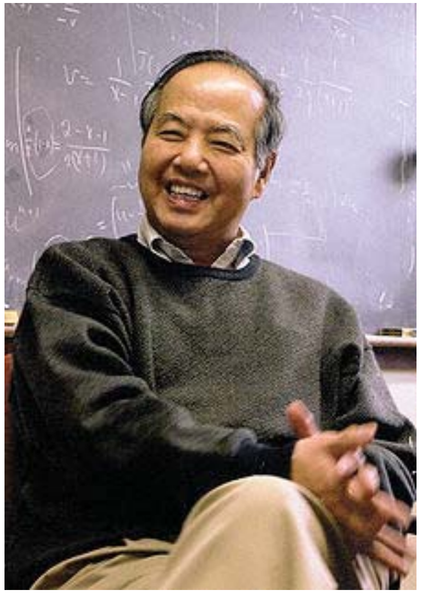
DK, Phys.Lett.B633(2006)260 [hep-ph/0406125]

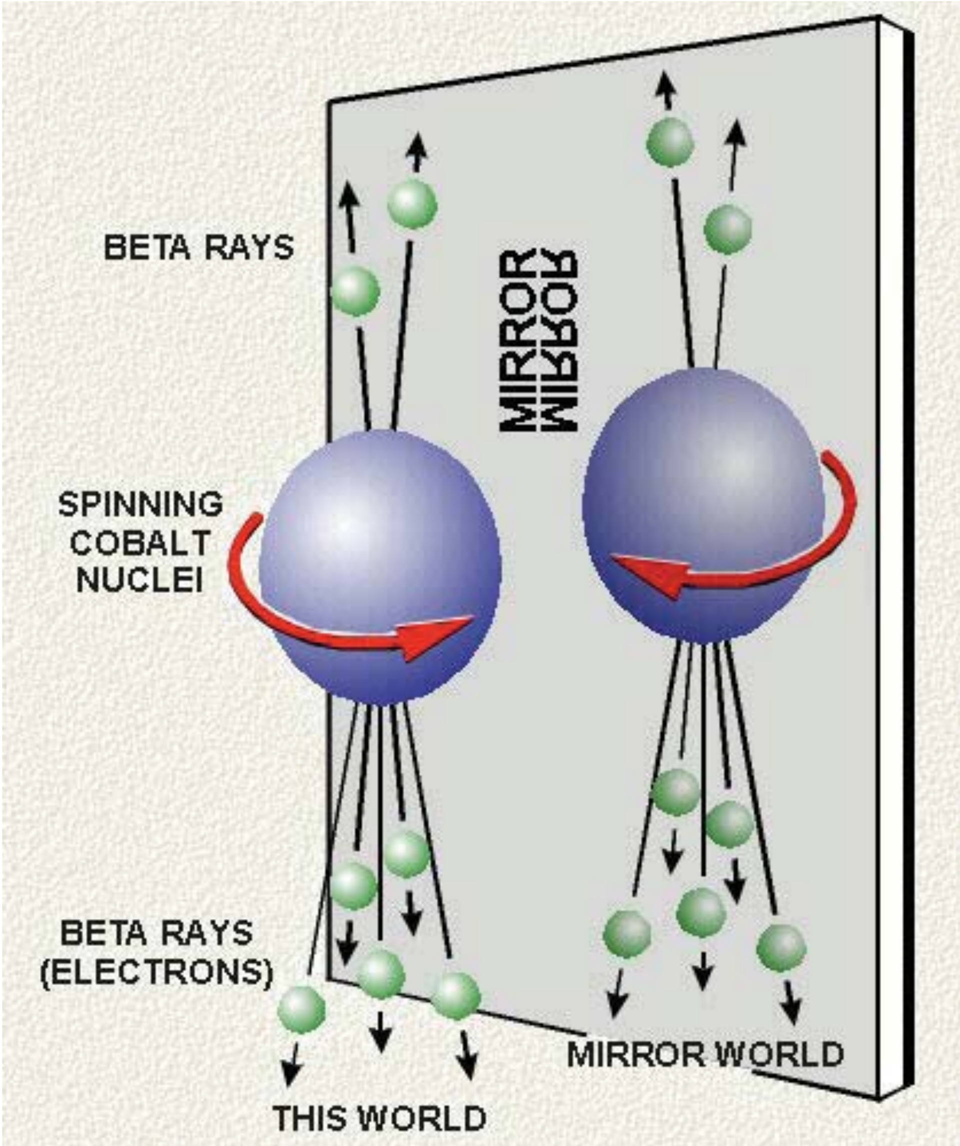
# Charge separation = parity violation:




$$\mathcal{P}: \quad \vec{p} \to -\vec{p}; \quad \vec{B} \to \vec{B}; \quad \vec{L} \to \vec{L}$$

# Analogy to P violation in weak interactions



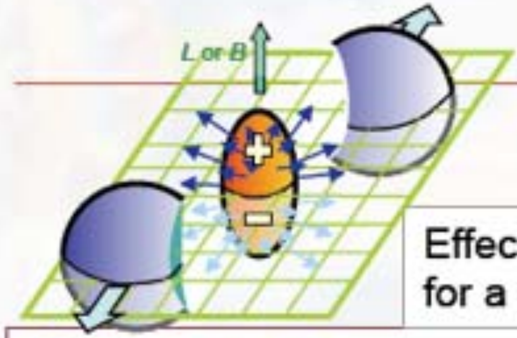



C.S. Wu, 1912-1997

**BUT**:
the sign of
the asymmetry
fluctuates
event by event

# Observable

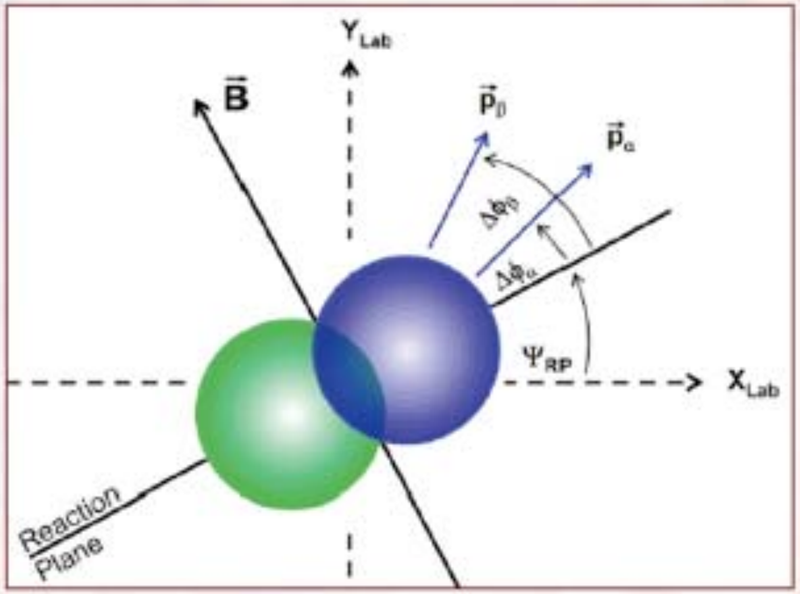
S.A. Voloshin, Phys. Rev. C 70 (2004) 057901




Effective particle distribution for a certain $Q$.

$$\frac{dN_\alpha}{d\phi} \propto 1 + 2v_{1,\alpha}\cos(\Delta\phi) + 2\,v_{2,\alpha}\cos(2\Delta\phi) + \ldots + 2a_{1,\alpha}\sin(\Delta\phi) + 2\,a_{2,\alpha}\sin(2\Delta\phi) + \ldots,$$

$$\Delta\phi = (\phi - \Psi_{RP})$$




- The effect is too small to observe in a single event
- The sign of $Q$ varies and $\langle a \rangle = 0$ (we consider only the leading, first harmonic) ➔ one has to measure correlations, $\langle a_\alpha a_\beta \rangle$, $\mathcal{P}$-even quantity (!)
- $\langle a_\alpha a_\beta \rangle$ is expected to be ~ $10^{-4}$
- $\langle a_\alpha a_\beta \rangle$ can not be measured as $\langle \sin\varphi_\alpha \sin\varphi_\beta \rangle$ due to large contribution from effects not related to the orientation of the reaction plane

➔ study the difference in corr's in- and out-of-plane

$$\langle\cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP})\rangle = \\ = \langle\cos\Delta\phi_\alpha\,\cos\Delta\phi_\beta\rangle - \langle\sin\Delta\phi_\alpha\,\sin\Delta\phi_\beta\rangle \\ = [\langle v_{1,\alpha}v_{1,\beta}\rangle + B^{in}] - [\langle a_\alpha a_\beta\rangle + B^{out}].$$

$$B^{in} \approx B^{out}, \quad v_1 = 0$$

A practical approach: three particle correlations: $\langle\cos(\phi_a + \phi_\beta - 2\phi_c)\rangle = \langle\cos(\phi_a + \phi_\beta - 2\Psi_{RP})\rangle\, v_{2,c}$

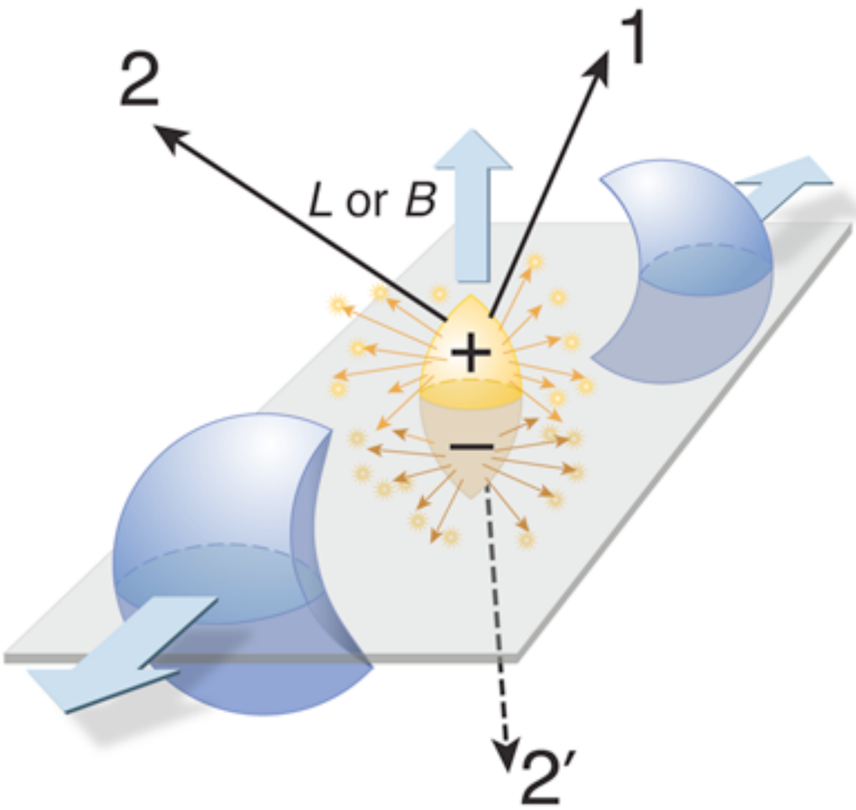
PRL **103,** 251601 (2009)

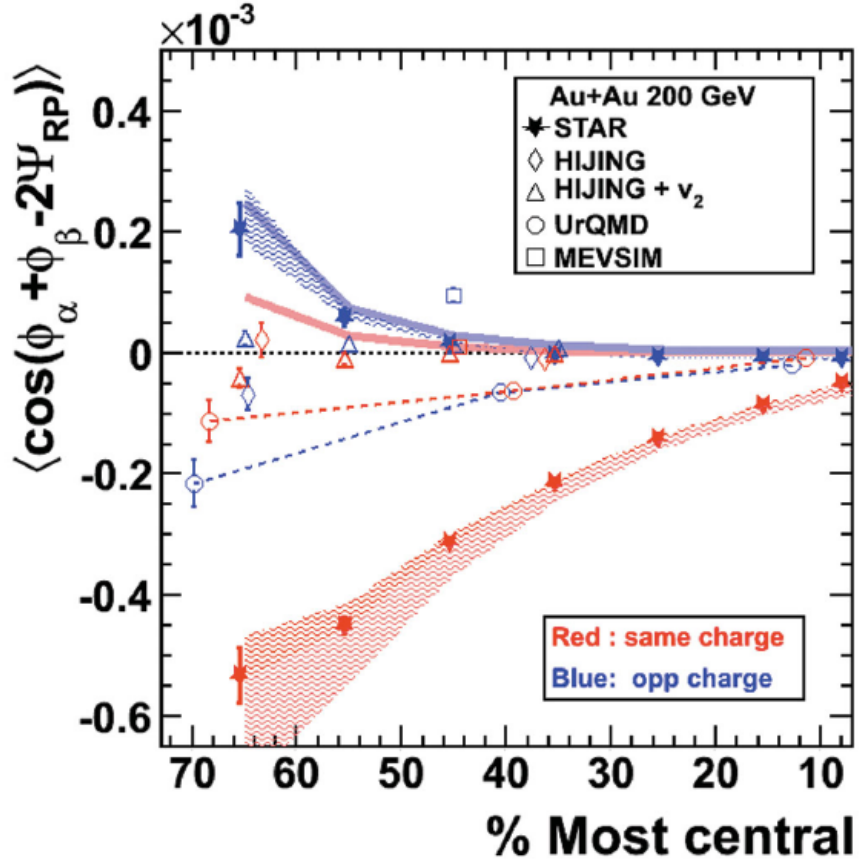
Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW LETTERS

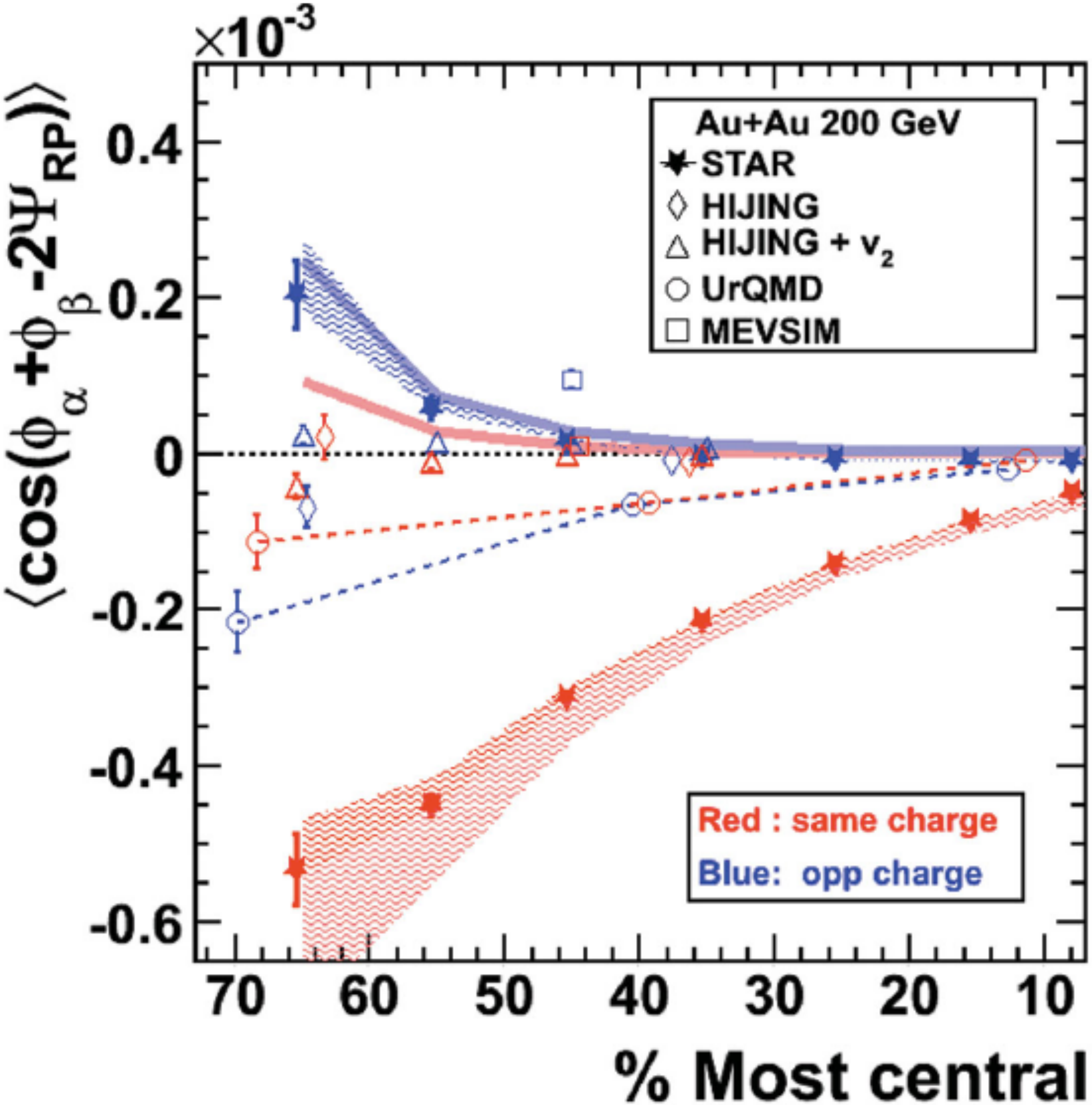
week ending
18 DECEMBER 2009

**Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation**

(STAR Collaboration)

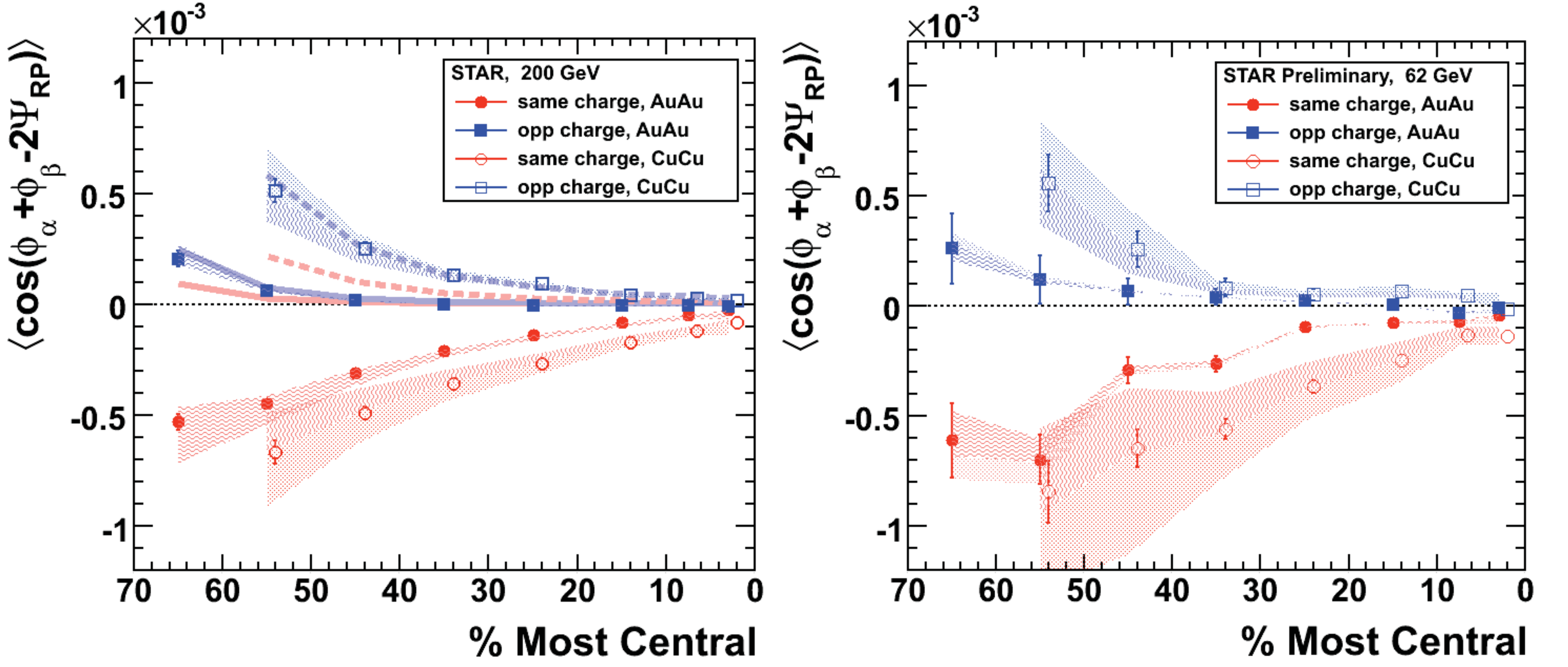





**P-even** observable; but: sensitive to P-odd fluctuations

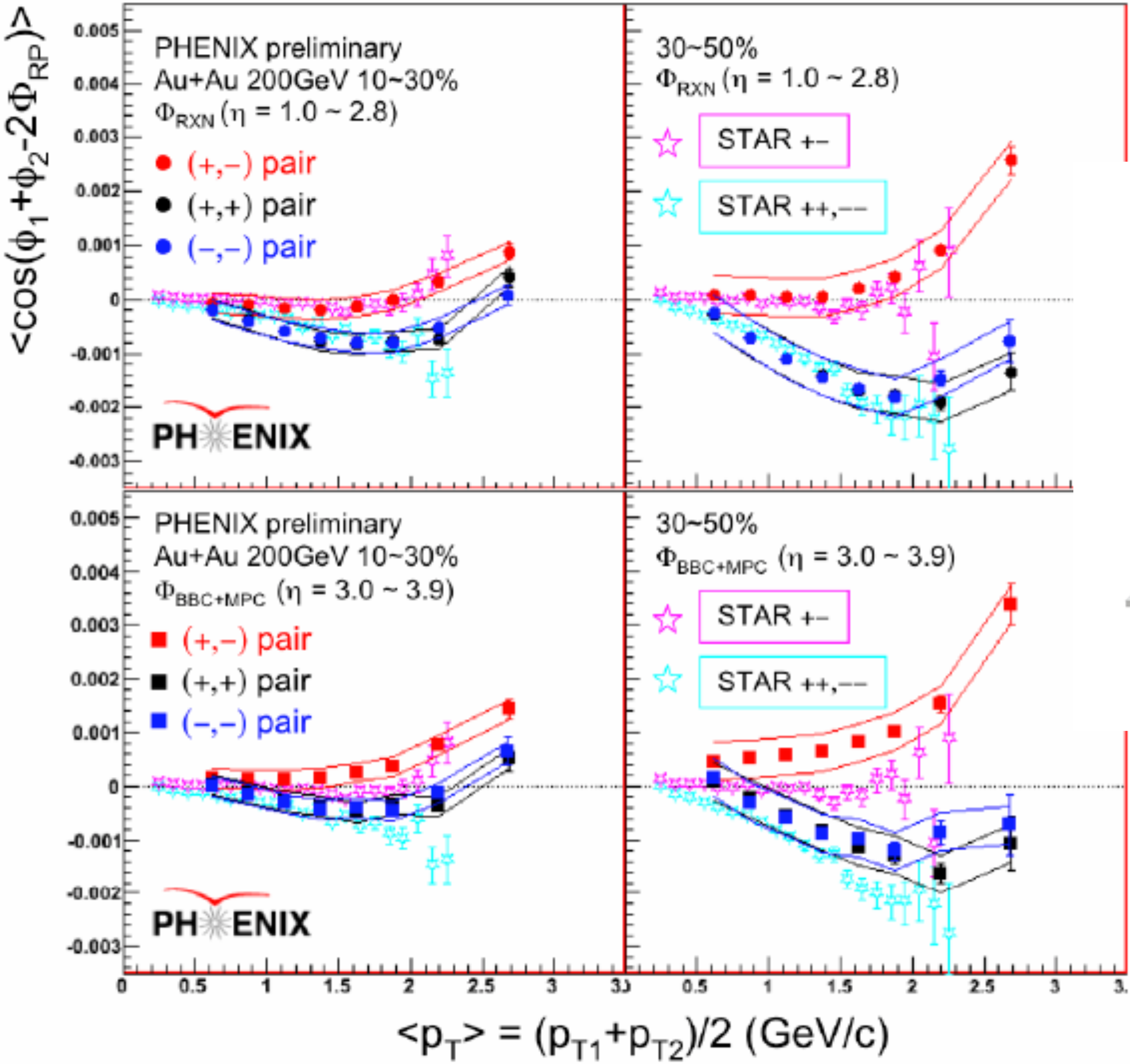
# Mass number and energy dependences

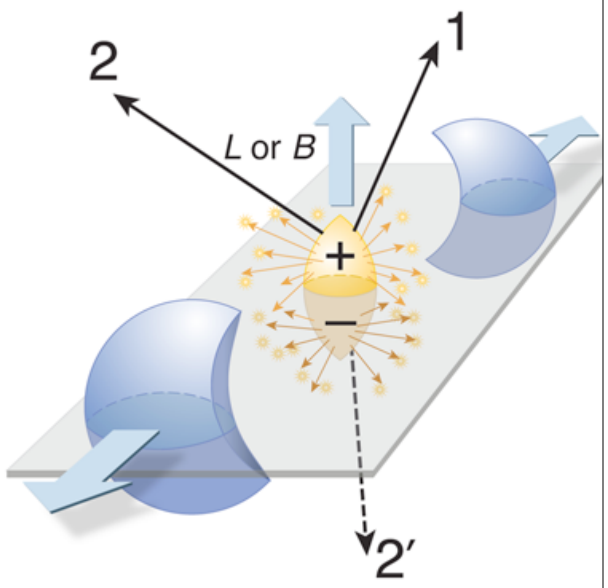



STAR Coll., arXiv:0909.1717 (Phys Rev C)

Expectations for the energy dependence: slow growth towards low energies reflecting longer-lived magnetic field, then gradual disappearance (no QGP): there has to be a maximum somewhere

S.Esumi et al
[PHENIX Coll]
April 2010




***Relatively good agreement between PHENIX & STAR***

# Summary

- The existence of topological solutions is an indispensable property of non-Abelian gauge theories that form the Standard Model
- Electric charge separation in the background magnetic field (CME) allows a **direct** observation of a topological effect in QCD
- The existence of the Chiral Magnetic Effect (CME) has been confirmed in first-principle lattice QCD calculations
- There is a recent observation of dynamical fluctuations in charge asymmetry at RHIC - an evidence for the CME?

RIKEN-BNL-CATHIE Workshop on

# P- and CP-odd Effects in Hot and Dense Matter

**April 26-30, 2010**

**Brookhaven National Laboratory**
**Long Island, New York, USA**

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P- and CP-odd effects in:
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Talks online at
http://quark.phy.bnl.gov/~kharzeev/cpodd/

Additional information and registration at
http://www.bnl.gov/riken/hdm/

**Registration deadline: March 1, 2010**

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**and Stony Brook University (Office of Vice-President for BNL Affairs)**