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Summer School in Cosmology

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Dark Emergy

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CANADA

What is Dark Energy?

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EITHER $\quad w_{d.e.}^{(2)} = -1$

$$\Big\downarrow$$

$$\rho_\Lambda = M_p^2 \Lambda$$

$$\Big\downarrow$$

no new degrees of freedom beyond GR + SM

(end of story)

$\Rightarrow$ Consistent low energy effective field theory is

$$S = \int \sqrt{-g} \Big[-\Lambda M_p^2 + \frac{1}{2} M_p^2 R + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu}$$

$$+ c_3 R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \cdots \frac{1}{M_p^2} \cdots \frac{1}{M_p^4} \cdots$$

$+ \quad \mathcal{L}_{\text{STANDARD MODEL}}$

$+ \quad$ STANDARD MODEL / GR COUPLINGS

Actual cutoff Λ_c may be lower $\Lambda_c < M_{PL}, c^2$

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OR $W_{d.e}(z) \neq -1$

$$\Big\downarrow$$

Dynamical $\rho_{d.e.}(z)$

$$\Big\downarrow$$

New degrees of freedom, particles (e.g. bosons or condensate of fermions) that lie beyond GR+SM but now at low energies

Theorem: $\boxed{GR + \Lambda + (R^2 + R^3 + \dots) \text{ is unique theory of massless spin two}}$

No way to modify gravity without introducing new degrees of freedom (unless break Lorentz invariance – see for example recent versions of Horava-Lifshitz theory)

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New degrees of freedom must necessarily be light $m_{d.e} \leq 10^{-33} eV$ (H)

Why? consider massive scalar in FRW

$$S = \int \sqrt{-g} \left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 \right]$$

Working in proper time, the equation for homogeneous solutions is

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

Two regimes of interest

$m \gg H$ (neglect friction) $\ddot{\phi} + m^2\phi \sim 0$

$$\phi \sim A\cos(mt)$$

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For a scalar $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$

$$p = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

$$\langle p \rangle_\phi = \frac{m^2}{2}A^2\langle \sin^2(mt) \rangle - \frac{1}{2}m^2A^2\langle \cos^2(mt) \rangle$$

average taken over time $T = \frac{2\pi}{m}$

$$= 0$$

$$\therefore \quad \langle w \rangle = 0 \Rightarrow \rho_\phi \sim \frac{1}{a^3}$$

behaves like matter.

Can alternatively be argued by noting that since

$$S = \int a^3\left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2\right]$$

$u = a^{3/2}\phi$ is natural canonically normalized variable

$$\therefore \rho \sim \dot{\phi}^2 + \ldots \sim \frac{1}{a^3}\dot{u}^2 + \ldots \sim \frac{1}{a^3}$$

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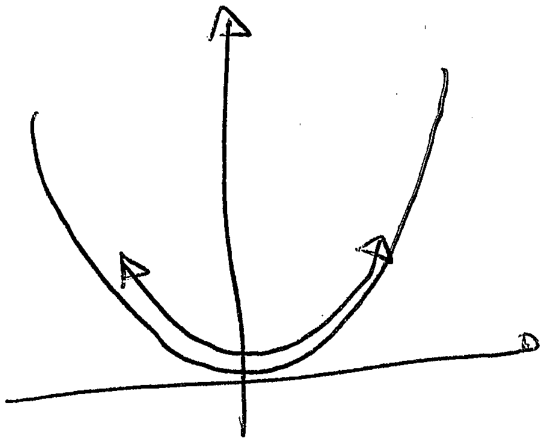
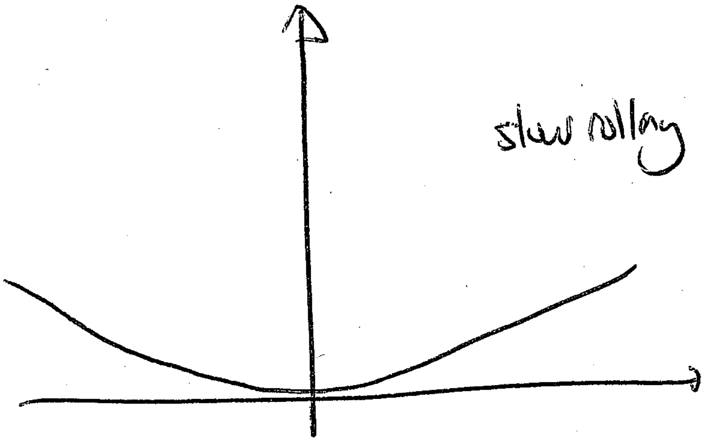


field oscillates back and forth many times in one Hubble time and so redshifts like matter

If $m \ll H$ friction dominates $3H\dot{\phi} = -m^2\phi$



slow rolling (like inflation)

$$(\ln\phi)^{\cdot} = -\frac{m^2}{3H}$$

$$\phi \sim A e^{-\frac{m^2}{3}\int \frac{1}{H} dt}$$

since $\int \frac{1}{H} dt \sim \frac{t}{H} \sim O(H^2) \overset{\text{no. of e-folds}}{N}$

for $m^2 \ll H^2$ variation of ϕ is very slow

In this case $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$

$$\rho = \frac{1}{2}m^2\left(\frac{m^2}{9H^2} + 1\right)A^2 \underbrace{e^{-\frac{2m^2}{3}\int\frac{1}{H}dt}}_{\text{largely constant}}$$

$$1 + w = \frac{p+\rho}{\rho} = \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2}$$

$$= \frac{2\times m^4\phi^2/9H^2}{m^4\phi^2/9H^2 + m^2\phi^2}$$

$$= \frac{2m^2/9H^2}{1 + m^2/9H^2} \approx \frac{2m^2}{9H^2} \overset{(\text{small})}{\approx} 0$$

if $m \ll H$

This is of course nothing more than text book slow roll inflation (chaotic)!

Thus to behave like dark energy we need new degrees of freedom to have $m_{d.e.} \lesssim H$.

This argument holds quite generally (after all its just about length scales.) eg in complicated condensate like models of d.e. it would hold for collective co-ordinate or goldstone like degrees of freedom.

In particular argument holds for both fundamental scalars or higher spin fields, and is independent of whether view dark energy as separate source of stress energy or modification of gravity.

If we believe in Lorentz invariance at high energies new degrees of freedom characterised by mass and spin

spin zero : extra massless or massive

$3 = 2 + 1$

spin one : extra massless or massive

↑ scalar hidden

spin two : extra ~~massless~~ or massive

(inconsistent theories)

spin ≥ 3 X X

(inconsistent theories)

Examples

Extra scalars

Quintessence, k-essence

Brans-Dicke $f(R)$ theories

$f(R, GB)$ theories

Gauss-Bonnet

Extra spin 1

TeVeS (mostly for dark matter alternative)

Vector inflation (mostly for inflation)

Extra spin 2

Extra dimensions (KK modes)

massive gravity

DGP model (cascading gravity)

Here strictly speaking not extra massive spin 2
but rather *entirely* massive spin 2!

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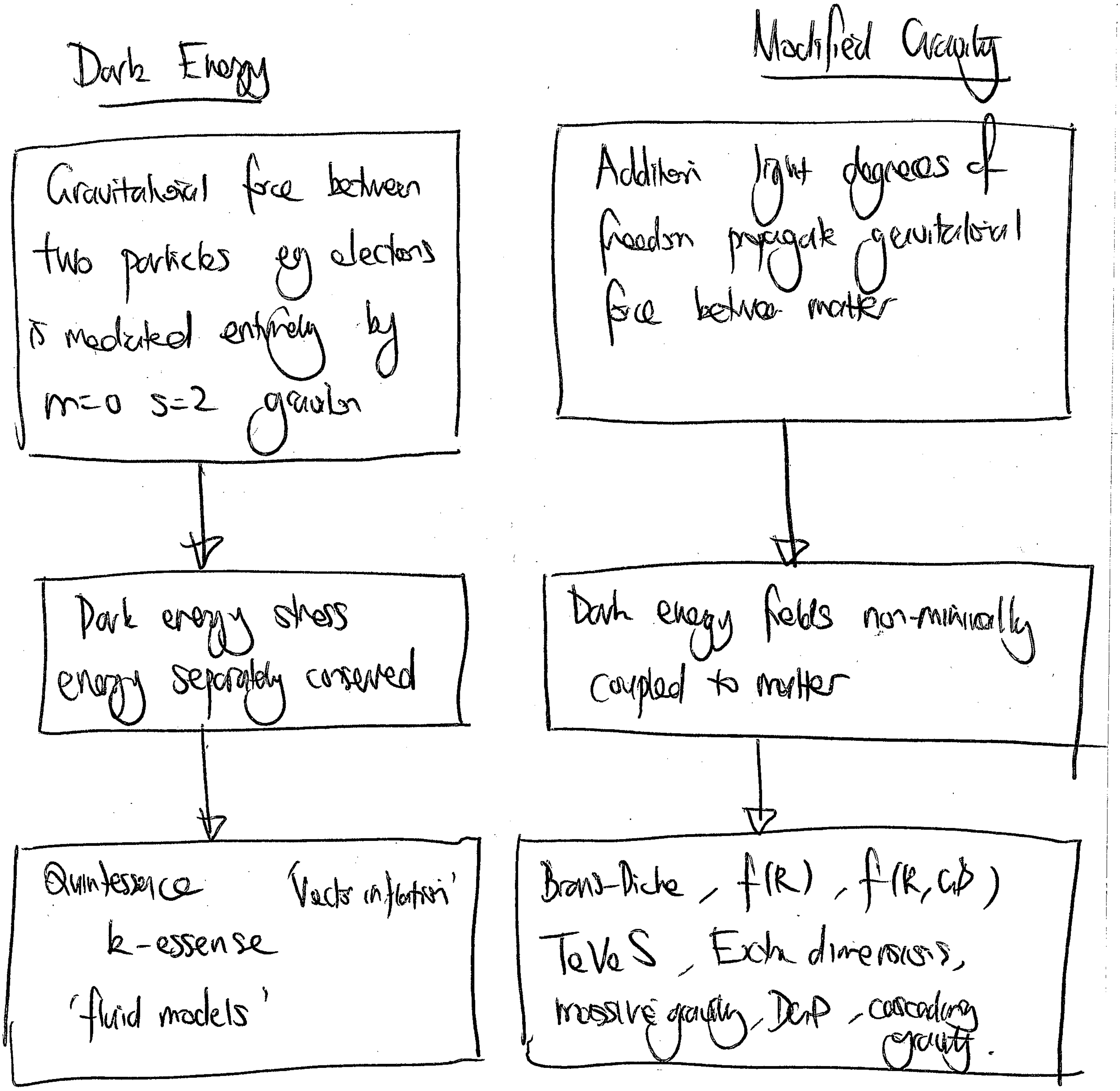
What is difference between Dark energy and modified gravity ?



Dark Energy

Gravitational force between two particles eg electrons is mediated entirely by $m=0$ $s=2$ graviton

Dark energy stress energy separately conserved

Quintessence 'Vector inflation'
k-essense
'fluid models'

Modified Gravity

Addition light degrees of freedom propagate gravitational force between matter

Dark energy fields non-minimally coupled to matter

Brans-Dicke, $f(R)$, $f(R,G)$
TeVeS, Extra dimensions,
massive gravity, DGP, cascading gravity.

Note that it is nothing to do with 'modifying the LHS or RHS of Einsteins equations' but rather to do with what degrees of freedom there are and what forces do they propagate.

Distinction largely overrated!

eg take example of coupled ~~[illegible]~~ dark energy, dark matter

$$S = \int \sqrt{-g} \Big[-\frac{1}{2}(\partial\phi)^2 - V(\phi) + \frac{M_p^2}{2} R + h^2(\phi)\,\mathcal{L}_{\text{baryonic matter}}[h(\phi)\,g_{\mu\nu}, \chi_i] + f^2(\phi)\,\mathcal{L}_{\text{dark matter}}[f\,g_{\mu\nu}, \psi_i] \Big]$$

for example

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If $h(\phi) = f(\phi) = 1$

we would call this quintessence - so ϕ describes dark energy

if $h(\phi) = f(\phi) \neq$ constant

we would call this a generalised Brans-Dicke theory which is equivalent to an $f(R)$ theory and so is in some sense a modified theory of gravity

if $h(\phi) \neq f(\phi)$ and neither are constant

we call this 'coupled dark energy'.

This illustrates the point that the namings are largely artificial.

N.B. 'modified gravity' assumes universal couplings to matter. Justified in case of braneworld constructions where matter lives on brane metric

Quick reminder on f(R) - Brans-Dicke equivalence

$$S = \frac{M_p^2}{2}\int d^4x\,\sqrt{-g}\left[f(R) + \mathcal{L}_m[\chi_i, g_{\mu\nu}] \right]$$

$$\equiv \frac{M_p^2}{2}\int d^4x\,\sqrt{-g}\left[\overset{\text{auxiliary field}}{\Phi} R + b(\Phi) + \mathcal{L}_m[\chi_i, g_{\mu\nu}] \right]$$

provided $\frac{\partial}{\partial \Phi} \Rightarrow R + b_{,\Phi} = 0$ (solve $\Phi = \Phi(R)$)

and $f(R) = \Phi(R) R + b(\Phi(R))$

since R contains $\ddot{g}$, $\dot{N}$ Φ is not strictly auxiliary but rather has dynamics

This is a Brans-Dicke theory in Jordan frame. It may be rewritten in Einstein frame as

$$g^E_{\mu\nu} = \Phi \, g_{\mu\nu}$$

$$\Phi = e^{\sqrt{\frac{2}{3}}\phi/M_{pl}}$$

$$\equiv \quad S = \frac{M_p^2}{2}\int d^4x \sqrt{-g_E}\Big[R_E - \frac{1}{2}(\nabla^E\phi)^2 - V(\phi)$$

$$+ \; e^{-2\sqrt{\frac{2}{3}}\phi/M_p} \mathcal{L}_m\big[x_i, e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{pl}}} g^E_{\mu\nu}\big]\Big]$$

who $V(\phi) = -\frac{M_{pl}^2}{2}\frac{b(\Phi)}{\Phi^2} = -\frac{M_{pl}^2}{2} e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_{pl}}} b\Big(e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{pl}}}\Big)$

Note that although f(R) is a functi of 10 variables

and Brans-Dicke is function of 11

No. of physical degrees of freedom is same

f(R) $10 - 4$ (gauge) $- 3$(constraints) $= 3$

Brans-Dicke $11 - 4$(gauge) $- 4$ (constraints) $= 3$

Thus these theories are identical as EFTs! even at quantum level

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What is sensible model of dark energy ?

Must be sensible low energy EFT !

Logic of EFT : specify particle content plus symmetries the write down every operator/interaction consistent with the symmetries

EFT is a 'double expansion'

- scales with which fields are suppressed
- scales with which derivatives are suppressed

For instance for a single scalar field a highly general (although not exhaustive form is) schematically

$$\mathcal{L}_{eff}^{int} = f^4 \sum_{n=0}^{\infty}\sum_{p=0}^{\infty} \overset{\text{O(1) co-efficients}}{c_{np}} \frac{\partial^p}{M^p}\left(\frac{\phi}{v}\right)^n \quad \left(\text{see Cliff Burgess reviews on EFT}\right)$$

NB – here 3 ~~length~~ mass scales M, v, f

M suppresses derivatives

v suppresses fields

f determines scale of theory $\left(\text{in a sense } \hbar \sim \frac{1}{f^4}\right)$

If we calculate some scattering amplitude with E external legs, L loops and V vertices with q representing a typical external momentum then we get

$$A_E(q) \sim f^4 \left(\frac{1}{v}\right)^E \left(\frac{Mq}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\underset{\text{related to } V}{\tilde{V}}}$$

For fixed E we are summing over loops and vertices

For validity we require $q \ll M$ i.e. $\partial \ll M$

$q \ll \frac{f^2}{M}$ i.e. $\partial \ll \frac{f^2}{M}$

N.B. loops are weighted by $\hbar \sim \frac{1}{f^4}(Mq)^2$ as expected

The fact that expansions are separate means that there are instances where we can resum expansion

eg allow $\phi \sim v$ (although $\sqrt{\delta\phi^2} \ll v$)

keeping $\partial \ll M$

$$S = -\frac{1}{2}(\partial\phi)^2 - \underbrace{f^4 \sum c_n \left(\frac{\phi}{v}\right)^n}_{V(\phi)}$$

(Quintessence !)

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An alternative extreme is where we resum as

k-essence

$$\mathcal{L}_{eff} = f^4 P\left(\frac{X}{f^4}\right) \qquad X = -\frac{1}{2}(\partial\phi)^2$$

where in effect $f \sim M \sim v$

EFT is still under control if $\partial \ll f$

However it is possible classically to have

$X \sim f^4$ i.e. $\partial\phi \sim f^2$

as long as $\sqrt{(\partial\delta\phi)^2} \ll f^2$

Allowing for modified kinetic term gives a wide array of possibilities for what quintessence may look like.