2156-6

Summer School in Cosmology

19 - 30 July 2010

Galaxy Formation

Houjun MO

Department of Astronomy
University of Massachusetts
Amherst, MA 01003
U.S.A.

Galaxy Formation

Houjun Mo (UMass)

Trieste Summer School 2010

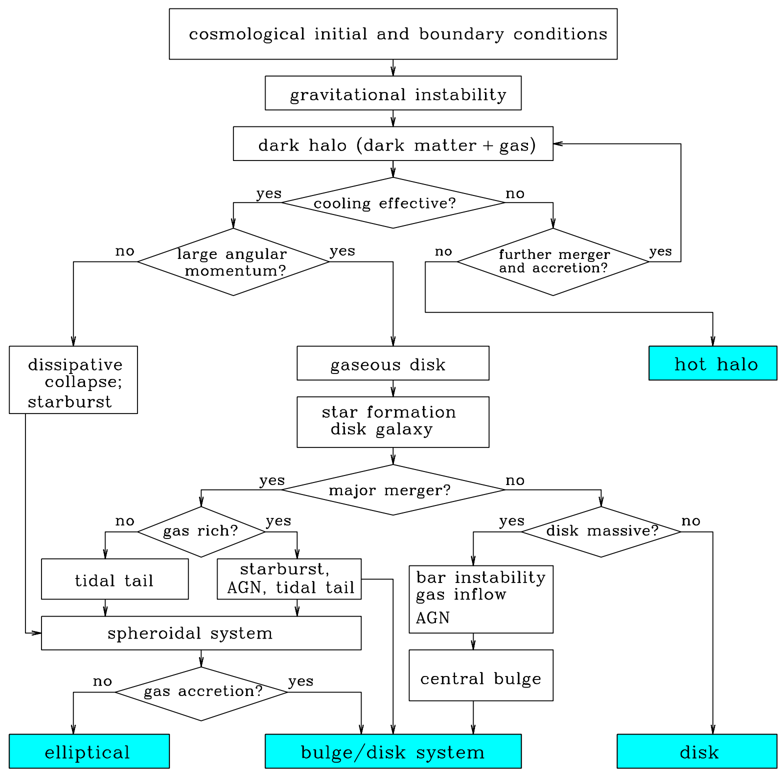
Reference: `Galaxy formation and evolution'
Mo, van den Bosch & White, 2010, Cambridge University Press
(see http://www.astro.yale.edu/vdbosch/book.html)

Outline

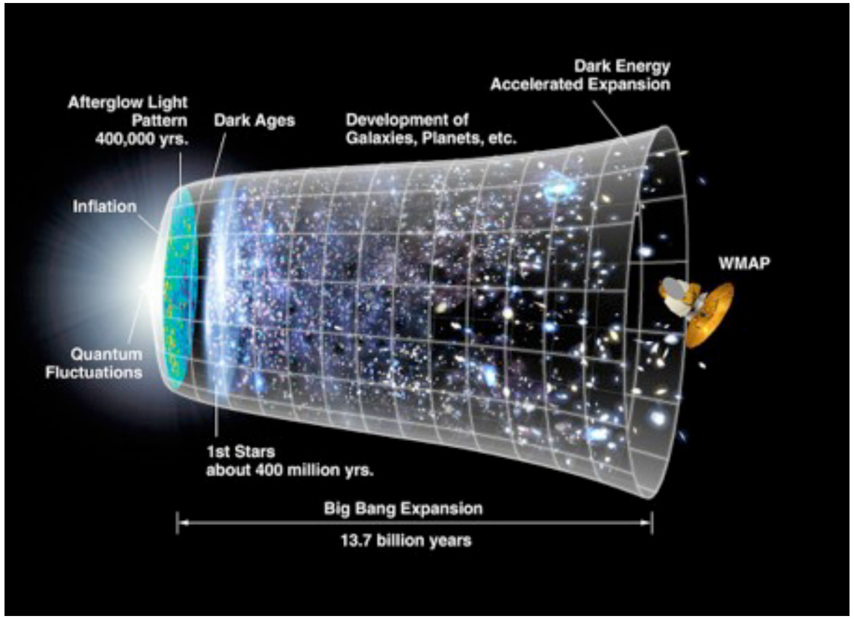
- The basic paradigm
- Formation and structure of CDM halos
- Formation of gaseous halos
- Formation of galaxies in CDM halos
- Statistical properties of the galaxy population

Galaxy formation Ingredients

- Cosmology: provides the initial and boundary conditions for galaxy formation
- Gravitational instability: drives large scale structure formation
- Formation of dark matter halos in the cosmic density field
- Formation of galaxies in dark matter halos: gas cooling, heating and accretion; star formation; feedback; galaxy merger
- Galaxy evolution: spectral synthesis; chemical evolution; dynamical evolution
- Galaxy populations: both low and high z



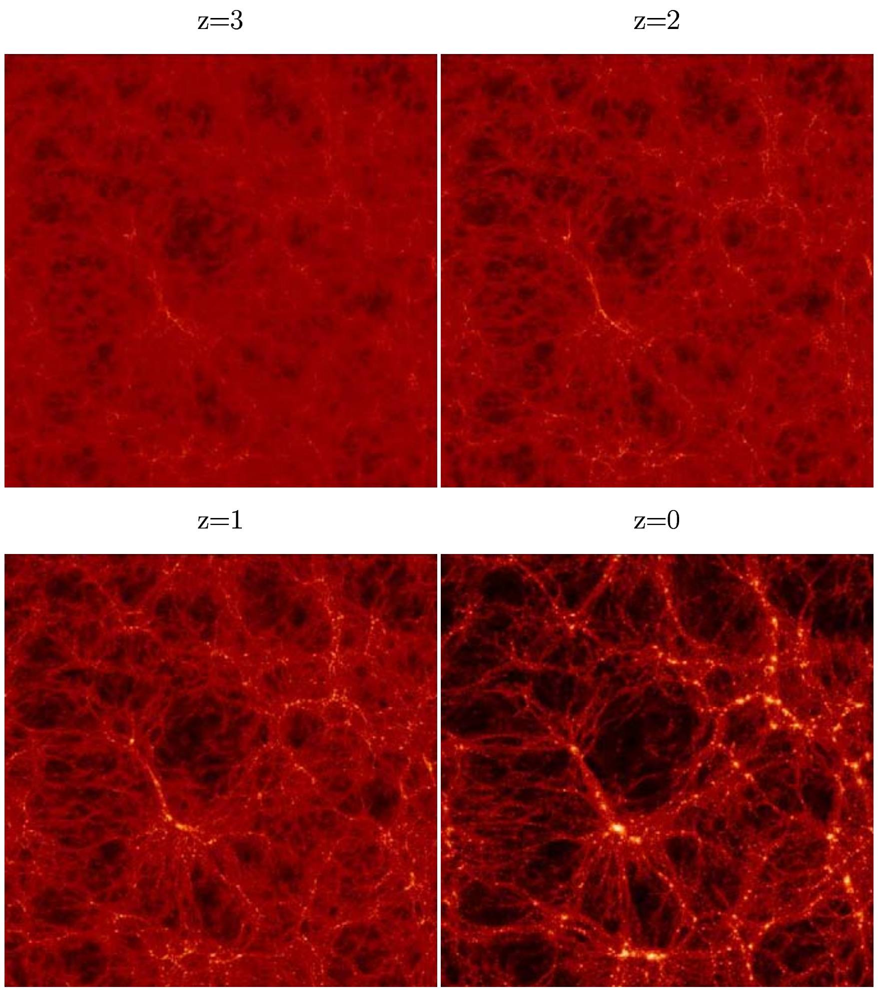
The standard paradigm

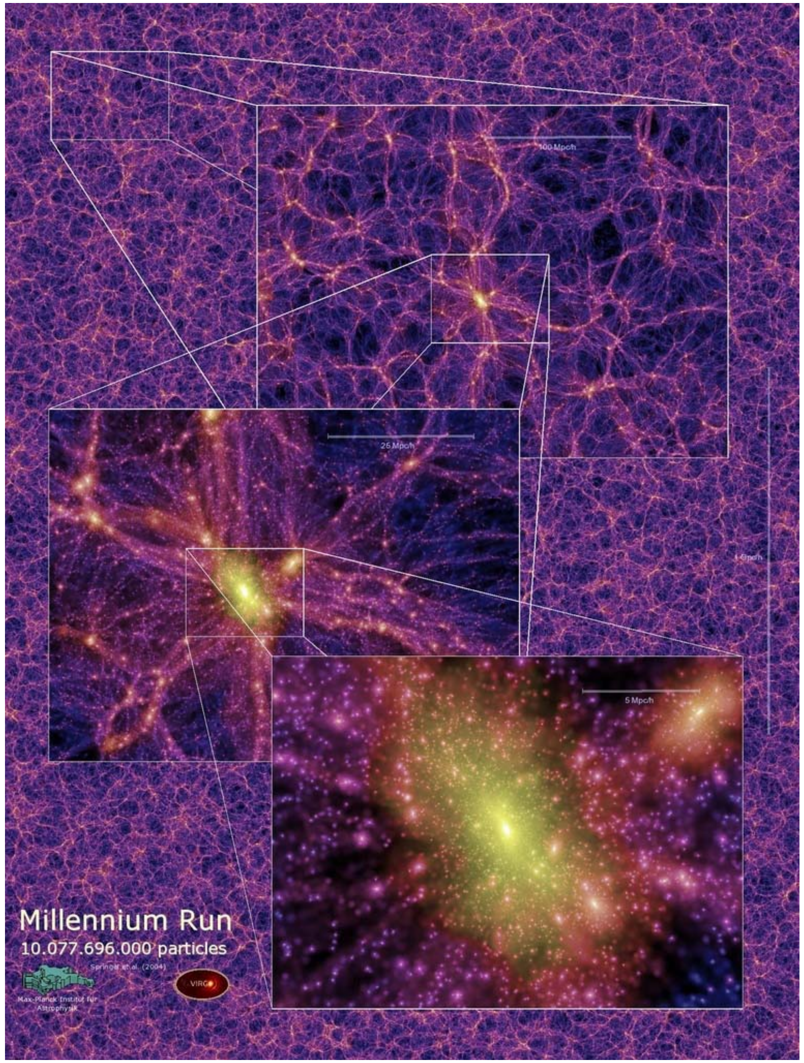


- Big Bang cosmology
- The Universe is dominated by cold dark matter and dark energy
- Primordial perturbations, probably generated by inflation, well constrained by observation (CMB; LSS etc)
- Cosmological models well constrained

$$k = 0;\ \Omega_m = 0.25;\ \Lambda = 0.75;\ h = 0.7;\ \Omega_b = 0.04;\ n = 1$$

Numerical simulation results





Brief review of structure formation

Gravitational instability: structure form from small perturbation due to action of gravity

Gaussian perturbations, completely specified by the linear spectrum $P(k)$:

$$\mathcal{P}_k(|\delta_k|,\phi_k)\, d|\delta_k|\, d\phi_k = \exp\left[-\frac{|\delta_k|^2}{2V_u^{-1}P(k)}\right]\frac{|\delta_k|\, d|\delta_k|}{V_u^{-1}P(k)}\frac{d\phi_k}{2\pi}.$$

Initial perturbation spectrum:

$$P_i(k) = Ak^n \quad (n\sim 1)\,.$$

Linear Power spectrum

$$P(k) = P_i(k)T^2(k)\,,$$

$T(k)$: linear transfer function describing linear evolution in EdS phase.

Power spectrum normalization: $\sigma_8 = \sigma(8h^{-1}\mathrm{Mpc})$, with

$$\sigma^2(R) = \frac{1}{2\pi^2}\int P(k)\hat{W}_R^2(k)k^2\mathrm{d}k\,;$$

$$\hat{W}_R(k) = \frac{3}{(kR)^2}\left[\sin(kR) - kR\cos(kR)\right]\,.$$

Evolution of perturbations

Linear Growth factor: $\delta(t) \propto D(t) = a(t)g(t)$:

$$g(z) \approx \frac{5}{2}\Omega_m(z)\left\{\Omega_m^{4/7}(z) - \Omega_\Lambda(z) + \left[1 + \frac{\Omega_m(z)}{2}\right]\left[1 + \frac{\Omega_\Lambda(z)}{70}\right]\right\}^{-1}$$

Nonlinear evolution: in general analytical solution difficult to obtain

For special cases:

spherical collapse model

ellipsoidal collapse model

Spherical collapse model

Consider $\Lambda = 0$ as an example. The radius of a mass shell obeys:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2} \,,$$

where M is the mass within r. Before shell crossing, M is constant:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 - \frac{GM}{r} = \mathcal{E} \,, \quad \text{(<0 for collapse)}$$

where $\mathcal{E}$ is the specific energy of the mass shell, determined by the initial condition of the mass shell,

$$r_i, \quad \text{and} \quad v_i \;\; (\text{or} \;\; \delta_i) \,,$$

or equivalently

$$r_l(t) = [a(t)/a_i] r_i, \quad \text{and} \quad \delta_l(t)) = [a(t)g(t)/a_i g_i] \delta_i \,.$$

The solution is:

$$\frac{r}{r_l(t)} = \frac{1}{2}\frac{1-\cos\theta}{\left[5\delta_l(t)/3g_t + (1-\Omega_t^{-1})\right]}\;;$$

$$H_t t = \frac{1}{2\Omega_t^{1/2}}\frac{\theta-\sin\theta}{\left[5\delta_l(t)/3g_t + (1-\Omega_t^{-1})\right]^{3/2}}\,,$$

where $H_t \equiv H(t)$, $\Omega_t = \Omega(t)$, $g_t = g(t)$.

Turn-around:

$$r_{\max}/r_l(t) = \left[5\delta_l(t)/3g_t + (1-\Omega_t^{-1})\right]^{-1}\;;\quad H_t t_{\max} = \frac{\pi}{2\Omega_t^{1/2}}\left[5\delta_l(t)/3g_t + (1-\Omega_t^{-1})\right]^{-3/2}\,.$$

If turn-around occur at $t_{\rm ta}$, then

$$\delta_l(t_{\rm ta}) = \frac{3g(t_{\rm ta})}{5}\left\{\left[\frac{\pi}{2\Omega^{1/2}(t_{\rm ta})H(t_{\rm ta})t_{\rm ta}}\right]^{2/3} - \left[1-\Omega^{-1}(t_{\rm ta})\right]\right\}\,.$$

For $\Omega = 1$, $\delta_l(t_{\rm ta}) = \frac{3}{5}\left(\frac{3\pi}{4}\right)^{2/3} \approx 1.06$, and the real density at this time is

$$\rho(t_{\rm ta}) = \overline{\rho}(t_{\rm ta})\left[\frac{r_l(t_{\rm ta})}{r_{\max}(t_{\rm ta})}\right]^3 = \left(\frac{3\pi}{4}\right)^2\overline{\rho}(t_{\rm ta}) \approx 5.55\overline{\rho}(t_{\rm ta})\,.$$

Gravitational Collapse

A mass shell is said to collapse at a time when $r = 0$, i.e. when at $t = 2t_{\rm max}$ (corresponding to $\theta = 2\pi$).

For a mass shell to collapse at time $t_{\rm col}$ requires

$$\delta_{\rm c}(t_{\rm col}) = \frac{3g(t_{\rm col})}{5}\left\{\left[\frac{\pi}{\Omega^{1/2}(t_{\rm col})H(t_{\rm col})t_{\rm col}}\right]^{2/3} - \left[1-\Omega^{-1}(t_{\rm col})\right]\right\}.$$

Since $H(t)t$ and $g(t)$ depend on t only through only on $\Omega(t)$ in the matter dominated epoch, we have

$$\delta_{\rm c}(t_{\rm col}) \approx \frac{3}{5}\left(\frac{3\pi}{2}\right)^{2/3}\left[\Omega(t_{\rm col})\right]^{0.0185} \approx 1.686\,.$$

This is also true for a flat universe with a cosmological constant.

Link non-linear collapse to linear theory

Virialization and dark halos

Virial theorem: for a steady state $2K + W = 0\,, \quad E = -K = \frac{W}{2}\,,$
where K: kinetic energy; W: gravitational potential energy, $E = K + W$.
Application to spherical collapse, assuming uniform sphere:

$$E = -\frac{3GM^2}{5r_{\rm max}}$$

After collapse suppose the radius settles at $r_{\rm vir}$, then

$$W = -\frac{3GM^2}{5r_{\rm vir}} = 2E\,, \quad \text{so that} \quad r_{\rm vir} = r_{\rm max}/2\,.$$

Assume collapse at $t_{\rm vir} = t_{\rm max}/2$, the mean over-density in $r_{\rm vir}$ is

$$1 + \Delta_{\rm vir} = \frac{\rho(t_{\rm max})(r_{\rm max}/r_{\rm vir})^3}{\bar\rho(t_{\rm vir})} = \frac{\rho(t_{\rm max})}{\bar\rho(t_{\rm max})}\frac{\bar\rho(t_{\rm max})}{\bar\rho(2t_{\rm max})}\left(\frac{r_{\rm max}}{r_{\rm vir}}\right)^3\,,$$

where $\bar\rho(t)$ is the background density at t. For $\Omega = 1$, $\Delta_{\rm vir} = 18\pi^2 \approx 178$.

$$\Delta_{\rm vir} \approx (18\pi^2 + 60x - 32x^2)/\Omega_{\rm m}(t_{\rm vir}) \quad (\Lambda = 0);$$

$$\Delta_{\rm vir} \approx (18\pi^2 + 82x - 39x^2)/\Omega_{\rm m}(t_{\rm vir}) \quad (\Omega_{\rm m} + \Lambda = 1)\,,$$

where $x = \Omega_{\rm m}(t_{\rm vir}) - 1$ (Bryan & Norman 1998).

Halo mass function: the Press-Schechter (PS) formalism

Spherical collapse model applied to Gaussian density field

We want $F(>M)$, the mass fraction in collapse objects (halos) with masses $>M$ at time t, because the mass function can then be obtained from

$$n(M,t)\,\mathrm{d}M = \frac{\overline{\rho}}{M}\,\frac{\partial F(>M)}{\partial M}\,\mathrm{d}M\,.$$

Now consider the smoothed density fluctuations:

$$\delta_s(\mathbf{x};R) \equiv \int \delta_0(\mathbf{x}')W(\mathbf{x}+\mathbf{x}';R)\,\mathrm{d}^3\mathbf{x}'\,.$$

For Gaussian filed:

$$\mathcal{P}(\delta_s) = \frac{1}{\sqrt{2\pi}\sigma(M)}\exp\left[-\frac{\delta_s^2}{2\sigma^2(M)}\right]\,,$$

where

$$\sigma^2(M) = \langle\delta_s^2(\mathbf{x};R)\rangle = \frac{1}{2\pi^2}\int_0^\infty P(k)\,\tilde{W}^2(\mathbf{k}R)\,k^2\,\mathrm{d}k\,,$$

with $M=\overline{\rho}V(R)$, is the mass variance of the smoothed field.

The fraction of regions with density contract $>\delta_c(t)$ is

$$\mathcal{P}[>\delta_c(t)] = \int_{\delta_c(t)}^\infty \mathcal{P}(\delta_s)\mathrm{d}\delta_s = \frac{1}{2}\,\mathrm{erfc}\left[\frac{\delta_c(t)}{\sqrt{2}\sigma(M)}\right]\,.$$

Can we set $F(>M) = \mathcal{P}[>\delta_c(t)]$? No, because $\mathcal{P}\to 1/2$ as $M\to 0$.

The PS ansatz

PS argued that material initially in under-dense will eventually be accreted by the collapsed objects, doubling their masses

So PS set $F(> M) = 2\mathcal{P}[> \delta_c(t)]$, obtaining the PS mass function

$$n(M,t)\,\mathrm{d}M = \frac{\overline{\rho}}{M^2}\,f_{\mathrm{PS}}(\nu)\,\mathrm{d}M\,,$$

where

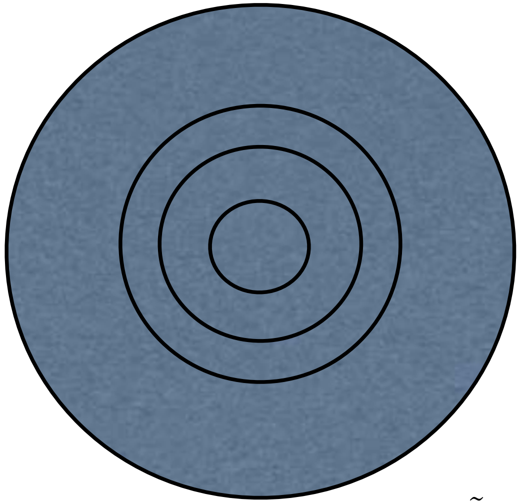
$$f_{\mathrm{PS}}(\nu) = \sqrt{\frac{2}{\pi}}\,\nu\,\exp(-\nu^2/2)\left|\frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln M}\right|\,,$$

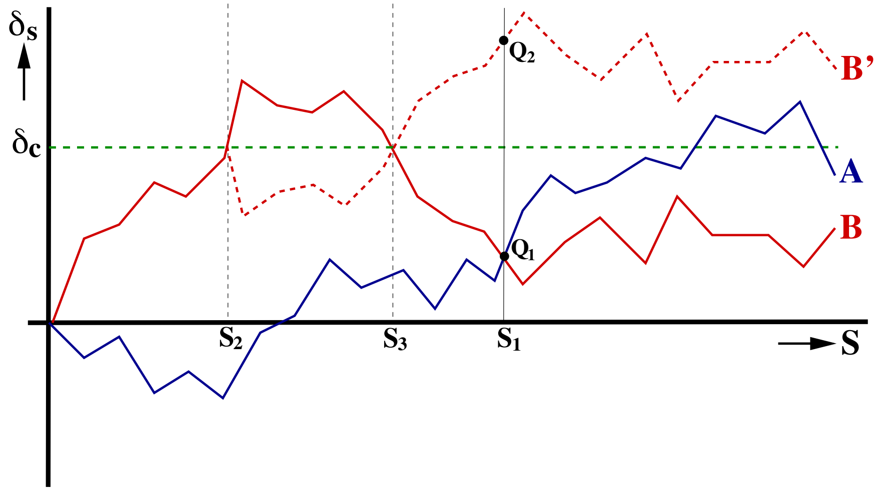
with

$$\nu = \delta_c(t)/\sigma(M)\,.$$

Time enters only through $\delta_c(t)$, and mass through $\sigma(M)$ and its derivative.

Excursion set derivation of the PS formula





Adopting top-hat filter in k-space: $\tilde{W}_k(k/k_c) = 1$ for $k \leq k_c$ and $= 0$ otherwise, where k_c is a cutoff wavenumver, corresponding to a smooth scale $R = 1/k_c$. Using $S \equiv \sigma^2(M)$,

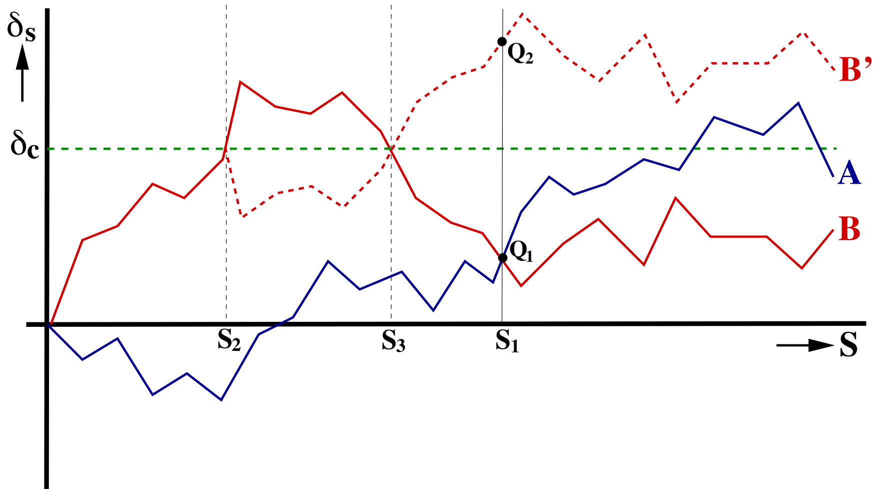
$$\langle(\Delta\delta_s)^2\rangle = \langle[\delta_s(\mathbf{x}; k_c + \Delta k_c) - \delta_s(\mathbf{x}; k_c)]^2\rangle = \frac{1}{2\pi^2}\int_{k=k_c}^{k=k_c+\Delta k_c} P(k)\, k^2\, \mathrm{d}k = \sigma^2(k_c{+}\Delta k_c){-}\sigma^2(k_c) = \Delta S$$

Thus, for a Gaussian field

$$P(\Delta\delta_s|\delta_s)\mathrm{d}(\Delta\delta_s) = \frac{1}{\sqrt{2\pi\Delta S}}\exp\left[-\frac{(\Delta\delta_s)^2}{2\Delta S}\right]\,\mathrm{d}(\Delta\delta_s)\,.$$

As S increases (or M decreases), δ_s executes an uncorrelated random walk.

The mass function from excursion set



We need $F(>M_1) = 1 - F(<M_1)$. Note that

$$F(<M_1) = F_{\rm FU}(>S_1)$$

where $F_{\rm FU}(>S_1)$ is the fraction of trajectories with first first up-crossing at $S > S_1$:

$$F_{\rm FU}(>S_1) = \int_{-\infty}^{\delta_c} \left[\mathcal{P}(\delta_s, S_1) - \mathcal{P}(2\delta_c - \delta_s, S_1)\right] \mathrm{d}\delta_s = \int_{-\infty}^{\nu_1} \frac{\mathrm{d}x}{\sqrt{2\pi}} e^{-x^2/2} - \int_{\nu_1}^{\infty} \frac{\mathrm{d}x}{\sqrt{2\pi}} e^{-x^2/2}\,.$$

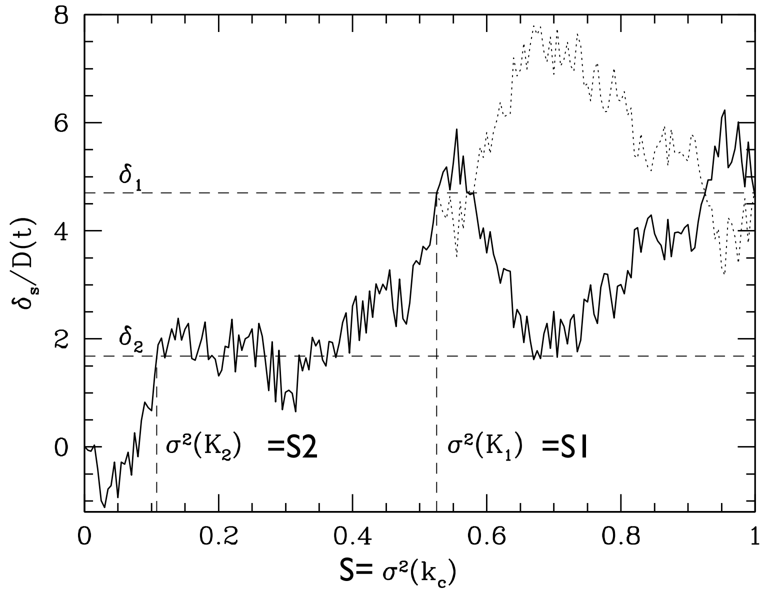
Thus

$$f_{\rm FU}(S,\delta_c)\,\mathrm{d}S = \frac{\partial F_{\rm FU}}{\partial S}\,\mathrm{d}S = \frac{1}{\sqrt{2\pi}}\frac{\delta_c}{S^{3/2}}\exp\left[-\frac{\delta_c^2}{2S}\right]\,\mathrm{d}S = f_{\rm PS}(\nu)\mathrm{d}\ln\nu$$

Halo Progenitors and Merger Trees

Application of excursion set model:

Progenitor mass distribution;
Halo merger trees (formation histories);
Halo spatial clustering.



Conditional random walk:

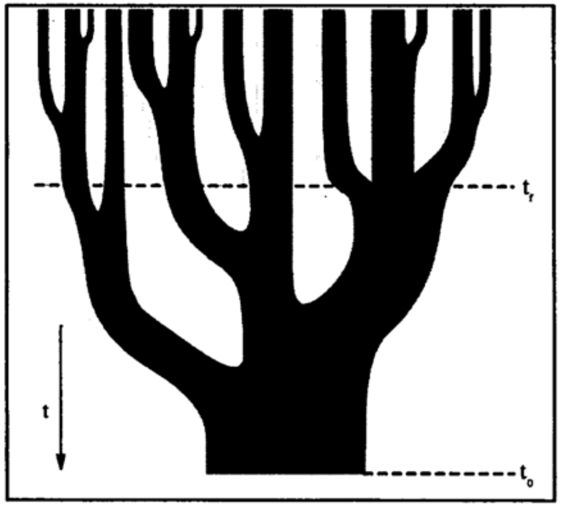
$$f_{\rm FU}(S_1,\delta_1|S_2,\delta_2)\,\mathrm{d}S_1=\frac{1}{\sqrt{2\pi}}\frac{\delta_1-\delta_2}{(S_1-S_2)^{3/2}}\exp\left[-\frac{(\delta_1-\delta_2)^2}{2\,(S_1-S_2)}\right]\,\mathrm{d}S_1\,.$$

Progenitor mass distribution:

$$n(M_1,t_1|M_2,t_2)\,\mathrm{d}M_1=\frac{M_2}{M_1}\,f_{\rm FU}(S_1,\delta_1|S_2,\delta_2)\,\left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right|\,\mathrm{d}M_1\,.$$

This provide a way to construct halo merger trees.

Constructing halo merger trees



A simple example, binary trees:

$$M \to M_p + \Delta M$$

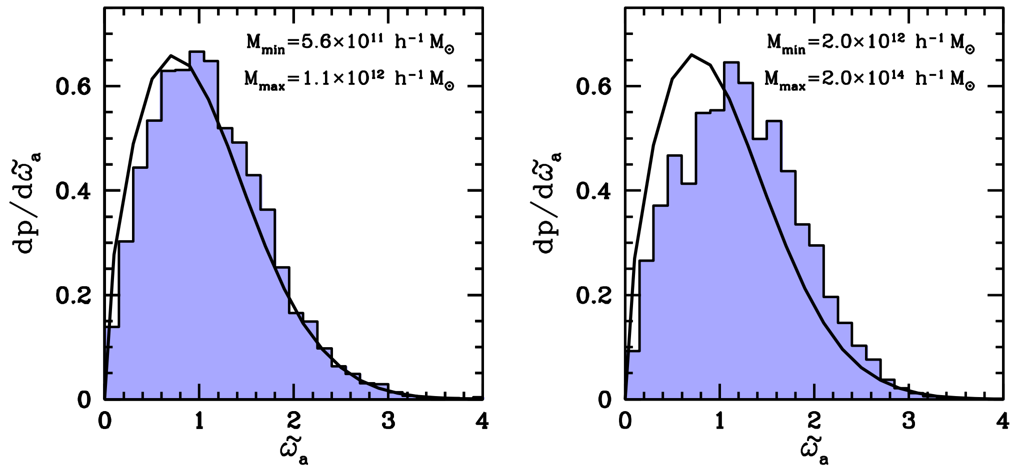
The probability of the splitting is given by

$$f_{\rm FU}(\Delta S, \Delta\delta)\,{\rm d}\Delta S = \frac{1}{\sqrt{2\pi}}\frac{\Delta\delta}{\Delta S^{3/2}}\,\exp\left[-\frac{\Delta\delta^2}{2\Delta S}\right]\,{\rm d}\Delta S\,,$$

$\Delta\delta = \delta_c(t-\Delta t) - \delta_c(t)$ reflects time step of the merger tree.

Halo assembly time t_f when a halo first obtain half of its mass:

$$\mathcal{P}(t_f < t_{\rm a}|M_0, t_0) = \int_{M_0/2}^{M_0} \frac{M_0}{M_1} f_{\rm FU}(S_1, \delta_1|S_0, \delta_0)\left|\frac{{\rm d}S_1}{{\rm d}M_1}\right|\,{\rm d}M_1\,.$$



Spatial clustering, halo bias

Average number of (M_1, t_1) halos in a spherical region (M_0, t_0):

$$N(1|0)\,\mathrm{d}M_1 = \frac{M_0}{M_1} f(1|0) \left| \frac{\mathrm{d}S_1}{\mathrm{d}M_1} \right| \mathrm{d}M_1\,,$$

where

$$f(1|0)\,\mathrm{d}S_1 \equiv \frac{1}{\sqrt{2\pi}} \frac{\delta_1 - \delta_0}{(S_1 - S_0)^{3/2}} \exp\left[-\frac{(\delta_1 - \delta_0)^2}{2\,(S_1 - S_0)} \right] \mathrm{d}S_1\,.$$

Define a bias factor:

$$\delta_{\mathrm{h}}^{\mathrm{L}}(1|0) = \frac{N(1|0)}{n(M_1, z_1) V_{\mathrm{L}}} - 1\,, \quad \text{where} \quad V_{\mathrm{L}} \equiv \frac{4\pi}{3} R_0^3\,.$$

For $M_0 \gg M_1$ ($S_0 \ll S_1$) and $|\delta_0| \ll \delta_1$:

$$\delta_{\mathrm{h}}^{\mathrm{L}}(1|0) = \frac{\nu_1^2 - 1}{\delta_1} \delta_0\,, \quad \text{where} \quad \nu_1 = \frac{\delta_1}{\sqrt{S_1}}\,.$$

In evolved space

$$\delta_{\mathrm{h}}(1|0) = \frac{N(1|0)}{n(M_1, z_1) V_{\mathrm{L}}} \frac{V_{\mathrm{L}}}{V_{\mathrm{E}}} - 1\,,$$

where $V_{\mathrm{L}}/V_{\mathrm{E}} = 1 + \delta(t)$, where δ is the average mass over-density of the evolved region. Thus

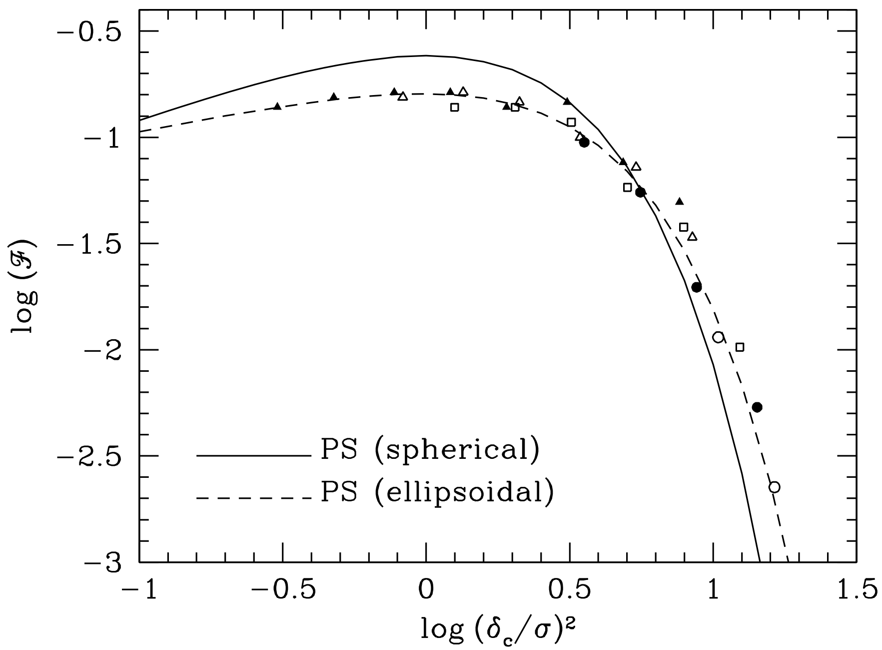
$$\delta_{\mathrm{h}}(1|0) = \delta(t) + \frac{\nu_1^2 - 1}{\delta_1} \delta_0 + \frac{\nu_1^2 - 1}{\delta_1} \delta_0 \delta(t)\,.$$

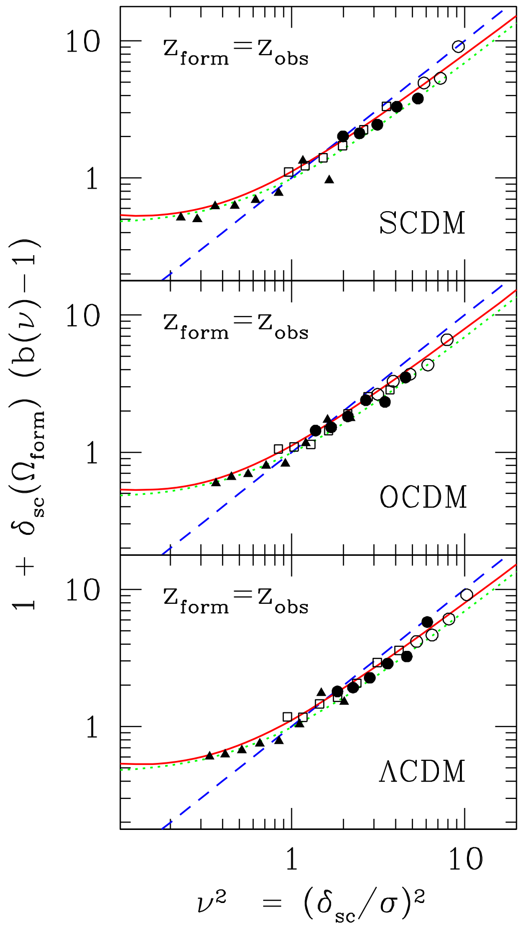
In the linear regime, $\delta(t) = D(t)\delta_0 \ll 1$:

$$\delta_{\mathrm{h}}(1|0) = b_{\mathrm{h}}(M_1, \delta_1; t)\delta(t)\,, \quad \text{where} \quad b_{\mathrm{h}}(M_1, \delta_1; t) = 1 + \frac{1}{D(t)} \left(\frac{\nu_1^2 - 1}{\delta_1} \right).$$

$b_{\mathrm{h}} = 1$ for $\nu_1 = 1$
i.e. $M = M^*$
where $\sigma(M^*) = \delta_c$
b_{h} increases with M

Comparisons with Numerical Simulations





Models clearly not perfect!

Elliptical collapse, an improvement

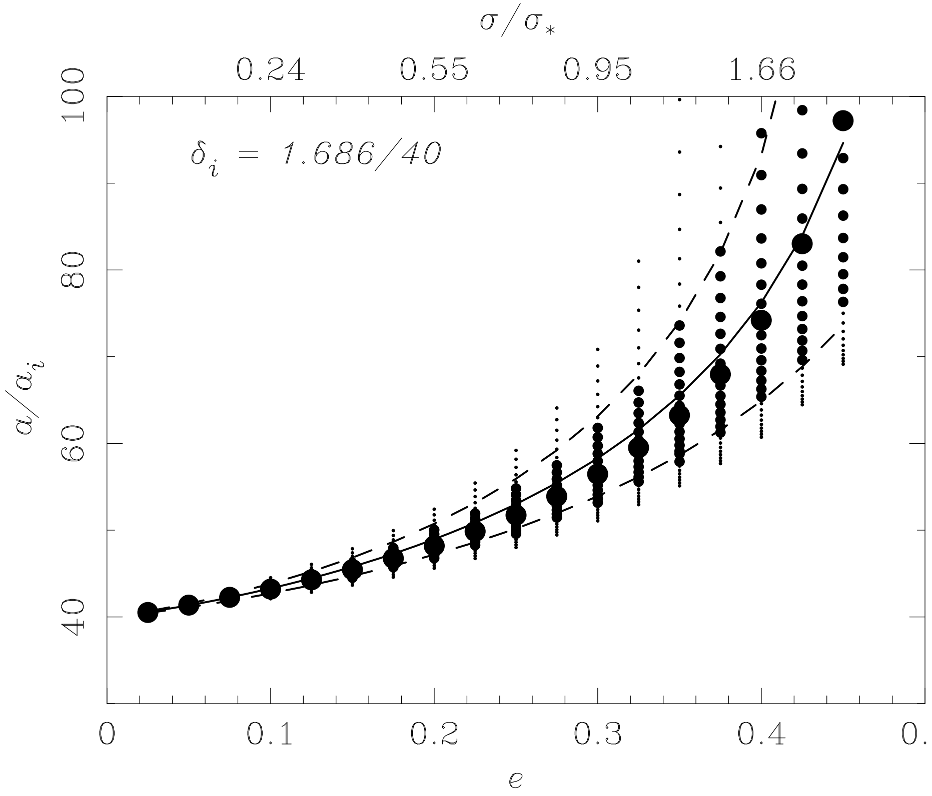
Basic idea (Sheth, Mo, Tormen 2001)

(i) Collapse of dark halos depends not only on over-density, but also on the shape of the tidal field: so ellipsoidal collapse instead of spherical collapse

(ii) In a Gaussian density field the shape (ellipticity) of the tidal field depends on the mass of the region in consideration

(iii) The threshold over-density for collapse depends on the mass scale in consideration

(iv) Excursion set with a moving barrier

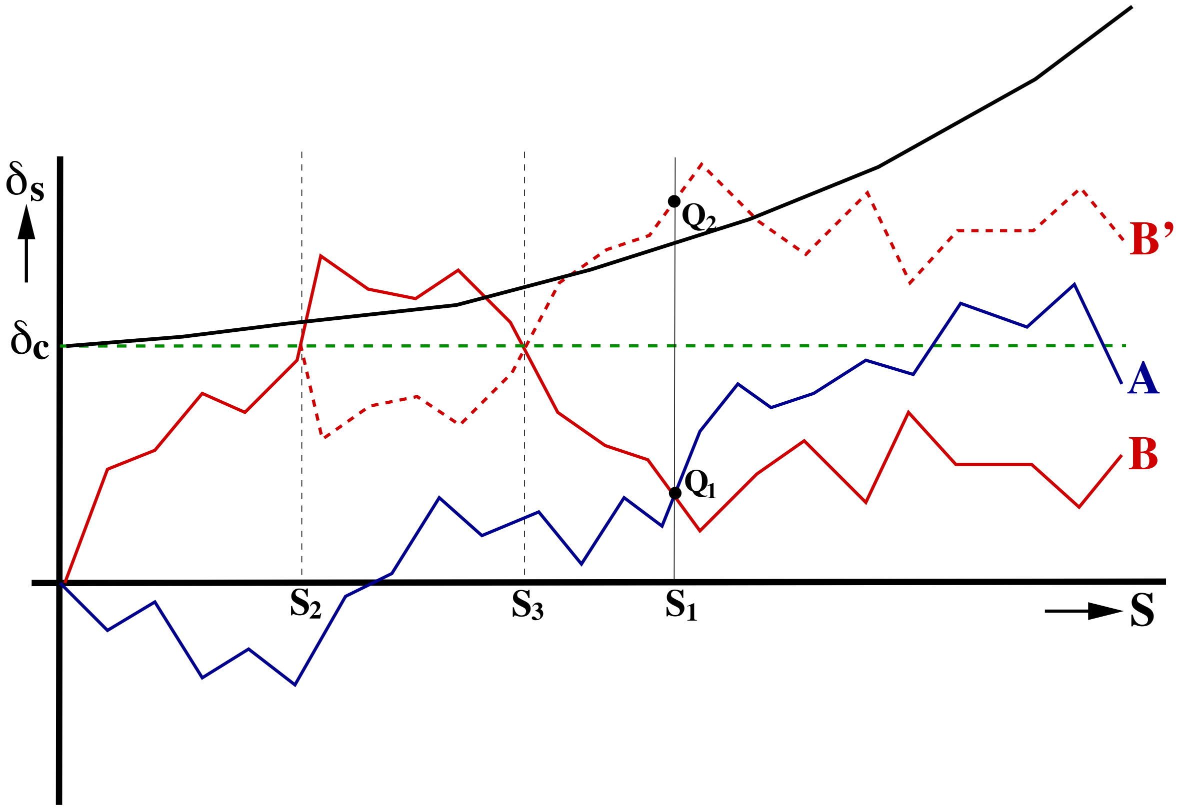


Mass-dependent collapse threshold:

$$\frac{\delta_{\rm ec}(M)}{\delta_{\rm sc}} = 1 + \beta \left[\frac{\sigma^2(M)}{\delta_{\rm sc}^2}\right]^\gamma$$

where $\beta \approx 0.5$, $\gamma \approx 0.6$.

Excursion set with moving barrier



Elliptical versus spherical model

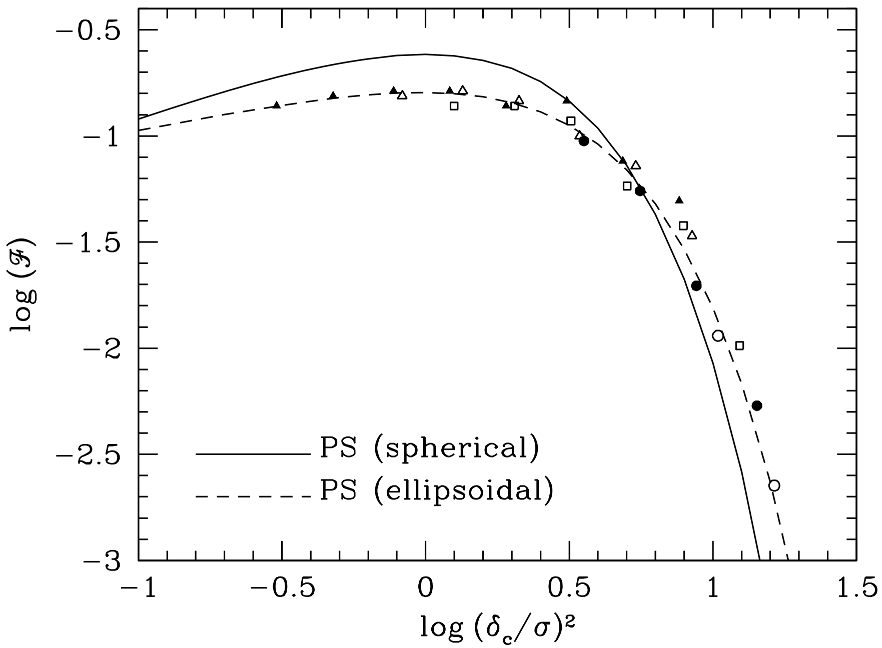
Mass function:

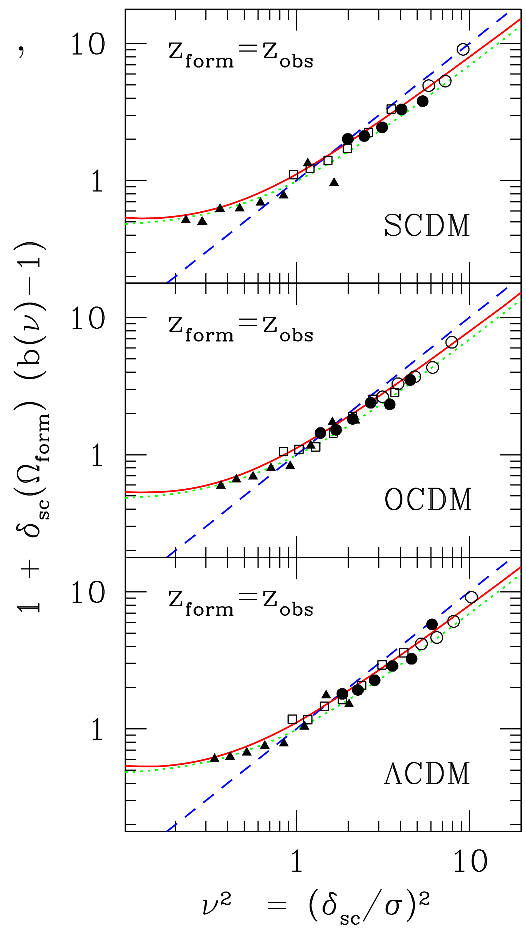
$$f_{\rm EC}(\nu) = A\left(1+\frac{1}{\tilde{\nu}^{2q}}\right) f_{\rm PS}(\tilde{\nu})\,,$$

where $\tilde{\nu} = 0.84\nu$ and $q = 0.3,\ A \approx 0.322$.
Halo bias:

$$b_{\rm h}(M_1,\delta_1,t) = 1 + \frac{1}{D(t)\delta_1}\left[{\nu_1'}^2 + b{\nu_1'}^{2(1-c)} - \frac{{\nu_1'}^{2c}/\sqrt{a}}{\nu'^{2c} + b(1-c)(1-c/2)}\right]\,,$$

where $\nu_1' = \sqrt{a}\nu_1$, $a = 0.707$, $b = 0.5$ and $c = 0.6$





Halo internal structure

Density profile
Angular momentum
Substructure
Shape

Most are based on N-body simulations

Dark matter halo profiles

Navarro, Frenk & White (NFW) profile:

$$\rho(r) = \rho_{\rm crit} \frac{\delta_{\rm char}}{(r/r_{\rm s})(1 + r/r_{\rm s})^2} \, ,$$

$r_{\rm s}$ a scale radius; $\delta_{\rm char}$ characteristic density contrast.

$$M(r) = 4\pi \overline{\rho} \delta_{\rm char} r_{\rm s}^3 \left[\ln(1 + c\,x) - \frac{c\,x}{1 + c\,x} \right] , \qquad V_c(r) = \sqrt{\frac{GM(r)}{r}} \, ,$$

where $x \equiv r/r_{\rm vir}$, and

$$c \equiv \frac{r_{\rm vir}}{r_{\rm s}}$$

is the halo concentration parameter.

$$\delta_{\rm char} = \frac{\Delta_{\rm vir}}{3} \frac{c^3}{\ln(1 + c) - c/(1 + c)} \, .$$

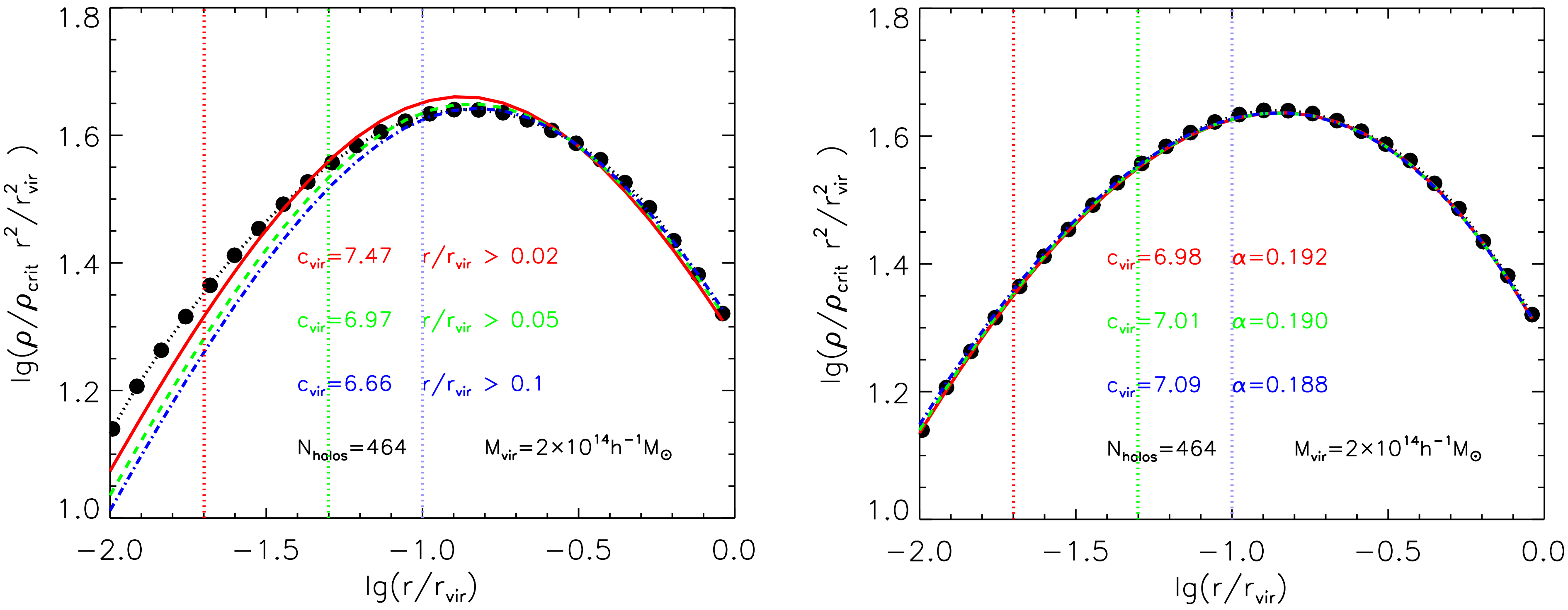
For given cosmology and M, NFW profile is characterized by c, which is found to be related to halo formation history. Zhao et al. (2009):

$$c(M,t) = 4 \times \left\{ 1 + \left[\frac{t}{3.75 t_{0.04}(M,t)} \right]^{8.4} \right\}^{1/8} \, ,$$

Compare with simulation results

NFW profile

Einasto profile



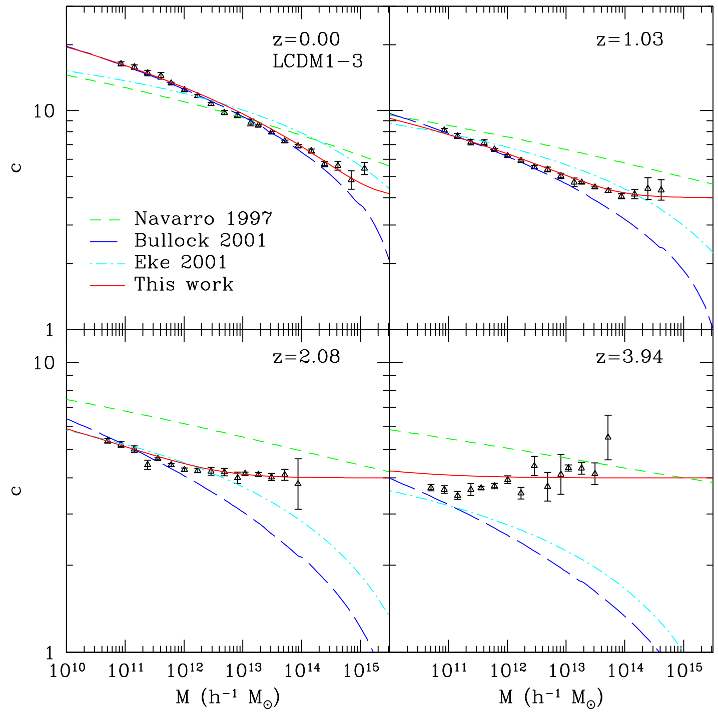
$$\rho(r) = \rho_{-2} \exp\left[\frac{-2}{\alpha}\left\{\left(\frac{r}{r_{-2}}\right)^{\alpha} - 1\right\}\right],$$

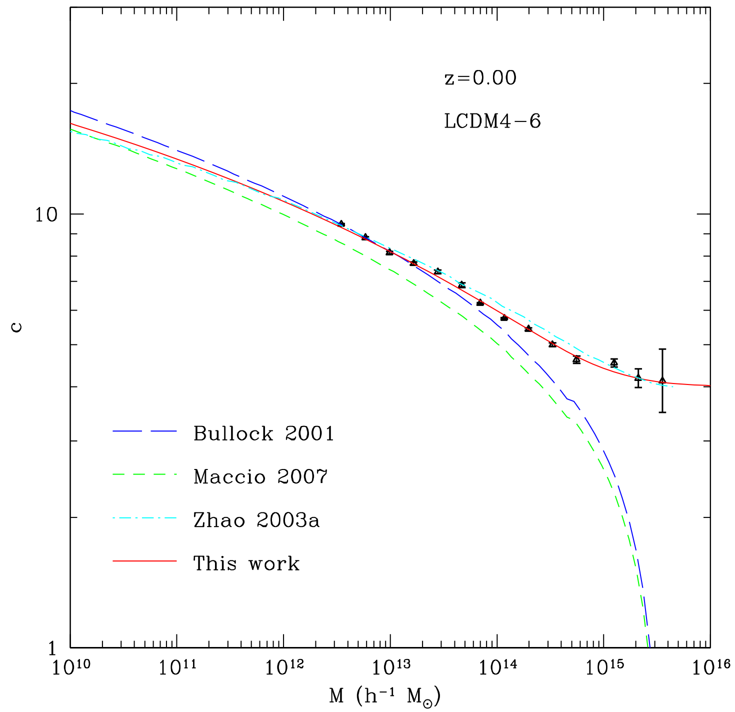
with $0.12 \leq \alpha \leq 0.25$

$c_{\rm vir} = r_{vir}/r_{-2}$

Gao et al. (2009)

Zhao et al. (2009)





Theoretical understanding of NFW profile and mass-dependence of c

Lu et al. (2006)

Halo angular momentum

Specified by a dimensionless spin parameter:

$$\lambda = \frac{J\,|E|^{1/2}}{G\,M^{5/2}}\,, \qquad \text{or alternatively} \qquad \lambda' = \frac{J}{\sqrt{2} M V_{\rm vir} r_{\rm vir}}\,,$$

where J is the total angular momentum, E is the total energy of the halo. For NFW profile with particles on circular orbits:

$$E = -4\pi \int_0^{r_{\rm vir}} \frac{\rho(r) V_c^2(r)}{2} r^2 {\rm d}r \equiv -\frac{M V_{\rm vir}^2}{2} F_{\rm E}\,,$$

$$F_{\rm E} = \frac{c}{2} \frac{\left[1 - 1/(1+c)^2 - 2\ln(1+c)/(1+c)\right]}{\left[c/(1+c) - \ln(1+c)\right]^2}\,.$$

So $\lambda' = \lambda F_{\rm E}^{-1/2}$ and $F_{\rm E} = 1$ for a singular isothermal profile $\rho(r) \propto r^{-2}$.

Spin parameter distribution:

$$p(\lambda)\,{\rm d}\lambda = \frac{1}{\sqrt{2\pi}\sigma_{\ln\lambda}} \exp\left[-\frac{\ln^2(\lambda/\overline{\lambda})}{2\sigma^2_{\ln\lambda}}\right] \frac{{\rm d}\lambda}{\lambda}\,.$$

with $\overline{\lambda} \approx 0.035$ and $\sigma_{\ln\lambda} \approx 0.5$.

Theoretical understanding: tidal torque by large-scale structure

$$\mathbf{J} = \int_{V_{\rm L}} {\rm d}^3\mathbf{x}_i\, \bar{\rho}_{\rm m} a^3 (a\mathbf{x} - a\overline{\mathbf{x}}) \times \mathbf{v} = \bar{\rho}_{\rm m} a^5 \int_{V_{\rm L}} {\rm d}^3\mathbf{x}_i\, (\mathbf{x} - \overline{\mathbf{x}}) \times \dot{\mathbf{x}}\,.$$

But not very successful.

Internal angular momentum distribution

Specific angular momentum (per unit mass) distribution within a halos (Bullock et al. 2001):

$$P(\mathcal{J}) = \begin{cases} \frac{\mu \mathcal{J}_0}{(\mathcal{J}_0+\mathcal{J})^2} & \text{if } \mathcal{J} \geq 0 \\ 0 & \text{if } \mathcal{J} < 0 \end{cases}$$

$$M(< \mathcal{J}) = \begin{cases} M_{vir} \frac{\mu \mathcal{J}}{\mathcal{J}_0+\mathcal{J}} & \text{if } \mathcal{J} \geq 0 \\ 0 & \text{if } \mathcal{J} < 0 \end{cases}$$

Define $\mathcal{J}_{\rm max}$ by $M(< \mathcal{J}_{\rm max}) = M_{\rm vir}$, then $\mathcal{J}_0 = (\mu - 1)\mathcal{J}_{\rm max}$. Note that

$$\mathcal{J}_{\rm tot} \equiv \int_0^{\mathcal{J}_{\rm max}} P(\mathcal{J}) {\rm d}\mathcal{J} = \zeta \mathcal{J}_{\rm max}\,; \quad \zeta = 1 - \mu \left[1 - (\mu - 1) \ln\left(\frac{\mu}{\mu - 1}\right)\right],$$

and that

$$\mathcal{J}_{\rm tot} = J/M = \sqrt{2} \lambda V_{\rm vir} r_{\rm vir} F_{\rm E}^{-1/2}\,.$$

So (λ, μ) specifies the angular momentum distribution. Bullock et al. (2001) found that μ has a lognormal distribution with $\overline{\mu} \simeq 1.25$ and $\sigma_{\ln \mu} \simeq 0.4$.

Some implications

For an isolated exponential disk, $\lambda \approx 0.425$. Consider a self-gravitating gas cloud of with initial spin $\lambda_i = 0.035$. Since M and J are conserved, and $E \propto 1/R$, we have

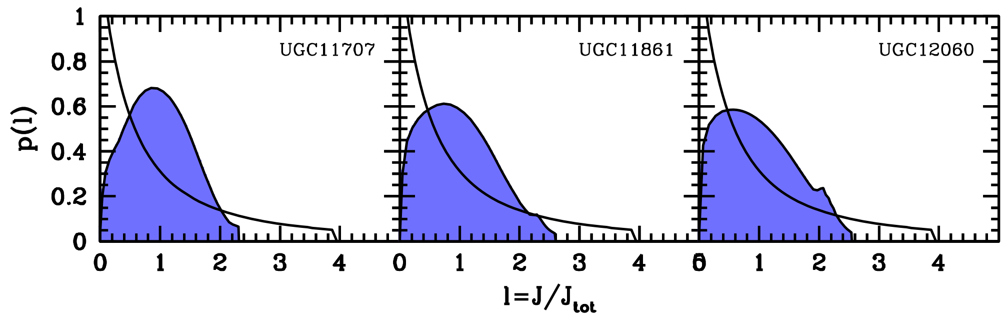
$$\lambda = \lambda_i (R/R_i)^{-1/2}, \qquad (R_i/R) \sim 70 \quad \text{to get } \lambda = 0.425 \text{ from } \lambda_i = 0.035.$$

Consider a disk of mass $M = 5 \times 10^{10} \mathrm{M}_\odot$ of radius $R = 10\mathrm{kpc}$. $R_i = 700\mathrm{kpc}$. The free fall time scale is

$$t_{\mathrm{ff}} = \sqrt{3\pi/32G\rho} \sim 43\mathrm{Gyr}\,.$$

This problem can be solved if disks form in dark matter halos

Angular momentum redistribution or selective assembly



Halo shape

In general, dark matter halos are not spherical, better described by elliptical shape (e.g. Jing & Suto 2002):

$$s = \frac{a_3}{a_1}, \quad q = \frac{a_2}{a_1}, \quad p = \frac{a_3}{a_2} \quad (a_1 \geq a_2 \geq a_3)\,.$$

The triaxiality parameter

$$T = \frac{a_1^2 - a_2^2}{a_1^2 - a_3^2} = \frac{1 - q^2}{1 - s^2}\,.$$

$T = 0$: oblate; $T = 1$: prolate. Simulations show:

$$0.5 < T < 0.85; \quad 0.5 < s < 0.75\,.$$

$$\langle s \rangle (M, z) = (0.54 \pm 0.03) \left[\frac{M}{M^*(z)} \right]^{-0.050 \pm 0.003},$$

with scatter in s is $\sigma_s \sim 0.1$.

The probability for p given s is

$$\mathcal{P}(p|s) = \frac{3}{2(1 - \tilde{s})} \left[1 - \left(\frac{2p - 1 - \tilde{s}}{1 - \tilde{s}} \right)^2 \right],$$

with $\tilde{s} = \max[s, 0.55]$.

Halo substructure; the sub-halo population

As a small halo merges into a larger halo it becomes a sub-halo orbiting within its host.

As it orbits, it is subjected to tidal forces from the host, causing it to lose mass

The orbit decays due to dynamical friction which causes the sub-halo to lose energy and angular momentum to the dark matter particles of its host.

Whether a sub-halo survives as a self-bound entity depends on its mass, density profile, and its orbit.

Mass function of survived halos:

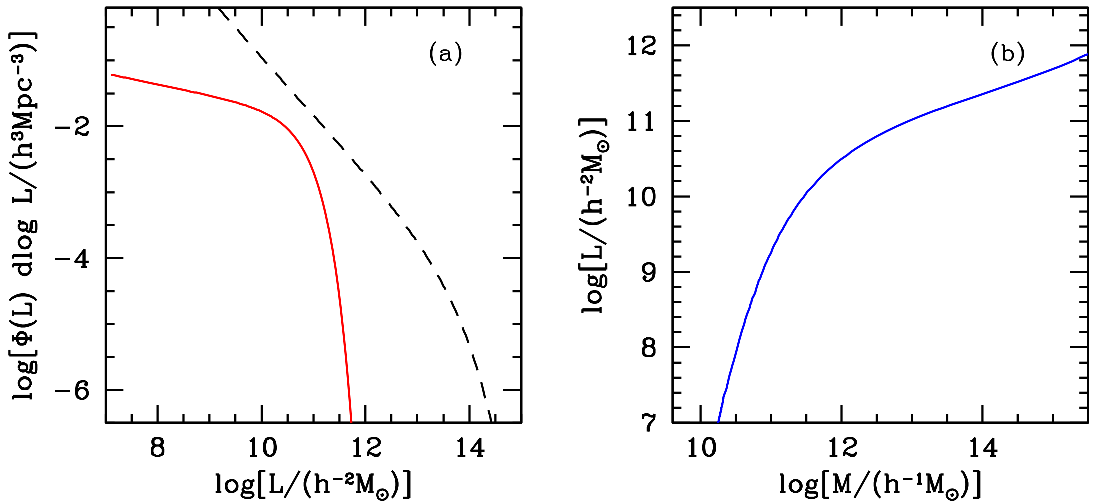
$$\frac{\mathrm{d}n}{\mathrm{d}\ln(m/M)} = \frac{f_0}{\beta\Gamma(1-\gamma)}\left(\frac{m}{\beta M}\right)^{-\gamma}\exp\left[-\left(\frac{m}{\beta M}\right)\right],$$

M: host mass; m: sub-halo mass; $\gamma = 0.9 \pm 0.1$, $0.1 < \beta < 0.5$, $f_0 \sim$ the mass fraction in sub-halos.
Un-evolved mass function (uses sub-halo mass at the time when it first becomes a sub-halo:

$$\frac{\mathrm{d}n}{\mathrm{d}\ln(m/M_0)} = A\left(\frac{m}{fM_0}\right)^{-p}\exp\left[-\left(\frac{m}{fM_0}\right)^{q}\right],$$

with $A \simeq 0.345$, $f \simeq 0.43$, $p \simeq 0.8$ and $q \simeq 3$.

Halo mass function and galaxy mass function



Although galaxies form in dark matter halos, the relation between galaxies and halos is not simple and can be affected by many processes.