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Multiscale Quantum Criticality: Nematic Instability in Metals

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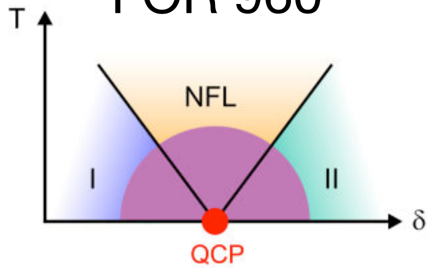


Outline

- Introduction:
  quantum phase transitions & problem of multiple scales

- Nematic instability of the Fermi liquid

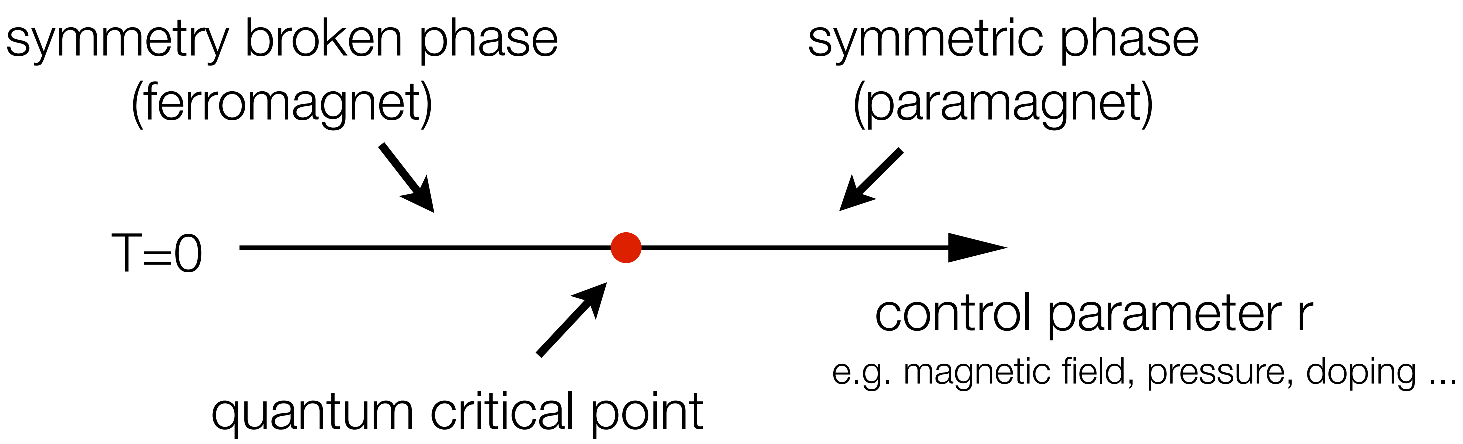
- effective bosonic theory — Mario Zacharias, Peter Wölfle, and MG, Phys. Rev. B **80**, 165116 (2009)

- electron spectral function — MG and Andrey V. Chubukov, Phys. Rev. B **81**, 235105 (2010)

Introduction:
quantum phase transitions &
problem of multiple scales

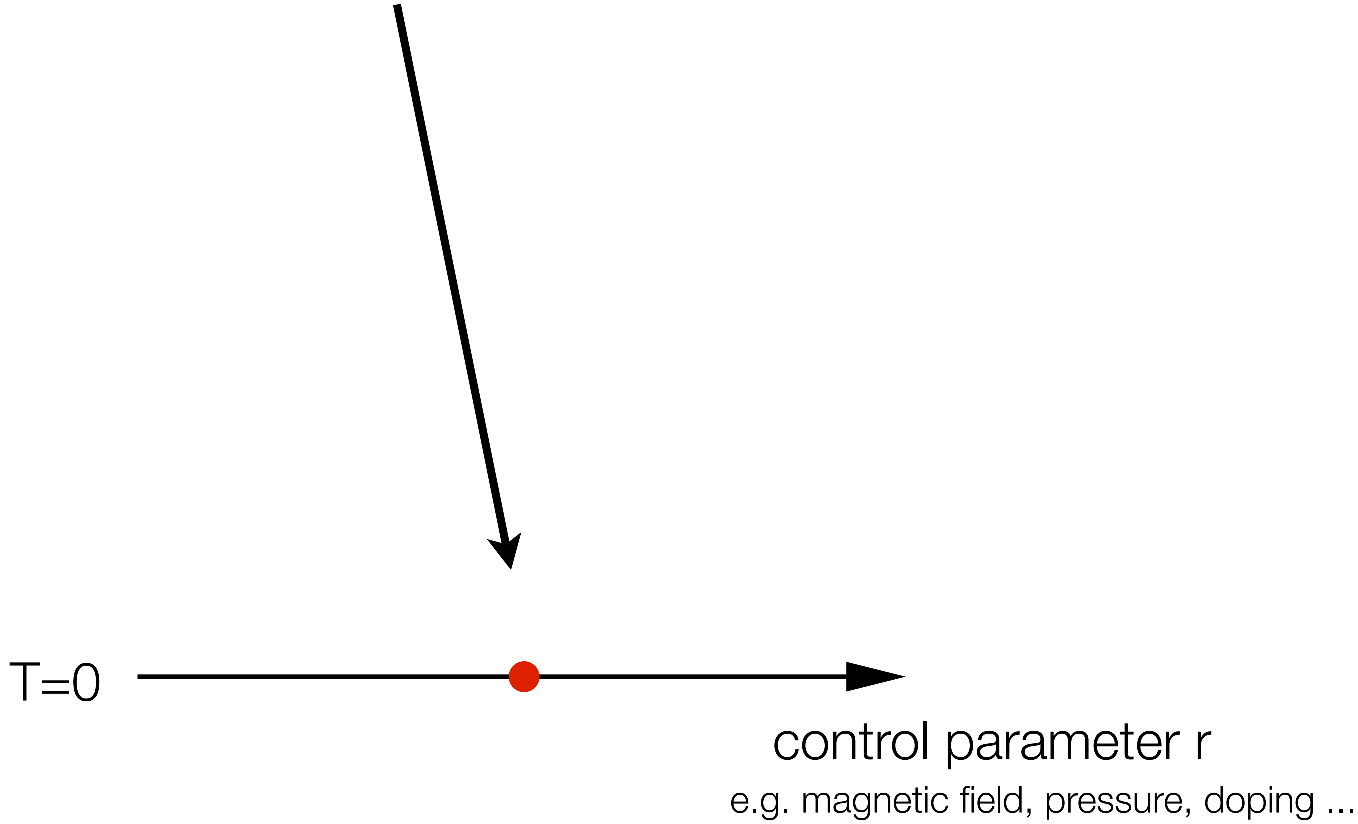
2nd order quantum phase transition

instability of the ground state at T=0 ⇨ quantum phase transition



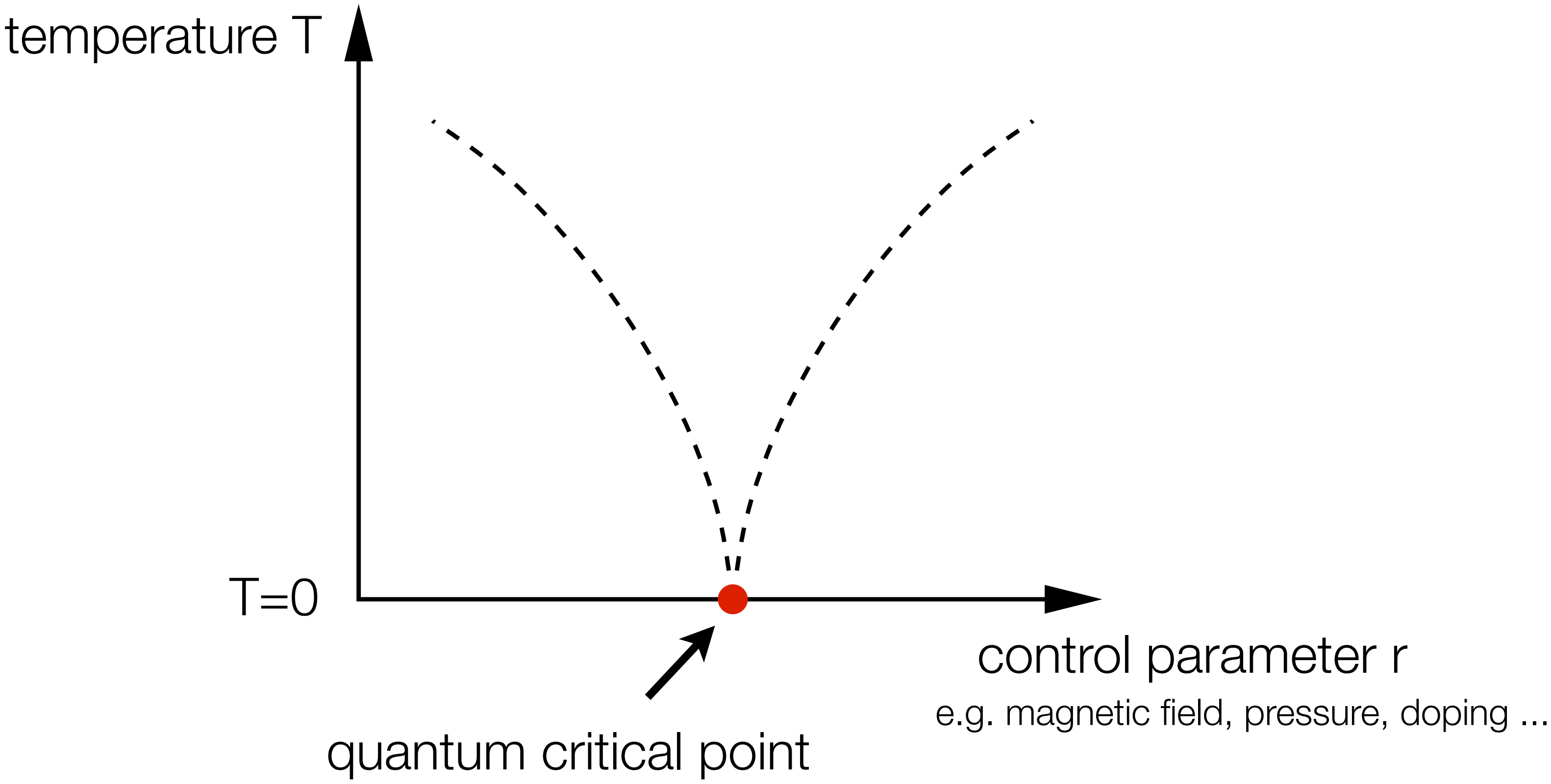
2^{nd} order quantum phase transition

abundance of low-energy fluctuations



2nd order quantum phase transition

abundance of low-energy fluctuations ⇨ determine properties at $T>0$



Critical exponents and effective dimensionality

- correlation length exponent ν $\quad \xi \sim |r|^{-\nu}$ control parameter r

- dynamical exponent z
  spectrum of critical fluctuations $\omega \sim k^z$

  vanishing characteristic energy scale $\boxed{\varepsilon \sim \xi^{-z}}$

- enhanced dimensionality
  correlation volume in space ξ^d and time ξ^z $\Rightarrow$ $d_{\text{eff}} = d + z$

Scaling close to quantum critical points

scaling ansatz of the critical free energy

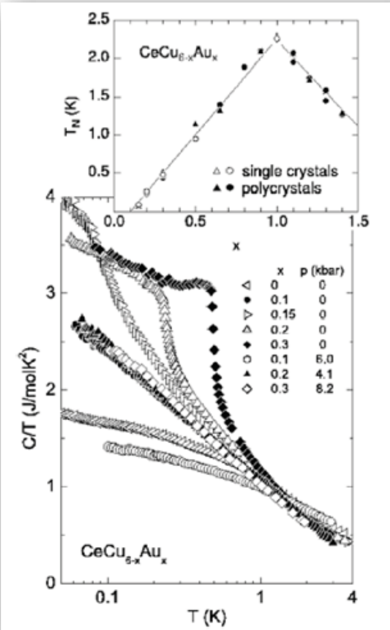
$$\mathcal{F}(r, T) = b^{-(d+z)} \mathcal{F}(r\, b^{1/\nu}, T\, b^{z})$$

exponents: effective dimensionality ($d+z$), correlation length exp. (ν), dynamical exp. (z)

with arbitrary scaling variable b

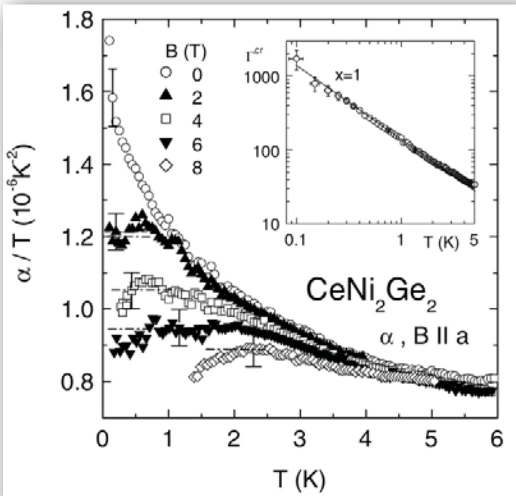
Scaling close to quantum critical points

basis for the interpretation of thermodynamic experiments:

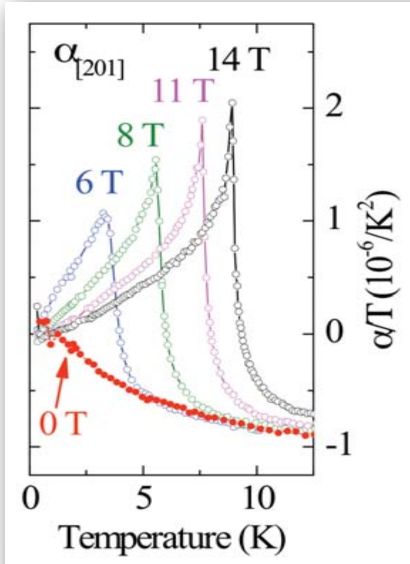


$CeCu_{6-x}Au_x$

v Löhneysen et al. (1996)



$CeNi_2Ge_2$ Küchler et al. (2003)



$TlCuCl_3$ Lorenz et al. (2006)

Quantum-to-classical crossover

at any small but finite temperature:

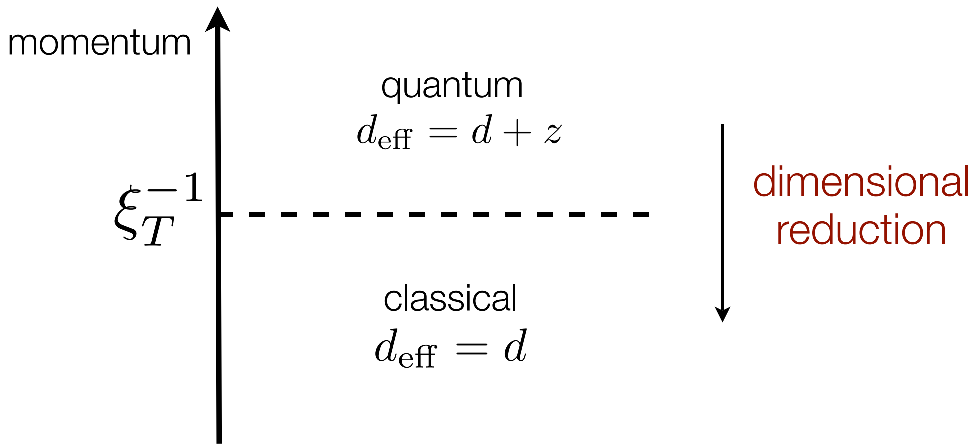
limited correlation volume in time

$$\xi^z \lesssim T^{-1}$$

$\Rightarrow$ thermal length $\quad \xi_T \sim T^{-\frac{1}{z}}$

quantum-to-classical crossover

critical fluctuations change their character as function of momentum

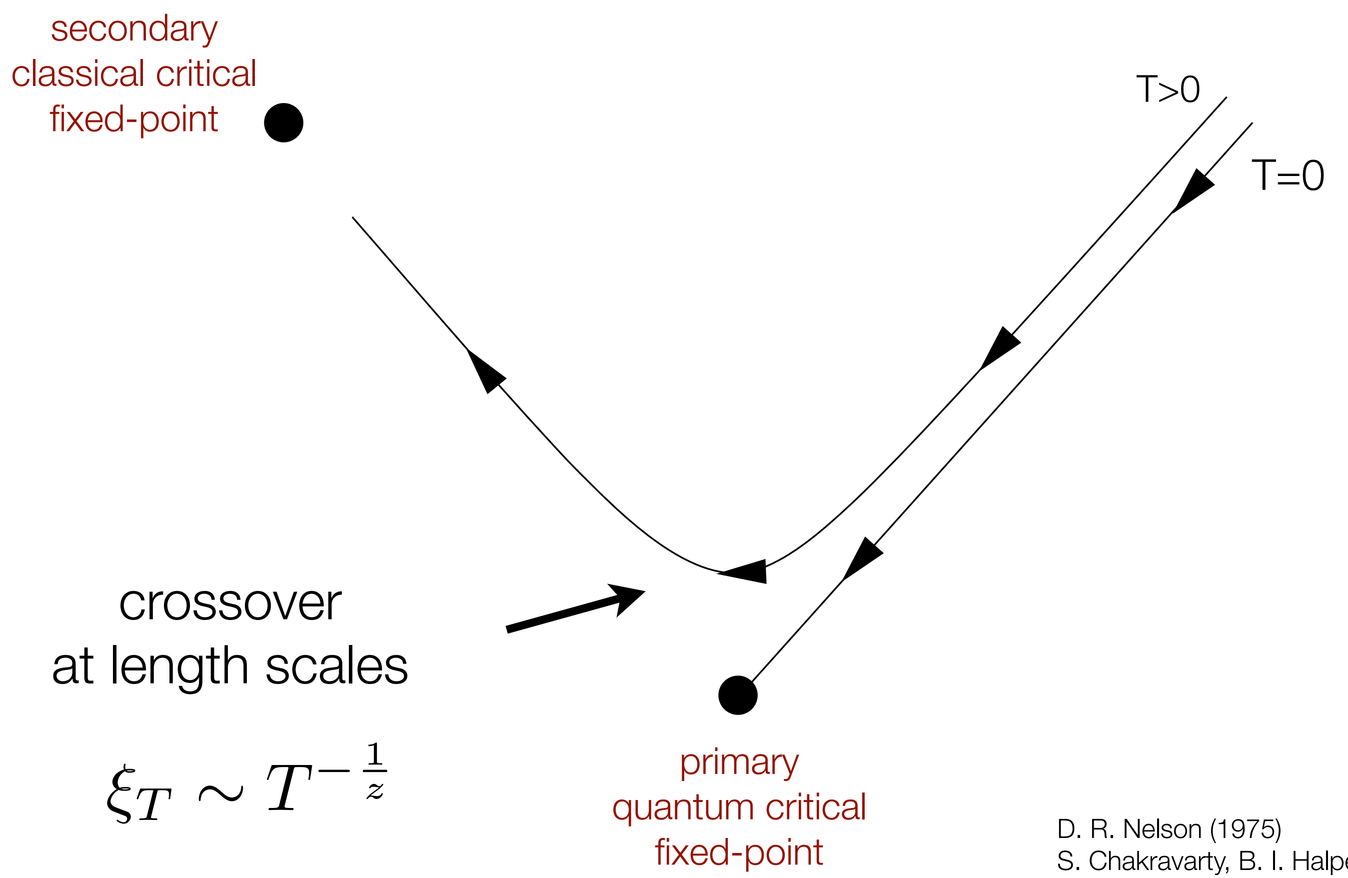


for small momenta: effective classical critical theory for the Matsubara zero mode

S. Sachdev (1997)
J. Zinn-Justin (2002)
....

Quantum-to-classical crossover

crossover between two critical fixed-points in RG flow



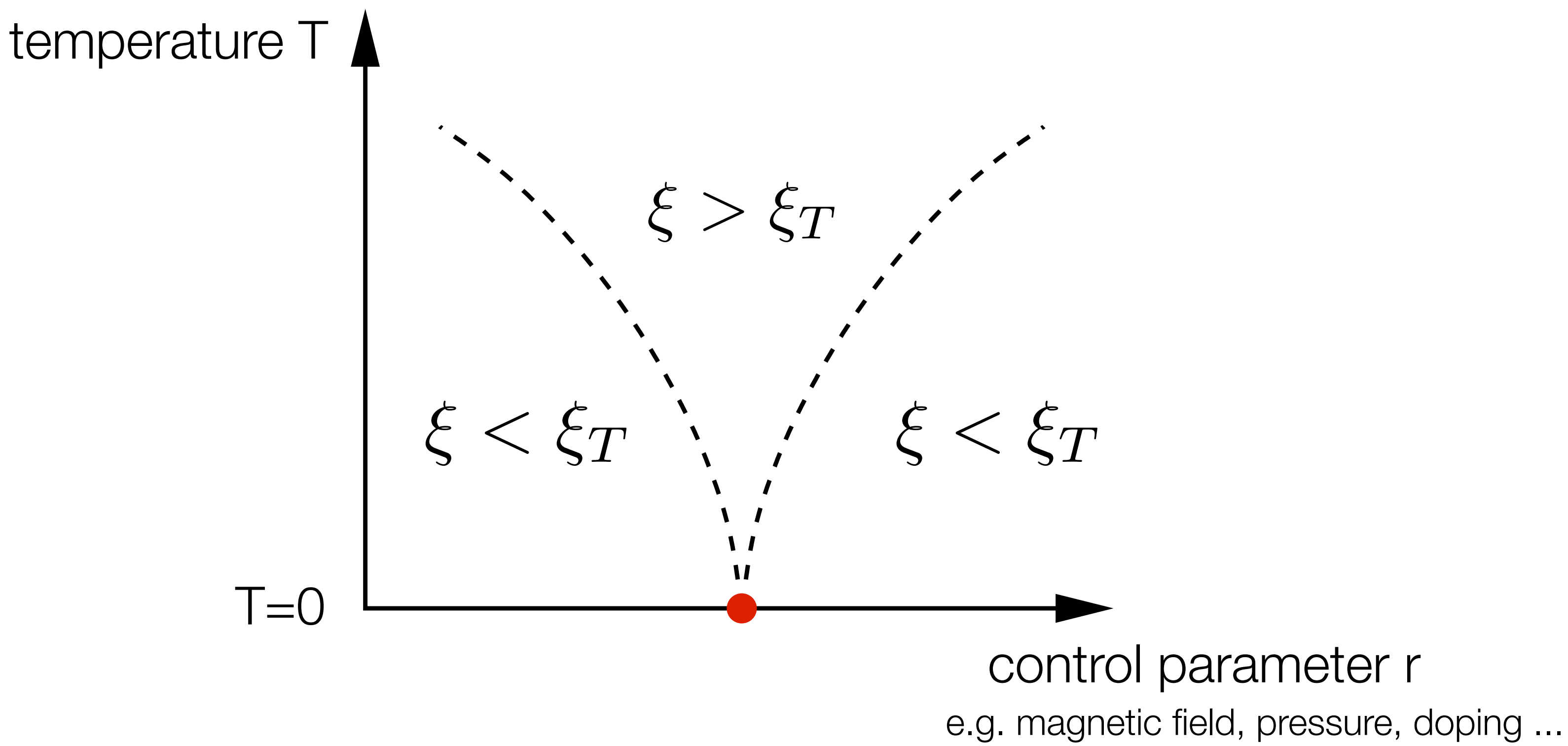
crossover
at length scales

$$\xi_T \sim T^{-\frac{1}{z}}$$

D. R. Nelson (1975)
S. Chakravarty, B. I. Halperin, D. R. Nelson (1989)
A. Millis (1993)
....

Quantum-to-classical crossover

thermal lenght ξ_T and correlation length ξ ⇨ crossover in phase diagram

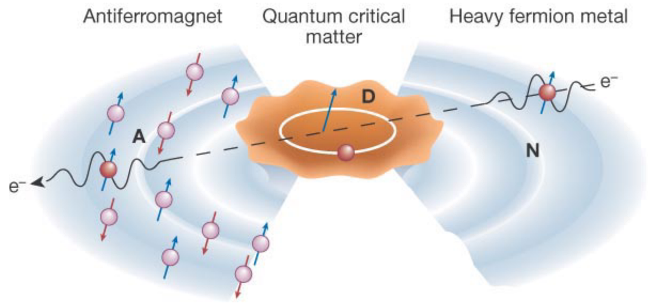


Problem of multiple scales

Coexistence of low-energy fluctuations

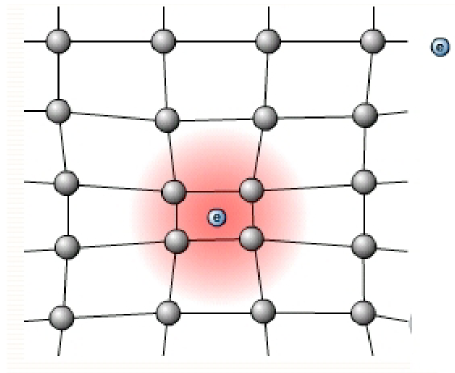
Example: magnetic metals

fluctuating magnetic domains
and ballistically moving electrons



Example: phonon-coupling

critical fluctuations and phonons



⇨ two dynamical exponents z_1 and z_2

Identification of quantum phase transition

- coexisting and interacting fluctuations $\longrightarrow$ entanglement!
- identification of proper critical degrees of freedom?
- two dynamical exponents $\longrightarrow$ scaling and power-laws?
- crossovers in the phase diagram?
- strong hybridization $\longrightarrow$ emergence of new universality classes?
  fermionic critical points?
- fluctuation driven first order transitions?

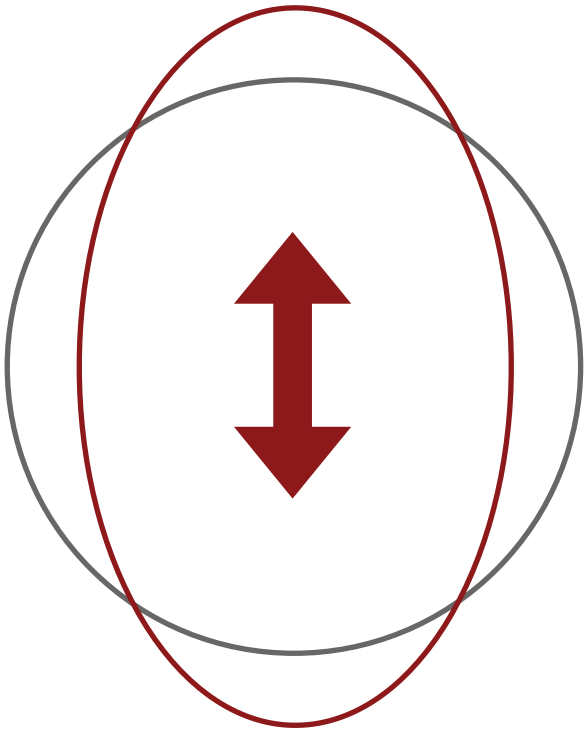
⇨ multiscale quantum criticality not well understood

⇨ study simple but representative examples

Nematic instability of the Fermi liquid

Pomeranchuk instability

instability of the Fermi surface ⇨ spontaneous quadrupole moment



Nematic order parameter

order parameter: tensor object similar as in nematics

$$\varepsilon_{ij}(q) = \sum_k \Psi^\dagger_{k+q/2} \left(d\, \hat{k}_i \hat{k}_j - \delta_{ij} \right) \Psi_{k-q/2}$$

traceless part of strain tensor ⇨ shear modes of the Fermi surface

in d=2 two shear modes: sin(2Φ) and cos(2Φ) components

model Hamiltonian

$$\mathcal{H} = \sum_k \epsilon_k \Psi^\dagger_k \Psi_k + \frac{F_2}{2} \sum_q \varepsilon_{ij}(q) \varepsilon_{ij}(-q)$$

Oganesyan, Kivelson and Fradkin (2001)

Ginzburg-Landau theory

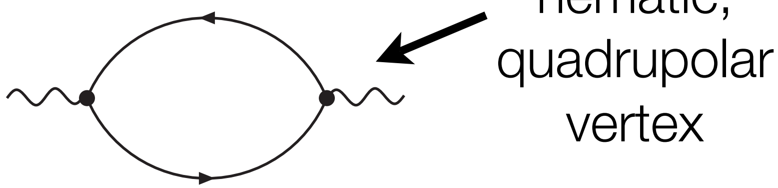
Hubbard-Stratonovich transformation
and perturbatively integrating out fermions

$$\mathcal{L} = \frac{1}{2}\sigma_{ij}\,\chi^{-1}_{ijkl}\,\sigma_{kl} + \frac{u}{12}\mathrm{tr}\{\sigma^4\}$$

effective elasticity theory for the Fermi surface,   (traceless) stress tensor σ_{ij}

in d=2 no cubic term $\mathrm{tr}\{\sigma^3\} = 0 \longrightarrow$ second order transition possible

susceptibility tensor χ_{ijkl} determined by quadrupolar polarizations of electrons



Susceptibility tensor

static part: $\chi^{-1}_{ijkl}(q) = \left[r + (aq)^2\right]\delta_{ik}\delta_{jl}$ diagonal!

tuning parameter of the transition $r = -\frac{1}{\nu F_2} - 1$

Pomeranchuk instability: $\boxed{F_2 \nu = -1}$

cf. Stoner criterion

dynamics depends on the polarization

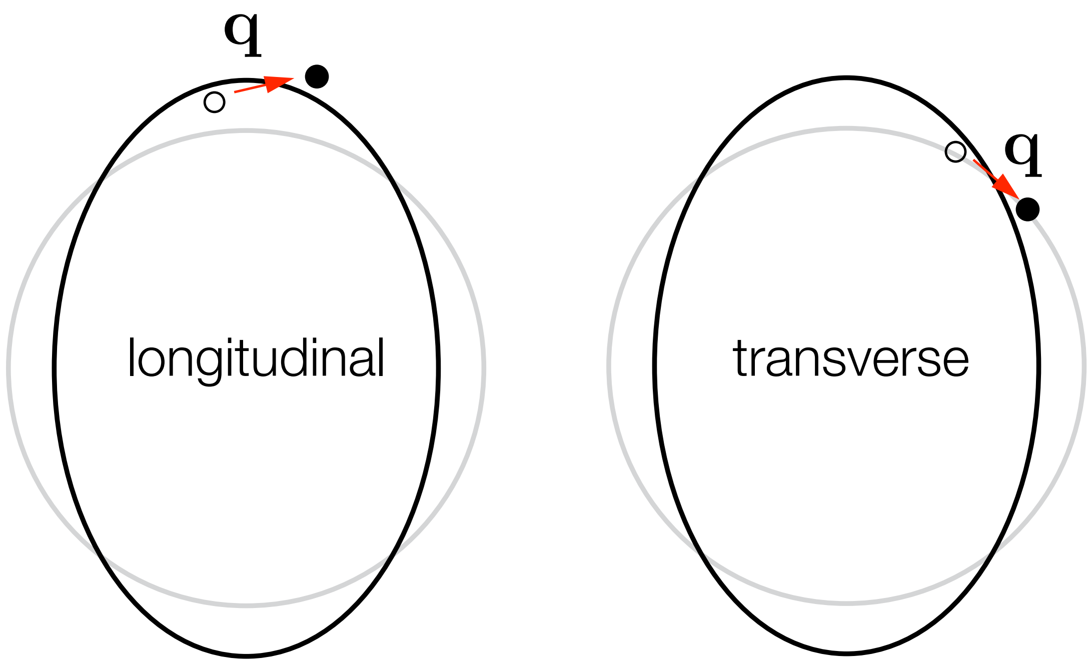
$$\sigma_{ij}(q) = \phi_{||}(q)\hat{Q}_{ij} + \phi_{\perp}(q)\hat{Q}^{\perp}_{ij}$$

longitudinal and transverse to the quadrupolar momentum tensor

$$\hat{Q}_{ij} = 2\hat{q}_i\hat{q}_j - \delta_{ij} \quad \text{and} \quad \hat{Q}_{ij}\hat{Q}^{\perp}_{ij} = 0$$

Multiscale dynamics

polarization longitudinal and transverse to quadrupolar momentum **q** tensor

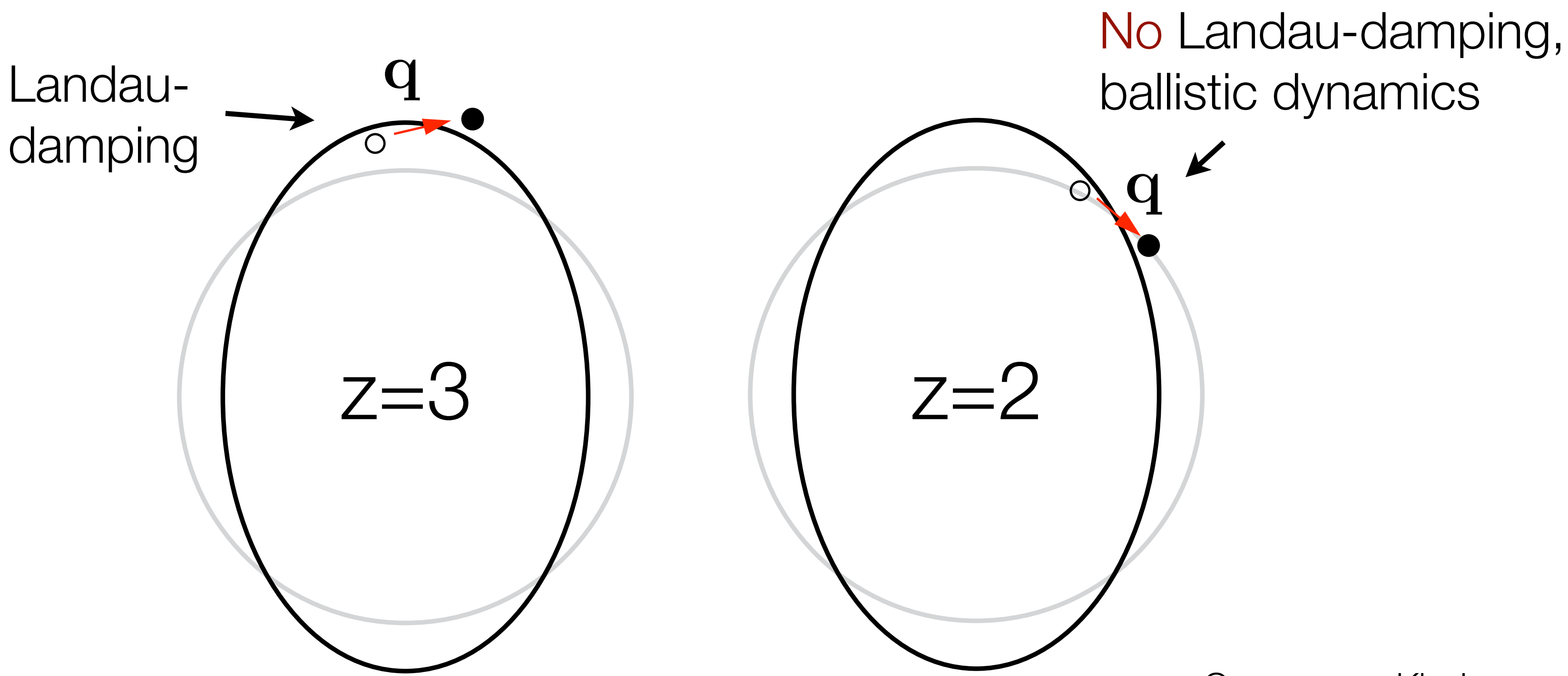


Oganesyan, Kivelson and Fradkin (2001)

different phase space for exciting particle-hole pairs in the metal

Multiscale dynamics

polarization longitudinal and transverse to quadrupolar momentum **q** tensor



Oganesyan, Kivelson
and Fradkin (2001)

different phase space for exciting particle-hole pairs in the metal

Multiscale dynamics

longitudinal polarization $\Rightarrow$ damped z=3 dynamics

$$\chi_{\parallel}^{-1}(q,\Omega_n) = r + (aq)^2 + 2\frac{|\Omega_n|}{v_F q}$$

transverse polarization $\Rightarrow$ ballistic z=2 dynamics

$$\chi_{\perp}^{-1}(q,\Omega_n) = r + (aq)^2 + 4\frac{\Omega_n^2}{(v_F q)^2}$$

multiple dynamics $\Rightarrow$ multiple energy scales $\varepsilon \sim \xi^{-z}$

Which mode is more important?

longitudinal z=3 mode has larger phase space $\Omega_n \sim q^z$

⇨ dominates the specific heat $C \sim T^{d/z}$

"z=2 mode plays no role in the critical theory"

Oganesyan, Kivelson and Fradkin (2001)

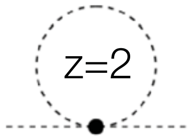
NO: transverse z=2 mode has smaller effective dimension d+z = 4

⇨ generates logarithmic singularities in loop corrections

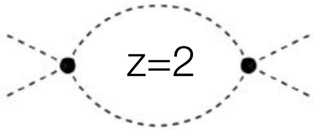
⇨ interplay of both modes determine critical properties

Theory at zero temperature T=0

transversal z=2 fluctuations $\Rightarrow$ logarithmic renormalization group flow



$$\frac{\partial r}{\partial \log D} = \left(2 - \frac{u}{24\pi}\right) r$$



$$\frac{\partial u}{\partial \log D} = -\frac{3}{32\pi} u^2$$

logarithmic scale dependence of the correlation length

$$\boxed{\xi^{-2}(\epsilon) \sim \frac{r}{(\log 1/\epsilon)^{4/9}}} \quad \text{for} \quad \epsilon > \xi^{-2}$$

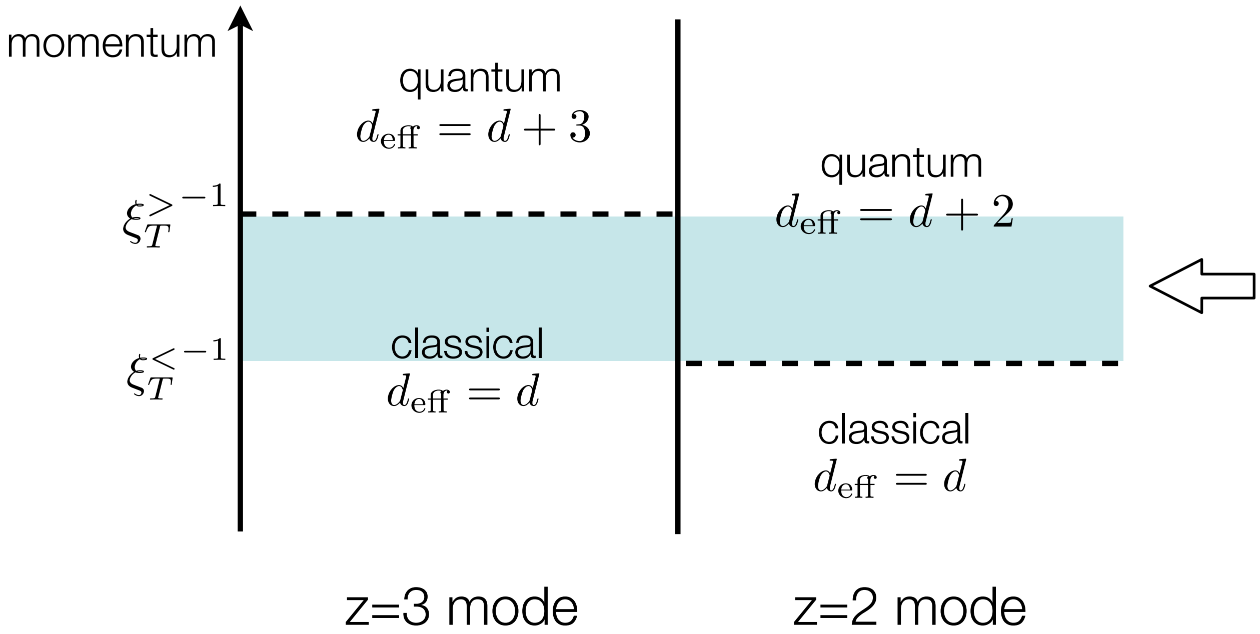
z=2 energy scale

exponent 4/9 differs from Ising and XY universality! characteristic for Pomeranchuk!

Theory at finite temperatures

multiple z $\Rightarrow$ multiple thermal lengths $\xi_T \sim T^{-\frac{1}{z}}$

Extended quantum-to-classical crossover

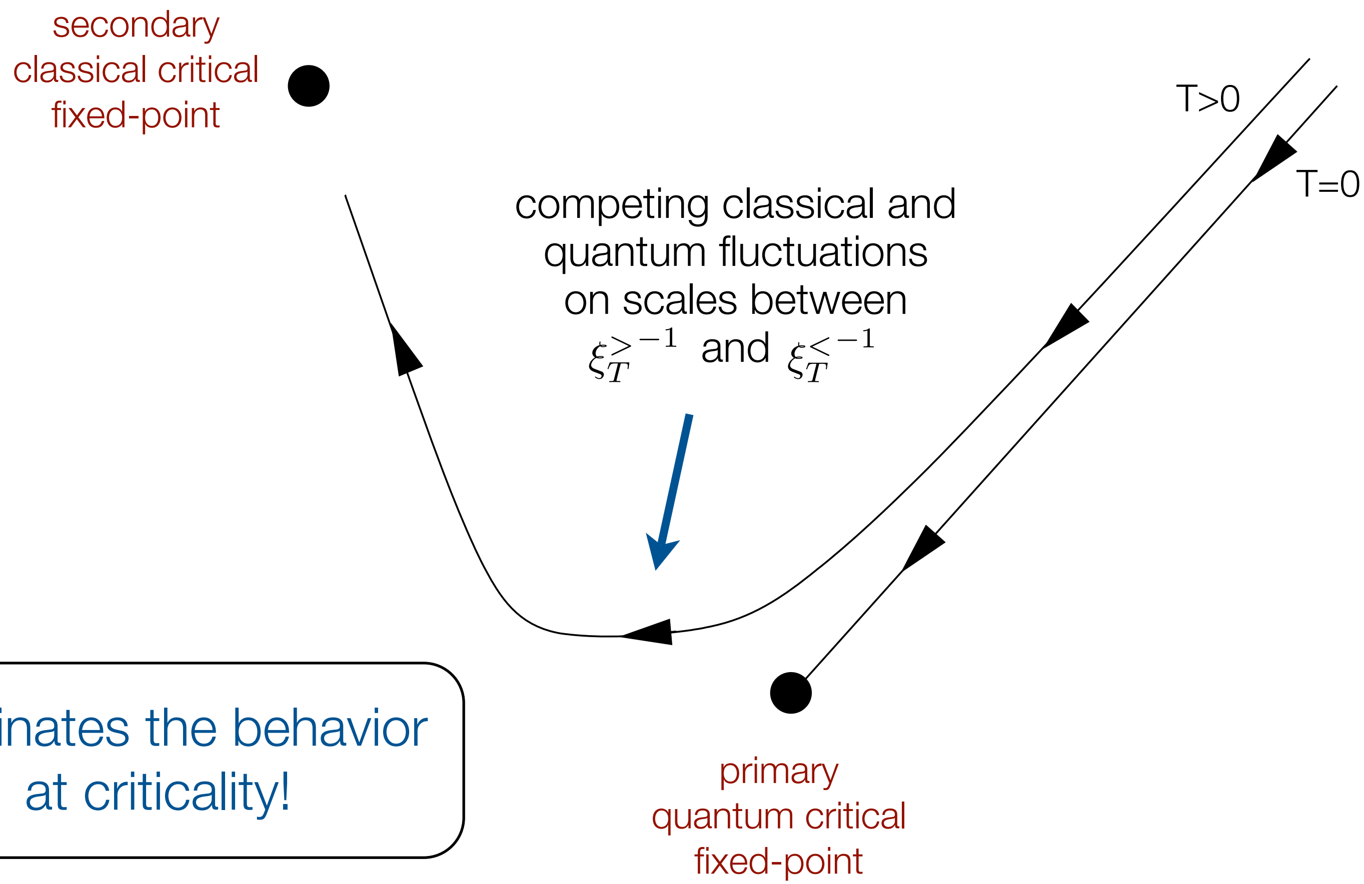


overlap regime in phase space with coexisting quantum and classical fluctuations

incomplete dimensional reduction!

Extended quantum-to-classical crossover

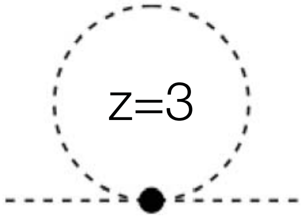
RG language: extended crossover regime



dominates the behavior at criticality!

Extended quantum-to-classical crossover

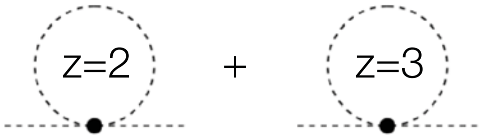
temperature boost of the RG flow:



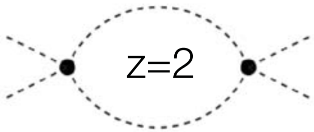
$$\delta r = \frac{uT}{3} \int_0^{{\xi_T^{>}}^{-1}} \frac{dqq}{2\pi} \chi_{\parallel}(q, \omega = 0)$$

Matsubara zero mode: log divergent in d=2

RG flow within the crossover regime



$$\frac{\partial r}{\partial \log D} = \left(2 - \frac{u}{24\pi}\right) r + \frac{u}{6\pi} T$$



$$\frac{\partial u}{\partial \log D} = -\frac{3}{32\pi} u^2$$

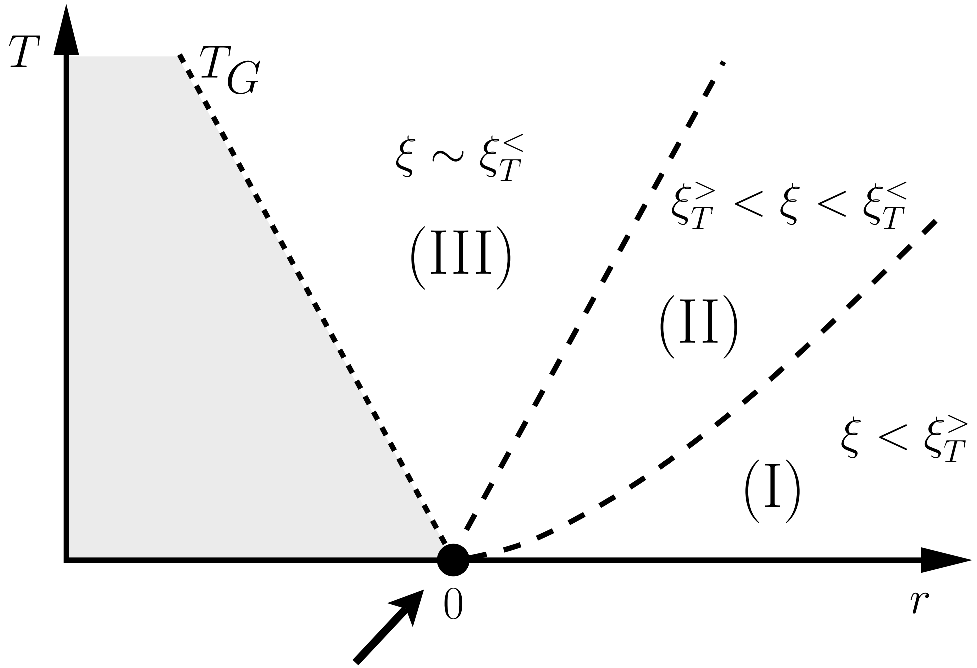
universal correlation length at criticality (r=0):

$$\boxed{\xi^{-2} = cT}$$

independent of the bare quartic coupling u

Multiple scales in thermodynamics

two thermal lengths ⇨ two crossover lines in the phase diagram

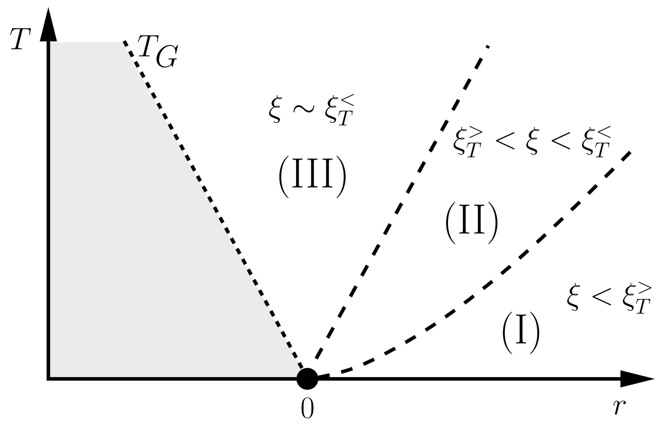


Pomeranchuk
quantum critical point

Sensitivity on crossover depends on specific thermodynamic quantity

No scaling in terms of a single dynamical exponent!

Multiple scales in thermodynamics



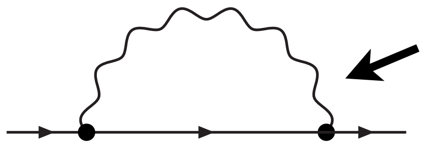
thermodynamics determined by the interplay of z=2 and z=3 modes

	specific heat coeff. $\gamma = -\frac{\partial^2 \mathcal{F}}{\partial T^2}$	thermal expansion $\alpha = \frac{\partial^2 \mathcal{F}}{\partial T \partial r}$	compressibility $\delta\kappa = \frac{\partial^2 \mathcal{F}}{\partial r^2}$
(I)	$\frac{\log^{2/9} \frac{1}{r}}{\sqrt{r}}$	$\frac{T}{r^{3/2}} \log^{2/9} \frac{1}{r}$	$\log^{1/9} \frac{1}{r}$
(II)	$\frac{1}{T^{1/3}}$	$\log^{5/9} \frac{1}{r}$	$\log^{1/9} \frac{1}{r}$
(III)	$\frac{1}{T^{1/3}}$	$\log^{5/9} \frac{1}{T}$	$\frac{1}{\log^{8/9} \frac{1}{T}}$

How do the critical boson modes affect the electron spectral function?

Longitudinal electron self-energy

one-loop self-energy



longitudinal z=3 mode

$$\Sigma(\omega_n) \sim \begin{cases} i\omega_n\xi & \text{for} \quad |\omega_n| \ll \xi^{-3} \\ i|\omega_n|^{2/3} & \text{for} \quad |\omega_n| \gg \xi^{-3} \end{cases}$$

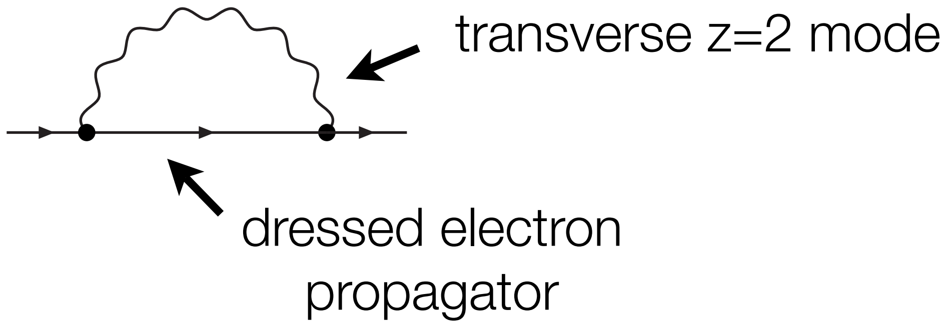
z=3 energy scale

Oganesyan, Kivelson and Fradkin (2001)
Metzner, Rohe, and Andergassen (2003)
....

singular correction $\Rightarrow$ consider in the following dressed electron propagators (Eliashberg-like theory)

Transverse electron self-energy

one-loop self-energy



off mass-shell part:

$$\delta\Sigma = -\frac{1}{2k_F a}\left(\Sigma_{||}(\omega_n) - \varepsilon_k\right)\log\left[\max\left\{\Sigma_{||}(\omega_n) - \varepsilon_k, v_F\xi^{-1}\right\}\right]$$

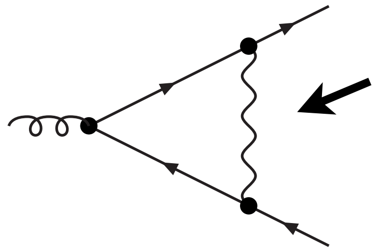
logarithmically singular correction to residue Z

develops if distance to mass-shell
exceeds fermionic z=1 energy scale

$$\varepsilon \sim \xi^{-1}$$

Transverse vertex correction

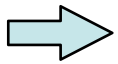
one-loop vertex correction



transverse z=2 mode

logarithmically singular
if internal electrons approach mass-shell

logs proliferate in higher orders,
explicitly checked up to two loop

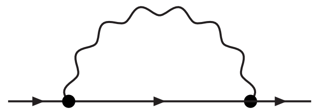


renormalization group

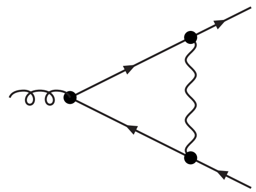
Renormalization group

renormalization group flow for residue and vertex

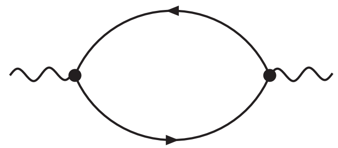
$$\frac{\partial(1/Z)}{\partial \log D} = \frac{1}{2k_F a} Z\Gamma^2$$



$$\frac{\partial \Gamma}{\partial \log D} = \frac{1}{2k_F a} Z^2\Gamma^3$$



combination $\Gamma Z = 1$ invariant!



polarizations unaffected! dynamics preserved!

Electron propagator

$$G(k,\omega_n) = \frac{Z[\Sigma_{||}(\omega_n) - \varepsilon_k]}{\Sigma_{||}(\omega_n) - \varepsilon_k}$$

$$\Sigma_{||}(\omega_n) \sim \begin{cases} i\omega_n \xi & \text{for} \quad |\omega_n| \ll \xi^{-3} \\ i|\omega_n|^{2/3} & \text{for} \quad |\omega_n| \gg \xi^{-3} \end{cases}$$

non-Fermi liquid frequency dependence at
z=3 energy scale

$$Z(\epsilon) \sim \begin{cases} \epsilon^{\eta} & \text{for} \quad |\epsilon| > \xi^{-1} \\ \xi^{-\eta} & \text{for} \quad |\epsilon| < \xi^{-1} \end{cases}$$

anomalous dimension at
z=1 energy scale

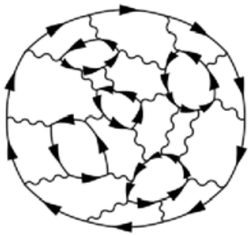
with $\eta = \dfrac{1}{2k_F a}$

$$\xi^{-2}(\epsilon) \sim \frac{r}{(\log 1/\epsilon)^{4/9}} \quad \text{for} \quad \epsilon > \xi^{-2}$$

z=2 energy scale

Further logarithms

planar diagrams of
Eliashberg theory dangerous



PHYSICAL REVIEW B **80**, 165102 (2009)

Low-energy effective theory of Fermi surface coupled with U(1) gauge field in 2+1 dimensions

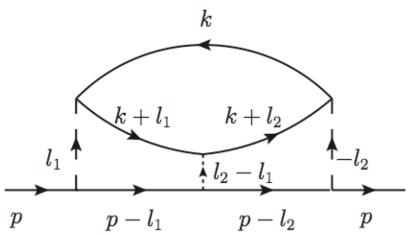
Sung-Sik Lee
Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 16 June 2009; published 1 October 2009)

longitudinal mode contributes
to anomalous dimension

**Quantum phase transitions of metals in two spatial dimensions:
I. Ising-nematic order**

Max A. Metlitski and Subir Sachdev
Department of Physics, Harvard University, Cambridge MA 02138
(Dated: January 6, 2010)

logarithmic singularity in
three loop order



see also Mross, McGreevy,
Liu and Senthil (2010)

interplay of various logs not established!

Summary

- nematic quantum criticality in metals: multiple energy scales
- extended quantum-to-classical crossover
  with interacting quantum and classical fluctuations
- in thermodynamics: flow of bosonic interaction,
  scale dependent ξ (z=2)
- in spectral function: nFL frequency dependence (z=3)
  and anomalous dimension (z=1)

Mario Zacharias. Peter Wölfle, and MG, Phys. Rev. B **80**, 165116 (2009)
MG and Andrey V. Chubukov, Phys. Rev. B **81**, 235105 (2010)