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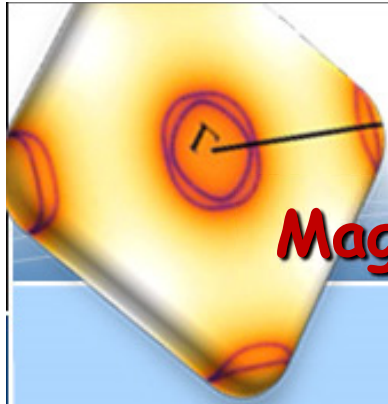
**2157-4**

**Workshop on Principles and Design of Strongly Correlated Electronic  
Systems**

*2 - 13 August 2010*

**Magnetism and Superconductivity in Pnictides**

Zlatko TESANOVIC  
*Johns Hopkins University  
Baltimore  
U.S.A.*



# INSTITUTE FOR QUANTUM MATTER

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and PRINCETON UNIVERSITY

## Magnetism and Superconductivity in Pnictides

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Funded by the U.S. Department of Energy



Swing flu space interest scientists most in 2009



\$ 100 question: What is the theory of iron-pnictides ?

- LDA + RPA: Mazin, Singh et al, Kuroki et al, Scalapino et al, Schmalian et al, ...
- Weak coupling<sup>++</sup>: Chubukov, Eremin et al, DH Lee, Vishwanath et al, Cvetkovic et al, Bernevig, Thomale et al, ...
- Mott limit: Si & Abrahams, Phillips et al, Sachdev et al, Kivelson et al, Zaanen & Sawatzky et al, Hu, Bernevig et al, Dagotto et al, ...
- Assorted insights: Haule & Kotliar, Hirschfeld et al, Benfatto, Castellani et al, PA Lee & Wen, Raghu, SC Zhang, et al, Nagaosa & Ng, Gor'kov et al, FC Zhang & Rice, ...



linked anti-angiogenesis to tumor growth.

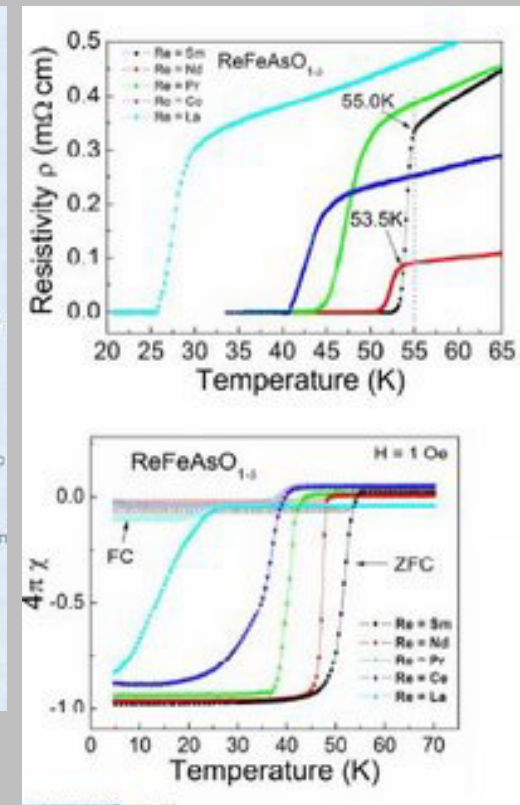
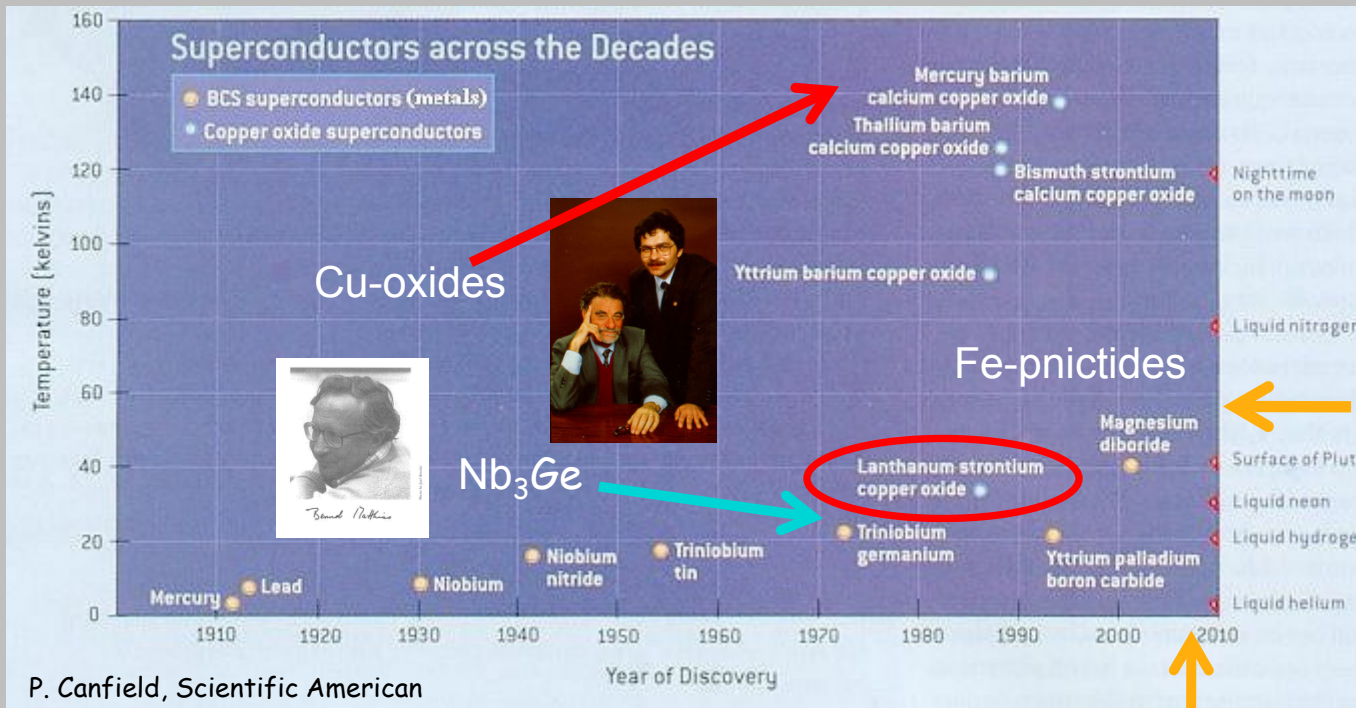
•Graphene, single-atom layers of carbon that have semiconductor properties. They "look like a coming revolution in electronics," Pendlebury says. *Science* magazine included graphene on its "Top Ten" list of breakthroughs for the year.

•Small RNA's, genetic materials that regulate genes in cells. They've emerged in "an astounding landscape" notes a highly-cited *Nature Reviews Molecular Cell Biology* survey led by V. Narry Kim of South Korea's Seoul National University. They have potential to treat diseases and reveal how genes work on a fundamental level inside cells. But not a big news item.

•Obesity gene, biology and diet studies. A *New England Journal of Medicine* report that found cutting calories, whatever their origin, mattered the most surprisingly high number of citations, considering it confirmed long-standing advice.



# Superconductors → Hg → Nb<sub>3</sub>Ge → cuprates → pnictides



Spring 2008

time



Greatest web-induced frenzy in history of condensed matter physics: 17 papers on arXiv in a single July '08 day. Comparable to the latest superstring "revolution" (Bagger-Lambert)

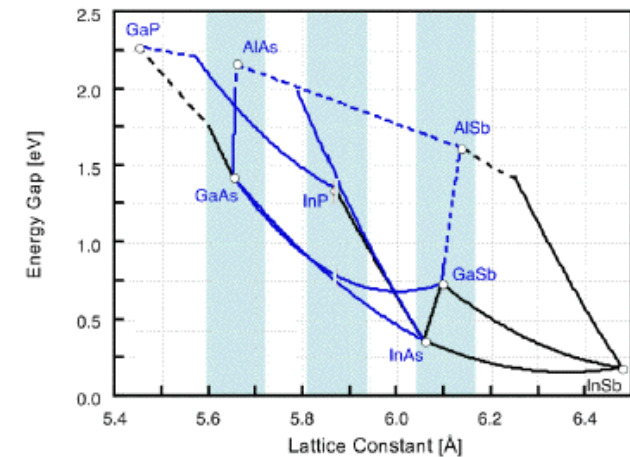


# Pnictides $\pi\nu\iota\gamma\epsilon\nu$ (Greek for choking, suffocation): Semiconductors $\rightarrow$ Semimetals $\rightarrow$ Superconductors

Periodic System of the Elements

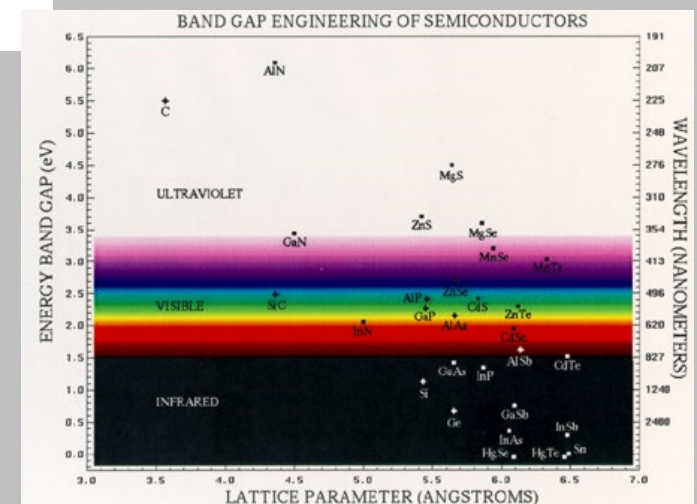
IA 1	IIA 2											IIIA 13	IVA 14	VA 15	VIA 16	VIIA 17	VIIIA 18
1 H	2 He											3 B	4 C	5 N	6 O	7 F	8 Ne
3 Li	4 Be											9 Al	10 Si	11 P	12 S	13 Cl	14 Ar
5 Na	6 Mg	7 Sc	8 Ti	9 V	10 Cr	11 Mn	12 Fe	13 Co	14 Ni	15 Cu	16 Zn	17 Ga	18 Ge	19 As	20 Se	21 Br	22 Kr
31 Rb	32 Sr	33 Y	34 Zr	35 Nb	36 Mo	37 Tc	38 Ru	39 Rh	40 Pd	41 Ag	42 Cd	43 In	44 Sn	45 Sb	46 Te	47 I	48 Xe
55 Cs	56 Ba	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
87 Fr	88 Ra	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

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Author: P.Menzel. copyright Ernst Klett Schulbuchverlag GmbH, Stuttgart 1991



**Pnictides** - elements from Group V of Periodic Table:  
nitrogen, phosphorus, arsenic, antimony and bismuth

**III-V Semiconductors** - formed by elements from Groups III and V: aluminium phosphide, aluminium arsenide, aluminium antimonide, gallium phosphide, gallium arsenide, gallium antimonide, indium phosphide, indium arsenide and indium antimonide plus numerous ternary and quaternary semiconductors.





# Fe-pnictides: Semimetals → Superconductors

May 2006

J|A|C|S  
COMMUNICATIONS

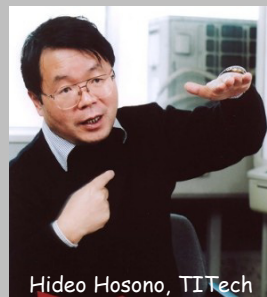
Published on Web 07/15/2006

## Iron-Based Layered Superconductor: LaOFeP

Yoichi Kamihara,<sup>†</sup> Hidenori Hiramatsu,<sup>†</sup> Masahiro Hirano,<sup>†,‡</sup> Ryuto Kawamura,<sup>§</sup> Hiroshi Yanagi,<sup>§</sup>  
Toshio Kamiya,<sup>†,§</sup> and Hideo Hosono<sup>\*,†,‡</sup>

ERATO-SORST, JST, Frontier Collaborative Research Center, Tokyo Institute of Technology, Mail Box S2-13,  
4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, Frontier Collaborative Research Center, Tokyo Institute of  
Technology, Mail Box S2-13, 4259 Nagatsuta, Midori-ku, Yokohama 226-8503, Japan, and Materials and Structures  
Laboratory, Tokyo Institute of Technology, Mail Box R3-4, 4259 Nagatsuta, Yokohama 226-8503, Japan

Received May 15, 2006; E-mail: hosono@msl.titech.ac.jp



Hideo Hosono, TITech



## Superconductivity at 43 K in Samarium-arsenide Oxides



X. H. Chen<sup>\*</sup> and T. Wu, G. Wu, R. H. Liu, H. Chen and D. F. Fang

Hefei National Laboratory for Physical Science at Microscale and Department of Physics,

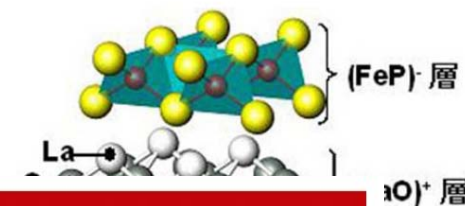
University of Science and Technology of China,

Hefei, Anhui 230026,

People's Republic of China

arXiv:0803.3603v1 [cond-mat.supr-con] 25 Mar 2008

(Dated: March 25, 2008)



### Letter

Nature 453, 761-762 (5 June 2008) |

### Superconductivity at

X. H. Chen<sup>1</sup> (✉), T. Wu<sup>1</sup> (✉)

<sup>1</sup> Hefei National Laboratory for Phys

Correspondence to: X. H. Chen<sup>1</sup> (✉)

Since the discovery of hig  
been devoted to exploring  
from Bardeen-Cooper-S  
copper oxide supercondu  
(ref. 2 ([/nature/journal/v453](#)  
La-Nd, Sm and Gd) are in  
LaO<sub>1-x</sub>F<sub>x</sub>FeAs (ref. 3 ([/nat](#)  
superconductivity in the  
measurements reveal a tr  
high-temperature supercc



International weekly journal of science

### Letter

Nature 459, 64-67 (7 May 2009) | doi:10.1038/nature07981; Received 4 November 2008; Accepted 13 March 2009

## A large iron isotope effect in SmFeAsO<sub>1-x</sub>F<sub>x</sub> and Ba<sub>1-x</sub>K<sub>x</sub>Fe<sub>2</sub>As<sub>2</sub>

R. H. Liu<sup>1</sup>, T. Wu<sup>1</sup>, G. Wu<sup>1</sup>, H. Chen<sup>1</sup>, X. F. Wang<sup>2</sup>, Y. L. Xie<sup>1</sup>, J. J. Ying<sup>1</sup>, Y. J. Yan<sup>1</sup>, Q. J. Li<sup>1</sup>, B. C. Shi<sup>1</sup>, W. S. Chu<sup>2,3</sup>, Z. Y. Wu<sup>2,3</sup> & X. H. Chen<sup>1</sup>

<sup>1</sup> Hefei National Laboratory for Physical Sciences at Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

<sup>2</sup> Beijing Synchrotron Radiation Facility, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

<sup>3</sup> National Synchrotron Radiation Laboratory, University of Science and Technology of China, Hefei 230026, China

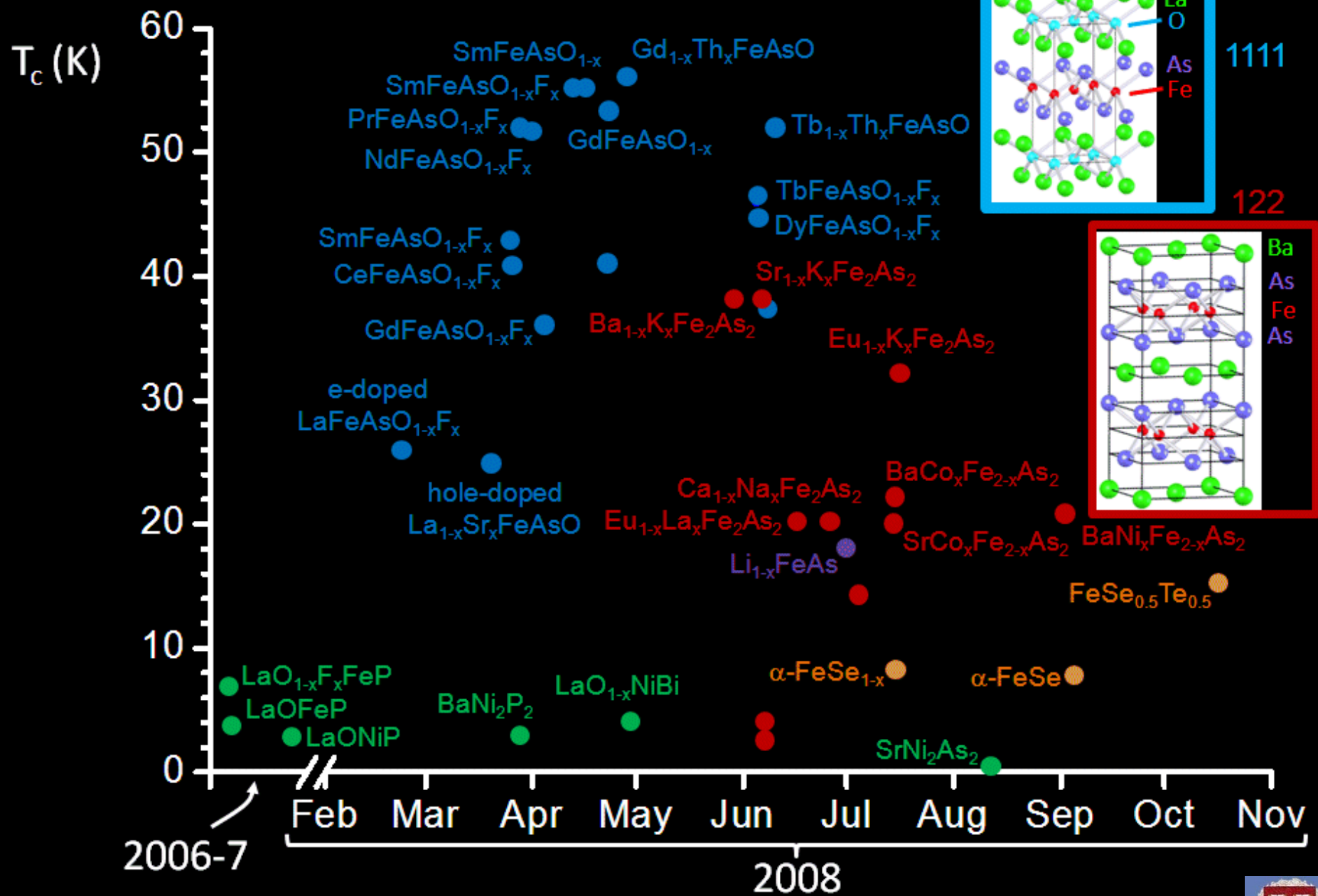
<sup>4</sup> Hefei National Laboratory for Physical Sciences at Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China

Accepted 5 May 2008; Published online 4 June 2008

Fe<sub>0.15</sub>

& C. L. Chien<sup>1</sup> (✉)

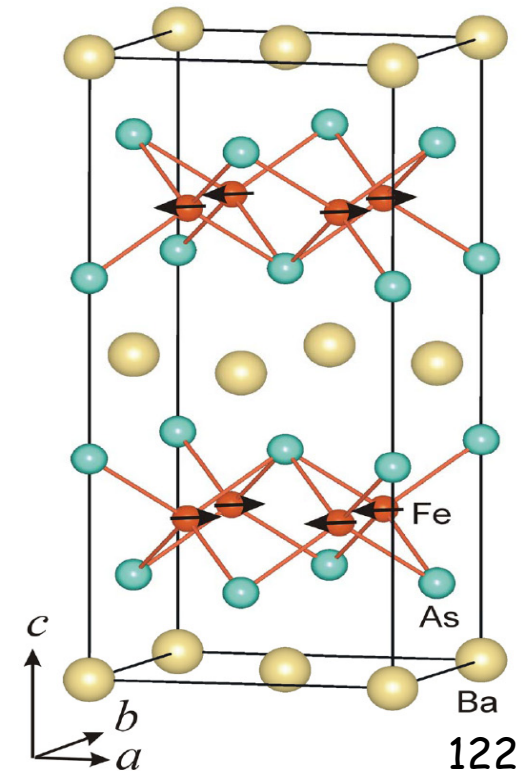
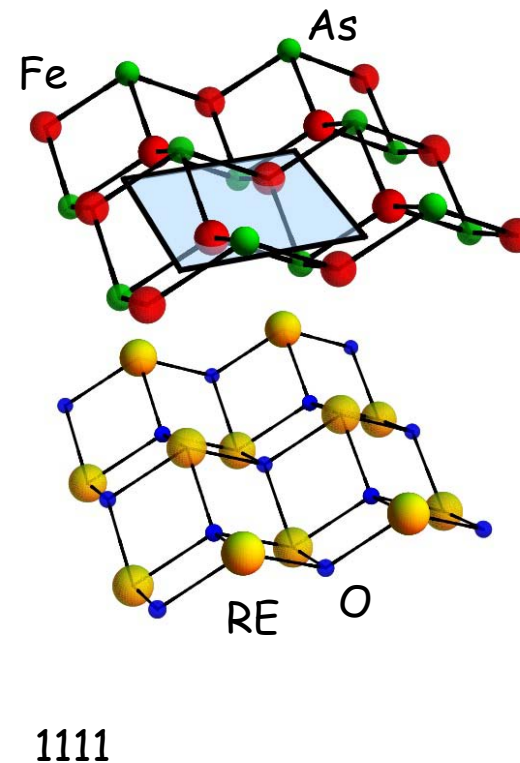
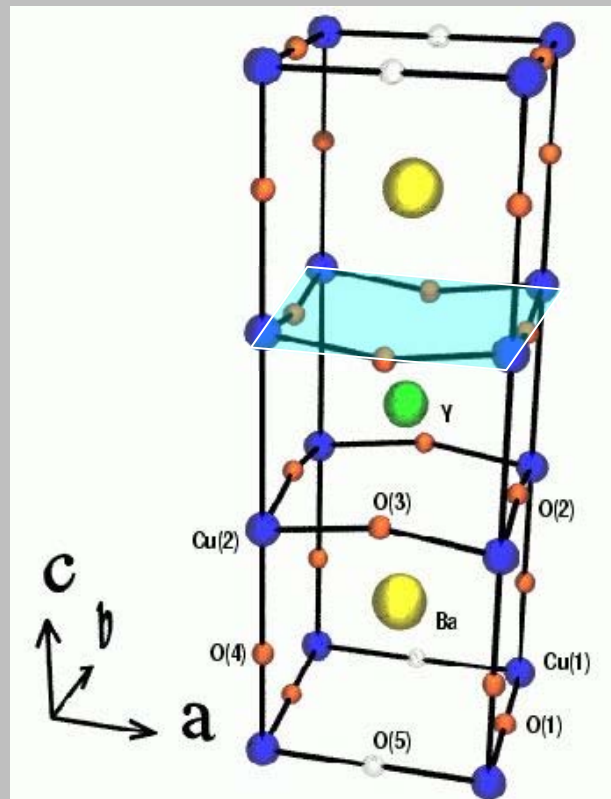
1218, USA



courtesy of J. Hoffman



# Cu-oxides versus Fe-pnictides



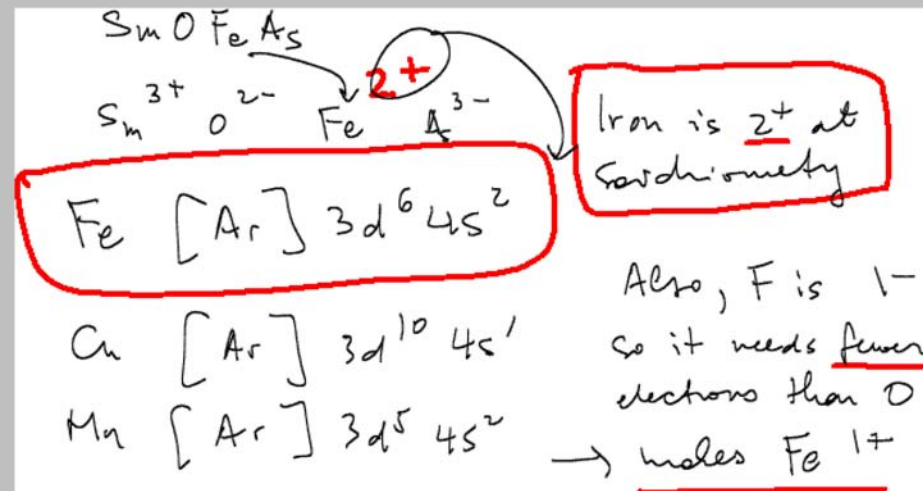
Both have d-electrons in key role (Cu vs Fe)  
Both are layered ( $\text{CuO}_2$  vs  $\text{FeAs}$ )  
Both have AF and SC in close proximity

However, there are also many differences! This may add up to new and interesting physics



# Key Difference: 9 versus 6 d-electrons

ZT, Physics 2, 60 (2009)



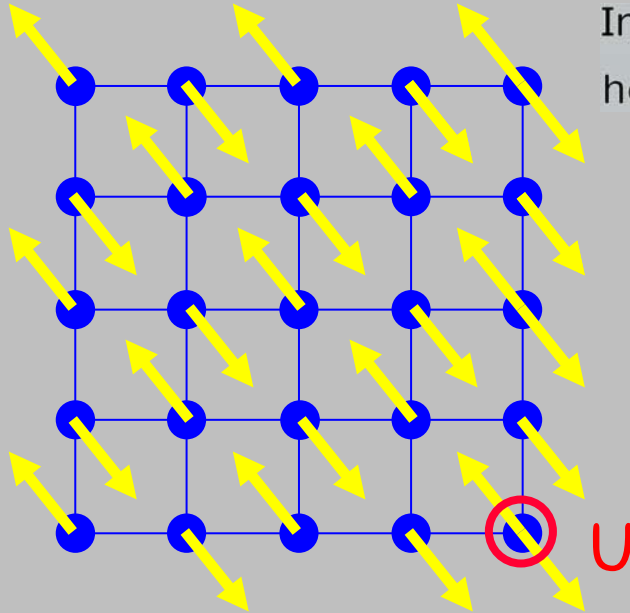
In CuO<sub>2</sub> a **single** hole in a filled 3d orbital shell

→ A suitable single band model might work

In FeAs large and **even** number of d-holes

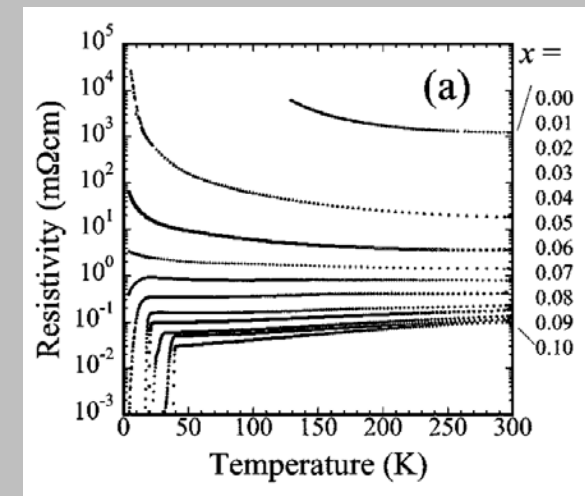
→ A multiband model is likely necessary

# Cu-oxides: Mott Insulators $\rightarrow$ Superconductors

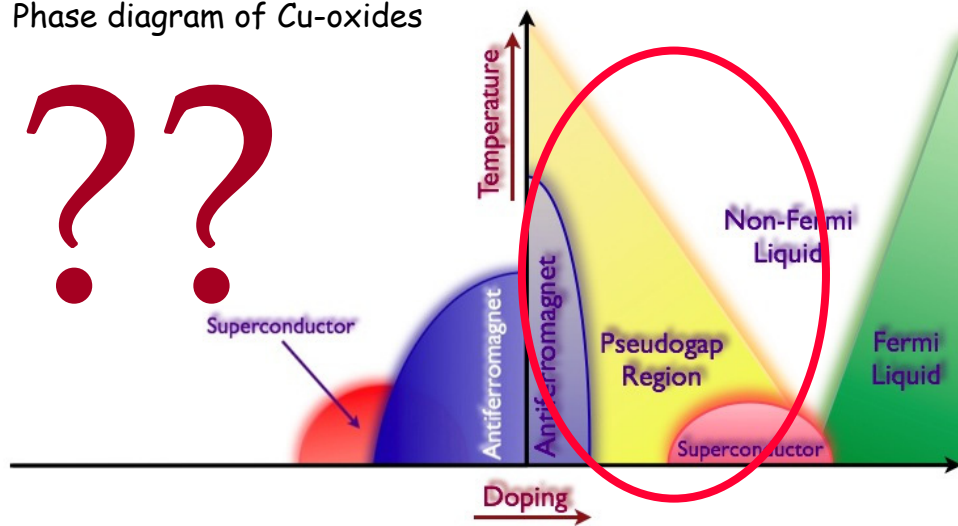


In a half-filled band Coulomb repulsion  $Un_{i\uparrow}n_{i\downarrow}$  ( $U \gg t$ ) keeps holes in place  $\Rightarrow$  Mott insulator + Neel antiferromagnet !!

Only when doped with holes (or electrons) do cuprates turn into superconductors



Phase diagram of Cu-oxides



How Mott insulators turn into superconductors, particularly in the pseudogap region, remains one of great intellectual challenges of condensed matter physics

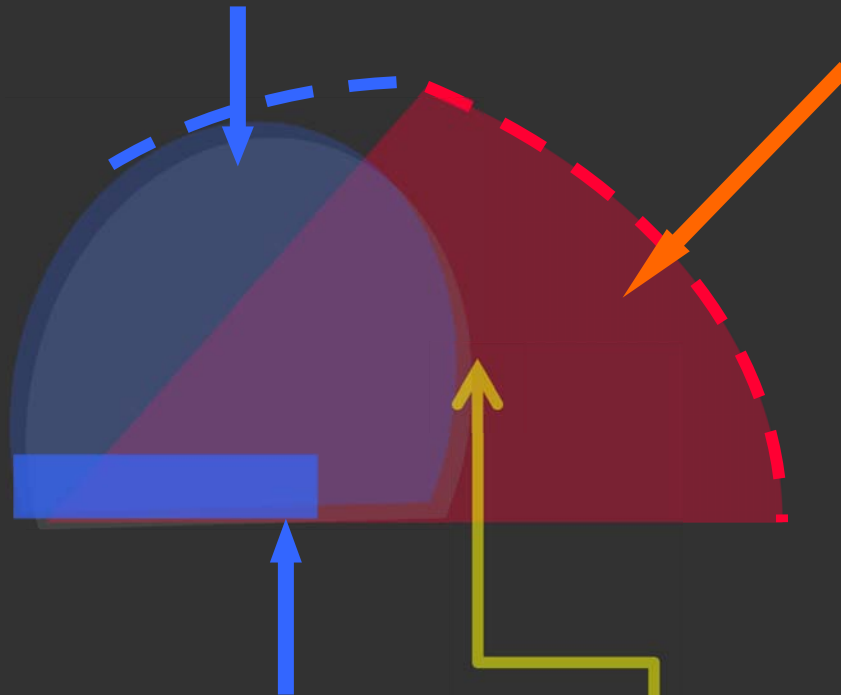
# How Correlated Superconductors turn into Mott Insulators

ZT, Nature Physics 4, 408 (2008)

Correlated superconductors have  
quantum (anti)vortex fluctuations

All superconductors have  
thermal fluctuations

Near  $T_c$  these are always  
phase fluctuations



Ground state with enhanced  
pairing correlations but no SC !!  
(gauge theories, QED<sub>3</sub>, chiral SB, AdS/CMT,...)

No such ground state in  
BCS theory (at weak coupling) !!

BCS-Eliashberg-Migdal

PW Anderson, Balents, MPA Fisher, Nayak, Franz, Vafeek, Melikyan, ZT, Senthil & PA Lee )

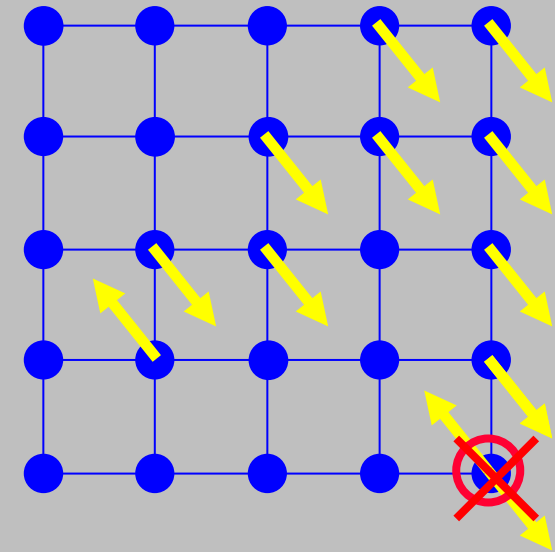
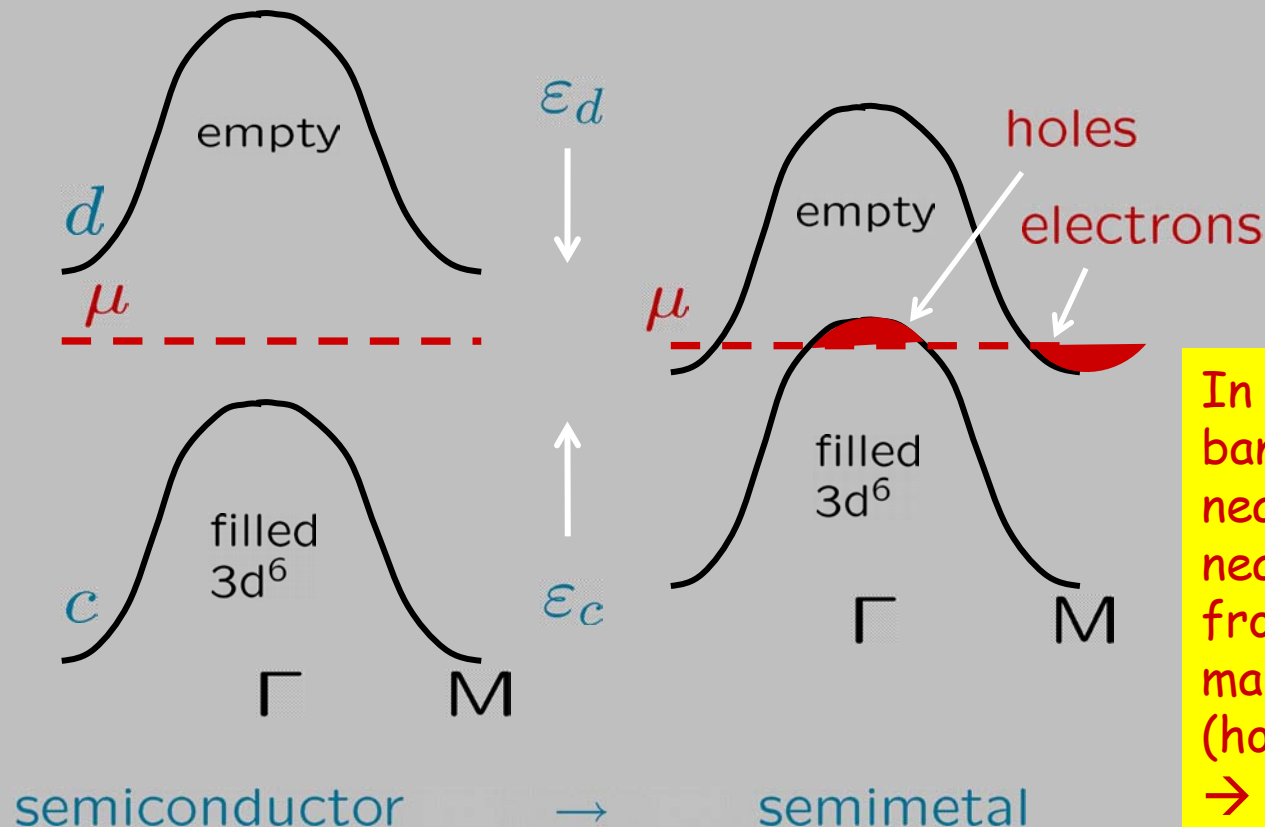
Optimal  $T_c$  in HTS is determined by quantum fluctuations



# Fe-pnictides: Semimetals $\rightarrow$ Superconductors

$$\epsilon_{\vec{k}}^c = \epsilon_c + t_c \cos(k_x a) + t_c \cos(k_y a)$$

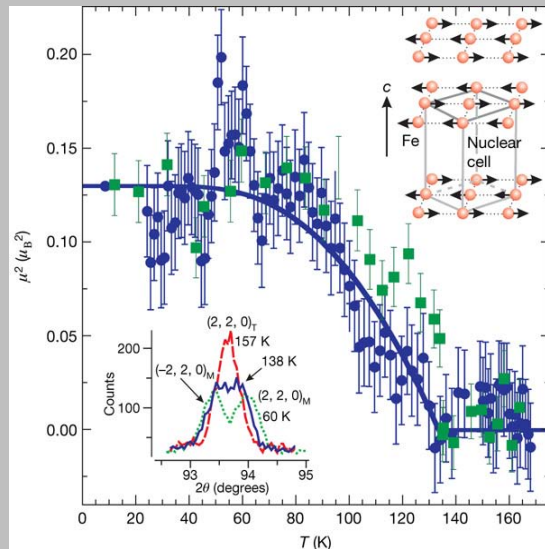
$$\epsilon_{\vec{k}}^d = \epsilon_d + t_d \cos(k_x a) + t_d \cos(k_y a)$$



In contrast to  $\text{CuO}_2$ , all d-bands in FeAs are either nearly **empty** (electrons) or nearly **full** (holes) and far from being **half-filled**. This makes it easier for electrons (holes) to avoid each other.  
 $\rightarrow$  **FeAs are less correlated than  $\text{CuO}_2$  (correlations are still important !! )**

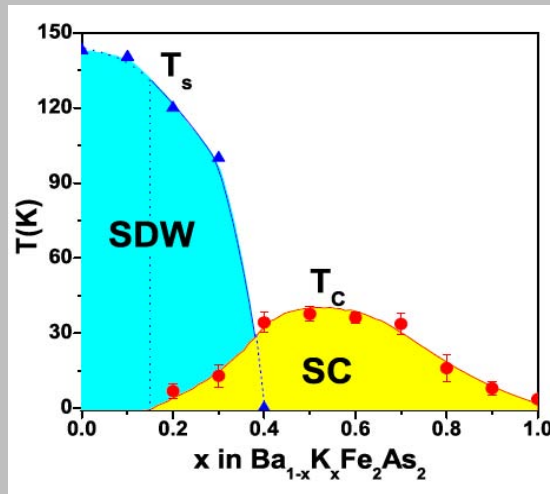
# Phase diagram of Fe-pnictides

C. de la Cruz, *et al.*, Nature **453**, 899 (2008)

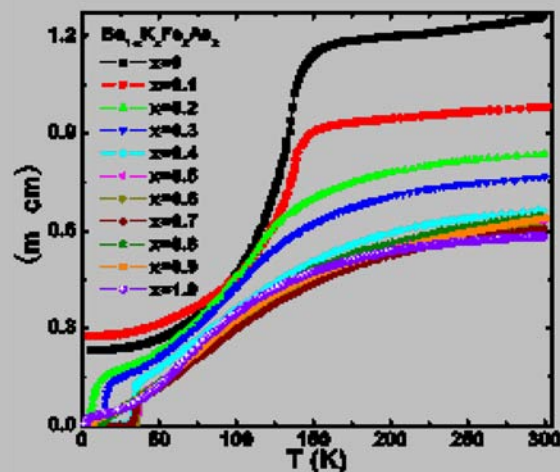


Like  $\text{CuO}_2$ , phase diagram of FeAs has SDW (AF) in proximity to the SC state.

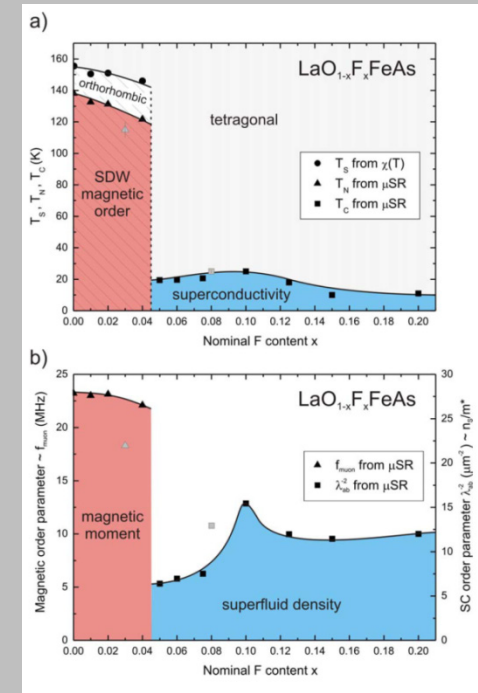
H. Chen, *et al.*, arXiv/0807.3950



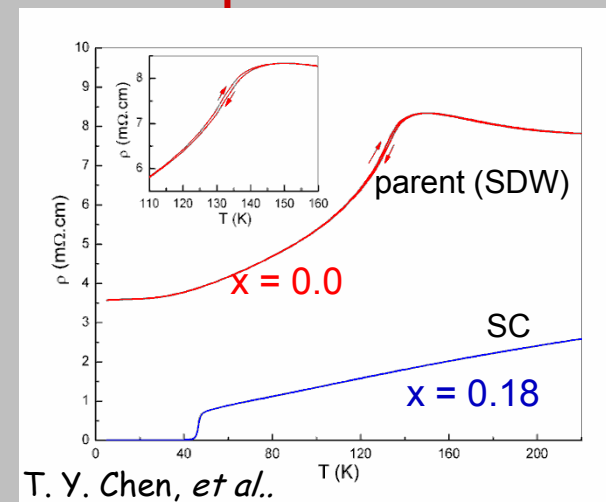
SC coexists with SDW (AF) in 122 compounds →



However, unlike  $\text{CuO}_2$ , all regions of FeAs phase diagram are (bad) metals !!



$\text{SmFeAsO}_{1-x}\text{F}_x$

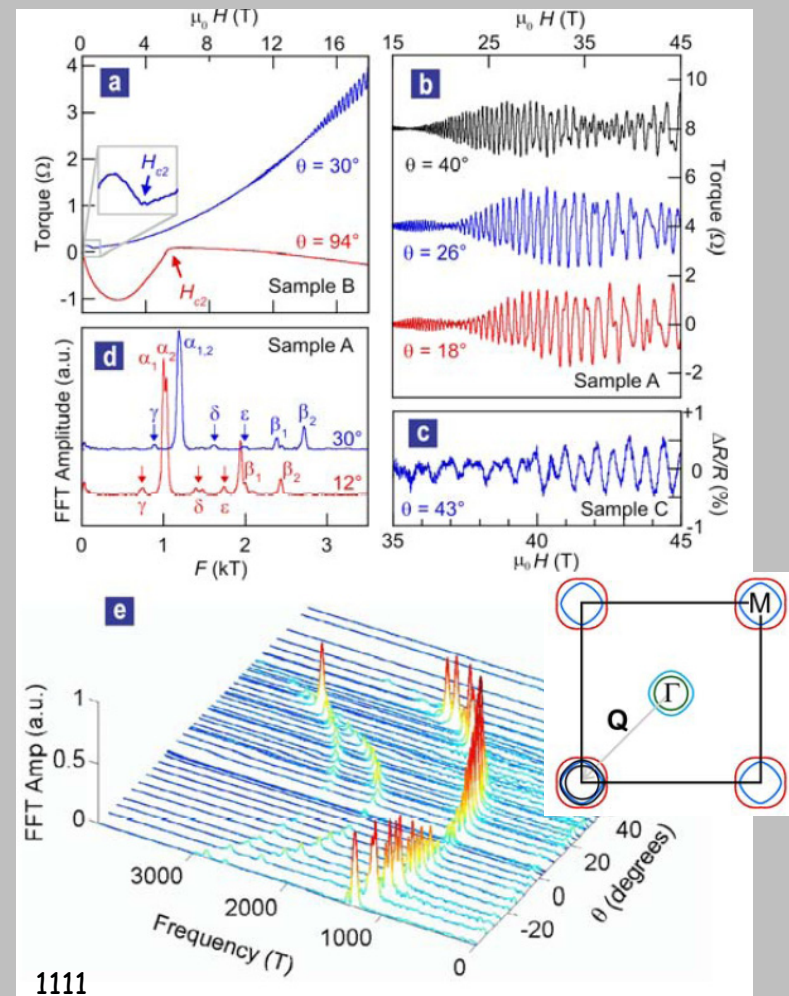
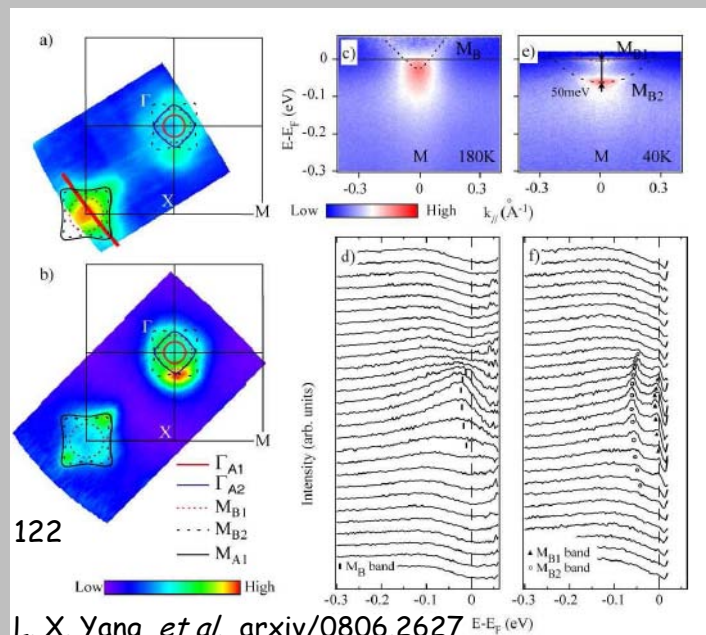
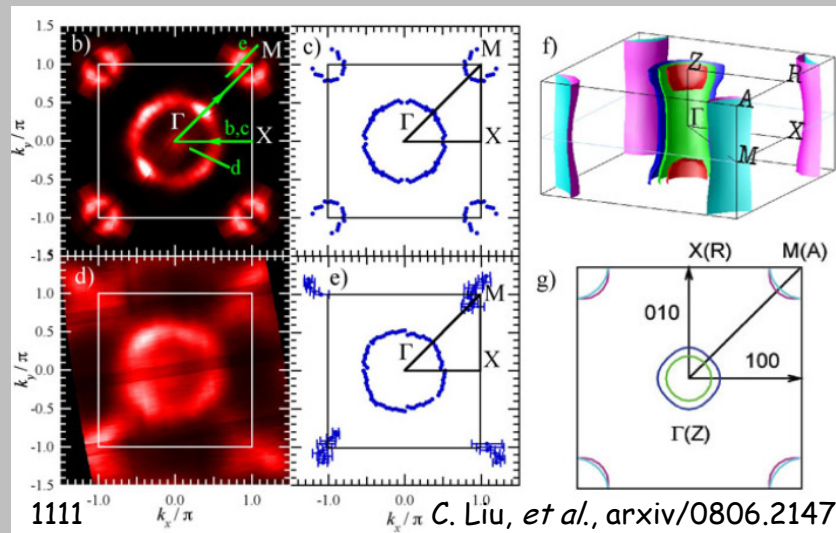


T. Y. Chen, *et al.*

# ARPES

+

# dHvA



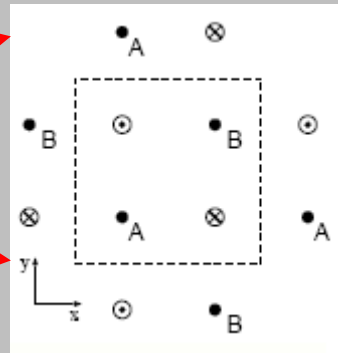
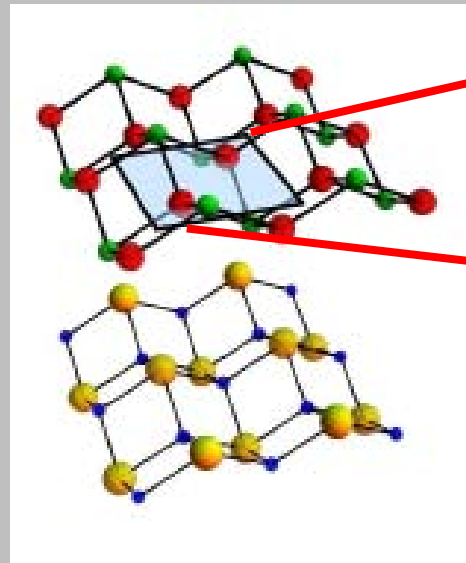
ARPES and dHvA see coherent (metallic) bands in rough agreement with LDA.



# Minimal Model of FeAs Layers I

V. Cvetkovic and ZT, EPL **85**, 37002 (2009)

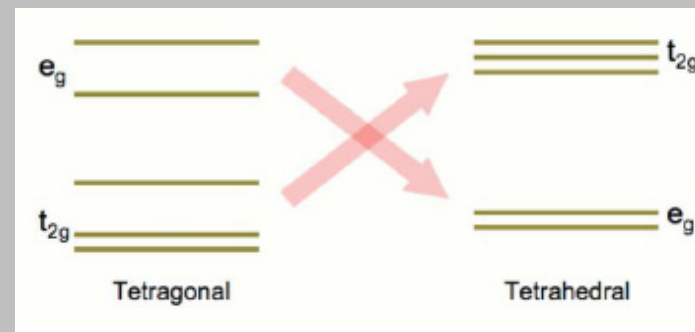
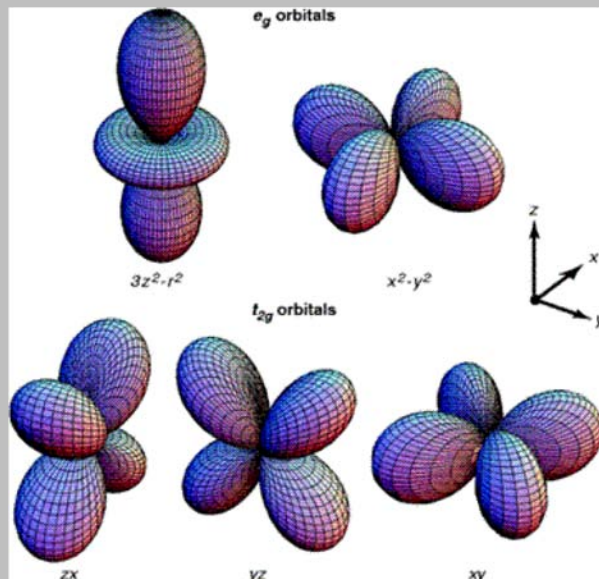
C. Cao, P. J. Hirschfeld, and H.-P. Cheng, PRB **77**, 220506 (2008)



We consider **effective 2D** Fe model, with **all 5 d-orbitals**. As bands are below  $E_F$  but they contribute **crucial** terms to the "minimal" model

"Puckering" of FeAs planes is essential:

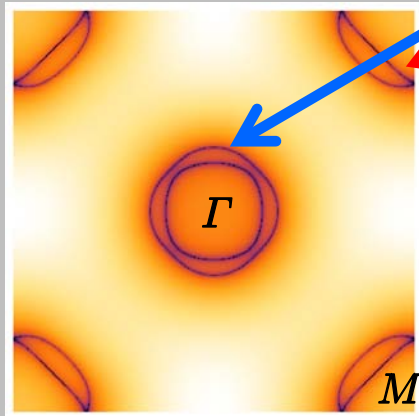
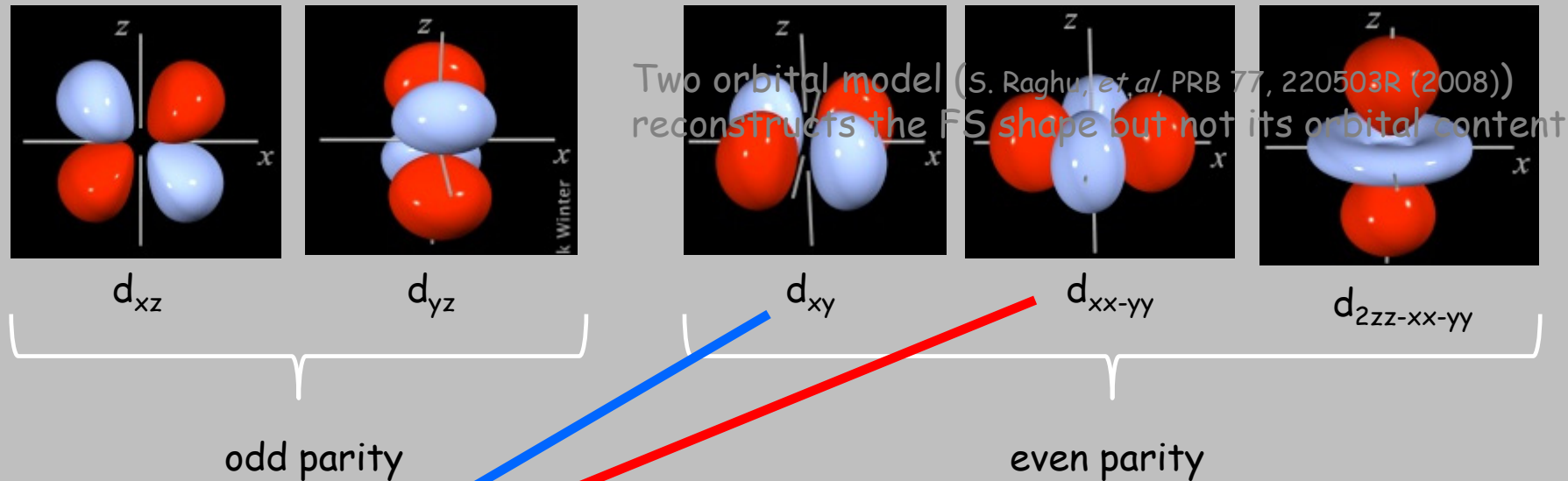
- i) All d-orbitals are near  $E_F$
- ii) Large overlap with As p-orbitals below  $E_F$   
 $\rightarrow$  enhanced itinerancy of d electrons  
 defeats Hund's rule and large local moment



$$(5 + 3) \times 2 = 16 \quad (\div 2) \text{ Wannier orbitals} \\ \Rightarrow \text{"minimal" model} \Rightarrow$$

# Minimal Model of FeAs Layers II

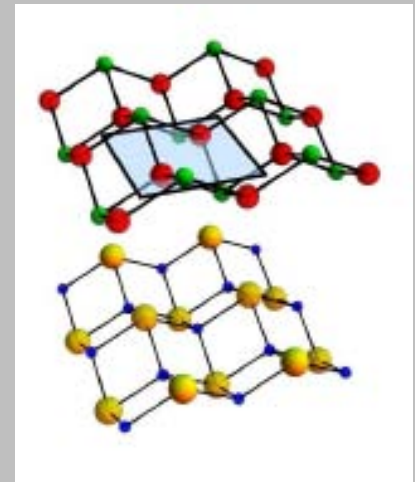
We consider an effective 2D model with 5 Fe + 3 As orbitals



**The importance of Fe 3d - As 4p hybridization:**

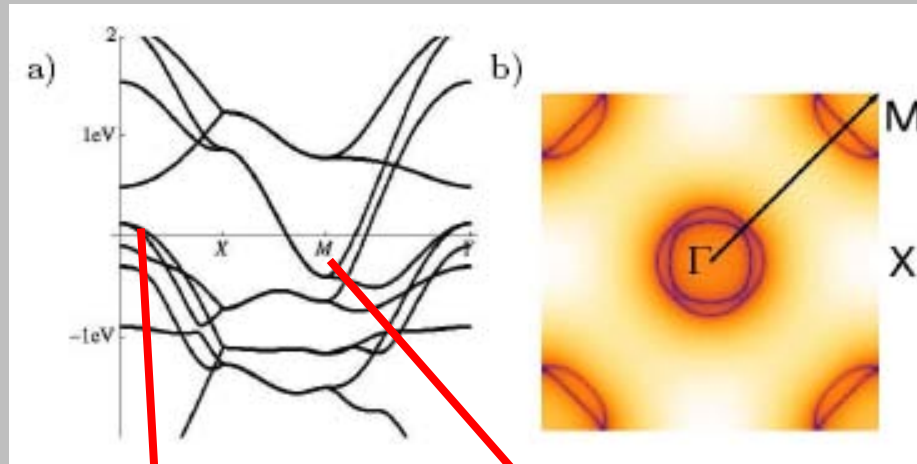
Without pnictide atoms many hopping processes would vanish by symmetry.

These symmetries are violated by pnictide puckering.



# Minimal Model of FeAs Layers III

V. Cvetkovic and ZT, EPL **85**, 37002 (2009)  
 C. Cao, P. J. Hirschfeld, and H.-P. Cheng, PRB **77**, 220506 (2008)  
 K. Kuroki *et al*, PRL **101**, 087004 (2008)



hole FS  
2 pockets (valleys)

electron FS  
2 pockets (valleys)

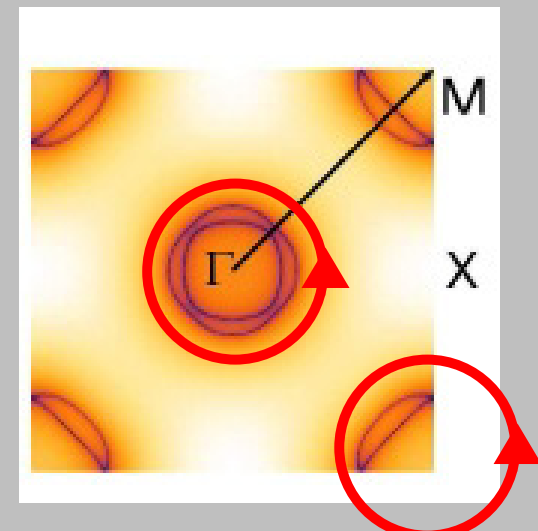
Tight-binding model optimized for band structure + exps.  
 Only nearest neighbor Fe-As, Fe-Fe, and As-As hoppings are used.

$\alpha$	$x^2 - y^2$	$z^2$	$xy$	$xz$	$\beta$	$x$	$z$
$\epsilon_\alpha$	-0.85	-1.4	-1.1	-1.15	$\epsilon_\beta$	-4.0	-4.0
$t_{\alpha,\alpha}^{Fe}$	-0.55	-0.5	-1.6	-0.55	$t_{\beta,\beta}^{As}$	-0.8	-0.45
$t_{\alpha,x/y}$	0.65	-1.4	1.5	3.2			
$t_{\alpha,z}$	2.1	1.25		0.7			

$t_{z^2,xy}^{Fe} = 0.1$ ,  $t_{xz,yz}^{Fe} = -0.75$ , and  $t_{x,y}^{As} = 0.8$ .

**Important:** Near  $E_F$  e and h bands contain significant admixture of **all** five Wannier d-orbitals,  $d_{xz}$  and  $d_{yz}$  of **odd** parity (in FeAs plane) and the remaining three d-orbitals of **even** parity in FeAs plane →

As one goes around the FS there is **strong** mixing of odd and even d-orbitals  
 ⇒ **no simple orbital "topology"**

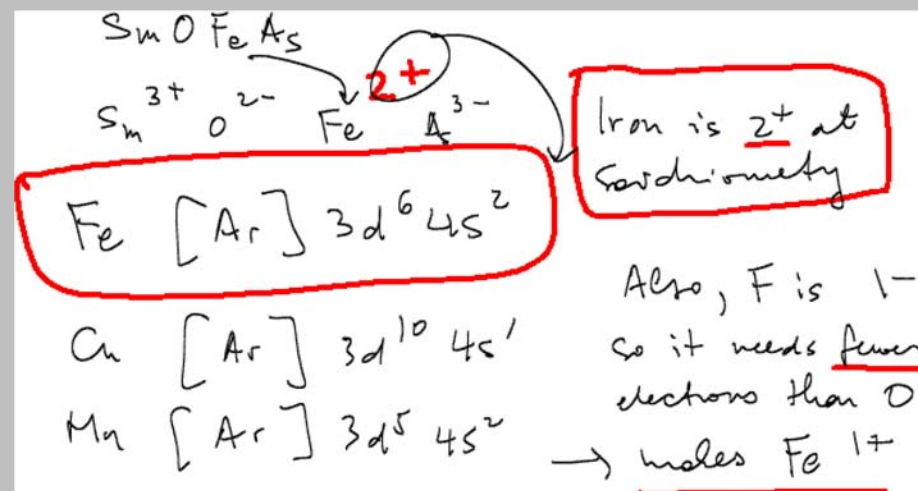
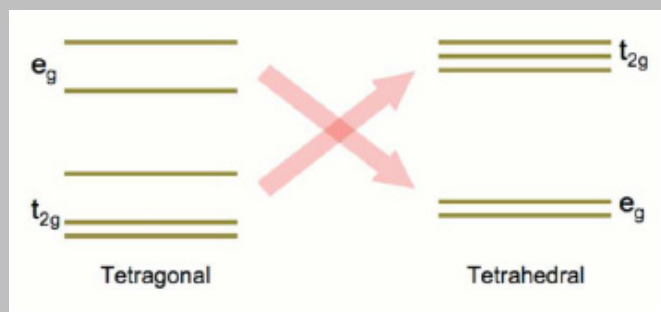
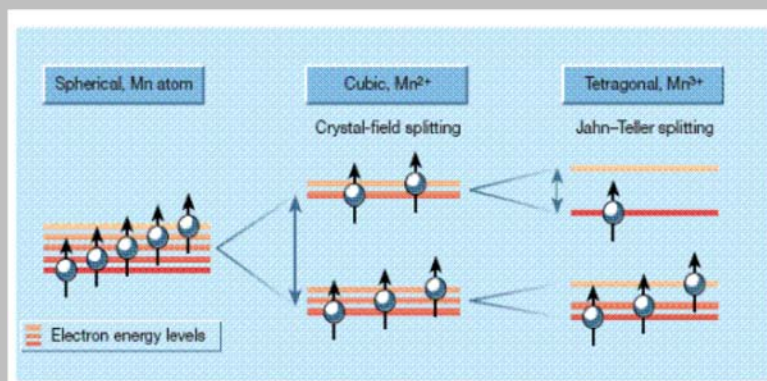




# Hund's Rule Defeated (but Lurking!)

Hund's rule rules for  $\text{Mn}^{2+}$  :  
all five d-electrons line up to minimize  
Coulomb repulsion  $\rightarrow S = 5/2$

Y. Singh *et al.*, arXiv/0907.4094 (MnAs)  
Y. Z. Zhang *et al.*, PRB **81**, 094505 (2010)

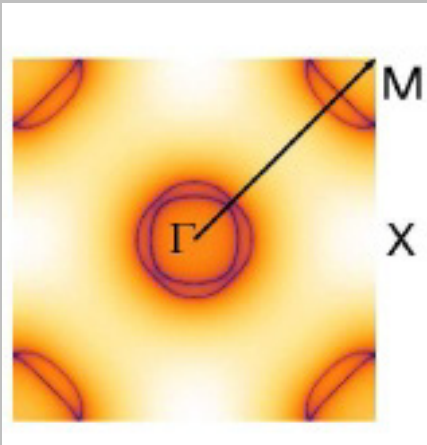


"Puckering" of FeAs planes is essential:

- All d-orbitals are near  $E_F$
- Large overlap with As p-orbitals below  $E_F \rightarrow$  enhanced itinerancy of d electrons defeats Hund's rule and large local moment

# Nesting and Valley Density-Wave (VDW) in Fe-pnictides I

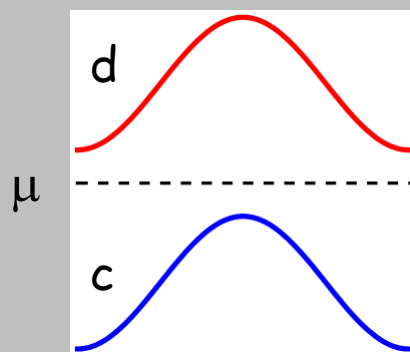
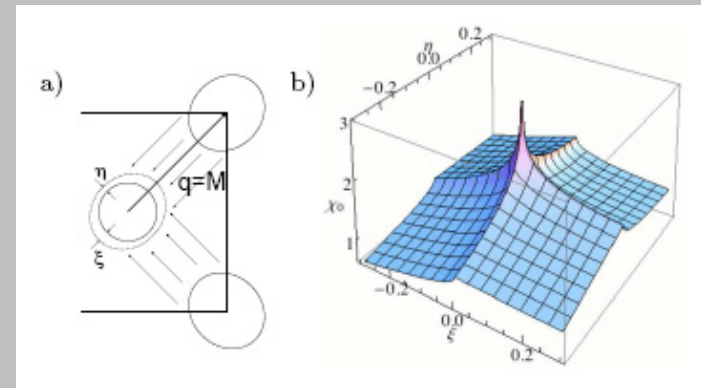
V. Cvetkovic and ZT, EPL **85**, 37002 (2009)  
 V. Cvetkovic and ZT, PRB **80**, 024512 (2009)  
 M. Korshunov and I. Eremin, PRB **78**, 140509 (2008)



If hole ( $\Gamma$ ) and electron bands ( $M$ ) are identical  
 $\Rightarrow$  perfect nesting at  $\mathbf{q} = \mathbf{M} = (\pi, \pi) \Rightarrow$   
 strongly enhanced electron-hole excitations

$$\chi'_0(\mathbf{q}, \omega = 0) = 2 \frac{m_e}{2\pi} \log \frac{\Lambda}{|\mathbf{q} - \mathbf{M}|},$$

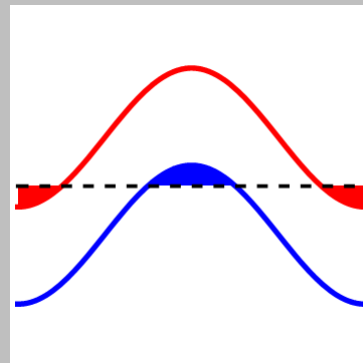
Turning on moderate interactions  $\rightarrow$   
**VDW** = itinerant multiband CDW (structural),  
 SDW (AF) and orbital orders at  $\mathbf{q} = \mathbf{M} = (\pi, \pi)$



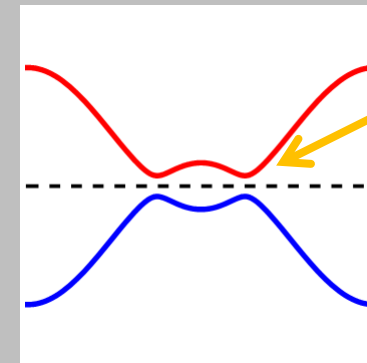
Semiconductor

$\epsilon_d \downarrow$

$\epsilon_c \uparrow$



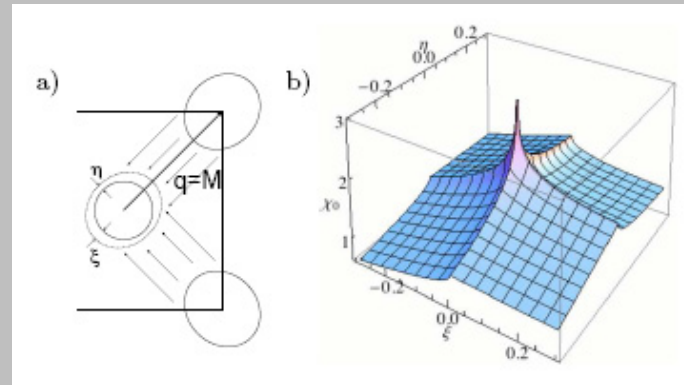
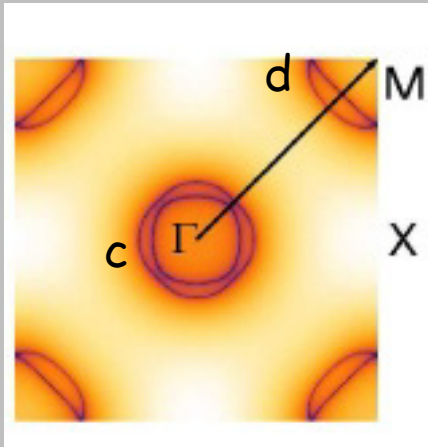
Semimetal



$\Delta_{\text{SDW}}$

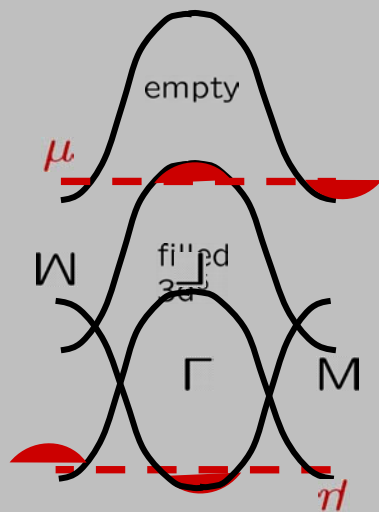
SDW, CDW, ODW or  
 combinations thereof  $\rightarrow$  VDW

# Nesting and Valley Density-Wave (VDW) in Fe-pnictides II



$$\chi'_0(\mathbf{q}, \omega = 0) = 2 \frac{m_e}{2\pi} \log \frac{\Lambda}{|\mathbf{q} - \mathbf{M}|},$$

Consider the bare-bones model for interactions:



$$H = -\varepsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \varepsilon_{\mathbf{k}} d_{\mathbf{k}}^{\dagger} d_{\mathbf{k}} + U n_c^2 + U n_d^2 + W n_c n_d + (\dots)$$

↑ ↑ ↑ ↑ ↑  
spinless fermions Intraband Interband

Particle-hole transformation

$$c \rightarrow h^{\dagger}, d \rightarrow e, n_c \rightarrow 1 - n_h, n_d \rightarrow n_e$$

$$H \rightarrow \varepsilon_{\mathbf{k}} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} + \varepsilon_{\mathbf{k}} e_{\mathbf{k}}^{\dagger} e_{\mathbf{k}} - W n_h n_e + (\dots)$$

Negative  $U$  Hubbard model:  $(h, e) \leftrightarrow (f_{\uparrow}, f_{\downarrow})$



# Many-Particle Problem in Quantum Matter

## i) BCS state & Cooper instability

Consider:  $\hat{H} = \sum_{\vec{k}\sigma} \xi_{\vec{k}} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} \xrightarrow{\text{non-interacting part}} +$

$$+ \frac{1}{2} \sum_{\vec{k}\vec{k}'\vec{q}\sigma\sigma'} V(\vec{k}, \vec{k}'; \vec{q}) c_{\vec{k}+\vec{q}\sigma}^\dagger c_{\vec{k}'-\vec{q}\sigma'}^\dagger c_{\vec{k}'\sigma'} c_{\vec{k}\sigma}$$

$\hookrightarrow$  interaction

The quartic form of interaction makes it difficult to solve  $\hat{H}$ .

Once quartic interaction term is present, we don't know how to solve  $H$ , except in **special cases**

Numerous approximate methods have been developed over decades

Hartree-Fock mean-field theory is **not trivial**

\* Hartree-Fock approximation:

$$c^\dagger c^\dagger c c \rightarrow \underbrace{c^\dagger c^\dagger c c}_{\langle c^\dagger c \rangle} + \underbrace{c^\dagger c^\dagger c c}_{\langle c^\dagger c \rangle} \quad \text{Hartree or direct} + \dots$$

\*  $\langle c_{\vec{k}\sigma}^\dagger c_{\vec{k}'\sigma'} \rangle$  can break a symmetry of  $\hat{H}$ !

# Quantum Phase Transitions

For example:

$$\langle c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}+\vec{Q}\sigma} \rangle \neq 0 \Rightarrow \text{CDW}$$

$$\langle c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}+\vec{Q}-\sigma} \rangle \neq 0 \Rightarrow \text{SDW}$$

... many other symmetry breaking states.

We can assume any form of the ground state, irrespective of whether it obeys the symmetries of  $H$ . As long as such Hartree-Fock state is determined **self-consistently to have the lowest energy**  $\rightarrow$

**We have the true ground state**

CDW breaks translational (lattice) invariance; SDW does the same plus, spin  $SU(2)$  symmetry. All such states are allowed and can be the ground state with a suitable interaction.

~~\*~~ How about  $\langle c^{\dagger} c^{\dagger} \rangle \neq 0$ ?

This breaks particle number conservation,  $[\hat{H}, \hat{N}] = 0$ !

Assume:

$$c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}'\sigma'}^{\dagger} \rightarrow \langle c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}'\sigma'}^{\dagger} \rangle + \underbrace{c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}'\sigma'}^{\dagger} - \langle c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}'\sigma'}^{\dagger} \rangle}_{\delta}$$

$$\downarrow$$

$$\langle c_{\vec{u}\sigma}^{\dagger} c_{\vec{u}-\sigma}^{\dagger} \rangle \delta_{\vec{u}, \vec{u}'} \delta_{\sigma, -\sigma'}$$

and keep only terms  $< O(\delta^2)$ . Also, assume  $V(\vec{u}, \vec{u}'; \vec{Q}) \rightarrow -U$  attractive, short-range interaction

Tinkham, Intro to SC

$$\langle c_{\vec{u}\sigma}^{\dagger} c_{-\vec{u}-\sigma}^{\dagger} \rangle \neq 0$$

[preserves spin  $SU(2)$  and translational invariance. It breaks  $U(1)$  particle number conservation]

# Hartree-Fock-BCS State

$$\hat{H} \rightarrow \hat{H}_{BCS} = \sum_{\vec{k}} \xi_{\vec{k}} c_{\vec{k}\uparrow}^{\dagger} c_{\vec{k}\uparrow} + \Delta \sum_{\vec{k}} c_{\vec{k}\uparrow}^{\dagger} c_{-\vec{k}\downarrow}^{\dagger} + \text{h.c.} + \left(\frac{1}{U}\right) \Delta^{\dagger} \Delta$$

Ground state of  $H_{BCS}$  is the famous BCS wavefunction

→

where

$$\Delta = U \sum_{\vec{k}} \langle c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow} \rangle \quad \text{self-consistency condition}$$

$$|\phi_0\rangle = \prod_k (u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger}) |0\rangle$$

Gap equation:  $\Delta_k = \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{\sqrt{\xi_{k'}^2 + |\Delta_{k'}|^2}}$

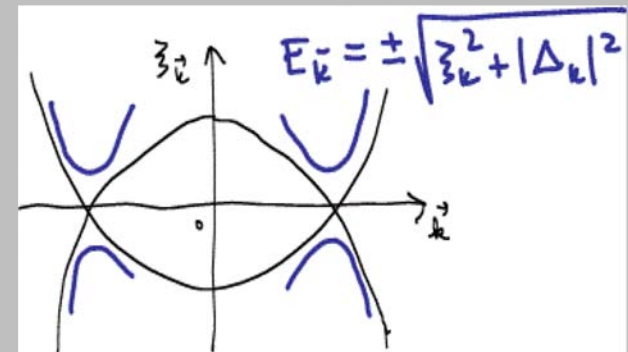
Ground state of  $H_{BCS}$ ,  $|\phi_0\rangle$  is a superconductor !!

Hamiltonian for new Bogoliubov-deGennes (BdG) spinors:  $[c_{\uparrow}^{\dagger}(\mathbf{r}); c_{\downarrow}(\mathbf{r})]$

$$\mathcal{H}_{BdG} = \begin{pmatrix} \frac{(\mathbf{p} - (e/c)\mathbf{A})^2}{2m} - \epsilon_F & \Delta \\ \Delta^* & \epsilon_F - \frac{(\mathbf{p} + (e/c)\mathbf{A})^2}{2m} \end{pmatrix}$$

BdG quasiparticle spectrum:

$$E_k = \pm \sqrt{\xi_k^2 + |\Delta_k|^2}$$

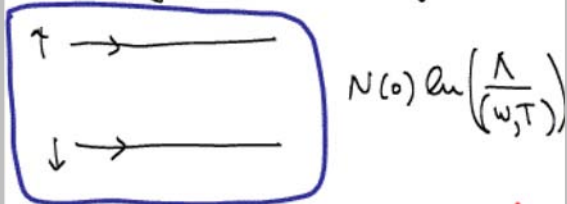




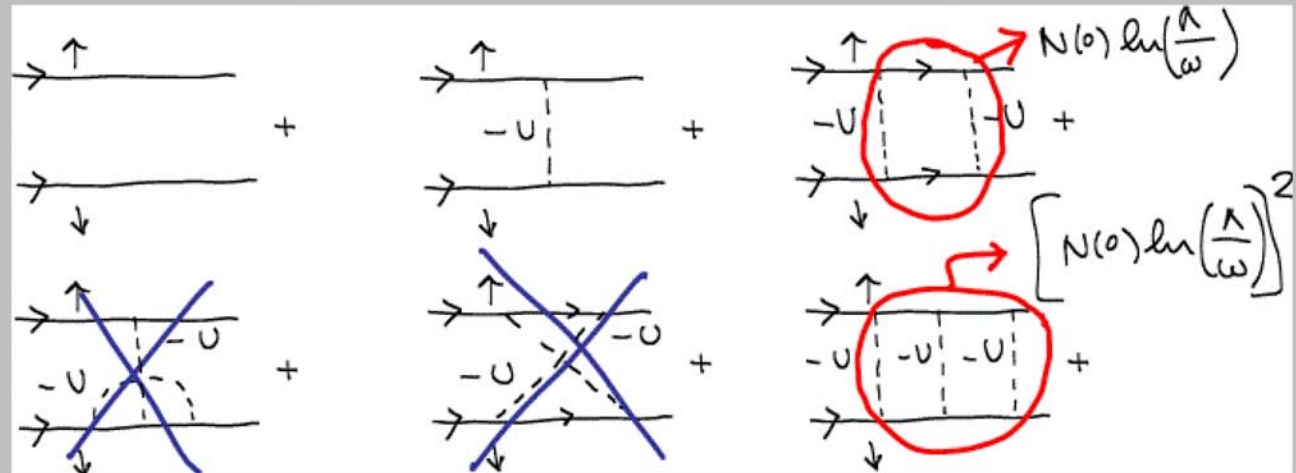
# Cooper Instability

Origin of Cooper instability and BCS ground state is repeated scattering of two electrons (p-p channel) →

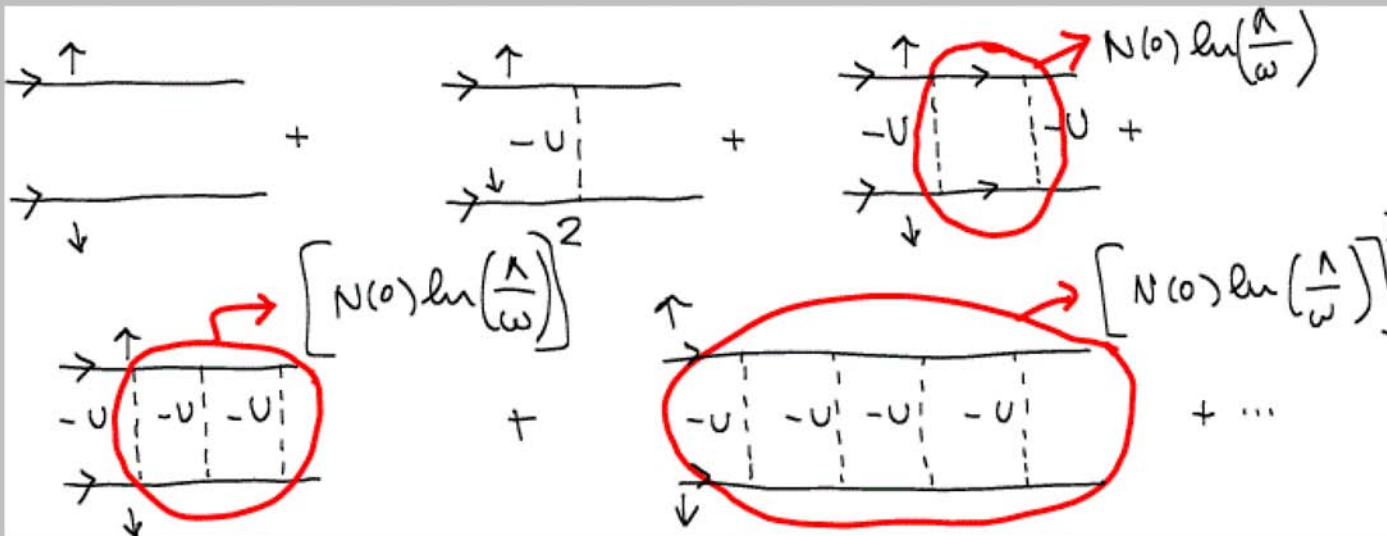
Pairing susceptibility



diverges as  $\omega, T \rightarrow 0$  !



Keep only **the most divergent diagrams** at any given order in perturbation theory:



This is like having a new dimensionless interaction  $g(\omega)$

$$g(\omega) = N(0)U \ln\left(\frac{\Lambda}{\omega}\right)$$

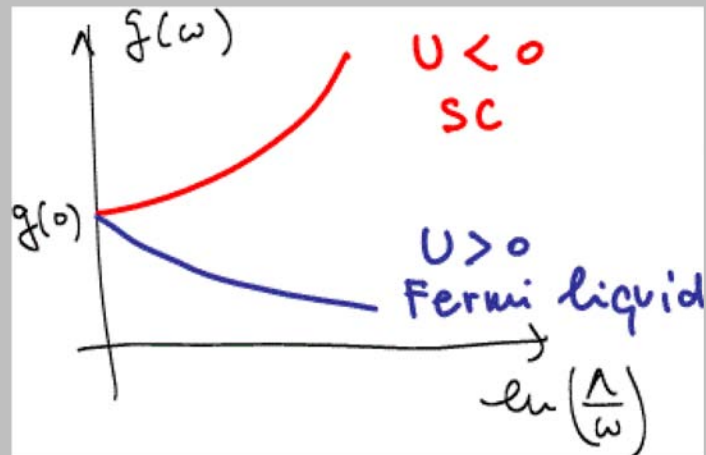
# Poor-woman Renormalization Group (RG)

This is like having  
a new dimensionless  
interaction  $g(\omega)$

$$g(\omega) = N(0)U \ln\left(\frac{\Lambda}{\omega}\right)$$

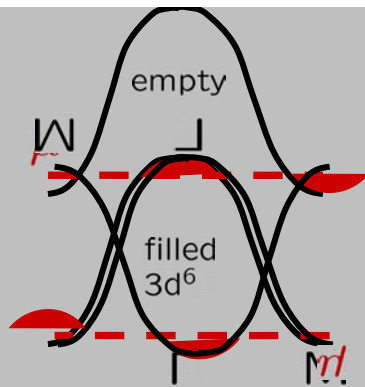
To the leading order:

$$\frac{dg(\omega)}{d \ln\left(\frac{\Lambda}{\omega}\right)} \equiv \dot{g}(\omega) = \begin{cases} +g^2(\omega) & \text{for } U < 0 \\ - & \text{for } U > 0 \end{cases}$$



So, **attractive** pairing interaction ( $U < 0$ )  
grows as  $\omega, T \rightarrow 0$  !!

For repulsive  $U > 0$ , the interaction  $\rightarrow 0$  and  
becomes irrelevant as  $\omega, T \rightarrow 0 \rightarrow$  Fermi liquid  
(normal) ground state



# Fictitious "Superconductor" → VDW in Fe-pnictides

$$H \rightarrow \varepsilon_k h_k^\dagger h_k + \varepsilon_k e_k^\dagger e_k - W n_h n_e + (\dots)$$

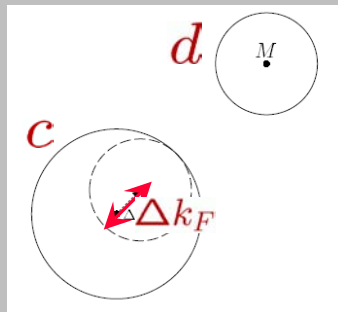
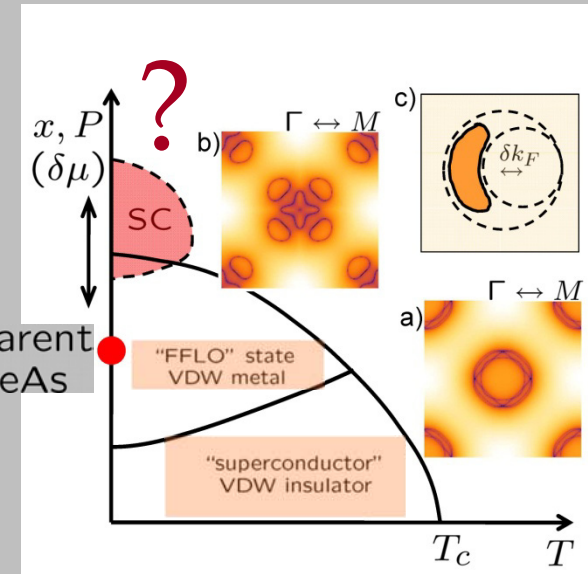
Negative  $U$  Hubbard model:  $(h, e) \leftrightarrow (f_\uparrow, f_\downarrow)$

$$\rightarrow H = \sum_\sigma \varepsilon_k f_{k,\sigma}^\dagger f_{k,\sigma} - W n_{f\uparrow} n_{f\downarrow} + (\dots)$$

Ground state is a "superconductor":

$$\langle f_\uparrow f_\downarrow \rangle \neq 0 \Rightarrow \langle c^\dagger d \rangle \neq 0 \Rightarrow$$

insulating VDW (SDW/ODW/CDW) at  $\mathbf{M} = (\pi, \pi)$



Generically, the  $c$  and  $d$  bands are different:

$$k_{Fc} - k_{Fd} = \Delta k_F \neq 0 \Rightarrow \text{fictitious Zeeman magnetic field}$$

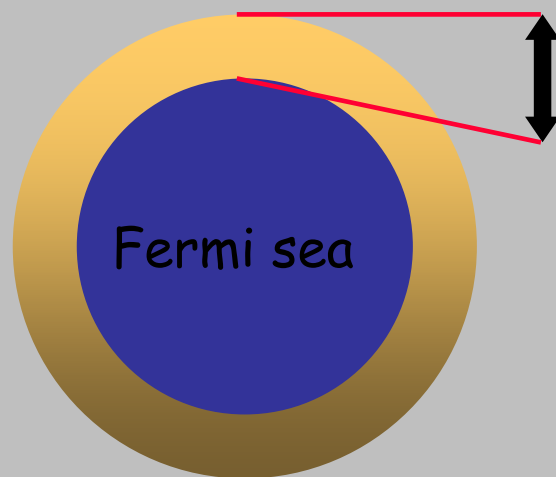
$\Rightarrow$  "Fulde-Ferrell-Larkin-Ovchinnikov", "breached pairing" states

$\Rightarrow$  metallic (incommensurate?) VDW (SDW/ODW/CDW)

What about real superconductivity?  $(\dots)$



# Pairing Gap $\Delta$ - Coastline of the Fermi Sea



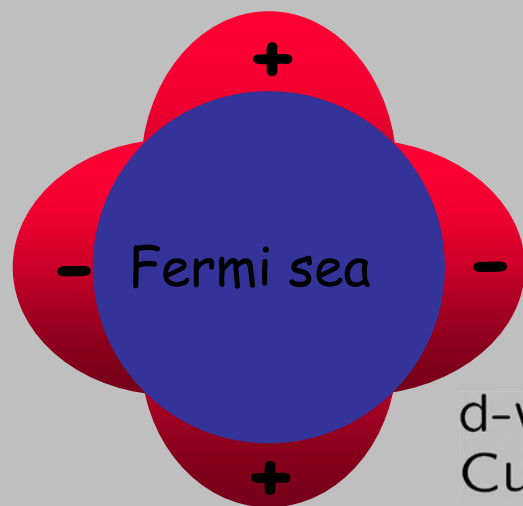
Superconducting gap  $\Delta$

Parting the waves  
of the Fermi Sea



s-wave (isotropic)  
Conventional SC (Nb, Pb, Al, ...)  
 $T_c < 25\text{K}$

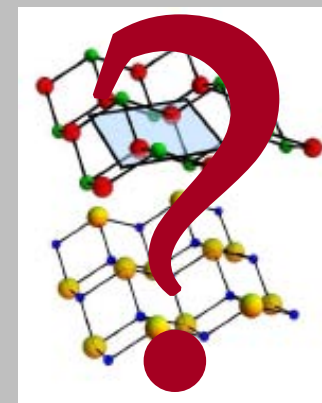
p-wave  
Superfluid  $^3\text{He}$ , SrRu (?)  
 $T_c \sim 1\text{mK} - 1\text{K}$



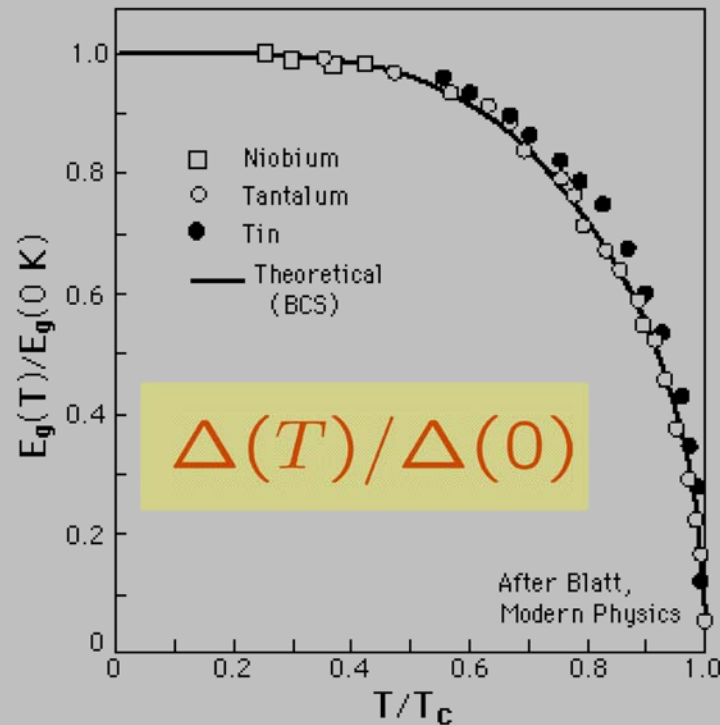
New REOFeAs SC  
 $T_c \sim 57\text{K}$

$\Delta = ?$

d-wave  
Cuprate SC  $T_c \sim 160\text{K}$



# What can $\Delta$ tell us about superconducting state ?



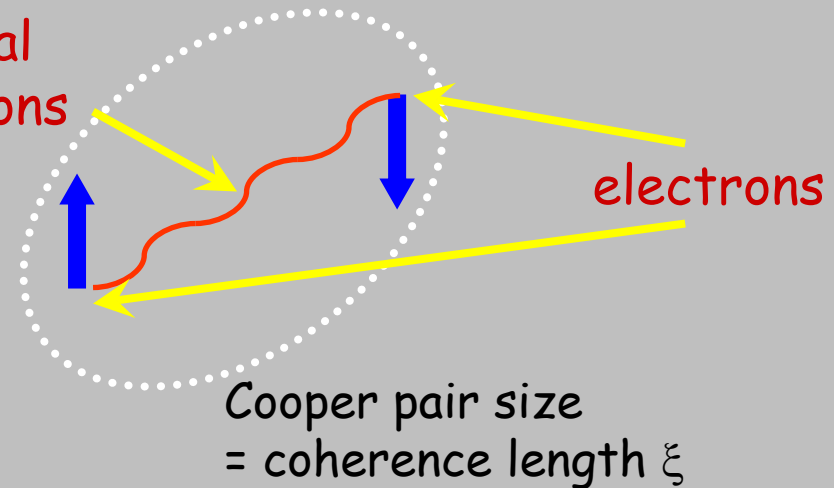
$$2\Delta(0)/k_B T_c = 3.53$$

Universal BCS behavior

In conventional superconductors  $\Delta$  is uniform along Fermi sea (s-wave)  
This reflects the pairing interaction being **attractive !!**  
Its origin is electron-electron interaction mediated by **phonons !!**

virtual  
phonons

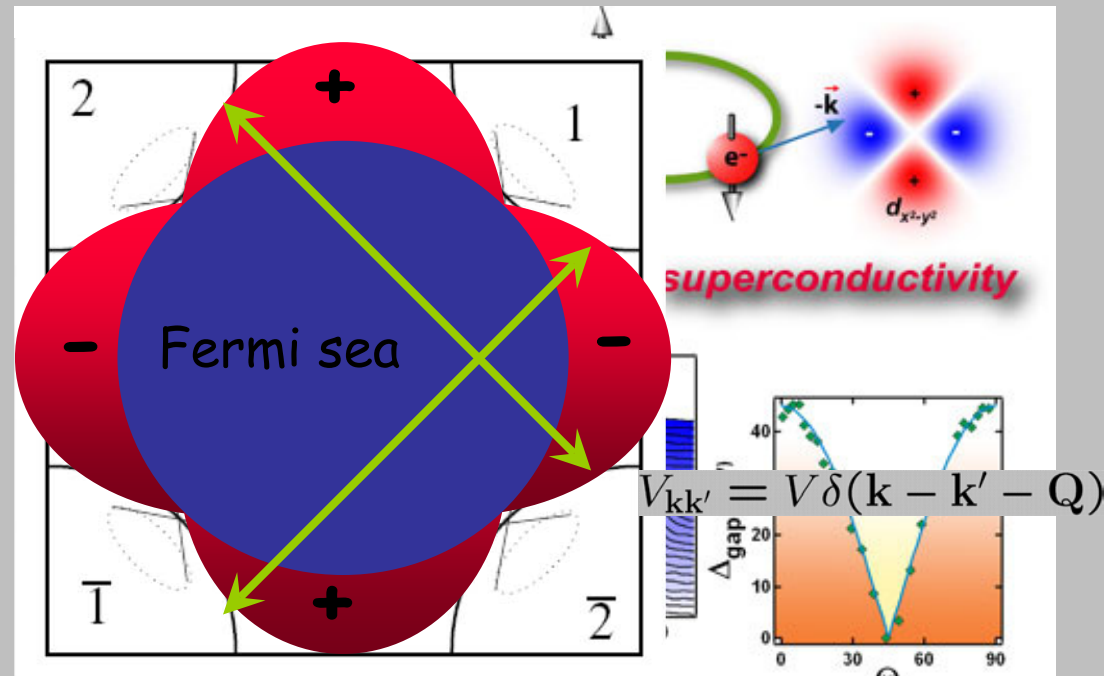
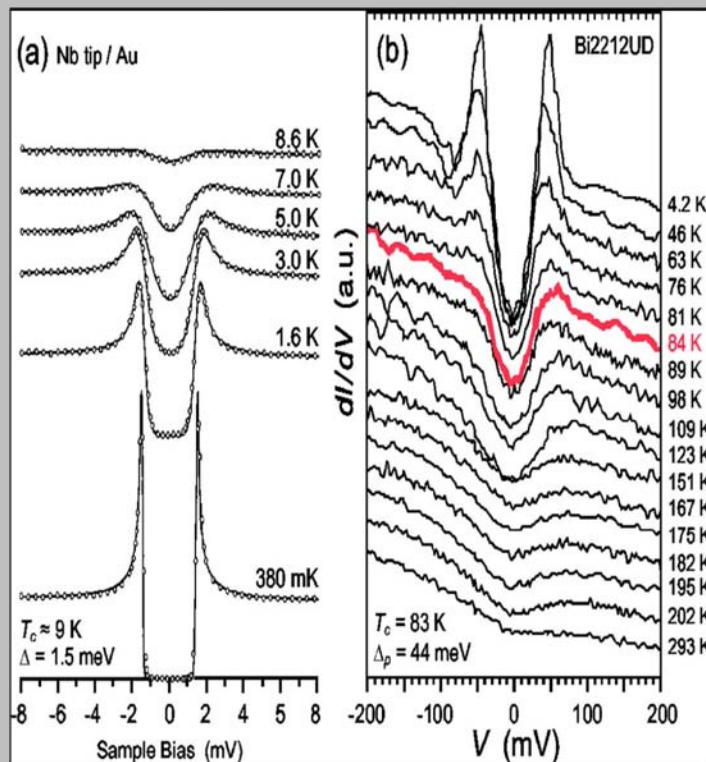
electrons



Standard BCS theory works well in materials like Nb, Sn or Hg. In Pb and more complex systems ( $\text{Nb}_3\text{Ge}$ ) one needs "strong coupling" theory ( $2\Delta/T_c \sim 4-6$ )

# What can $\Delta$ tell us about superconducting state ?

In cuprate superconductors  $\Delta$  has **nodes** and  $d_{x^2-y^2}$  symmetry. This suggests the basic interaction is **repulsive !!** The same is true of other nodal SCs



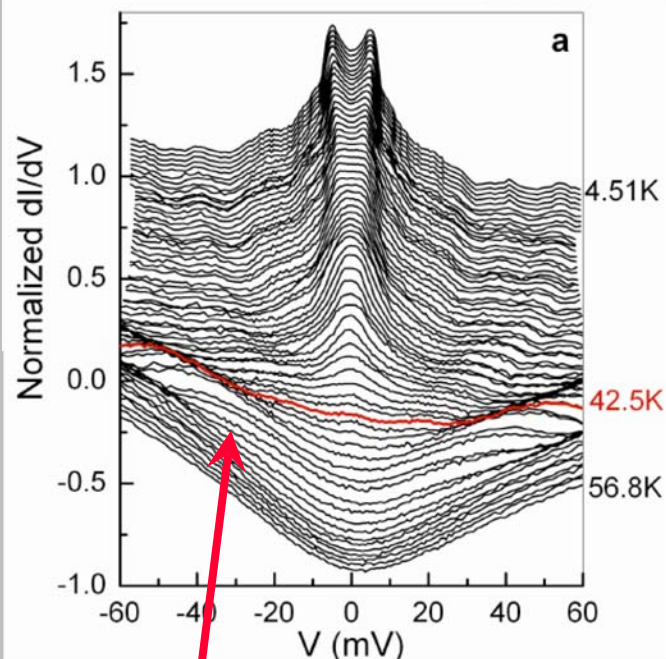
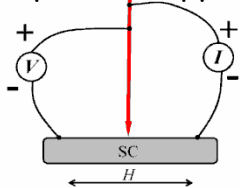
$$\text{Gap equation } \Delta_{\mathbf{k}} = - \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}}$$

Furthermore, high temperature cuprate SCs exhibit a **pseudogap** behavior:  
 $\Delta$  remains finite even for  $T > T_c$  !  
 This reflects strong **fluctuations** !

**Results for FeAs mostly appear inconsistent with these features**



Andreev spectroscopy



No pseudogap !

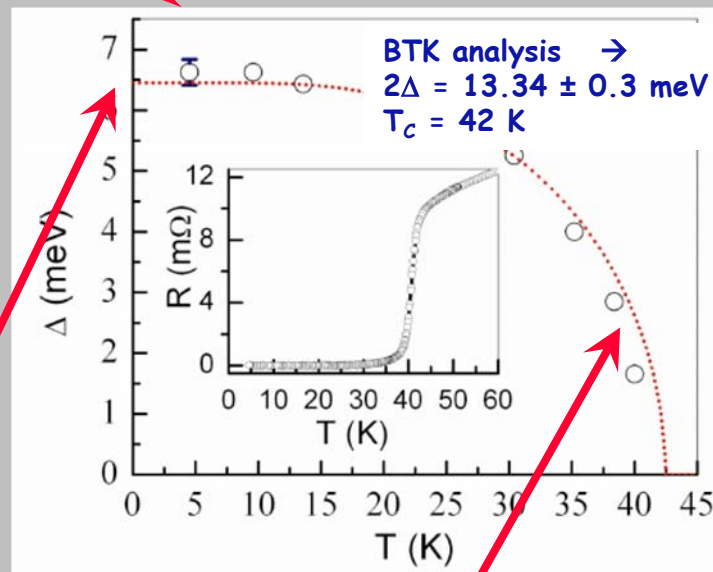
$\Delta$  disappears at  $T = T_c$

$$2\Delta(0)/k_B T_c = 3.68 \sim 3.53$$

# $\Delta$ in FeAs superconductors I

T. Y. Chen *et al.*, Nature **453**, 1224 (2008)

$$\frac{2\Delta_{d(p)}(0)}{k_B T_c} = 4.28$$



Nodeless isotropic gap !

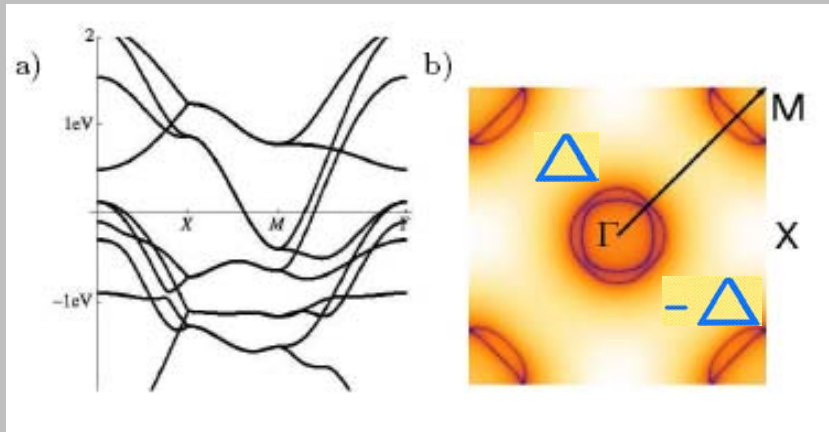
$\Delta(T)$  consistent with BCS theory  
 (sign/phase variation still possible!)

**Conclusions: Nodeless superconducting gap and no pseudogap behavior. Very different from high  $T_c$  cuprate superconductors !!**



# $\Delta$ in FeAs superconductors II

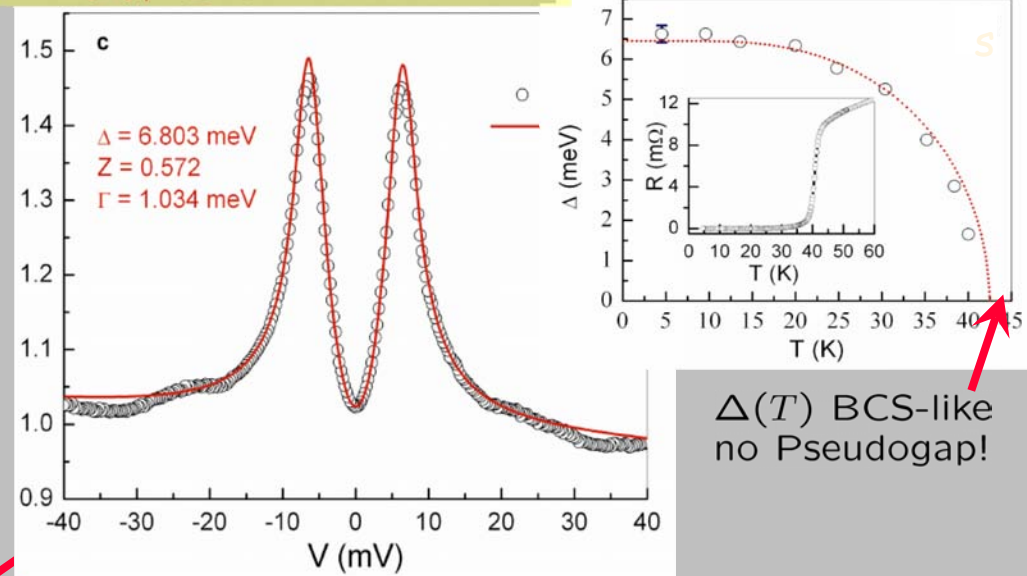
T. Y. Chen *et al.*, Nature **453**, 1224 (2008)



Fermi sea in FeAs materials is more like **Land o'Lakes**. It is multiply-connected.

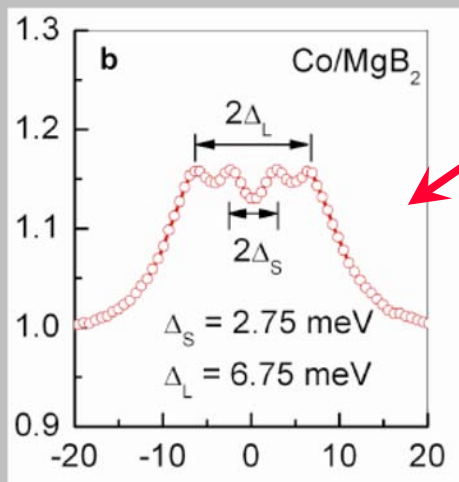
$$2\Delta(0)/k_B T_c = 3.68 \sim 3.53$$

$$\frac{2\Delta_{d(p)}(0)}{k_B T_c} = 4.28$$



$\Delta(T)$  BCS-like  
no Pseudogap!

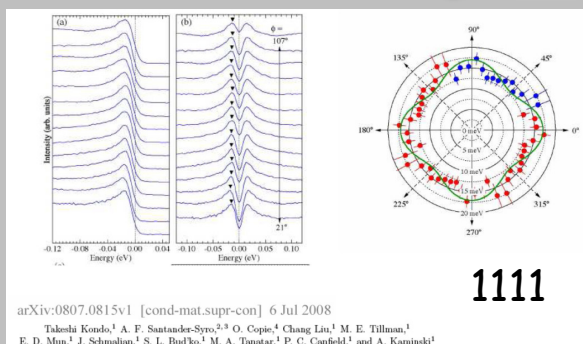
We see only a **single isotropic BCS-like gap**  
If different Fermi "lakes" had significantly different  $\Delta$  we would see them



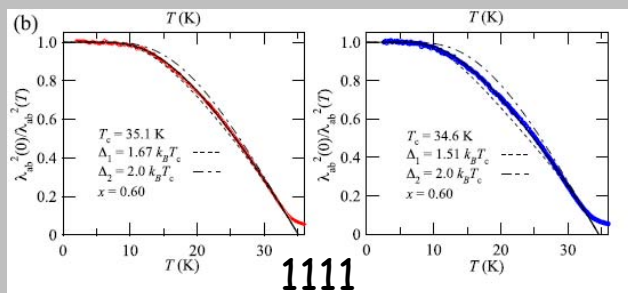
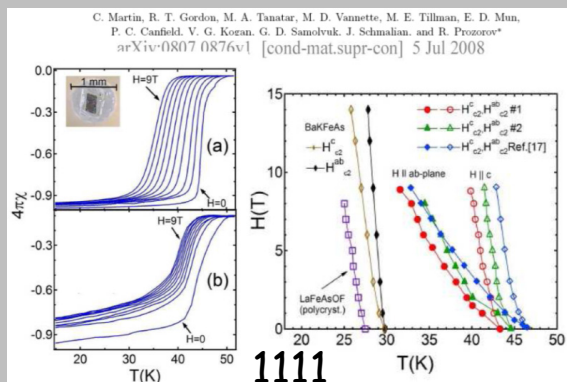
Only a "single" superconducting gap - but sign/phase could be different for holes and electrons. No pseudogap!

**Conclusions: Conventional phonon-mechanism is unlikely but so is Mott limit-induced repulsion of the cuprate d-wave kind. We have something new !!**

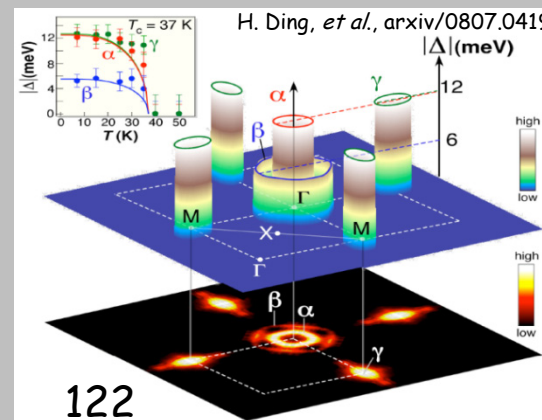
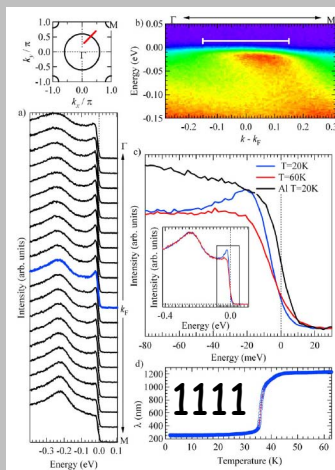
# Emerging consensus (PCAR, ARPES, STM, $\mu$ w, SQUID, ...): nodeless "single" $\Delta$ in 1111, "two" $\Delta$ 's in 122, nodes in lower $T_c$ SC ??



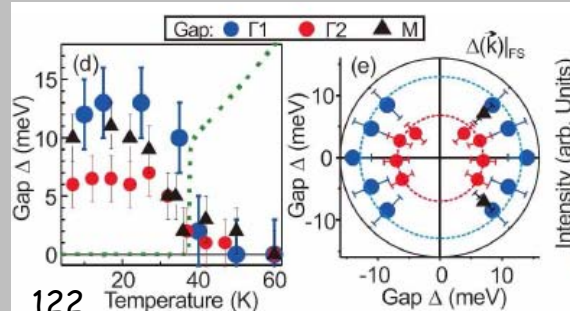
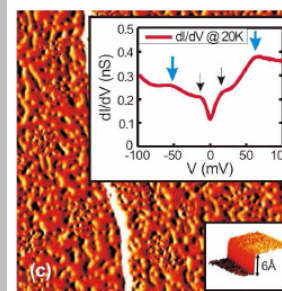
C. Liu, et al. arxiv/0806.2147



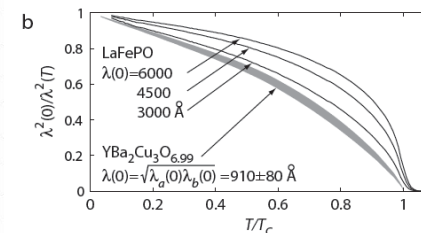
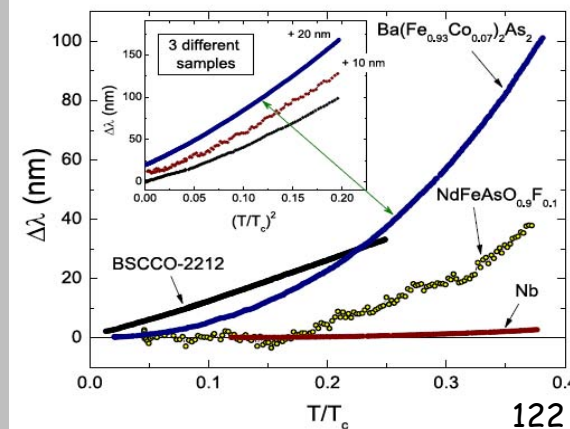
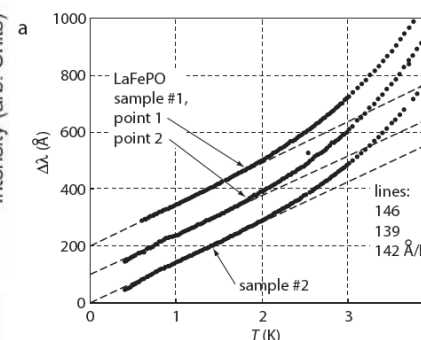
Multiband superconductivity  
in Fe-pnictides !?



NMR sees  
nodal behavior  
( $\sim T^2$ ) in 1111



L. Wray, et al., PRB 78 184508 (2008),

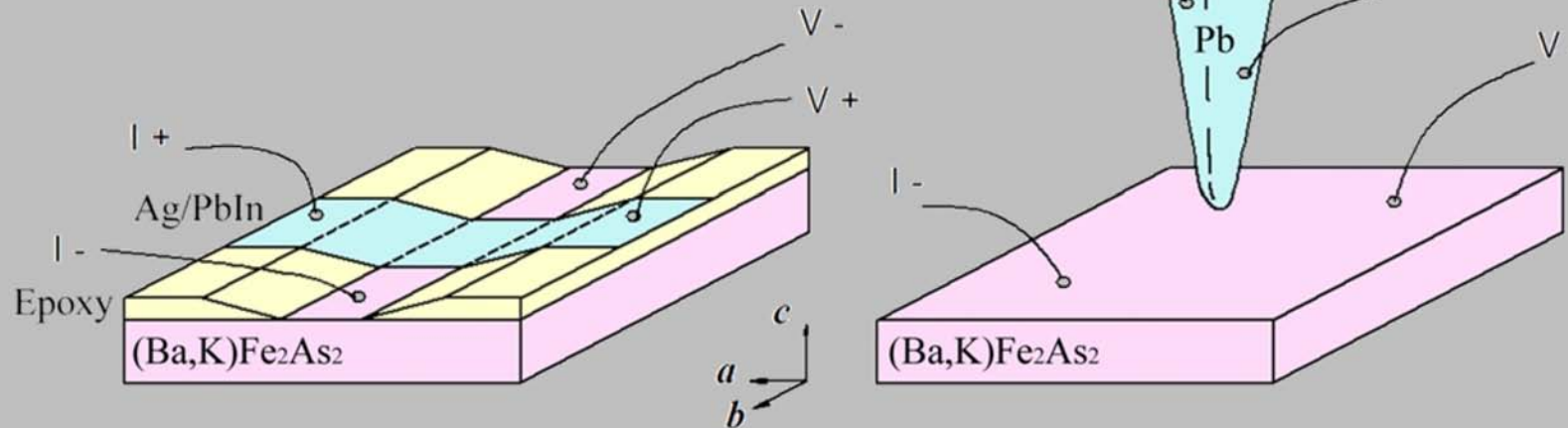


C. Hicks, et al., arxiv/0903.5260

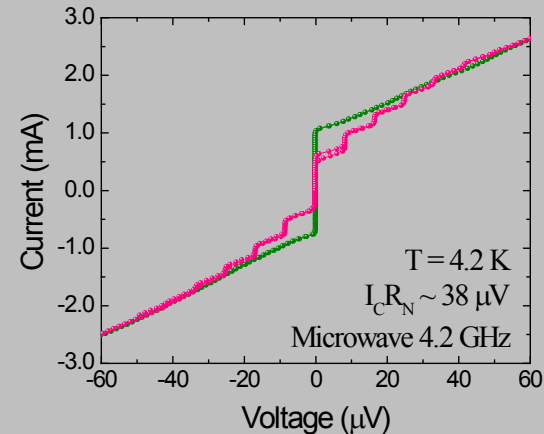
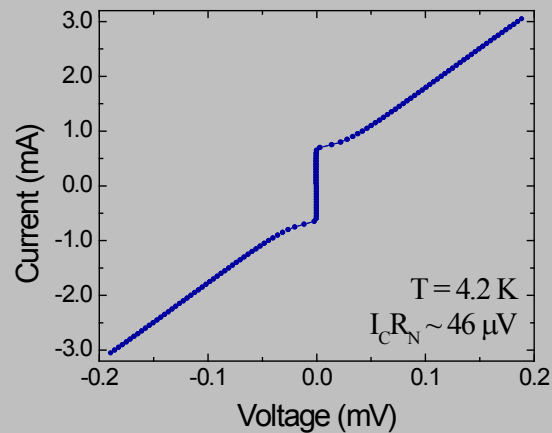
R. T. Gordon et al., arxiv/0810.2295

# Josephson Effect Between FeAs and Pb

X. Zhang *et al.*, PRL **102**, 147002 (2009)



## Point Contact Junction

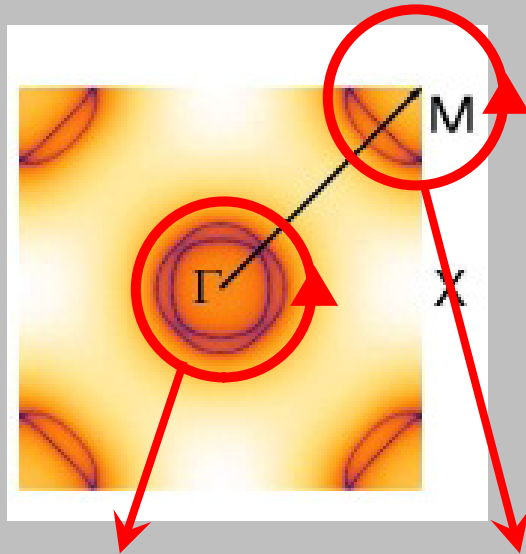


- I-V characteristics are resistively like a shunted junction
- Shapiro steps were observed under microwave irradiation

**Strong indication of s-wave like SC state**



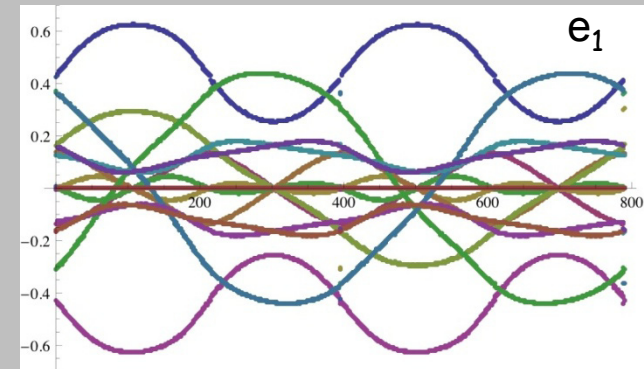
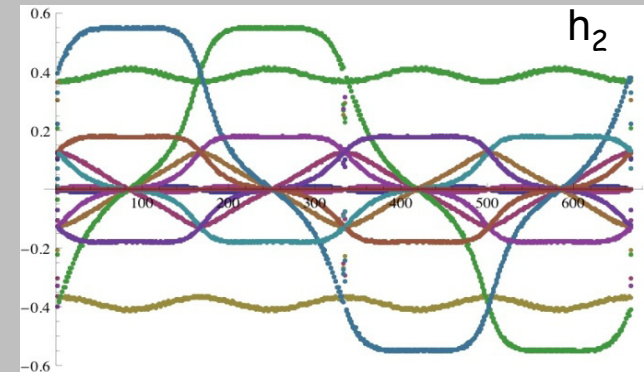
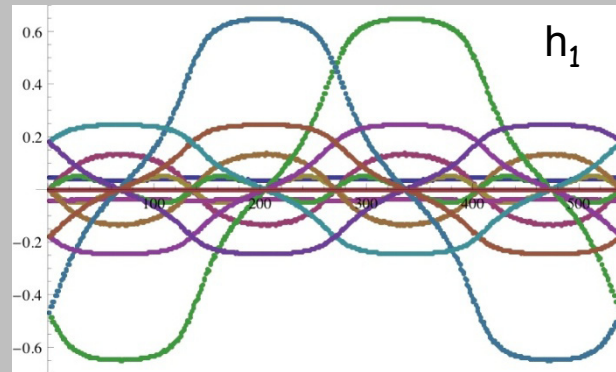
# Minimal Model of FeAs Layers IV



hole FS  
2 pockets (valleys)

electron FS  
2 pockets (valleys)

As one goes around the FS there is  
**strong** mixing of odd and even d-orbitals  
 $\Rightarrow$  **rich orbital content**



**FeAs are different from  $\text{CuO}_2$**

Charge carriers are more itinerant and less localized on atomic sites. Multiband description is necessary, unlike an effective single band model of cuprates



# Interactions in FeAs I

High multiband itinerancy implies significant metallic screening

Yang *et al*, PRB **80**, 014508 (2009):

$U_d$  not larger than  $\sim 2$  eV,  $J_{\text{Hund}} \sim 0.8$  eV

from X-ray absorption

$\Rightarrow$  moderate correlations  $U_d \sim t$ ,  $J_{\text{Hund}} < U_d$

Consider  $\frac{1}{2} \int d^2r d^2r' V(\mathbf{r}, \mathbf{r}') n(\mathbf{r}) n(\mathbf{r}')$ , where  $V(\mathbf{r}, \mathbf{r}')$  is the screened Coulomb repulsion  $\Leftrightarrow$  Hubbard-like Hamiltonian with  $U_d$  and  $J_{\text{Hund}}$  reflecting atomic limit Coulomb correlations

$$H_{\text{FeAs}} = - \sum_{ij, \alpha\beta} t_{ij}^{\alpha\beta} c_{i\alpha}^\dagger c_{j\beta} + \sum_{i, \alpha} \epsilon_i^\alpha c_{i\alpha}^\dagger c_{i\alpha} + \frac{1}{2} U_d \sum_i n_{di}^2 - J_{\text{Hund}} \sum_i \mathbf{S}_{di}^2 + (\dots)$$

Sawatzky *et al* discuss various interorbital interactions ( $\dots$ )

Effective interaction at the Fermi surface:

$$\sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \Gamma_{\alpha, \beta, \gamma, \delta}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) f_{\mathbf{k}+\mathbf{q}, \alpha}^\dagger f_{\mathbf{k}'-\mathbf{q}, \beta}^\dagger f_{\mathbf{k}', \delta} f_{\mathbf{k}, \gamma}$$

$$\Gamma_{\alpha, \beta, \gamma, \delta}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) \rightarrow U, W, G_1, G_2$$

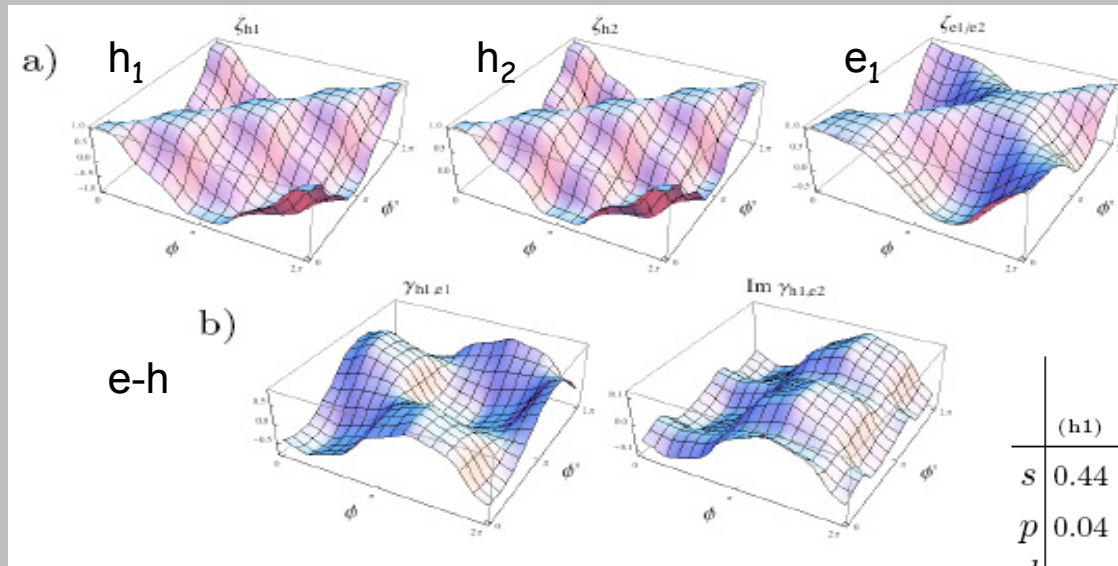
All flavor conserving ( $U, W$ ) and flavor mixing ( $G_1, G_2$ ) vertices

# Interactions in FeAs II

V. Cvetkovic and ZT, PRB **80**, 024512 (2009);  
arXiv:0808.3742

A. V. Chubukov *et al*, PRB **78**, 134512 (2008)

vertices change significantly around Fermi surface:



**k-space "Josephson" terms:**

$$\rightarrow G_2 c^\dagger c^\dagger dd + h.c.$$

All interaction vertices @ FS:  
interband, intraband, mixed  
(typical sizes  $U, W \gg G_1, G_2$ )

	$U$				$W$				$G_1$	$G_2$
	(h1)	(h2)	(e1)	(e2)	(h1,e1)	(h1,e2)	(h2,e1)	(h2,e2)	(h1,e1)	(h1,e1)
$s$	0.44	0.31	0.35	0.35	0.21	0.25	0.27	0.29	0.14	0.14
$p$	0.04	0.21	0.17	0.20	0.22	0.21	0.22	0.22	0.01	0.01
$d$	0.22	0.12	0.09	0.10	0.11	0.13	0.09	0.11	0.03	0.02

Effective vertex at the FS:

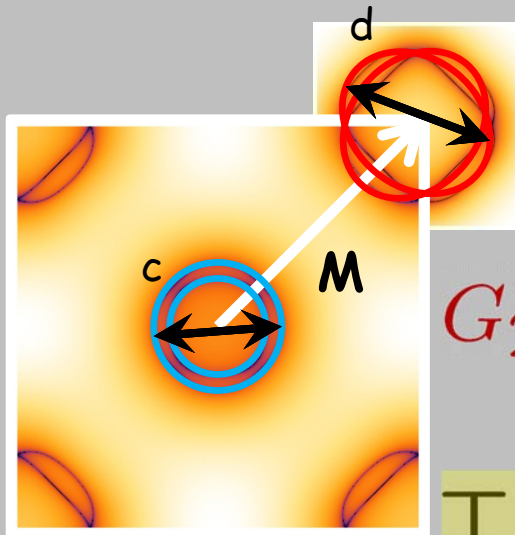
$$\Gamma_{\alpha,\beta,\gamma,\delta}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) \quad (\mathbf{k}, \mathbf{k}' \in \text{FS}) \rightarrow V_s + V_p p_4(\varphi) p_4(\varphi') + V_d d_4(\varphi) d_4(\varphi')$$

$V_s$ ,  $V_p$ , and  $V_d$  are  $C_4$  version of  $s$ -,  $p$ - and  $d$ -wave coupling constants

Typically, we find  $W_s$  is dominant  $\rightarrow$  Valley density-wave(s) (VDW) in FeAs  $G_2 c^\dagger c^\dagger dd$  These "Josephson" terms are not crucial for SDW  $\rightarrow$  Could they be the cause of SC?

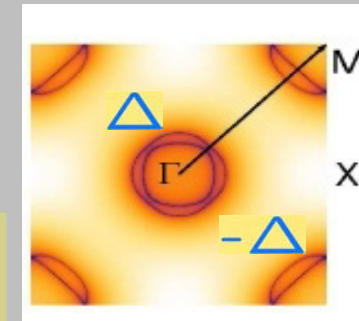
# Two Kinds of Interband Superconductivity

ZT, Physics 2, 60 (2009)



Interband pairing acts like Josephson coupling in k-space.  
If  $G_2$  is repulsive  $\rightarrow$  antibound Cooper pairs ( $s'$ SC)

$$G_2 c^\dagger c^\dagger d d$$



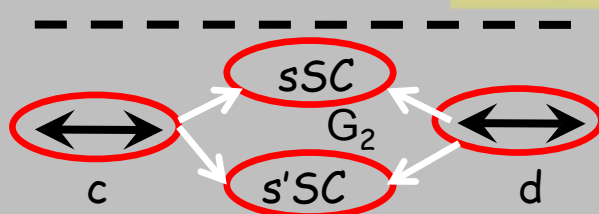
The Problem: Need

$$G_2 > \sqrt{U_c U_d}$$

Most unlikely!

Type-A interband SC

intrinsic) interband SC:



$$(c_\uparrow c_\downarrow, d_\uparrow d_\downarrow) \rightarrow \psi_c, \psi_d$$

intraband Cooper pairing  
further enhanced by  $G_2$

$$(c_\uparrow c_\downarrow - d_\uparrow d_\downarrow)$$

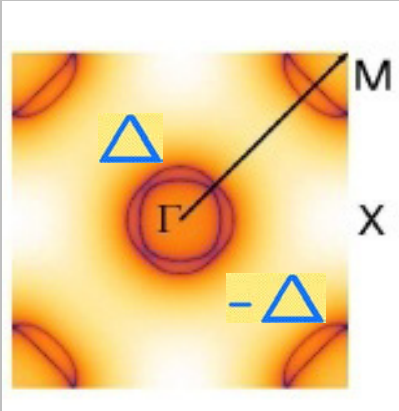
intrinsic interband Cooper pairs !

Single SC order parameter  $\psi_{s'}$  !!



# Interplay of Valley Density-Wave (VDW) and SC in FeAs I

V. Stanev, J. Kang, ZT, PRB **78**, 184509 (2008)



The condition for interband SC is actually milder:

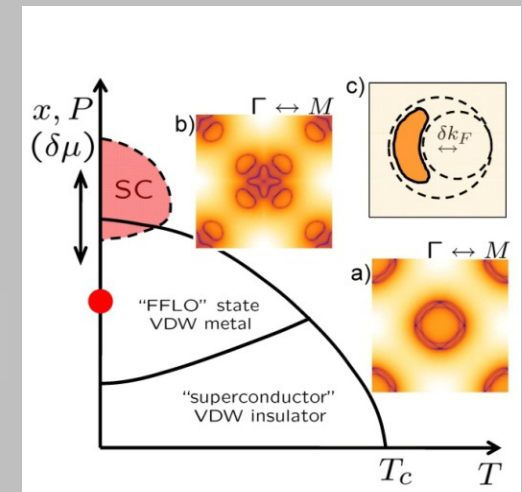
$$G_2 < \sqrt{U_c U_d} \quad \text{but} \quad G_2^* > \sqrt{U_c^* U_d^*}$$

$$U_{c,d}^* = \frac{U_{c,d}}{1 + U_{c,d} \log(\omega_{C2}/\omega_{C1})}$$

$\omega_{C1}$  ( $\omega_{C2}$ ) - Inter (intra) band energy scales

RG calculations indicate, near a VDW state:

$G_2$  grows, while  $U_{c(d)}$  is suppressed



A. V. Chubukov *et al*, PRB **78**, 134512 (2008)

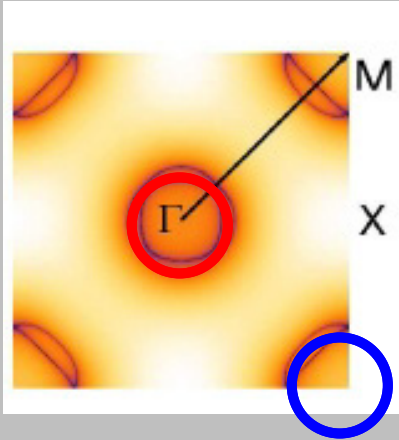
In Fe-pnictides interband superconductivity (s' or s+- state) is a strong possibility (perhaps with little help from phonons)

I. I. Mazin *et al*, PRL **101**, 057003 (2008); M. Parish, J. Hu, and B. A. Bernevig, PRB **78**, 144514 (2008)



# Hierarchy of Energy Scales $U, W \gg G_1, G_2$ → Unified Model of Valley Density-Wave (VDW)

Unified model  $N = 4 \rightarrow SU(4) \times SU(4)$   
 V. Cvetkovic and ZT, PRB **80**, 024512 (2009)



$$H_0 = \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}}^{(0)} [\sum_{\alpha} h_{\mathbf{k}}^{(\alpha)\dagger} h_{\mathbf{k}}^{(\alpha)} + \sum_{\beta} e_{\mathbf{k}}^{(\beta)\dagger} e_{\mathbf{k}}^{(\beta)}]$$

All  $e$  and  $h$  bands are identical  $\Rightarrow$   
 $H_0$  has  $SU(8)$  internal symmetry

Orbital flavor-conserving vertices ( $U, W$ )  
 reduce this to  $U(4) \times U(4)$ :

	$T_s$ (K)	$T_N$ (K)	$m_{\text{ord}} (\mu_B)$
LaFeAsO	155	137	0.36
CeFeAsO	155	140	0.83
PrFeAsO	153	127	0.48
NdFeAsO	150	141	0.9
CaFeAsF	134	114	0.49
SrFeAsF	175	120	
CaFe <sub>2</sub> As <sub>2</sub>	173	173	0.8
SrFe <sub>2</sub> As <sub>2</sub>	220	220	0.94-1.0
BaFe <sub>2</sub> As <sub>2</sub>	140	140	0.9

$$H_{\text{int}} \rightarrow U^{(h)} \sum_{\alpha\alpha'\sigma\sigma'} h_{\sigma}^{(\alpha)\dagger} h_{\sigma'}^{(\alpha')\dagger} h_{\sigma'}^{(\alpha')} h_{\sigma}^{(\alpha)} + U^{(e)} \sum_{\beta\beta'\sigma\sigma'} e_{\sigma}^{(\beta)\dagger} e_{\sigma'}^{(\beta')\dagger} e_{\sigma'}^{(\beta')} e_{\sigma}^{(\beta)} + 2W \sum_{\alpha\beta\sigma\sigma'} e_{\sigma'}^{(\beta)\dagger} h_{\sigma}^{(\alpha)\dagger} h_{\sigma}^{(\alpha)} e_{\sigma'}^{(\beta)} + (\dots)$$

$U(4) \times U(4)$  symmetry is reasonable since  $U$  and  $W$  do not vary much in different ( $e, h$ ) channels

	(h1)	(h2)	(e1)	(e2)	(h1,e1)	(h1,e2)	(h2,e1)	(h2,e2)	(h1,e1)	(h1,e1)
s	0.44	0.31	0.35	0.35	0.21	0.25	0.27	0.29	0.14	0.14
p	0.04	0.21	0.17	0.20	0.22	0.21	0.22	0.22	0.01	0.01
d	0.22	0.12	0.09	0.10	0.11	0.13	0.09	0.11	0.03	0.02

Finally, flavor-mixing vertices  $G_{1,2} (\ll U, W)$  have the highest symmetry that physics will allow:

$$(\dots) \rightarrow 2G_1 \sum_{\alpha\beta\sigma\sigma'} (\sigma\sigma') e_{\sigma}^{(\beta)\dagger} h_{-\sigma}^{(\alpha)\dagger} h_{-\sigma'}^{(\alpha)} e_{\sigma'}^{(\beta)} + G_2 \sum_{\alpha\alpha'\beta\beta'\sigma\sigma'} (\sigma\sigma') h_{-\sigma}^{(\alpha)} e_{\sigma}^{(\beta)} h_{-\sigma'}^{(\alpha')} e_{\sigma'}^{(\beta')} \epsilon^{\alpha\alpha'} \epsilon^{\beta\beta'} + h.c.$$

VDW in Fe-pnictides  
 is a (nearly) highly  
 symmetric combination:  
 SDW/CDW/ODW

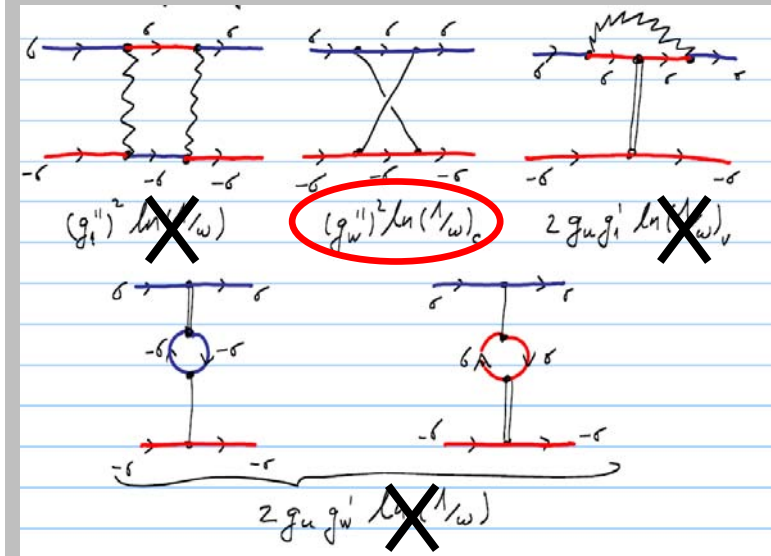
# Interactions in FeAs III

V. Cvetkovic & ZT (RG) ; A. V. Chubukov, I. Eremin *et al* (parquet);  
F. Wang, H. Zhai, Y. Ran, A. Vishwanath & DH Lee (fRG)  
R. Thomale, C. Platt, J. Hu, C. Honerkamp & A. Bernevig (fRG)

Effective interaction at the Fermi surface:

$$\sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \Gamma_{\alpha, \beta, \gamma, \delta}(\mathbf{k}, \mathbf{k}'; \mathbf{q}) f_{\mathbf{k}+\mathbf{q}, \alpha}^{\dagger} f_{\mathbf{k}'-\mathbf{q}, \beta}^{\dagger} f_{\mathbf{k}', \delta} f_{\mathbf{k}, \gamma} \rightarrow \boxed{U, W, G_1, G_2}$$

$$\begin{aligned} g_U(\omega) &= g_U - g_U^2 \ln\left(\frac{\Lambda}{\omega}\right)_{pp} - g_2^2 \ln\left(\frac{\Lambda}{\omega}\right)_{pp}, \\ g_2(\omega) &= g_2 - 2g_2 g_U \ln\left(\frac{\Lambda}{\omega}\right)_{pp} + 2g_2 g'_W \ln\left(\frac{\Lambda}{\omega}\right)_{ph}^c + \\ &\quad 2g_2 g''_W \ln\left(\frac{\Lambda}{\omega}\right)_{ph}^v - 2g_2 g_1'' \ln\left(\frac{\Lambda}{\omega}\right)_{ph}, \\ g'_W(\omega) &= g'_W + (g'_W)^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph} + g_2^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph}, \\ g''_W(\omega) &= g''_W + (g''_W)^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph}, \\ g'_1(\omega) &= g'_1 - 2g'_1 g_1'' \ln\left(\frac{\Lambda}{\omega}\right)_{ph} + 2g'_1 g''_W \ln\left(\frac{\Lambda}{\omega}\right)_{ph}^v, \\ g''_1(\omega) &= g''_1 - (g'_1)^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph} - (g''_1)^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph} - \\ &\quad g_2^2 \ln\left(\frac{\Lambda}{\omega}\right)_{ph} + 2g_1'' g''_W \ln\left(\frac{\Lambda}{\omega}\right)_{ph}^v, \end{aligned} \quad (15)$$

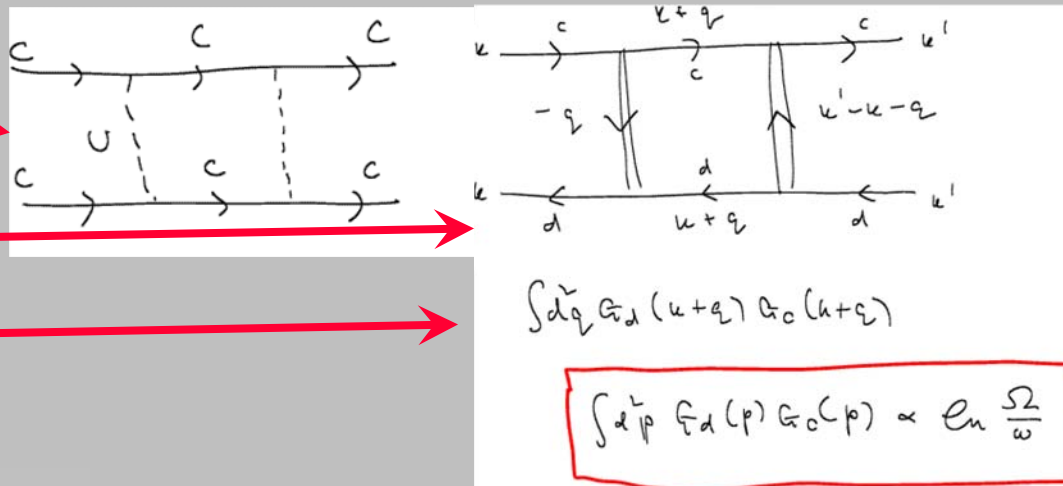


If  $G_1, G_2 \ll U, W$   
 $\rightarrow$   
 relevant vertices:  $U, W$ , &  $G_2$

# Interplay of VDW and SC in FeAs II

RG (near VDW):

$$\begin{aligned}\dot{U} &= -U^2 - G_2^2 \\ \dot{G}_2 &= -2G_2U + 4G_2W \\ \dot{W} &= +W^2 + G_2^2 \quad (G_1 \rightarrow 0)\end{aligned}$$



Proximity to VDW is crucial:

$$\begin{aligned}\dot{U} \pm \dot{G}_2 &= -(U \pm G_2)^2 \pm 4WG_2 \\ \dot{W} &= W^2 + G_2^2\end{aligned}$$

If  $W \rightarrow 0$  then  $\dot{U} \pm \dot{G}_2 = -(U \pm G_2)^2$

SC only for  $G_2 > U$  !!

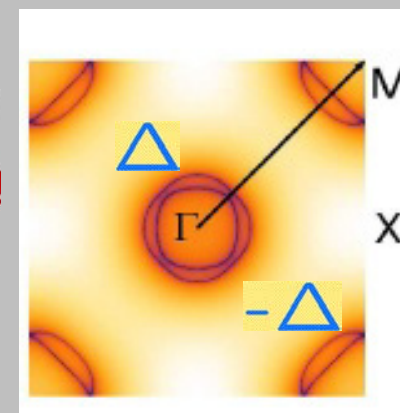
**This is true interband SC since  $U > 0$  - different from  $U < 0$ :**

$$(c_{\uparrow}c_{\downarrow} - d_{\uparrow}d_{\downarrow})$$

interband Cooper pairs !

$$(c_{\uparrow}c_{\downarrow}, d_{\uparrow}d_{\downarrow})$$

intraband Cooper pairs



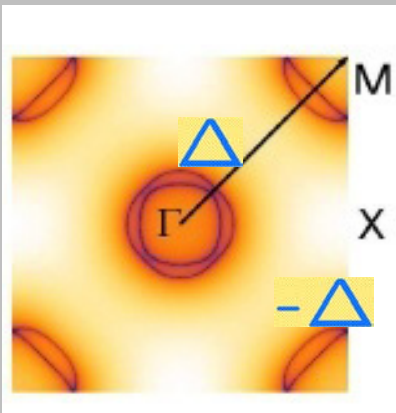
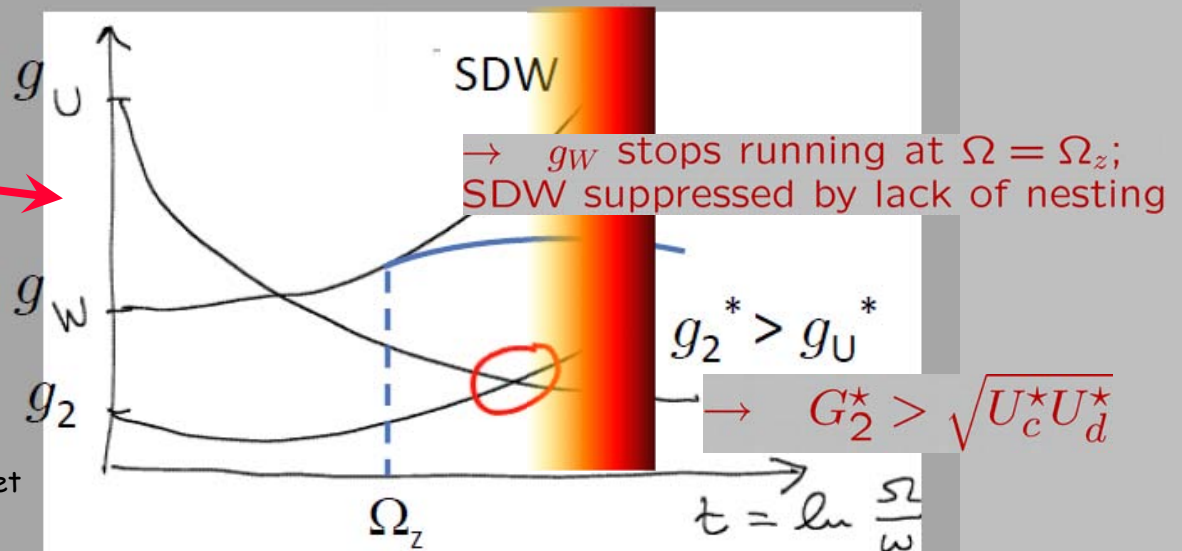
# RG Theory of Interband Mechanism of SC in FeAs

V. Cvetkovic and ZT, PRB **80**, 024512 (2009)

RG flows (near SDW):

$$\begin{aligned}\dot{U} &= -U^2 - G_z^2 \\ \dot{G}_z &= -2G_z U + 4G_z W \\ \dot{W} &= +W^2 + G_z^2 \quad (G_1 \rightarrow 0)\end{aligned}$$

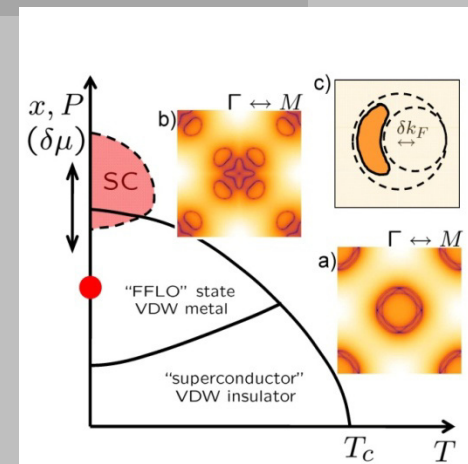
A. V. Chubukov *et al*, PRB **78**, 134512 (2008) parquet  
F. Wang *et al*, PRL **102**, 047005 (2009) fRG  
R. Thomale *et al*, PRB **80**, 180505(R) (2009) fRG



$$G_2 < \sqrt{U_c U_d}$$

$$G_2^* > \sqrt{U_c^* U_d^*}$$

$$\rightarrow T_c \sim \Omega_z \exp(-1/(g_2^* - \sqrt{\mu_c^* \mu_d^*}))$$



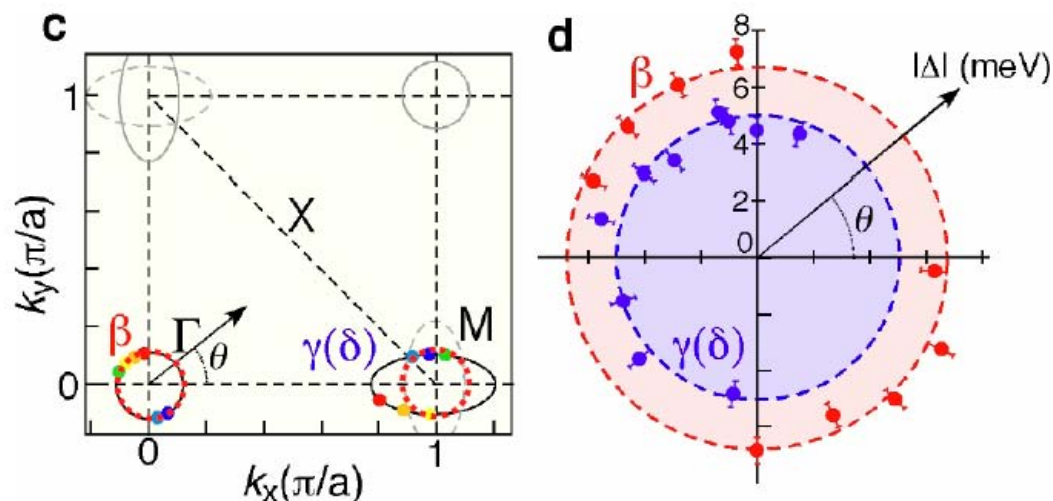
In Fe-pnictides interband superconductivity ( $s'$  or  $s_{+-}$  state) is a strong possibility but there is some fine tuning with SDW/CDW/ODW



# Correlation between SC and Nesting (ARPES)

H. Ding group (CAS Beijing & BC)

In optimally **electron** doped samples, quasi FS nesting between the **outer** ( $\beta$ ) hole pocket and the electron pockets



- Large SC gaps for **well-nested** hole and electron pockets
- SC **collapses** as one dopes **away** from near nesting
- Holds for **both** hole and electron doping

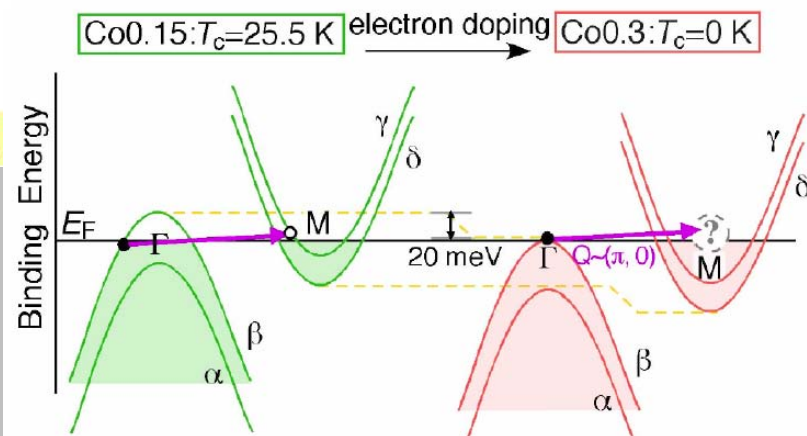
Strong pairing also happens to these FSs!

$$\frac{2\Delta}{k_B T_c} = 6, 4.5$$

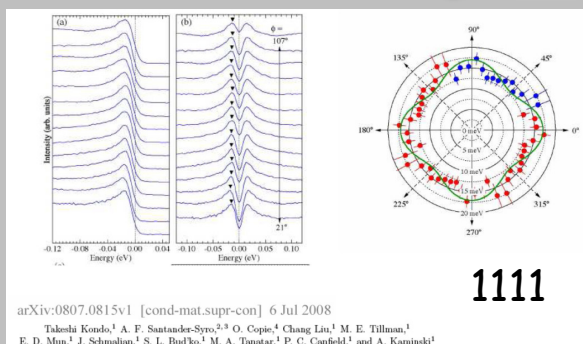
for  $\beta, \gamma(\delta)$

K. Terashima *et al.*, PNAS 106,

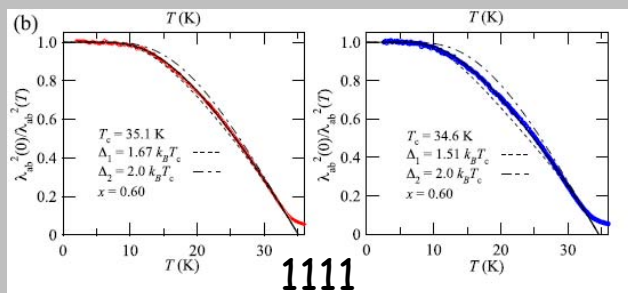
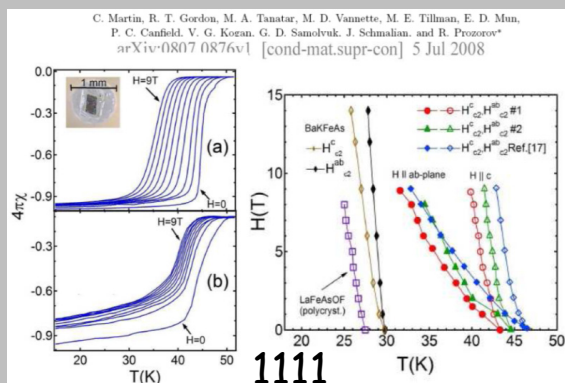
Nodeless SC correlates with degree of nesting and SC disappears as  $h(e)$  Fermi pockets are doped away.



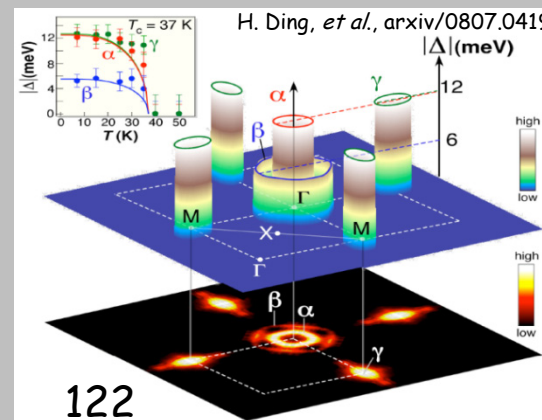
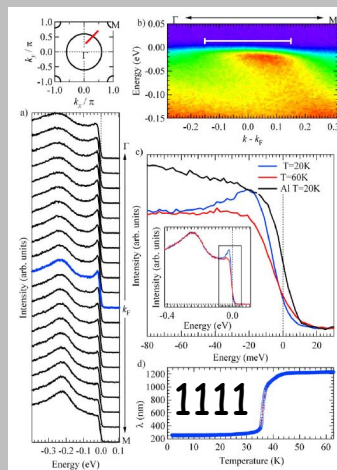
# Emerging consensus (PCAR, ARPES, STM, $\mu$ w, SQUID, ...): nodeless "single" $\Delta$ in 1111, "two" $\Delta$ 's in 122, nodes in lower $T_c$ SC ??



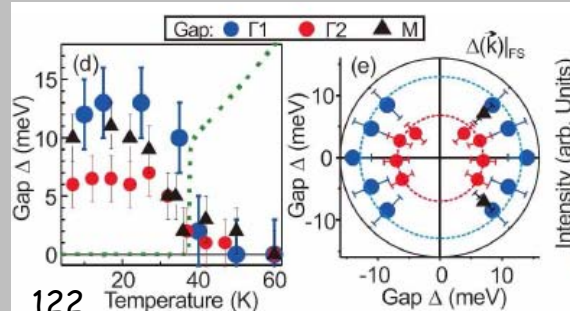
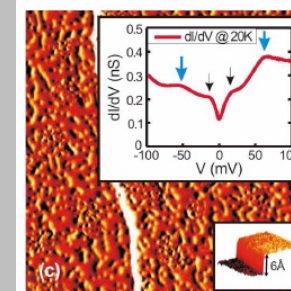
C. Liu, et al. arxiv/0806.2147



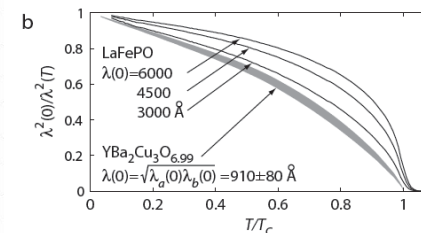
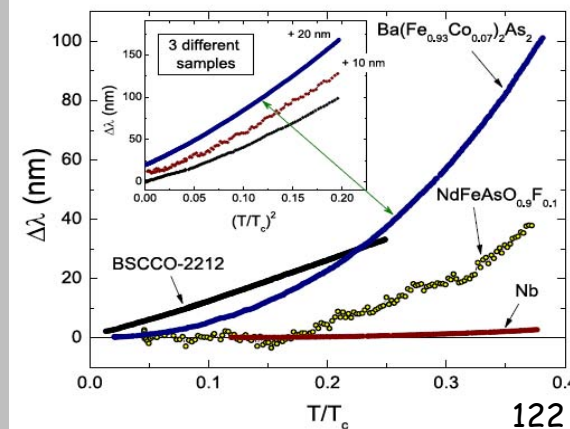
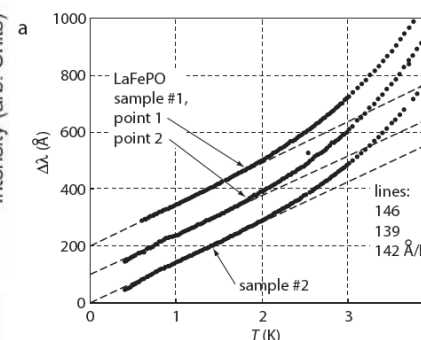
Multiband superconductivity  
in Fe-pnictides !?



NMR sees  
nodal behavior  
( $\sim T^2$ ) in 1111



L. Wray, et al., PRB 78 184508 (2008),

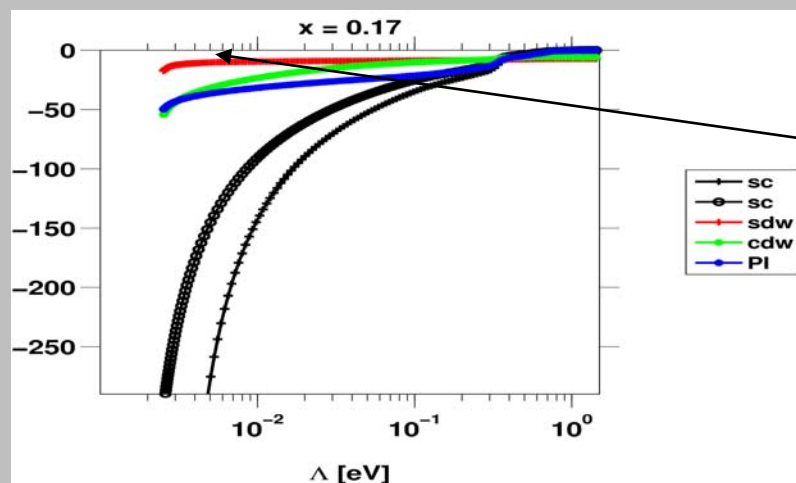
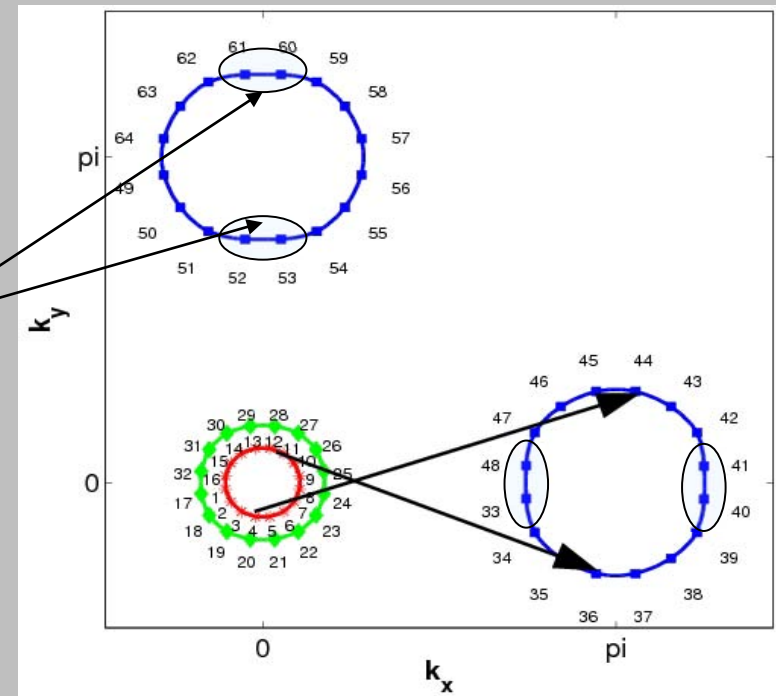
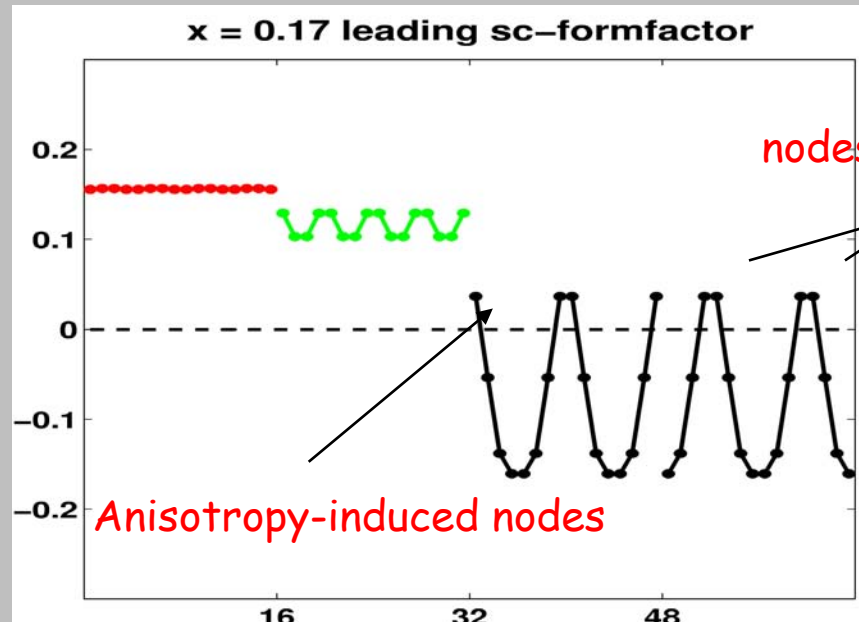


C. Hicks, et al., arxiv/0903.5260

R. T. Gordon et al., arxiv/0810.2295

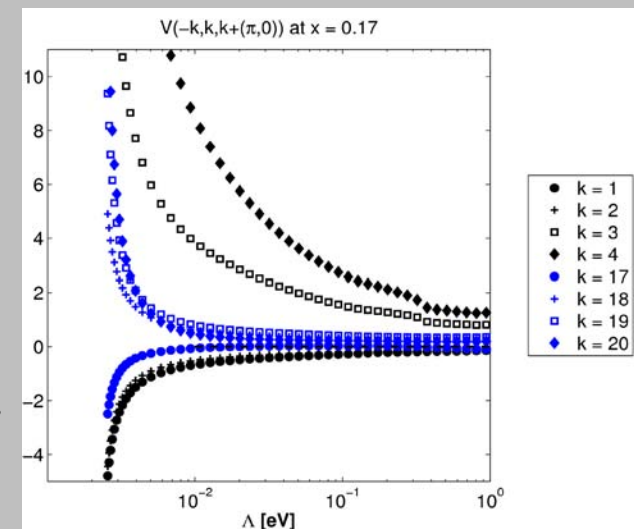
# Nodal SC from Orbital Structure (fRG)

R. Thomale, C. Platt, W. Hanke & A. Bernevig, arXiv:1002.3599;  
also, F. Wang, D. H. Lee *et al*; K. Kuroki *et al*



Change of orbital  
character in  
electron pockets  
**reduces** SDW  
fluctuations

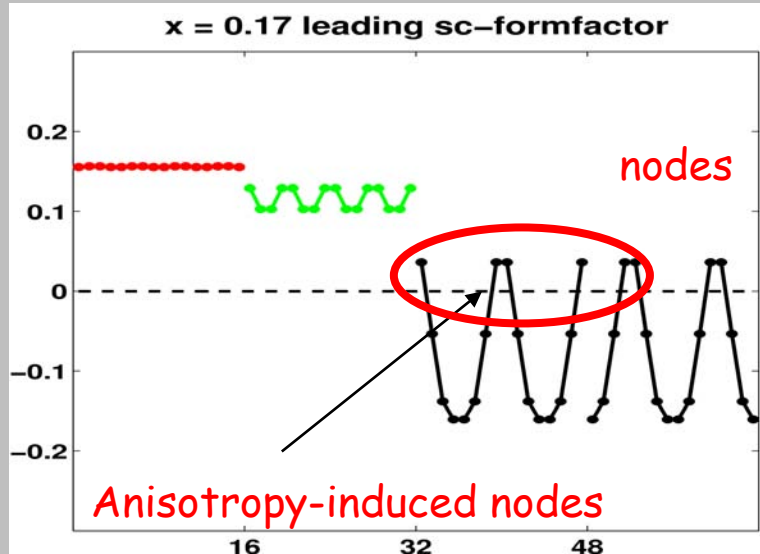
**Nodal s+-** is the  
dominant instability  
as  $T \rightarrow T_c$





# Accidental Gap Nodes and Zeroes in Iron-Pnictides I

V. Stanev *et al.*, arXiv:1006.0447



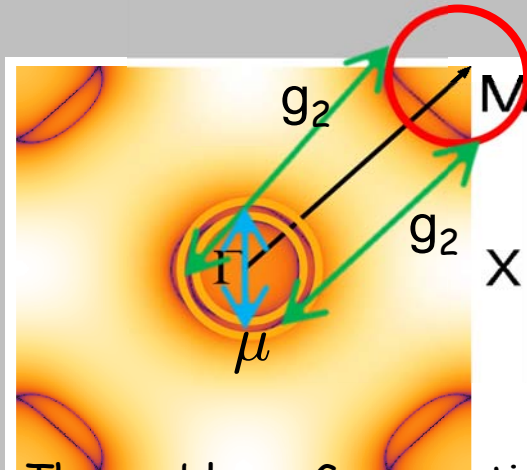
These nodes are **accidental** and NOT protected by any symmetry - SC state is still  $s_{\pm}$  or  $s'$ . They are caused by strong **orbital anisotropy** of pairing interaction.

Such nodal  $s'$  states arise in various calculations (fRG, RG, etc) near  $T = T_c$ . But do these **accidental nodes survive as  $T \rightarrow 0$  ?!**

Consider interband interaction:

$$g_2(k, k') = \lambda_0 + \lambda_n \cos 4\theta$$

$$\rightarrow \Delta_e(\theta) \equiv \Delta_0 + \Delta' \cos 4\theta$$



The problem: Gap equations can wipe out accidental nodes for  $T \ll T_c \rightarrow$

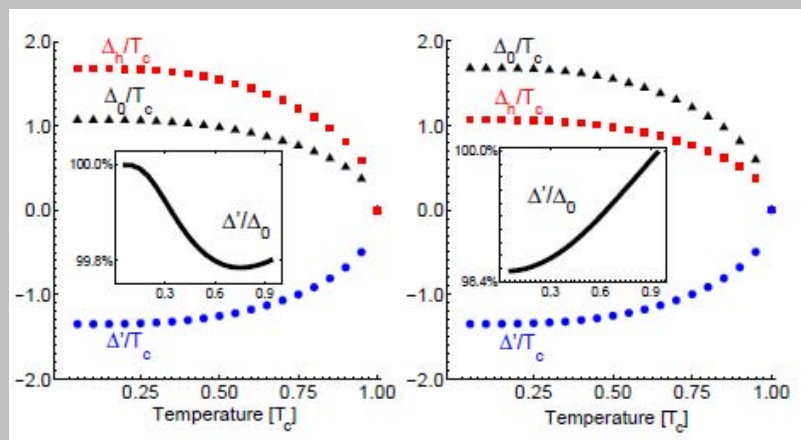
$$\begin{aligned} \Delta_h &= - \int_0^\Lambda d\xi \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{\tanh(E_e/2T)}{E_e} (\lambda_0 + \lambda_n \cos 4\theta) \Delta_e \\ &\quad - \mu \int_0^\Lambda d\xi \frac{\tanh(E_h/2T)}{E_h} \Delta_h \\ \Delta_0 &= - \mu \int_0^\Lambda d\xi \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{\tanh(E_e/2T)}{E_e} \Delta_e \\ &\quad - \lambda_0 \int_0^\Lambda d\xi \frac{\tanh(E_h/2T)}{E_h} \Delta_h \\ \Delta' &= - \lambda_n \int_0^\Lambda d\xi \frac{\tanh(E_h/2T)}{E_h} \Delta_h, \end{aligned} \quad (1)$$



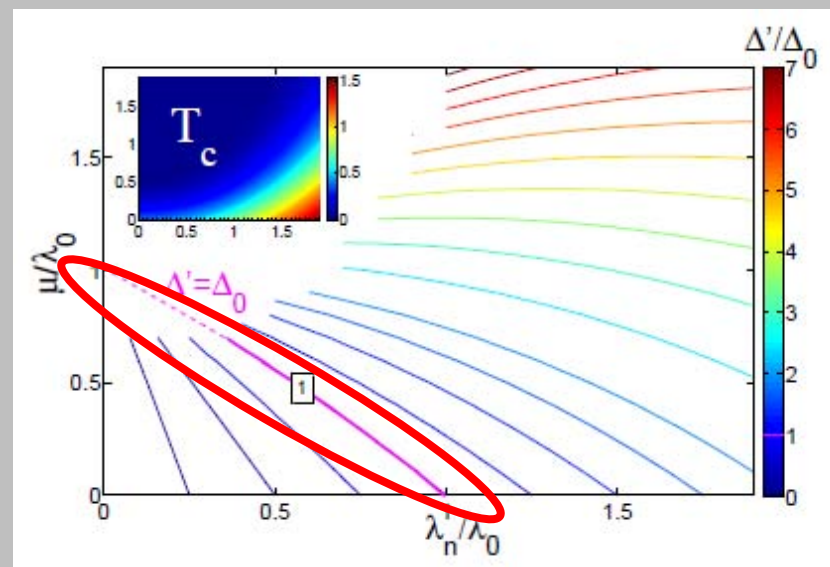
# Accidental Gap Nodes and Zeroes in Iron-Pnictides II

V. Stanev *et al.*, arXiv:1006.0447

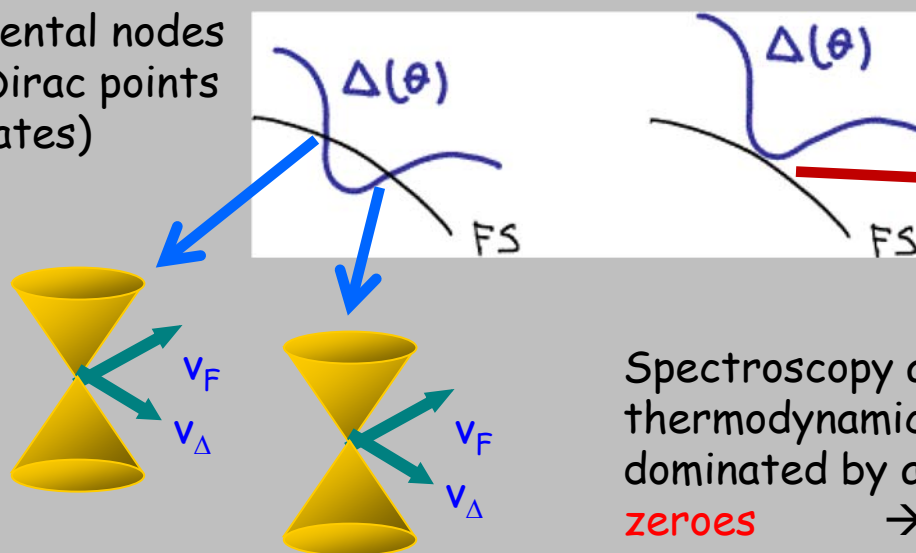
These accidental nodes appear "robust" as  $T \rightarrow 0$



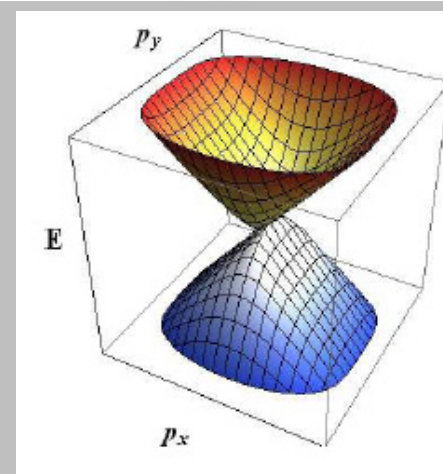
$$\Delta_e(\theta) \equiv \Delta_0 + \Delta' \cos 4\theta$$



Accidental nodes  
→ Dirac points  
(cuprates)



Spectroscopy and low T  
thermodynamics  
dominated by accidental  
**zeroes** →



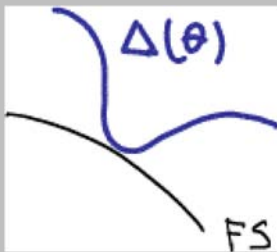
Zero-point BdG  
quasiparticles  $\Delta(\theta) \sim \theta^2$

# Critical "Zero-Point" Quasiparticle Scaling

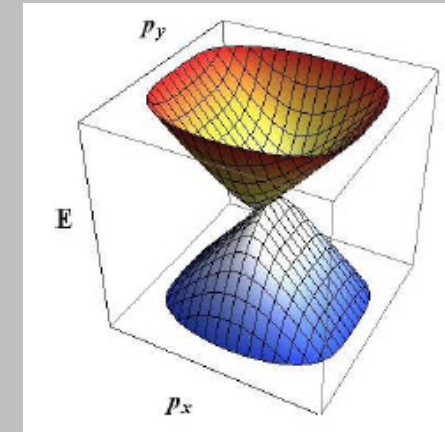
V. Stanev *et al.*, arXiv:1006.0447

$\Delta' = \Delta_0$  defines a **line of quantum phase transitions** in (T, H) phase diagram  $\rightarrow$   
**Critical scaling of zero-point BdG quasiparticles**

$$\mathcal{H}_{\text{BdG}} = \begin{bmatrix} \frac{p^2}{2m} - \epsilon_F & \hat{\Delta}(\mathbf{r}) \\ \hat{\Delta}(\mathbf{r}) & -\frac{p^2}{2m} + \epsilon_F \end{bmatrix} \approx \begin{bmatrix} v_F p_x & \frac{8\Delta_0}{p_F^2} p_y^2 \\ \frac{8\Delta_0}{p_F^2} p_y^2 & -v_F p_x \end{bmatrix}$$



$$\Rightarrow E = \pm \sqrt{(v_F p_x)^2 + (8\Delta_0 p_y^2 / p_F^2)^2}$$



Spectroscopy and thermodynamics dominated by **zero-point scaling**  
 $\leftrightarrow$  Different from Dirac and Simon-Lee scaling in cuprates

$$N(E) \sim \text{Re} \left( \int d\theta \frac{E}{\sqrt{E^2 - \Delta(\theta)^2}} \right) \sim \sqrt{E}$$

$$C(T) \approx \frac{1}{T^2} \int_0^\infty dE E^2 N(E) \frac{1}{\cosh^2(E/2T)} \propto T^{3/2}$$

$$\chi_s(T)/\chi_n \approx \frac{1}{T} \int_0^\infty dE N(E) \frac{1}{\cosh^2(E/2T)} \propto T^{1/2}$$

$$\frac{(T_1)_n}{(T_1)_s} \approx \frac{1}{T} \int_0^\infty dE N(E)^2 \frac{1}{\cosh^2(E/2T)} \propto T$$

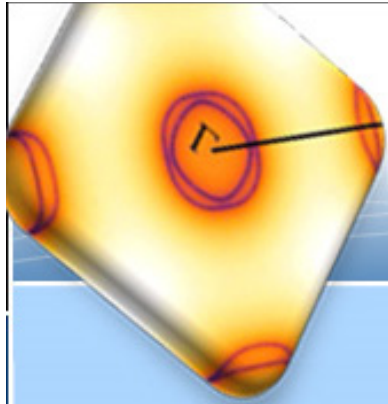
$$\lambda(T) = \sqrt{\frac{c^2 m}{4\pi e^2 \rho_s(T)}} \approx \lambda_0 + \lambda_1 T^{1/2}$$

At finite H new form of scaling replaces nodal Simon-Lee scaling:

$$N(E \rightarrow 0, H) \sim H^{1/3+\eta}$$

$$N(0) \sim H^{1/2}$$

$$C(H, T) \sim TH^{1/3+\eta}$$



## Conclusions

- Iron pnictides are semimetals turned superconductors
- Correlations are significant, hence a SDW in parent compounds, but weaker than in cuprates
- Both magnetism and superconductivity are intrinsically multiband in nature –  $s'$  interband SC is a likely possibility near a nesting-driven SDW
- Orbital anisotropy of pairing interaction can lead to anisotropic SC gap with accidental nodes/zeros  $\rightarrow$  quantum critical “zero-point” scaling replaces Dirac-SL scaling seen in cuprates  
 **$\rightarrow$  new physics, beyond the “standard” model?**

