



The Abdus Salam  
International Centre for Theoretical Physics



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## Summer School in Cosmology

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### Galaxy Formation

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# Formation of Gaseous Disks

- Suppose collapse timescale is shorter than star formation timescale, so that gas first collapses to form a gaseous object before forming stars.
- Consider a gas cloud with some initial angular momentum, which collapses and dissipates energy effectively.
- Collapsed structure: a state in which energy is as low as possible, but with total angular momentum conserved.
- Preferred end-state is a rotating disk, in which angular momenta of all mass elements point in the same direction.
- Why not an infinitesimal amount of gas ( $dM$ ) orbiting a black-hole of mass  $M$ , keeping  $J = dM (G M R)^{1/2}$ ? This requires a very effective transfer of angular momentum from inside out, which is not achievable.

# Non-Self-Gravitating Disks in Isothermal Spheres

Assuming the disk mass is a fraction,  $m_d$ , of the halo mass,

$$M_d \approx 1.3 \times 10^{11} h^{-1} M_\odot \left( \frac{m_d}{0.05} \right) \left( \frac{V_{vir}}{200 \text{ km s}^{-1}} \right)^3 \mathcal{Q}^{-1}(z), \quad \mathcal{Q}(z) \equiv \left[ \frac{\Delta_{vir}(z)}{100} \right]^{1/2} \left[ \frac{H(z)}{H_0} \right].$$

Assuming an exponential disk with surface density profile

$$\Sigma(R) = \Sigma_0 \exp(-R/R_d),$$

$R_d$ : scalelength;  $\Sigma_0$ : central surface density. Neglecting disk gravity, then

$$J_d = 2\pi \int_0^\infty V_{vir} \Sigma(R) R^2 dR = 2M_d R_d V_{vir}.$$

Assuming  $J_d = j_d J$ , then

$$R_d = \frac{\lambda G M^{3/2}}{2 V_{vir} |E|^{1/2}} \left( \frac{j_d}{m_d} \right).$$

Assuming all particles to be on circular orbits:  $E = -\frac{MV_{vir}^2}{2}$ , one obtains

$$R_d = \frac{1}{\sqrt{2}} \left( \frac{j_d}{m_d} \right) \lambda r_{vir} \approx 10h^{-1} \text{kpc} \left( \frac{j_d}{m_d} \right) \left( \frac{\lambda}{0.05} \right) \left( \frac{V_{vir}}{200 \text{km s}^{-1}} \right) \mathcal{Q}^{-1}(z);$$

$$\Sigma_0 \approx 207h M_{\odot} \text{pc}^{-2} \left( \frac{m_d}{0.05} \right) \left( \frac{j_d}{m_d} \right)^{-2} \left( \frac{\lambda}{0.05} \right)^{-2} \left( \frac{V_{vir}}{200 \text{km s}^{-1}} \right) \mathcal{Q}(z).$$

For the Milky Way,  $V_{rot} \approx 220 \text{km/s}$ ,  $M_d \approx 5 \times 10^{10} M_{\odot}$ , and  $R_d \approx 3.5 \text{kpc}$ . Assuming  $h = 0.7$  and  $V_{rot} = V_{vir}$  we have  $m_d \sim 0.01$  and  $\lambda \sim 0.011$ .

Disk evolution through  $\mathcal{Q}(z)$ : for a given  $V_{vir}$  disks are smaller and denser at higher  $z$ .

# Self-Gravitating Disks in Halos with Realistic Profiles

Consider a halo with some unperturbed density profile

$$\rho(r) = \frac{1}{4\pi r^2} \frac{dM(r)}{dr},$$

where  $M(r)$  is the halo mass within radius  $r$ . We can write

$$E = -\frac{MV_{vir}^2}{2} F_E,$$

where  $F_E$  is a factor depending on the form of  $\rho(r)$ .

Assuming disk material to move on circular orbits,

$$J_d = 2M_d R_d V_{vir} F_R; \quad F_R = \frac{1}{2} \int_0^{r_{vir}/R_d} u^2 e^{-u} \frac{V_c(uR_d)}{V_{vir}} du,$$

where  $V_c(R)$  is the rotation curve. Using  $r_{vir} \gg R_d$ , we have

$$R_d = \frac{1}{\sqrt{2}} \left( \frac{j_d}{m_d} \right) \lambda r_{vir} F_R^{-1} F_E^{-1/2}.$$

Computation of  $F_R$  needs both  $R_d$  and  $V_c(r)$ , the above set of equations has to be solved iteratively.

# Disk effect on halo: adiabatic contraction

Due to disk self-gravity,  $V_c(r)$  is the sum in quadrature of the contributions from the disk and from the dark matter halo modified by the growth of the disk

$$V_c^2(r) = V_{c,d}^2(r) + \frac{GM_{h,ac}(r)}{r},$$

where the halo mass profile  $M_{h,ac}(r)$  is different from the original profile due to disk gravity.

This effect is usually modeled with adiabatic contraction. Under adiabatic evolution, the action defined through a canonical coordinate,  $q_i$ , and its conjugate momentum,  $p_i$ ,  $J_i = \frac{1}{2\pi} \oint p_i dq_i$ , is a conserved quantity.

For a spherical halo with particles on circular orbits,  $J = r V(r)$ . Thus,

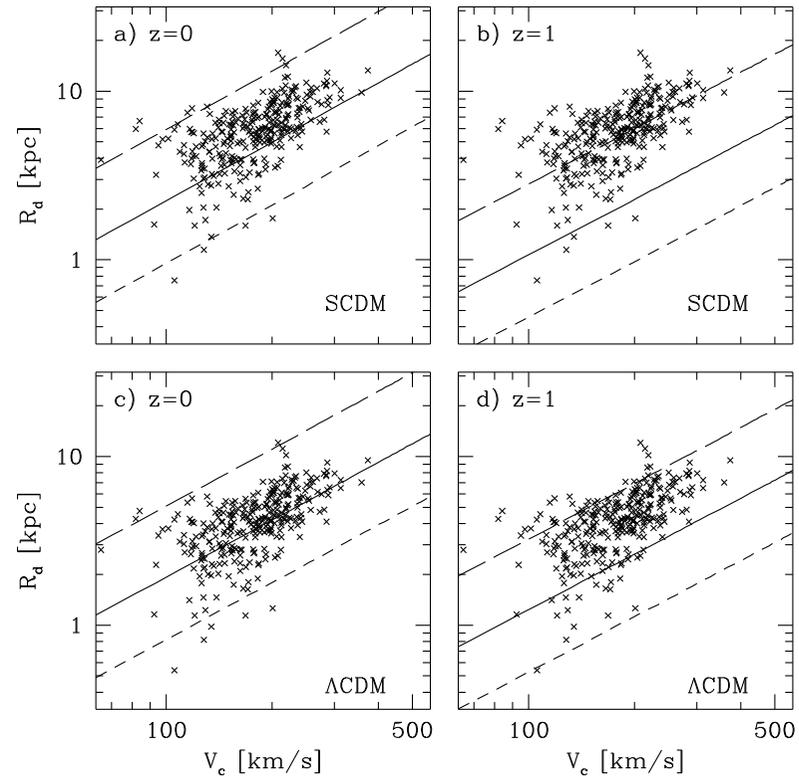
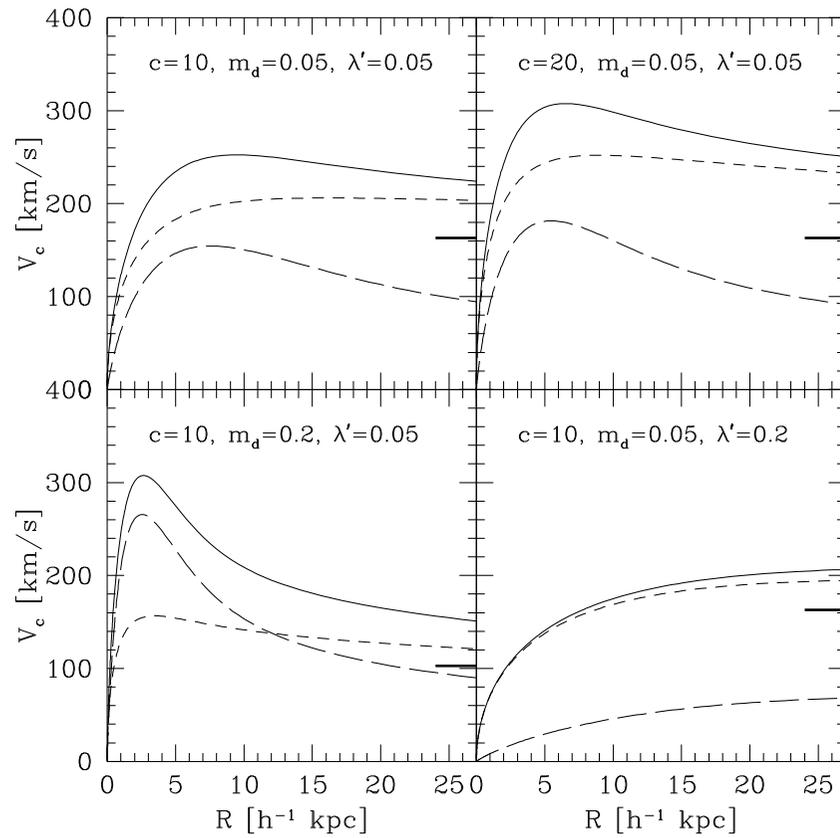
$$r_f M_f(r_f) = r_i M_i(r_i).$$

The contracted profile can be obtained through combining the above equation with

$$M_f(r_f) = M_d(r_f) + (1 - m_d)M_i(r_i),$$

by solving for  $r_f$  and  $M_f(r_f)$ .

# Model predictions



Assuming NFW halos (Mo, Mao & White 1998)

# Disk Assembly

Consider a disk with surface density  $\Sigma(R, t)$  at  $t$ , embedded in a halo of  $M(t)$ . Suppose gas accretion rate  $\dot{M}_d(t)$ , and the newly accreted material has a specific angular momentum distribution  $P(\mathcal{J}, t)d\mathcal{J}$ . If gas settles conserving angular momentum, then

$$2\pi\dot{\Sigma}(R, t)RdR = \dot{M}_d(t)P(\mathcal{J}, t)d\mathcal{J},$$

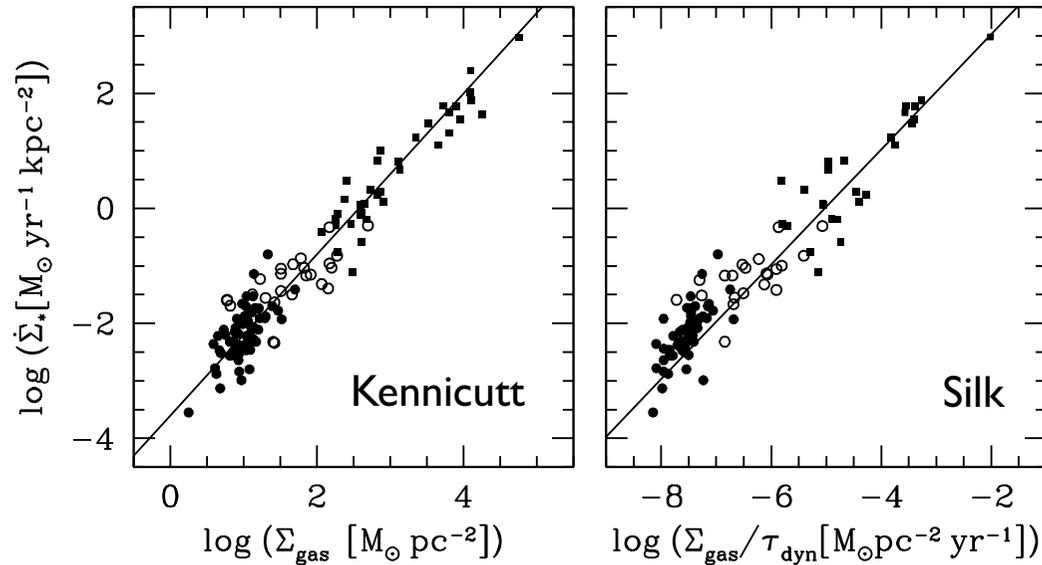
where  $\mathcal{J} = RV_c(R, t)$  with  $V_c(R, t)$  the rotation curve at  $t$ . Thus

$$\dot{\Sigma}(R, t) = \frac{\dot{M}_d(t)}{2\pi R^2} P(\mathcal{J}, t)RV_c(R, t) \left[ 1 + \frac{\partial \ln V_c(R, t)}{\partial \ln R} \right].$$

For a given halo,  $V_c(R, t)$  can be obtained from  $\Sigma(R, t)$ , and so disk growth is determined once  $\dot{M}_d(t)$  and  $P(\mathcal{J}, t)$  are given.  $\dot{M}_d(t)$  is set by  $\dot{M}(t)$  and cooling rate; modeling  $P(\mathcal{J}, t)$  is more uncertain, but may be obtained from simulation.

# Disk star formation

Star formation not well understood; currently modeled with empirical prescription:



Kennicutt-Schmidt model:

$$\dot{\Sigma}_* = (2.5 \pm 0.7) \times 10^{-4} \left( \frac{\Sigma_{\text{gas}}}{M_\odot \text{ pc}^{-2}} \right)^{1.4 \pm 0.15} M_\odot \text{ yr}^{-1} \text{ kpc}^{-2},$$

where  $\Sigma_{\text{gas}} = \Sigma_{\text{HI}} + \Sigma_{\text{H}_2}$ .

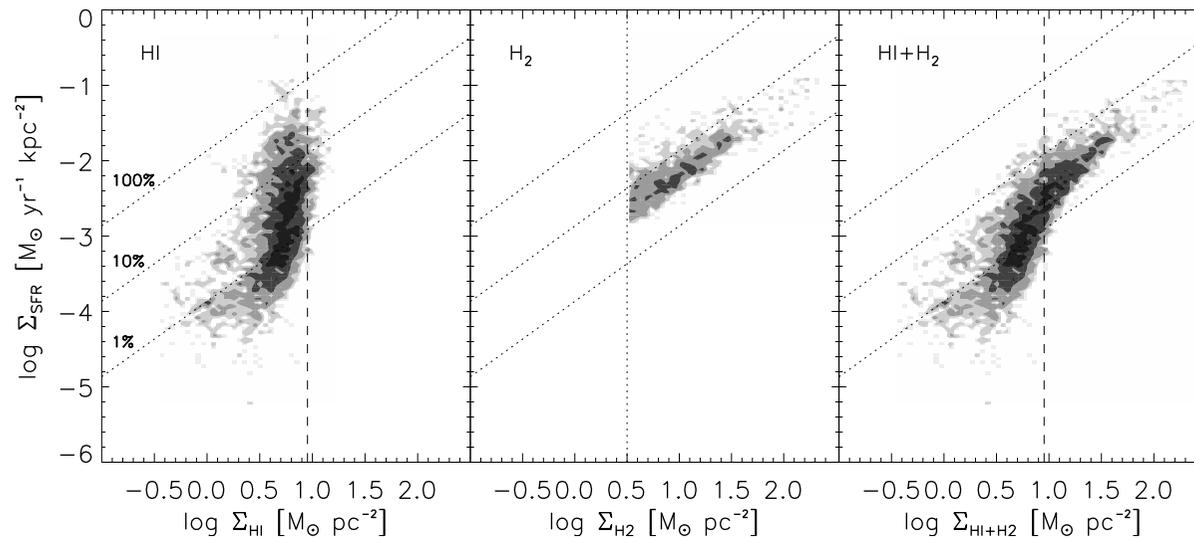
Silk model:

$$\dot{\Sigma}_* \approx 0.017 \Sigma_{\text{gas}} \Omega,$$

with  $\Omega$  the circular frequency.

# Local star formation law

Observation: star formation occurs in dense cores of molecular clouds. Locally star formation directly proportional to molecular gas density:



$$\dot{\Sigma}_\star = (7 \pm 3) \times 10^{-4} \left( \frac{\Sigma_{\text{H}_2}}{\text{M}_\odot \text{pc}^{-2}} \right)^{1.0 \pm 0.2} \text{M}_\odot \text{yr}^{-1} \text{kpc}^{-2},$$

Relation to Kennicutt law:

$$\Sigma_\star \propto \Sigma_{\text{gas}}^N, \quad N = \frac{d \log \dot{\Sigma}_\star}{d \log \Sigma_{\text{gas}}} = 1 + \frac{d \log f_{\text{mol}}}{d \log \Sigma_{\text{gas}}},$$

Model of Krumholz, McKee & Tumlinson (2009):

(i) Molecular fraction  $f_{\text{H}_2}$  determined by balance of dissociation by FUV radiation and formation on dust grains surfaces;

(ii) Amount of star formation per free-fall time in a molecular cloud,  $\epsilon_{\text{ff}} \sim 0.01$ .

The KMT star formation prescription:

$$\dot{\Sigma}_{\star} = f_{\text{H}_2}(\Sigma_{\text{gas}}, c, Z) \frac{\Sigma_{\text{gas}}}{2.6 \text{Gyr}} \times \begin{cases} (\Sigma_{\text{gas}}/[85\text{M}_{\odot}\text{pc}^{-2}])^{-0.33} & (\Sigma_{\text{gas}} < 85\text{M}_{\odot}\text{pc}^{-2}) \\ (\Sigma_{\text{gas}}/[85\text{M}_{\odot}\text{pc}^{-2}])^{0.33} & (\Sigma_{\text{gas}} > 85\text{M}_{\odot}\text{pc}^{-2}) \end{cases},$$

$$f_{\text{H}_2}(\Sigma_{\text{gas}}, c, Z) \approx 1 - \left[ 1 + \left( \frac{3}{4} \frac{s}{1 + \delta} \right)^{-5} \right]^{-1/5},$$

with

$$s = \ln(1 + 0.6\chi)/(0.04\Sigma_{c,1}Z);$$

$$\chi = 0.77(1 + 3.1Z^{0.365});$$

$$\delta = 0.0712(0.1s^{-1} + 0.675)^{-2.8};$$

$$\Sigma_{c,1} = \Sigma_c/(1\text{M}_{\odot}\text{pc}^{-2}).$$

$\Sigma_c$  is the gas surface density smoothed on 100 pc scale, related to  $\Sigma_c = c\Sigma_{\text{gas}}$ , with  $c$  the clumpiness factor of the disk.

The threshold density:  $\Sigma_{\text{gas}} \sim 10\text{M}_{\odot}\text{pc}^{-2}$ , below which the gas is too thin to shield dissociating photons and to form molecules.

# Supernova feedback effects

Assuming spherical, static flow, the fluid equations can be combined to give two first order differential equations for the fluid velocity,  $v$ , and the adiabatic sound speed of the gas,  $w$ :

$$\frac{r}{v^2} \frac{dv^2}{dr} = \frac{-w^2}{2\pi(v^2 - w^2)} \left[ 4\pi \left( 2 - \frac{V_c^2}{w^2} \right) + \frac{2}{3}A - \left( \frac{4}{3} + \frac{w_i^2}{v^2} \right) B \right],$$

$$\frac{r}{w^2} \frac{dw^2}{dr} = \frac{-v^2}{6\pi(v^2 - w^2)} \left[ 4\pi \left( 2 - \frac{V_c^2}{v^2} \right) + \left( \frac{5}{3} - \frac{w^2}{v^2} \right) A + \left( \frac{w^2}{v^2} - \frac{5w_i^2}{2v^2} - \frac{3w^4}{2v^4} + \frac{3w_i^2w^2}{2v^4} - \frac{5}{6} \right) B \right],$$

where  $V_c^2 \equiv r(d\Phi/dr)$  specifies the shape of the gravitational potential well, and the injected gas is assumed to have an initial *isothermal* sound speed,  $w_i \equiv (k_B T_i / \mu m_p)^{1/2}$ , with  $T_i$  the initial temperature of the injected gas. The quantities  $A$  and  $B$  are given by

$$A = \frac{\dot{M} \Lambda(T) n_{\text{H}}^2}{r(\rho w v)^2}, \quad B = \frac{\dot{\rho}_{\text{inj}} \dot{M}}{r(\rho w)^2}, \quad \dot{M} = 4\pi \int_0^r \dot{\rho}_{\text{inj}}(r') r'^2 dr',$$

The sonic point at  $r_1$  (where  $v = w$ ) separates the heating base ( $atr_{<}r_1$ ) from the supersonic wind at  $r > r_1$ .

The wind can reach a velocity  $\sqrt{2.5}w_1$  (Efstathiou 2000). If  $w_1 > V_{\text{esc}}/\sqrt{2.5}$ , wind escapes; otherwise a hot corona. If  $A(r_1) \gg 1$ , the wind can cool. (cold wind)

Gas temperature at the heating base specified by  $w_1$  is determined by the balance between supernova heating and radiative cooling. This defines a critical value of  $V_c$ ,

$$V_{\text{crit}} \sim 100 \text{km s}^{-1},$$

so that gas removal occurs in halos with  $V_c < V_{\text{crit}}$ .

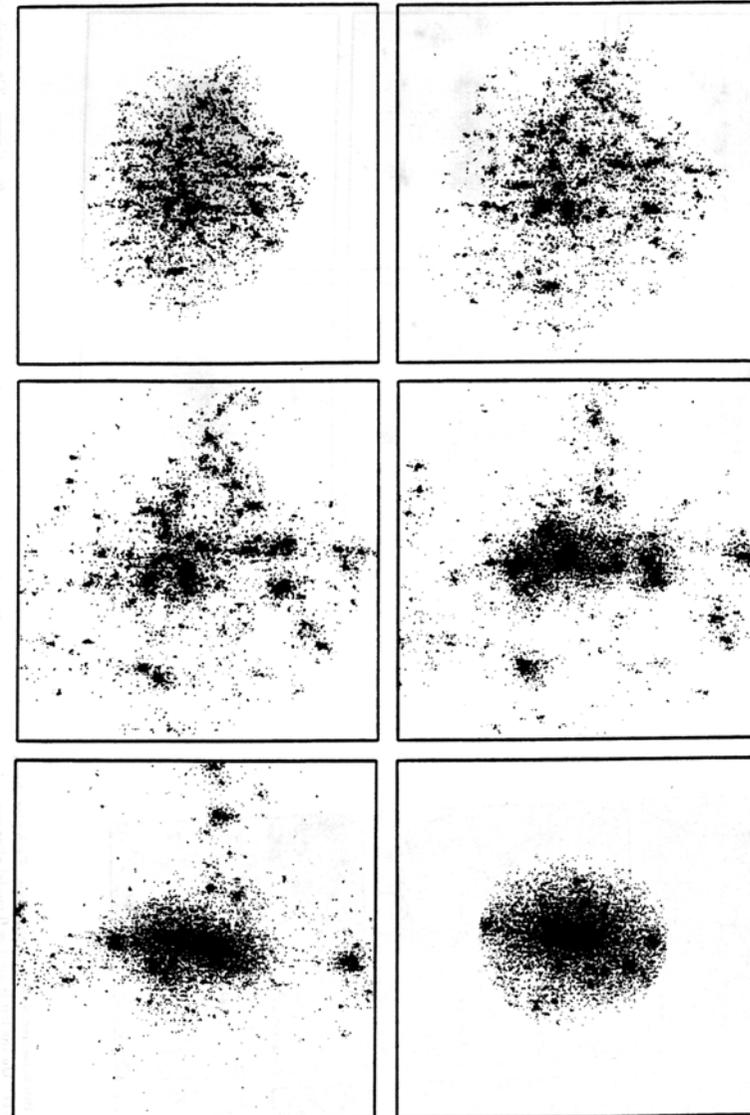
If the energy input from star formation is equal to the binding energy of the cold gas, then

$$\mathcal{E}_0 \dot{M}_\star = (\dot{M}_g - \dot{M}_\star) V_c^2 / 2; \quad \dot{M}_\star = \frac{\dot{M}_g}{1 + (V_0/V_c)^2},$$

where  $\mathcal{E}_0$  is the energy feedback per unit mass of formed stars, and  $V_0 \sim V_{\text{crit}}$ .

# Galaxy interaction and transformation

In hierarchical model, dark halos merge constantly, bringing their galaxies in a common halo; galaxies in a common halo can interact with each other or merge, transforming a new galaxy



# Merger Criterion

Consider a simple case: two identical spherical galaxies of mass  $M$  and median radius  $r_{\text{med}}$ . The internal mean-square velocity is  $\langle v^2 \rangle \approx 0.4GM/r_{\text{med}}$ . Encounter is specified by  $E_{\text{orb}}$  (the specific orbital energy) and  $L$  (the specific angular momentum) in units derived from  $\langle v^2 \rangle$  and  $r_{\text{med}}$ :

$$\hat{E} \equiv \frac{E_{\text{orb}}}{(1/2)\langle v^2 \rangle} \quad \text{and} \quad \hat{L} \equiv \frac{L}{\langle v^2 \rangle^{1/2} r_{\text{med}}}.$$

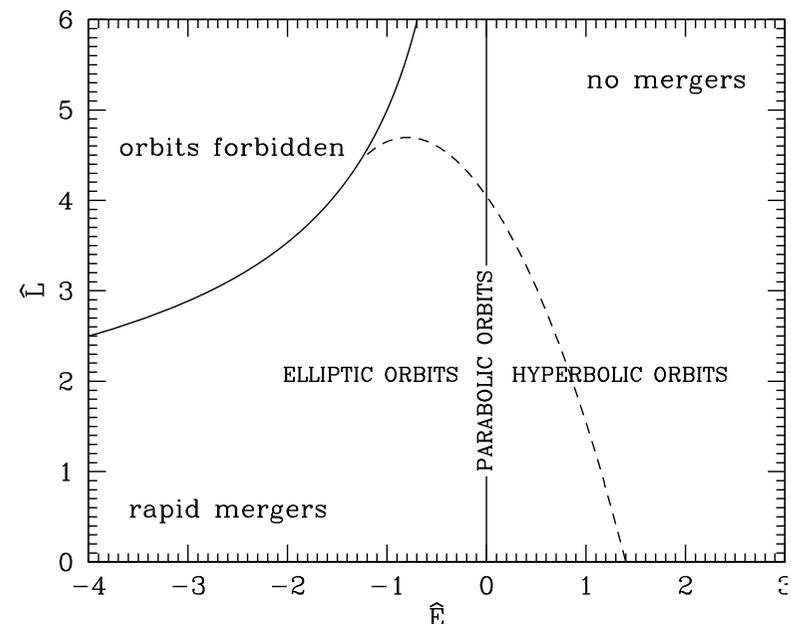
Each encounter is then associated with a point in the  $(\hat{E}, \hat{L})$  plane which can be divided into different regions

Merger needs low  $\hat{E}$  ( $E_{\text{orb}} < \langle v^2 \rangle/2$  or  $\sigma^2 < \langle v^2 \rangle/2$ ) and low  $\hat{L}$  (large  $r_{\text{med}}$ )

- (i) Orbits in the upper-left region are forbidden: for a given orbital energy the largest possible angular momentum is that of circular orbit.
- (ii) Encounters with too high orbital energy and too high orbital angular momentum cannot lead to merger.
- (iii) Mildly hyperbolic orbits can lead to a merger if the orbital angular momentum is sufficiently low

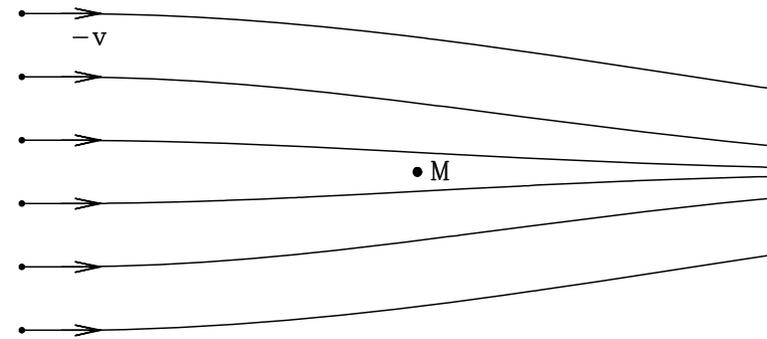
Conclusion 1: Galaxies can merge quickly if they are in systems with velocity dispersion comparable to the internal velocity dispersion of the individual galaxies.

Conclusion 2: Massive, extended halos can merger more easily than their central galaxies

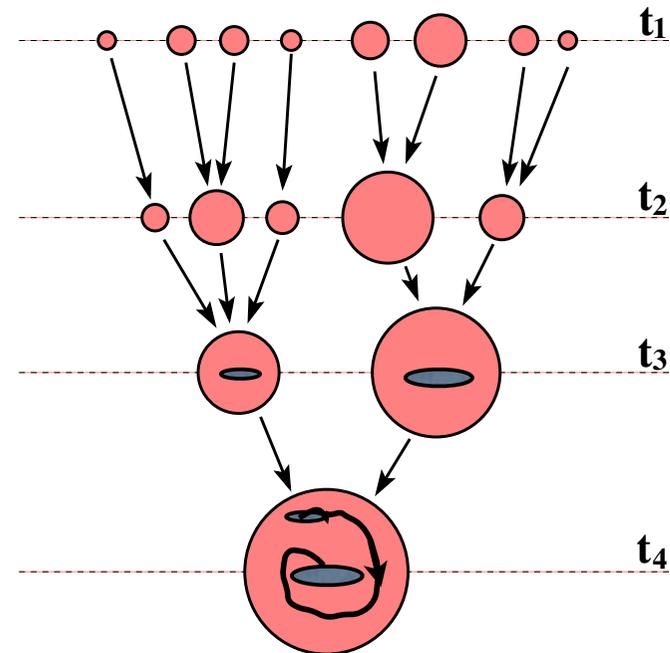


# Dynamical friction

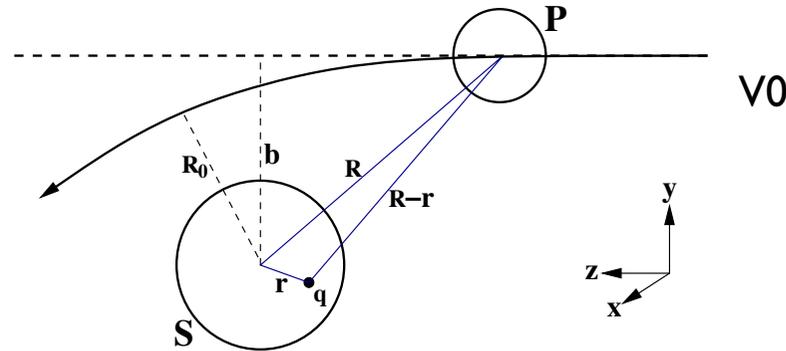
As an object moves through a sea of particles, it accelerates the surrounding particles, the number density of particles down-stream is higher than up-stream, experiencing a net drag force (dynamic friction).



A satellite galaxy in a halo experiences dynamical friction of dark matter particles that causes its orbit to decay, leading to merger at the halo center.



# Chandrasekhar dynamical friction formula



Consider the encounter of an object of mass  $M$  with a particle with mass  $m$  (the standard gravitational scattering problem). The total change in velocity is given by

$$\delta \mathbf{V}_M = \delta \mathbf{V}_{M\parallel} + \delta \mathbf{V}_{M\perp}.$$

If 'M' goes through a homogeneous sea of particles,  $\sum \delta \mathbf{V}_{M\perp} = 0$ . For each scattering with impact parameter  $b$  and velocity  $V_0$ , we have

$$\delta \mathbf{V}_{M\parallel} = \frac{2mV_0}{m+M} \left[ 1 + \frac{b^2 V_0^4}{G^2(M+m)^2} \right]^{-1}.$$

If the number density of particles with velocity  $\mathbf{v}_m$  is  $f(v_m) d^3 \mathbf{v}_m$ , the rate of encounters of 'M' with such particles and with impact parameters in the range  $b \rightarrow b + db$  is  $2\pi b db \times v_0 \times f(v_m) d^3 \mathbf{v}_m$ . The rate of change in  $\mathbf{v}_M$  due to encounters with these particles is then

$$\begin{aligned} \left( \frac{d\mathbf{v}_M}{dt} \right)_{v_m} d^3 \mathbf{v}_m &= V_0 f(\mathbf{v}_m) d^3 \mathbf{v}_m \int_0^{b_{\max}} |\Delta v_{M\perp}| \cdot 2\pi b db \\ &= 2\pi \ln(1 + \Lambda^2) G^2 m(m+M) f(\mathbf{v}_m) d^3 \mathbf{v}_m \frac{(\mathbf{v}_m - \mathbf{v}_M)}{|\mathbf{v}_m - \mathbf{v}_M|^3}, \end{aligned}$$

where  $\Lambda \equiv \frac{b_{\max} v_0^2}{G(M+m)}$ ,  $b_{\max}$  is the largest impact parameter to be considered.

Assuming isotropic velocity distribution, the rate of change in  $\mathbf{v}_M$  can be obtained by integrating over  $\mathbf{v}_m$ . This is equivalent to finding the ‘gravitational field’ at ‘position’  $\mathbf{v}_M$  generated by the ‘mass density’  $4\pi \ln(\Lambda) G^2 m(m + M) f(\mathbf{v}_m)$ . Thus

$$\frac{d\mathbf{v}_M}{dt} = -16\pi^2 \ln \Lambda G^2 m(m + M) \frac{\mathbf{v}_M}{v_M^3} \int f(v_m) v_m^2 dv_m .$$

This is the Chandrasekhar dynamical friction formula.

If  $f(v_m)$  is Maxwellian, then

$$\frac{d\mathbf{v}_M}{dt} = -\frac{4\pi \ln \Lambda G^2 (m + M) \rho}{v_M^3} \left[ \operatorname{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \mathbf{v}_M ,$$

with  $X \equiv v_M/(\sqrt{2}\sigma)$  and  $\sigma$  the velocity dispersion.

# Application to galaxies in dark matter

Consider a satellite on circular orbit in an isothermal halo with  $\rho_0(r) = V_c^2/(4\pi Gr^2)$ ;  $\sigma = V_c/\sqrt{2}$  and so  $X = 1$ . Assuming  $M \gg m$  the friction force experienced by the satellite at radius  $r$  is

$$F = -0.428 \ln \Lambda \frac{GM^2}{r^2}.$$

This force is tangential, and the rate of change in the angular momentum  $L$  is  $\frac{dL}{dt} = \frac{Fr}{M}$ . Using  $L = rV_c$ , then

$$r \frac{dr}{dt} = -0.428 \frac{GM}{V_c} \ln \Lambda.$$

For an initial orbit with radius  $r_i$ , the time for 'M' to sink to the halo center is

$$t_{\text{df}} = \frac{1.17 r_i^2 V_c}{\ln \Lambda GM} = \frac{1.17}{\ln \Lambda} \left( \frac{r_i}{R_h} \right)^2 \left( \frac{M_h}{M} \right) \frac{R_h}{V_c}.$$

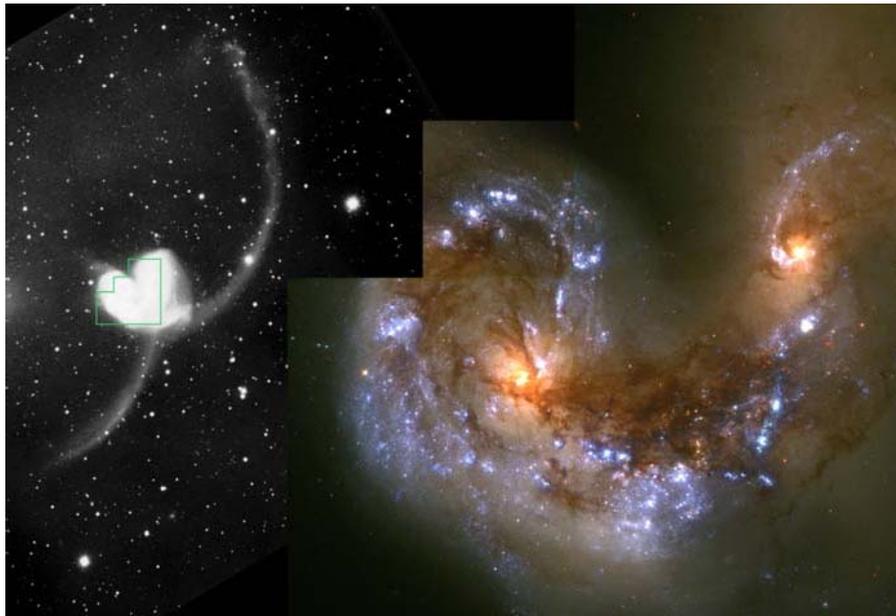
If  $r_i \sim R_h$ ,

$$t_{\text{df}} \approx \frac{1.17}{\ln(M_h/M)} \left( \frac{M_h}{M} \right) \frac{1}{10H(z)},$$

where we have used  $R_h/V_c = 1/10H(z)$ . Thus, the dynamical friction timescale is longer than the age of the universe for  $M/M_h < 30$ .



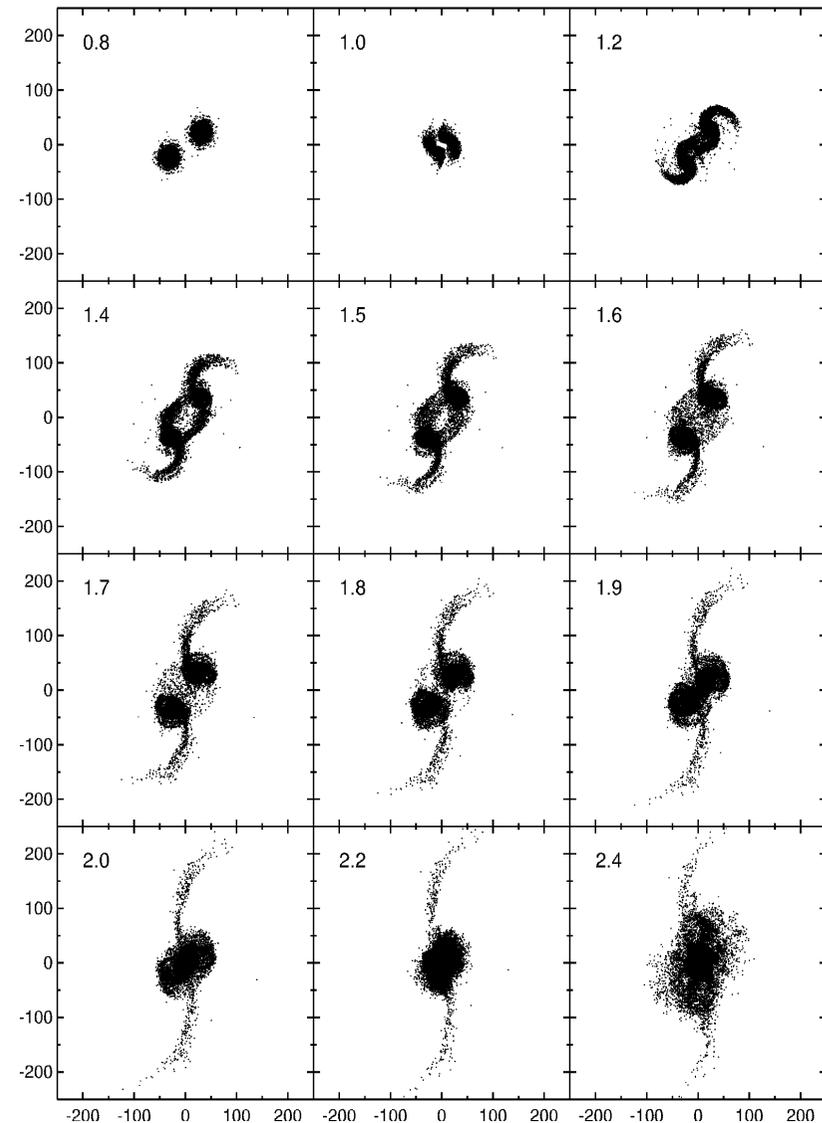
# Tidal tails and streams



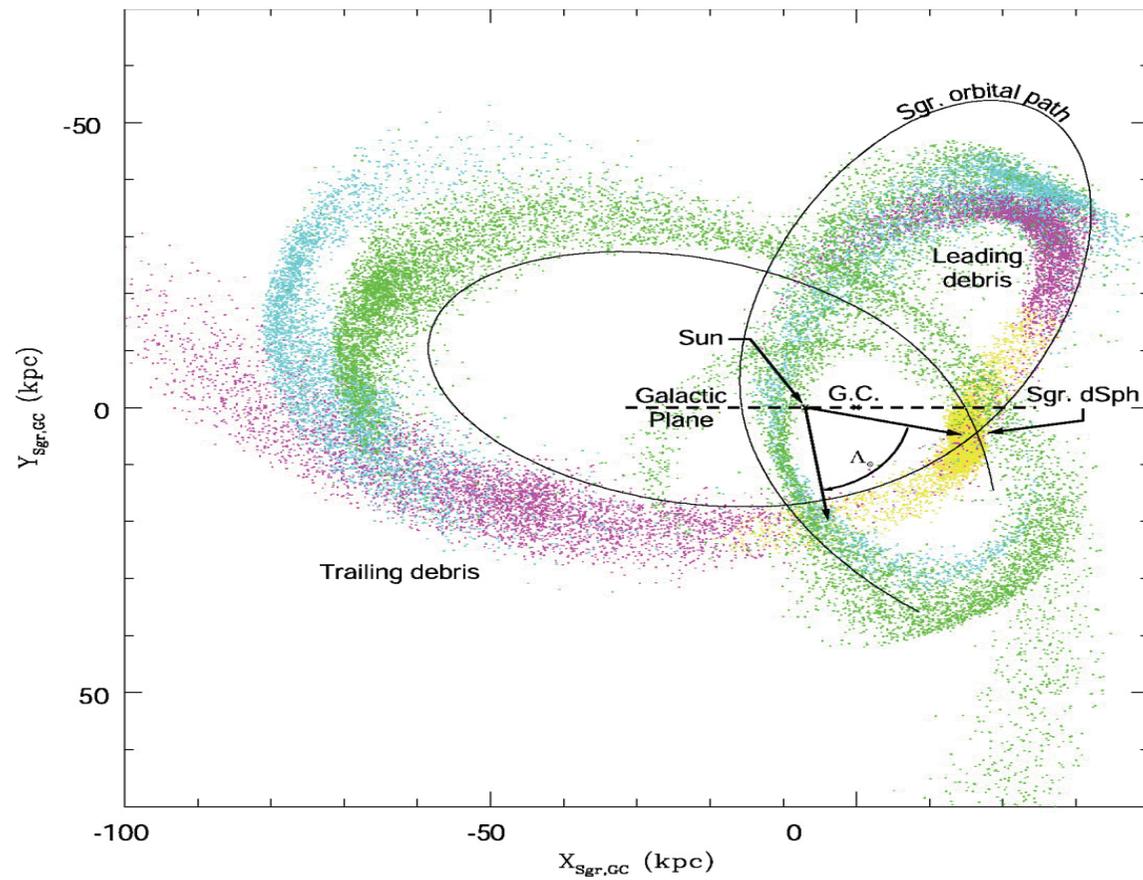
$$\mathcal{E} \equiv \frac{V_{\text{esc}}^2(2R_d)}{V_c^2(2R_d)}$$

characterizes the ability of producing extended tidal tails (Mo, Mao, White 1998; Springel & White 1999; Dubinski et al. 1999).

Prominent tidal tails are produced for  $\mathcal{E} < 6$  (assuming prograde merger).



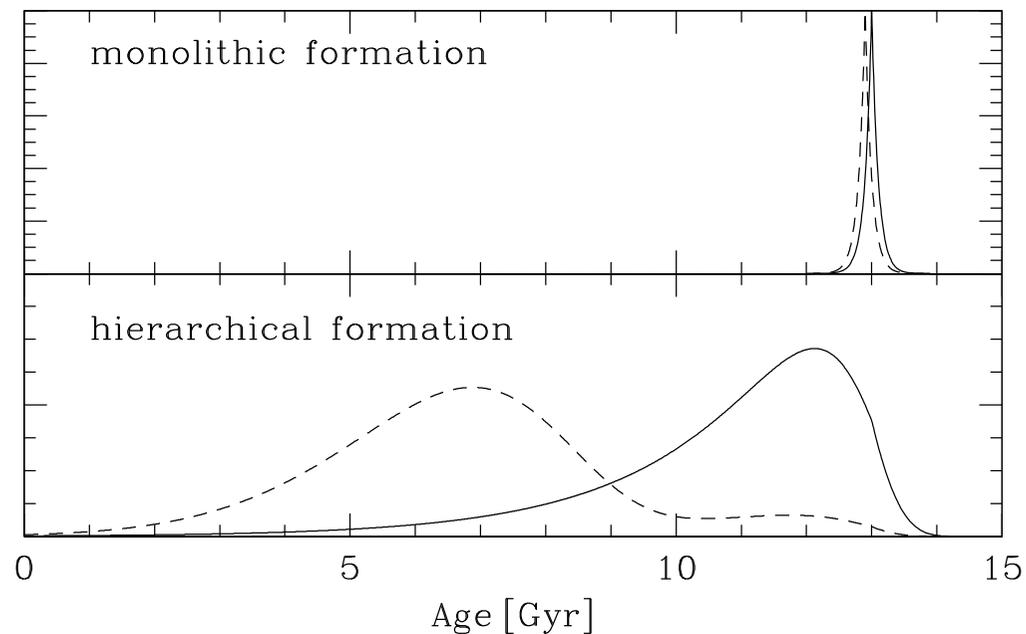
# Tidal streams



*The Sagittarius Dwarf Tidal Stream*

# Formation of elliptical galaxies

- Monolithic collapse scenario: not based on any model of structure formation.
- Merger scenario: well motivated in hierarchical models of structure formation





Major mergers can make elliptical-like objects

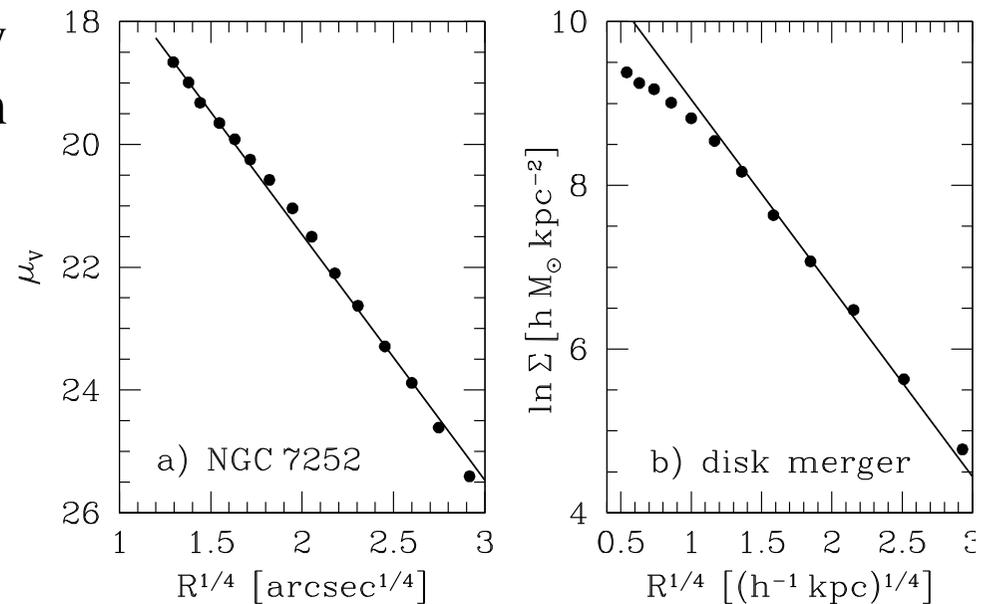
# Merger remnants

- Major mergers of galaxies are expected to be accompanied by violent changes in the gravitational potential of the system.
- Because of violent relaxation, the merged system generally relax to form a smooth object near the center of the system, with some irregular structure at large radii.

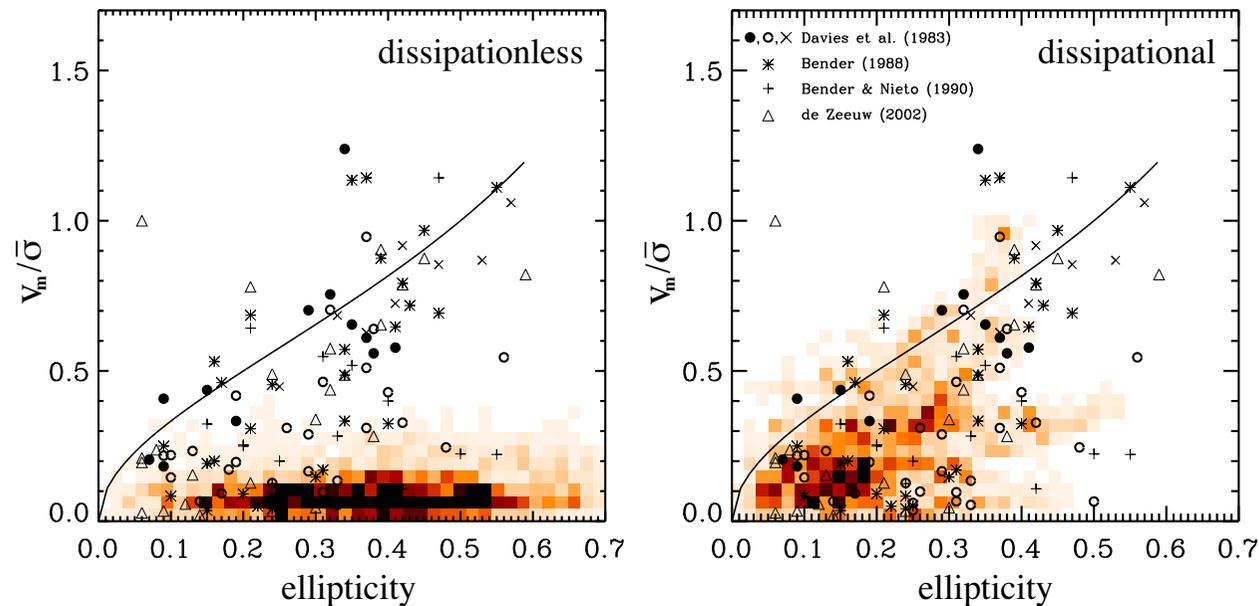
## Simulation results

Major mergers of galaxies generally lead to elliptical-like remnants, with some irregular structures in the outer regions.

The final density profiles of merger remnants in projection are well fitted by the R-quarter profiles over a large radial interval.



The remnant of a major merger generally rotates slowly in the inner region, because dynamic friction transfer angular momentum from particles with high binding energy to the ones with low binding energy. The effective transfer of angular momentum from the merging galaxies to dark matter leads to slowly-rotating remnants, and the inner part of such a remnant is supported by velocity dispersion.



Major merger remnants have properties similar to elliptical galaxies

# Summary of simulation results

- Mergers of two stellar disks: need massive halos to reduce angular momentum to produce slowly rotating remnants. Such mergers produce too large ellipticity;
- Multiple mergers of stellar disks produce rounder remnants but too extended cores;
- Mergers of disk galaxies containing cold gas can produce relatively round and more centrally concentrated remnants like intermediate mass elliptical, but cannot produce giant ellipticals that have boxy orbits;
- Mergers of elliptical progenitors (dry mergers) can produce boxy elliptical galaxies.

Can CDM model generate the right conditions to produce the right numbers of wet/dry mergers? Yes, if some processes can suppress cold gas accretion by massive galaxies (AGN feedback?)