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Screening in Strongly Coupled Plasmas: Universal Properties from Strings in Curved Space

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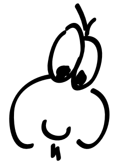
Screening in Strongly Coupled Plasmas –
Universal Properties from Strings in Curved Space



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Gribov-80 Memorial Workshop

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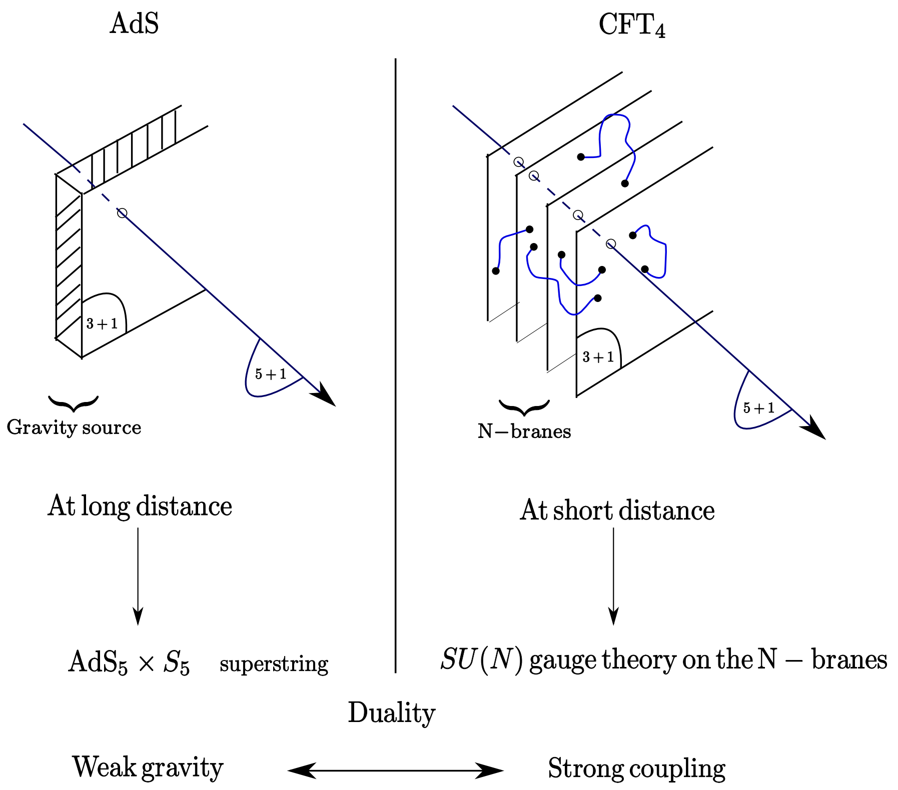
Motivation

- Aim: understand the dynamics of the quark-gluon plasma (QGP)
- Data hint at strongly coupled QGP
- Method needed for strongly coupled gauge theory,
  especially for dynamical quantities
- Promising candidate: AdS/CFT correspondence Maldacena

AdS / CFT

- Duality between supergravity on $AdS_5 \times S^5$
  and
  $\mathcal{N} = 4$ supersymmetric SU(N_c) Yang-Mills theory in $3+1$ dimensions for $N_c \to \infty$



- Metric of $AdS_5 \times S^5$ is

$$
\begin{aligned}
ds^2 &= \frac{r^2}{R^2}\left(-dt^2 + \sum_{i=1}^{3} dx_i^2\right) + \frac{R^2}{r^2}\,dr^2 + R^2\,d\Omega_5 \\
&= \frac{1}{z^2}\left(-dt^2 + \sum_{i=1}^{3} dx_i^2 + dz^2\right) + R^2\,d\Omega_5\,,
\end{aligned}
$$

with AdS curvature radius $R = 4\pi g_{\mathrm{YM}}^2\, l_s^4 N_c$ and

$$
z = \frac{R}{r}
$$

- Classical gravity requires large 't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N_c$.

$\mathcal{N} = 4$ SYM is not QCD, but closer to it at high T

- $\mathcal{N} = 4$ SYM very different from QCD:
  - ⋆ maximally supersymmetric
  - ⋆ conformal theory, coupling is constant
  - ⋆ no confinement, no chiral symmetry breaking
  - ⋆ no particles in fundamental representation
  - ⋆ $N_c \to \infty$ for duality

- At finite T, differences are smaller:
  - ⋆ Above $\sim 2\,T_c$ QCD thermodynamics looks almost conformal (on the lattice).
  - ⋆ no confinement in QCD above T_c
  - ⋆ Finite T breaks supersymmetry.
  - ⋆ Strongly coupled plasma is maybe not too sensitive to microscopic d.o.f.

How to extract information (possibly) applicable to QCD

- Gravity dual of QCD is not known (... if there is one).
- Attempt to come closer to QCD: Break conformal invariance!
- How do observables depend on non-conformality? Look for:
  - ⋆ Robustness: relatively small change
  - ⋆ Universality: no change at all, or systematically in one direction
- $\mathcal{N} = 4$ SYM can give good approximation or a bound on observable (at least in some class of non-conformal theories)
- Famous example: $\eta/s = \frac{1}{4\pi}$ in a large class of theories, possibly a lower bound for all possible theories — Kovtun, Son, Starinets

  Other example: $c_s^2 \leq \frac{1}{3}$ in a wide class of theories — Cherman, Cohen, Nellore; Hohler, Stephanov

Finite T: AdS_5 black hole

- At finite T $\mathcal{N}=4$ SYM corresponds to AdS_5 black hole background:

$$
\begin{aligned}
ds^2 &= -f dt^2 + \frac{r^2}{R^2}(dx_1^2 + dx_2^2 + dx_3^2) + \frac{1}{f} dr^2 + R^2\, d\Omega_5 \\
&= G_{\mu\nu} dx^\mu dx^\nu + R^2\, d\Omega_5
\end{aligned}
$$

with

$$
f = \frac{r^2}{R^2}\left(1 - \frac{r_0^4}{r^4}\right)
$$

- T of gauge theory is Hawking temperature of BH, with horizon position r_0

$$
T = T_H = \frac{r_0}{\pi R^2}
$$

- S^5 part not relevant for our considerations

Non-conformal deformations of AdS_5 black hole

- KTY model (again $T = \frac{r_0}{\pi R^2}$) Kajantie, Tahkokallio, Yee

$$ds^2 = e^{\frac{29}{20}c\frac{R^4}{r^2}}\left[-\frac{r^2}{R^2}\left(1-\frac{r_0^4}{r^4}\right)dt^2 + \frac{r^2}{R^2}d\boldsymbol{x}^2 + \frac{R^2}{r^2}\frac{dr^2}{1-\frac{r_0^4}{r^4}}\right]$$

$$ds^2 = \frac{R^2 e^{\frac{29cz^2}{20}}}{z^2}\left[-\left(1-\frac{z^4}{z_0^4}\right)dt^2 + d\boldsymbol{x}^2 + \frac{dz^2}{1-\frac{z^4}{z_0^4}}\right]$$

- Relevant parameter of KTY model is $\frac{c}{T^2}$, reasonable range is $0 \leq \frac{c}{T^2} \leq 4$.

- 'Realistic' thermodynamics is obtained for $c = 0.127\,\text{GeV}^2$.

- KTY is not a solution to supergravity equations of motion.
  $\longrightarrow$ Consistency questionable. Thermodynamics?!

- Potential V in supergravity action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2}(\partial_\mu \Phi)^2 - V(\Phi) \right)$$

can be chosen such that one obtains a 2-parameter model DeWolfe & Rosen

$$ds^2 = e^{2A(r)}(-h(r)dt^2 + d\boldsymbol{x}^2) + \frac{e^{2B(r)}}{h(r)} dr^2$$

with parameters

$$\frac{c}{T^2}, \qquad \alpha = \frac{c}{\phi} \qquad \left(\Phi = \sqrt{\frac{3}{2}} \phi \frac{R^2}{r^2} \right)$$

and temperature

$$T = \frac{e^{A(\Phi_h) - B(\Phi_h)} |h'(\Phi_h)|}{4\pi}$$

- Defining A and B as

$$
\begin{aligned}
A(\Phi) &= \frac{1}{2}\ln\left(\sqrt{\frac{3}{2}}c\frac{R^2}{\alpha}\right) - \frac{1}{2}\ln\Phi - \frac{\alpha}{\sqrt{6}}\Phi \\
B(\Phi) &= \ln\left(\frac{R}{2}\right) + \frac{1+2\alpha^2}{2\alpha^2}\ln\left(1+\alpha\sqrt{\frac{2}{3}}\Phi\right) - \ln\Phi - \frac{1}{\alpha\sqrt{6}}\Phi
\end{aligned}
$$

one can calculate h from supergravity equations of motion. Gubser et al

- Exponential factor of KTY metric is obtained for $\alpha_{\mathrm{KTY}} = \frac{20}{49}$.

- Two (different) versions of this model by treating ϕ as dilaton or not: 'string frame' and 'Einstein frame' metric.

- 2-parameter model solves supergravity equations of motion
$\longrightarrow$ consistent thermodynamics

Static potential in hot moving plasma

- Static potential of heavy quark-antiquark pair is obtained via temporal Wegner-Wilson loop:

$$W(\mathcal{C}) = Tr\,\mathcal{P}\,\exp\left[i\oint_{\mathcal{C}} dx_\mu\,A^\mu(x)\right]$$

- Potential $E(L)$ is

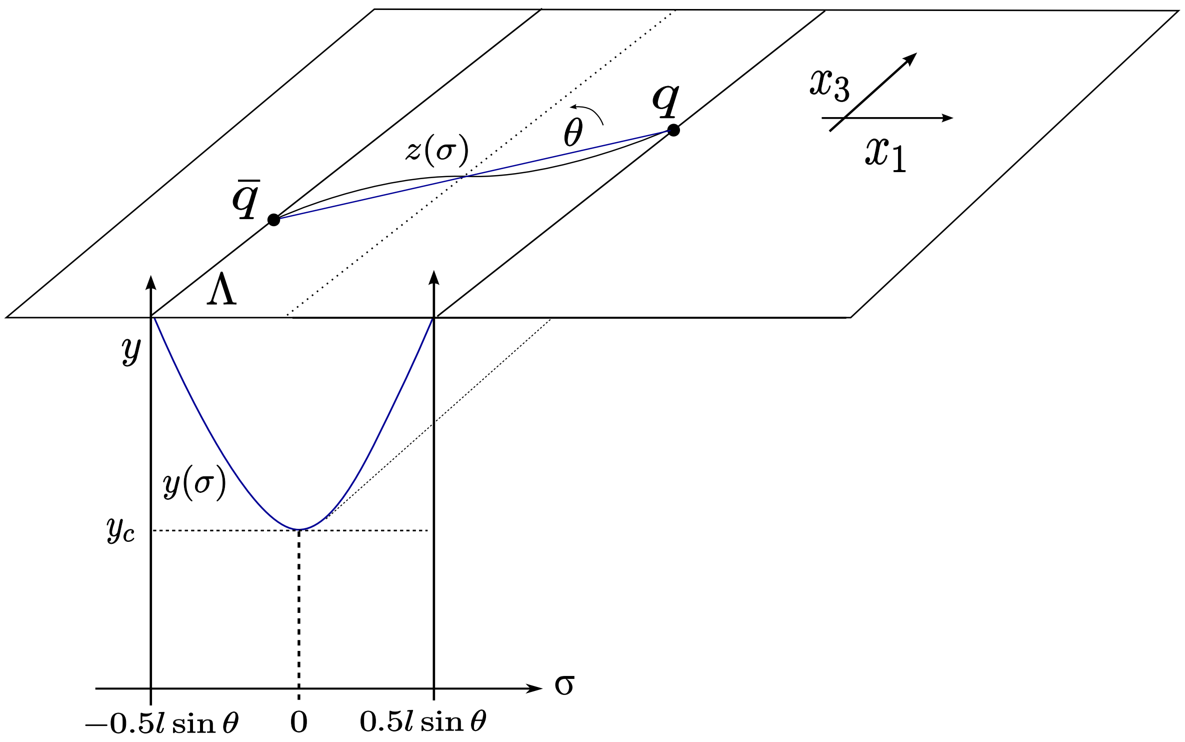
$$\langle W(\mathcal{C})\rangle = \exp\left[-i\mathcal{T}\,E(L)\right]$$

- On gravity side

$$\langle W(\mathcal{C})\rangle \propto e^{-i(S-S_0)}$$

with Nambu-Goto action S for string hanging down in radial direction of AdS_5. S_0 is twice action for open string hanging down from single quark.

- For plasma moving in x_3-direction:



- We calculate the potential for $\mathcal{N}=4$ and the KTY and 2-parameter models, for all plasma velocities and all orientation angles w.r.t. plasma wind.

$\mathcal{N}=4$: Liu, Rajagopal, Wiedemann

KTY: Liu, Rajagopal, Shi

Calculation in $\mathcal{N} = 4$

- Calculation in the other models similar but more complicated.

- Boost metric for moving plasma with velocity $v = \tanh \eta$.
  Parametrize string world sheet and extremize Nambu-Goto action

$$S = \frac{\mathcal{T}}{2\pi\alpha'} \int_{-\frac{L}{2}}^{\frac{L}{2}} d\sigma \sqrt{A \left(\frac{(\partial_\sigma r)^2}{f} + \frac{r^2}{R^2} \right)} \qquad A = \frac{r^2}{R^2} \left[1 - \frac{r_0^4 \cosh^2 \eta}{r^4} \right]$$

- Obtain conserved Hamiltonian ($\longrightarrow$ constant of motion q) and solve for coordinate functions ($r = r_0 y$),

$$\mathcal{H} \equiv \mathcal{L} - y' \frac{\partial \mathcal{L}}{\partial y'} = \frac{y^4 - \cosh^2 \eta}{\mathcal{L}} = q$$

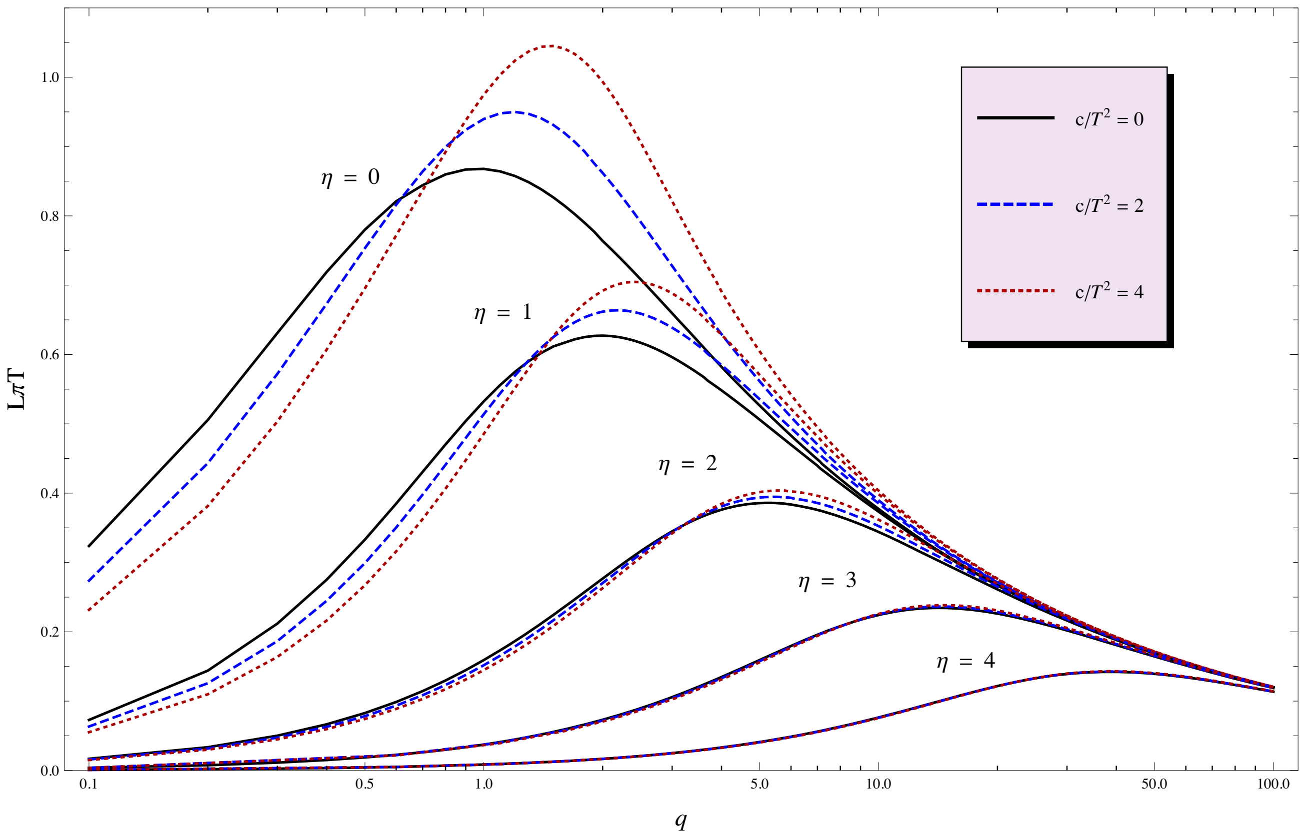
$$y' = \frac{1}{q} \sqrt{(y^4 - 1)(y^4 - y_c^4)} \quad \text{with} \quad y_c^4 \equiv \cosh^2 \eta + q^2$$

- Obtain quark-antiquark distance (using boundary conditions) as function of q

$$\frac{L\pi T}{2} = \int_0^{\frac{L\pi T}{2}} d\sigma = q \int_{y_c}^{\Lambda} dy \frac{1}{\sqrt{(y^4-1)(y^4-y_c^4)}}$$

- Perform similar calculation in KTY and 2-parameter models.

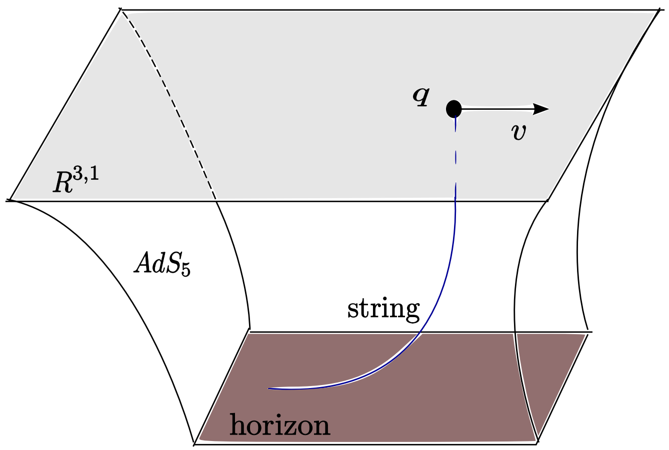
- In KTY model ($c/T^2 = 0$ gives $\mathcal{N} = 4$):



- Two solutions for each distance L up to a maximal $L_{\max}$.
  We call $L_{\max}$ the screening length.

Action for single string

- Needed for finding the static potential.



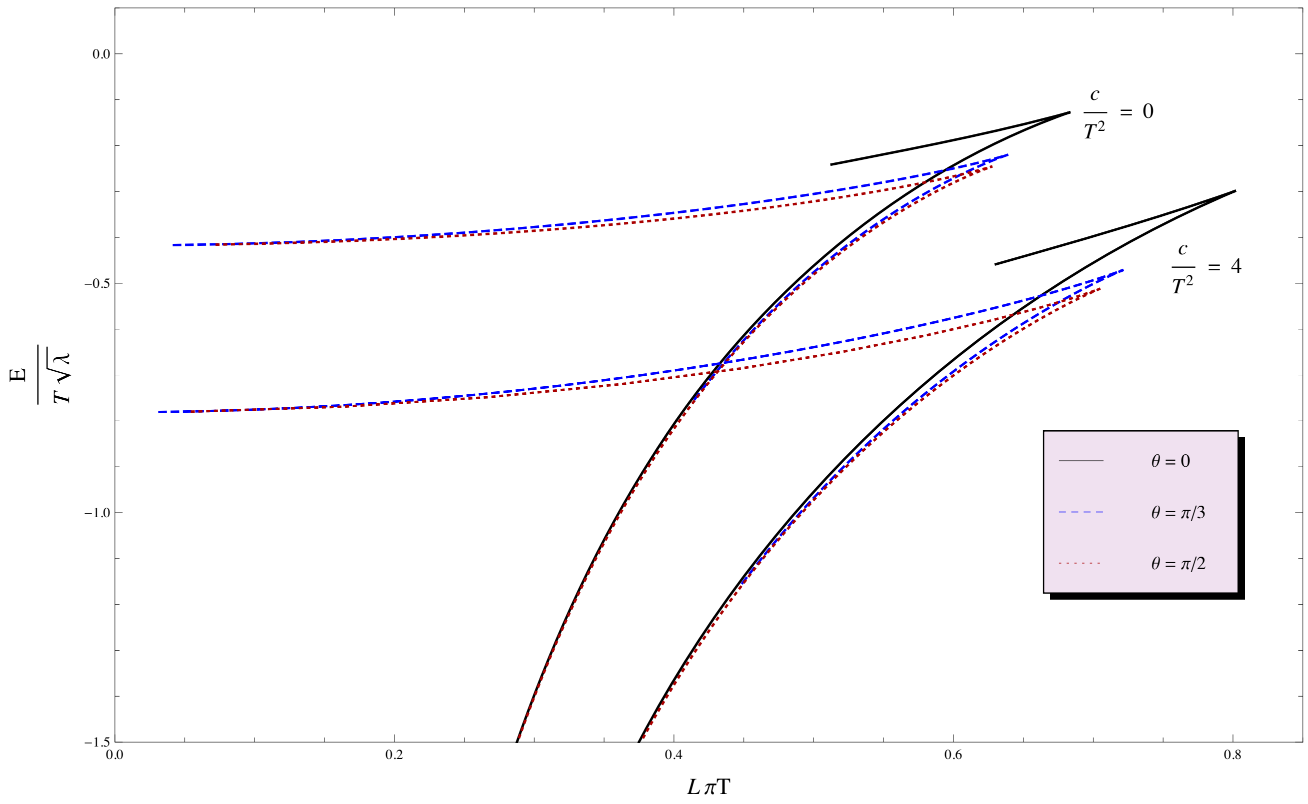
- In $\mathcal{N} = 4$, for example, single string action $\frac{1}{2}S_0$ is

$$\frac{1}{2}S_0 = \frac{1}{2}\sqrt{\lambda}\mathcal{T}T\int_1^\infty dy$$

- Single string action gives drag force, see later.

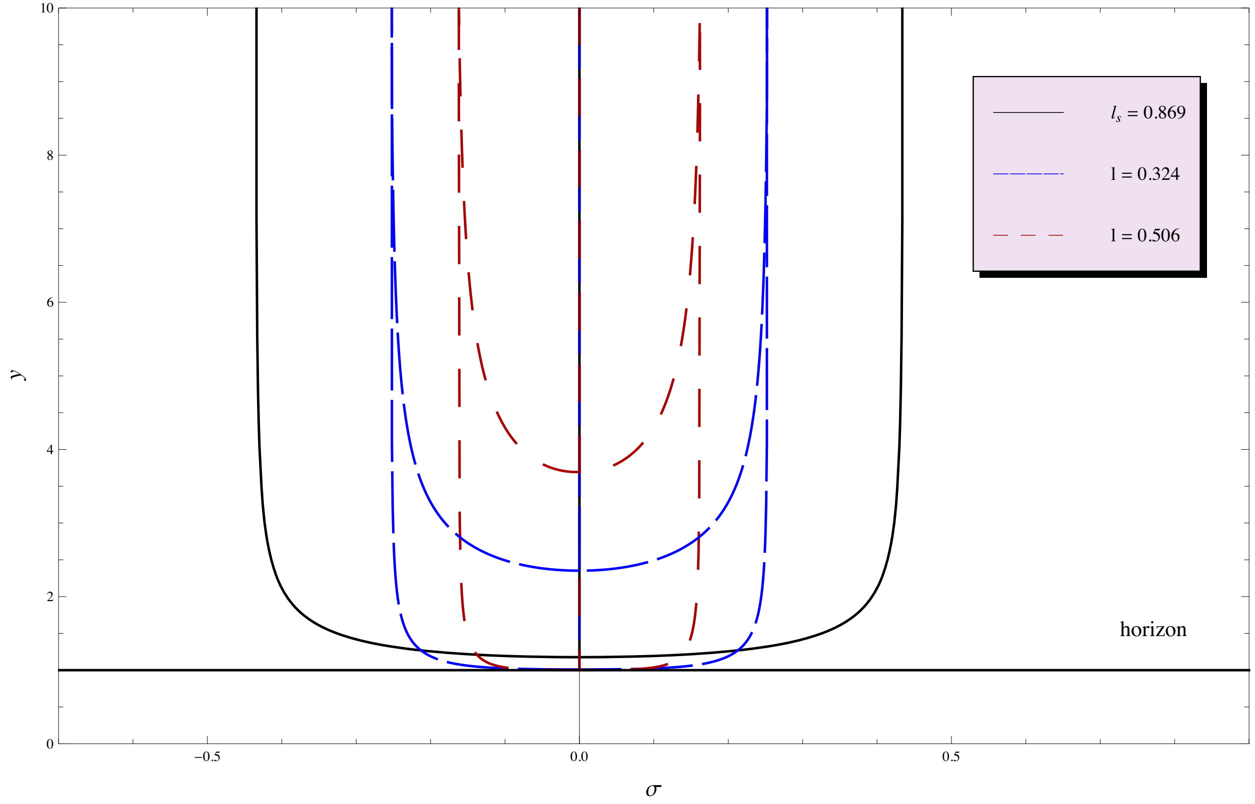
Static potential in plasma wind

- Potential for $\eta = 1$ and different orientation angles



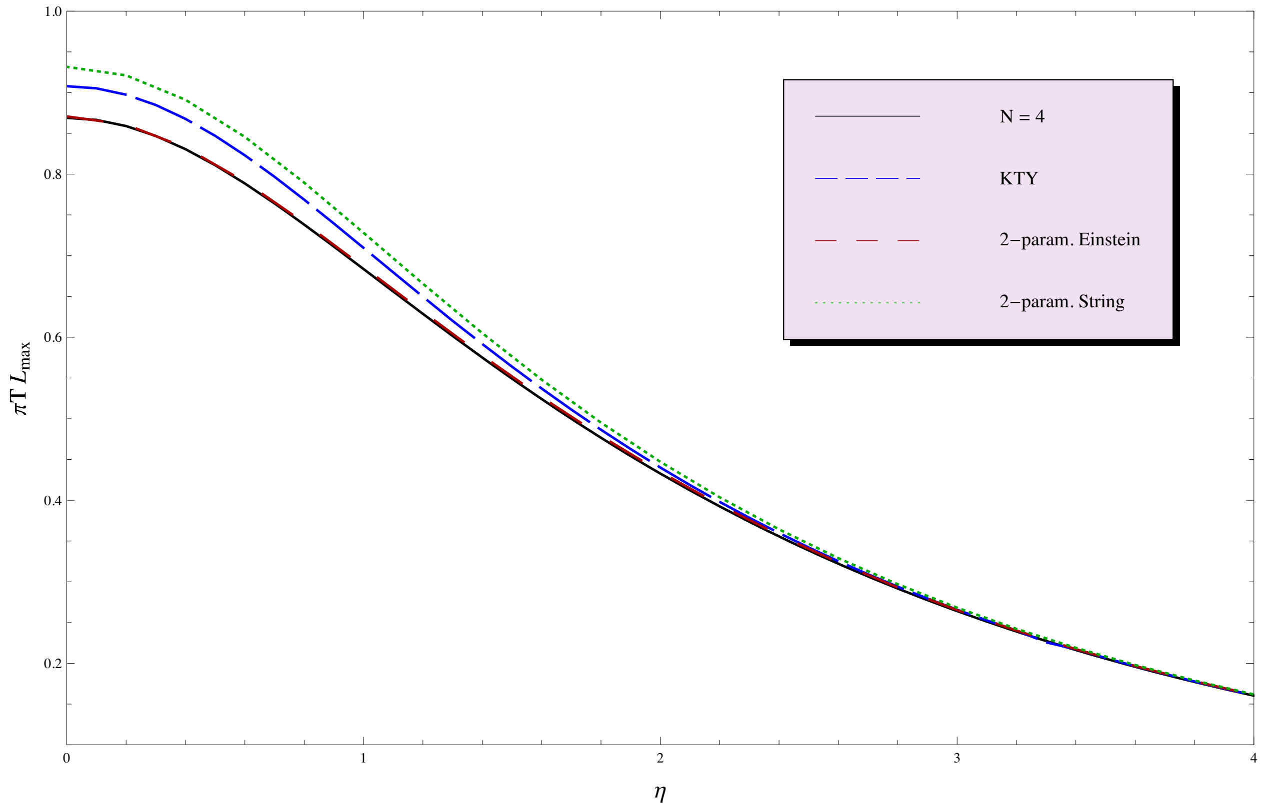
String configurations

- Two string configurations for each $L < L_{\max}$.
  The one coming closer to the horizon is unstable.



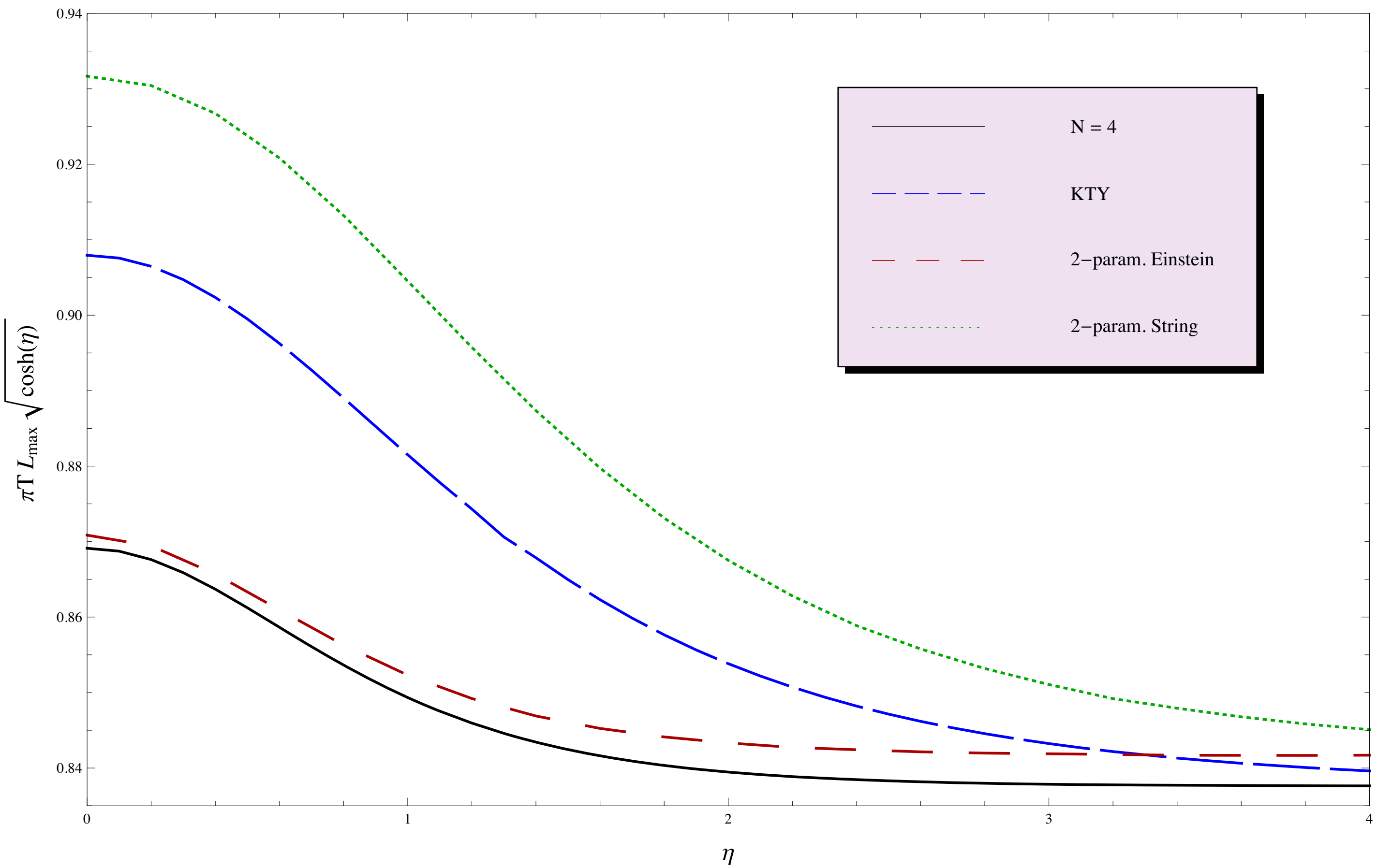
Screening length in all four models

- Velocity dependence of screening length $L_{\rm max}\pi T$ for $\theta = 0$, $c/T^2 = 1$, $\alpha = \alpha_{\rm KTY}$



- $L_{\rm max}$ is minimal in $\mathcal{N} = 4$. Holds for all choices of parameters in the other models.

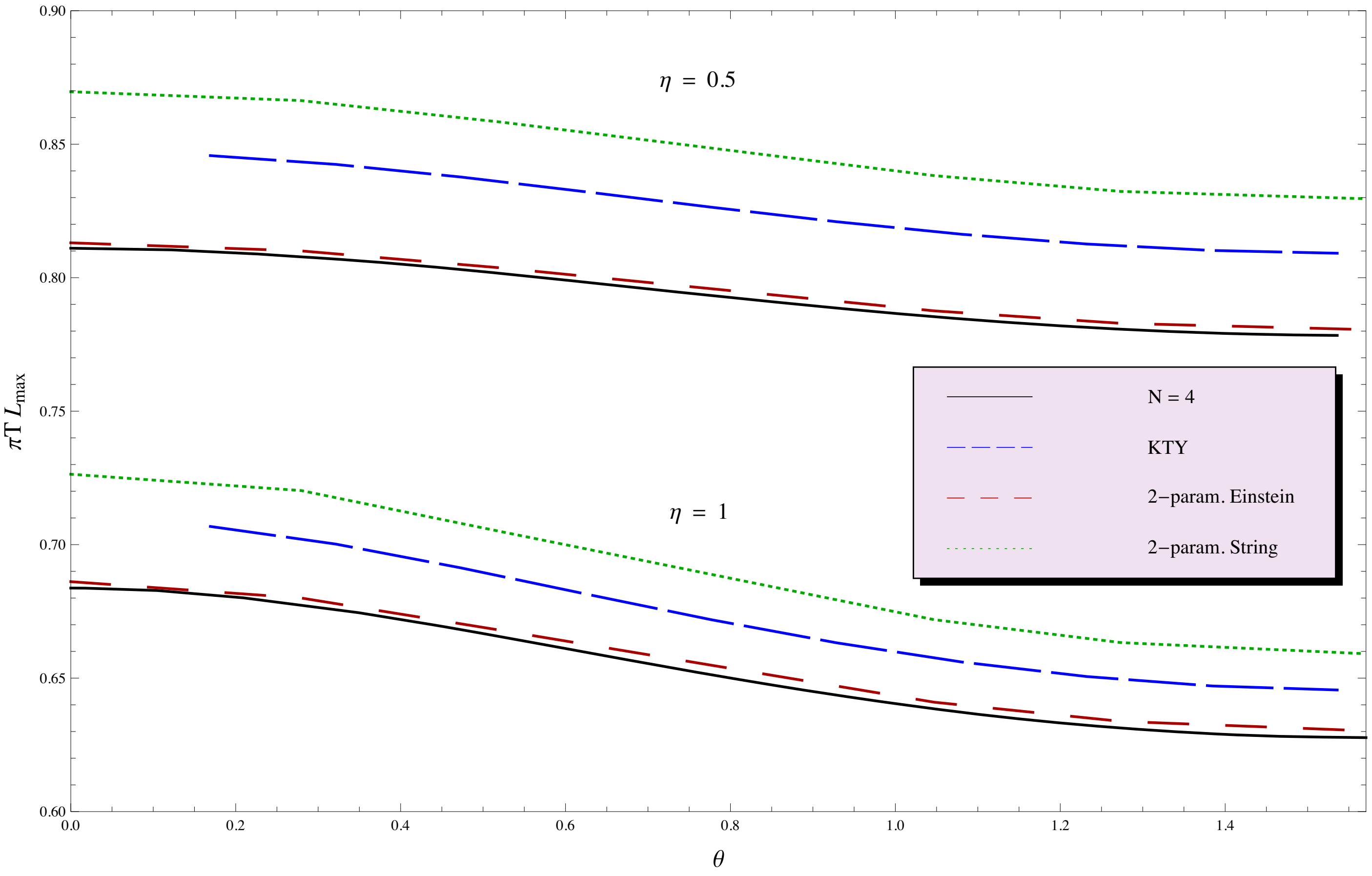
- Screening length scaled with dominant velocity dependence $\sim \frac{1}{\sqrt{\cosh\eta}}$



- Does not meet expected relation $L_{\max} \sim (\cosh^2\eta)^{-\nu}$ with $4\nu = 1 - \frac{3}{4}(1 - 3c_s^2)$.

Natsuume, Okamura

- Screening length depends only weakly on orientation angle θ.

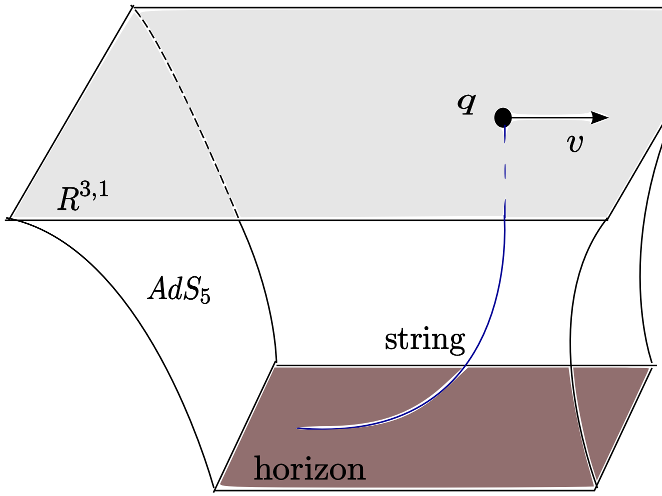


- Angular dependence similar to abelian plasma.

Chu, Matsui

Drag force on single quark moving in plasma

- Consider single quark being pulled through medium with velocity v.



Drag force: force required to keep quark moving at constant velocity.

- In $\mathcal{N} = 4$ we have

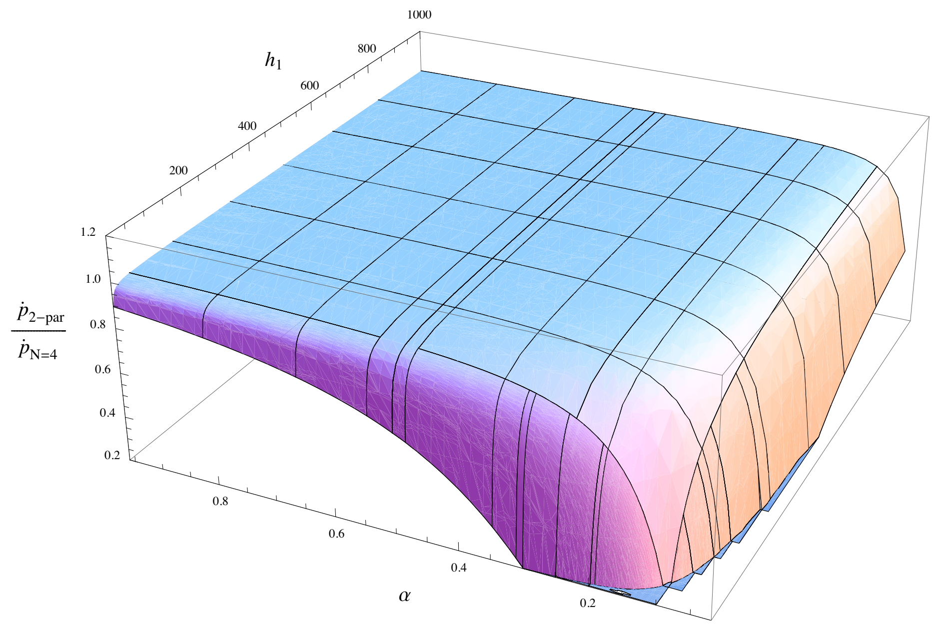
$$\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}$$

- In KTY one finds

$$\frac{dp}{dt} = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}\exp\left(\frac{29c}{20\pi T^2}\sqrt{1-v^2}\right)$$

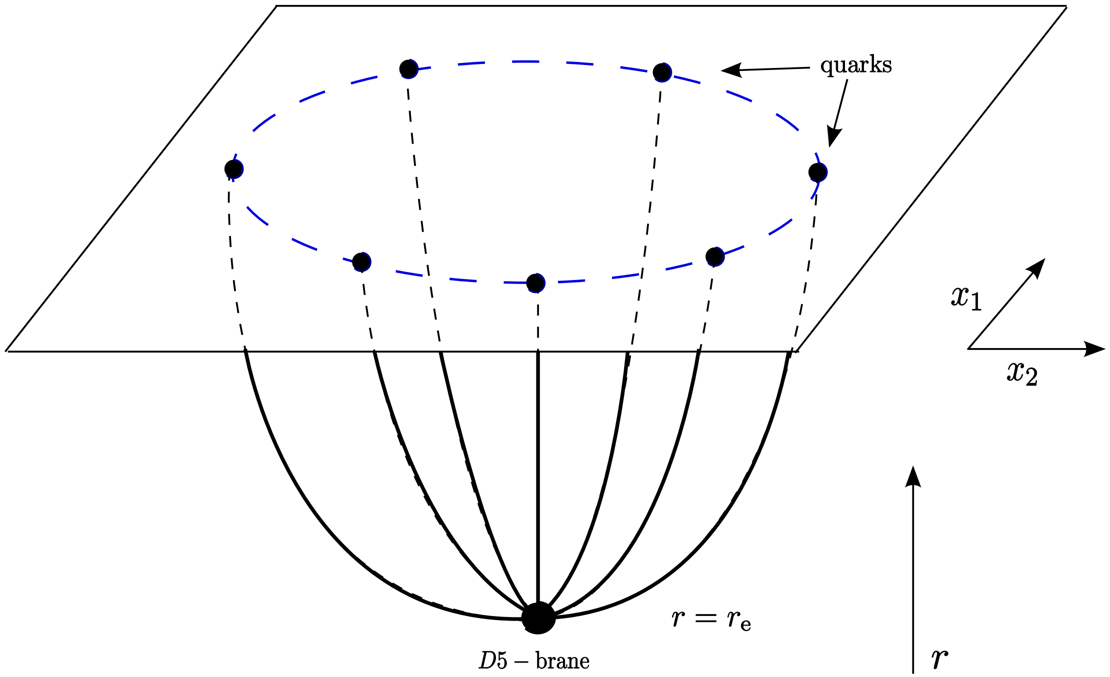
bigger than in $\mathcal{N}=4$?! (Contradicting expectations...)

- In thermodynamically consistent 2-parameter model, we find drag force smaller than in $\mathcal{N}=4$ for all choices of the two parameters.



Heavy baryon screening

- Consider baryon configuration with N_c quarks arranged on a circle



- Very simple model for baryon.

- Technically convenient: introduce density of quarks along circle.

- D5 brane extended in S^5 directions only. Its action is (for all models)

$$S_{D5} = \frac{\sqrt{-g_{00}}\,\mathcal{T}\,V_5}{(2\pi)^5 \alpha'^3}$$

- Action for each of the strings:

$$S_{\rm str} = \frac{\mathcal{T}}{2\pi\alpha'} \int_{r_e}^{\infty} dr \mathcal{L}_{\rm str}$$

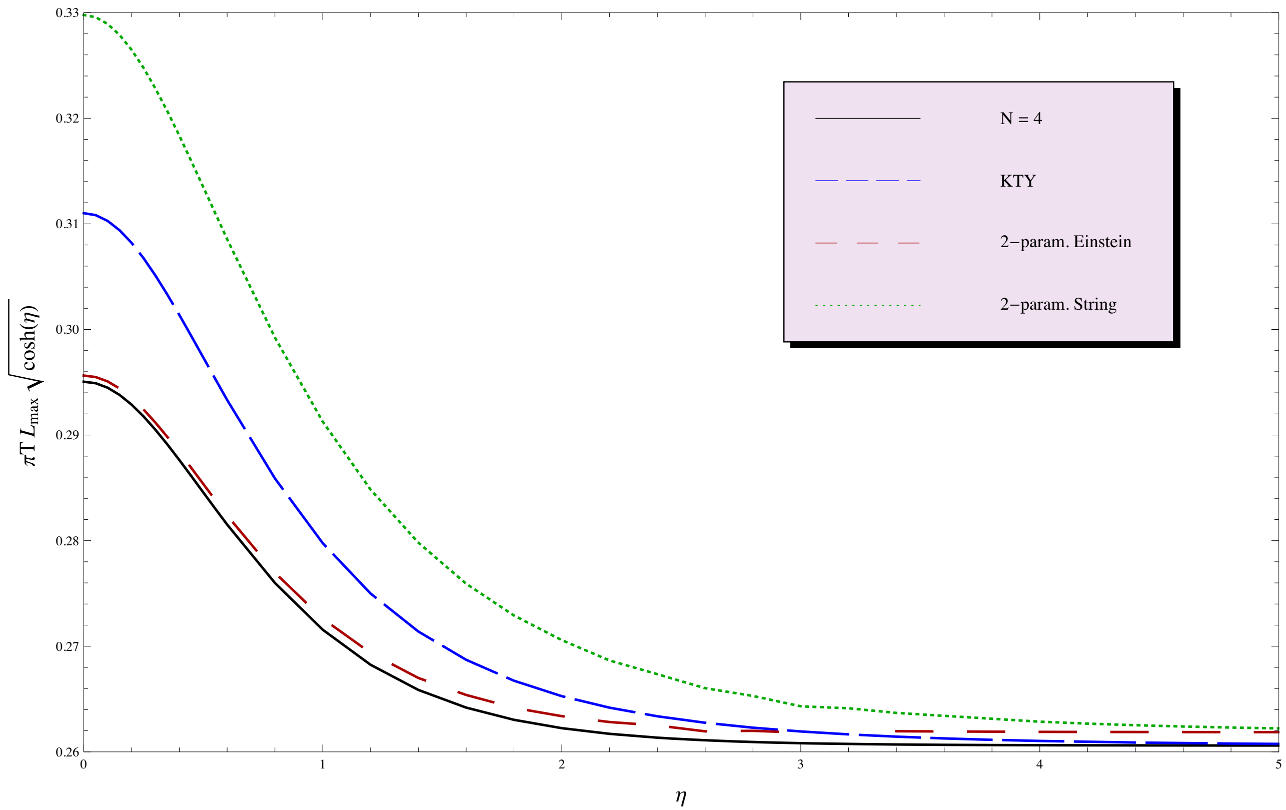
- Total action:

$$S = \sum_{a=1}^{N_c} S_{\rm str}^a + S_{D5} - S_{\rm mass}$$

- Condition: forces on D5 brane have to cancel for stable configuration.

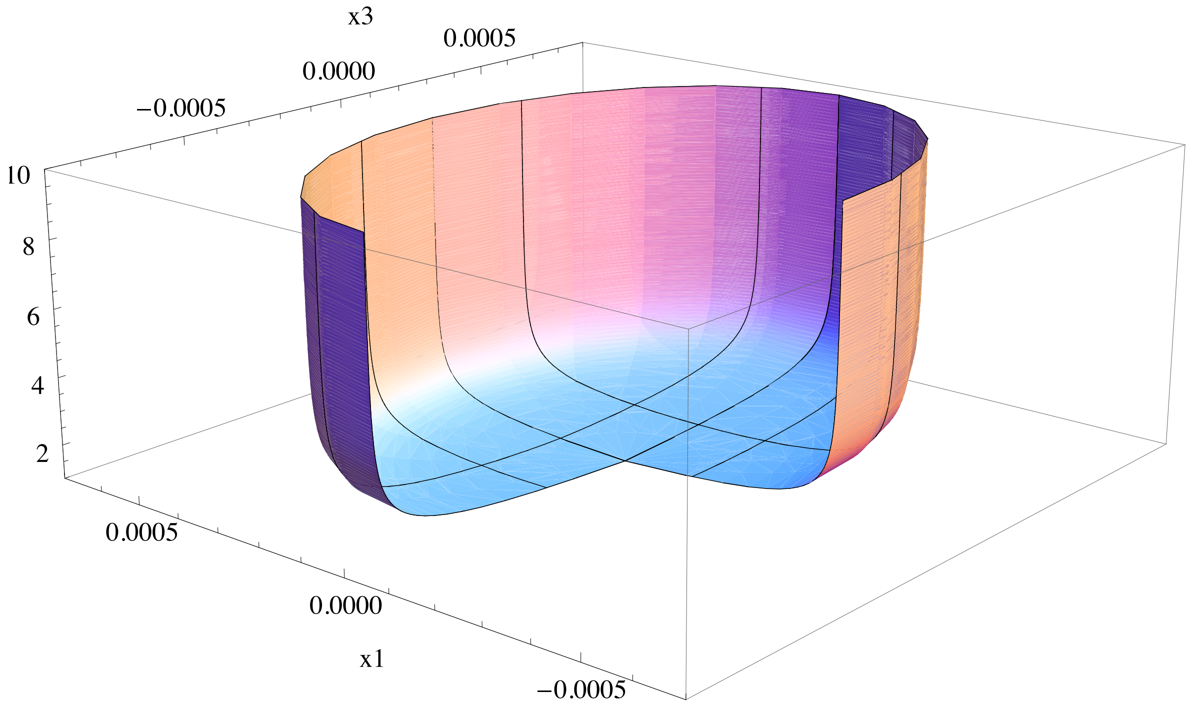
- Baryon screening length in the different models for perpendicular wind

$\mathcal{N}=4$: Athanasiou, Liu, Rajagopal



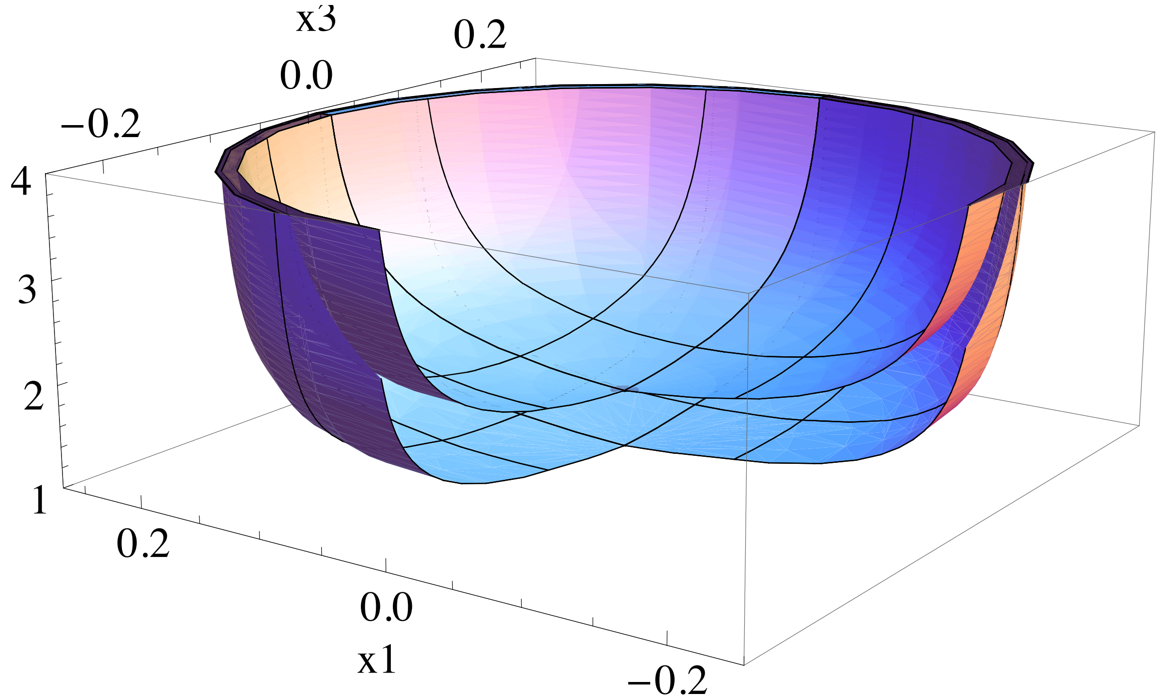
- Again, $L_{\rm max}$ is minimal in $\mathcal{N}=4$. Holds for all choices of parameters in the other models.

- Shape of baryon in plasma wind in scaled fifth dimension, $\frac{r}{r_e}$ (here for the example of $\mathcal{N} = 4$)



for $\eta = 2$, $\rho = \frac{r_0}{r_e} = 0.455$.

- As in meson case, there are two configurations.
  The one closer to the horizon is unstable.



$\eta = 0$, $\rho = 0.855$ and $\rho = 0.581$ – both corresponding to same baryon radius $L\pi T = 0.262$. (Horizon scaled to $r = 1$ since solutions have different 5-brane positions.)

Summary

- We have calculated heavy meson and baryon screening in the wind of hot strongly coupled plasmas, in particular the dependence on velocity and orientation angle.

- The screening length is a robust quantity.

- The screening length in $\mathcal{N} = 4$ SYM is minimal for all kinematic parameters in a large class of theories.

- We conjecture that it is a universal lower bound for an even wider range of theories. $\mathcal{N} = 4$ SYM might be the most strongly coupled gauge theory.

- Outlook: analytic study of screening length for general deformation of metric.