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# Summer School in Cosmology

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## Dark Energy

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# Dark Energy / Modified Gravity Notes

Conventions $M_p^2 = \frac{1}{8\pi G}$ $\hbar = c = 1$ $- + + +$

* Dark energy is name given to physical stress energy that drives current accelerated expansion of universe

* Modified Gravity is a name given to the idea that the apparent cosmic acceleration is due either directly or indirectly due to a modification of GR.

## Central Issue : Cosmic Acceleration

Observations tell us that if we assume FRW is a good approximation on large scales the universe appears to be accelerating.
By this we mean the Hubble flow (i.e. ignoring peculiar velocity contributions) of galaxies, represented by the scale factor $a(t)$

satisfies $\boxed{\ddot{a} > 0}$ $\dot{} = \frac{d}{dt}$ 

2ab

↑ proper time

$ds^2 = -dt^2 + a^2(t) \ldots$

Traditionally people define 'deceleration parameter'

$$q = -\frac{\left(\ddot{a}/a\right)}{H^2} = -\frac{a\ddot{a}}{\dot{a}^2}$$

N.B. to be physical must be as many a's on top and bottom, to be dimensionless must be as many dots $\dot{} = d/dt$.

$q > 0$ universe is decelerating

$q < 0$ " " accelerating

(N.B. $\dot{a}^2 > 0$, $a > 0$ by choice)

860

Friedman equation

+1 closed
0 flat
−1 open

$$H^2 = \frac{1}{3M_p^2}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

not include as define

$$\rho = \rho_{d.e} + \dots$$

where $\rho_{d.e}$ includes possible

$$\rho_\Lambda = M_{pl}^2 \Lambda \text{ contribution}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$\frac{d\rho}{da} = -3(\rho + p)$$

$$\ddot{a} = \frac{1}{2}\frac{d(\dot{a}^2)}{da} = \frac{1}{2}\frac{d}{da}\left(H^2 a^2\right)$$

$$= \frac{1}{6M_p^2}\frac{d}{da}\left[\rho a^2\right]$$

N.B. curvature does not enter formula for acceleration!

$$\ddot{a} = \frac{1}{6M_p^2} a \left[2\rho - 3(\rho + p)\right] = -\frac{a}{6M_p^2}(\rho + 3p)$$

In an FRW universe,

4 400

$$q = -\frac{\ddot{a}\,a}{\dot{a}^2}$$

$$= \frac{\frac{1}{6M_{pl}^2}(\rho + 3p)}{\frac{1}{3M_{pl}^2}\rho - \frac{k}{a^2}}$$

Defining $w = \frac{p}{\rho}$ (equation of state, not necessarily constant)

$$\boxed{q = \frac{1}{2}(1 + 3w)\left(1 - \frac{3M_{pl}^2 k}{\rho a^2}\right)^{-1}}$$

(obvious simplest for $k=0$!)

~~[illegible]~~

universe will accelerate if $w < -1/3$

decelerate if $w > -\frac{1}{3}$

500

Matter / dust / non-relativistic matter has $w = 0$ (pressureless fluid)

Radiation has $w = +\frac{1}{3}$ (or ultrarelativistic matter)

Thus stress energy made out of particulate matter (i.e. from fermions) or massless spin one fields (radiation) will always lead to deceleration.

We would like to know

| | | |
|---|---|---|
| Scale factor $a(t)$ | AS FUNCTION OF | time $t$ |

What we observe is

| | | |
|---|---|---|
| Distances of e.g. galaxies, supernovae | AS FUNCTION OF | redshift $z$ |

There are 2 principal ways of measuring distances

STANDARD CANDLES (choose objects of known luminosity) $\Rightarrow$ Luminosity distance

STANDARD RULERS (choose objects of known width) $\Rightarrow$ Angular distance

Consider an luminous object in an FRW cosmology

$$ds^2 = -dt^2 + a^2(t)\left[d^2\chi^2 + b^2(\chi)\, d^2\Omega_{S_2}\right]$$

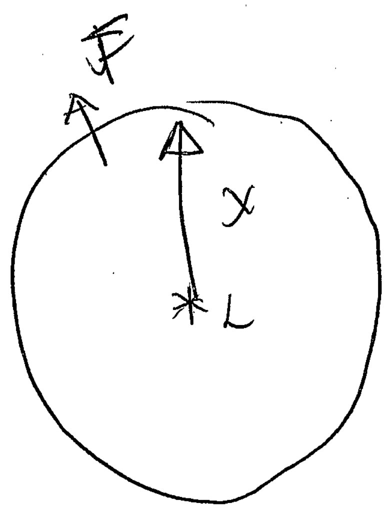
$$b(\chi) = \begin{cases} \sin\chi & k=+1 \\ \chi & k=0 \\ \sinh\chi & k=-1 \end{cases}$$

At some time $t_0$, consider a sphere of size $\chi_0$.
The induced metric on the sphere is

$$ds^2 = a^2(t)\, b^2(\chi)\, d^2\Omega_{S_2}$$

So the Area is
$$A = a^2 b^2(\chi) \iint d^2\Omega_{S_2}$$

$$A = 4\pi a^2 b^2(\chi)$$



The flux $\mathcal{F}$ of light at $x$ is given by

$$\mathcal{F} = \frac{L}{A} \quad \text{where} \quad L = \frac{\Delta E}{\Delta t}$$

The luminosity distance is defined by

$$d_L = \sqrt{\frac{L_S}{4\pi F_0}}$$ (distance inferred in Euclidean geometry)

$$F_0 = \frac{L_0}{a^2 b^2(x)} \qquad d_L = \sqrt{\frac{L_S}{L_0}}\; a\, b(x)$$

Because of the redshift

$$\frac{\Delta E_S}{\Delta E_0} = \frac{\lambda_0}{\lambda_S} = (1+z) \qquad \boxed{\frac{L_S}{L_0} = (1+z)^2}$$

$$\frac{\Delta t_S}{\Delta t_0} = \frac{\lambda_S}{\lambda_0} = \frac{1}{(1+z)}$$

Net result is that

$$\boxed{d_L = (1+z)\, a_0\, b(\chi)}$$

This should be compared with the comoving distance which is simply $\chi$

$$d_c = \chi$$

This equation is still slightly useless — we need to know $\chi$. Photons travel along

$$dt = a(t)\, d\chi$$

$$\chi = \int_{t_s}^{t_0} \frac{1}{a(t)}\, dt = \int_{a_s}^{a_0} \frac{1}{a^2 H}\, da$$

Now $\frac{a(z)}{a_0} = \frac{1}{(1+z)}$

$$= \boxed{\frac{1}{a_0}\int_0^z \frac{dz}{H(z)}}$$

∴ Putting this together

$$\boxed{d_L = (1+z)\, a_0\, b \left( \frac{1}{a_0} \int_0^z \frac{dz}{H(z)} \right)}$$

This is simplest for a flat universe for which

$$\boxed{d_L(z) = (1+z) \int_0^z \frac{dz}{H(z)}}$$

(N.B. $a_0$ drops out)

Alternatively we can rewrite this as

$$\boxed{H(z) = \left[ \frac{d}{dz} \left( \frac{D_L(z)}{1+z} \right) \right]^{-1}}$$

1080

Supernovae are only observed out to a few in redshift $z \sim 3$ consequently, we may neglect radiation ~~and~~ (also lets assume $k=0$)

$$H^2 = \frac{1}{3M_p^2}\left[\rho_{d.e.} + \rho_m\right]$$

Formally since ~~$p$~~ $w_{d.e.} = \frac{p_{d.e}}{\rho_{d.e}}$.

$$\frac{d\rho}{d\ln a} = -3\rho(1+w)$$

$$\frac{d\ln\rho}{d\ln(1+z)} = +3(1+w)$$

$$\Rightarrow \boxed{\rho = \rho_0\, e^{\int_0^z 3(1+w)\frac{dz}{(1+z)}}}$$

This is simplest if $w \sim$ constant

$$\Rightarrow \quad \rho = \rho_0 (1+z)^{3(1+w)} = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+w)}$$

Important point : smaller $w$ dominates at late times

larger $w$ dominates at early times.

Since matter has $w = 0$, any accelerating component with $w = -\frac{1}{3}$ will dominate at late times.

$\boxed{\text{THE FATE OF OUR UNIVERSE IS INEVITABLY TIED TO THE NATURE OF DARK ENERGY (OR WHAT DRIVES ACCELERATION}}$

Corollary

As the universe expands and cools it increasingly begins to probe lower ~~energy~~ and lower energies.

Nature of dark energy is tied to what is the correct $\boxed{\text{low energy effective field theory for gravity + matter}}$

Dark energy is unique among current problems in that it tells us that we do not understand physics at lower energy scales $\underline{E \sim H}$ (more tomorrow)

$\sim 10^{-33}\,\mathrm{eV}$ !

Another way to write Friedman equation is

$$H^2_{(z)} = H^2_0 \left[ \Omega_{m,0} (1+z)^3 + \Omega_{d.e.,0} (1+z)^{3(1+w_{d.e.})} \right]$$

$H(0) = H_0$

$$\boxed{\Omega^{(z)}_m = \frac{\rho^{(z)}_m}{3 M_p^2 H^2}}$$

$\Omega_m(z0) = \Omega_{m,0}$

$$d_L(z) = (1+z) \int_0^z \frac{dz}{H(z)}$$

Conventional to rewrite as

$$H_0 d_L(z) = (1+z) \int_0^z dz \frac{1}{\sqrt{\Omega_{m,0} (1+z)^3 + \Omega_{d.e.0} (1+z)^{3(1+W_{d.e})}}}$$

If there were only matter!

$$H_0 d_L(z) = \frac{(1+z)}{1 = \sqrt{\Omega_{m,0}}} \int_0^z dz \; (1+z)^{-3/2}$$

$$\boxed{H_0 d_L(z) = \frac{2(1+z)}{1 = \sqrt{\Omega_{m,0}}} \left[ 1 - (1+z)^{-1/2} \right]}$$

For small $z$ $\quad H_0 d_L(z) \sim \dfrac{z}{1 = \sqrt{\Omega_{m,0}}} \sim z$

In fact this is always true to first order in $z$

1500

$$d_L(z) = (1+z)\left[\frac{z}{H_0} + \frac{1}{2}z^2 \frac{d}{dz}\left(\frac{1}{H}\right)\Bigg|_{z=0} + \cdots\right]$$

$$H_0 d_L(z) \sim \cancel{\frac{1}{H_0}} z + z^2\left[\frac{1}{\cancel{H_0}} - \frac{1}{2}\frac{H'_0}{H_0}\right]$$

Lets write this more conventionally ! $a = a_0 \frac{1}{(1+z)}$

$$\frac{dH}{dz}\bigg|_{z=0} = -a_0 \frac{dH}{da} = -\frac{1}{H}\frac{dH}{dt}$$

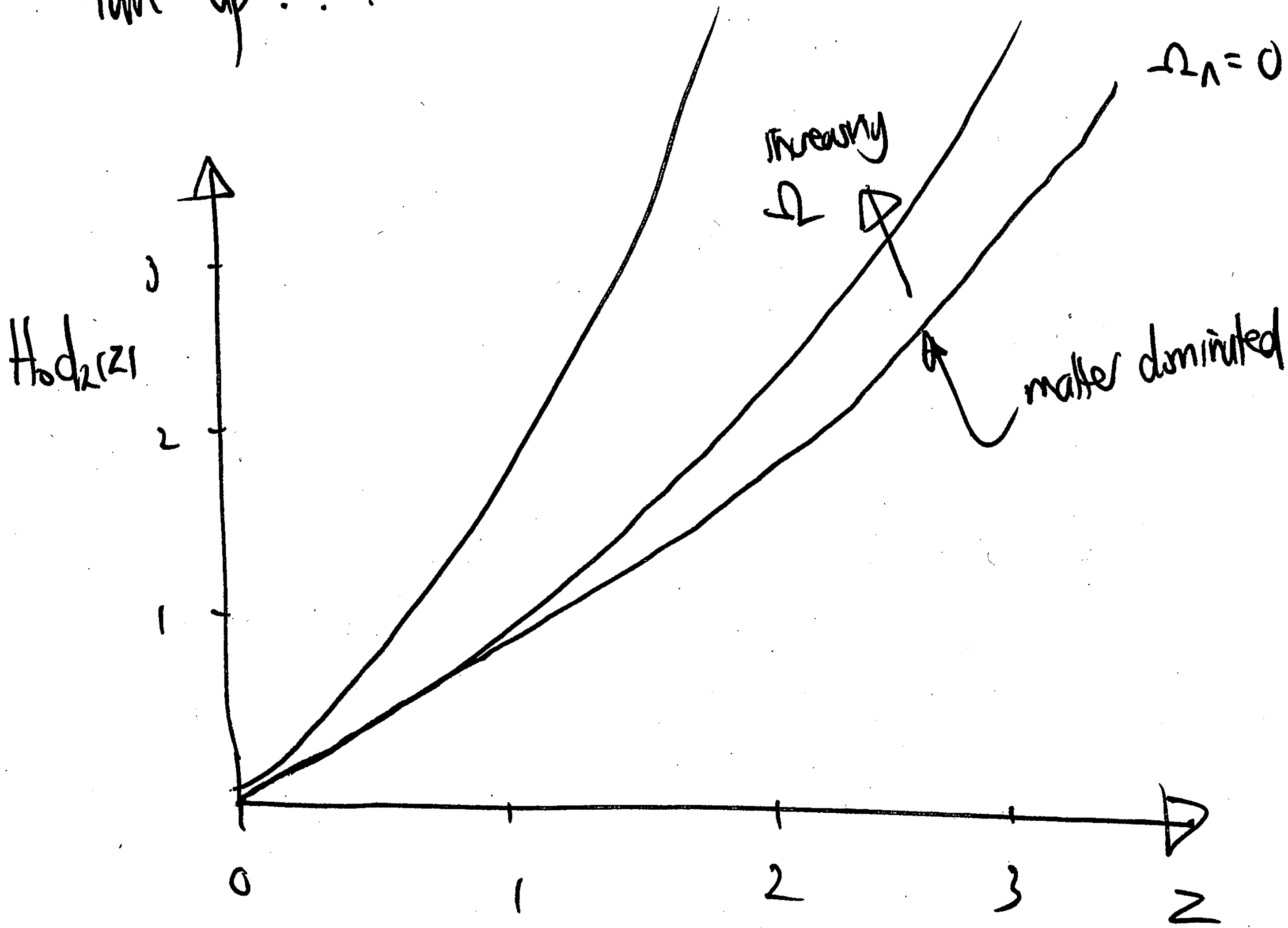
$$H = \frac{\dot{a}}{a} \qquad \dot{H} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} - H^2$$

Since $q = \dfrac{-\dfrac{\ddot{a}}{a}}{H^2}$ $\quad -q_0 = +|q_0|$ !

$$\boxed{H_0 d_L(z) \sim z + \frac{1}{2}(1 - q_0)z^2 + \cdots}$$

1600

The larger the acceleration, the longer the turn up . . .




More dark energy implies higher redshift supernovae appear dimmer than you would expect in a matter dominated universe.

1700

Note that these curves get modified by d.e. at large $z$ only because we have chosen to <u>fix</u> the normalization at $z=0$.

The physics is the other way around ~~to~~ d.e. dominates at small $z$

Assuming $w_{d.e.} \sim -1$ $\Omega_{d.e} \sim 0.7$
$\Omega_m \sim 0.3$

Transition is when

$$0.3 \times (1+z)^3 \sim 0.7$$

$$\boxed{z \sim 1.3}$$

Thus it is only in extremely recent history that d.e. has come to dominate.

This leads to something

If we are confident of $\Omega_{m,0}$ and $H_0$ we can directly reconstruct the equation of state of dark energy from the supernovae data

$$\rho_{d.e} = 3M_{pl}^2 \left[ H^2 - H_0^2 (1+z)^3 \Omega_{m,0} \right]$$

$$W_{d.e.} = \frac{P}{\rho}$$

$$\frac{d\rho}{d\ln a} = -3\rho(1+W) \qquad 3(1+W) = \frac{d\rho}{d\ln(1+z)}$$

$$= \frac{(1+z)}{\rho}\frac{d\rho}{dz}$$

$$\boxed{1+W_{d.e.}(z) = \frac{1}{3}(1+z)\frac{d\ln\rho_{d.e.}}{dz}}$$

1900

$$H\, W_{d.e.}(z) = \frac{1}{3}(1+z)\left[\frac{2\frac{dH}{dz}H - 3H_0^2(1+z)^2\Omega_{m,0}}{H^2 - H_0^2(1+z)^3\Omega_{m,0}}\right]$$

Finally remember that $H = \left[\frac{d}{dz}\left(\frac{D_L(z)}{1+z}\right)\right]^{-1}$

substituting in gives an expression for $W_{d.e.}(z)$

assuming $D_L(z)$, $\frac{d}{dz}D_L(z)$, $\frac{d^2}{dz^2}D_L(z)$,

$H_0$, $\Omega_m(0)$

All ~~conceptually~~ straightforward things to determine observationally?

In practice however this is rarely what is done. It is usual to parameterize $W(z)$

say as $W(z) = W_0 + W'z$ or $W_0 + b\ln(1+z)$

If $z \ll 1$, distinction unimportant, however (2000)
at $z \sim 1$ make a difference

$W(z) = W_0 + W' z$ is arguably pathological
at large $z$

A more common one is (Linder)

$$W(a) = W_0 + W_a(1-a) = W_0 + \frac{W_a z}{(1+z)}$$

(should really do principal component analysis)
Implicitly in this is that d.e. undergoes a transition
from e.g. state $W_0 + W_a$ at large $z$ to $W_0$ at
small $z$ (around $z = 1$).

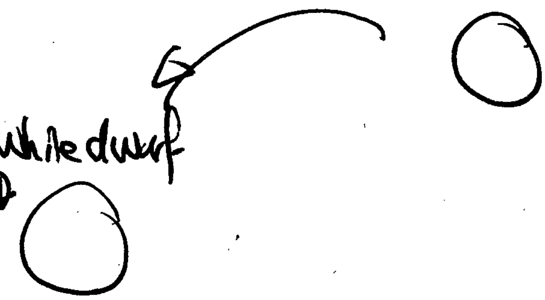
It need not be said that the theoretical justification
of any of these parameterizations is extremely poor.

Sometimes people look at the jerk parameter

$$j = \frac{\dddot{a}}{a H^3}$$ (Rapetti 2007, Visser 2004)

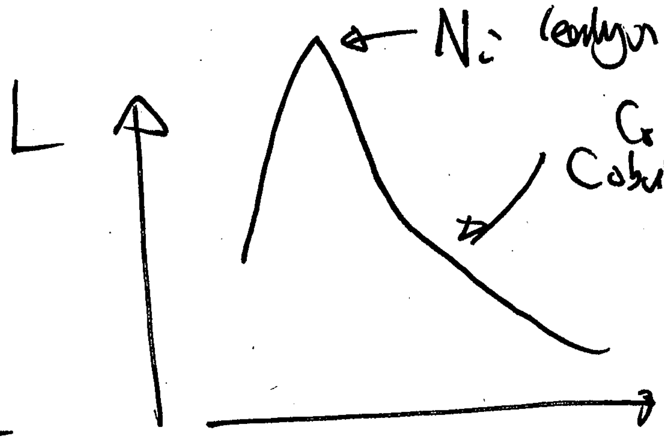
Type IA supernovae are a prime candidate as they are very bright! standard candles

Stellar explosion




mass increases to Chandrasekhar limit $m \sim 1.4 M_\odot$

As this occurs core gets hot enough for carbon fusion $C + C \to$ heavier elements (fusion) to occur




Peak luminosity is determined by amount of Nickel.
(Philips relation)

$M_V(\text{supernova}) = -19.3$

$M_V^\odot = -4.8$

$L_{SN} \sim 10^{9-10} L_{sun}$

(typically $L_{galaxy} \sim 10^{11} L_{sun}$)

22(b)

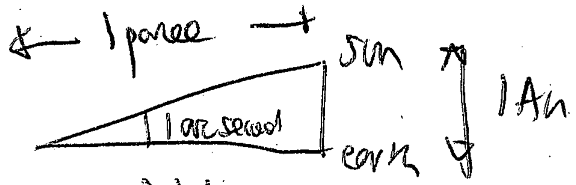
More Nickel + higher peak slower ~~can~~ decay.
This changes shape and can be factored in.

Astronomers use apparent magnitudes (defined on a log scale)

$$m_1 - m_2 = 2.5 \log\left(\frac{F_1}{F_2}\right)$$

$m(\text{Vega}) = 0$

Vega = bright star in Lyra constellation




1 parsec ~ 3.2 light years.

Absolute magnitude is related to absolute luminosity

$$m - M = 5 \log\left(\frac{d_L}{10\,pc}\right)$$

$M(\text{sun}) \sim 4.83$

Sometimes people use the distance modulus

$$\mu = 5 \log\left(H_0 d_L\right)$$

(clearly more useful)

First done by $\boxed{\text{Supernova Cosmology Project and High-z Supernova Team}}$

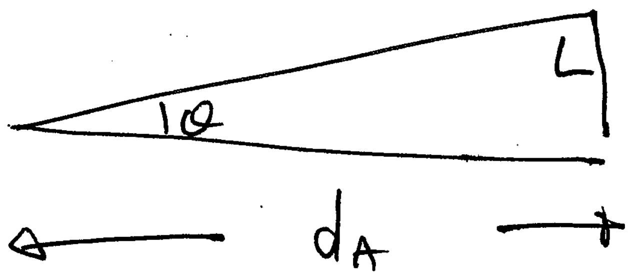
## Other probes

Cosmic microwave anisotropies are angular fluctuations in the temperature $\delta T(\theta, \phi)$ of the would be black body spectrum left over from decoupling of photon / baryon fluids around the time of recombination at $z \sim 1100$

Because these are angular fluctuations they tell us about angular distances albeit at one given redshift.

Given a standard ruler of length $L$, subtending and angle $\theta$ on the sky we define

$$d_A = \frac{L}{\theta}$$



2400

The angle subtended by a source at redshift $z$ in FRW will be

expansion of space

$$L = a(z)\, b(\chi)\, \Delta\theta$$

takes into account curvature of space

$$\Rightarrow \quad d_A = a(z)\, b(\chi)$$

$$\frac{d_A}{d_L} = \frac{a_0\, b(\chi) / (1+z)}{a_0\, b(\chi)\, (1+z)} = \underline{\underline{\frac{1}{(1+z)^2}}}$$

In practice this relationship is not quite true since scattering of photons tends to reduce the luminosity giving ~~a smaller~~ larger than true $d_L$ but largely unaffecting $d_A$.

2500

In the case of CMB, peaks/troughs are determined (acoustic oscillations)

are determined by oscillations in the coupled baryon/photon fluid as the pressure and gravity try to oppose each other. These stop at decoupling and so the position of the first peak is determined by the sound horizon (maximum distance photons could have travelled)

$$r_s = \int c_s \, d\tau = \int \frac{1}{\sqrt{3}} \, d\tau \sim \frac{(1+z_{rec})\, c_s}{H(z_{rec})}$$

at last scattering

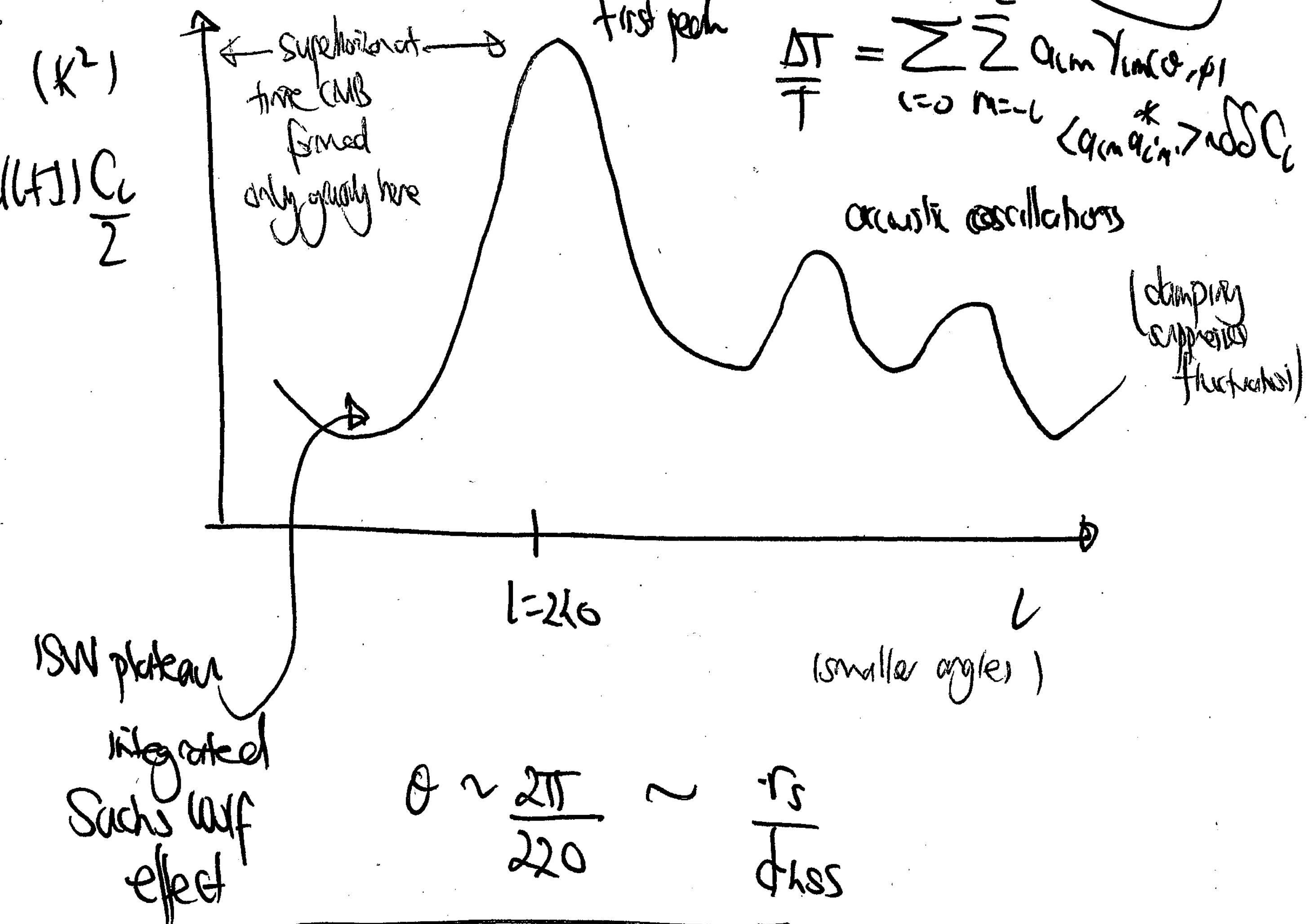
$$1^\circ \longrightarrow \theta \sim \frac{r_s}{d_A} \longleftarrow$$ angular distance to (14 Gpc) last scattering surface.

$r_s$ is essentially correlation length for temperature fluctuations

(2600)

$$\frac{\Delta T}{T} = \sum_{l=0} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi)$$

$$\langle a_{lm} a^*_{l'm'} \rangle \delta\delta C_l$$




$$\theta \sim \frac{2\pi}{220} \sim \frac{r_s}{d_{LSS}}$$

$$\boxed{r_s \sim 145 \text{ Mpc}}$$

These same peaks can be seen in large scale structure where they are known as Baryon Acoustic Oscillations - Even more promising as sensitive to

$$D_V = \left[ (1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3}$$

$$D_A(z) = \frac{1}{H_0} \frac{\sin_k \left( H_0 \sqrt{|\Omega_k|} \int_0^z dz/H(z) \right)}{(1+z)\sqrt{\Omega_k}}$$

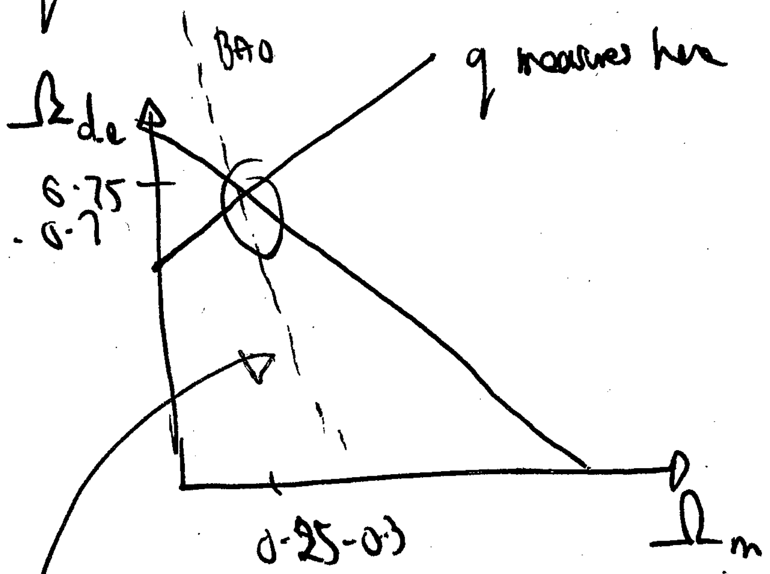
(2601)

CMB measure well flatness

$\Omega_m + \Omega_{de} \sim 1$ (neglecting radiation)

Supernova measures $q$ which is

$q \sim \frac{1}{2}\Omega_m - \Omega_{de}$




BAO gives 3rd independent probe more or less confirming the existence of dark energy.

## review of probes

2700

Luminosity distance — SN Ia

Angular distance — BAO, [CMB] A0

Age of universe — Globular cluster ages

( $\Omega_\Lambda = 0$ $t = \frac{2}{3H_0} \sim 8-10$ Gyr ) (~13 Gyr)

Growth of structure (eg ISW, DM distribution, Mass (z) distribution) — Galaxy surveys, Weak Lensing, CMB + cross correlation with LSS

Precision measurements of Gravity

Deviations from equivalence principle - Ground and Space-based EP tests

Solar system tests (perihelion) - LLR

Large extra dimensions
Existence of extra scalars } Lab experiments, colliders, WIMP + neutrino searches

Cosmological birefringence — CMB / galaxy polarisation measurements

2800

WMAP 7

Allowing for w (constant $\neq -1$)

$-0.86 < 1 + w < 0.59$ (95% CL)

$\Omega_{de} = 0.741^{+0.095}_{-0.099}$

$\Omega_c = 0.215^{+0.082}_{-0.078}$

$\Omega_b = 0.044^{+0.017}_{-0.022}$

$r_s(z_d) = 153.1 \pm 1.7$ Mpc

$\cancel{\dfrac{r_s(z_d)}{D_v(z=0.35)}}$

# Cosmological Constant

What is it ? Why must it be included ?

Gravity is a non-renormalizable theory .

Many years people thought NR does not make sense
(due to infinite no. of renormalizations)
∴ treat gravity classically , field theory quantum

Modern perspective = Wrong !

NR theories make sense ! at least below
some energy scale $\Lambda_c$ = cutoff theory.

idea every physical quantity can be expanded
as a power series in energy over $\Lambda_c$

eg $$\sigma(E) = E^\alpha \left( 1 + \frac{E}{\Lambda_c} + \frac{E^2}{\Lambda_c^2} + \dots \right)$$

3000

As long as we restrict to a finite order in expansion we only have a finite no. of co-efficients to renormalize $\therefore$ predictable

eg for gravity $\Lambda_c = M_{pl}$ and natural expansion is (see Burgess gr-qc/0311082) Quantum Gravity in Everyday life

$$S = \int \frac{1}{2} M_p^2 \sqrt{-g} \left[ R + c_1 \frac{R^2}{M_p^2} + c_2 \frac{R^{\mu\nu} R_{\mu\nu}}{M_p^2} + c_3 \frac{R^4}{M_p^4} + \dots \right]$$

Infinite number of co-efficients $c_1, c_2 \dots$ but if $R \ll M_p$ corrections become increasingly smaller. Thus in many case no need to go beyond leading terms to get quantum corrections

eg. we may use this method to compute

quantum corrections to Newton's law

tree level part

$$V_{Newton + quantum} = -\frac{M_1 M_2 G}{r}\left[1 + \frac{\lambda G(M_1 + M_2)}{r c^2} + \frac{\varepsilon G \hbar}{r^2 c^3} \cdots\right]$$

(Donoghue) 95

quantum one-loop part

Gravity should (always) be treated as low energy effective field theory.

Logic of effective field theory is

Write down every local operator consistent with symmetries and particle content of theory. Co-efficients of operators are parameters which get renormalized

For gravity we can always add a c.c.

$$S = \int \sqrt{-g}\left[ -\Lambda + \frac{1}{2} M_{pl}^2 R + \frac{c_1 R^2}{M_{pl}^2} + \ldots \right]$$

operator must be included to define EFT.

No problem occurs when we couple gravity to SM as long as we continue to work in EFT framework.

Note that we cannot in this framework calculate $\Lambda$! It is a parameter ~~just~~ that must be fit by experiment. Thus statements like "SM predicts a cosmological constant of order $TeV^4$" are wrong.

3300

$$\Lambda = \Lambda_{bare} + \Lambda_{quantum}$$

$$\Lambda_{quantum} = \sum_{S} (2S+1)(-1)^{S} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2+m^2}$$

Separation artificial! only $\Lambda$ counts.

Why then do we worry about ~~technical naturalness~~? cosmological constant problem?

It is a problem of technical naturalness. We cannot predict the actual value in the context of EFT, but we can see individual contributions of order

$m^4$ $\qquad \rho_\Lambda \sim 10^{-48}\,\text{GeV}^4 \qquad m_e \sim 511\,\text{keV} \sim \frac{1}{2}\,\text{MeV}$

$m_{Proton}^4 \sim \text{GeV}^4$

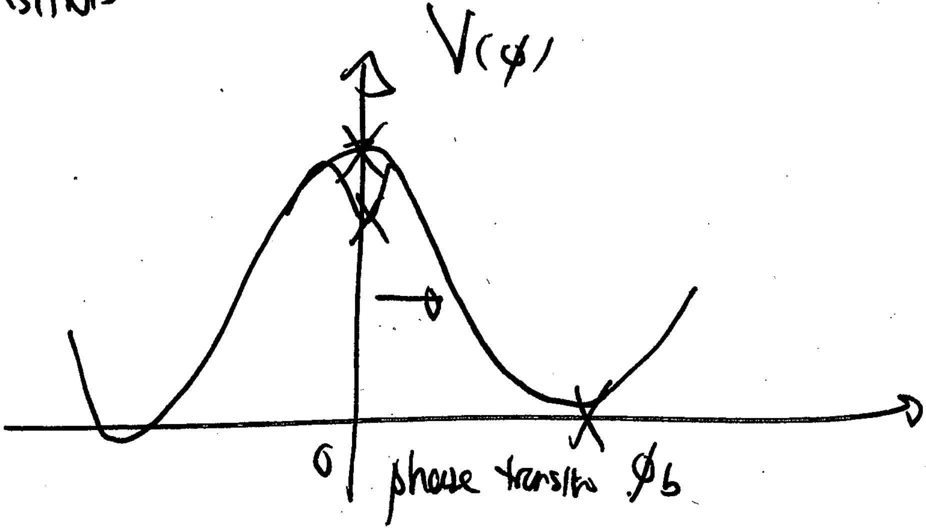
$\frac{m_e^4}{\rho_\Lambda} \sim 10^{34} \qquad M_{new}^4 \sim \text{TeV}^4 \sim 10^{12}\,\text{GeV}^4$

$\frac{M_{pl}^4}{\rho_\Lambda} \sim 10^{120} \qquad M_{pl}^4 \sim (10^{18}\,\text{GeV})^4 \sim 10^{72}\,\text{GeV}^4$

$M_{GUT}^4 \sim (10^{14}\,\text{GeV})^4 \sim 10^{56}\,\text{GeV}^4$

3400

Point - although we cannot predict $\Lambda$ in LEEFT, we can calculate changes in $\Lambda$, either through integrating out heavy particles or eg via phase transitions




$$\Lambda_{eff} = V_{min}$$

$$\Delta \Lambda_{eff} = V(\phi_b) - V(0)$$

eg

Potential energy of Higgs field $V \sim (100\,GeV)^4$

QCD condensate energy in presence of $q\bar{q}$ bilinears (chiral symmetry breaking) $V \sim (100\,MeV)^4$