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Discovery of the Perfect Fluidity of Quark-Gluon Plasma at RHIC and hydrodynamical analysis of recent RHIC data

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Exploring the Quark Gluon Plasma at the Relativistic Heavy Ion Collider

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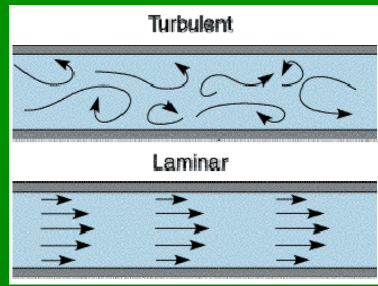
- Milestones at RHIC
- Hydrodynamics
- Direct photon results

Milestones of RHIC results

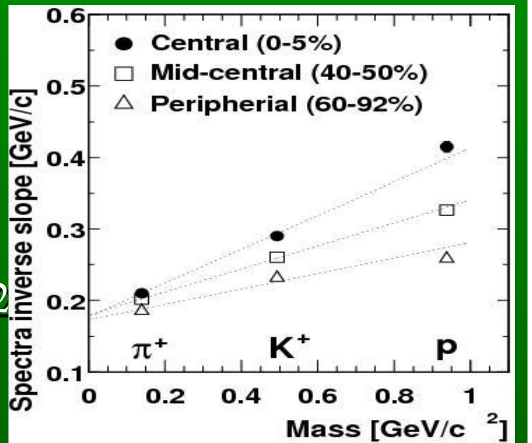
- Jet suppression in A+A: new phenomenon
  - Phys. Rev. Lett. 88, 022301 (2002)
- No jet suppression in d+A: new form of matter
  - Phys. Rev. Lett. 91, 072303 (2003)
- Summary of the results: matter is a liquid
  - Nucl. Phys. A 757, 184-283 (2005)
- Elliptic flow scaling: quark degrees of freedom
  - Phys. Rev. Lett. 98, 162301 (2007)
- Heavy quark flow: viscosity near lower limit
  - Phys. Rev. Lett. 98, 172301 (2007)
- Initial temperature, state of matter: no final answer yet

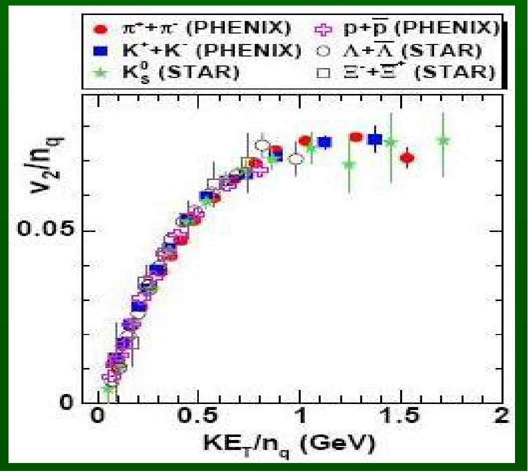
Hydrodynamic scaling predictions

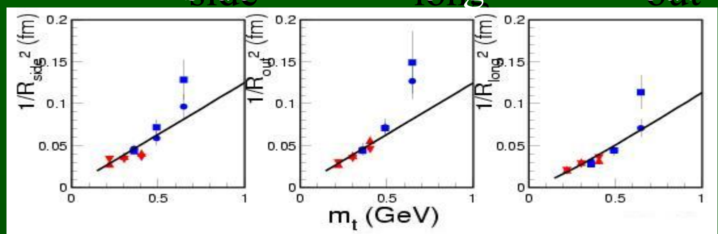
- Hydro predicts scaling (also viscous)
- What does a scaling mean?
  - For example Reynolds number ρvr/η
  - Only a combination of parameters matters



- Collective, thermal behavior → Loss of information
- Spectra slopes: $N_1 \sim e^{-\frac{m_t}{T_{\rm eff}}};\ T_{\rm eff} = T_0 + m u_t^2$
- Elliptic flow: $v_2 = \frac{I_1(w)}{I_0(w)};\ w \sim KE_T$
- HBT radii: $R_{\rm side}^2 \approx R_{\rm long}^2 \approx R_{\rm out}^2 \sim \frac{1}{m_t}$

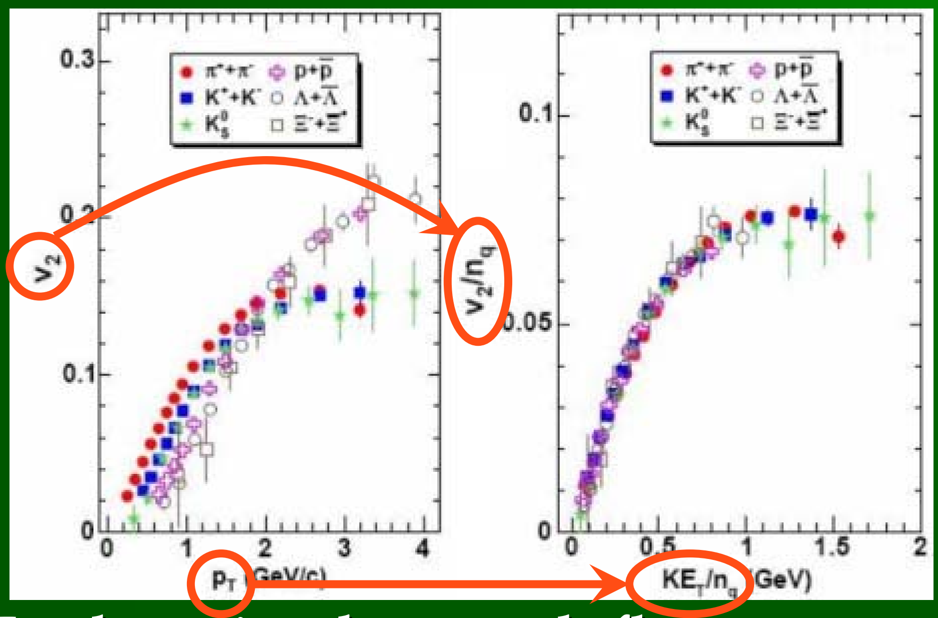


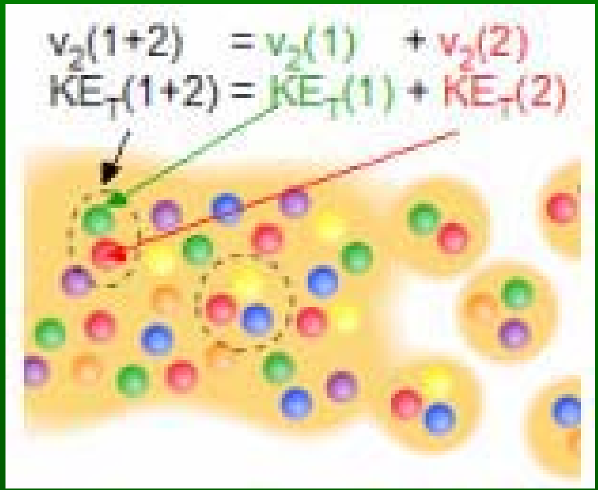




Quark degrees of freedom

- Elliptic flow (v_2) is a central observable
  - measure of momentum space asymmetry
  - sign of collective behavior
- Measured for several types of particles
- Scaling variable suggested by hydro: KE_t
- Both v_2 and KE_t scale by the number of quarks

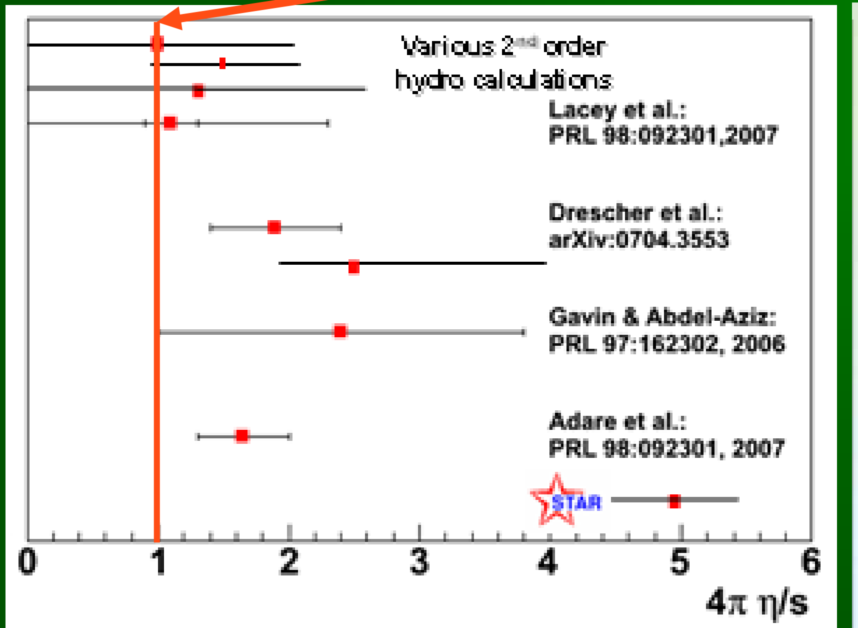


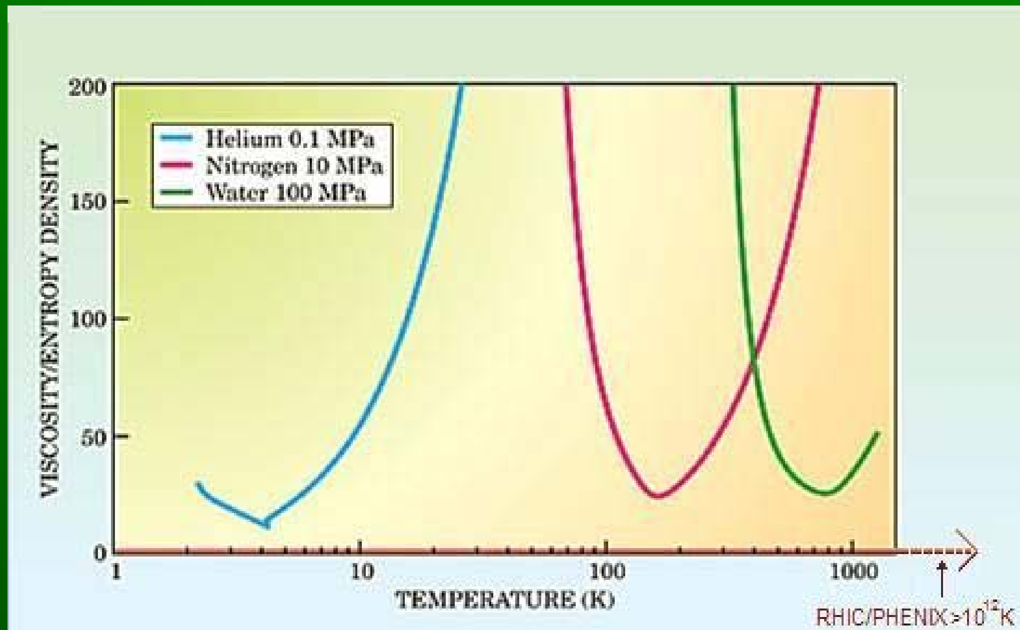


- Explanation by quark flow
- Indirect sign of quark degrees of freedom

Superfluidity

- Kinematic viscosity η/s lower than that of superfluid helium
- Conjectured limit: 1/4π (via AdS/CFT)





Perfect hydro solutions

Solution	Basic prop's	EoS	Observables
Csörgő, Nagy, Csanád Phys.Lett.B 663:306-311, 2008 Phys.Rev.C77:024908,2008	Ellipsoidal, 1D accelerating	$\varepsilon - B = \kappa\,(p + B)$	dn/dy, ε
Landau Izv. Acad. Nauk SSSR 81 (1953) 51	Cylindr., 1D, accelerating	$\varepsilon = \kappa n T$	None
Hwa-Björken R.C. Hwa, PRD10, 2260,1974 J.D. Bjorken, PRD27, 40(1983)	Cylindr., 1D, non-accelerating	$\varepsilon = \kappa n T$	dn/dy, ε
Bialas et al. A. Bialas, R. A. Janik, and R. B. Peschanski, Phys. Rev. C76, 054901 (2007).	1D, betweend Landau and Hwa-Björken	$\varepsilon = \kappa n T$	dn/dy
Csörgő, Csernai, Hama, Kodama Heavy Ion Phys. A 21, 73 (2004))	Ellipsoidal, 3D, non-accelerating	$\varepsilon = \kappa n T$	This work does the calculation
???	Ell, 3D, accel.	realistic	???

Where we are

- Revival of interest, new solutions
  - Sinyukov, Karpenko, nucl-th/0505041
  - Pratt, nucl-th/0612010
  - Bialas et al.: Phys.Rev.C76:054901,2007
  - Borsch, Zhdanov: SIGMA 3:116,2007
  - Nagy et al.: J.Phys.G35:104128,2008 and arXiv/0909.4285
  - Liao et al.: arXiv/09092284 and Phys.Rev.C80:034904,2009
  - Mizoguchi et al.: Eur.Phys.J.A40:99-108,2009
  - Beuf et al.:Phys.Rev.C78:064909,2008 (dS/dy as well!)
- Need for solutions that are:
  - accelerating + relativistic+ 3 dimensional
  - explicit + simple + compatible with the data

A 3D relativistic solution

- The hydro fields are these:
  - ν(s) arbitrary, but realistic to choose Gaussian $\nu(s) = e^{-bs/2}$ b<0 is realistic
- Ellipsoidal symmetry

$$s = \frac{x^2}{X(t)^2} + \frac{y^2}{Y(t)^2} + \frac{z^2}{Z(t)^2}$$

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{(3/\kappa)} \frac{1}{\nu(s)}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\left(3+\frac{3}{\kappa}\right)}$$

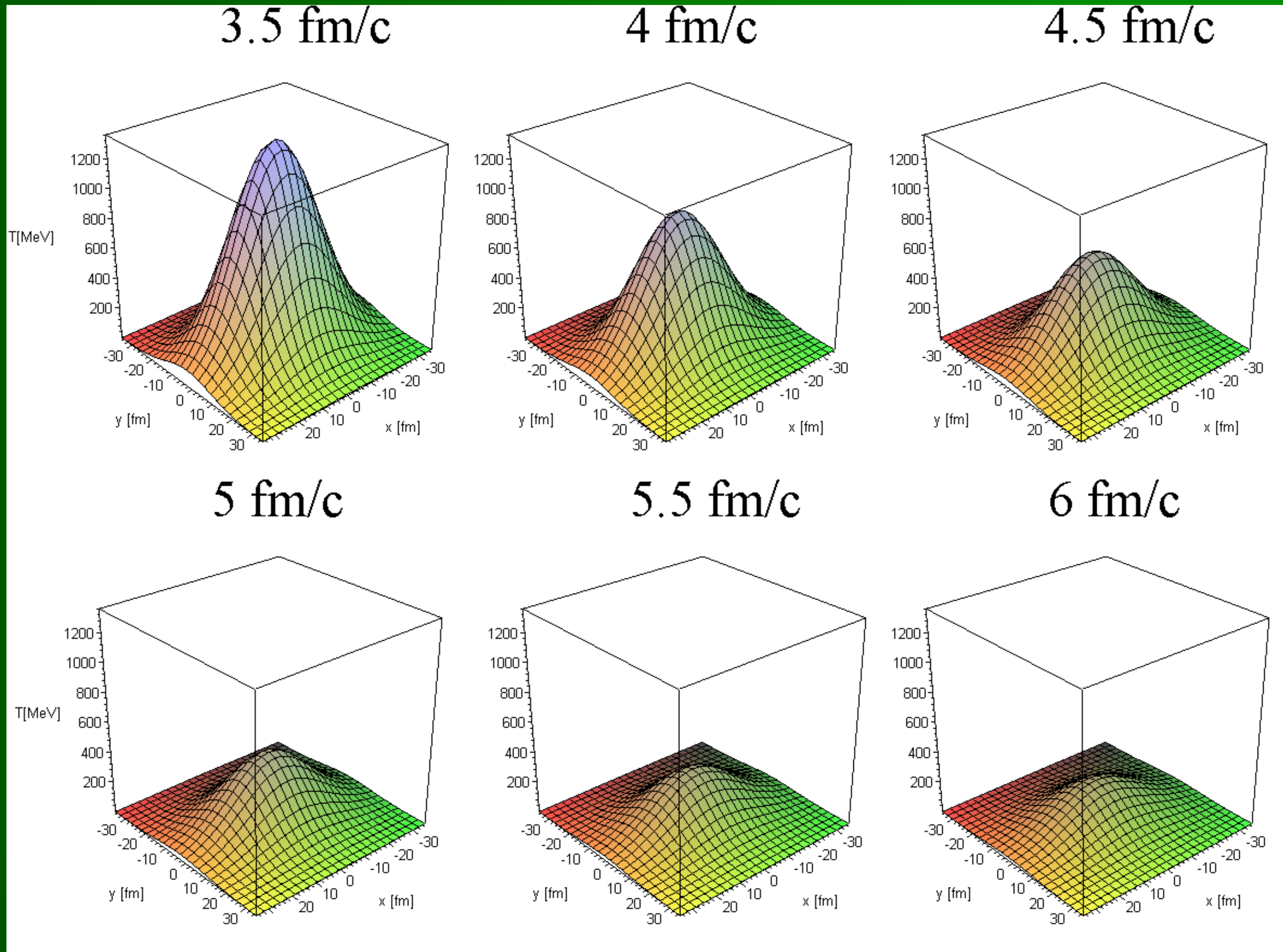
$$u^\mu = \gamma \left(1, \frac{\dot{X}(t)}{X(t)} x, \frac{\dot{Y}(t)}{Y(t)} y, \frac{\dot{Z}(t)}{Z(t)} z\right)$$

  - (thermodynamic quantities const. on the s=const. ellipsoid)
- Directional Hubble-flow (no acceleration)
  - v=Hr or H=v/r, the Hubble-constants: $\frac{\dot{X}(t)}{X(t)}, \frac{\dot{Y}(t)}{Y(t)}, \frac{\dot{Z}(t)}{Z(t)}$
  - $\dot{X}(t), \dot{Y}(t), \dot{Z}(t) : const.; \left(\dot{X}, \dot{Y}\right) \Leftrightarrow \left(u_t, \varepsilon\right)$

(T. Csörgő, L. P. Csernai, Y. Hama és T. Kodama, Heavy Ion Phys. A 21, 73 (2004))

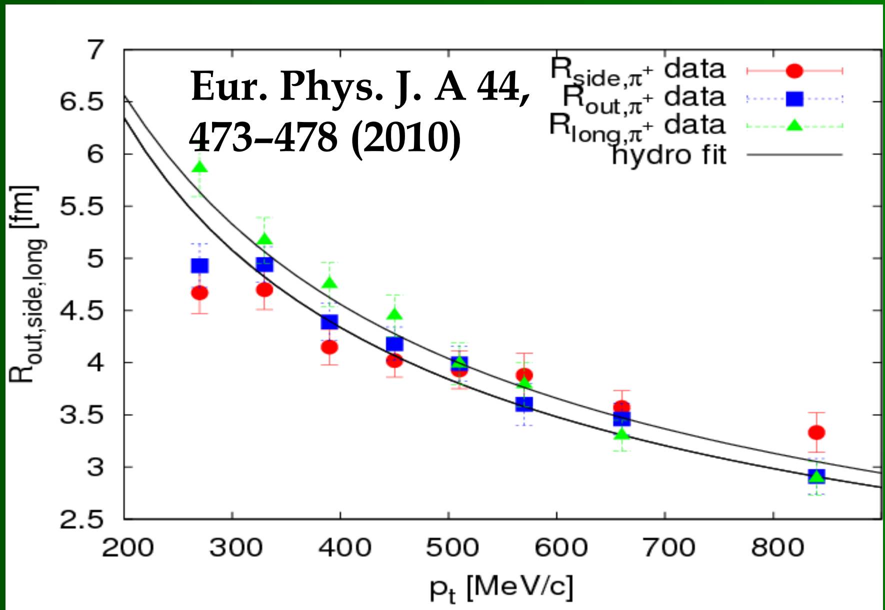
Temperature profile

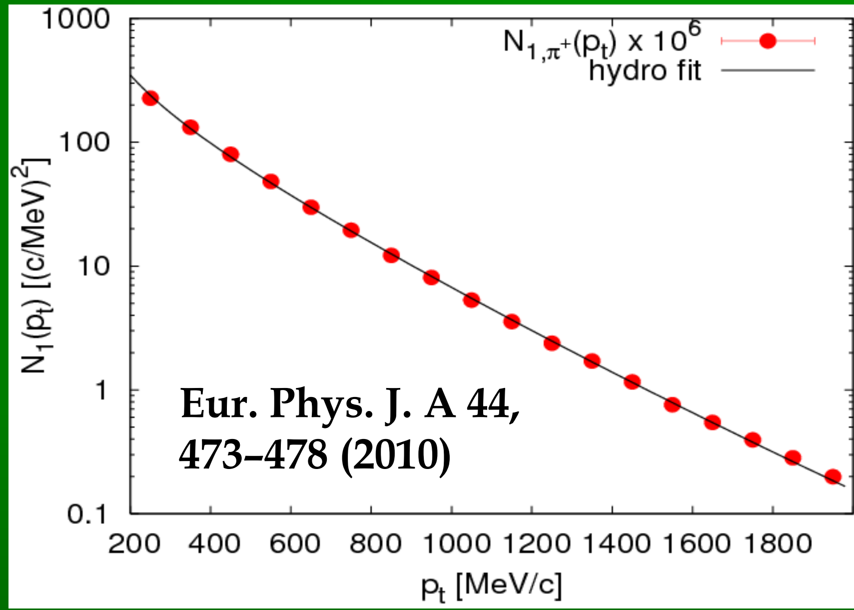
Transverse temperature profile as a function of time
with an example parameter set:



Single pion spectum with HBT radii

- 0-30% centrality, Au+Au, PHENIX

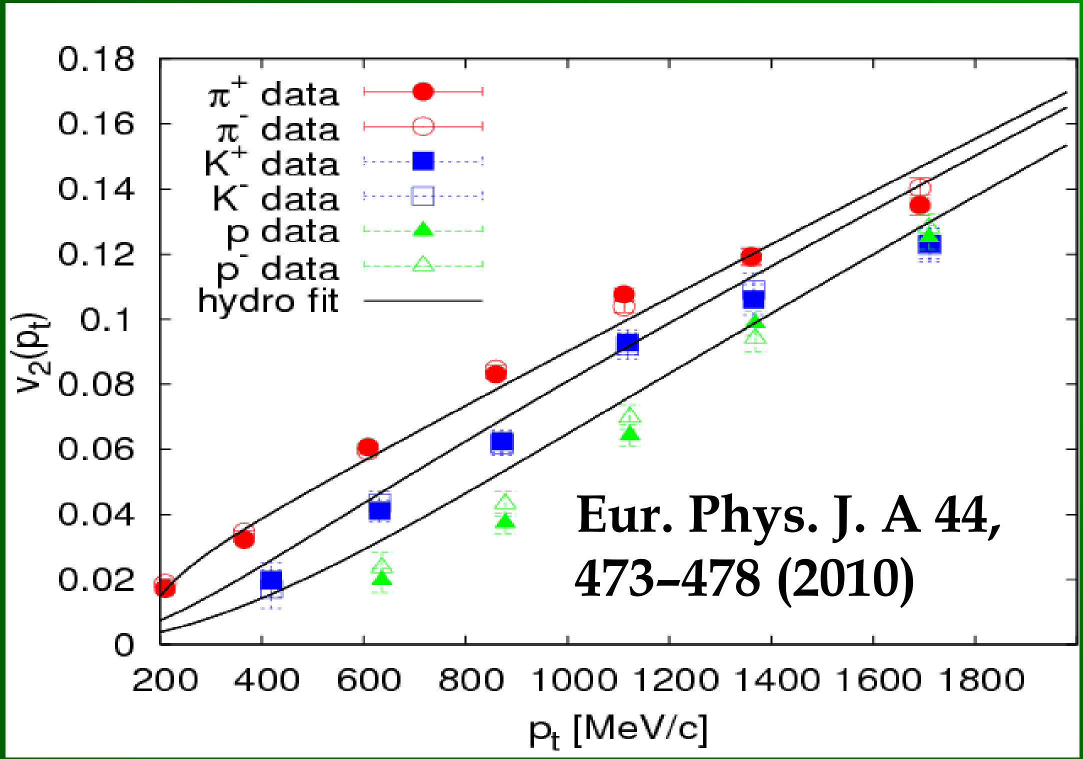




• T_0	199 ± 3 MeV	central freeze-out temp.
• ε	0.80 ± 0.02	momentum space ecc.
• u_t^2/b	-0.84± 0.1 (b<0)	transv. flow/temp. grad
• τ_0	7.7 ± 0.1	freeze-out proper time
• χ^2	171 (24 with theory error)	

Elliptic flow

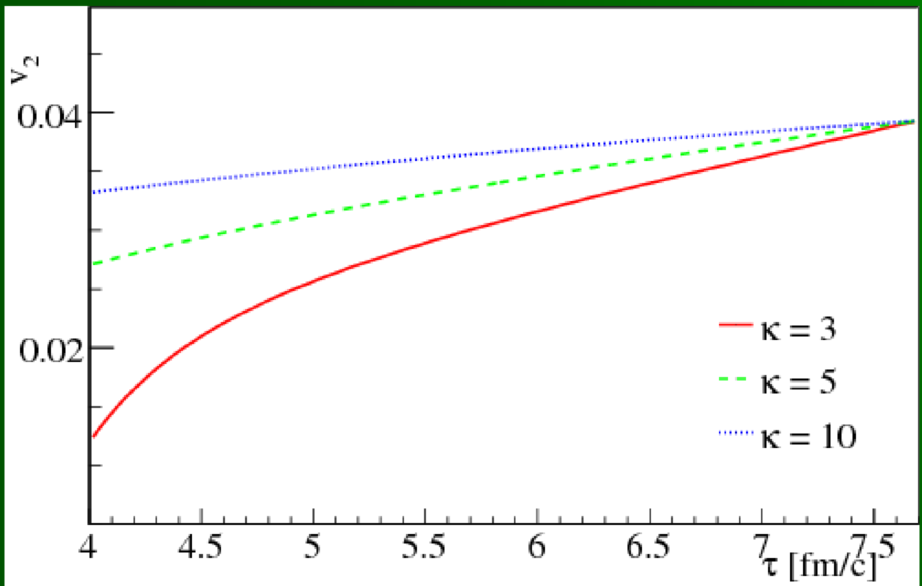
- 0-92% centrality, Au+Au, PHENIX (R.P. method technique)

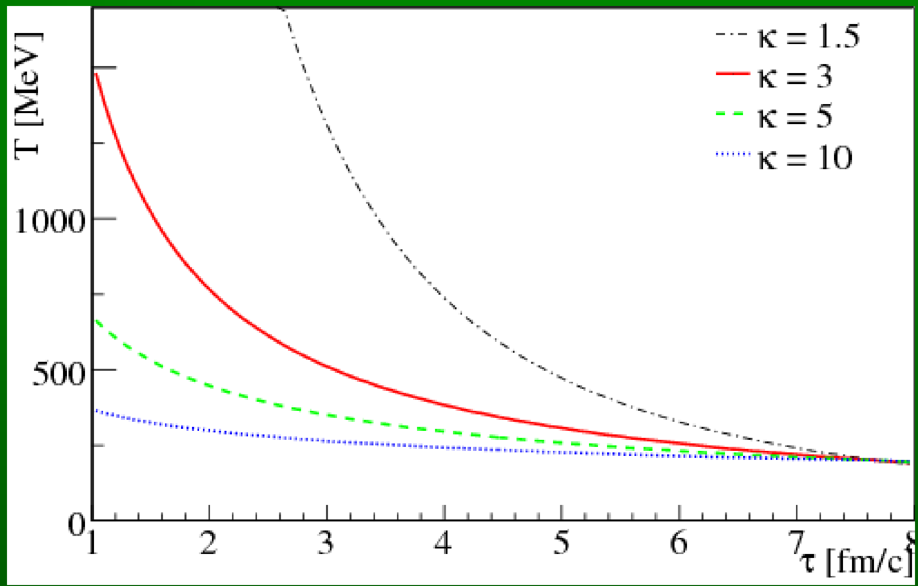


- T_0 204 ± 7 MeV f.o. temperature
- ε 0.34 ± 0.01 eccentricity
- u_t^2/b -0.34 ± 0.07 (b<0) transv. flow/temp. grad
- χ^2 256 (66 with theory error)

Insensitivity on the EoS

- Hadronic data corresponds to the initial state
- Not directly sensitive to
  - Equation of state
  - Initial condition





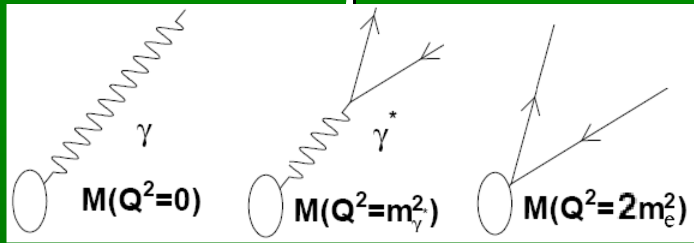
- Look for other data…
- Photon-emission rather continuous

Summary of the fit results

- Freeze-out central temperature ~ 200 MeV
- Hadronic fits don't depend on:
  - EoS (κ or sound velocity)
  - initial temperature
- Other data needed to determine IC and EoS
- After thermalization:
  - thermal/direct emission will play a significant role
  - direct photons 'feel' the evolution of the fireball
- Can we measure direct photons?
- Yes we can!

How to measure direct photons?

- PHENIX measurement done: PRL 104, 132301 (2010)
- Problem: huge background from h → γγ
- Idea: thermal + virtual photon production parralel
  - $X \rightarrow e^+e^-$, $X \rightarrow \gamma$ and $X \rightarrow \gamma^* \rightarrow e^+e^-$ from the same process



- Dielectron and real photon production related as:

$$\frac{d^2 n_{ee}}{dm_{ee}dp_t} = \frac{2\alpha}{3\pi}\frac{1}{m_{ee}} L(m_{ee}) S(m_{ee}) \frac{dn_\gamma}{dp_t},\quad L(m_{ee}) = \sqrt{1-\frac{4m_e^2}{m_{ee}^2}}\left(1+\frac{2m_e^2}{m_{ee}^2}\right)$$

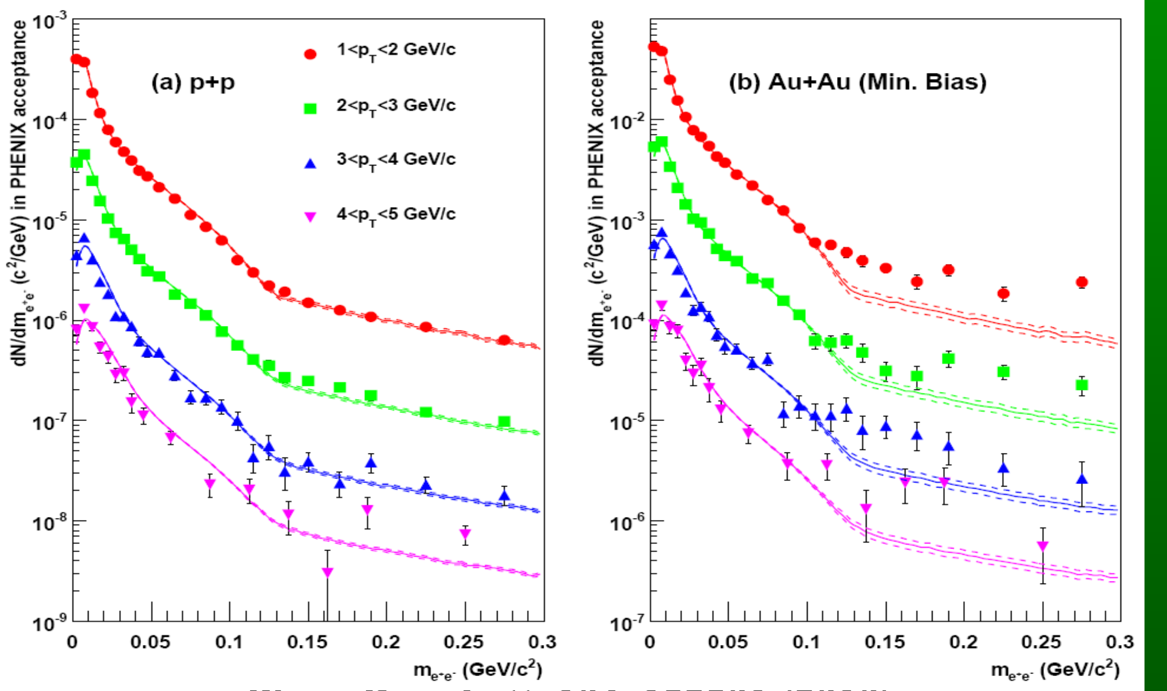
- S process dependent, dn_{γ^*}/dn_γ, for π_0 and η e.g.:

$$S(m_{ee}) = \left|F(m_{ee}^2)\right|^2 \left(1-m_{ee}^2/M_h^2\right)^3,\ S(m_{ee}) = 0 \text{ for } m_{ee} > M_h$$

- For p_t » m_{ee} » m_e: L, S → 1

Dielectron spectrum measurement

- Measured electron pairs with p_t of 1-5 GeV
- Easy via electron ID capabilities
- Compare to dielectrons from hadronic cocktail

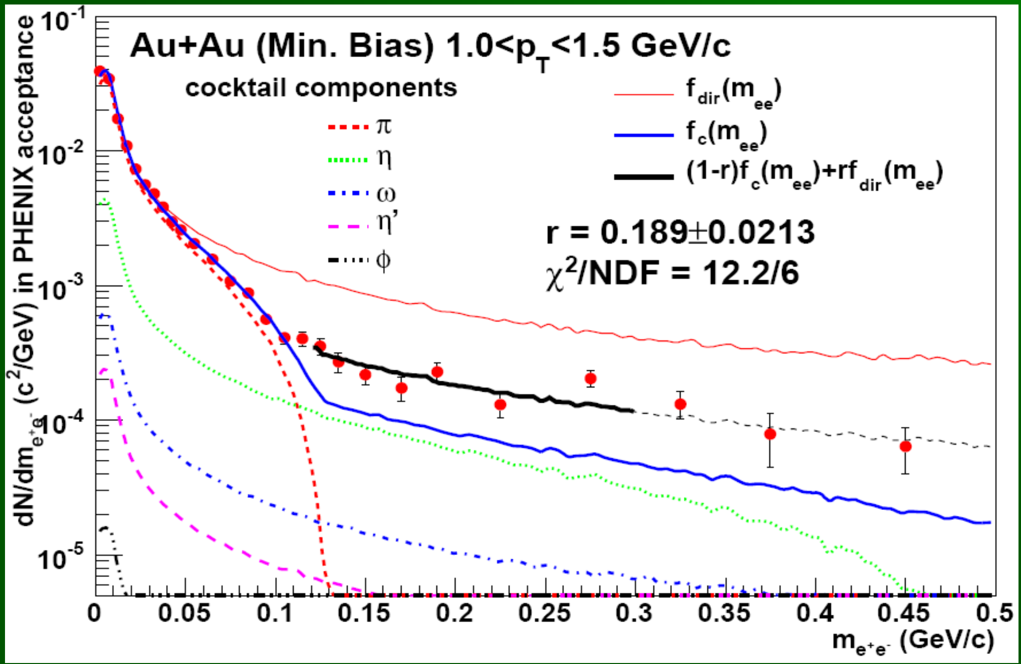


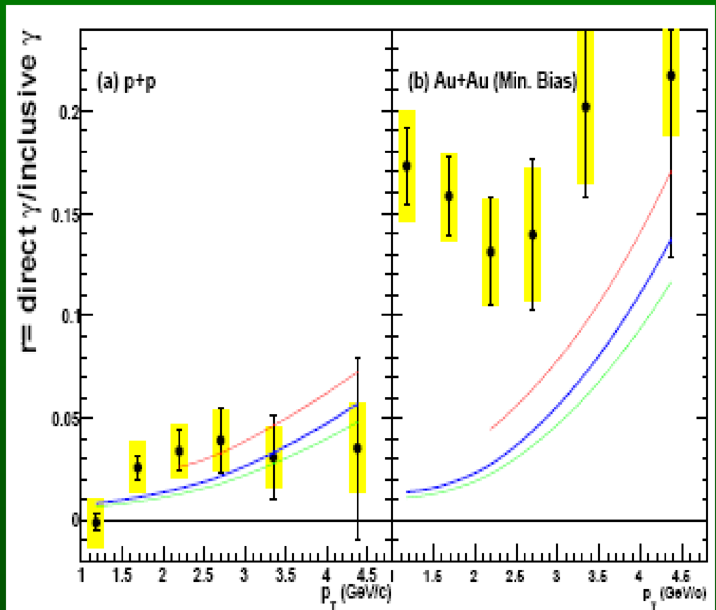
Phys. Rev. Lett. 104, 132301 (2010)

- Excess seen above pion mass due to virtual γ

Direct photons versus decay photons

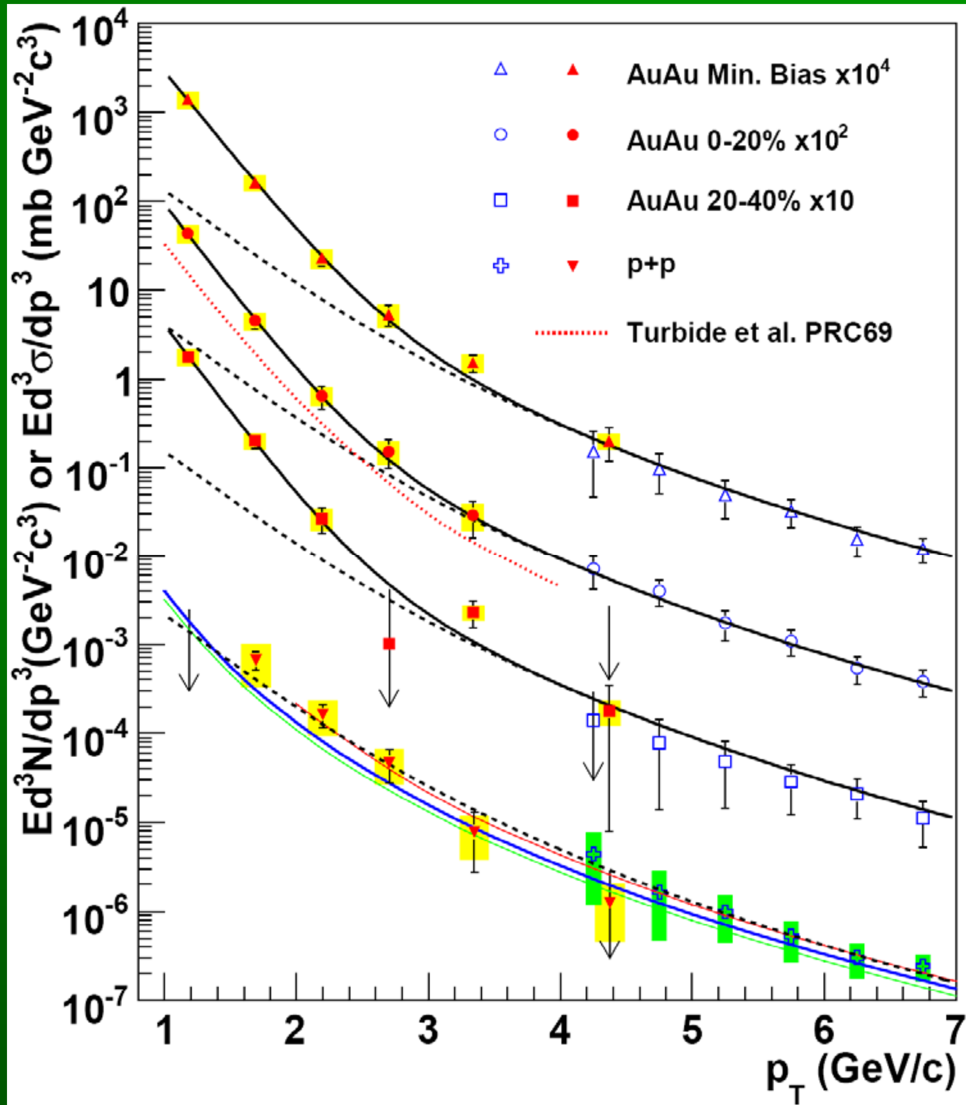
- Excess: virtual direct photons (decaying into e^+e^- pairs)
- Inclusive e^+e^-: hadronic + dir. virtual photon components
  - Hadronic electron pairs (f_c), calculated from cocktail: π, η, ω, η′, ϕ
  - Electron pairs from direct virtual photons (f_{dir}) calculated from f_c via previous formula
- Determine ratio r by fit for separate p_t bins
- Use r to scale inclusive photon spectra





Direct photon spectra measured

- Inclusive photons: from the m_{ee}<0.03 GeV/c^2 inclusive e^+e^- yield (S=1 here)
- Direct photon spect.: $N_{dir}(p_t) = r \times N_{incl}(p_t)$
- Thermal below roughly 3 GeV
- What temperature does this correspond to?



Initial temperature

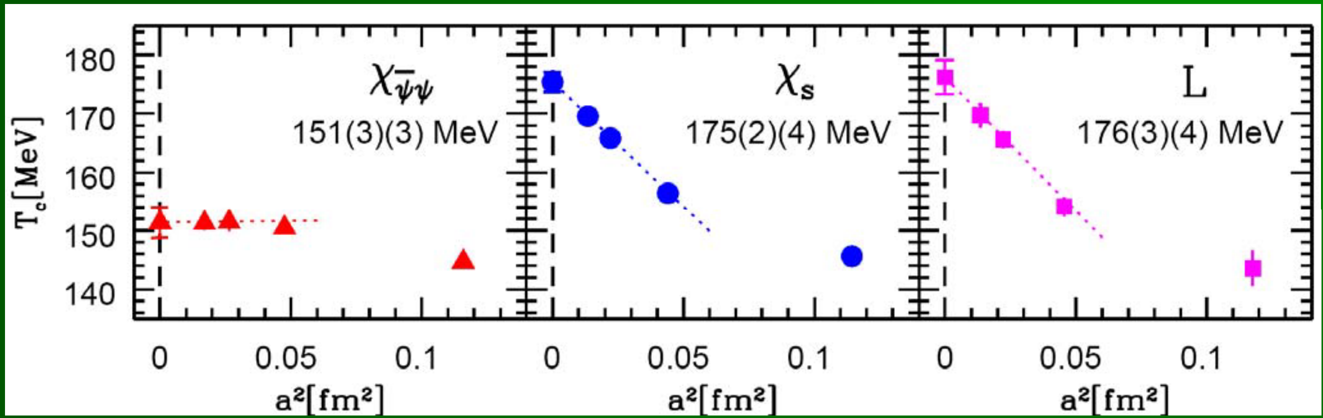
- Direct photon spectra can be fitted with a single average temperature
- 233 ± 14 ± 19 MeV for minimum bias
- Initial temperature definitely higher
- More sophisticated method: using hydro
- Photon yield would then be:

$$\int \frac{p_\mu d^3\Sigma_\mu}{e^{\frac{p_\mu u^\mu}{T}}-1} dt$$

- Integration over the whole time evolution

Transition temperature

- Budapest-Wuppertal group (arXiv:0908.3341) :
  - Chiral susceptibility: 151(3)(3) MeV
  - Strange quark number susceptibility: 175(2)(4) MeV
  - Polyakov loops: 176(3)(4) MeV



- MILC group, PRD71, 034504 (2005):
  - chiral susceptibility: 169(12)(4) MeV
- RBC-Bielefeld, PRD74 , 054507 (2006):
  - chiral susceptibility and Polyakov loops: 192(7)(4) MeV
- Some disagreement, but well below 200 MeV
- Experimental data: T_i at least 2-3σ above T_c

Summary

- Hydro working at RHIC
- Indirect signs of quark DoF seen
- T_c calculated via lattice QCD
- Hadronic data at RHIC + hydro: $T_{final} \simeq T_c$
- Photon data at RHIC + hydro: $T_{initial}$ » T_c

Thank you for your attention

Why we use hydrodynamics?

- Hadron & parton cascade models
  - Noneqilibrium models
  - Don't describe most of particle creation at low p_t
- Flow + hadronic cascade and resonance corrections
  - Small correction at RHIC energies (arXiv:0903.1863)
- Most of particle creation according to hydro
  - Pion and kaon HBT radii m_t scaling (Acta Phys.Polon.Supp.1:521-524,2008)
- After ~10 fm/c noneqilibrium expanding hadron gas
  - Anomal diffusion, no correction for spectra or flow (Braz.J.Phys.37:1002-1013,2007)
- American Institute of Physics: success of hydro is the physics story 2005

Little vocabulary of hydrodynamics

- Exact/parametric solution
  - Solution of hydro equations analytically, without approximation
  - Usually has free parameters
- Hydro inspired parameterization
  - Distribution determined at freeze-out only, their time dependence is not considered
- Numerical solution
  - Solution of hydro equations numerically
  - Start from arbitrary initial state

Nonrelativistic solutions

Solution	Symmetry	Density prof.	EoS	Observables
Csizmadia et al. Phys. Lett. B443:21-25, 1998	Sphere	Gaussian	$\varepsilon = \frac{3}{2} nT$	Calculated
Csörgő Central Eur.J.Phys.2: 556-565,2004	Sphere	Arbitrary	$\varepsilon = \frac{3}{2} nT$	Not calculated
Akkelin et al. Phys.Rev. C67,2003	Ellipsoid	Gaussian (T=T(t))	$\varepsilon = \kappa(T) nT$	Calculated
Csörgő Acta Phys.Polon. B37:483-494,2006	Ellipsoid	Arbitrary (T=T(r,t))	$\varepsilon = \kappa nT$	Not calculated
Csörgő, Zimányi Heavy Ion Phys.17:281-293,2003	Ellipsoid	Gaussian	$\varepsilon = \kappa_e nT - B$	Calculated

Equations of relativistic hydro

- Assuming local thermal equilibrium
- For a perfect fluid: $T^{\mu\nu} = wu^\mu u^\nu - pg^{\mu\nu},\ w = \varepsilon + p \Rightarrow \partial_\nu T^{\mu\nu} = 0$
- Equations in four-vector form and nonrelativistic notation
  - Euler equation:

$$wu^\nu \partial_\nu u^\mu = (g^{\mu\eta} - u^\mu u^\eta)\partial_\eta p \qquad \frac{w}{1-v^2}\frac{d\mathbf{v}}{dt} = -\left(\nabla p + \mathbf{v}\frac{\partial p}{\partial t}\right)$$

  - Energy conservation:

$$w\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0 \qquad \frac{1}{w}\frac{d\varepsilon}{dt} = -\nabla\mathbf{v} - \frac{1}{1-v^2}\frac{d}{dt}\frac{v^2}{2}$$

  - Charge conservation:

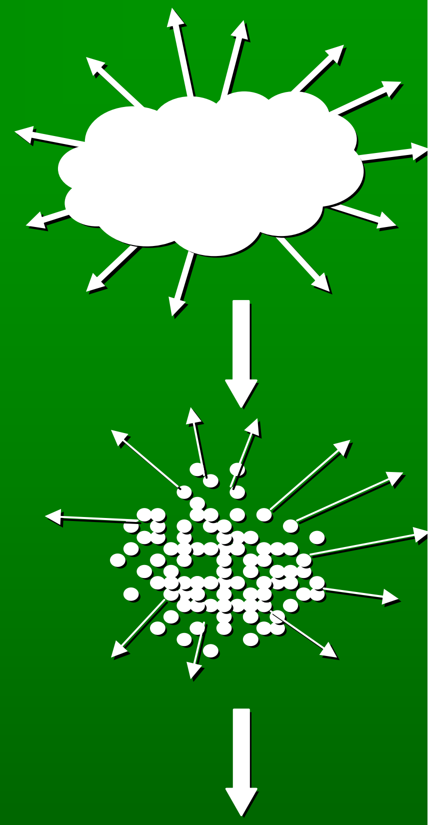
$$\partial_\mu (nu^\mu) = 0 \qquad \frac{d}{dt}\ln\frac{n}{\sqrt{1-v^2}} = -\nabla\mathbf{v}$$

$$u^\mu \partial_\mu = \frac{d}{d\tau}$$ comoving proper-time derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\nabla$$ comoving derivative

How analytic hydro works

- Take hydro equations and EoS
- Find a solution
  - Will contain parameters (like Friedmann, Schwarzschild etc.)
  - Will use a possible set of initial conditions
- Use a freeze-out condition
  - Eg fixed proper time or fixed temperature
  - Generally a hyper-surface
- Calculate the hadron source function
- Calculate observables
  - E.g. spectra, flow, correlations
  - Straightforward calculation
- Hydrodynamics: Initial conditions ⊗ dynamical equations ⊗ freeze-out conditions



$N_1(p_t,\varphi,\eta)$

$\mathrm{HBT}(p_t,\varphi,\eta)$

How analytic hydro works

Hydro equations + EoS

$$(\partial_t + \nabla \vec{v})n = 0$$

$$(\partial_t + \nabla \vec{v})T = -p\nabla \vec{v}$$

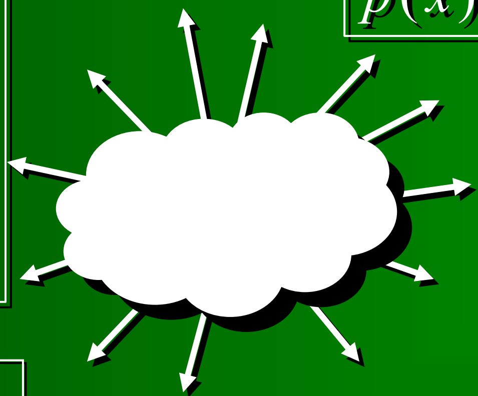
$$(\partial_t + \vec{v}\nabla)\vec{v} = -(1/n)\nabla p$$

$$T = \tfrac{3}{2}\, p$$

PLB505:64-70,2001
hep-ph/0012127

Self-similar solution:

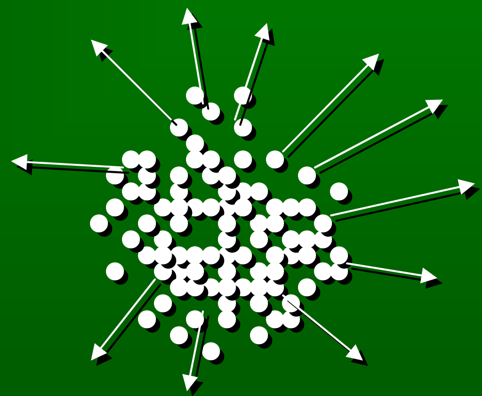
$$p(x), n(x), T(x), \vec{v}(x)$$



Phase-space distribution
Boltzmann-equation

$$f(x,p) \sim n(x)\exp\left\{-\frac{(p-mv)^2}{kT}\right\}$$

$$(\partial_t + \vec{v}\nabla) f(x,p) = S(x,p)$$



PRC67:034904,2003
hep-ph/0108067

Source

$S(x,p)$

PRC54:1390-1403,1996
hep-ph/9509213

Observables

$N_1(p)$, $C_2(p_1,p_2)$, $v_2(p)$

Scheme works also backwords

The source function

- Source function: probability of a particle created at x with p
- Maxwell-Boltzmann distribution + extra terms

$$S(x,p)d^4x = \mathcal{N} n(x) \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right]\frac{p_\mu u^\mu}{u^0} H(\tau) d^4x$$

- $\mathcal{N}$ normalization
- $H(\tau)d\tau$ freeze-out distribution
  if sudden: $H(\tau) = \delta(\tau - \tau_0)$
- $\frac{p_\mu u^\mu}{u^0} d^3x$ Cooper-Fry prefactor (flux term)
- Validity: $\tau_0 > R_{HBT}$, $m_t > T_0$

Single particle spectrum

- Source function: spatial origin and momentum
- Momentum distribution

→ integrate on spatial coordinates: $N_1(p) = \int_{R^4} S(x,p) d^4x$

- Second order Gaussian approximation around emission maximum
- After integration:

$$N_1(p) = \bar{N} \cdot \bar{E} \cdot \bar{V} \cdot \exp\left[\frac{p^2}{2ET_0} - \frac{E}{T_0} - \frac{p_x^2}{2ET_x} - \frac{p_y^2}{2ET_y} - \frac{p_z^2}{2ET_z} \right]$$

- Directional slope parameter:

$$T_x = T_0 + \frac{ET_0 \dot{X}_0^2}{b(T_0 - E)}$$

Transverse momentum spectrum

- Go to mid-rapidity (y=0)
- Integrate on transverse angle ϕ

$$N_1(p_t) = \bar{N}\,\bar{V}\left(m_t - \frac{p_t^2(T_{\rm eff}-T_0)}{m_t T_{\rm eff}}\right)\exp\left[-\frac{p_t^2}{2m_t T_{\rm eff}} + \frac{p_t^2}{2m_t T_0} - \frac{m_t}{T_0}\right]$$

- – The effective temperature is from the slopes:

$$T_x = T_0 + m_t \dot{X}^2 \frac{T_0}{b(T_0 - E)},\quad T_y = T_0 + m_t \dot{Y}^2 \frac{T_0}{b(T_0 - E)},$$

$$\frac{1}{T_{\rm eff}} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right)$$

The elliptic flow

- The elliptic flow can be calculated as:

$$v_2 = \frac{\int_0^{2\pi} d\phi N_1(p_t,\phi)\cos(2\phi)}{\int_0^{2\pi} d\phi N_1(p_t,\phi)}$$

- Result (similar to other models): $v_2 = \frac{I_1(w)}{I_0(w)}$

- $I_n(w)$: modified Bessel functions

$$I_n(w) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{w\cos(2\phi)}\cos(2n\phi)d\phi$$

- Where w is:

$$w = \frac{p_t^2}{4m_t}\left(\frac{1}{T_y}-\frac{1}{T_x}\right) \sim E_K \frac{\varepsilon}{T_{eff}}$$

Two-particle correlation radii

- Definition: $$C_2(p_1,p_2)=\frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_1)}$$
- From the source function: $$C_2(q,K)=1+\lambda\left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2$$
- Changing coordinates

$$q=p_1-p_2,\ K=0.5\left(p_1+p_2\right)\Rightarrow C_2(q,K)=1+\lambda\exp\left(-R_x^2q_x^2-R_y^2q_y^2-R_z^2q_z^2\right)$$

- Result: $$R_i=\frac{T_0\tau_0(T_i-T_0)}{m_tT_i}$$
- The usual scaling (same for kaons!): $$R_i^2\sim\frac{1}{m_t}$$
- Bertsch-Pratt coordinates: $$R_{out}=R_{side}=0.5\left(R_x^2+R_y^2\right)$$
- Freeze-out: τ = const. $\leftrightarrow$ $\Delta\tau = 0 \rightarrow R_{out} = R_{side}$

Famous solutions

- Landau's solution (1D, developed for p+p):
  - Accelerating, implicit, complicated, 1D
  - L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51
  - I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529
  - L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309
- Hwa-Bjorken solution:
  - Non-accelerating, explicit, simple, 1D, boost-invariant
  - R.C. Hwa, Phys. Rev. D10, 2260 (1974)
  - J.D. Bjorken, Phys. Rev. D27, 40(1983)
- Others
  - Chiu, Sudarshan and Wang
  - Baym, Friman, Blaizot, Soyeur and Czyz
  - Srivastava, Alam, Chakrabarty, Raha and Sinha

Virtual to real photon ratio

The yields of virtual photons dN_{γ^*} and electron pairs dN_{ee} are related:

$$\frac{d^2N_{ee}}{dM^2} = \frac{\alpha}{3\pi}\frac{L(M)}{M^2}dN_{\gamma^*}, \tag{B1}$$

$$L(M) = \sqrt{1-\frac{4m_e^2}{M^2}}\left(1+\frac{2m_e^2}{M^2}\right), \tag{B2}$$

$$\frac{d^2N_{ee}}{dM^2} = \frac{\alpha}{3\pi}\frac{L(M)}{M^2}S(M,q)dN_{\gamma}. \tag{B3}$$

Here we have introduced $S(M,q) = dN_{\gamma^*}(M)/dN_\gamma$ to factor out the difference between real photon emission and virtual photon emission. The factor $S(M,q)$ is pro-

tors, phase space, and spectral functions. $S(M,q)$ approaches 1 for small M, $S(M,q)\to 1$ for $M\to 0$. Additionally, since $L(M)\simeq 1-6m_e^4/M^4$ for $m_e \ll M$, $L(M)=1$ is a very good approximation. Thus the relationship between the electron pair yield and the direct photon yield simplifies to

$$\frac{d^2N_{ee}}{dM^2} \simeq \frac{\alpha}{3\pi}\frac{1}{M^2}dN_\gamma, \tag{B4}$$

$$\frac{d^2N_{ee}}{dM} \simeq \frac{2\alpha}{3\pi}\frac{1}{M}dN_\gamma. \tag{B5}$$

$$\frac{d^2n_{ee}}{dm_{ee}} = \frac{2\alpha}{3\pi}\frac{1}{m_{ee}}\sqrt{1-\frac{4m_e^2}{m_{ee}^2}}\left(1+\frac{2m_e^2}{m_{ee}^2}\right)S\,dn_\gamma \tag{1}$$