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Dark Energy

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Higher spins

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We have already been introduced to a large class of scalar field models

eg quintessence $-\frac{1}{2}(\partial\phi)^2 - V(\phi)$

k-essence $p(X) = f^4 F\left(\frac{-\frac{1}{2}(\partial\phi)^2}{f^4}\right)$

Brans-Dicke / f(R) type Non-minimal coupling to matter.

Another quite different class arises when we have theories based on higher spins - specifically spin 2. To see this, consider the 10 component symmetric tensor $h_{\mu\nu}$ which is intended to describe a massive spin 2 field.

A massive spin 2-field should have 5 d.o.f.

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To see $h_{\mu\nu} = H_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$

$$+ 2\partial_\mu \partial_\nu \phi$$

Now we have 15 components $H_{\mu\nu}^{(10)}$ $A_\mu^{(4)}$ $\phi^{(1)}$

But we also have symmetries

(4) $H_{\mu\nu} \to H_{\mu\nu} + \partial_\mu X_\nu + \partial_\nu X_\mu$ $\quad A_\mu \to A_\mu - X_\mu$

(1) $A_\mu \to A_\mu + \partial_\mu \Psi$ $\quad \phi \to \phi - \Psi$

Thus total no. of physical propagating d.o.f. will be

$$15 - 4 - 4 - 1 - 1 = 5 \, !$$

(X_μ gauge, X_μ constraint, Ψ gauge, Ψ constraint)

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Crucially inside every spin 2 field $h_{\mu\nu}$, there is a scalar ϕ

$$h_{\mu\nu} = \dots + 2\partial_\mu \partial_\nu \phi$$

which only enters with 2 derivatives acting on it.

There is an additional hidden global symmetry

$$\phi \to \phi + v^\mu x_\mu + c$$

since $\partial_a \partial_b (v^\mu x_\mu + c) = 0$ $h_{\mu\nu} \to h_{\mu\nu}$

This is called the Galilean symmetry. It is only realized approximately in flat space.

Essentially any theory of 'massive gravity' will have a light scalar ϕ which admits this galilean symmetry.

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Physically this scalar acts very much like Brans-Dicke scalar.

In particular, the effective theory can in certain case be expressed as

$$S = \int \sqrt{-g} \left[\frac{M_p^2 e^{-\phi}}{2} R - \frac{1}{\Lambda^3} \Box\phi (\partial\phi)^2 + \mathcal{L}_{matter}(g_{\mu\nu}) \right]$$

like Brans-Dicke but with a non-trivial interaction

Although not immediately apparent, if we take the above action to second order, expanding around flat space

$$\phi = 0 + \delta\phi$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_p}$$

Then the quadratic action is invariant under

$h_{\mu\nu} \to h_{\mu\nu}$

~~$\delta\phi$~~ $\delta\phi \to \delta\phi + v_\mu x^\mu + c$

Galileon as EFT

Nicolis et al 0811.2197 (5600)

As before we can follow our recipe

$$\mathcal{L}_{int} = f^4 \sum c_{np} \left(\frac{\partial}{M}\right)^p \left(\frac{\phi}{\mu}\right)^n$$

except now we add requirement that under

$\phi \to \phi + v^\mu x_\mu + c$ $\quad \mathcal{L}_{int} \to \mathcal{L}_{int} + \partial_\mu J^\mu$

adding requirement that e.o.ms are second order

linear tadpole

$$\Rightarrow \mathcal{L}_{int} = \lambda \phi - \frac{1}{\Lambda^3} \Box\phi (\partial\phi)^2$$

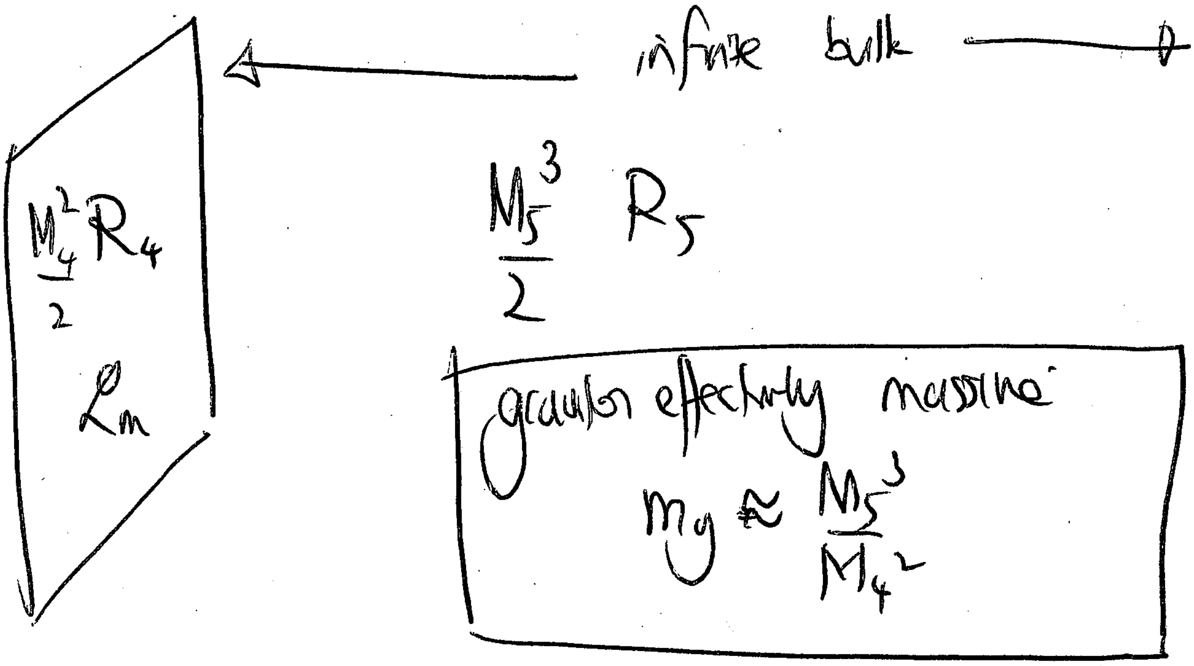
$$+ \text{ schematically } \frac{1}{\Lambda^6} \partial\partial\phi \, \partial\partial\phi \, \partial\phi \, \partial\phi$$

$$+ \frac{1}{\Lambda^9} \partial\partial\phi \, \partial\partial\phi \, \partial\partial\phi \, \partial\phi \, \partial\phi$$

contractions over indices

Particular realization

DGP model (Dvali – Gabadadze – Porrati)



graviton effectively massive

$$m_g \approx \frac{M_5^3}{M_4^2}$$

$$S = \int d^5x \sqrt{-g_5} \left[\frac{M_5^3}{2} R_5 \right] + \int d^4x \sqrt{-g_4} \left[\frac{1}{2} M_4^2 R_4 + \mathcal{L}_m \right]$$

Israel matching condition gives

extrinsic curvature of brane

$$M_4^2 G^4_{\mu\nu} = T^m_{\mu\nu} + M_5^3 \left(K_{\mu\nu} - g_{\mu\nu} K \right)$$

In a particular low energy limit known as decoupling limit, effective 4d action is (5800)

$M_4^2 R$ to second order → $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_4$

$$S = \int -\frac{1}{2}(\partial h)^2 - \phi\left(\Box h - \partial_\mu \partial_\nu h^{\mu\nu}\right) - \frac{\Box\phi}{\Lambda^3}(\partial\phi)^2 + \frac{1}{2} h_{\mu\nu} T_m^{\mu\nu}$$

$$\Lambda^3 \sim M_4 m_g^2 \sim \frac{M_4 M_5^6}{M_4^4} \qquad \Lambda \sim \frac{M_5^2}{M_4}$$

going to Einstein frame $h_{\mu\nu}^E = h_{\mu\nu} - \eta_{\mu\nu}\phi$

$$\Rightarrow S = \int -\frac{1}{2}(\partial h^E)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{\Box\phi}{\Lambda^3}(\partial\phi)^2 + \frac{1}{2} h_{\mu\nu} T^{\mu\nu} + \frac{1}{2}\phi T$$

classic Brans-Dicke type coupling (for $\frac{1}{2}\phi T$)

Friedman equation gets modified to

$$H^2 + \frac{k}{a^2} \pm \frac{2M_5^3}{M_4^2}\sqrt{H^2+\frac{k}{a^2}} = \frac{1}{3M_4^2}\rho$$

extrinsic curvature contribution

In more general massive theories gravity we expect something qualitatively like

$$H^2 + \frac{k}{a^2} \pm \left(H^2+\frac{k}{a^2}\right)^{\alpha} = \frac{\rho}{3M_4^2}$$

exponent gets modified by extent two which graviton mass is a resonance

Back to observations / tests of D.E.

Primary probe Modified expansion history due to additional degrees of freedom

– in each case we must work out effective Friedman equation and match with observations

SN, CMB, LSS (BAO)

Since D.E. only comes to dominate at late times redshift ~ 1.3 or so, it is recent probes of expansion history which are most constraining SN, LSS

Problem: In many of models considered there were free functions $V(\phi)$ (potential) or couplings $h(\phi)$, $f(\phi)$ – these free functions can always be tuned to fit observed expansion history.

N.B. not true in case of massive gravity theories

Secondary

Evolution of dark energy perturbations and their effect on the growth of structure.

Now we must distinguish between

Ⓐ **Traditional Dark energy**

Gravitational force between matter mediated by massless spin 2 field

Ⓑ **Modified Gravity / Coupled Dark Energy**

Gravitational force mediated by additional d.e. degrees of freedom eg scalars

In case Ⓐ the most straightforward thing is to describe dark energy as a fluid

$$T^{\mu\nu} = (\rho + p) u^{\mu} u^{\nu} + P g^{\mu\nu}$$

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For instance (just concentrating on scalar perturbations (in Newtonian / longitudinal gauge)

conventions can differ

$$ds^2 = a^2(\eta)\left[-(1+2\phi)\,d\eta^2 + (1-2\psi)\,d\vec{x}^2\right]$$

Write

$$\delta T^0{}_0 = -\delta\rho$$

$$\delta T^0{}_j = (\bar{\rho} + \bar{p})\, v_j \qquad (\text{velocity} \quad \theta = \vec{\nabla}\cdot\vec{v} = ik^i v_i$$

$$\delta T^i{}_j = \delta p\, \delta^i{}_j$$

$$+ (\bar{\rho} + \bar{p})\frac{3}{2}\left[\frac{\nabla_i \nabla_j}{\nabla^2} - \frac{1}{3}\delta_{ij}\right]\sigma$$

anisotropic stress

The full system of equations ends up being 630d

$$\mathcal{H} = \frac{a'}{a}$$

~~$k^2\phi + 3\mathcal{H}(\phi' + \mathcal{H}$~~

$$k^2\psi + 3\mathcal{H}(\psi' + \mathcal{H}\phi) = \frac{1}{2M_{pl}^2} a^2(-\delta\rho)$$

$$k^2(\psi' + \mathcal{H}\phi) = \frac{1}{2M_p^2} a^2(\rho + P)\,\theta$$

$$\psi'' + \mathcal{H}(\phi' + 2\psi') + \left(2\frac{a''}{a} - \mathcal{H}^2\right)\phi + \frac{k^2}{3}(\psi - \phi) = \frac{1}{2M_p^2} a^2\left[+\delta p\right]$$

$$\boxed{k^2(\psi - \phi) = \frac{3}{2M_p^2} a^2(\rho + p)\,\sigma}$$ N.B. in particular

Defining $\delta = \frac{\delta\rho}{\rho}$

$$\delta' = -(1+w)(\theta - 3\psi') - 3\mathcal{H}\left(\frac{\delta p}{\rho} - w\delta\right)$$

$$\theta' = -\mathcal{H}(1-3w)\theta - \frac{w'}{(1+w)}\theta + \frac{k^2\,\delta p/\bar{\rho}}{1+w} + k^2\psi - k^2\phi$$

Missing info is

$$\delta p = c_s^2 \delta\rho \qquad (c_s = 1 \text{ in quintessence}, \neq 1 \text{ in k-essence})$$

and σ (this is zero in quintessence models)

Perturbations adiabatic if

$$\frac{\delta p}{\dot{p}} = \frac{\delta\rho}{\dot{\rho}} \Rightarrow c_s^2 = \frac{dp}{d\rho}$$

In general no reason for this

entropy perturbation

$$\Gamma = \frac{\delta p}{p} - \frac{c_s^2}{w}\frac{\delta\rho}{\rho}$$

$\Gamma = 0$ for adiabatic perts, Γ is gauge invariant.

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(B) Situation quite different!

Presence of scalar fields coupled to matter can give rise to

- violations of equivalence principle
- solar system constraints / perihelion effects
- pulsar constraints (eg additional radiative modes)
- highly modified linear growth of structure
- gravitationally non-linear growth of structure even in the Newtonian regime.

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eg for traditional Brans-Dicke theory

$$S = \int \frac{M_p^2}{2}\left(\frac{\phi}{M_{pl}} R - \frac{w (\partial \phi)^2 M_{pl}}{\phi} \right) + \mathcal{L}_m$$

Current constraints $w > 40000$ (Cassini-Huygens experiment)
(measure Saturn and its satellites orbits)

Physically this is equivalent to requiring coupling to gravity to be very small!

eg define $\psi = 2\sqrt{w}\, \phi^{1/2}$

$$S = \int \frac{M_p^2}{2}\left(\frac{1}{M_{pl}}\left(\frac{\psi}{2\sqrt{w}}\right)^2 R - (\partial \psi)^2 M_{pl} \right) + \mathcal{L}_m$$

Then set $\psi = 2\sqrt{w}\, M_{pl}^{1/2} + \frac{\delta\psi}{M_{pl}^{1/2}}$

(2/100)

$$S = \int \frac{M_{pl}^2}{2}\left(1 + \frac{\delta\varphi}{\sqrt{W} M_{pl}}\right) R - \frac{1}{2}(\partial\delta\varphi)^2 + \mathcal{L}_m$$

requires W large for coupling to be small.

A direct application of this constraint to many f(R) and all galileon models would completely rule them out.

However there is more subtle physics at work

Chameleon effect (Khoury + Weltman)

Work in Einstein's frame

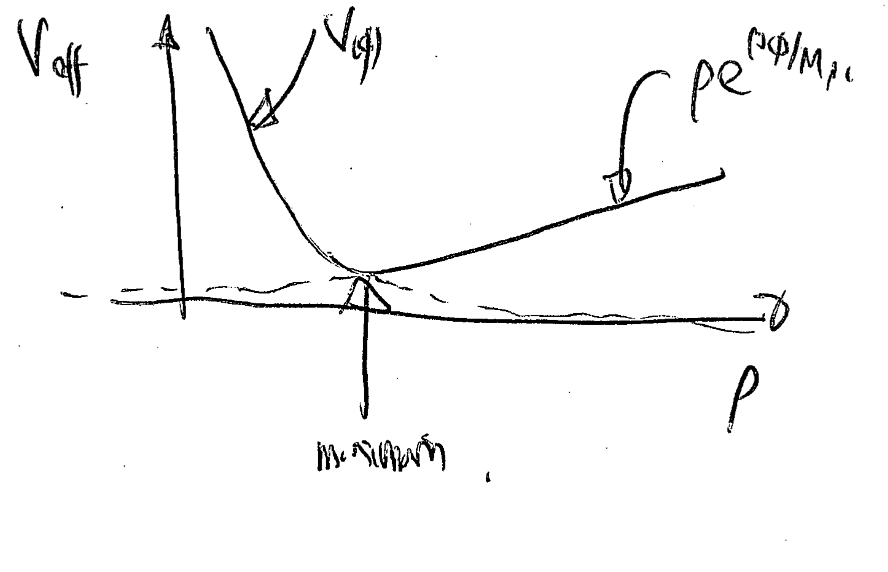
$$S_{cham} = \int d^4x \sqrt{-g}\left[\frac{M_{pl}R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + \mathcal{L}_m\left[g e^{2\beta\phi/M_{pl}}, \chi\right]$$

The effective potential is

$$V_{eff}(\phi) = V(\phi) + \rho e^{\beta\phi/M_p}$$

And effective mass

$$m^2_{eff}(\phi) = V_{,\phi\phi} + \frac{\beta^2}{M_{pl}^2}\rho e^{\beta\phi/M_{pl}}$$



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The conditions necessary for the mechanism to take place are $\beta > 0$

Balance $V_{,\phi} < 0$

Stability $V_{,\phi\phi} > 0$

m increases with density $V_{,\phi\phi\phi} < 0$

easy to satisfy eg $V(\phi) \sim \frac{M^{4+n}}{\phi^n}$ $M < 1\text{meV}$

Cosmologically chameleon behaves like matter at early times and c.c. at late times (Like a freezing model of quintessence)

Allows us to evade 5th force and Weak Equivalence principle tests.

Vainshtein effect

An extremely similar effect occurs (Vainshtein) in massive gravity & galileon like theories

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 + \frac{\phi}{M_{pl}}\rho \right]$$

Expanding around a background solution generates a large kinetic term

$$\Box\phi \sim \frac{\rho}{M_{pl}}.$$

$$S_{eff} = \int d^4x \left(-\frac{1}{2g^2}(\partial\phi)^2 + \frac{\phi}{M_{pl}}\rho \right)$$

$$\frac{1}{g^2} \sim 1 + \frac{\rho}{\Lambda^3 M_{pl}}$$

rescaling gives $\phi \to g\hat{\phi}$

$$S_{eff} = \int d^4x \left(-\frac{1}{2}(\partial\hat{\phi})^2 + g\phi/M_{pl}\,\rho \right)$$

So for example constraints from Cassini-Huygens would give $g \ll 10^{-4} - 10^{-5}$

Thus either through chameleon or Vainshtein effect these new models can survive.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\omega} \hat{g}_{\mu\nu} dx^\mu dx^\nu$$

anisotropic stress

$$ds^2 = -(1+2\phi)dt^2 + a^2(1-2\psi)d\vec{x}^2$$

$$= -(1+2(\hat{\phi}+\omega))dt^2 + a^2(1-2(\hat{\psi}-\omega))d\vec{x}^2$$

in absence of anisotropic source

$$\hat{\phi} - \hat{\psi} = 0 \Rightarrow \phi - \psi \neq 0 \; !$$

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Phenomenological parametizations

$$\nabla^2 \psi = \frac{1}{2M_p^2} a^2 Q \, \delta_M \rho$$

two new parameter ω, Q

$$\frac{\phi}{\psi} = 1 + \omega$$

(gravitational slip)

Different physics probes different things

Lensing ISW $\phi + \psi$

Galaxy peculiar velocities ψ

Galaxy surveys ψ

Also

$$\frac{d \ln \delta_m}{d \ln a} = [\Omega_m(a)]^\gamma$$

Linder parameterization.

γ = growth factor.