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# Galaxy Formation

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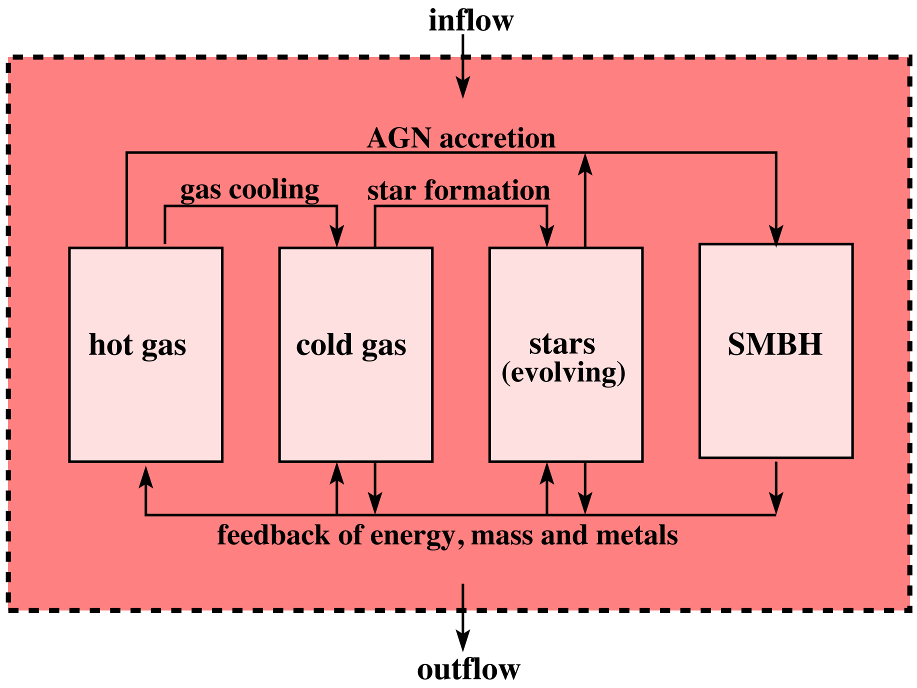
# Evolution of gas component

- Galaxy formation involves baryonic gas
- Gas heating and cooling
- Star formation and feedback
- Assembly of gas into galaxies



# Basic equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \text{(continuity)}\,;$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\, \mathbf{v} = -\left(\nabla \Phi + \frac{\nabla P}{\rho}\right); \qquad \text{(Euler)}$$

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{v^2}{2} + \mathcal{E}\right)\right] + \nabla \cdot \left[\rho\left(\frac{v^2}{2} + \frac{P}{\rho} + \mathcal{E}\right)\mathbf{v}\right] - \rho \mathbf{v} \cdot \nabla \Phi = \mathcal{H} - \mathcal{C} \qquad \text{(Energy)}\,.$$

$\rho$, $\mathbf{v}$, $P$, $\mathcal{E}$ are density, velocity, pressure and specific internal energy of the fluid; $\mathcal{H}$ and $\mathcal{C}$ are the heating and cooling rates per unit volume.

For an ideal gas with an adiabatic index $\gamma$, $P = \rho(\gamma - 1)\mathcal{E}$, and the energy can be replaced by

$$\frac{P}{\gamma - 1}\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \ln\left(\frac{P}{\rho^\gamma}\right) = \mathcal{H} - \mathcal{C} \qquad \text{(Entropy)}\,.$$

The gravitational potential $\Phi$ satisfies the Poisson equation:

$$\nabla^2 \Phi = 4\pi G \rho_{\rm tot}\,,$$

where $\rho_{\rm tot}$ is the total mass density of the universe including dark matter.

In the linear regime:

$$\delta_{\rm b}(\mathbf{k}, t) = \frac{\delta_{\rm dm}(\mathbf{k}, t)}{1 + k^2/k_{\rm J}^2}, \qquad \text{with} \qquad k_{\rm J}^2 = \frac{3a^2 H^2}{2c_{\rm s}^2}\,.$$

# Hydrostatic equilibrium

In this case gravitational forces are balanced by pressure gradients:

$$\nabla P(\mathbf{r}) = -\rho(\mathbf{r})\nabla\Phi(\mathbf{r})$$

where the gravitational potential satisfies the Poisson equation

$$\nabla^2\Phi = 4\pi G\left(\rho_{\rm dm} + \rho\right) .$$

The iso-potential surfaces are the same as the isobaric surfaces.

In spherical symmetry and for an ideal gas:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{\mathrm{d}(k_{\rm B}T\rho/\mu m_{\rm p})}{\mathrm{d}r}, \qquad \frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM(r)}{r^2},$$

$M(r)$: total mass within $r$; $\mu = \rho/(nm_{\rm p})$ is the mean molecular weight of the gas. The hydrostatic equation is then

$$M(r) = -\frac{k_{\rm B}T(r)r}{\mu m_{\rm p}G}\left[\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T}{\mathrm{d}\ln r}\right] .$$

This provides an estimate of the *total* mass of a halo from measurements of the density and temperature profiles, $\rho(r)$ and $T(r)$, of the gas.

# Gas density profile in a potential well

Hydrostatic equilibrium alone is not sufficient to determine the density distribution of the gas, because the dependence of the temperature on radius remains unspecified. Further assumptions about the gas is needed. Examples: (i) polytropic equation of state; T(r)=T.

Polytropic gas:

$$P = A\rho^{\Gamma} \,,$$

where $A$ and $\Gamma$ are constant.

Hydrostatic equilibrium gives

$$k_{\rm B}T(\mathbf{r}) = \frac{(1-\Gamma)}{\Gamma}\mu m_{\rm p}\,\Phi(\mathbf{r}) \,,$$

and other quantities follow from the equation of state:

$$\rho \propto T^{1/(\Gamma-1)} \,; \qquad P \propto T\rho \,.$$

# Isothermal spheres

In this case $T$ is independent of $r$, and hydrostatic equilibrium gives

$$\rho(r) = \rho_0 \exp\left(-\frac{\Phi}{c_T^2}\right) , \qquad c_T^2 \equiv \frac{k_{\rm B}T}{\mu m_{\rm p}} ,$$

and the Poisson equation gives

$$\frac{1}{r^2}\frac{{\rm d}}{{\rm d}r}\left(r^2\frac{{\rm d}\Phi}{{\rm d}r}\right) = 4\pi G\left[\rho_{\rm dm}(r) + \rho_0 \exp\left(-\frac{\Phi}{c_T^2}\right)\right] ,$$

which can be solved for given $\rho_{\rm dm}(r)$.

If $\rho_{\rm dm} = 0$, this is the Lane-Emden equation. Assume $\rho(r)$ to be a power law, the solution is:

$$\Phi(r) = \frac{2k_{\rm B}T}{\mu m_{\rm p}} \ln\frac{r}{r_0} , \qquad \rho(r) = \frac{2k_{\rm B}T}{\mu m_{\rm p}}\frac{1}{4\pi G r^2} , \qquad M(r) = \frac{2k_{\rm B}T}{\mu m_{\rm p}}\frac{r}{G} ,$$

with $r_0$ defined by $\Phi(r_0) = 0$. Defining the circular velocity of the gaseous sphere as $V_c \equiv [GM(r)/r]^{1/2}$, then

$$V_c^2 = \frac{2k_{\rm B}T}{\mu m_{\rm p}} , \qquad \rho(r) = \frac{V_c^2}{4\pi G r^2} , \qquad M(r) = \frac{V_c^2 r}{G} .$$

These are the properties of a singular isothermal sphere. Useful but not physical: $\rho(r=0) = \infty$ and $[{\rm d}\Phi/{\rm d}r](r=0) = \infty$.

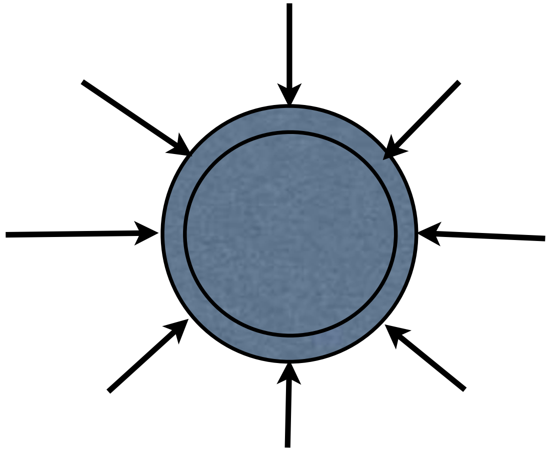
Assuming boundary conditions $\rho(r=0) = \rho_0$ and $[{\rm d}\Phi/{\rm d}r](r=0) = 0$, the solution can by approximated by the King profile:

$$\rho(r) = \frac{\rho_0}{[1+(r/r_0)^2]^{3/2}} , \qquad r_0 = \frac{3c_T}{\sqrt{4\pi G\rho_0}} ,$$

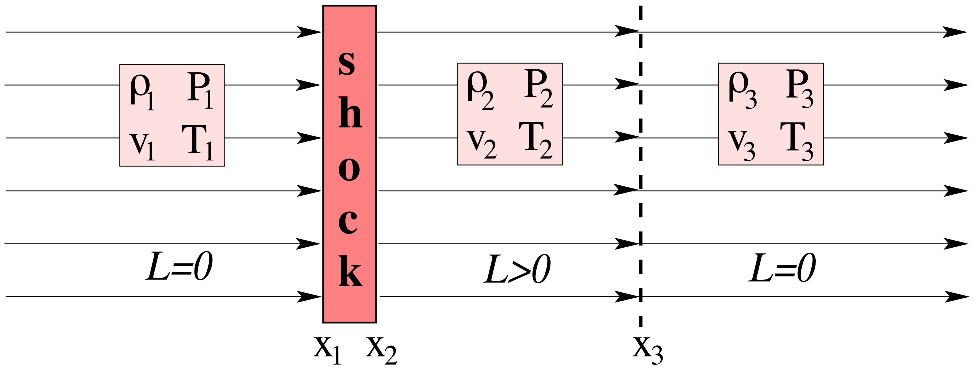
for $r < 2r_0$. For $r > 10r_0$, $\rho(r)$ approaches that of a singular isothermal sphere.

# Formation of Hot Gaseous Halos

Accretion shock:



Shock front:




Jump conditions:

$$\rho_2 v_2 = \rho_1 v_1\,; \qquad \rho_2 v_2^2 + P_2 = \rho_1 v_1^2 + P_1\,; \qquad \frac{1}{2}v_2^2 + \frac{P_2}{\rho_2} + \mathcal{E}_2 = \frac{1}{2}v_1^2 + \frac{P_1}{\rho_1} + \mathcal{E}_1\,,$$

where subscripts '1' and '2' denote quantities for the upstream and downstream, respectively.

These jump conditions (Rankine-Hugoniot jump conditions) can be written in terms of the Mach number of the upstream gas $\hat{M}_1 \equiv v_1/c_{\mathrm{s},1}$ (where $c_{\mathrm{s},1}^2 = \gamma P/\rho$ is the sound speed)

$$\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \left[\frac{1}{\hat{M}_1^2} + \frac{\gamma-1}{\gamma+1}\left(1 - \frac{1}{\hat{M}_1^2}\right)\right]^{-1}, \qquad \frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}\hat{M}_1^2 - \frac{\gamma-1}{\gamma+1};$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = \frac{\gamma-1}{\gamma+1}\left[\frac{2}{\gamma+1}\left(\gamma\hat{M}_1^2 - \frac{1}{\hat{M}_1^2}\right) + \frac{4\gamma}{\gamma-1} - \frac{\gamma-1}{\gamma+1}\right].$$

If $\hat{M}_1 > 1$, gas is compressed ($\rho_2 > \rho_1$ and $P_2 > P_1$), decelerated ($v_2 < v_1$) and heated ($T_2 > T_1$).

For strong shock where $\hat{M}_1 \gg, 1$

$$\rho_2/\rho_1 \to (\gamma+1)/(\gamma-1) \quad (= 4 \text{ for } \gamma = 5/3)\,;$$

$$T_2 \to [2(\gamma-1)/(\gamma+1)^2]\,(\mu m_\mathrm{p}/k_\mathrm{B})\,v_1^2 \quad (= 3\mu m_\mathrm{p} v_1^2/16 k_\mathrm{B} \text{ for } \gamma = 5/3).$$

# Heating by accretion shocks

Suppose infall velocity of accreted gas is $v_{\rm in}$ and shocked gas has zero velocity, then the upstream velocity $v_1 = v_{\rm in} + v_{\rm sh}$ and the downstream velocity $v_2 = v_{\rm sh}$, with $v_{\rm sh}$ the velocity of the shock front. Thus, the jump conditions give

$$\frac{k_{\rm B}T_2}{\mu m_{\rm p}} = \frac{v_{\rm in}^2}{16\gamma}\left[2\gamma\left(1+\sqrt{1+\epsilon}\right)^2 - (\gamma-1)\epsilon\right]\left[\frac{2\epsilon}{(1+\sqrt{1+\epsilon})^2} + (\gamma-1)\right],$$

where

$$\epsilon \equiv \frac{\gamma}{v_{\rm in}^2}\left(\frac{4}{\gamma+1}\right)^2 \frac{k_{\rm B}T_1}{\mu m_{\rm p}}.$$

If $v_1^2 \gg k_{\rm B}T_1/(\mu m_{\rm p})$ (i.e. $\epsilon \to 0$ and $T_1/T_2 \to 0$), we have

$$T_2 = (\gamma-1)\,T_{\rm vir}\left(\frac{v_{\rm in}}{V_c}\right)^2 \quad \text{and} \quad \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}.$$

Thus, if $v_{\rm in}$ is comparable to the circular velocity at the radius of the shock, $V_c$, the temperature of the shocked gas will be comparable to the virial temperature.

In general, one can write

$$\frac{1}{2}v_{\rm in}^2 = \frac{1}{2}v_{\rm ff}^2 + \Delta W - \frac{c_{\rm s1}^{\,2}}{\gamma-1}\left[1-\left(\frac{\rho_{\rm ta}}{\rho_{\rm sh}}\right)^{\gamma-1}\right].$$

If shock at the virial radius, then

$$\frac{1}{2}v_{\rm ff}^2 \equiv \frac{GM}{r_{\rm sh}} - \frac{GM}{r_{\rm ta}} \approx \frac{1}{2}V_c^2.$$

For cold accretion ($c_{\rm s1} \sim 0$) and no shell crossing:

$$T_2 = (\gamma-1)T_{\rm vir}.$$

# Formation of gaseous halos

Accretion shocks generate entropy for each mass shell:

$$S(M_{\rm gas}) = \frac{P(M_{\rm gas})}{\rho^{\gamma}(M_{\rm gas})} = \frac{k_{\rm B}}{\mu m_{\rm p}} \frac{T(M_{\rm gas})}{\rho^{\gamma-1}(M_{\rm gas})} \, .$$

If no cooling, this quantity is conserved. This can be combined with hydrostatic equilibrium:

$$\frac{{\rm d}P}{{\rm d}M_{\rm gas}} = \frac{G M_{\rm CDM}}{4\pi r^4} \quad \text{and} \quad \frac{{\rm d}r}{{\rm d}M_{\rm gas}} = \frac{1}{4\pi \rho_{\rm gas} r^2} \, ,$$

to solve for gas profile.

# Gas cooling and heating

## Radiative processes:

Free-free:

$\mathrm{p + e \to p' + e' + \gamma}$

Bound-free:

$\mathrm{H + \gamma \to p + e}$ (photoionization)

$\mathrm{p + e \to H + \gamma}$ (recombination)

Bound-bound:

$\mathrm{H + \gamma \to H^*}$ (photo-excitation)

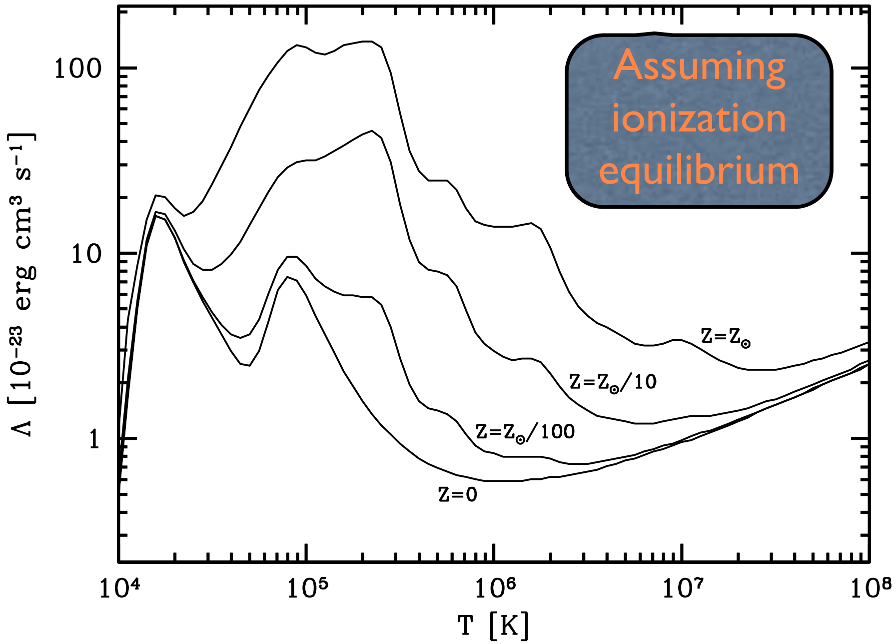
$\mathrm{H + e \to H^* + e'}$ (collisional excitation)

$\mathrm{H^* \to H + \gamma}$ (spontaneous decay)

Compton scattering:

$\mathrm{e + \gamma \to e' + \gamma'}$ (e.g. cooling against CMB)

## Cooling function: $\Lambda \equiv \mathcal{C}/n_\mathrm{H}^2$




Free-free cooling rate:

$$\mathcal{C}_\mathrm{ff} = \int \epsilon_\mathrm{ff}(\nu)\,\mathrm{d}\nu \approx 1.4\times10^{-23} T_8^{1/2}\left(\frac{n_\mathrm{e}}{\mathrm{cm}^{-3}}\right)^2 \mathrm{erg\,s^{-1}cm^{-3}}$$

Recombination cooling rate:

$$\mathcal{C}_a(T) = \frac{g_a}{g_{a+1}} n_\mathrm{e} n_{a+1}\left(\frac{2\pi m_\mathrm{e} k_\mathrm{B} T}{h_\mathrm{P}^2}\right)^{-3/2}\frac{4\pi}{c^2}\int_{\nu_a}^{\infty}\nu^2\sigma_\mathrm{pi}(\nu,a)h_\mathrm{P}(\nu-\nu_a)\exp\left[-\frac{h_\mathrm{P}(\nu-\nu_a)}{k_\mathrm{B}T}\right]\mathrm{d}\nu\,.$$

Due to collisional excitation and and de-excitation:

$$\mathcal{C}_{X,Y} = n_Y n_X \sum_{b<a}(E_a - E_b)\left[x_b\gamma_{ba}(X,Y) - x_a\gamma_{ab}(X,Y)\right]\,,$$

$x_a = n_a/n_X$; $\gamma_{ba}$ and $\gamma_{ab}$ are the excitation and de-excitation rate coefficients.

# Photoionization heating

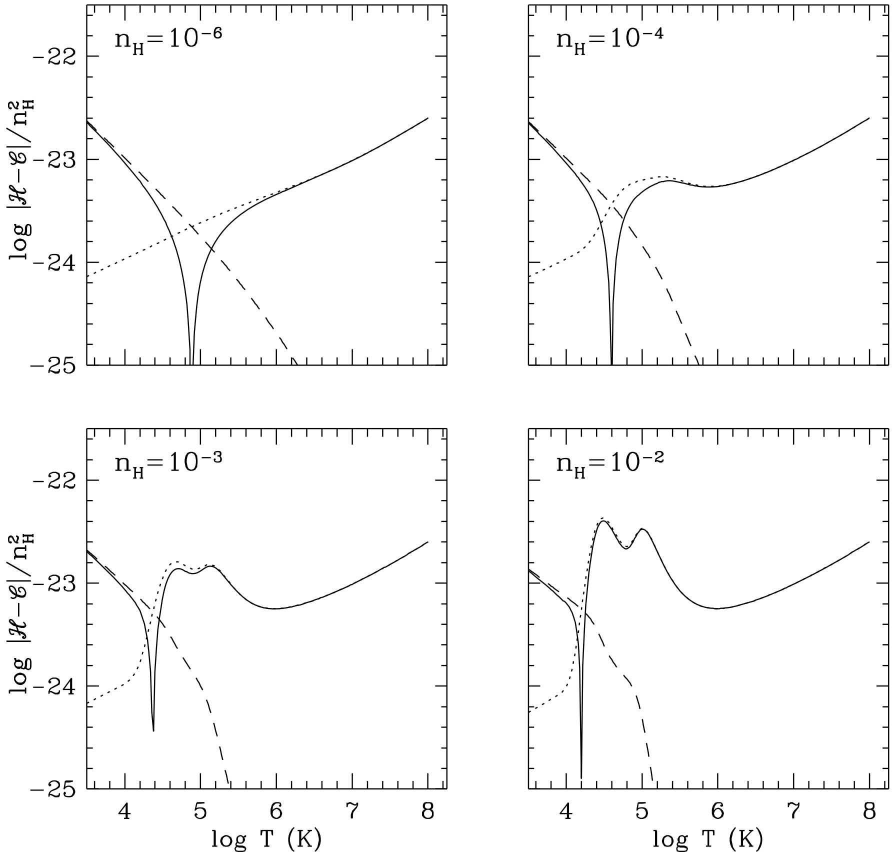
Heating rate:

$$\mathcal{H} = \sum_i n_i \epsilon_i \,, \qquad \epsilon_i = \int_{\nu_i}^{\infty} \frac{4\pi J(\nu)}{h_{\rm P}\nu} \sigma_{{\rm phot},i}(\nu) \left(h_{\rm P}\nu - h_{\rm P}\nu_i\right) \, {\rm d}\nu \,.$$

This does not include the energy loss due to recombinations, which is included in the recombination cooling.

In a static state, photoionization is balanced by recombination.

However, the loss of energy due to recombination is smaller than the gain from photoionization, because the recombination rate is in general higher for lower-energy electrons, causing a net heating.



# Radiative cooling time scales

Consider a uniform spherical cloud in virial equilibrium:

$$\frac{3k_{\rm B}T}{\mu m_{\rm p}} = \frac{3}{5}\frac{GM}{r} = \frac{3}{5}\frac{GM_{\rm gas}}{f_{\rm gas}r} \, .$$

Solving for $M_{\rm gas}$ gives

$$M_{\rm gas} \approx 8.4 \times 10^{12} T_6^{3/2} f_{\rm gas}^{3/2} n_{-3}^{-1/2} {\rm M}_\odot \, .$$

Suppose the cloud has a over-density $\delta$ at redshift $z$, then

$$n_{-3} \approx 1.9 \times 10^{-2} f_{\rm gas}(1+\delta)(\Omega_{\rm m,0}h^2)(1+z)^3$$

and thus

$$M_{\rm gas} \approx 6.1 \times 10^{13} T_6^{3/2} f_{\rm gas}(1+\delta)^{-1/2}(\Omega_{\rm m,0}h^2)^{-1/2}(1+z)^{-3/2}{\rm M}_\odot \, .$$

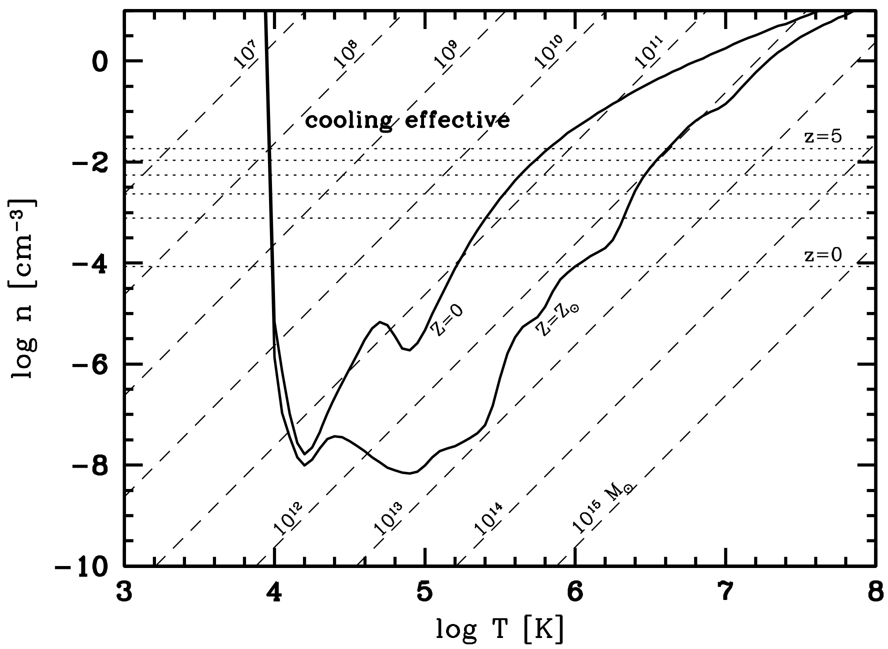
The cooling timescale:

$$t_{\rm cool} \equiv \frac{\rho\mathcal{E}}{\mathcal{C}} = \frac{3nk_{\rm B}T}{2n_{\rm H}^2\Lambda(T)} \approx 3.3\times10^9 \frac{T_6}{n_{-3}\Lambda_{-23}(T)}{\rm yr}\,,$$

$\mathcal{E}$: internal energy per unit mass. This should be compared with free-fall time scale of the cloud:

$$t_{\rm ff} = \sqrt{\frac{3\pi}{32G\rho}} = \sqrt{\frac{3\pi f_{\rm gas}}{32Gn\mu m_{\rm p}}} \approx 2.1\times10^9 f_{\rm gas}^{1/2} n_{-3}^{-1/2}{\rm yr}\,.$$

Cooling is effective if $t_{\rm cool} \ll t_{\rm ff}$




## Overcooling problem:

In a hierarchical model of structure formation, smaller halos are expected to form earlier. Thus, at high redshift most of the cosmic mass is expected to be in low-mass halos in which gas can cool effectively. If all the cold gas formed stars, there would be no gas left today to formed the IGM.

# Cooling in gaseous halos

Consider a model where the density and pressure profiles have power-law forms:

$$\rho_{\rm ad}(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha} , \qquad P_{\rm ad}(r) = P_0 \left(\frac{r}{r_0}\right)^{-\beta} .$$

For an idea gas

$$T_{\rm ad}(r) = T_0 \left(\frac{r}{r_0}\right)^{\alpha-\beta} , \qquad \text{with} \qquad T_0 = \frac{\mu m_{\rm p}}{k_{\rm B}} \frac{P_0}{\rho_0} .$$

Piecewisely, the cooling function may be written as a power law of $T$:

$$\Lambda(T) = \Lambda_0 \left(\frac{T}{T_0}\right)^{\nu} ,$$

$\nu \in (-1, 0)$ in $10^5{\rm K} < T < 10^7{\rm K}$ for cosmic composition. The cooling time is

$$t_{\rm cool}(r) = \frac{3n(r)k_{\rm B}T(r)}{2n_{\rm H}^2(r)\Lambda(T)} = t_0 \left(\frac{r}{r_0}\right)^{1/\tau} ,$$

where

$$t_0 \equiv \frac{3}{2}\frac{k_{\rm B}T_0}{\Lambda_0}\left(\frac{\mu m_{\rm p}}{\rho_0}\right)\left(\frac{n}{n_{\rm H}}\right)^2 , \qquad \tau \equiv [\alpha + (\alpha - \beta)(1-\nu)]^{-1} .$$

# Cooling radius and mass cooling rate

Define the cooling radius, $r_{\rm cool}$, at which the cooling time is equal to the age, $t$, then

$$r_{\rm cool}(t) = r_0 \left(\frac{t}{t_0}\right)^{\tau} .$$

The mass cooling rate:

$$\dot{M}_{\rm cool}(t) = 4\pi\rho(r_{\rm cool}) r_{\rm cool}^2 \frac{{\rm d}r_{\rm cool}}{{\rm d}t} = \frac{4\pi\rho_0 r_0^3}{t_0}\tau \left(\frac{t}{t_0}\right)^{\tau(3-\alpha)-1} ,$$

which implies that

$$M_{\rm cool}(t) = \begin{cases} \frac{4\pi\rho_0 r_0^3}{3-\alpha}\left(\frac{t}{t_0}\right)^{\tau(3-\alpha)} & \text{if } \alpha \neq 3 \\ \frac{4\pi\rho_0 r_0^3}{3+(3-\beta)(1-\nu)} \ln\left(\frac{t}{t_0}\right) & \text{if } \alpha = 3 \end{cases} .$$

For an isothermal sphere, $\alpha = \beta = 2$, so that $r_{\rm cool} \propto M_{\rm cool} \propto t^{1/2}$. I.e. cooling region expands with time.

In a growing halo, this is valid only for $r_{\rm cool} \ll r_{\rm vir}$.

# Hot-mode versus cold-mode accretion

In an growing halo, there are therefore two length-scales: $r_{\rm vir}$ and $r_{\rm cool}$:

- $r_{\rm cool} \ll r_{\rm vir}$, accreted gas can be shocked and form a hydrostatic gas halo. The accretion of gas by galaxy in the halo center is through cooling of hot halo gas (hot-mode accretion);

- $r_{\rm cool} > r_{\rm vir}$, gas can cool as soon as it is accreted, no shocks and no hydrostatic gas halo. The accretion of gas by galaxy in the halo center is through direct cold gas accretion (cold-mode accretion).

Since $r_{\rm vir} \propto V_c t$ and for $n(r) \propto r^{-2}$ we have $r_{\rm cool} \propto \Lambda^{1/2} t^{1/2}$, $r_{\rm vir} = r_{\rm cool}$ defines a critical time for given $V_c$, $t_{\rm crit} \propto \Lambda(T)/V_c^2$, so that

Hot-mode accretion at $t > t_{\rm crit}$

Cold-mode accretion at $t < t_{\rm crit}$

Equivalently, at a given time, there is a critical halo mass $M_{\rm crit}(t)$,

Hot-mode accretion at $M > M_{\rm crit}$

Cold-mode accretion at $M < M_{\rm crit}$

What is the value of $M_{\rm crit}(t)$?

# Hot-mode versus cold-mode accretion the critical mass

## A simple model (Birnboim & Dekel 2003)

Gas is gravitationally stable as long as

$$\gamma_{\rm eff} \equiv \frac{{\rm d}\ln P}{{\rm d}\ln\rho} = \frac{\dot{P}}{P}\,\frac{\rho}{\dot{\rho}} > \frac{2\gamma}{\gamma + \frac{2}{3}}\,.$$

In adiabatic case $\gamma_{\rm eff} = \gamma$, the criterion reduces to $\gamma > 4/3$.
We need to calculate $r_{\rm eff}$ in the presence of cooling:

$$\dot{P} = (\gamma - 1)\left[\rho\dot{\mathcal{E}} + \dot{\rho}\mathcal{E}\right]\;; \qquad \dot{\mathcal{E}} = -P\dot{V} - \mathcal{L} = \frac{P\dot{\rho}}{\rho^2} - \mathcal{L}\,,$$

with $V$ the specific volume. Then

$$\gamma_{\rm eff} = \gamma - \frac{\rho}{\dot{\rho}}\,\frac{\mathcal{L}}{\mathcal{E}}\,.$$

Apply the stability criterion to the post-shock gas. Rankine-Hugoniot jump conditions for a strong shock:

$$\rho_2 = \frac{\gamma+1}{\gamma-1}\rho_1\,, \qquad v_2 = \frac{\gamma-1}{\gamma+1}v_1\,, \qquad P_2 = \frac{2\rho_1 v_1^2}{\gamma+1}\,.$$

Since $v^2 = GM/r = (4\pi G/3)\overline{\rho}\Delta_{\rm vir} r^2$, we have $v(r) = v_2(r/r_{\rm sh})$. Thus

$$\frac{\dot{\rho}}{\rho} = -\nabla\cdot\mathbf{v} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,v\right) = -\frac{3v_2}{r_{\rm sh}}\,.$$

Using

$$\mathcal{L} = \frac{\rho_2\Lambda(T)}{\mu^2 m_{\rm p}^2}\left(\frac{n_{\rm H}}{n}\right)^2\,,$$

the stability criterion assuming $\gamma = 5/3$ reduces to

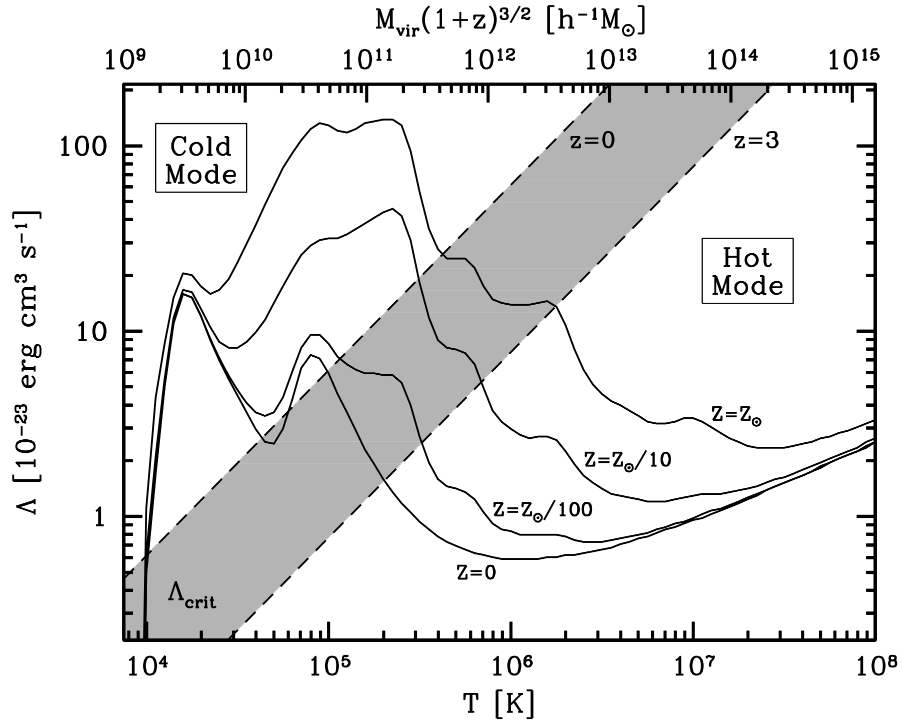
$$\Lambda(T_2) < \Lambda_{\rm crit} = 0.022\frac{m_{\rm p}^2\,|v_1|^3}{\rho_1\,r_{\rm sh}}\,.$$

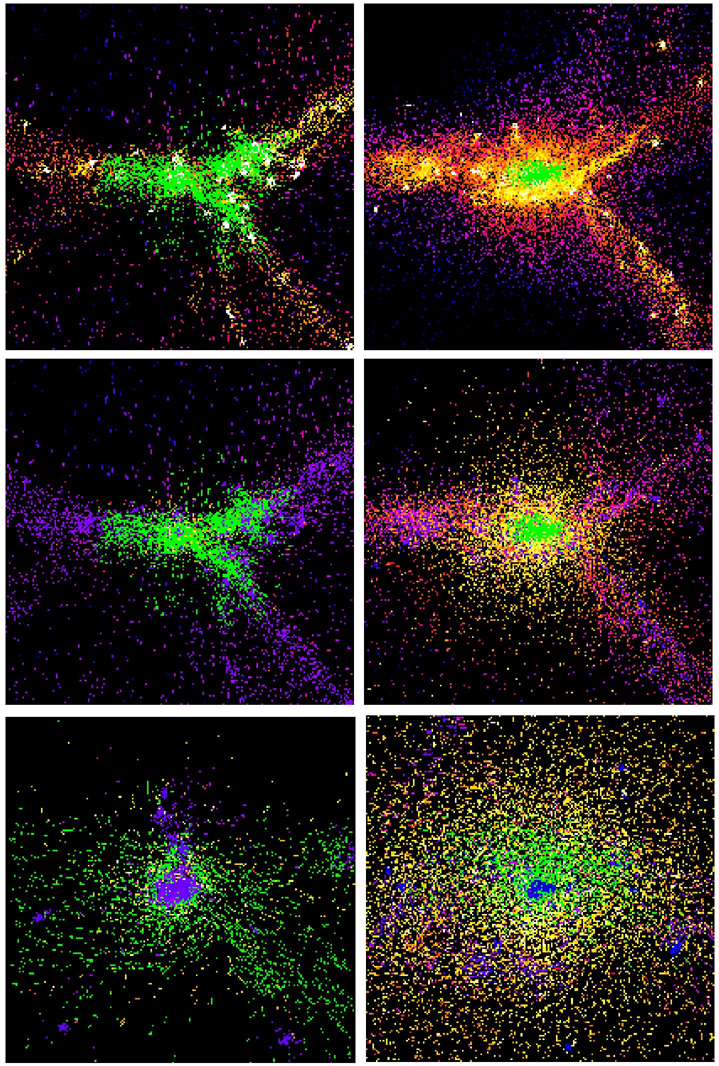
Assuming $\rho_1 = \overline{\rho}_{\rm b}$, $v_1 = V_{\rm vir}$, and $r_{\rm sh} = r_{\rm vir}$ gives:

$$\Lambda_{\rm crit} = 61.8\times10^{-23}{\rm erg\,cm^3\,sec^{-1}}\left(\frac{T_{\rm vir}}{10^6{\rm K}}\right)\left(\frac{\delta_{\rm vir}}{100}\right)^{1/2}\left(\frac{\Omega_{\rm b,0}h^2}{0.024}\right)^{-1}\left(\frac{h}{0.7}\right)(1{+}z)^{-3/2}.$$

Note that

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2k_{\rm B}}V_{\rm vir}^2 = 7.5\times10^5{\rm K}\left(\frac{M_{\rm vir}}{10^{12}h^{-1}{\rm M}_\odot}\right)^{2/3}\left(\frac{\delta_{\rm vir}}{100}\right)^{1/3}(1+z)\,.$$



Note that $10^{11}h^{-1}\mathrm{M}_\odot < M_{\mathrm{crit}} < 10^{12}h^{-1}\mathrm{M}_\odot$, with a remarkably weak dependence on redshift.

Cold-mode accretion: $\dot{M}_{\mathrm{acc}} \sim f_{\mathrm{b}}\dot{M}_{\mathrm{vir}}$;
Hot-mode accretion: $\dot{M}_{\mathrm{acc}} \sim \dot{M}_{\mathrm{cool}}$.