The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
IAEA
International Atomic Energy Agency

2146-5

Gribov-80 Memorial Workshop on Quantum Chromodynamics and Beyond'

26 - 28 May 2010

The QCD rotator in the chiral limit

Peter Hasenfratz
University of Bern
Switzerland

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

The QCD Rotator in the Chiral Limit

Peter Hasenfratz

Institute of Theoretical Physics, University of Bern, Switzerland

Gribov-80 Memorial Workshop on QCD and Beyond, 2010

Introduction and summary

analytic results on the low lying spectrum of QCD
next-to-next-to-leading (NNL) chiral perturbation theory (ChPT)
special environment: delta-regime
created by the 'would be Goldstone bosons'
in a box of size $L_s \times L_s \times L_s \times (L_t \to \infty)$, $\quad L_s \gtrsim 2.5 fm$

The low lying spectrum is a quantum mechanical rotator
whose inertia recieves small, calculable corrections.
leading order(L):
Fisher, Privman, 1983; Brezen, Zinn-Justin 1983;
Leutwyler, 1987

next-to-leading(NL)
P.H., Niedermayer, 1993

next-to-next-to-leading(NNL)
P.H., 2009

Up to NNL order the low lying spectrum is expressed in terms of only 3 constants of ChPT in the chiral limit.

The same low lying spectrum can be studied in numerical experiments (lattice QCD)

$\longrightarrow$ precise constraints on the low energy constants.

Note: the low lying stable energy spectrum is the simplest and cleanest numerical problem on the lattice;

The condition $L_s \gtrsim 2.5fm$ is not trivial. The lattice community is close to that today and will be there tomorrow.

2-flavor QCD in the chiral limit; $SU(2) \times SU(2) \sim O(4)$
dimensional regularization (DR) is used in this work
the low lying spectrum up to NNL order in ChPT reads:

$$E_j = \frac{1}{2\Theta} j(j+2)\,,\ j = 0, 1, 2, \dots\,,$$

where the inertia Θ depends on the low energy constants F, Λ_1, Λ_2:

$$\Theta = F^2 L_s^3 \Big\{ 1 - \frac{2}{F^2 L_s^2} G^* + \frac{1}{(F^2 L_s^2)^2} \Big[0.088431628 + \partial_0 \partial_0 G^* \frac{1}{3\pi^2} \left(\frac{1}{4} \ln(\Lambda_1 L_s)^2 + \ln(\Lambda_2 L_s)^2 \right) \Big] \Big\}$$

$$G^* = -0.2257849591\,; \qquad \partial_0 \partial_0 G^* = -0.8375369106\ .$$

The result is simple, the underlying ChPT is, however, not.
Is the result correct?
F. Niedermayer and Ch. Weyermann (PhD):
result with a different technique using lattice regularization;
the connection between DR vs. lattice regularization is missing;
effective field theory, untouched problem
interested?

The chiral action

Use 'magnetic language': we have a field with 4 components in the internal space. The field is described in terms of microscopic magnets. The lagrangean up to NNL order reads:

$$L = L^2_{\mathrm{eff}} + L^4_{\mathrm{eff}} \,,$$

where

$$L^2_{\mathrm{eff}} = \frac{F^2}{2} \partial_\mu \mathbf{S} \, \partial_\mu \mathbf{S} \;,$$
$$L^4_{\mathrm{eff}} = -l_1 \, (\partial_\mu \mathbf{S} \, \partial_\mu \mathbf{S})(\partial_\nu \mathbf{S} \, \partial_\nu \mathbf{S}) - l_2 \, (\partial_\mu \mathbf{S} \, \partial_\nu \mathbf{S})(\partial_\mu \mathbf{S} \, \partial_\nu \mathbf{S}) \,.$$

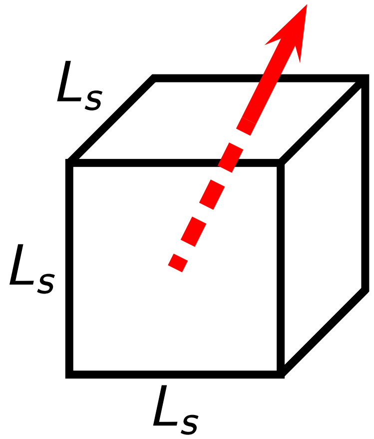
Here F, l_1, l_2 are the bare low energy constants. Further,

$$\mathbf{S}(x) = (S_0(x), S_1(x), S_2(x), S_3(x)), \;\; \mathbf{S}^2(x) = 1 \,,$$

and x lives in $d = 4 = (d-1) + 1$ (space and euclidean time)

The leading (L) rotator

The microscopic magnets are closely parallel in the $L_s \times L_s \times L_s$ box. In leading order we ignore the small fluctuations.

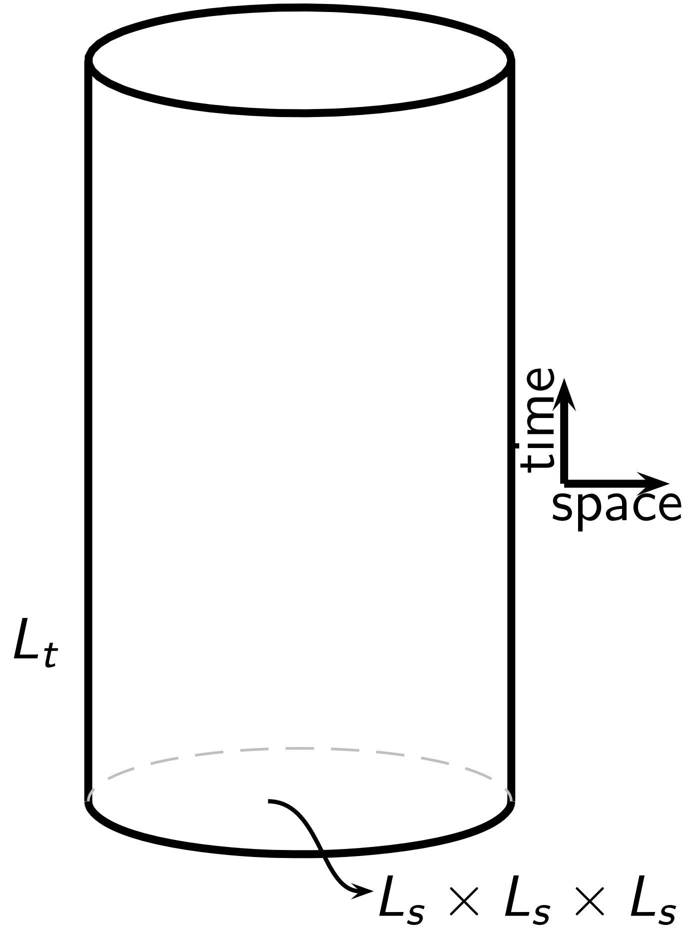


In leading order, on each time slice, the length of the magnetisation is constant, but the direction is changing slowly. Let $\mathbf{e}(t)$ the direction of the total magnetization at t. The leading action reads

$$A^2_{\mathrm{eff}} = \frac{F^2}{2} \int \; dx \partial_\mu \mathbf{S}(x)\, \partial_\mu \mathbf{S}(x) \;\rightarrow$$

$$A_{\mathrm{rot}} = \frac{F^2 V_s}{2} \int dt\, \dot{\mathbf{e}}(t) \dot{\mathbf{e}}(t)\,, \quad \mathbf{e}(t)^2 = 1$$

This is a quantum mechanical rotator with inertia $\Theta = F^2 V_s$.



Separating the slow and fast modes

The diraction of the magnetization $\mathbf{e}(t)$ moves much slower than the single microscopic magnets. We integrate out these fast modes and obtain a generalized rotator in terms of the slow modes $\mathbf{e}(t)$. Then remains a simple problem in quantum mechanics.
We start with the path integral

$$Z = \prod_x \int d\mathbf{S}(x)\delta(\mathbf{S}^2(x) - 1)\exp\left(-A_{\text{eff}}(\mathbf{S})\right) ,$$

where A_{eff} is built from the lagrangean $L^2_{\text{eff}} + L^4_{\text{eff}}$.
Insert '1' in the path integral

$$1 = \prod_t \int d\mathbf{m}(t)\delta(\mathbf{m}(t) - \frac{1}{V_s}\sum_{\mathbf{x}} \mathbf{S}(t,\mathbf{x})) , \quad \mathbf{m}(t) = m(t)\,\mathbf{e}(t) .$$

The vector $\mathbf{e}(t)$ is the direction of the 'magnetisation' on the time slice t. These are the slow modes.

The remaining modes are the fast modes

$$\mathbf{R}(x) = \left((1 - \mathbf{\Pi}^2(x))^{\frac{1}{2}}, \mathbf{\Pi}(x)\right)$$

which can be treated in perturbation theory. In the pairing

$$< \Pi(x)_i, \Pi(0)_j >= \delta_{i,j} \frac{1}{F^2} D^*(x)$$

the $k = (k_0, \mathbf{k} = \mathbf{0})$ part is subtracted, since those are the slow modes. The constrained Green's function D^* is related to G^* and $\partial_0 \partial_0 G^*$ which enter the NNL result for Θ:

$$D^*(0) = \frac{1}{L_s^2} G^* , \;\; \partial_0 \partial_0 D^*(0) = \frac{1}{L_s^4} \partial_0 \partial_0 G^* \, .$$

The inertia Θ up to NNL order

The standard $O(4)$ rotator is obtained, where only the inertia is modified

$$\Theta = F^2 V_s \Big\{ 1 - \tfrac{N-2}{F^2} D^*(0) + \tfrac{N-2}{F^4}\Big(D^*(0) D^*(0) + 2\int_x \partial_0\partial_0 D^*(x) D^*(x) D^*(x)\Big) + \tfrac{1}{F^4}(8l_1 + 16 l_2)\partial_0\partial_0 D^*(0)\Big\}$$

The only unknown part is the integral above. This integral is singular and needs some work. The result reads

$$\int dx\, \partial_0\partial_0 D^*(x) D^*(x) D^*(x) = -\tfrac{1}{L_s^4}\Big\{ d0d0G^* \tfrac{1}{8\pi^2}\tfrac{5}{3}\Big[\tfrac{1}{d-4} + \ln(\tfrac{1}{L_s})\Big] + 0.029492025146\,.\Big\}$$

The singularities in the low energy constants l_1, l_2 cancel the singularities above. We obtain the result on page 4.

$$E_j = \frac{1}{2\Theta} j(j+2)\,,\ j = 0, 1, 2, \ldots\ ,$$

$$\begin{aligned}\Theta = F^2 L_s^3 \Big\{ 1 - \frac{2}{F^2 L_s^2} G^* \\ + \frac{1}{(F^2 L_s^2)^2} \Big[0.088431628 \\ + \partial_0 \partial_0 G^* \frac{1}{3\pi^2} \left(\frac{1}{4} \ln(\Lambda_1 L_s)^2 + \ln(\Lambda_2 L_s)^2 \right) \Big] \Big\}\end{aligned}$$

Corrections to the first excitation

The total corrections to Θ are 50, 30 and 20 percent for
$L_s = 2.0, 2.5$ and 3.0 fermi, respectively.
The NNL corrections are ten times smaller than that of the NL corrections.