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26 - 28 May 2010

Transport coefficients for hadron matter, holography and phase transitions

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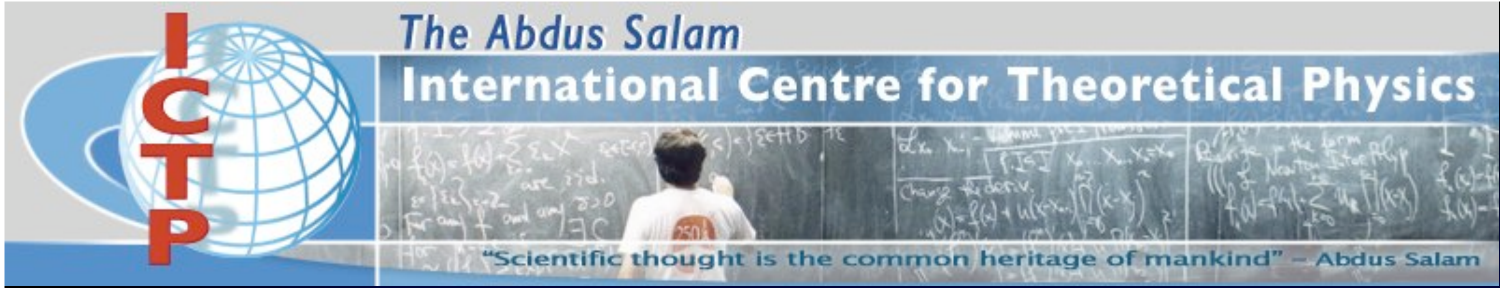

Transport coefficients for hadron matter, holography and phase transitions

Antonio Dobado

with J. Llanes-Estrada and J. M. Torres-Rincon

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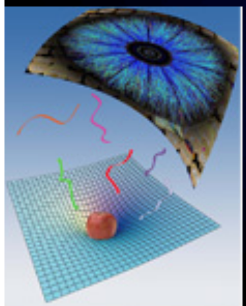
GRIBOV-80 MEMORIAL WORKSHOP ON QUANTUM CHROMODYNAMICS AND BEYOND



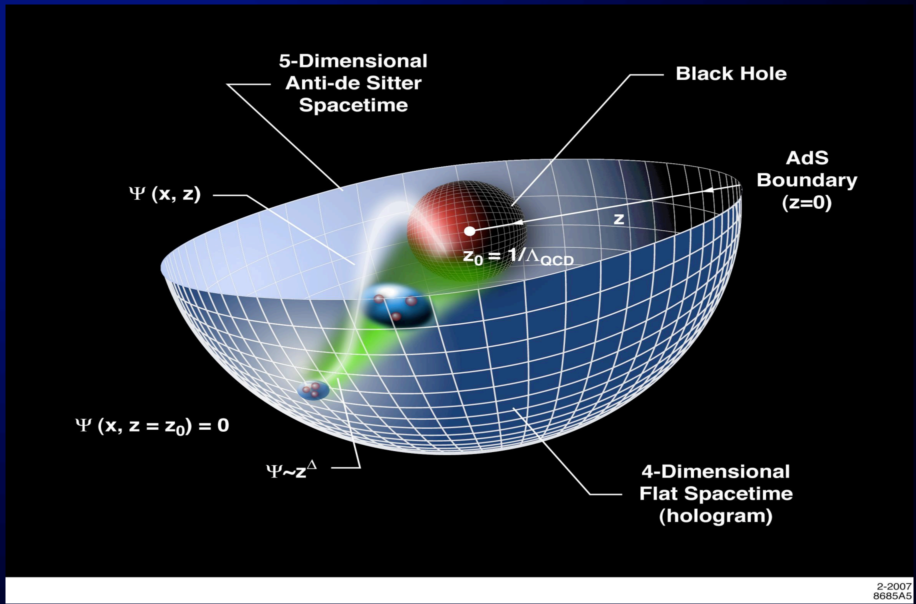
26 May-28 May 2010

Miramare, Trieste, Italy

Outline

- Holography and the Maldacena conjecture
- Computation of η/s from the AdS/CFT correspondence
- The Kovtun, Son, Starinets bound
- The RHIC case
- Could the KSS bound be violated?
- η/s and the phase transition
- Conclussions and open questions

- Holography and the Maldacena conjecture



Estates the equivalence of theories defined in different dimensions:

Field Theory

Gravity Theory

N=4 SU(N) Yang Mills theory in 3+1 dimensions

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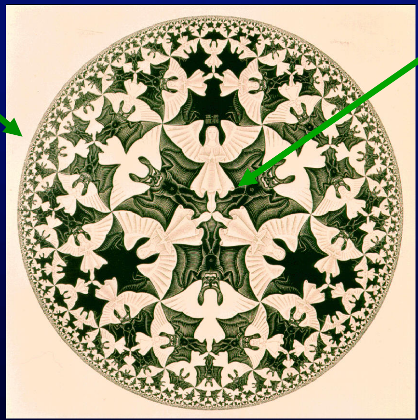
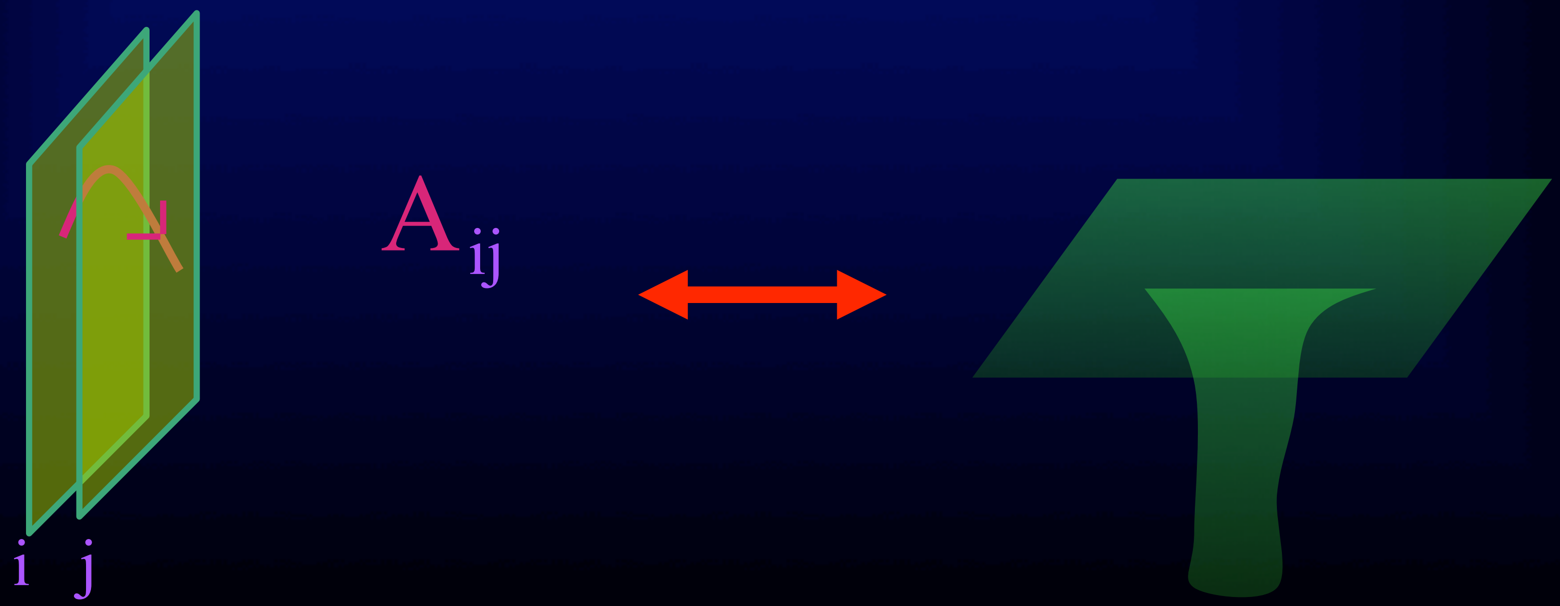
Type IIB string theory in $AdS_5 \times S_5$



Dp-branes in string theory

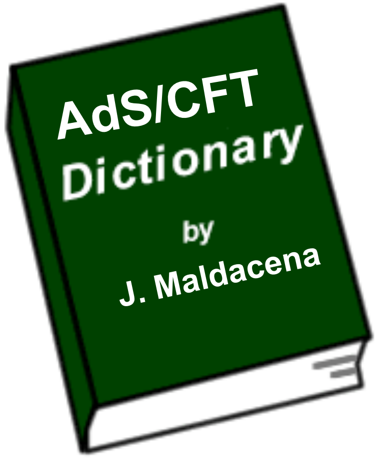
Polchinski 95

- The low energy excitations of a Type IIB string theory in the presence of a set of *N* 3D-branes can be described in two very different ways



Mapping of parameters

- Parameters of gauge theory g, N; 't Hooft coupling g^2N.
- String theory side has three parameters
  - String length ℓ_s:
  - String coupling $g_{\rm st}$
  - Curvature of space R



Mapping between parameters:

$$
\begin{aligned}
g^2 &= 4\pi g_{\rm st} \\
g^2 N_c &= \frac{R^4}{\ell_s^4}
\end{aligned}
$$

$g^2N_c \gg 1 \Leftrightarrow \ell_s \ll R$

Einstein gravity instead of string theory

Reliable calculation in a strongly coupled field theory through its gravity dual.

Gauge/gravity duality at finite temperature

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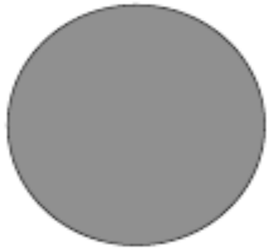
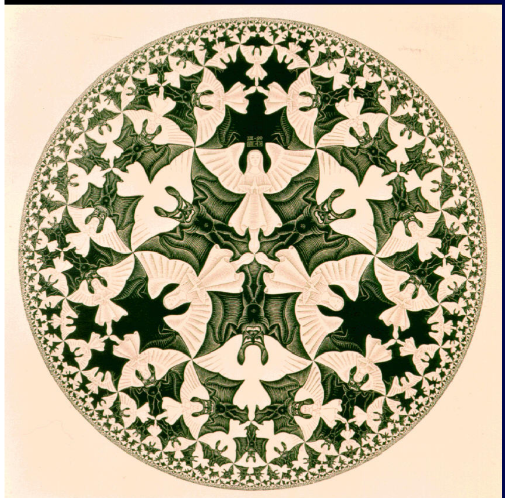


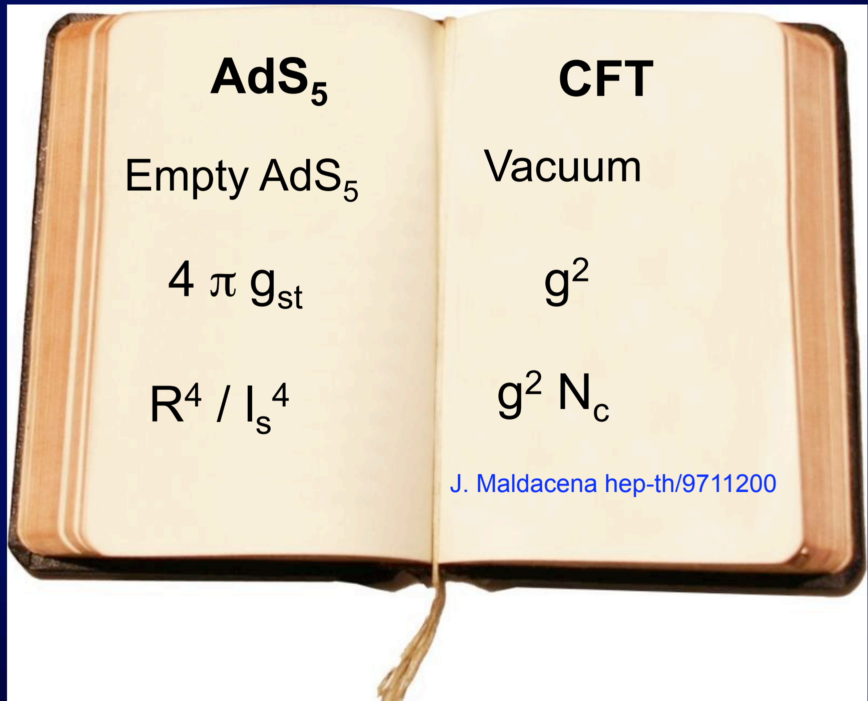
"Quark gluon plasma" = black hole (in anti de-Sitter space)

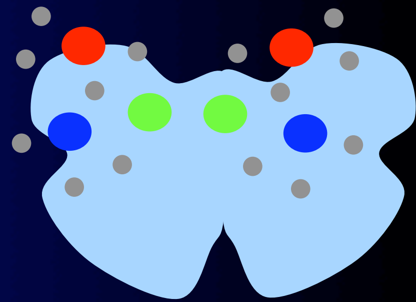
$$ds^2 = \frac{r^2}{R^2}[-f(r)dt^2 + d\mathbf{x}^2] + \frac{R^2}{r^2 f}dr^2 + R^2 d\Omega_5^2$$

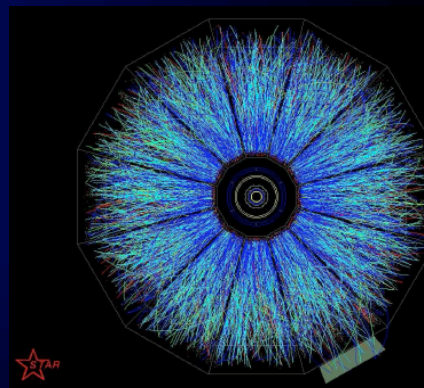
$f(r) = 1 - r_0^4/r^4, T = r_0/\pi R^2$

N=4 SYM versus AdS/CFT (T=0)

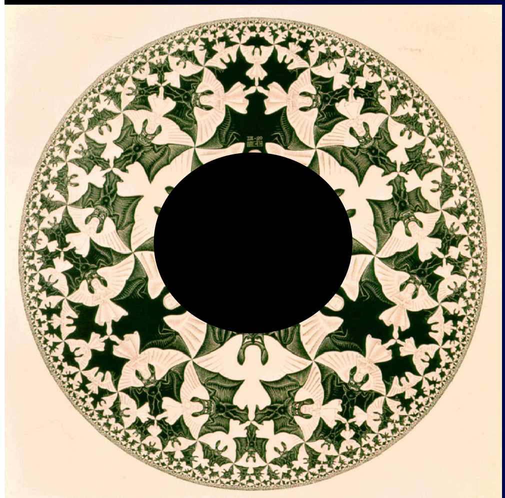


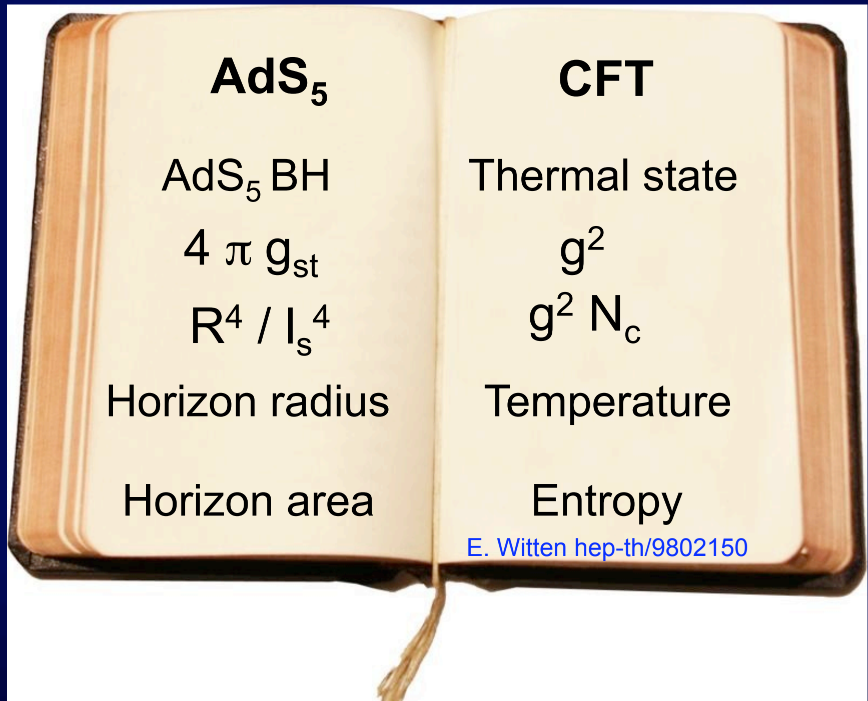


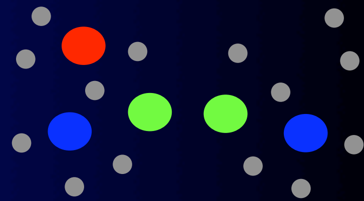


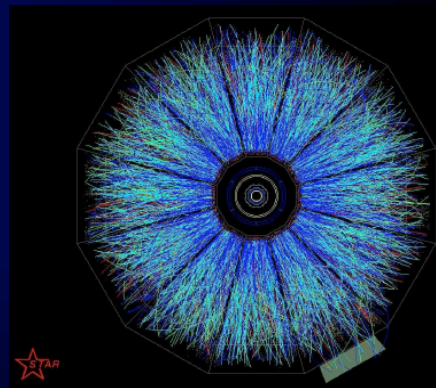


N=4 SYM versus AdS/CFT (T>0)

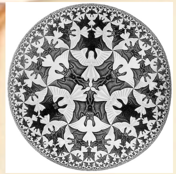


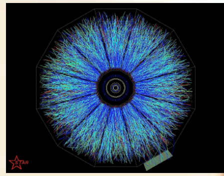






N=4 SYM versus AdS/CFT (Hydrodynamics)





AdS_5	CFT
Effective description:	
Einstein equations	Relativistic fluid dynamics
Static solutions:	
Black branes in AdS_5	Perfect fluids configurations
Perturbations:	
Non-uniformly evolving BB	Dissipative fluid flow

How about QCD?

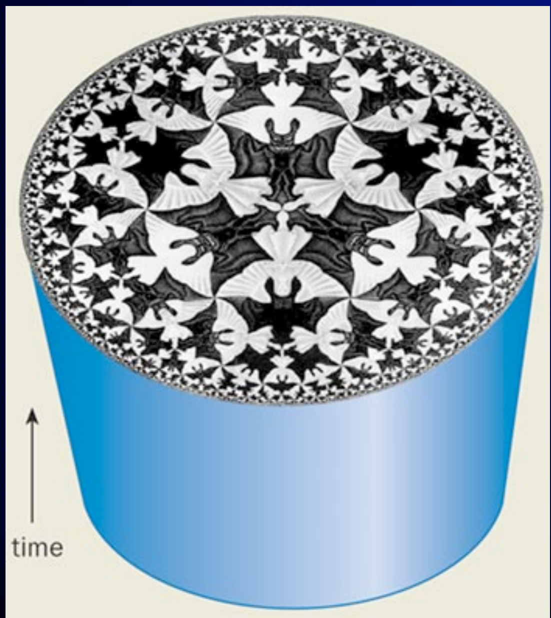
- No gravity dual for QCD
- There exist theories with gravity duals which are similar to QCD: confinement, chiral symmetry breaking
- but no asymptotic freedom

What can we do?

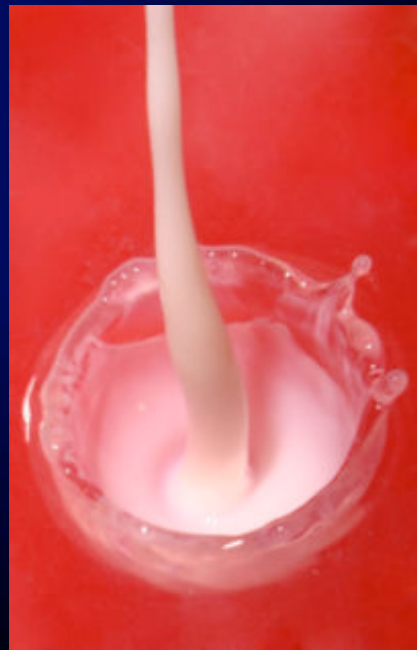
- Use the $\mathcal{N} = 4$ SYM plasma as a simplest model of a strongly coupled plasma
- Many similarities to real quark-gluon plasma: deconfinement, Debye screening
- Recently has shed light on the behavior of viscosity at strong coupling

- Computation of η/s from the AdS/CFT correspondence

Holography



Viscosity



Viscosity in Relativistic Hydrodynamics:

The effective theory desribing the dymnamics of a system (or a QFT) at large distances and time

Perfect fluids:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu j^\mu = 0$$

$$\partial_\mu (s u^\mu) = 0$$

Entropy conservation

$$d\epsilon = TdS,\ dP = sdT, \text{ and } \epsilon + P = Ts$$

for $\mu = 0$

Including dissipation:

local rest frame (Landau-Lifshitz)

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

$$u^i(x) = 0 \qquad T^{00} = \epsilon, \qquad T^{0i} = 0$$

$$\sigma_{ij} = \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u^k \right) + \zeta \delta_{ij} \partial_k u^k$$

$$\sigma^{00} = \sigma^{0i} = 0$$

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \left[\eta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \partial_\lambda u^\lambda \right) + \zeta g_{\alpha\beta} \partial_\lambda u^\lambda \right]$$

$$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

projector

conserved currents

$$j^\mu = \rho u^\mu - D P^{\mu\nu} \partial_\nu \alpha$$

Fick's Law

Hydrodynamic modes:

Look for normal modes of linerized hydrodynamics } $\omega(\mathbf{k})$

complex because of dissipation

Examples: difusion law

$$\partial_t \rho - D\nabla^2 \rho = 0 \quad \Rightarrow \quad \omega = -iDk^2$$

pole in the current-current Green function

From the energy-momentum tensor equation:

$$\omega = -i\frac{\eta}{\epsilon + P}k^2$$

Shear modes (transverse)

$$\epsilon + P = Ts \quad \Rightarrow$$

η/s is a good way to characterize the intrinsic ability of a system to relax towards equilibrium

$$\omega = c_s k - \frac{i}{2}\left(\frac{4}{3}\eta + \zeta\right)\frac{k^2}{\epsilon + P}$$

Sound mode (longitudinal)

$$c_s = \sqrt{dP/d\epsilon}$$

speed of sound

Notice that for conformal fluids $\zeta = 0$

Kinetic theory computation of viscosity:

$$f = f(\mathbf{x}, \mathbf{p}, t) \equiv f_p(x)$$

distribtuion function

$$\frac{p^\mu}{E_p}\partial_\mu f_p(x) = \frac{g_\pi}{2}\int_{123} d\Gamma_{12;3p}\left\{f_1 f_2(1+f_3)(1+f_p) - (1+f_1)(1+f_2)f_3 f_p\right\}$$

Boltzmann (Uehling-Uhlenbenck) Equation

$$1, 2 \to 3, p$$

$$f_p(x) = f_p^{(0)}(x)\left[1 - \left\{1 + f_p^{(0)}(x)\right\}\chi_p(x)\right]$$

Chapman-Enskog

$$f_p^{(0)}(x) = \left(e^{\beta(x)V_\mu(x)p^\mu} - 1\right)^{-1}$$

thermal distribution

$$\chi_p(x) = \beta(x)A(p)\nabla\cdot\mathbf{V}(x) + \beta(x)B(p)\left(\hat{p}_i\hat{p}_j - \frac{1}{3}\delta_{ij}\right)\left(\frac{\nabla_i V_j(x) + \nabla_j V_i(x)}{2} - \frac{1}{3}\delta_{ij}\nabla\cdot\mathbf{V}(x)\right)$$

$$\begin{aligned}\eta &= \frac{g_\pi\beta}{10}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_p}n_p(1+n_p)B_{ij}(p)\left(p_i p_j - \frac{1}{3}\delta_{ij}\mathbf{p}^2\right)\\ &= \frac{g_\pi^2\beta}{10}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_p}n_p(1+n_p)B_{ij}(p)\hat{F}_{ij}[B] \equiv g_\pi^2\langle B|\hat{F}[B]\rangle\end{aligned}$$

When can we apply kinetic theory?

The mean free path must be much larger than the interaction distance (two well defined lengh scales)

$$\ell_{\rm mfp} \sim \frac{1}{n\sigma v}$$

mean free path

Typically low density, weak interacting, systems

Examples:

a) For a NR system like a hard sphere gas

$$\eta = \frac{1}{3} m n \bar{v}\, \ell_{\rm mfp} = \frac{1}{3}\frac{m\bar{v}}{\sqrt{2}\sigma}$$

$$\bar{v} = \sqrt{\frac{8RT}{\pi m}} \longrightarrow \eta \;\alpha\; \sqrt{T}$$

Maxwell's Law:

b) For a relativistic system like massless $\lambda\phi^4$ QFT

$$\eta \sim \epsilon \ell_{\rm mfp}$$

$$n \sim T^3,\ \sigma \sim \lambda^2 T^{-2},\ \text{and}\ v \sim 1$$

$$\epsilon \sim T^4$$

$$\eta \sim \frac{T^3}{\lambda^2}$$

The stronger the interaction the lesser the viscosity

The Kubo formula for viscosity in QFT:

Linear response theory

$$S = S_0 + \int_x J_a(x) O_a(x)$$ coupling operators to an external source

then $$\langle O_a(x) \rangle = -\int_y G^R_{ab}(x-y) J_b(y)$$ $$iG^R_{ab}(x-y) = \theta(x^0 - y^0) \langle [O_a(x), O_b(y)] \rangle$$

retarded Green function

Consider the energy-momentum tensor as our operator coupled to the metric:

$$\begin{aligned} g_{ij}(t, \mathbf{x}) &= \delta_{ij} + h_{ij}(t), \qquad h_{ij} \ll 1 \\ g_{00}(t, \mathbf{x}) &= -1, \qquad g_{0i}(t, \mathbf{x}) = 0\,. \end{aligned}$$

for curved space-time

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \left[\eta (\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) + \left(\zeta - \frac{2}{3} \eta \right) g_{\alpha\beta} \nabla \cdot u \right] \Rightarrow \sigma_{xy} = 2\eta \Gamma^0_{xy} = \eta \partial_0 h_{xy}$$

by comparison with linear response theory

$$G^R_{xy,xy}(\omega, 0) = \int dt\, d\mathbf{x}\, e^{i\omega t} \theta(t) \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle = -i\eta\omega + O(\omega^2)$$

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, 0)] \rangle$$

The Kovtun-Son-Starinets (KSS) holographic computation

Consider a QFT with gravitational dual

$$ds^2 = g^{(0)}_{MN} dx^M dx^N$$
$$= f(\xi)(dx^2 + dy^2) + g_{\mu\nu}(\xi) d\xi^\mu d\xi^\nu$$

For example for $\mathcal{N} = 4$ $SU(N_C)$ SYM

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + dx^2 + dy^2 + dz^2\right] + \frac{R^2}{r^2(1 - r_0^4/r^4)} dr^2,$$

The dual theory is a QFT at a temperature *T* which equals the Hawking temperature of the black-brane.

$$S = \frac{A}{4G}$$

$$\hbar = c = k_B = 1$$

natural units

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle$$

Consider a graviton polarized in the x-y direction and propagating perpendiculary to the brane

Klebanov

In the dual QFT the absorption cross-section of the graviton by the brane measures the imaginary part of the retarded Green function of the operator coupled to the metric i.e. the energy-momentum tensor

$$\sigma_{\mathrm{abs}}(\omega) = -\frac{2\kappa^2}{\omega} \mathrm{Im}\, G^{\mathrm{R}}(\omega)$$
$$= \frac{\kappa^2}{\omega} \int dt d\mathbf{x} e^{i\omega t} \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle,$$

Then we have

$$\eta = \frac{\sigma_{\mathrm{abs}}(0)}{2\kappa^2} = \frac{\sigma_{\mathrm{abs}}(0)}{16\pi G}$$

The absorption cross-section can be computed classically

$$g_{MN} = g^{(0)}_{MN} + h_{MN}$$

$h_{xy} = h_{xy}(\xi)$ only non-vanishing component, x and y independent

$$R_{MN} = T_{MN} - \frac{T}{D-2} g_{MN}$$

Einstein equations

$$T_{\alpha\beta} - \frac{T}{D-2} g_{\alpha\beta} = -\left(\mathcal{L} + \frac{T^{(0)}}{D-2}\right)(\delta_{\alpha\beta} f + h_{\alpha\beta})$$

$(\alpha, \beta = x, y)$

$$T_{MN} = -g_{MN}\mathcal{L} + \cdots$$

$$R_{xy} = -\frac{1}{2}\Box h_{xy} + \frac{1}{f}\partial^\mu f \partial_\mu h_{xy} - \frac{(\partial f)^2}{2f^2} h_{xy}$$
$$= -\left(\mathcal{L} + \frac{T^{(0)}}{D-2}\right) h_{xy}.$$

$$h^x_y = h_{xy}/f$$

$$\Box h^x_y = 0$$

Linearized Einstein equations

$$\Box h_{xy} - 2\frac{\partial^\mu f}{f}\partial_\mu h_{xy} + 2\frac{(\partial f)^2}{f^2} h_{xy} - \frac{\Box f}{f} h_{xy} = 0$$

Equation for a minimally coupled scalar

Theorem: Das, Gibbons, Mathur, Emparan

For $\omega \to 0$ The scalar cross-section is equal to the area of the horizon $\sigma_{\rm abs} = a$

but $s = a/4G$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

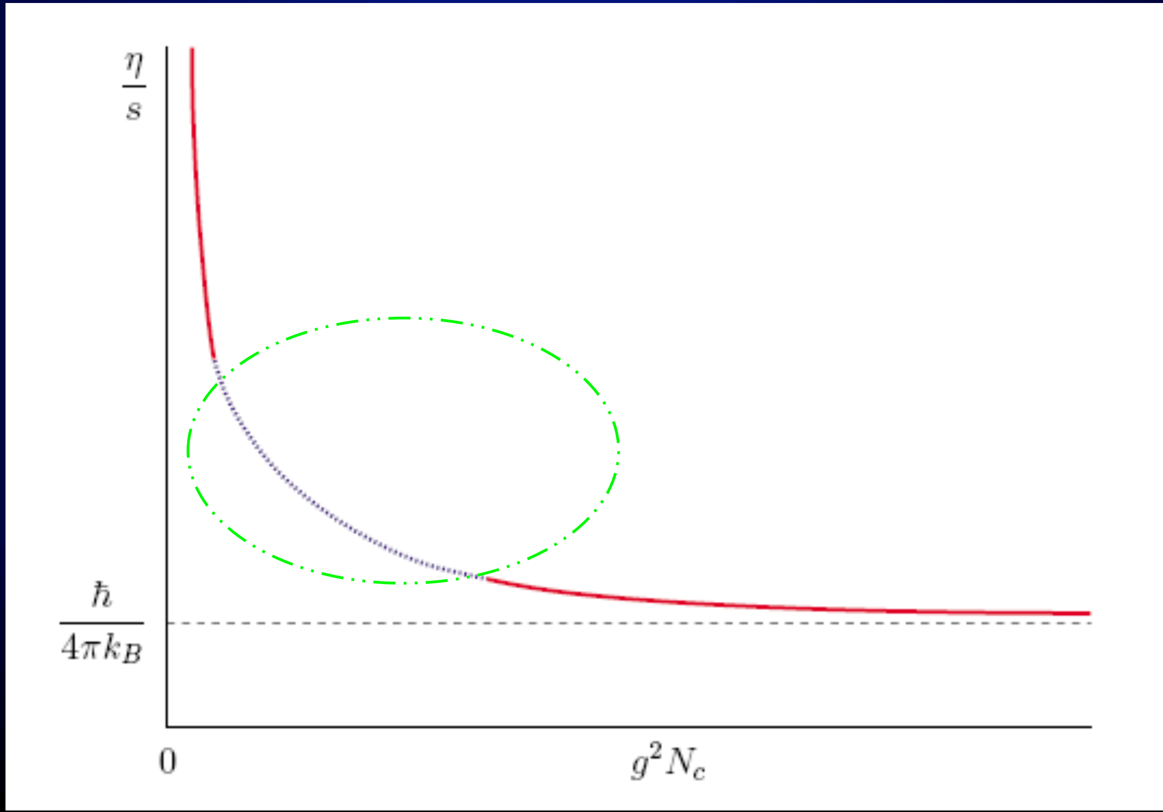
Notice that the result does not depend on the particular form of the metric. It is the same for Dp, M2 and M5 branes. Basically the reason is the universality of the graviton absortion cross-section.

Computation of the leading corrections in inverse powers of the 't Hooft coupling for $\mathcal{N} = 4$ $SU(N_C)$ SYM

Buchel, Liu and Starinets

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}\left[1 + \frac{135\zeta(3)}{8(2g^2N_c)^{3/2}} + \cdots\right],$$

$\zeta(3) \approx 1.2020\,569\ldots$ Apéry´s constant



- The Kovtun, Son, Starinets bound

$\eta/s \geq 1/4\pi$?

An interesting conjecture: The KSS bound

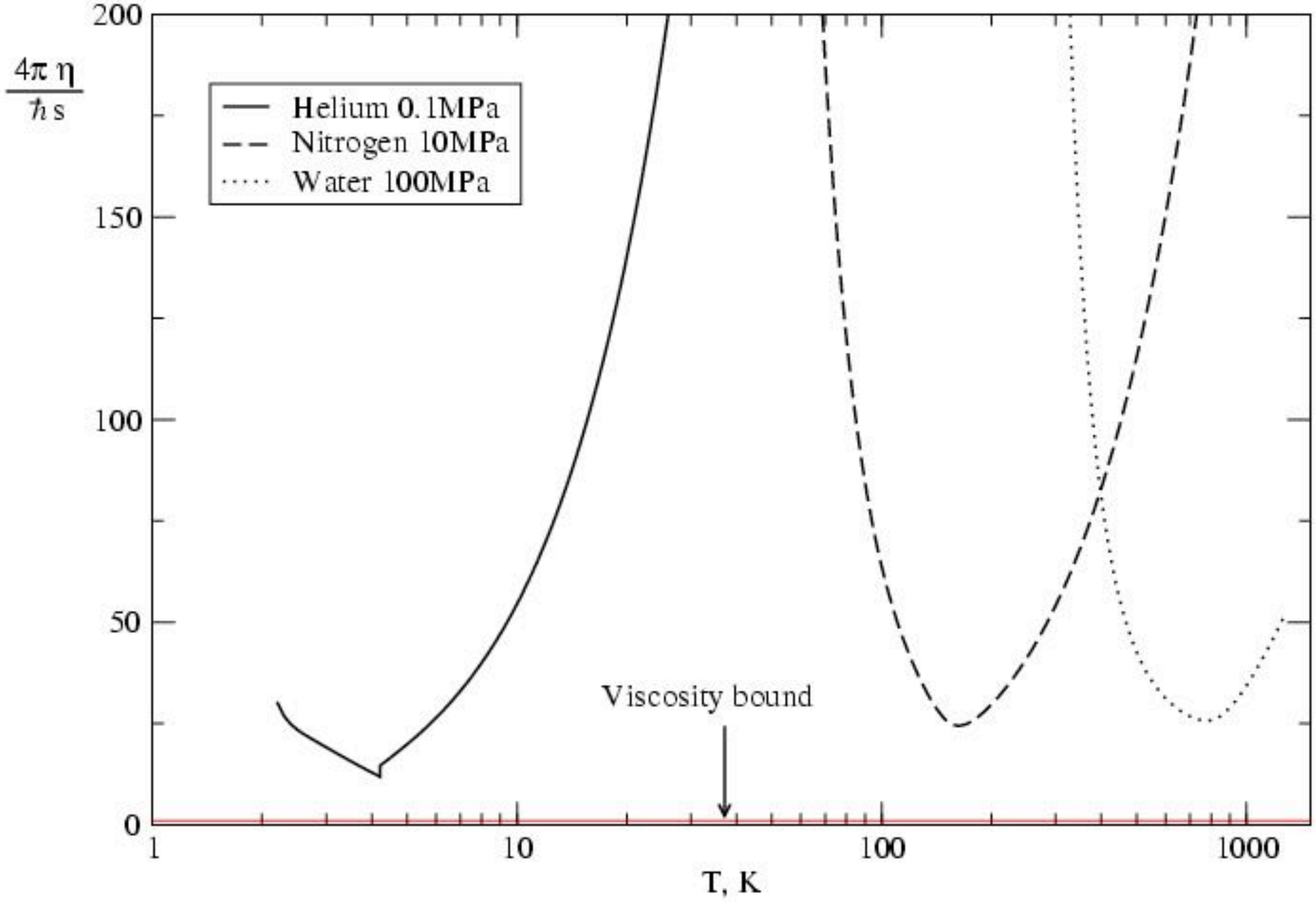
$$\eta/s \geq 1/4\pi$$

For any system described by a sensible* QFT

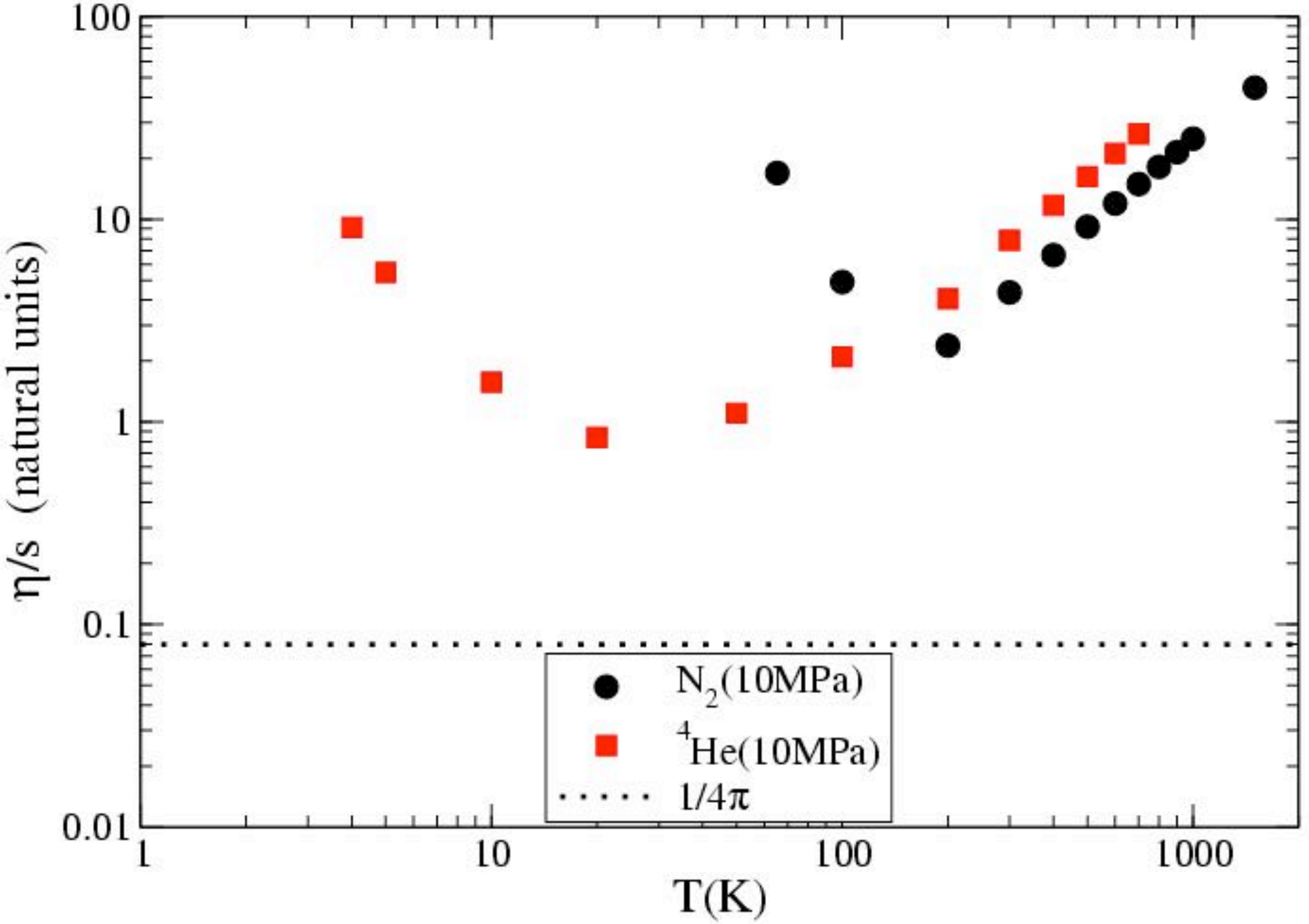
Kovtun, Son and Starinets, PRL111601(2005)

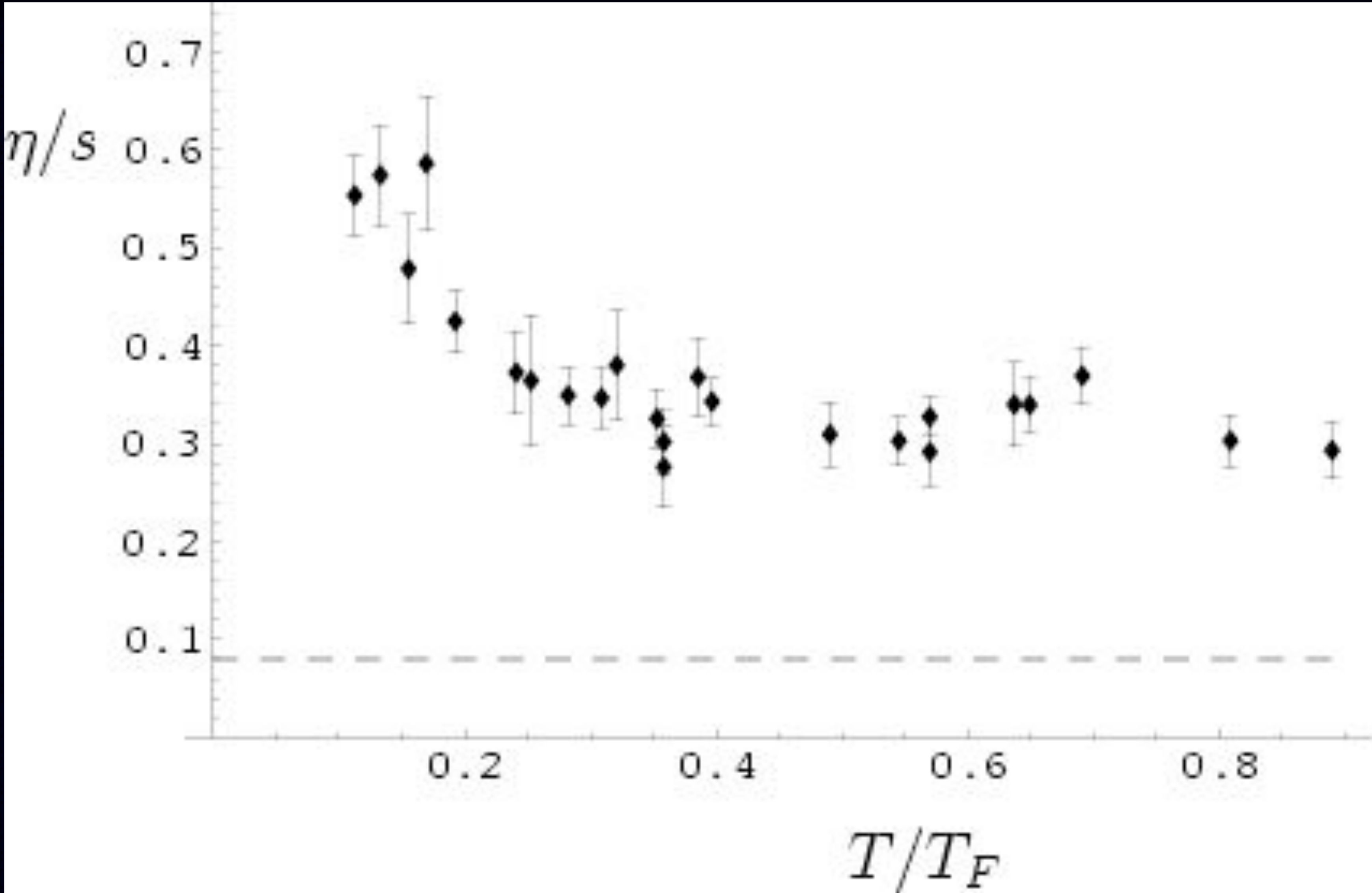
*Consistent and UV complete.

This bound applies for common laboratory fluids



η/s for various systems

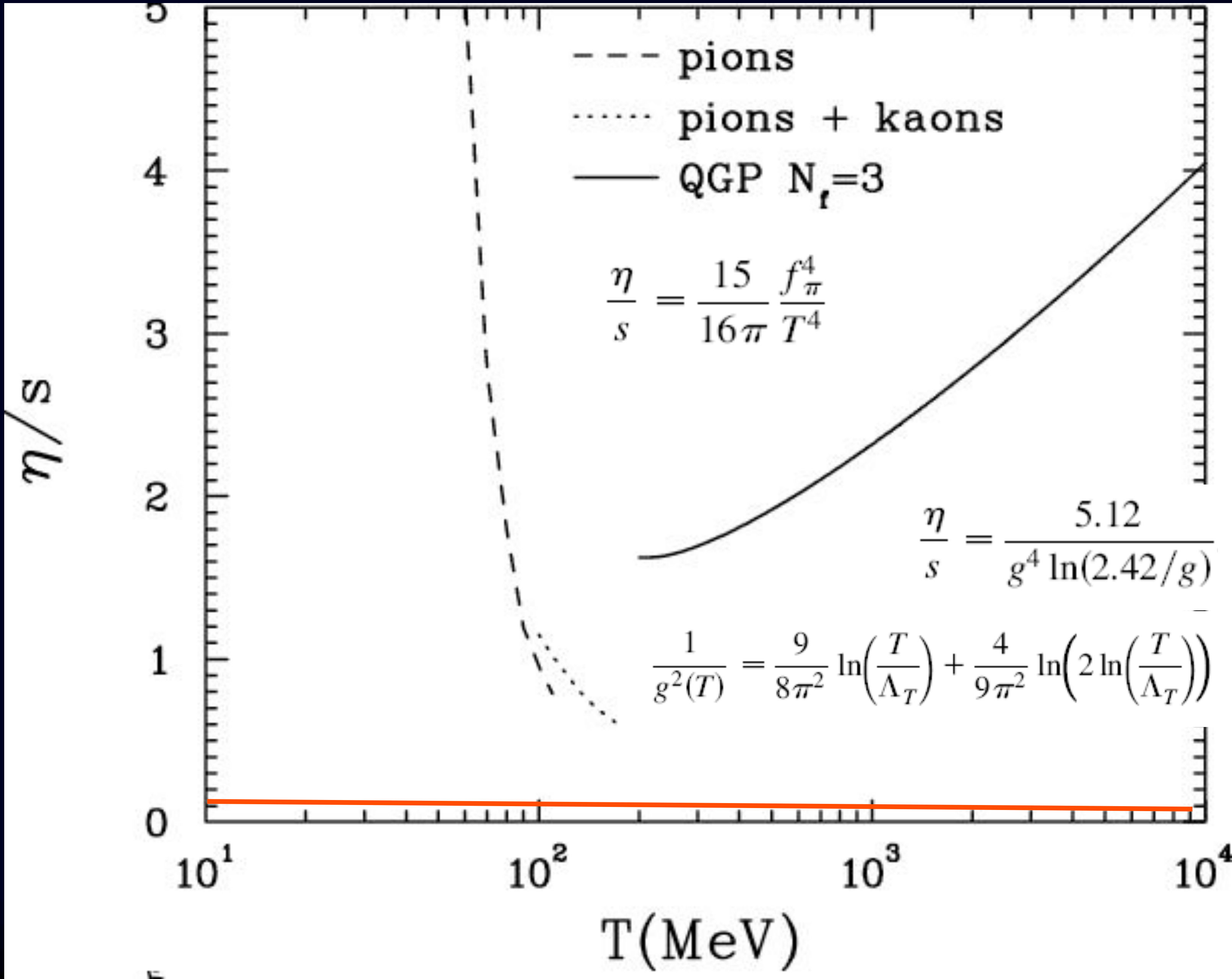




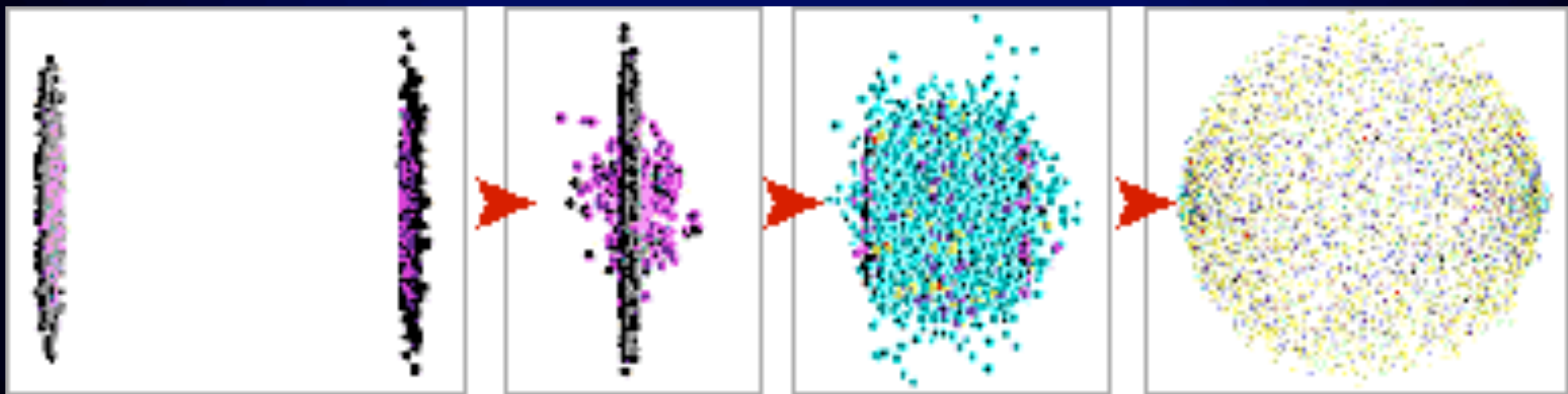
Trapped atomic gas

T. Schaefer, cond-mat/0701251

Hadronic matter

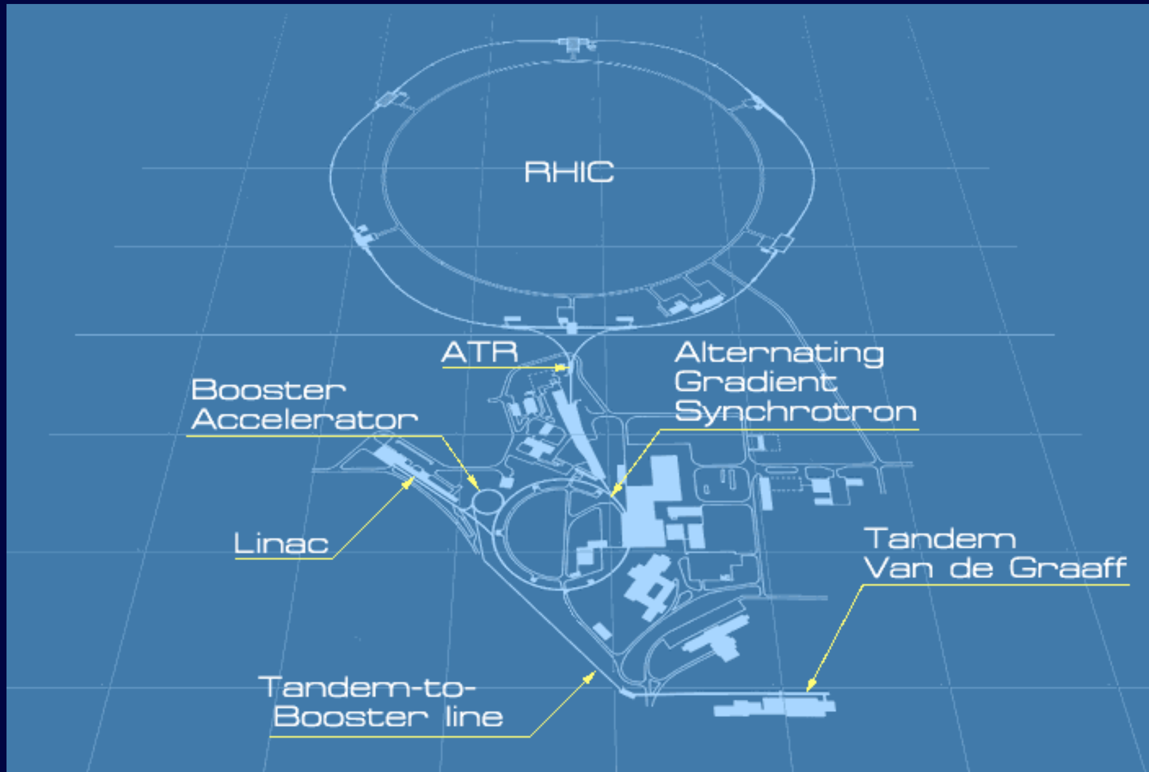


• The RHIC case



RHIC: Relativistic Heavy Ion Collider

(The largest viscosimeter)



Length = 3.834 m

E_{CM} = 200 GeV A

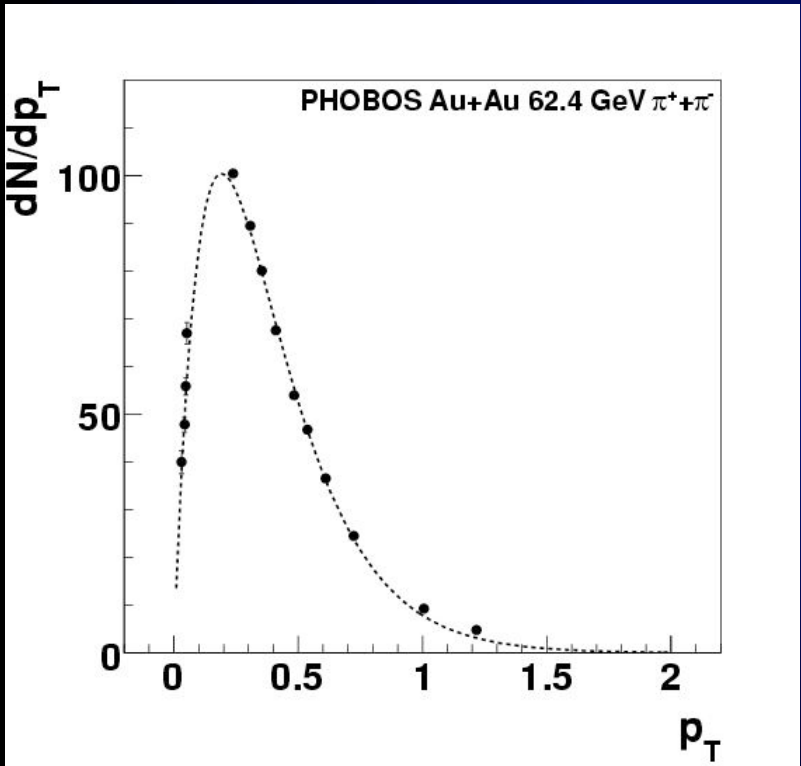
$L = 2\ 10^{26}$ cm^{-2} s^{-1}

(for Au+Au (A=197))

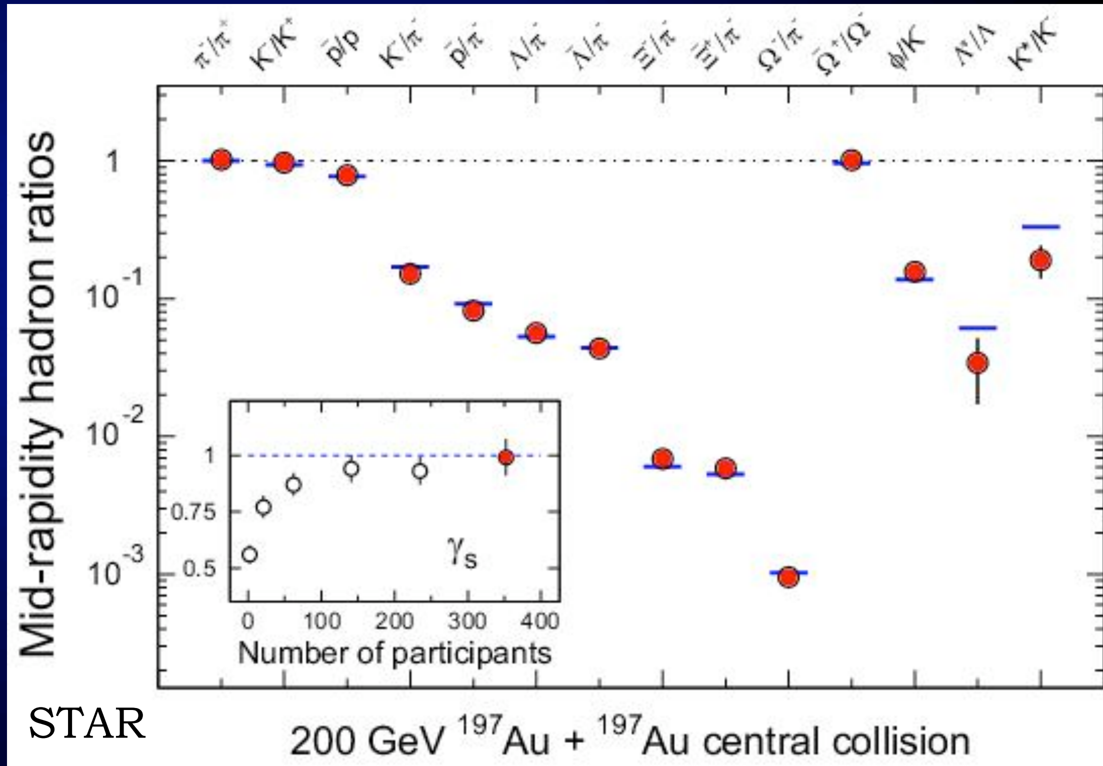
Experiments:

Some important results from RHIC

Thermochemical models describes well the different particle yields for T=177 MeV, μ_B = 29 Mev for E_{CM} = 200 GeV

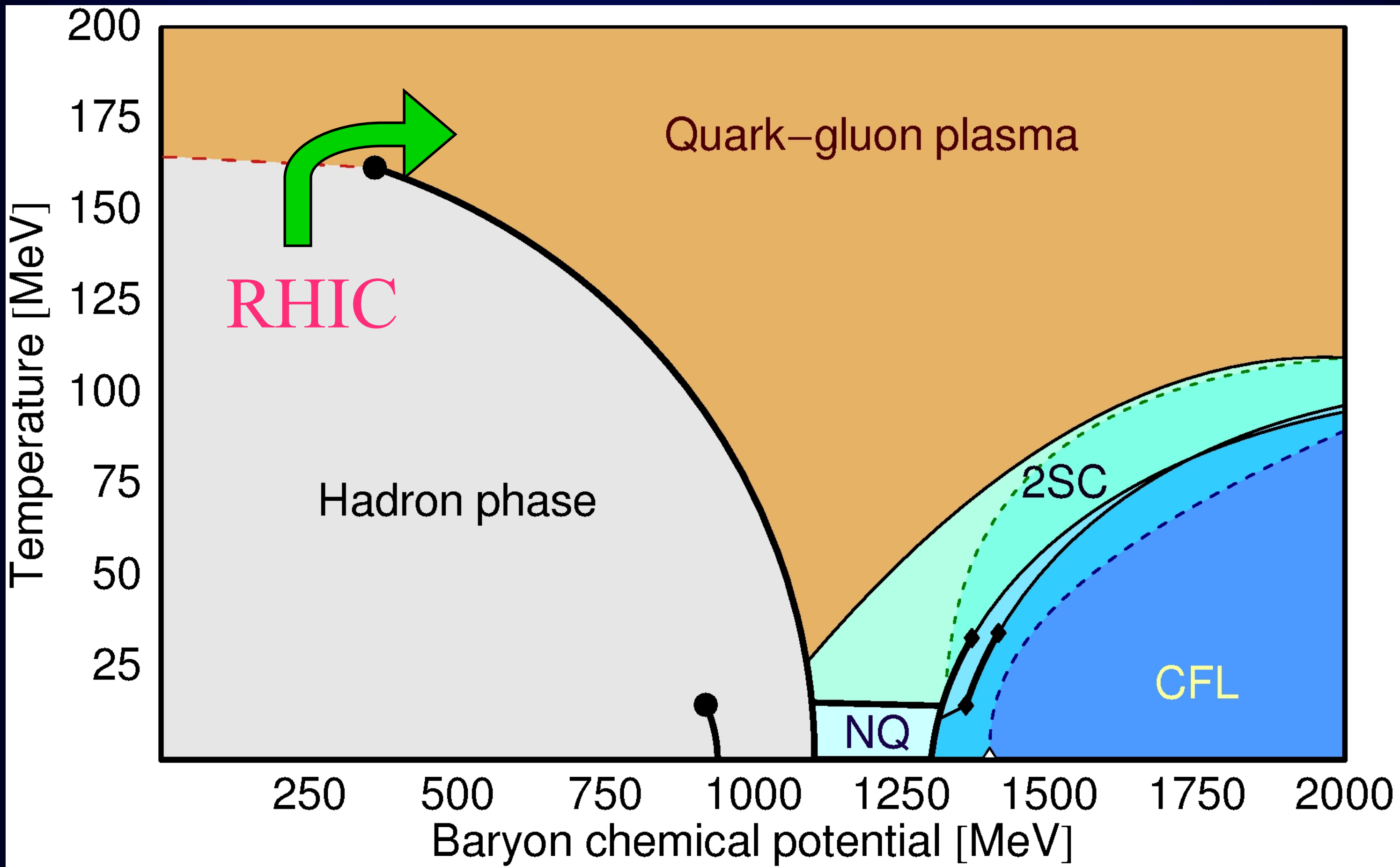


Bose-Einstein spectrum as an indication of thermal equilibrium



Thermochemical model of hadron ratios

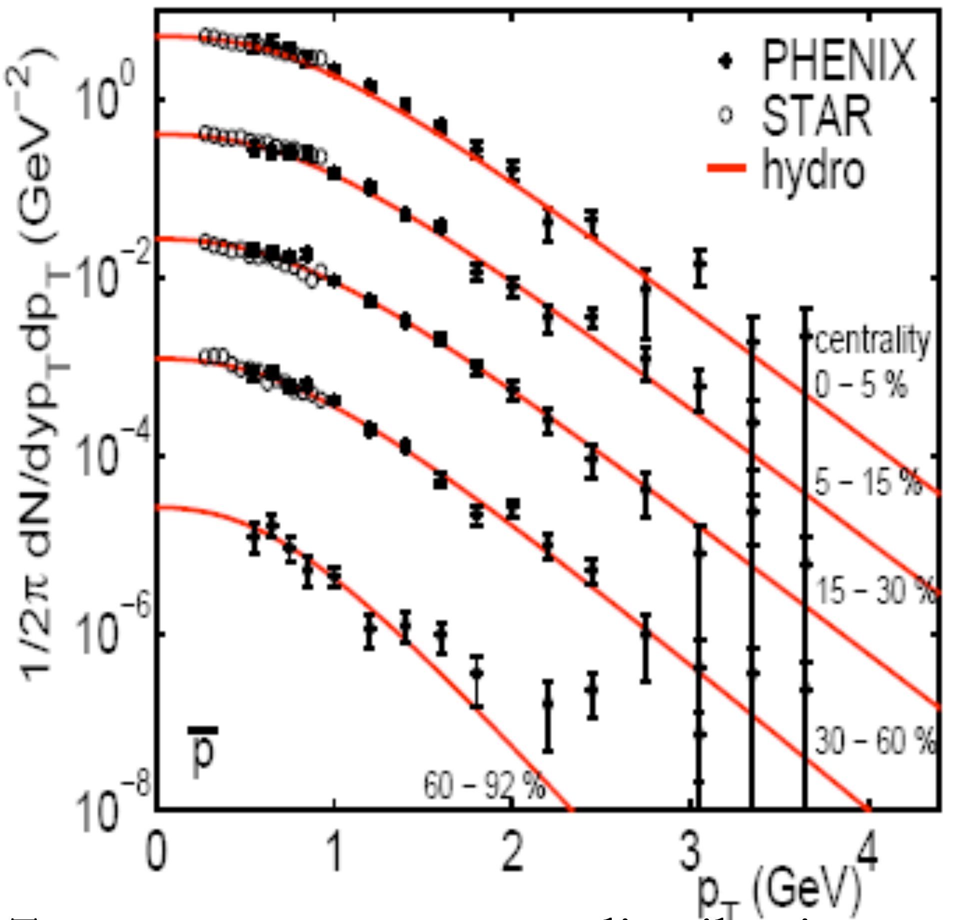
From the observed transverse/rapidity distribution the Bjorken model predicts an energy density at $\tau_0 = 1$ fm of 4 GeV fm^{-3} whereas the critical density is about 0.7 GeV fm^{-3}, i.e. matter created may be well above the threshold for QGP formation.



Expected phase diagram for hadron matter

Ruester et al

A surprising amount of collective flow is observed in the outgoing hadrons, both in the single particle transverse momentum distribution (radial flow) and in the asymmetric azimuthal distribution around the beam axis (elliptic flow).

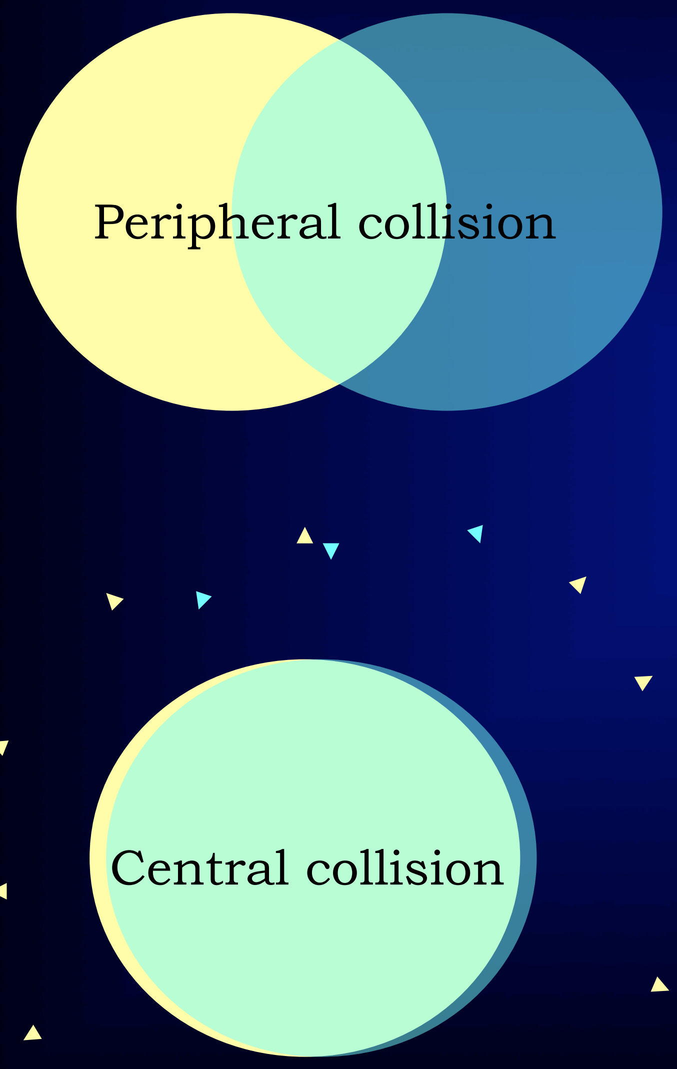


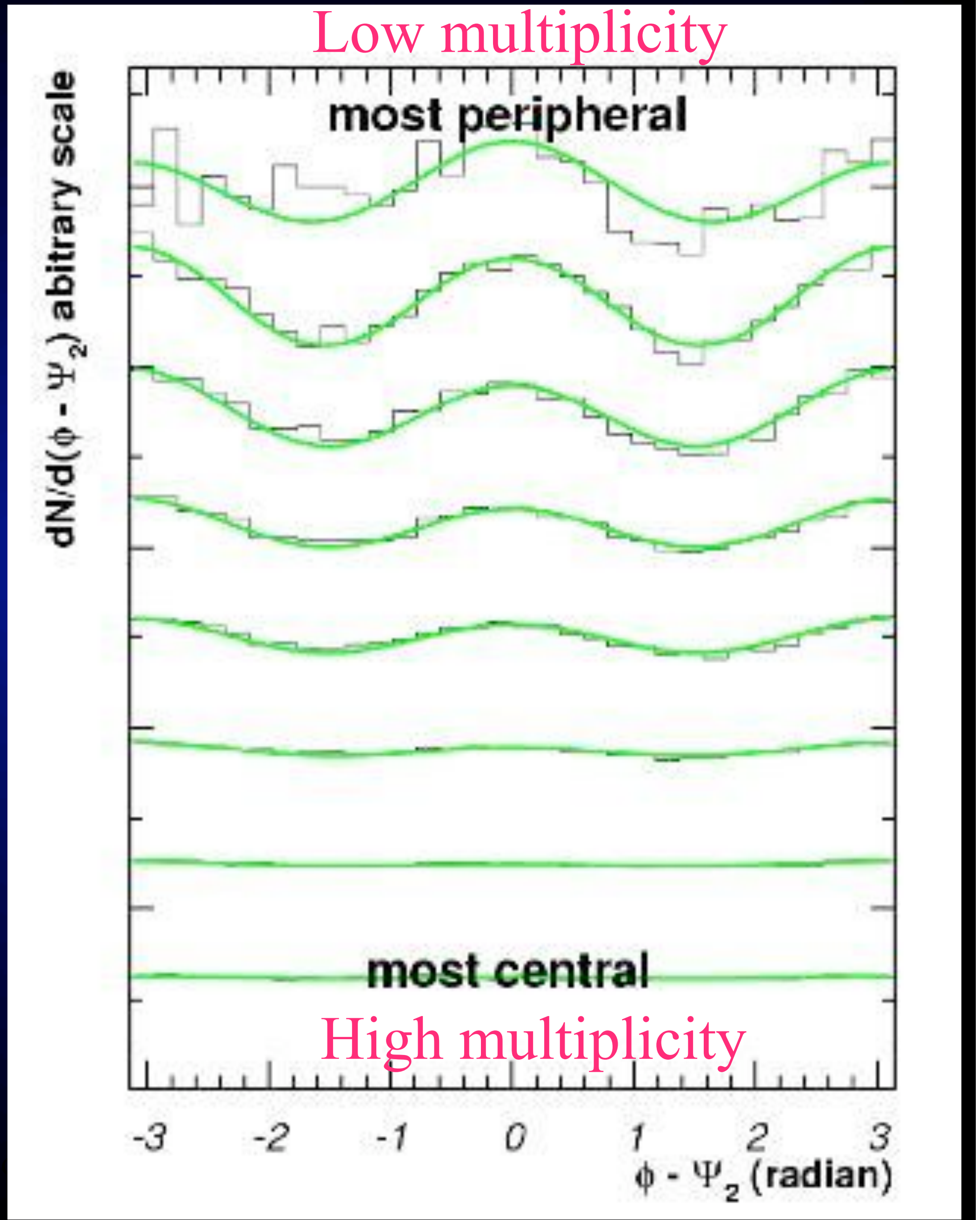
Transverse momentum distribution

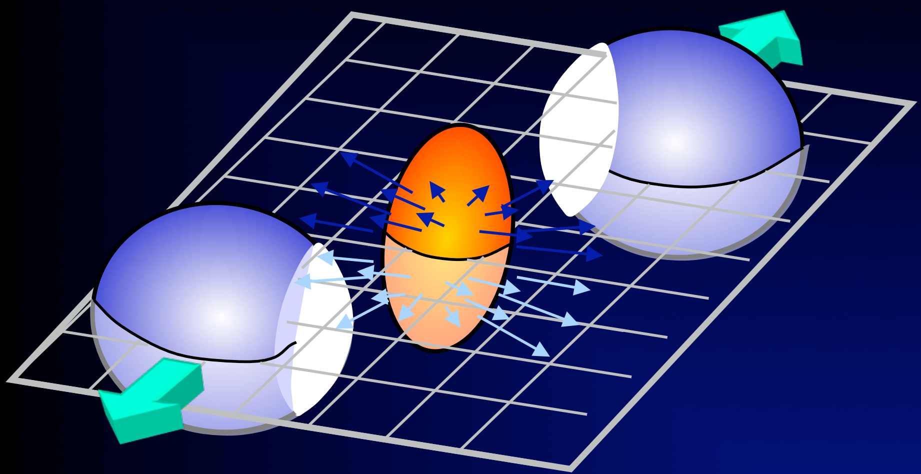
Kolb and Rapp

Hydrodynamical model for proton production

Elliptic flow







Peripheral collision:
In order to have anisotropy (elliptic flow) the hydrodynamical regime has to be stablished in the overlaping region.

$$l \ll L$$

Expect Large Pressure Gadients → Hydro Flow

Viscosity would smooth the pressure gradient and reduce elliptic flow

$$\frac{dN}{d\phi} = \frac{v_0}{2\pi} + \frac{v_2}{\pi}\cos(2\phi) + \frac{v_4}{\pi}\cos(4\phi) + \cdots$$

$$\frac{\langle P_x\rangle^2 - \langle P_y\rangle^2}{\langle P_x\rangle^2 + \langle P_y\rangle^2} = v_2$$

Found to be much larger than expected at RICH → **Low Viscosity!**

Main conclussions from RHIC

Fluid dynamics with very low viscosity reproduces the measurements of radial and elliptic flow up to transverse momenta of 1.5 GeV.

Collective flow is probably generated early in the collision probably in the QGP phase before hadronization.

The QGP seems to be more strongly interacting than expected on the basis of pQCD and asymptotic freedom (hence low viscosity).

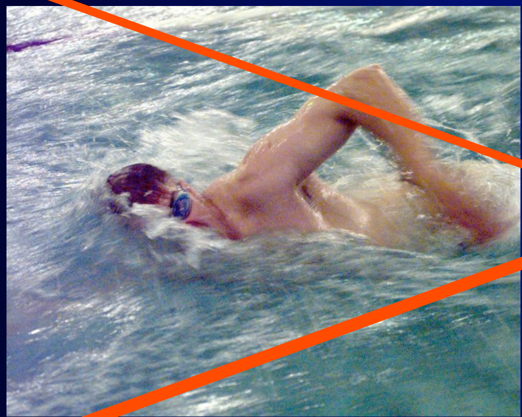
Some estimations of η/s based for example on elliptic flow (Teaney, Shuryak) and transverse momentum correlations (Gavin and Abdel-Aziz) seems to be compatible with value close to 0.08 (the KSS bound)

- Could the KSS bound be violated?

$$\eta/s \geq 1/4\pi \ ?$$

If the KSS conjecture is correct there is no perfect fluids in Nature. Is this physically acceptable?

In non relativistic fluid dynamics is well known the d'Alembert paradox (an ideal fluid with no boundaries exerts no force on a body moving through it, i.e. there is no lift force. Swimming or flying impossible).



More recently, Bekenstein et al pointed out that the accreation of an ideal fluid onto a black hole could violate the Generalized Second Law of Thermodynamics suggesting a possible connection between this law and the KSS bound.

Is it possible to violate the KSS bound?

Lower η ⟵ η/s ⟶ Increase s

↓ Larger σ

↓ Larger N

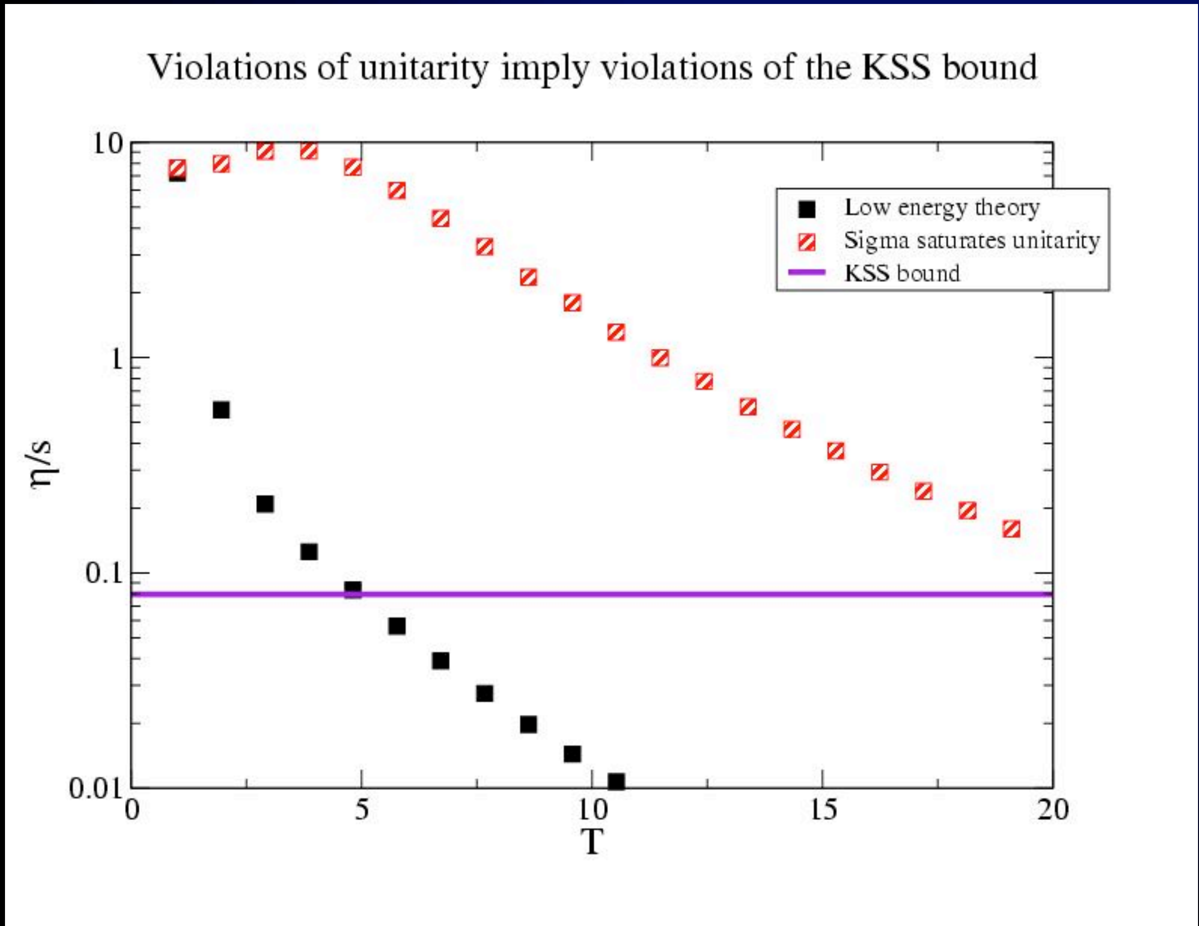
Or modifying the low energy effective theory for gravity on the bulk

Increasing the cross-sections is forbidden by unitarity:

Example: LSM

$$\mathcal{L} = \frac{1}{2}\pi^a_{,\mu}\pi^{a,\mu} + \frac{1}{2}h_{,\mu}h^{\mu} - \frac{M^2}{2}h^2$$
$$-\lambda(\pi^2 + h^2)^2 - 4\lambda v h(\pi^2 + h^2))$$

Kinetic computation



$$|\bar{T}|^2 = \frac{1}{9v^4}\left(4s^2 + 8(t^2 + u^2) + 4tu\right)$$

Low-energy-approximation (violates unitarity at higher energies)

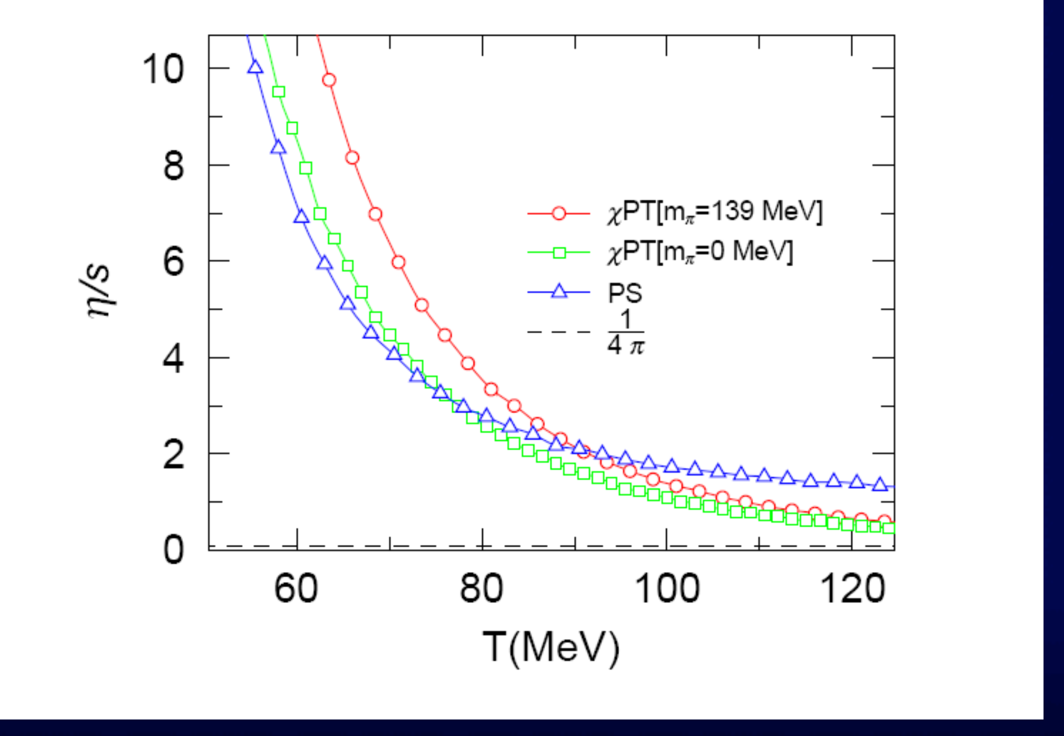
$$s/v^2 \rightarrow \frac{8\lambda s}{M^2 - iM\Gamma - s}$$

Reintroducing the Higgs reestablishes unitarity and the KSS bound

Example: Hadronic matter ($\mu_B = 0$)

Lowest order Quiral Perturbation Theory (Weinberg theorems)

$$|\mathcal{T}|^2 = \frac{1}{9}\sum_{I=0,1,2}(2I+1)|\mathcal{T}^{(I)}|^2 = \frac{1}{9f_\pi^4}\left\{21m_\pi^4 + 9s^2 - 24M_\pi^2 s + 3(t-u)^2\right\}$$



Cheng and Nakano

Violation of the bound about T = 200 MeV

Quiral Perturbation Theory (Momentum and mass expansion)

$$t_{IJ} \simeq t^{(0)}_{IJ} + t^{(1)}_{IJ} + t^{(2)}_{IJ} + \ldots$$

$$\begin{aligned} \mathrm{Im} t^{(0)}_{IJ} &= 0 \\ \mathrm{Im} t^{(1)}_{IJ} &= \sigma_{\alpha\beta} t^{(0)2}_{IJ} \\ \mathrm{Im}(t^{(2)}_{IJ} + t^{(1)}_{IJ}) &= \sigma_{\alpha\beta}\left(t^{(0)2}_{IJ} + 2t^{(0)}_{IJ}\mathrm{Re}t^{(1)}_{IJ}\right) \simeq \sigma_{\alpha\beta} \mid t^{(0)}_{IJ} + t^{(1)}_{IJ} \mid^2 \end{aligned}$$

The Inverse Amplitude Method

$$t_{IJ}(s) = C_0 + C_1 s + C_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t_{IJ}(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(t_{IJ})$$

$$\begin{aligned} t^{(0)}_{IJ} &= a_0 + a_1 s \\ t^{(1)}_{IJ} &= b_0 + b_1 s + b_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t^{(1)}_{IJ}(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(t^{(1)}_{IJ}) \end{aligned}$$

Dobado and Peláez

$$G(s) = t^{(0)2}_{IJ}/t_{IJ}$$

$$\mathrm{Im}G = -t^{(0)2}_{IJ}\frac{\mathrm{Im}t_{IJ}}{\mid t_{IJ}\mid^2} = -t^{(0)2}_{IJ}\sigma = -\mathrm{Im}t^{(1)}_{IJ}$$

$$G(s) = G_0 + G_1 s + G_2 s^2 + \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im}G(s')ds'}{s'^3(s'-s-i\epsilon)} + LC(G) + PC$$

$$\begin{aligned} \frac{t^{(0)2}_{IJ}}{t_{IJ}} &\simeq a_0 + a_1 s - b_0 - b_1 s - b_2 s^2 \\ &- \frac{s^3}{\pi}\int_{(M_\alpha+M_\beta)^2}^{\infty} \frac{\mathrm{Im} t^{(1)}_{IJ}(s')ds'}{s'^3(s'-s-i\epsilon)} - LC(t^{(1)}_{IJ}) + PC \simeq t^{(0)}_{IJ} - t^{(1)}_{IJ} \end{aligned}$$

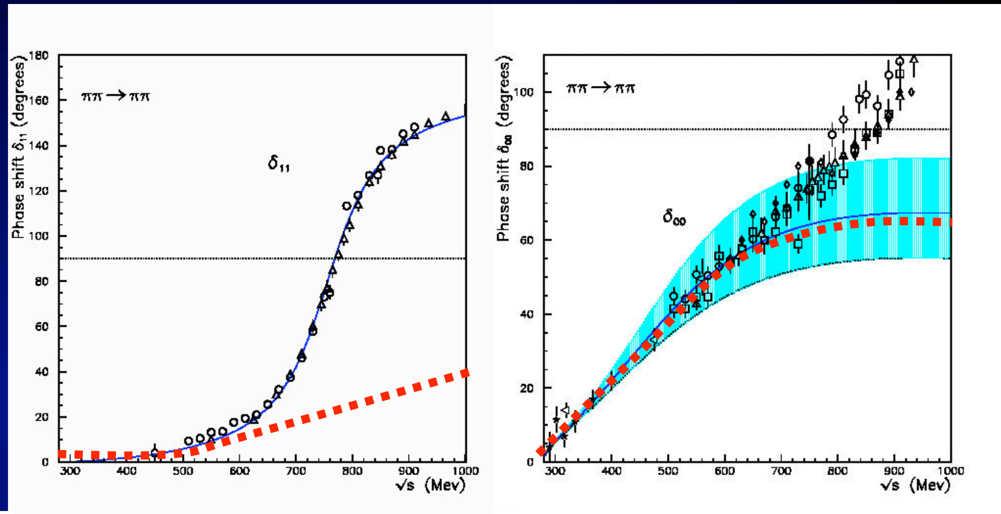
$$t_{IJ} \simeq \frac{t^{(0)2}_{IJ}}{t^{(0)}_{IJ} - t^{(1)}_{IJ}}$$

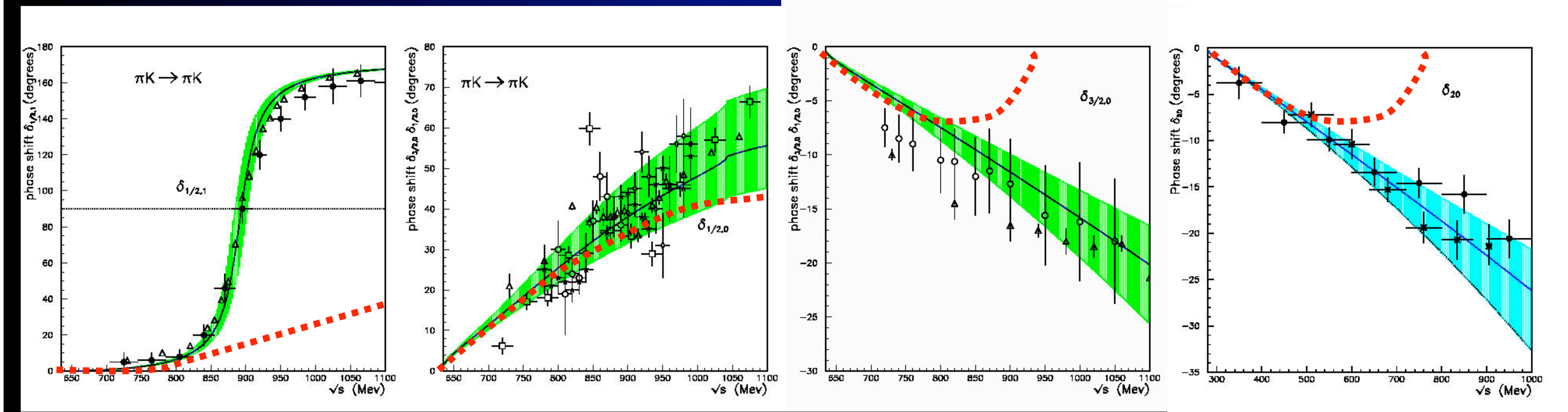
$$\mathrm{Im}t_{IJ} = \sigma_{\alpha\beta} \mid t_{IJ} \mid^2$$

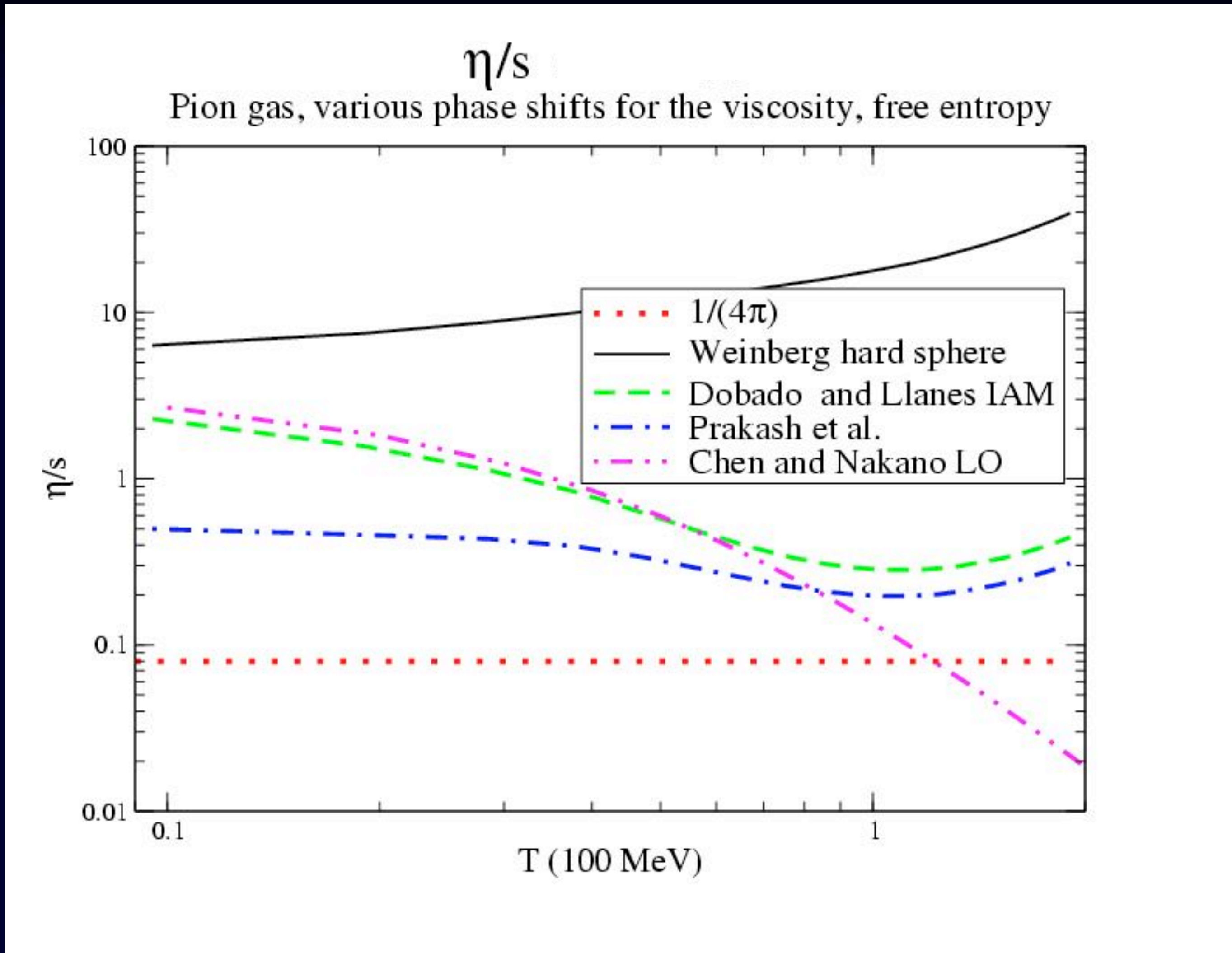
The Inverse Amplitude Method

Lowest order ChPT (Weinberg Theorems)
is only valid at very low energies.

However second order ChPT suplemented with Dispersion Relations (the Inverse amplitude method) makes it possible a simultaneous description of ππ→ππ and πK→πK up to 800-1000 MeV including resonances







Unitarity reestablishes the KSS bound!

Violation of the KSS bound in non-relativistic highly degenerated system:

In principle it could be possible to avoid the KKS bound in a NR system with constant cross section and a large number g of non-identical degenerated particles by increasing the Gibbs mixing entropy

Kovtun, Son, Starinets, Cohen..

However the KKS bound is expected to apply only to systems that can be obtained from a sensible (UV complete) QFT

Is it possible to find a non-relativistic system coming from a sensible QFT that violates the KKS bound for large degeneration?

To explore this possibility we start from the Non-Linear Sigma Model

$SO(g+1)/SO(g) = S^g$

coset space

$$\mathcal{L}_\chi = \frac{1}{2} g_{ab} \partial_\mu \pi^a \partial^\mu \pi^b + m^2 f^2 \sqrt{1 - \pi^2/f^2}$$

$a, b = 1$ to $a, b = g$

$$\sigma = \frac{23 m^2}{384 \pi f^4} = \pi R^2$$

total cross section

$$\eta_\chi = \frac{120 \pi^{3/2} f^4}{23 m^{3/2}} \sqrt{T}$$

viscosity

$$\lambda = \sqrt{\frac{2\pi}{mT}}$$

De Broglie thermal wave length

$$\eta = \frac{5\sqrt{mT}}{16\sqrt{\pi} R^2}$$

hard sphere gas viscosity

$$s = n \left(\log \frac{g}{n\lambda^3} + \frac{5}{2} \right)$$

entropy density

$$n = \frac{gz}{\lambda^3}$$

number density

$T << m$ non relativistic limit

$$\frac{\eta_\chi}{s} = \frac{240\sqrt{2}\pi^3}{23} \frac{f^4}{m^4} \frac{m}{T} \frac{1}{n\lambda^3 \left(\log \frac{g}{n\lambda^3} + \frac{5}{2} \right)}$$

KSS bound violation for exponentially large g!

$$\frac{\eta_\chi}{s} = \frac{240\sqrt{2}\pi^3}{23}\frac{f^4}{m^4}\frac{m}{T}\frac{1}{n\lambda^3\left(\log\frac{g}{n\lambda^3}+\frac{5}{2}\right)}$$

Now we can complete the NLSM in at least two different ways

The first one just QCD since the NLSM is the lagrangian of CHPT at the lowest order with g=3

$$N_f = 2 \text{ and } N_c = 3$$

Two flavors QCD

$$T << m,\ n\lambda^3 << g = 3 \text{ and } m \sim f$$

$$\Rightarrow \quad \eta_\chi/s \gg 1/4\pi$$

Low temperature and low density regime

$$\beta(g) = -g^3\left(11 - 2N_f/3\right)/16\pi^2$$

QCD beta function for N_C=3

$$SU(N_f)_L \times SU(N_f)_R/SU(N_f)_{L+R}$$

Chiral coset

different from

$$SO(g+1)/SO(g) = S^g$$

NLSM coset

The second one is the LSM

$$SO(g+1)/SO(g) = S^g$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi^T - V(\Phi) + \alpha\sqrt{g} F \sigma$$

$$V(\Phi) = -\mu^2 \mid \Phi \mid^2 + \lambda \mid \Phi \mid^4$$

$$\Phi = (\pi_1, \pi_2, \ldots, \pi_g, \sigma)$$

multiplet

$$h = \sigma - \sigma_0$$

Higgs field

$$m_h^2 = 8\lambda g F^2 + 3m^2$$

Higgs mass

$$\Phi_{vac} = (0, 0, \ldots, 0, \sigma_0)$$

vacuum state

$$\kappa = 4\lambda\sqrt{gF^2 + m^2/4\lambda}$$

coset space

$$\mathcal{L} = \frac{1}{2}\partial_\mu \pi^a \partial^\mu \pi^a - \frac{1}{2} m^2 \pi^2 + \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - \kappa h(\pi^2 + h^2) - \lambda(\pi^2 + h^2)^2,$$

$$\sigma_g = \frac{11 m^2}{128\pi(g^2 F^4)} + O(1/g)$$

Cross-section

$$\frac{\eta_g}{s} = \frac{80\sqrt{2}\pi^3}{11} \frac{(g^2 F^4)}{m^4} \frac{m}{T} \frac{1}{n\lambda^3 \left(\log \frac{g}{n\lambda^3} + \frac{5}{2}.\right)}$$

For large enough g KSS is violated!

However, due to the Landau pole, the LSM is thought to be a trivial theory that can only be used as an effective theory at low energies i.e. it is more likely a non UV complete theory.

Einstein gravity corrections:

The KSS result is obtained by considering only GR as the low energy theory for gravity on the AdS space:

$$S_{GB} = \frac{1}{16\pi G_N} = \int dx^5 \sqrt{-g}\left[R - 2\Lambda + \frac{\lambda_{GB}}{2} L^2 (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right]$$

$$\Lambda = -6/L^2$$

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{GB})$$

~~KSS~~

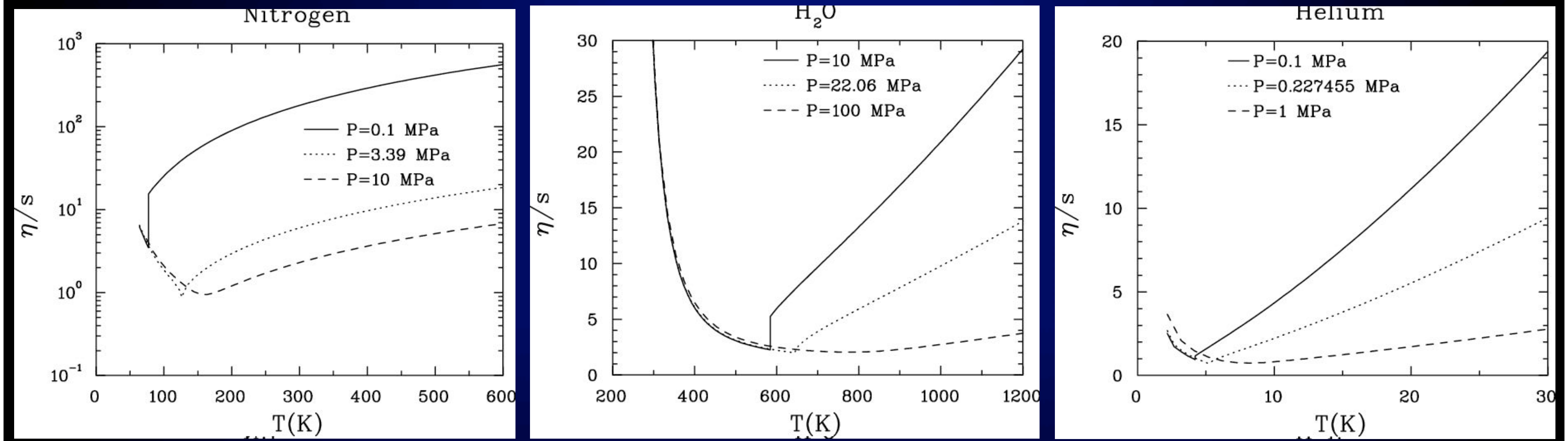
Non perturbative $\lambda_{GB} > 4$?

Causality $$\frac{\eta}{s} \geq \frac{16}{25}\left(\frac{1}{4\pi}\right)$$ $\lambda_{GB} \leq 9/100$

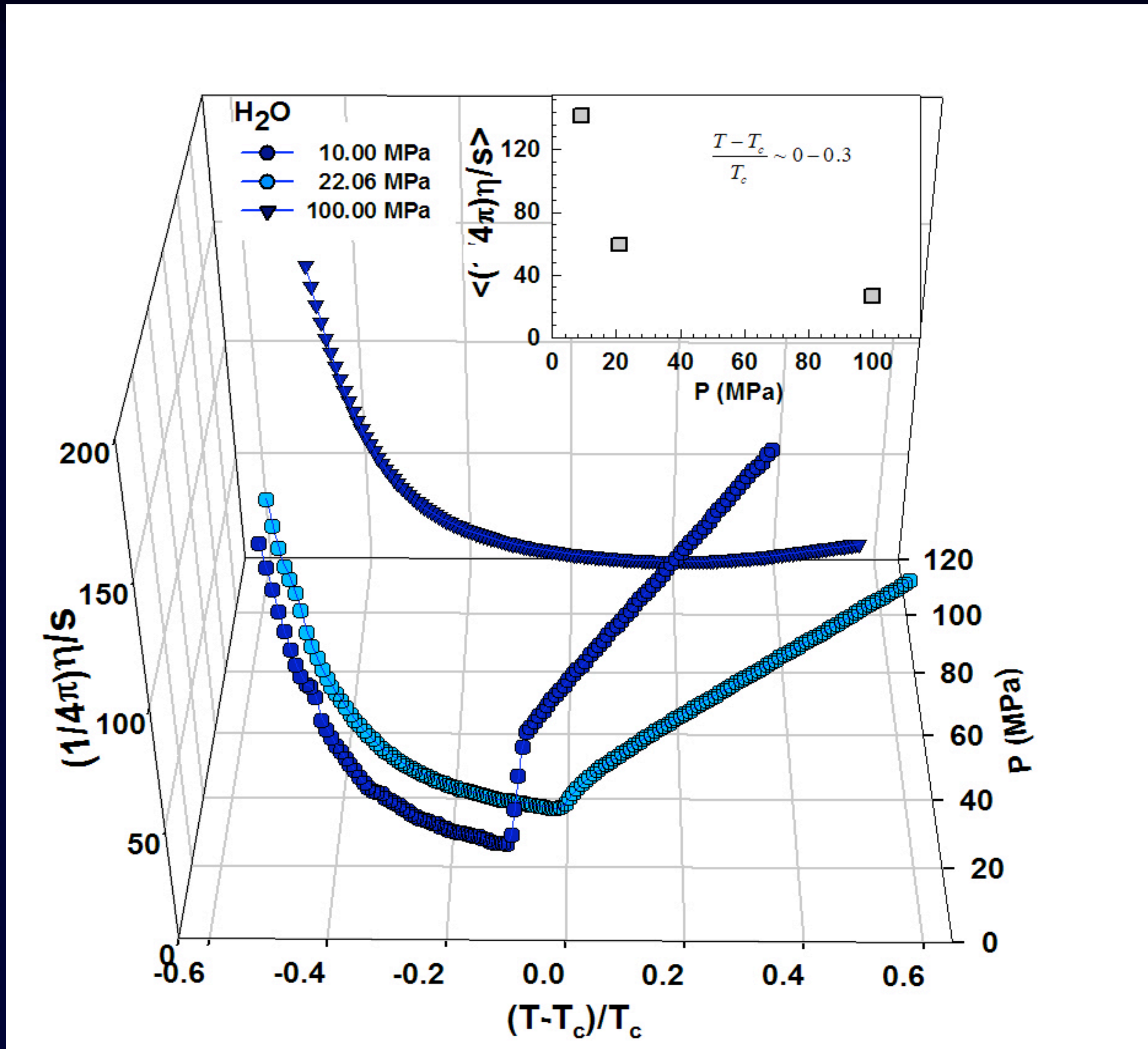
- η/s and the phase transition

Minimun of η/s and phase transition

Recently Csernai, Kapusta and McLerran made the observation that in all known systems both happen at the same point.

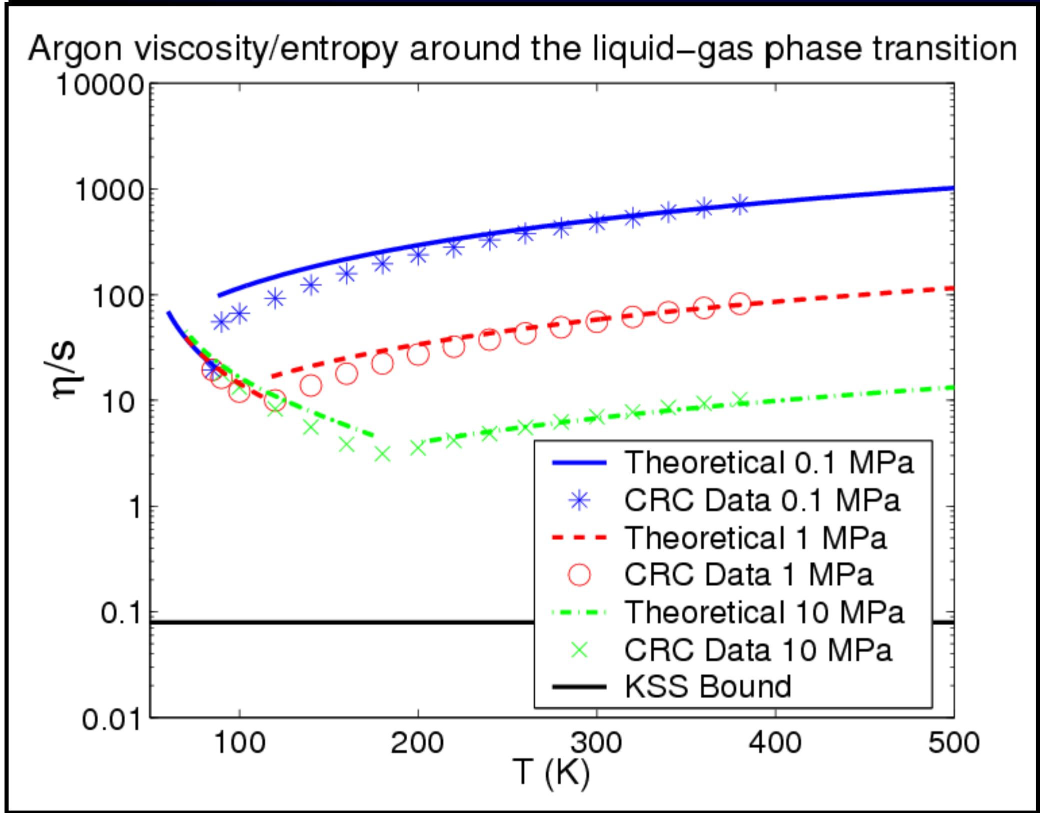


Csernai, Kapusta, McLerran nucl-th/0604032

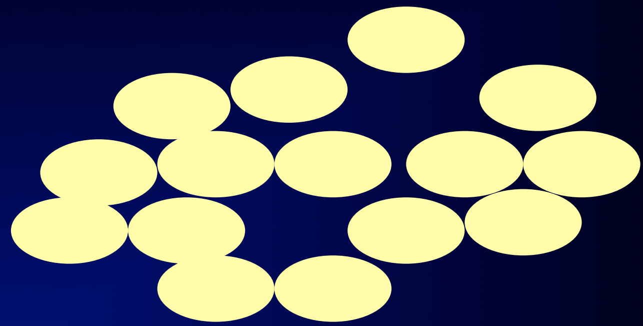


Water end point

Argon

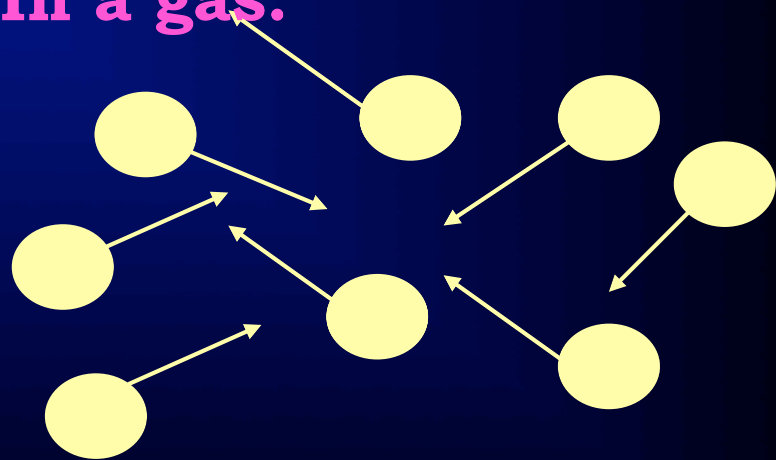


In a liquid: (Mixture of clusters and voids) atoms push the others to fill the voids



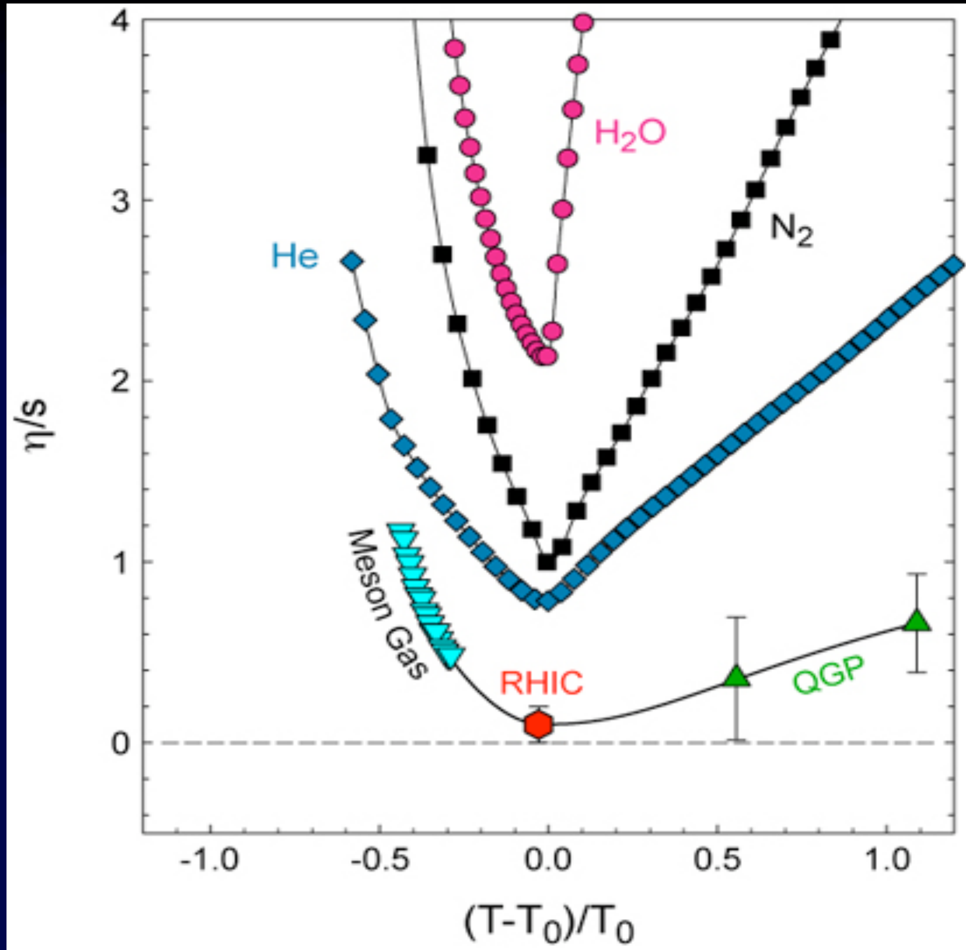
As $T \to 0$, less voids, less p transport $\eta \to \infty$

In a gas:



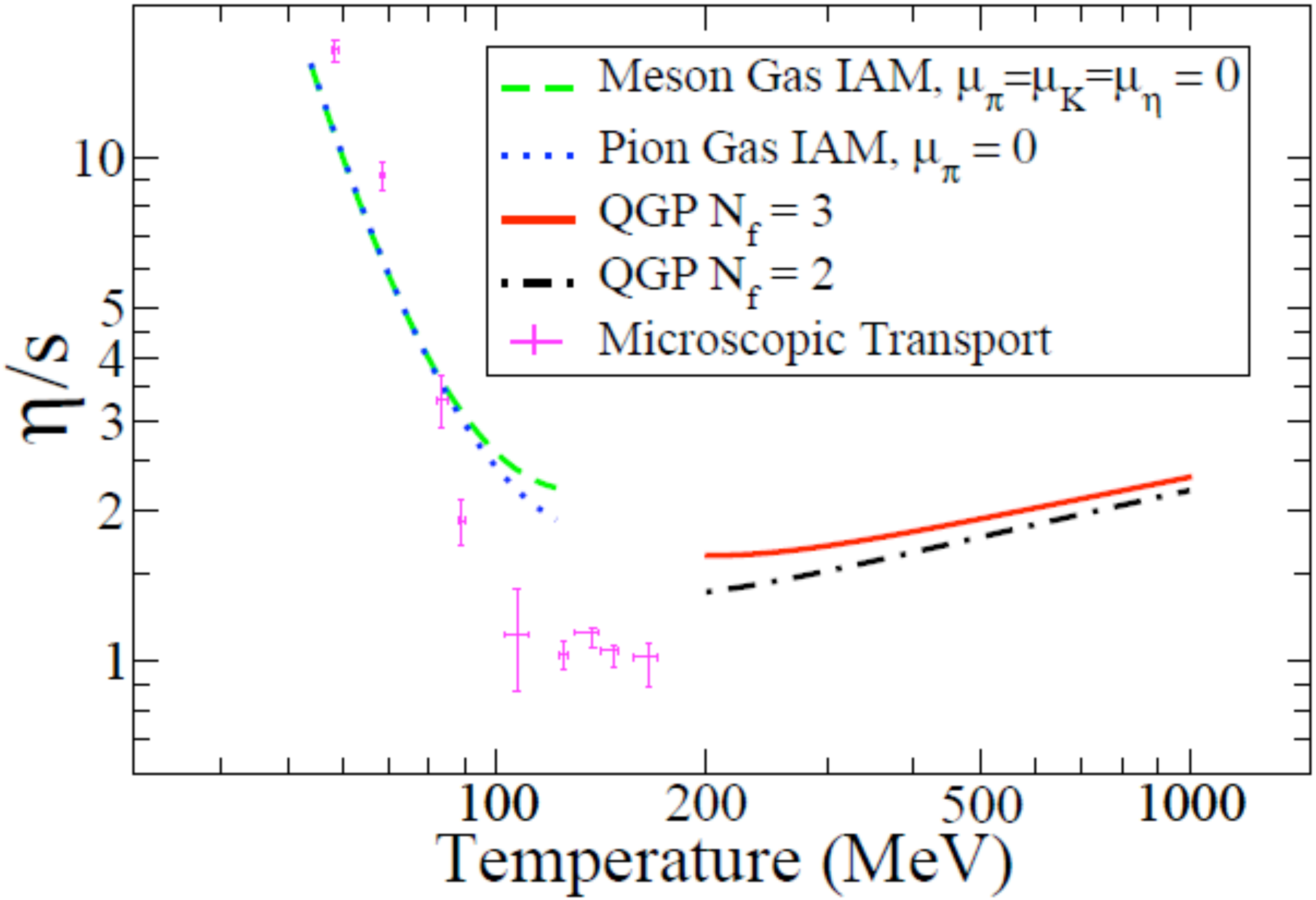
Eyring liquid theory and billiard ball gas

As $T \to \infty$, less p transport, $\eta \to \infty$



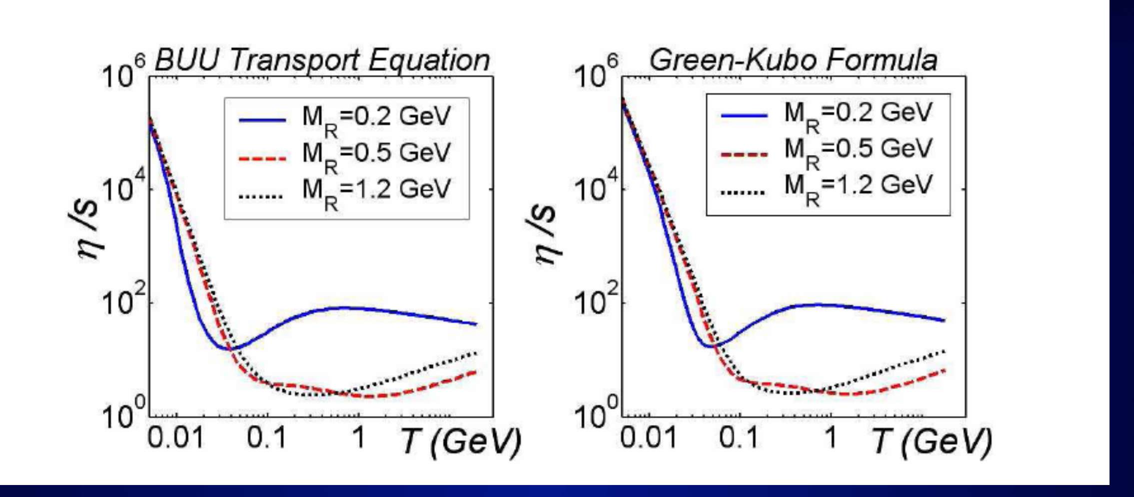
Empirically η/s is observed to reach its minimun at or near the critical temperature for standard fluids.

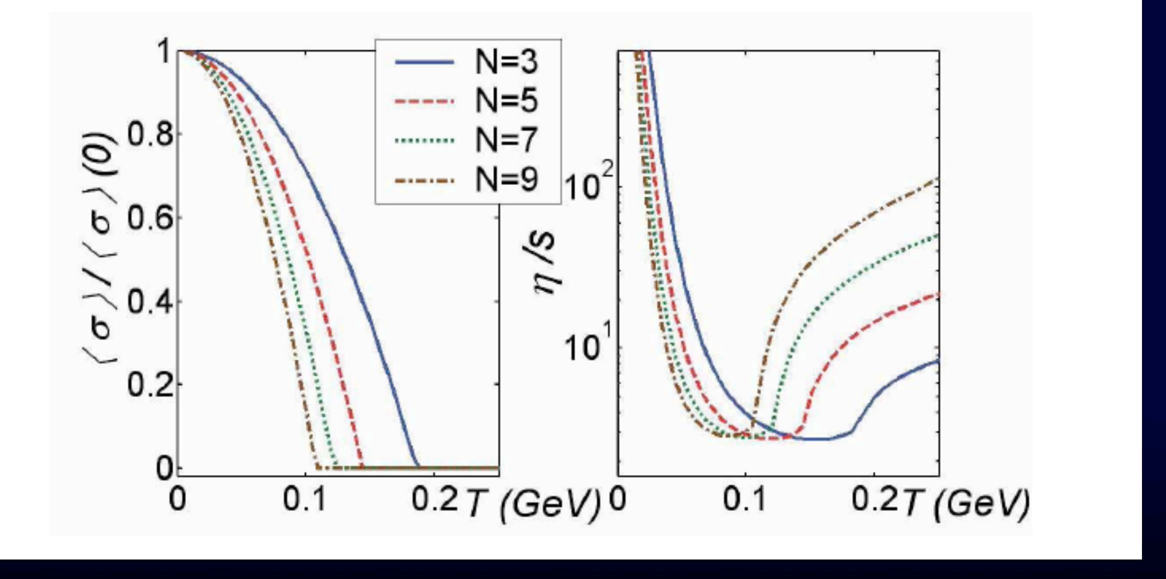
Apparently there is a connection between η/s and the phase transition but we do not have any theory about that (universal critical exponents?)



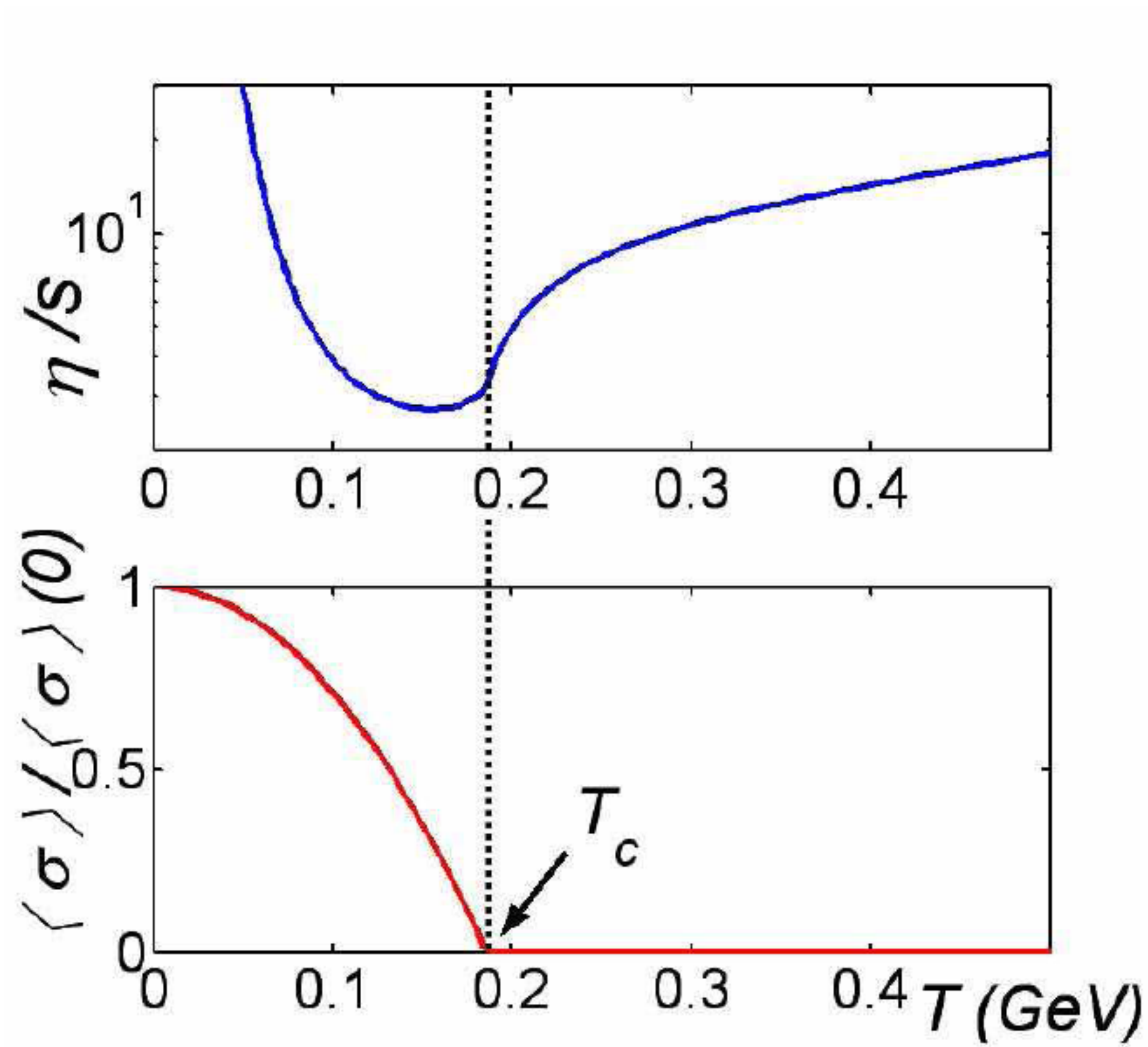
An interesting possibility is that the same could happen in QCD too

The Linear Sigma Model $SO(g+1)/SO(g) = S^g$

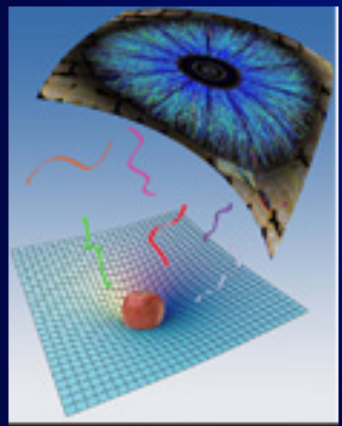




The Linear Sigma Model (large N limit)



- Conclussions and open questions



Summary and open questions

The AdS/CFT correspondence makes possible to study new aspects of QFTs such as viscosity and other hydrodynamic behavior.

The KSS bound set a new limit on how perfect a fluid can be coming from holography which was completely unexpected.

From the experimental point there is no counter examples for this bound.

From the RHIC data we observe a large amount of collective flow that can be properly described by hydrodynamic models with low viscosity compatible with the KSS bound.

Some theoretical models suggest that unitarity could be related in some way with the KSS bound.

There is a theoretical counter example of the bound in a non-relativistic model with large degeneracy. However possibly the model is not UV complete because of the triviality of the LSM. This could be an indication that a more precise formulation of the bound is needed. There are other counter examples coming from higher derivative gravity corrections

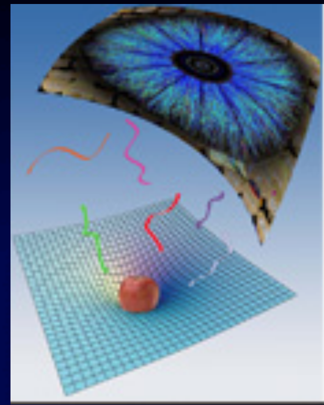
Some open questions:

Is the bound correct for some well defined formulation?

Could it be possible to really measure η/s at RHIC with precision enough to check the KSS bound?

Are there any connections between the KSS and the entropy or the Bekenstein bounds?

How are related the minima of η/s with phase transitions? Could it be considered an order parameter?



«La nostra bella Trieste! I have often said that angrily but tonight I feel it true. I long to see the lights twinkling along the Riva as the train passes Miramar. After all, Nora, it is the city which has sheltered us»*.

* James Joyce, from a letter to Nora, (September 1909)

Thank you very much for your attention