2157-10

Workshop on Principles and Design of Strongly Correlated Electronic Systems

2 - 13 August 2010

How Spins Become Pairs: Composite Pairing and Magnetism in the 115 Heavy Fermion Superconductors

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composite pairing and magnetism in the 115 heavy fermion superconductors

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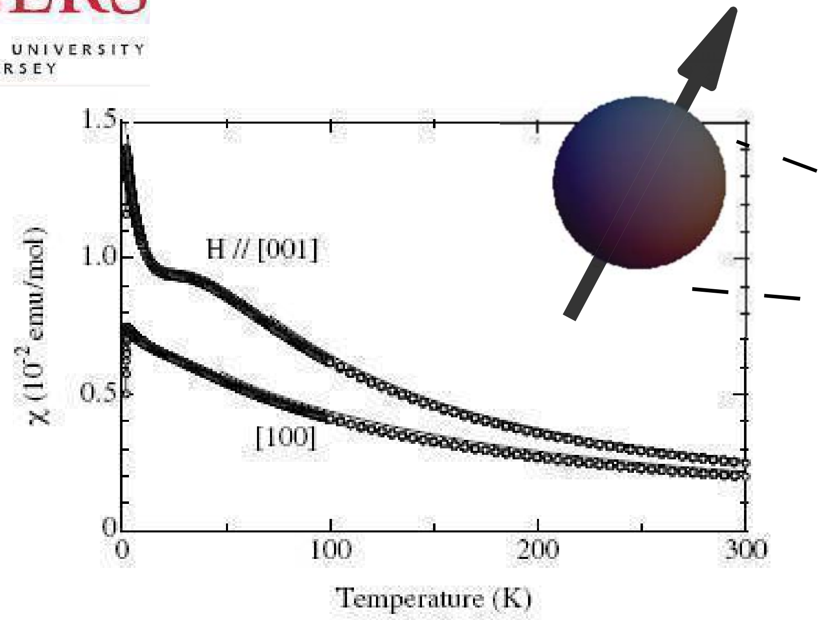
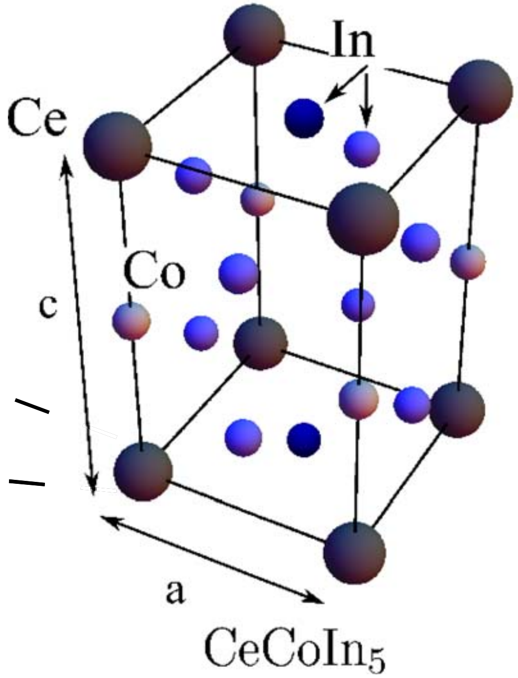


Shishido *et al.* JPSJ **71**, 162 (2002)



R. Flint, **M. Dzero and P. Coleman**, Nature Physics 4, 643(2008)
R. Flint and **P. Coleman**, arXiv:0912.2339(2009)

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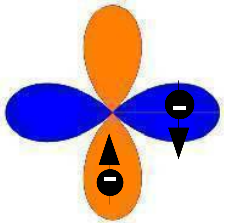
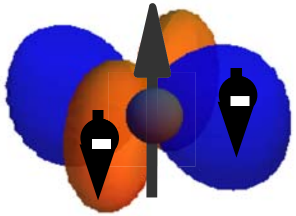
Magnetic Pairing



Composite Pairing



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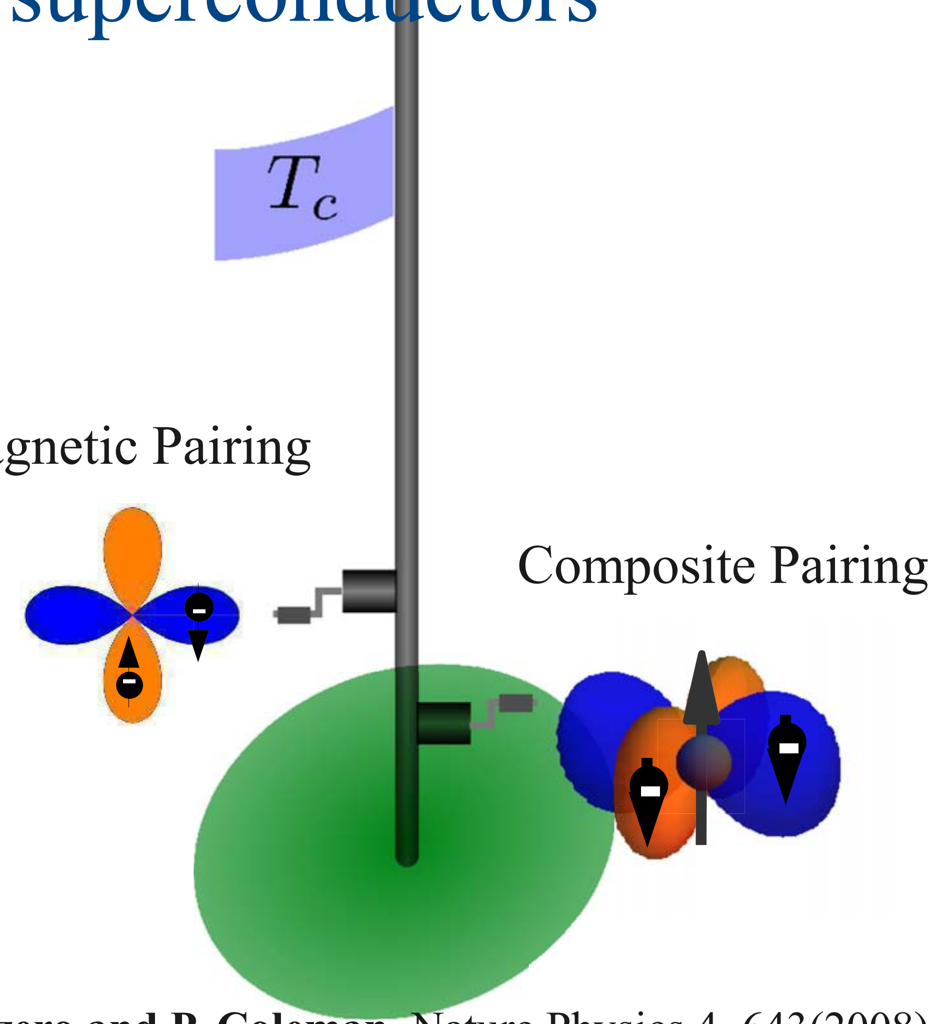

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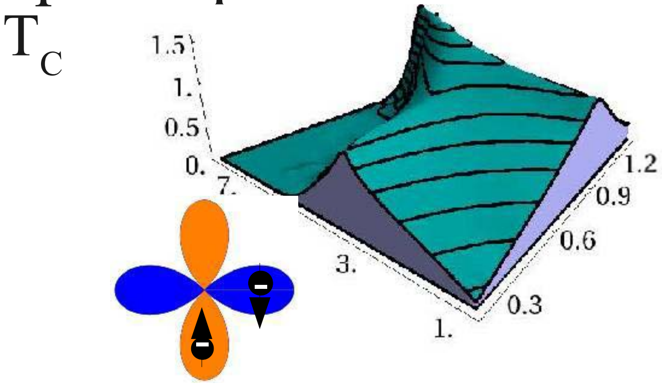
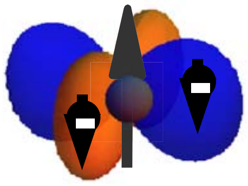
Outline

- The materials: how are the 115 superconductors special?
- How do spins form pairs?
  - Magnetically mediated pairing
  - Composite pairing
- The tool: symplectic-N
- Illustration: the two-channel Kondo-Heisenberg model
- Experimental consequences
  - Condensate quadrupole moment





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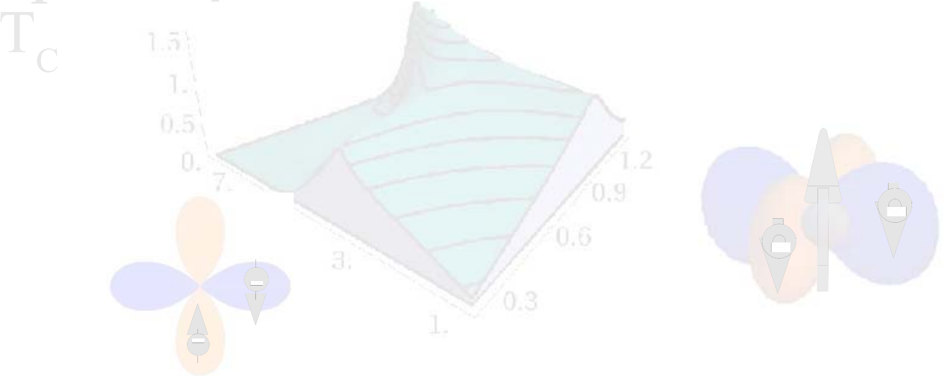
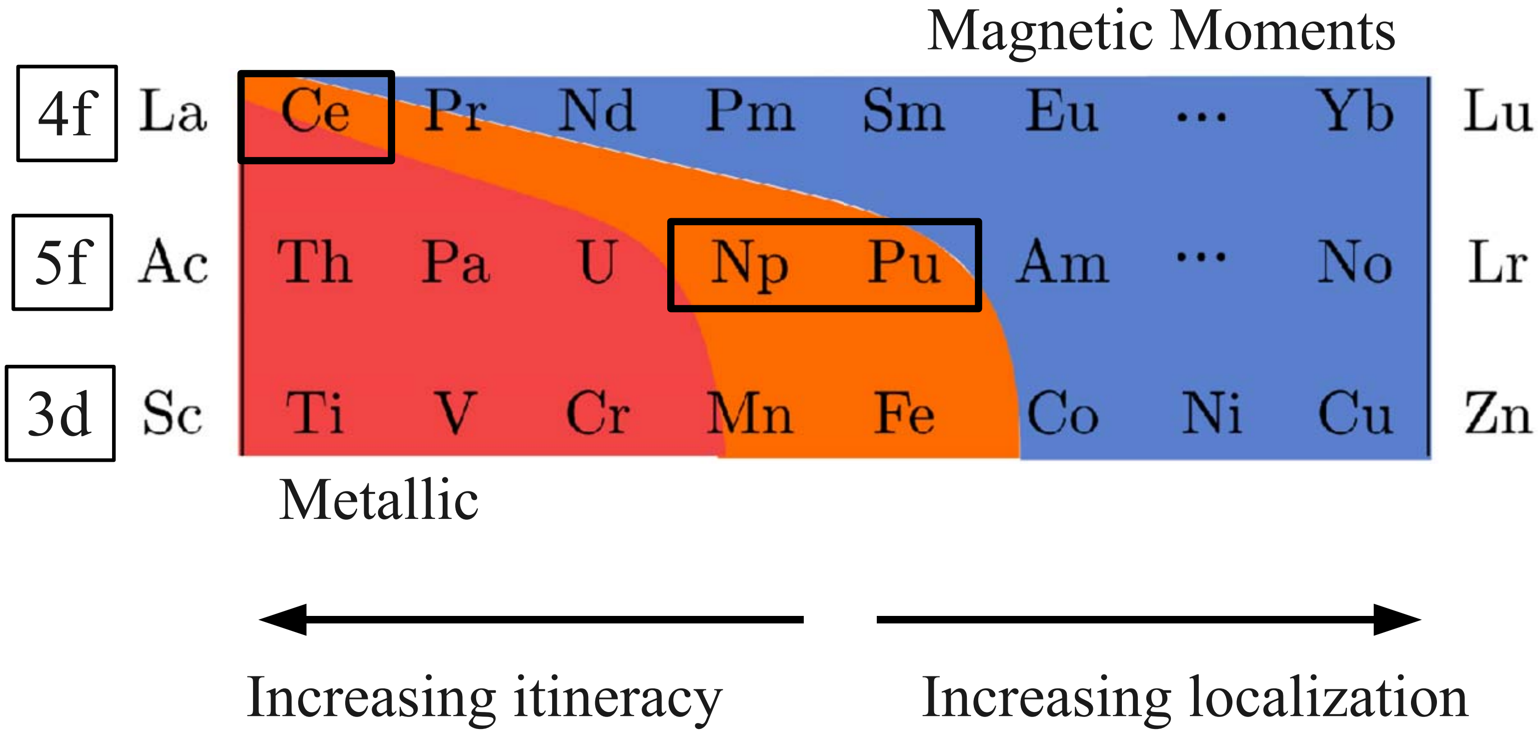


Localization vs. Itineracy

In the cuprates, this competition is induced by doping

In heavy fermions, this competition is built into the **atom**



Kmetko and Smith, 1983

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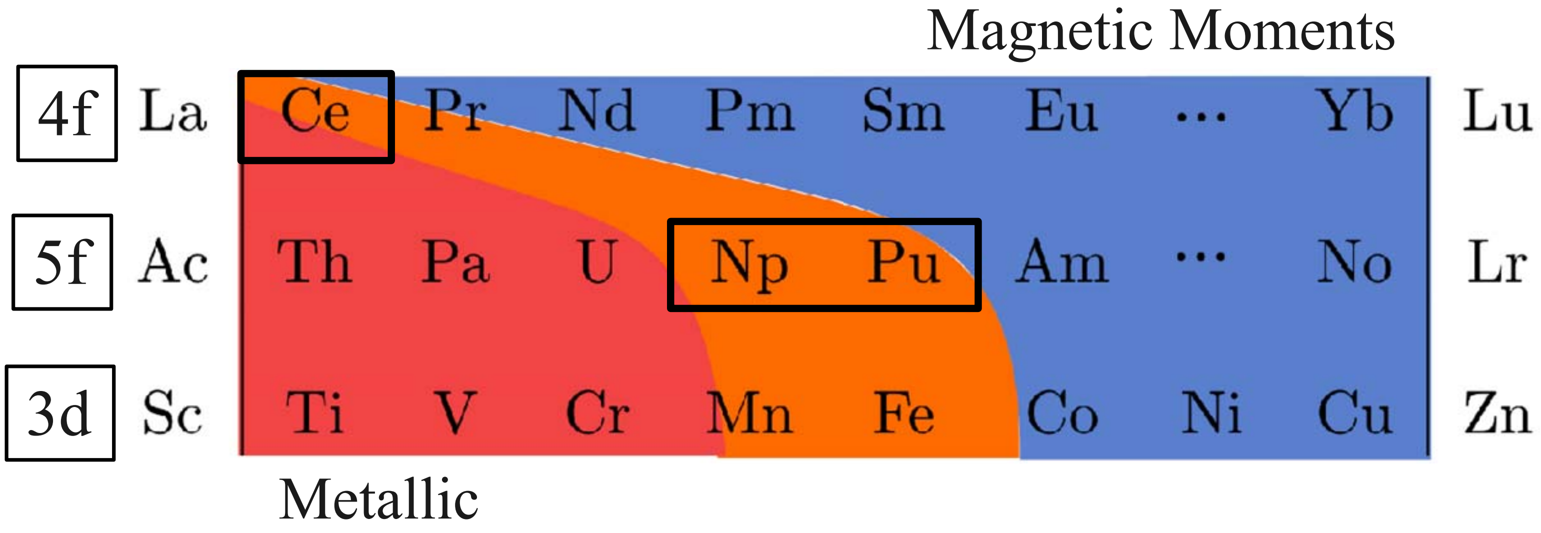


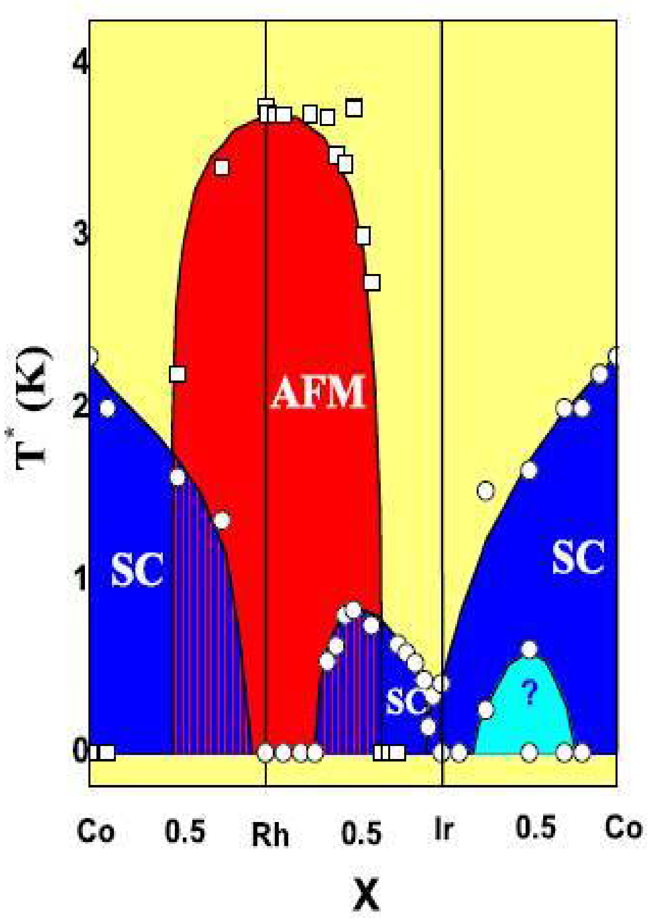
4f	La	Ce	Pr	Nd	Pm	Sm	Eu	⋯	Yb	Lu
5f	Ac	Th	Pa	U	Np	Pu	Am	⋯	No	Lr
3d	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn

- Heavy Fermi liquids
- Magnetism
- Superconductivity
- Quantum criticality

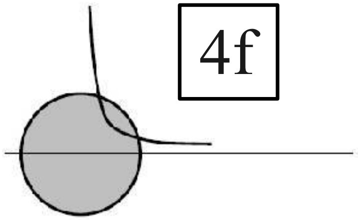
Kmetko and Smith, 1983

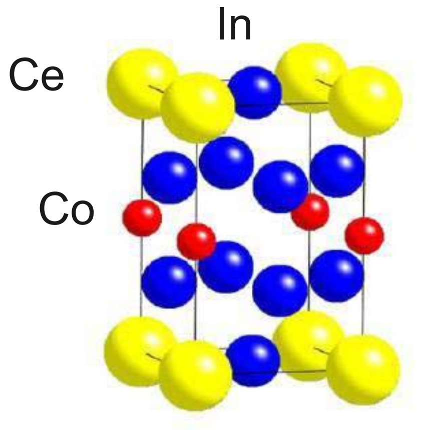
A new family of unconventional superconductors

Ce*M*In$_5$

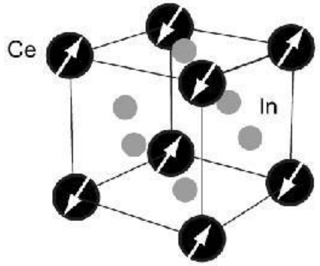


Sarrao and Thompson 2007



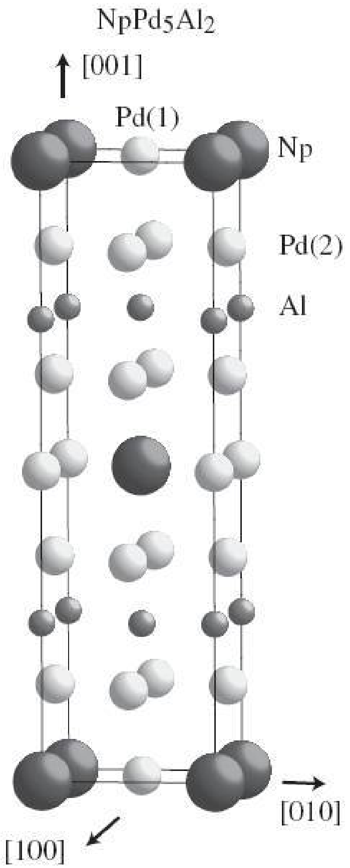


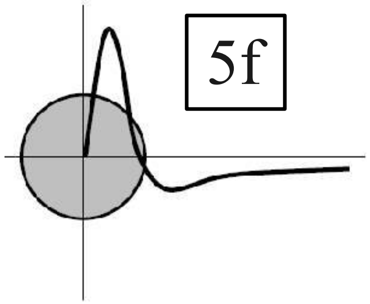
$CeCoIn_5$ 2.3K 2001

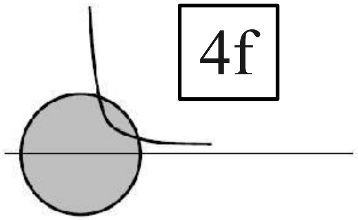


$CeIn_3$ .2K 1998

A new family of unconventional superconductors

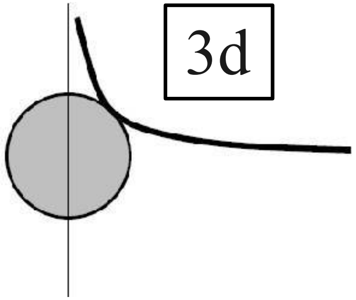


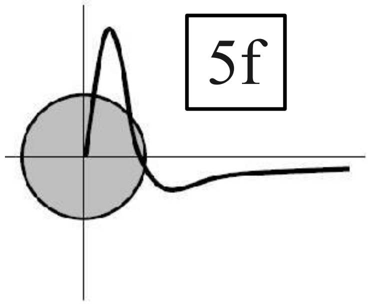


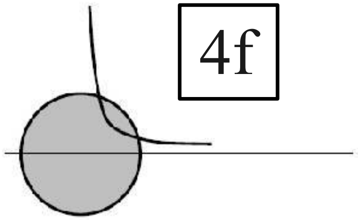


$PuCoGa_5$	18.5K	2003
$NpPd_5Al_2$	4.5K	2007
$CeCoIn_5$	2.3K	2001
$CeIn_3$	.2K	1998

A new family of unconventional superconductors







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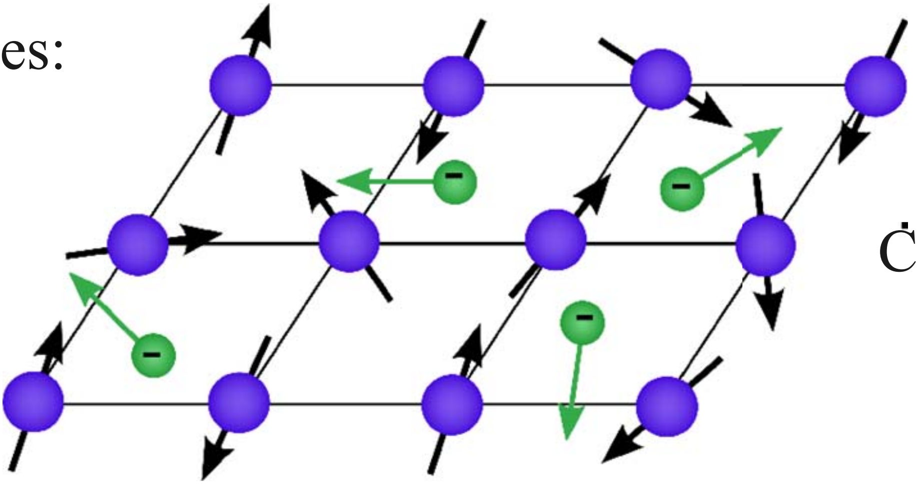
“Conventional” heavy fermion superconductivity

At high temperatures:



Local moments

Conduction electrons

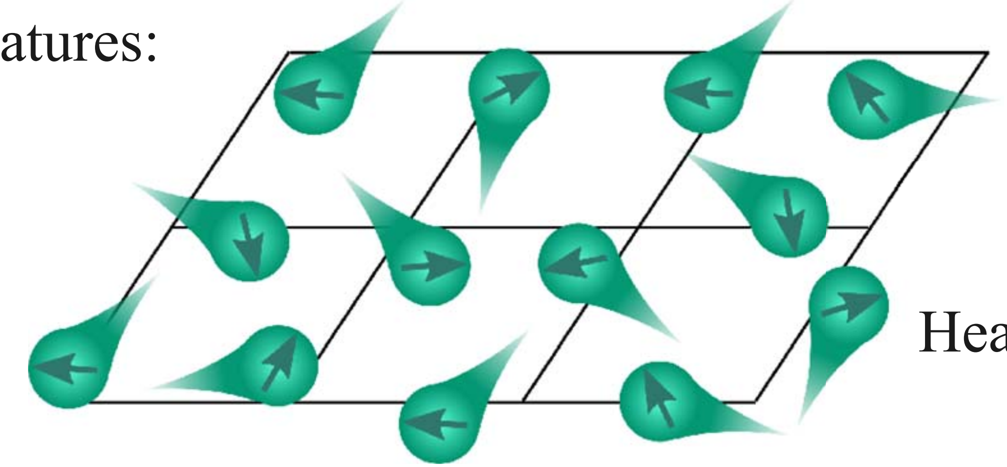
How do we get from here to heavy Cooper pairs?

Beal-Monod, Bourbonnais and Emery (1986)
Scalapino, Loh and Hirsch (1986)
Miyake, Schmitt-Rink and Varma (1986)

“Conventional” heavy fermion superconductivity

At lower temperatures:

$T < T^*$



Heavy quasiparticles

How do we get from here to heavy Cooper pairs?

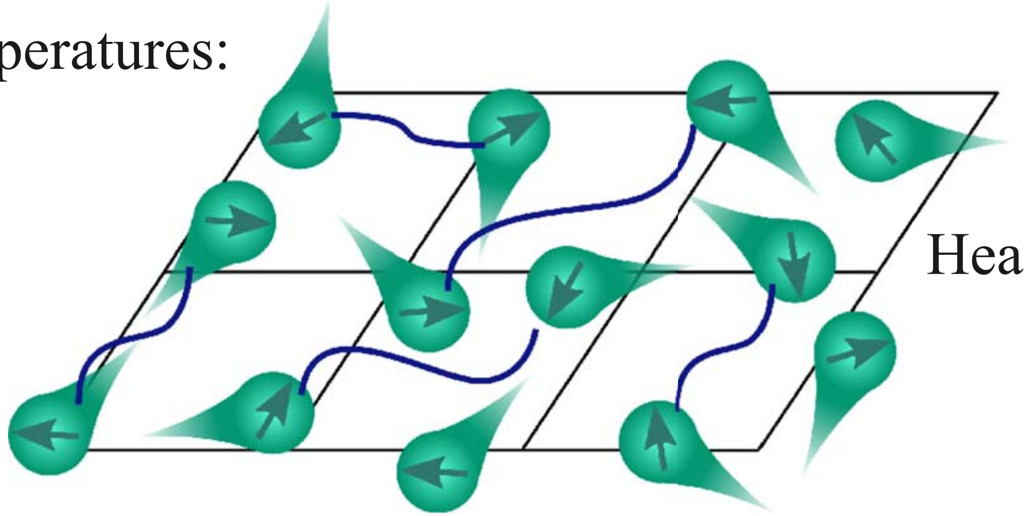
1. The local moments quench [via the Kondo effect], forming heavy quasiparticles

Beal-Monod, Bourbonnais and Emery (1986)
Scalapino, Loh and Hirsch (1986)
Miyake, Schmitt-Rink and Varma (1986)

“Conventional” heavy fermion superconductivity

At very low temperatures:

$T < T_C$



Heavy Cooper Pairs

How do we get from here to heavy Cooper pairs?

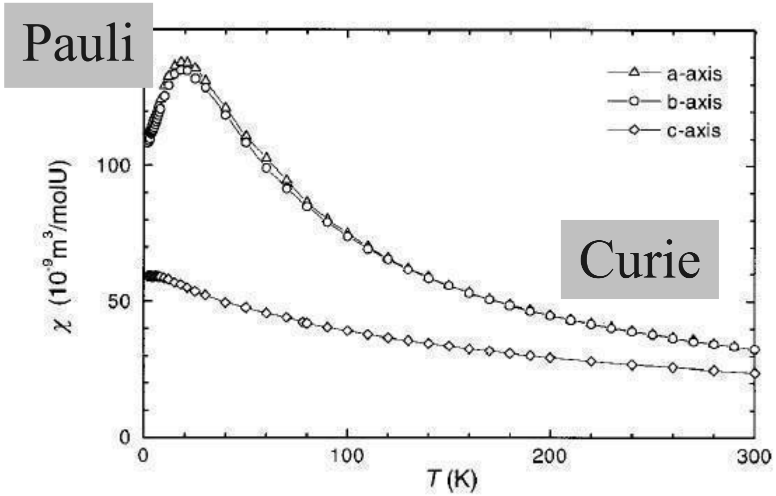
1. The local moments quench [via the Kondo effect], forming heavy quasiparticles

2. The heavy quasiparticles pair [via residual spin fluctuations]

These two stages are well separated.

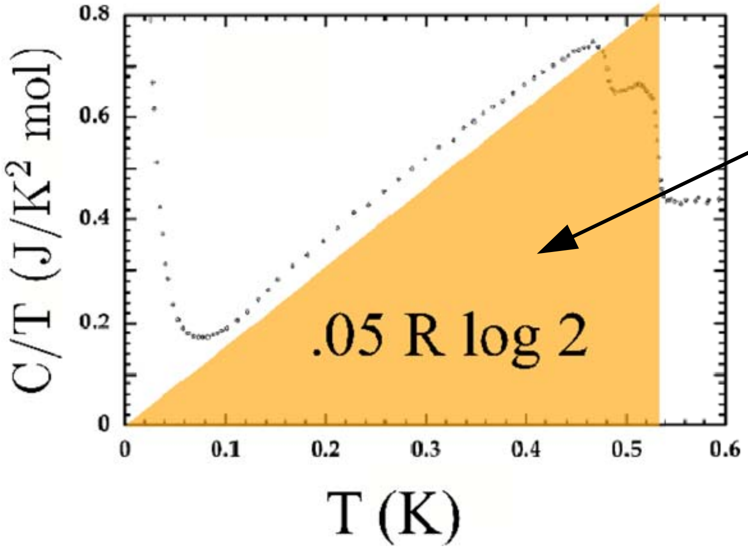
Beal-Monod, Bourbonnais and Emery (1986)
Scalapino, Loh and Hirsch (1986)
Miyake, Schmitt-Rink and Varma (1986)

UPt_3



Pauli paramagnetic by 30K

$T_c = 0.5K$

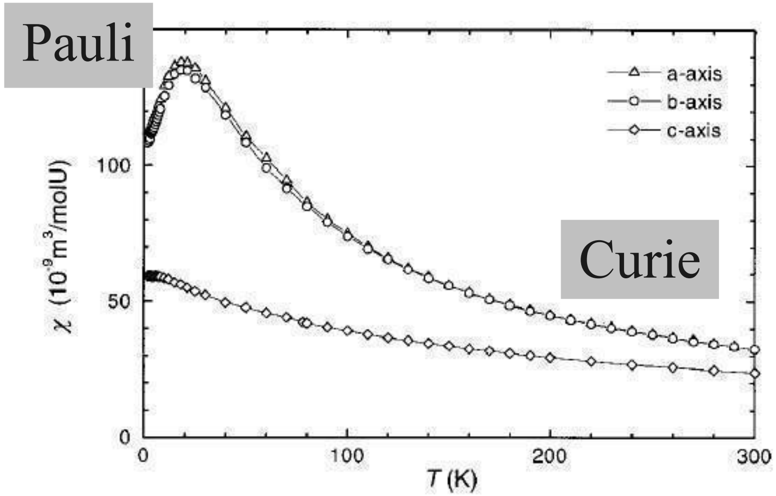


$$S = \int_0^T \frac{C(T')}{T'} dT'$$

Small condensation entropy
Spins are quenched
Two stages are well separated

Frings *et al.* J. Magn. Magn. Mater. **31**, 240(1983)
Brison *et al.* J. Low Temp. Phys. **95**, 145(1994)

UPt_3



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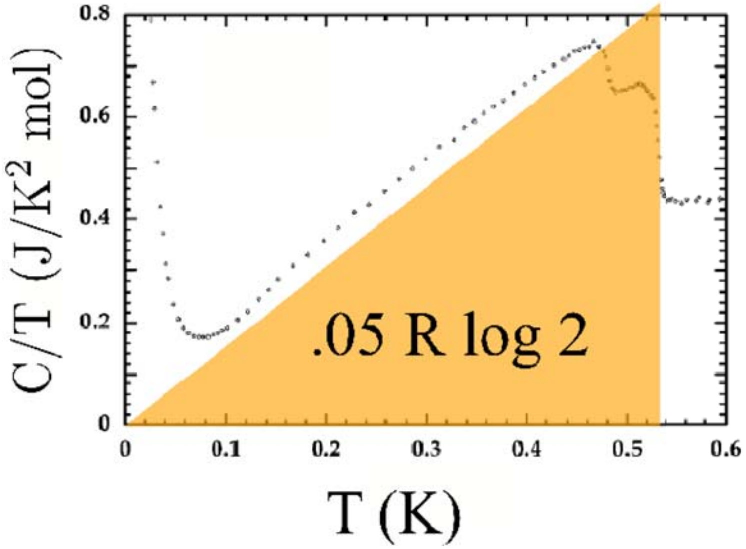

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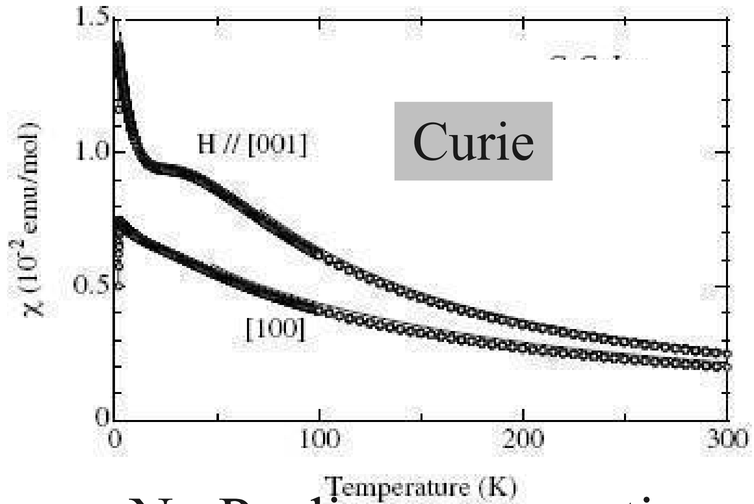
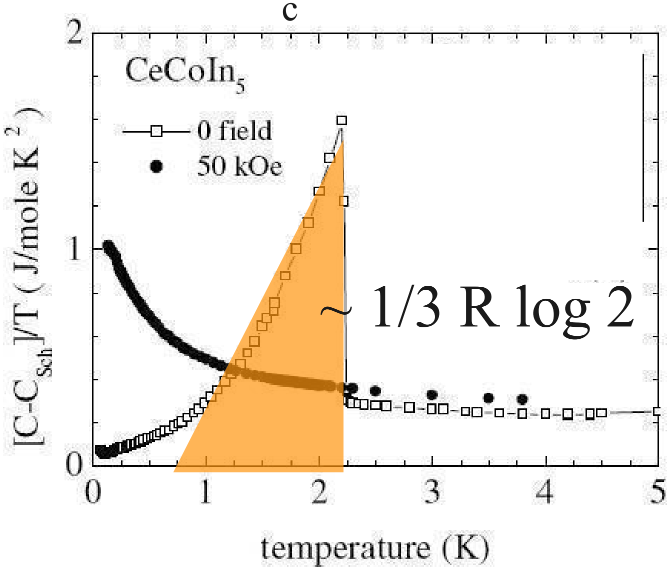


No Pauli paramagnetism

$T_c = 2.3K$



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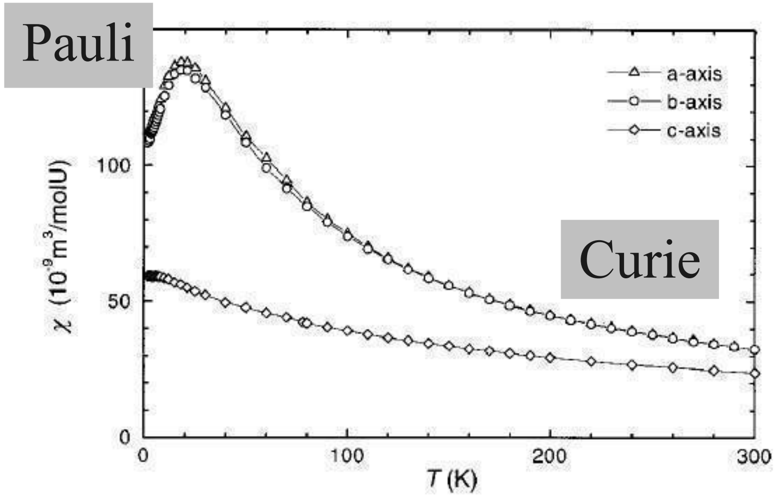
UPt_3



Pauli paramagnetic by 30K

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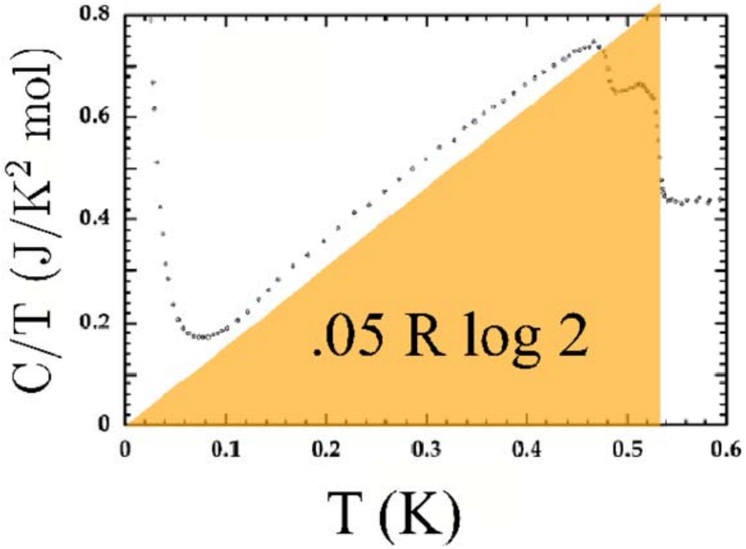

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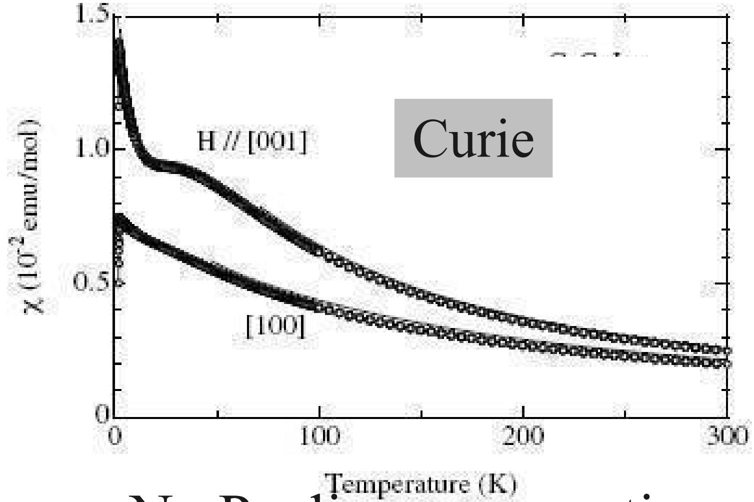
$CeCoIn_5$



No Pauli paramagnetism

$T_c = 2.3K$

Large condensation entropy
Spins quench directly into
the superconductor: **how?**

Shishido *et al.* JPSJ **71**, 162 (2002)
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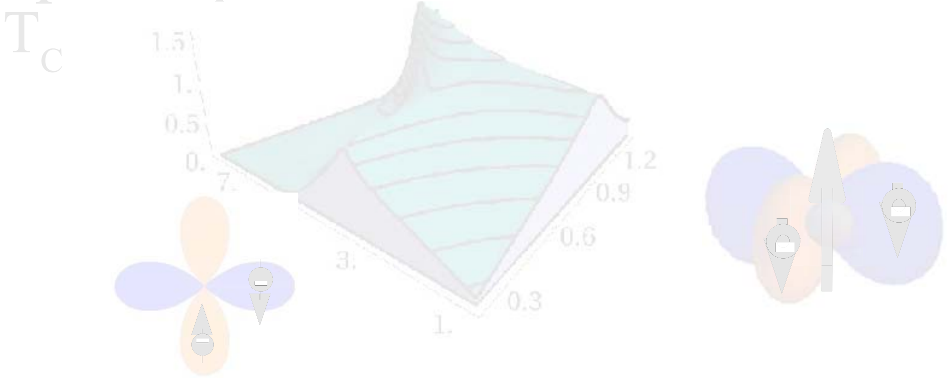
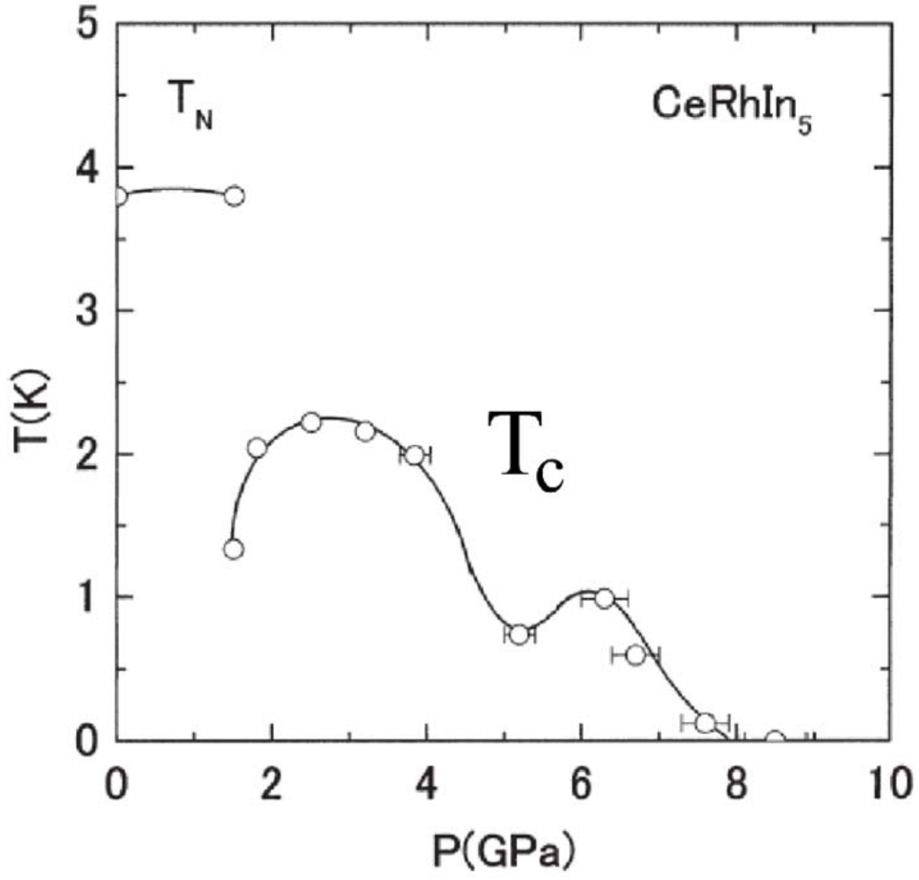
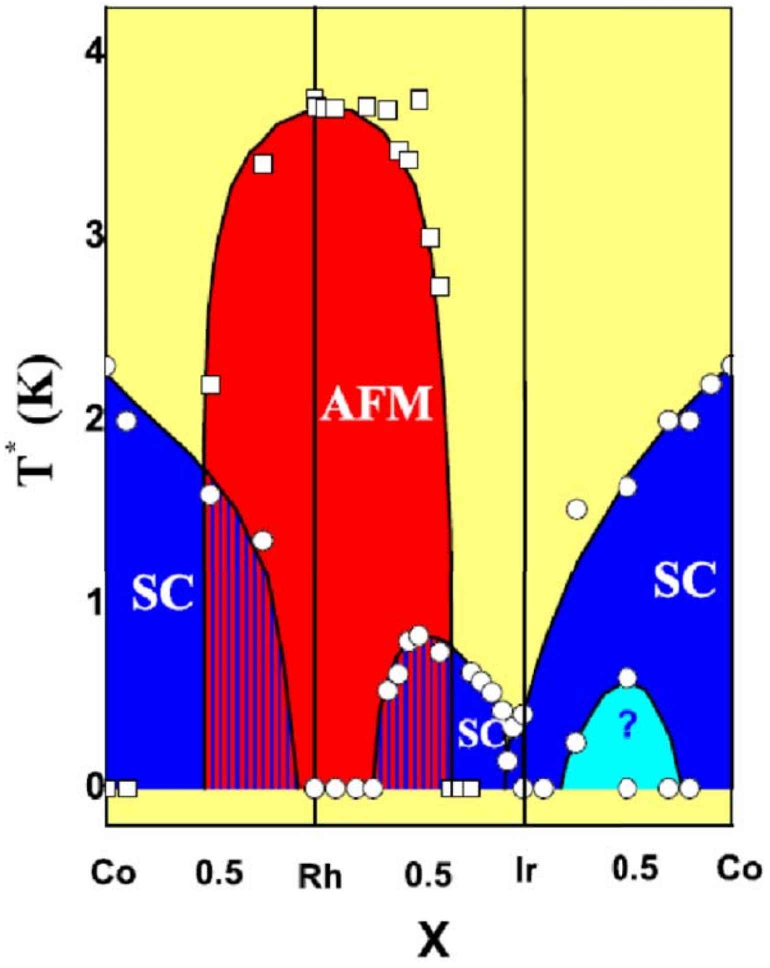


Internal structure of a heavy Cooper pair



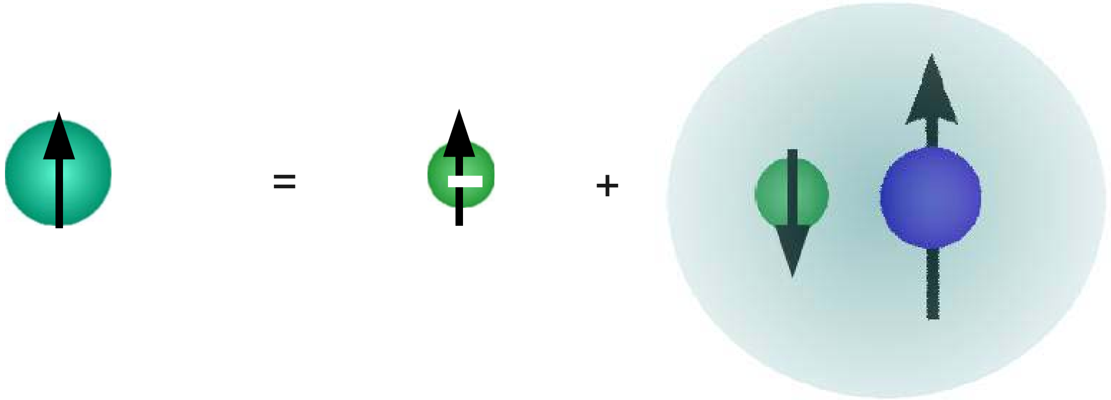


Muramatsu *et al.* JPSJ **70**, 3362(2001)

Sarrao and Thompson JPSJ **76**, 051013(2007)

Internal structure of a heavy quasiparticle

$$a_{k\uparrow}^{\dagger} = u_k c_{k\uparrow}^{\dagger} + v_k f_{k\uparrow}^{\dagger}$$



Heavy quasiparticles

Conduction electrons

Localized f-electrons

$$P_G f_{j\uparrow}^{\dagger} \sim c_{j\downarrow}^{\dagger} S_{j+}$$

Internal structure of a heavy Cooper pair

$$a^{\dagger}_{k\uparrow} = u_k c^{\dagger}_{k\uparrow} + v_k f^{\dagger}_{k\uparrow}$$

$$\Delta_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow}$$

Internal structure of a heavy Cooper pair

$$a^\dagger_{k\uparrow} = u_k c^\dagger_{k\uparrow} + v_k f^\dagger_{k\uparrow}$$

$$\Delta_{\mathbf{k}} \left(u_k c^\dagger_{k\uparrow} + v_k f^\dagger_{k\uparrow} \right) \left(u_k c^\dagger_{-k\downarrow} + v_k f^\dagger_{-k\downarrow} \right)$$

Internal structure of a heavy Cooper pair

$$\Delta_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow}$$

$$a^{\dagger}_{k\uparrow} = u_k c^{\dagger}_{k\uparrow} + v_k f^{\dagger}_{k\uparrow}$$

$$\sim \Delta^{cond}_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \Delta^{C}_{\mathbf{k}} \left(c^{\dagger}_{\mathbf{k}\uparrow} f^{\dagger}_{-\mathbf{k}\downarrow} + f^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \right) + \Delta^{M}_{\mathbf{k}} f^{\dagger}_{\mathbf{k}\uparrow} f^{\dagger}_{-\mathbf{k}\downarrow}$$

Composite pairs

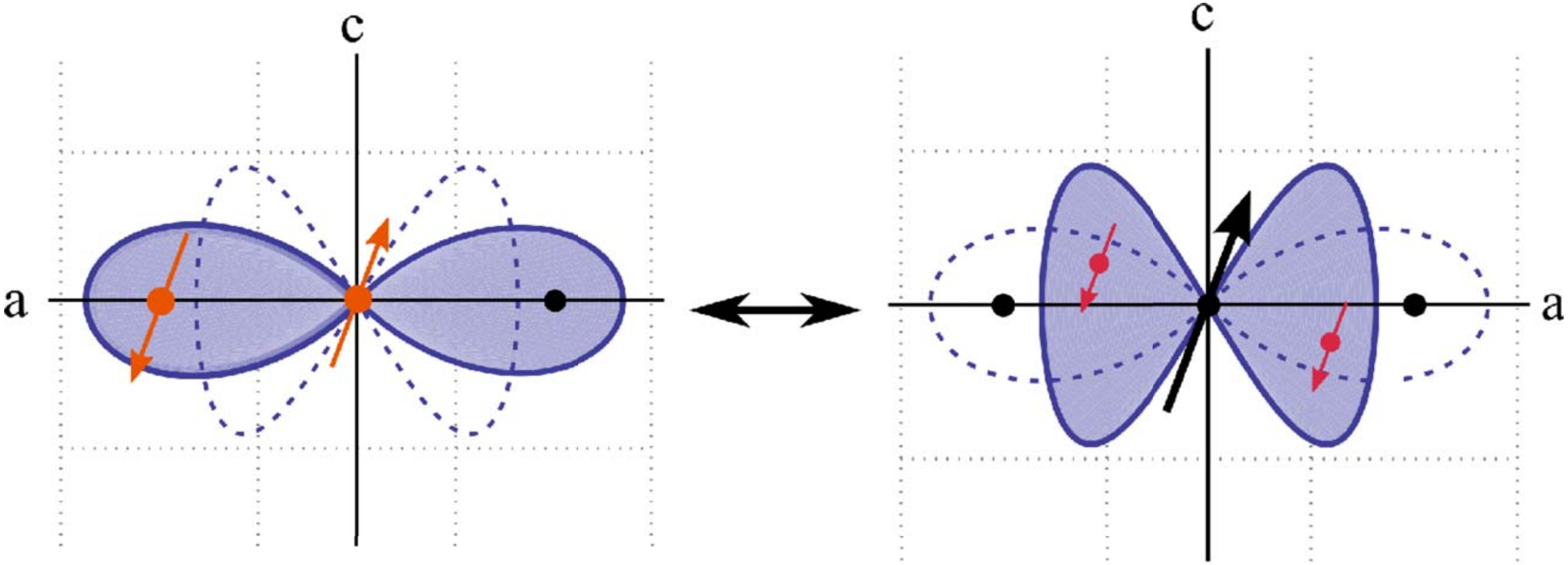
Magnetic pairs

Both magnetic and composite components will **always** be present

Internal structure of a heavy Cooper pair

$$\Delta_{\mathbf{k}} a^\dagger_{\mathbf{k}\uparrow} a^\dagger_{-\mathbf{k}\downarrow}$$

$$a^\dagger_{k\uparrow} = u_k c^\dagger_{k\uparrow} + v_k f^\dagger_{k\uparrow}$$



Magnetic Pair

$$\Delta^M_{ij} f^\dagger_{i\uparrow} f^\dagger_{j\downarrow}$$

Composite Pair

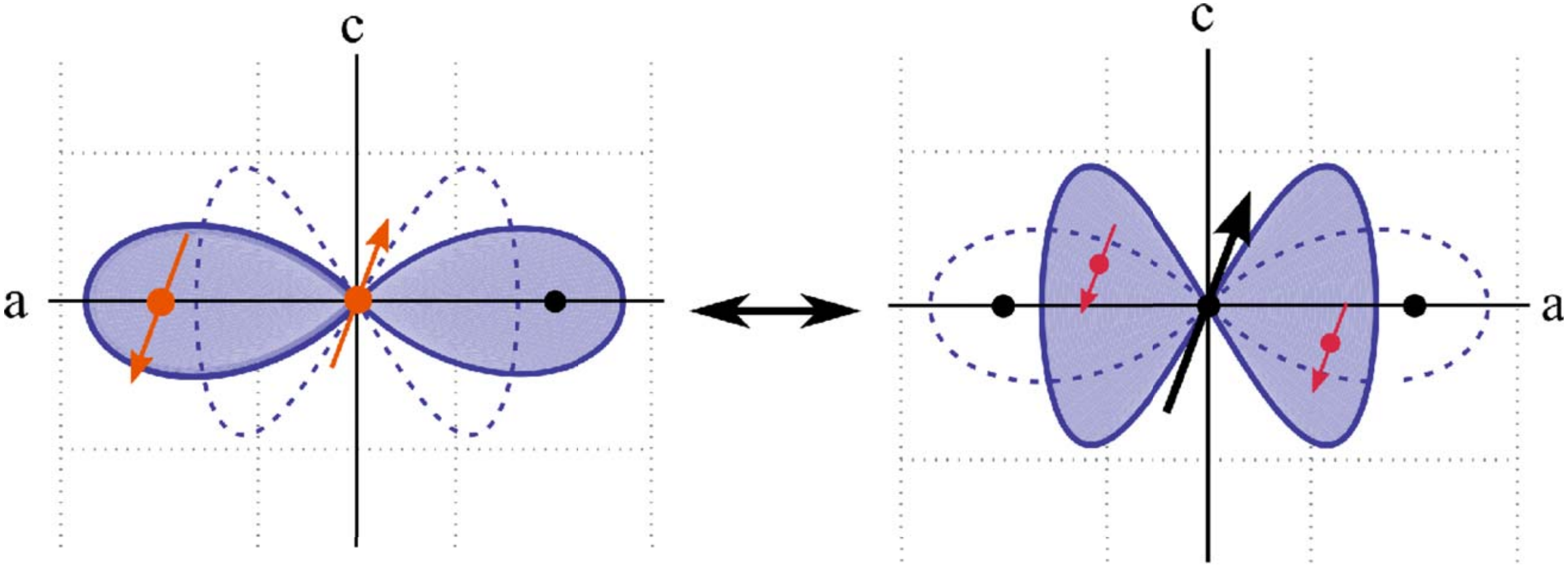
$$\Delta^C_{ij} c^\dagger_{i\uparrow} f^\dagger_{j\downarrow}$$

Two components, both with d-wave symmetry

Internal structure of a heavy Cooper pair

$$\Delta_{\mathbf{k}} a^{\dagger}_{\mathbf{k}\uparrow} a^{\dagger}_{-\mathbf{k}\downarrow}$$

$$a^{\dagger}_{k\uparrow} = u_k c^{\dagger}_{k\uparrow} + v_k f^{\dagger}_{k\uparrow}$$



Magnetic Pair

$$\Delta^{M}_{ij} (c^{\dagger}_{i\downarrow} S_{i+})(c^{\dagger}_{j\uparrow} S_{j-})$$

Favored by:
spin fluctuations

Composite Pair

$$\Delta^{C}_{ij} c^{\dagger}_{i\uparrow} (c^{\dagger}_{j\uparrow} S_{j-})$$

Favored by:
two channel Kondo physics

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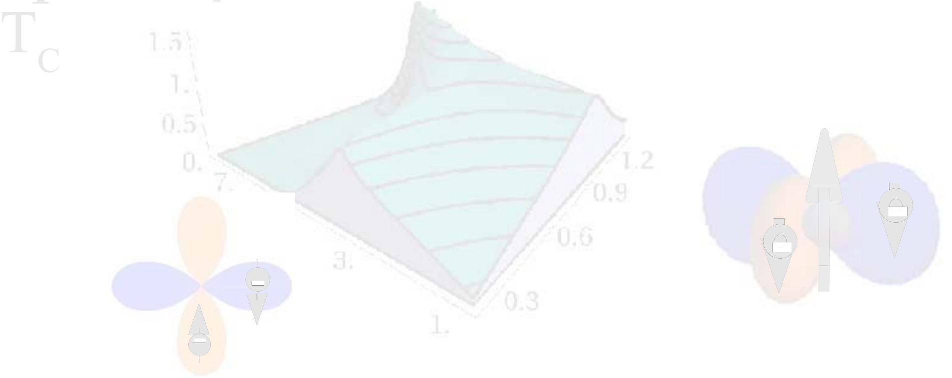
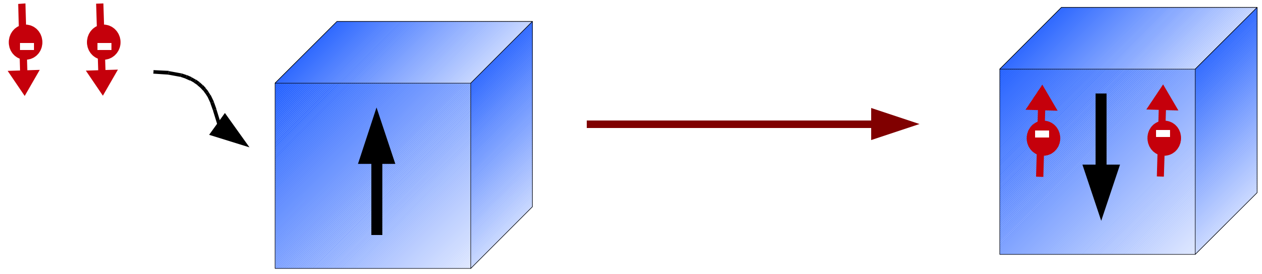


Composite Pairing

Copairing



$c^{\dagger}_{A\uparrow}c^{\dagger}_{B\uparrow}S_{-}$

Composite pair

The product of a conduction
electron pair and a spin flip

The composite pair is a **singlet**
– requiring a **triplet** pair of conduction electrons

→ Antisymmetric spatially – two electrons, two orthogonal channels

Abrahams, Balatsky, Scalapino, Schrieffer 1995

The Two Channel Kondo Model

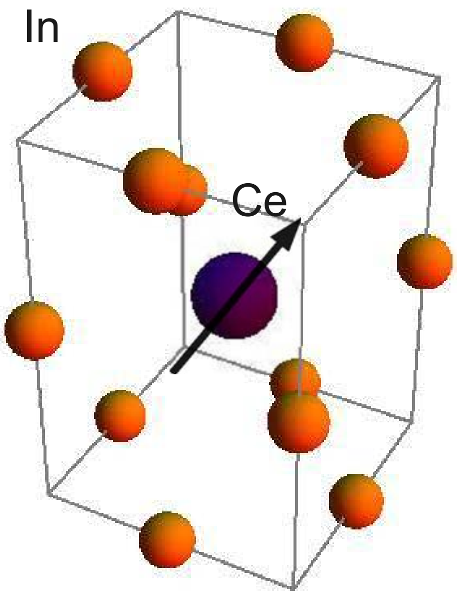
$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j + J_2 \sum_j \psi_{2j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2k\beta} \cdot \vec{S}_j$$

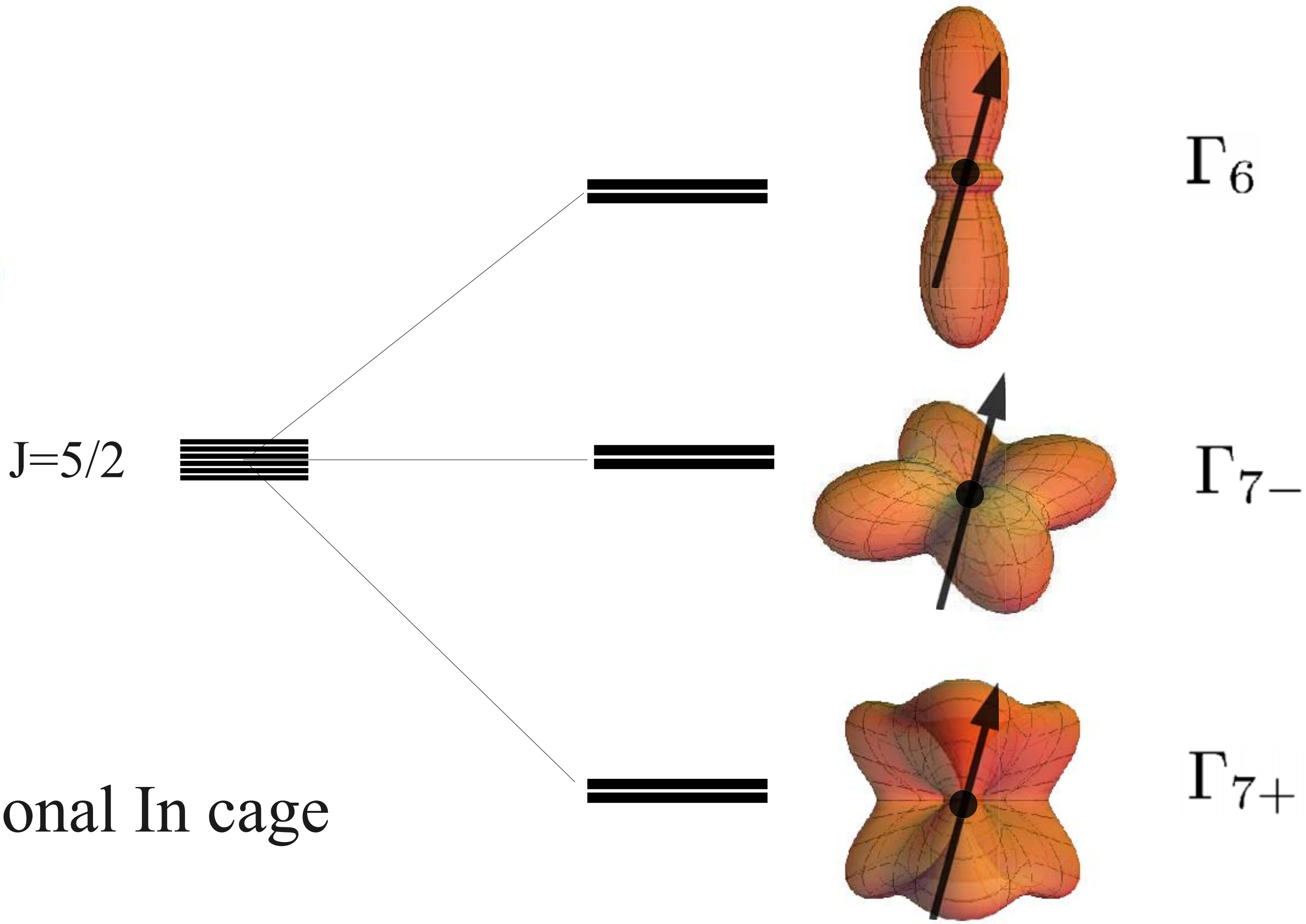
Wannier functions at site j:

$$\psi_{\Gamma j}^\dagger = \sum_k \Phi_{\Gamma k} \mathrm{e}^{i\vec{k}\cdot\vec{R}_j} c_k$$

The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2k\beta} \cdot \vec{S}_j$$





Ce^{3+}(4f^1) in tetragonal In cage

The Two Channel Kondo Model

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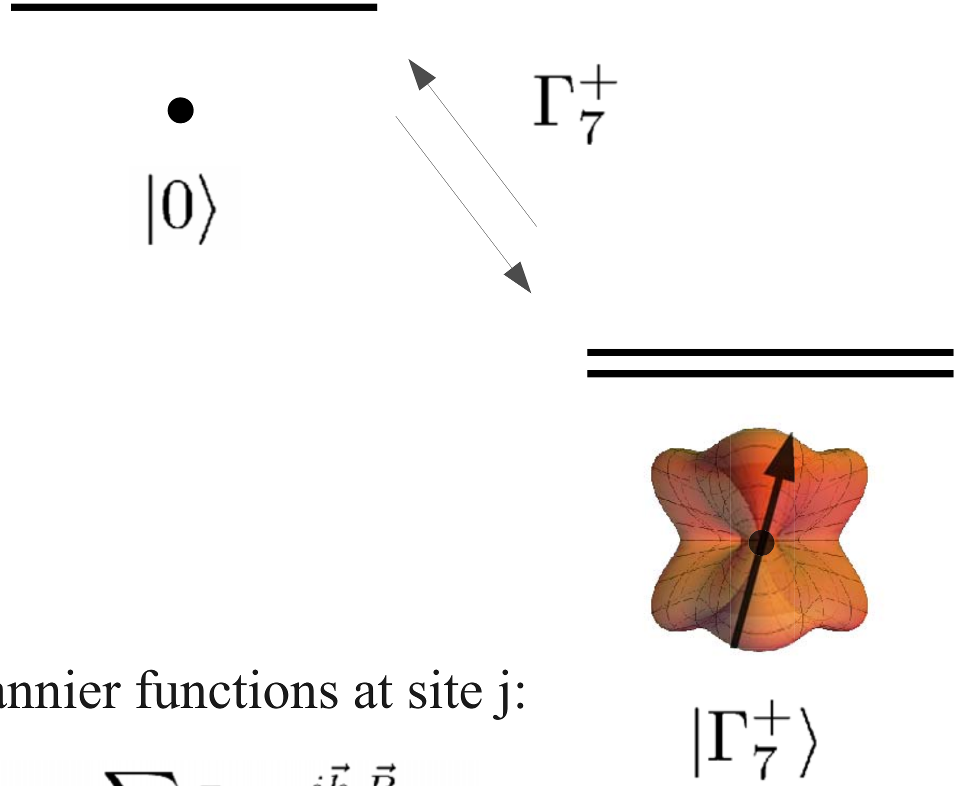

- $f^0 + e^- \rightleftharpoons f^1$

Wannier functions at site j:

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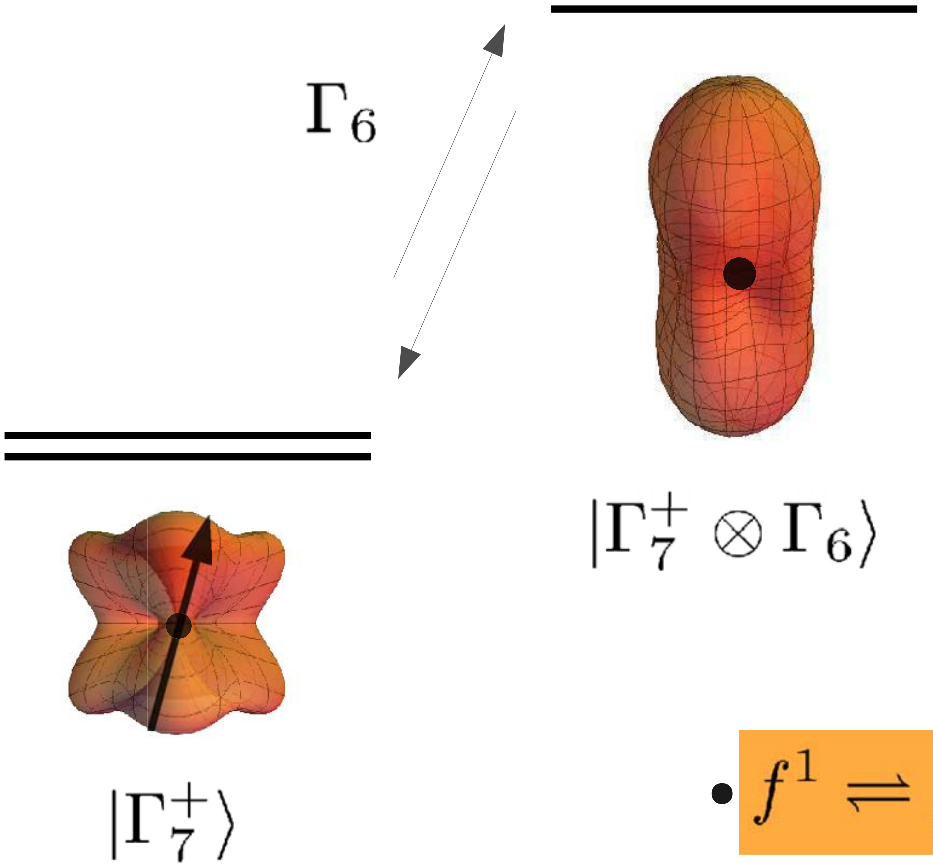
The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j + J_2 \sum_j \psi_{2j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2k\beta} \cdot \vec{S}_j$$



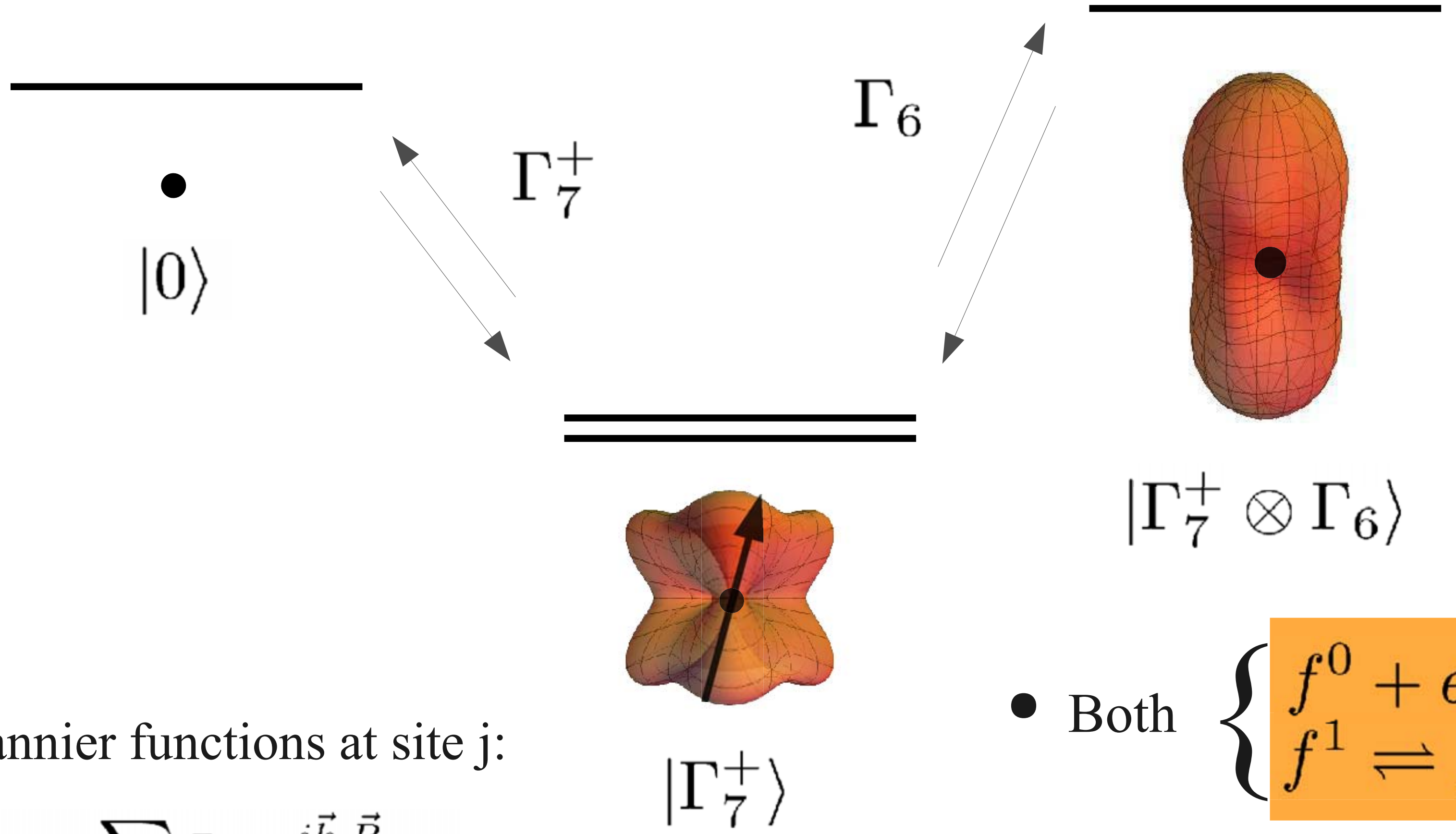
Wannier functions at site j:

$$\psi_{\Gamma j}^\dagger = \sum_k \Phi_{\Gamma k} \mathrm{e}^{i\vec{k}\cdot\vec{R}_j} c_k$$

- $f^1 \rightleftharpoons f^2 + h^+$

The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1k\beta}\cdot\vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2k\beta}\cdot\vec{S}_j$$



- Both $\left\{ \begin{array}{l} f^0 + e^- \rightleftharpoons f^1 \\ f^1 \rightleftharpoons f^2 + h^+ \end{array} \right.$

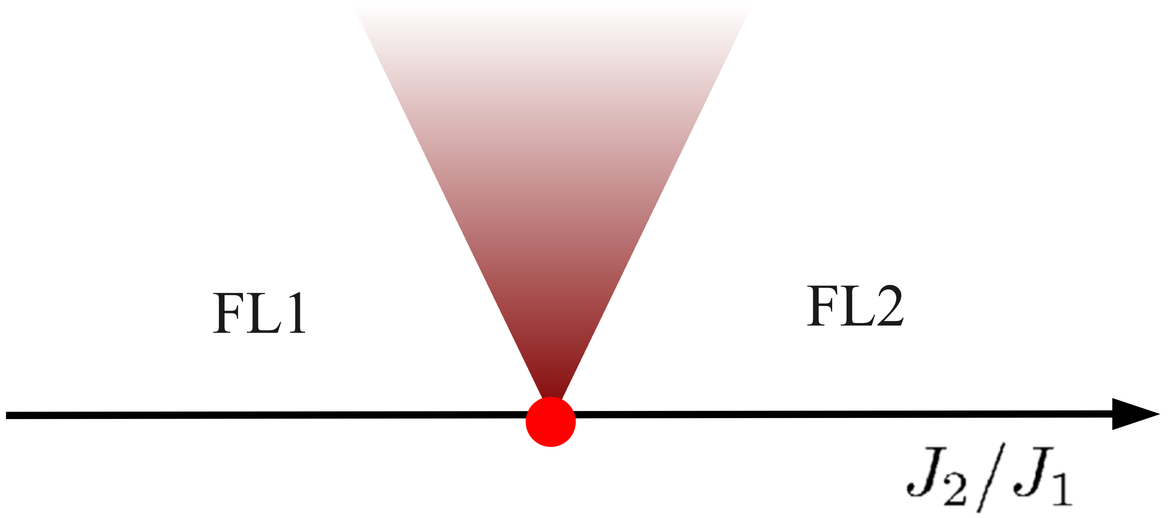
Wannier functions at site j:

$$\psi^\dagger_{\Gamma j} = \sum_k \Phi_{\Gamma k} e^{i\vec{k}\cdot\vec{R}_j} c_k$$

The Two Channel Kondo Model

The impurity has a quantum critical point for $J_1 = J_2$

Singular composite pair fluctuations
– expect to be hidden in the lattice

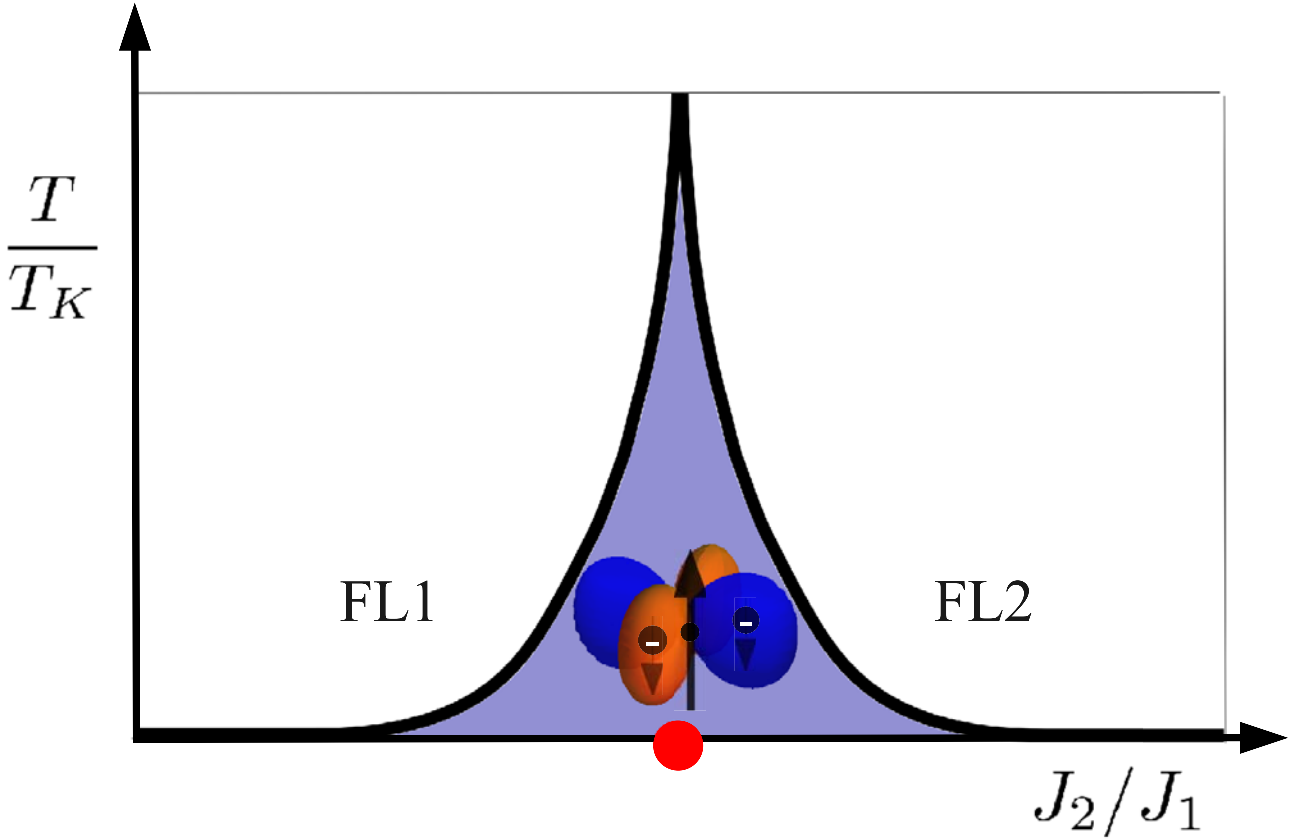


Emery and Kivelson 1992

Coleman, Andrei, Tsvelik and Kee 1999
Jarrell, Pang, and Cox 1997

The Two Channel Kondo Model

The impurity has a quantum critical point for J1 = J2



In the lattice, this QCP is thought to be avoided by composite pair superconductivity – I will show this in the large N limit

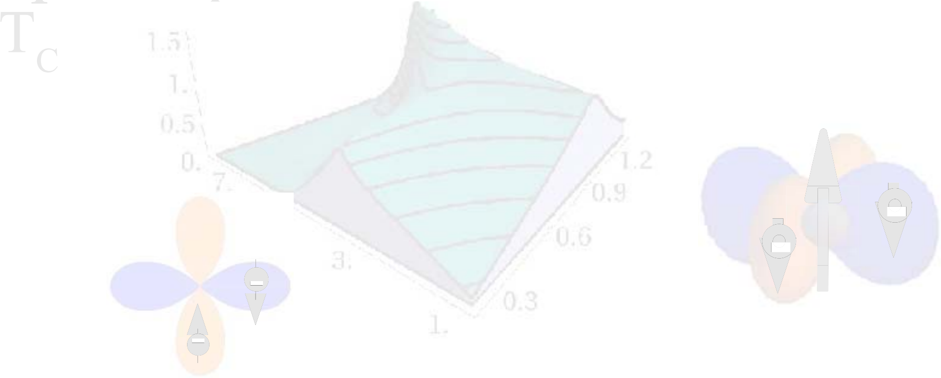
The Two Channel Kondo Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi_{1j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1k\beta} \cdot \vec{S}_j + J_2 \sum_j \psi_{2j\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2k\beta} \cdot \vec{S}_j$$

How can we solve this model?

Outline

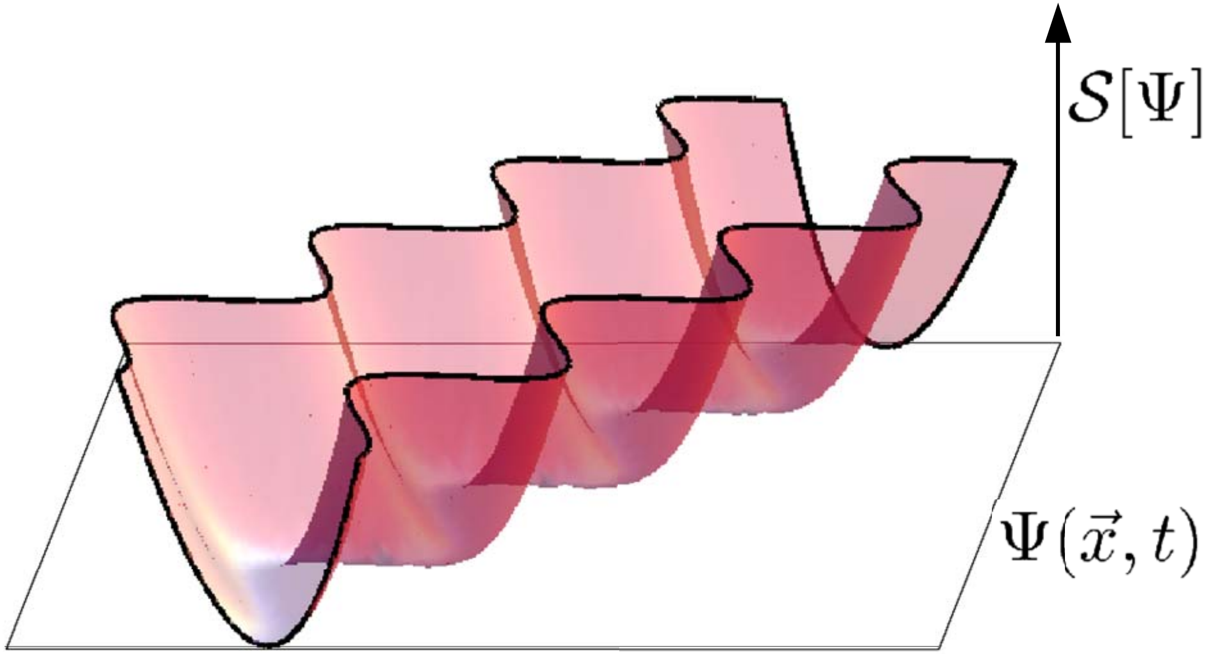
- The materials: how are the 115 superconductors special?
- How do spins form pairs?
  - Magnetically mediated pairing
  - Composite pairing
- The tool: symplectic-N
- Illustration: the two-channel Kondo-Heisenberg model
- Experimental consequences
  - Condensate quadrupole moment



How to solve a strongly correlated problem

The problem: no small parameters

$$\mathcal{Z} = \sum \mathrm{e}^{-\mathcal{S}[\Psi]}$$



The action fluctuates wildly

How to solve a strongly correlated problem

The problem: no small parameters

The solution: extend the spin group

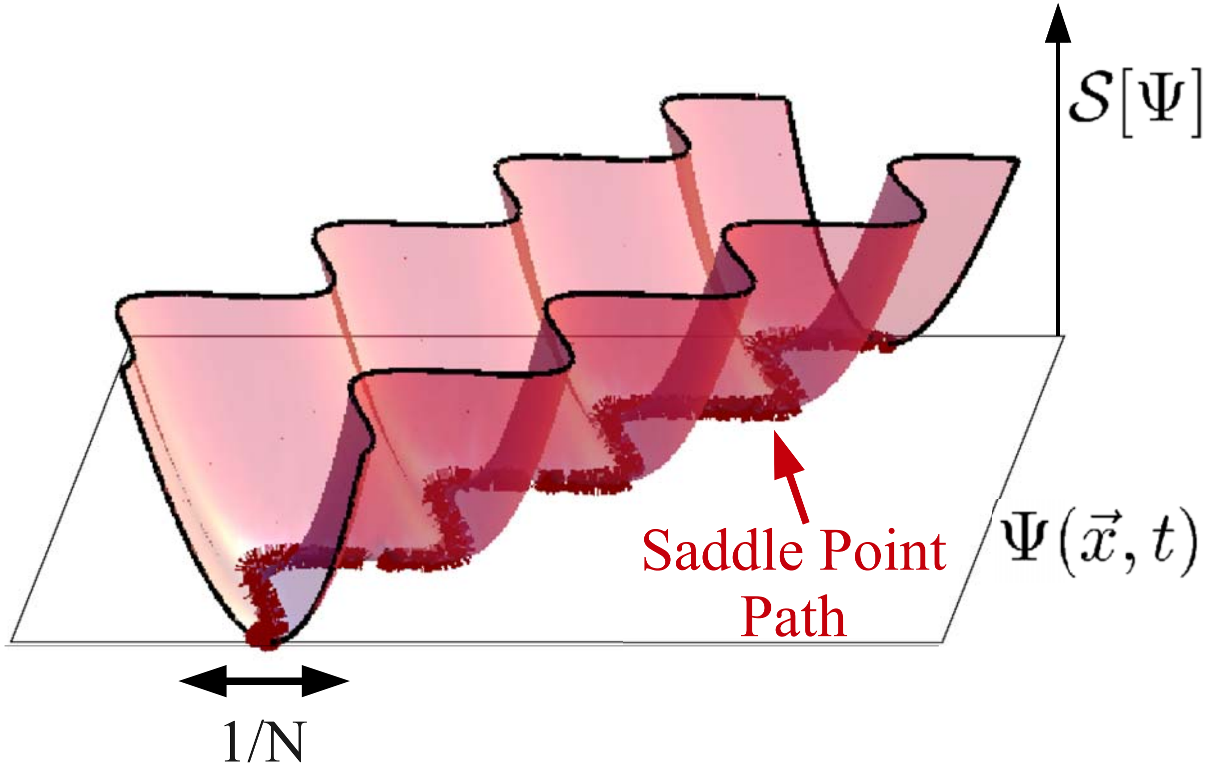
N = 2 $\sigma \in \{-\frac{1}{2}, \frac{1}{2}\}$

$$\Big\downarrow$$

Arbitrary N $\{-\frac{N}{2}, \frac{N}{2}\}$

1/N becomes a small parameter

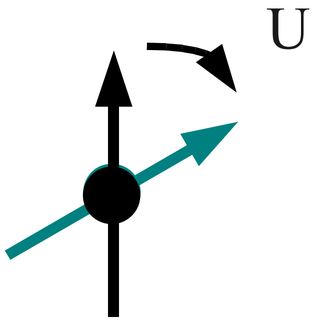
$$\mathcal{Z} = \sum e^{-N S[\Psi]}$$



How to choose the **right** large N limit

How do we ensure the large N limit captures the essential physics?

What defines a spin?



Continuous spin rotations

$$U = \exp i \frac{\vec{\alpha}}{2} \cdot \vec{\sigma}$$

Pauli
Matrices

How to choose the **right** large N limit

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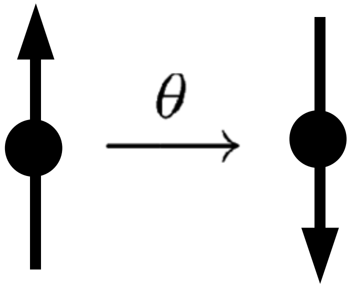

Continuous spin rotations

$$U = \exp i\frac{\vec{\alpha}}{2}\cdot\vec{\sigma}$$

Time reversal Symmetry

$$\vec{S} \xrightarrow{\theta} -\vec{S}$$

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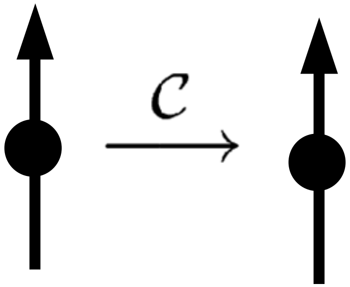

Continuous spin rotations

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Neutrality

$$\vec{S} \xrightarrow{\mathcal{C}} +\vec{S}$$

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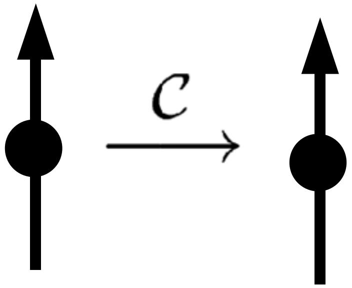

Continuous spin rotations

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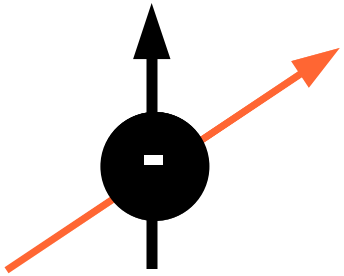
SU(N) misses these discrete symmetries

How to choose the **right** large N limit

How do we ensure the large N limit captures the essential physics?

Time reversal is essential to Cooper pairing:



$|\vec{p},\uparrow\rangle$

Continuous spin rotations

$$U = \exp i\frac{\vec{\alpha}}{2}\cdot\vec{\sigma}$$

Time reversal Symmetry

$$\vec{S} \xrightarrow{\theta} -\vec{S}$$

Neutrality

$$\vec{S} \xrightarrow{C} +\vec{S}$$

Symplectic-N includes these discrete symmetries

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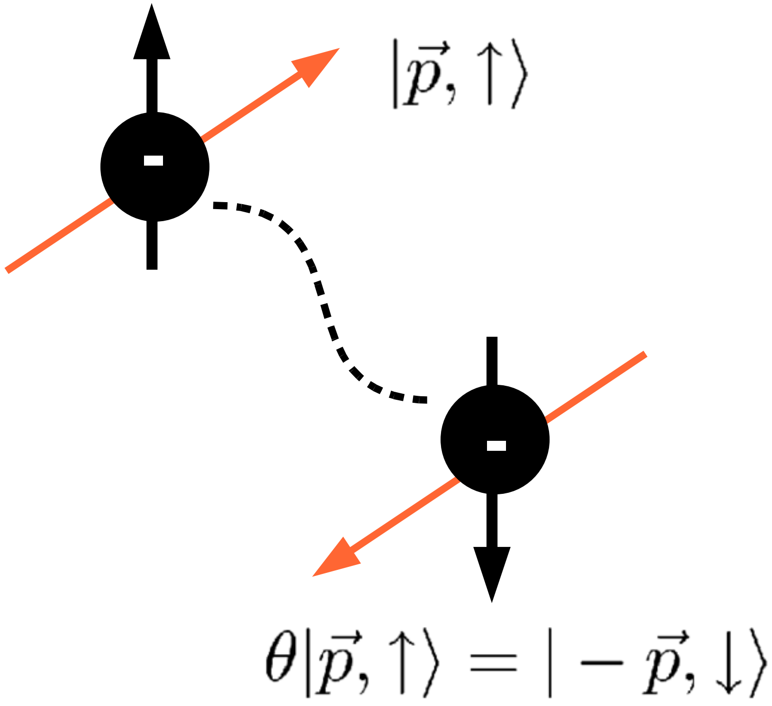


Continuous spin rotations

$$U = \exp i\frac{\vec{\alpha}}{2}\cdot\vec{\sigma}$$

Time reversal Symmetry

$$\vec{S} \xrightarrow{\theta} -\vec{S}$$

Neutrality

$$\vec{S} \xrightarrow{C} +\vec{S}$$

Symplectic-N includes these discrete symmetries

Discrete Symmetries and the Symplectic condition

Continuous spin rotations:

$$U = \exp i \frac{\vec{\alpha}}{2} \cdot \vec{\sigma}$$

Pauli Matrices

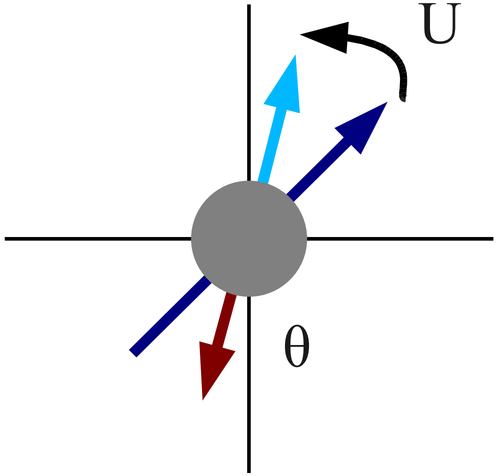
Discrete time-reversal symmetry:

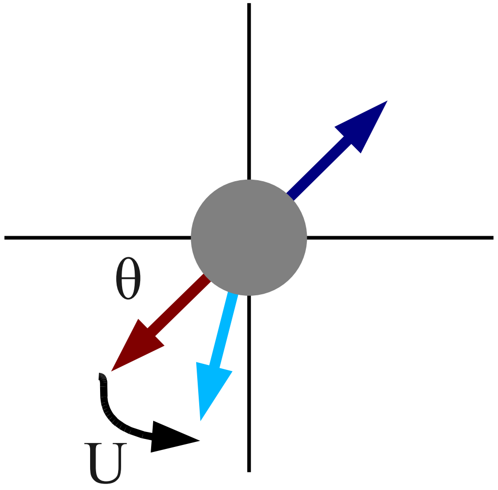
$$\theta = \hat{\epsilon} K \qquad \hat{\epsilon} = i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Complex conjugation

A consistent definition of time reversal requires:

$$U\theta = \theta U$$





This is the symplectic condition: $U \hat{\epsilon} U^T = \hat{\epsilon}$

$$SU(2) \cong SP(2)$$

Symmetries and Large N

The SU(N) spins can be written:

ψ can be either fermionic or bosonic

$$T_{\alpha\beta} = \psi^\dagger_\alpha \psi_\beta - \frac{n_\psi}{N}$$

$$U = \exp i \frac{\vec{\alpha}}{2} \cdot \vec{T} \longrightarrow \begin{cases} U\hat{\epsilon}U^T \neq \hat{\epsilon} \\ \boxed{U\hat{\epsilon}U^T = \hat{\epsilon}} \end{cases}$$

SP(N)

The symplectic spins are given by:

$$S_{\alpha\beta} = \psi^\dagger_\alpha \psi_\beta - \tilde{\alpha}\tilde{\beta}\psi^\dagger_{-\beta}\psi_{-\alpha}$$

Time Reversal — Invert

Charge Conjugation — Neutral

$$\tilde{\alpha} = \text{sign}(\alpha)$$

The one channel Kondo model in Symplectic-N

Use symplectic spins to decouple:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

"Hybridization"

"Pairing"

$$S_{\alpha\beta} = f^\dagger_\alpha f_\beta - \tilde{\alpha}\tilde{\beta}f^\dagger_{-\beta}f_{-\alpha}$$

The one channel Kondo model in Symplectic-N

Use symplectic spins to decouple:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right) + \left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

"Hybridization"

"Pairing"

Does symplectic-N reproduce
the SU(N) one channel Kondo Lattice model?

The one channel Kondo model in Symplectic-N

Again, two quartic terms:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\xrightarrow[\text{Hubbard-Stratonovich}]{} \left[\left(f^\dagger_\alpha V_\Gamma+\tilde{\alpha} f_{-\alpha}\Delta_\Gamma\right)\psi_{\Gamma\alpha}+\text{h.c.}\right]+N\left(\frac{|V_\Gamma|^2+|\Delta_\Gamma|^2}{J_\Gamma}\right)$$

$\uparrow$ “Hybridization” $\uparrow$ “Pairing”

The one channel Kondo model in Symplectic-N

Again, two quartic terms:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\underbrace{\left(f^\dagger_\alpha V_\Gamma + \tilde{\alpha} f_{-\alpha}\Delta_\Gamma\right)}_{}\psi_{\Gamma\alpha} + \text{h.c.}\right] + N\left(\frac{|V_\Gamma|^2+|\Delta_\Gamma|^2}{J_\Gamma}\right)$$

Superconductivity?!

The one channel Kondo model in Symplectic-N

Again, two quartic terms:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

"Hybridization"

"Pairing"

Aside: Continuous particle-hole symmetry

$$f^\dagger_\alpha \longrightarrow \cos\theta f^\dagger_\alpha + \sin\theta\tilde{\alpha}f_{-\alpha}$$

Reflects the neutrality of the spin

N= 2:
Affleck, Zou, Hsu and Anderson 1988

$$S_{\alpha\beta} = f^\dagger_\alpha f_\beta - \tilde{\alpha}\tilde{\beta}f^\dagger_{-\beta}f_{-\alpha}$$

The one channel Kondo model in Symplectic-N

Again, two quartic terms:

$$\sum_j \frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\underbrace{\left(f^\dagger_\alpha V_\Gamma+\tilde{\alpha}f_{-\alpha}\Delta_\Gamma\right)}_{\tilde{V}_\Gamma\tilde{f}^\dagger_\alpha}\psi_{\Gamma\alpha}+\mathrm{h.c.}\right]+N\left(\frac{|V_\Gamma|^2+|\Delta_\Gamma|^2}{J_\Gamma}\right)$$

Fix SU(2) gauge

Superconductivity?!

No.

$$\tilde{V}_\Gamma=\sqrt{|V_\Gamma|^2+|\Delta_\Gamma|^2}$$

The one channel Kondo model in Symplectic-N

Again, two quartic terms:

$$\sum_j \frac{J_1}{N}\psi^\dagger_{j1}\vec{\sigma}\psi_{j1}\cdot\vec{S}_j = -\frac{J_1}{N}\left[\left(\psi_1^\dagger f\right)\left(f^\dagger\psi_1\right)+\left(\psi_1^\dagger\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_1\right)\right]$$

$$\longrightarrow \left[\tilde{V}_1\tilde{f}^\dagger_\alpha\psi_{\Gamma\alpha}+\mathrm{h.c.}\right]+N\left(\frac{|\tilde{V}_1|^2}{J_1}\right)$$

Recover the SU(N) one-channel Kondo model

Newns and Read 1983
Auerbach and Levin 1986

The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\tilde{V}_1 \tilde{f}^\dagger_\alpha \psi_{\Gamma\alpha} + \text{h.c.}\right] + N\left(\frac{|\tilde{V}_1|^2}{J_1}\right)$$

$$+ \quad \left[\left(f^\dagger_\alpha\psi_{2\alpha}\right)V_2 + \left(\tilde{\alpha} f_{-\alpha}\psi_{2\alpha}\right)\Delta_2 + \text{h.c.}\right] + N\left(\frac{|V_2|^2+|\Delta_2|^2}{J_2}\right)$$

Neglect

The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right) + \left(\psi^\dagger_\Gamma \hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\underset{\substack{\uparrow\\ \text{Hybridization}}}{\left(f^\dagger_\alpha \psi_{2\alpha}\right)} V_2 + \underset{\substack{\uparrow\\ \text{Pairing}}}{\left(\tilde{\alpha} f_{-\alpha}\psi_{2\alpha}\right)} \Delta_2 + \text{h.c.}\right] + N\left(\frac{|V_2|^2+|\Delta_2|^2}{J_2}\right)$$

The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j}\frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma}\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\left(f^\dagger_\alpha \underset{\overset{\uparrow}{\text{Hybridization}}}{\psi_{1\alpha}}\right)V_1+\left(\tilde{\alpha}\underset{\overset{\uparrow}{\text{Pairing}}}{f_{-\alpha}}\psi_{2\alpha}\right)\Delta_2+\text{h.c.}\right]+N\left(\frac{|V_1|^2}{J_1}+\frac{|\Delta_2|^2}{J_2}\right)$$

Superconductivity requires both channels:

SU(2) gauge invariant order parameter

$$V_1\Delta_2-\Delta_1V_2$$

The two channel Kondo model in Symplectic-N

Now include both channels:

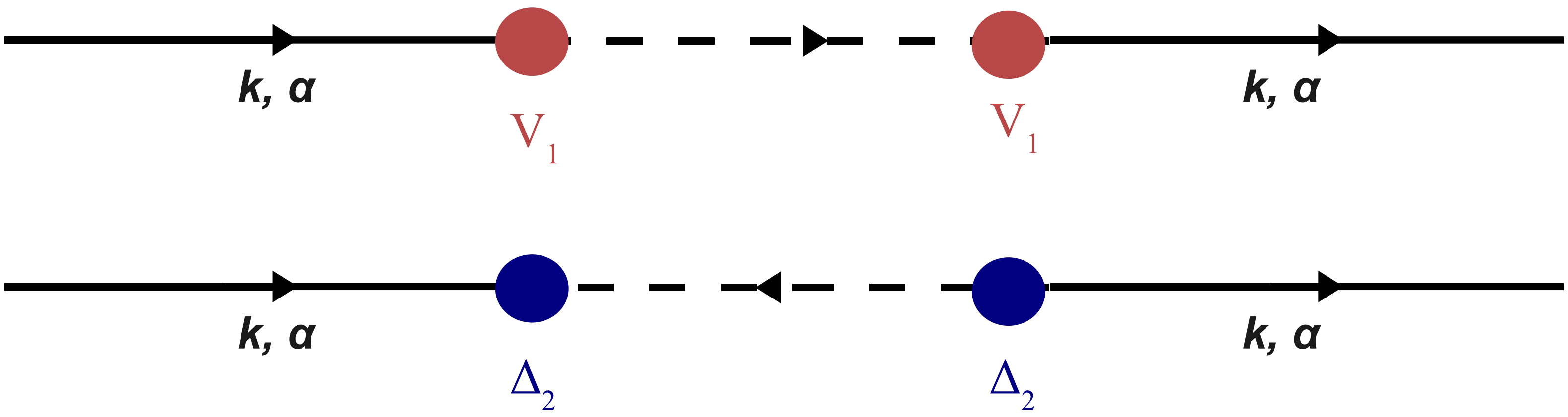
$$\sum_{\Gamma j}\frac{J_\Gamma}{N}\psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma}\frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right)+\left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\left(f^\dagger_\alpha\psi_{1\alpha}\right)V_1+\left(\tilde{\alpha}f_{-\alpha}\psi_{2\alpha}\right)\Delta_2+\text{h.c.}\right]+N\left(\frac{|V_1|^2}{J_1}+\frac{|\Delta_2|^2}{J_2}\right)$$

Hybridization

Pairing

Kondo scattering is an intra-channel effect:



The two channel Kondo model in Symplectic-N

Now include both channels:

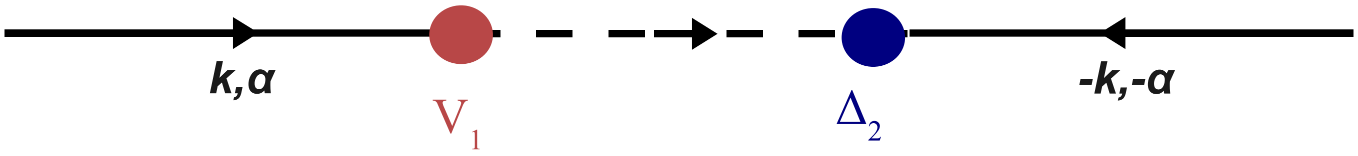
$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma} \vec{\sigma} \psi_{j\Gamma} \cdot \vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N} \left[\left(\psi^\dagger_\Gamma f \right) \left(f^\dagger \psi_\Gamma \right) + \left(\psi^\dagger_\Gamma \hat{\epsilon}^\dagger f^\dagger \right) \left(f \hat{\epsilon} \psi_\Gamma \right) \right]$$

$$\longrightarrow \left[\left(f^\dagger_\alpha \psi_{1\alpha} \right) V_1 + \left(\tilde{\alpha} f_{-\alpha} \psi_{2\alpha} \right) \Delta_2 + \mathrm{h.c.} \right] + N \left(\frac{|V_1|^2}{J_1} + \frac{|\Delta_2|^2}{J_2} \right)$$

Hybridization

Pairing

Superconductivity is inter-channel:



Arises from **resonant Andreev scattering**

The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_\Gamma \frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right) + \left(\psi^\dagger_\Gamma \hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\left(\underset{\substack{\uparrow \\ \text{Hybridization}}}{f^\dagger_\alpha\psi_{1\alpha}}\right)V_1 + \left(\underset{\substack{\uparrow \\ \text{Pairing}}}{\tilde{\alpha} f_{-\alpha}\psi_{2\alpha}}\right)\Delta_2 + \text{h.c.}\right] + N\left(\frac{|V_1|^2}{J_1} + \frac{|\Delta_2|^2}{J_2}\right)$$

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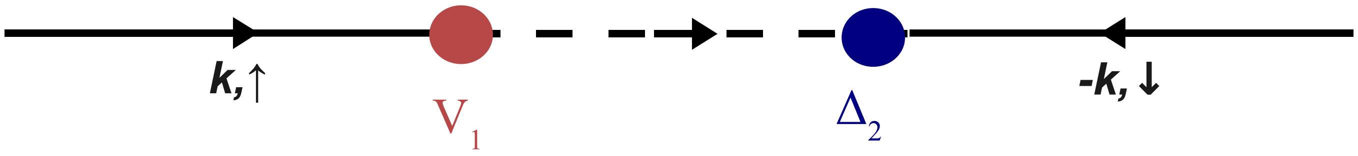
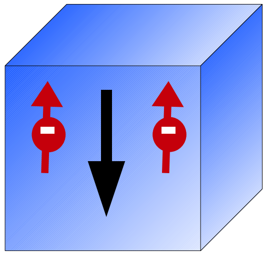


$\psi_{1\uparrow} f^\dagger_\uparrow$ $\qquad$ $f_\downarrow \psi_{2\uparrow}$



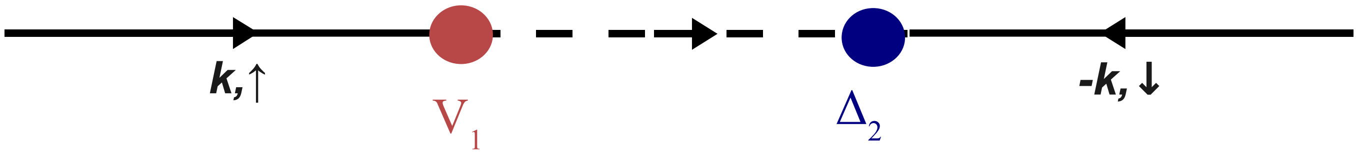
The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right) + \left(\psi^\dagger_\Gamma\hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

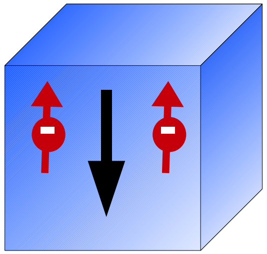
$$\longrightarrow \left[\underset{\substack{\uparrow\\ \text{Hybridization}}}{\left(f^\dagger_\alpha\psi_{1\alpha}\right)} V_1 + \underset{\substack{\uparrow\\ \text{Pairing}}}{\left(\tilde{\alpha} f_{-\alpha}\psi_{2\alpha}\right)}\Delta_2 + \text{h.c.}\right] + N\left(\frac{|V_1|^2}{J_1}+\frac{|\Delta_2|^2}{J_2}\right)$$

Superconductivity is inter-channel:



Annihilating a
composite pair:

$$\psi_{1\uparrow}\underbrace{f^\dagger_\uparrow f_\downarrow}_{S_+}\psi_{2\uparrow}$$



The two channel Kondo model in Symplectic-N

Now include both channels:

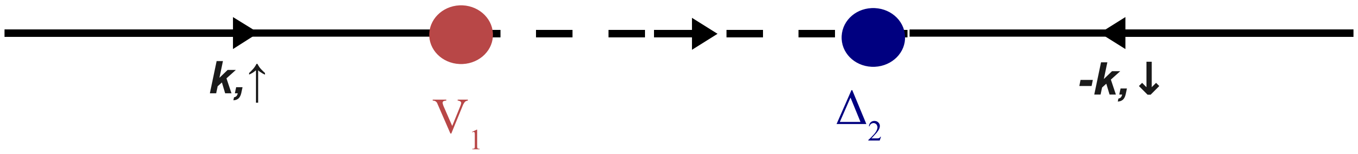
$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma}\vec{\sigma}\psi_{j\Gamma}\cdot\vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N}\left[\left(\psi^\dagger_\Gamma f\right)\left(f^\dagger\psi_\Gamma\right) + \left(\psi^\dagger_\Gamma \hat{\epsilon}^\dagger f^\dagger\right)\left(f\hat{\epsilon}\psi_\Gamma\right)\right]$$

$$\longrightarrow \left[\left(f^\dagger_\alpha \psi_{1\alpha}\right) V_1 + \left(\tilde{\alpha} f_{-\alpha}\psi_{2\alpha}\right)\Delta_2 + \mathrm{h.c.}\right] + N\left(\frac{|V_1|^2}{J_1} + \frac{|\Delta_2|^2}{J_2}\right)$$

Hybridization

Pairing

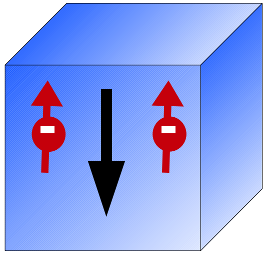
Superconductivity is inter-channel:



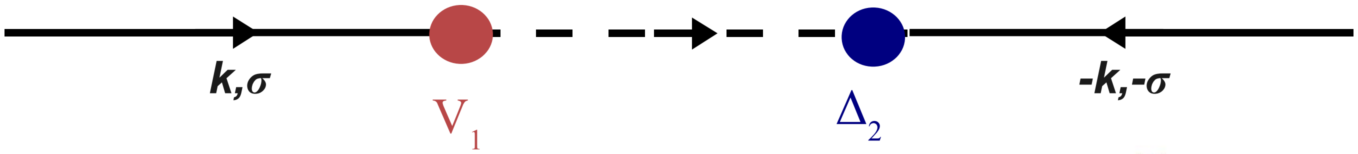
Annihilating a **composite pair**:

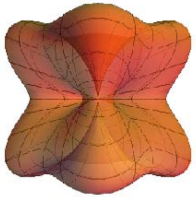
$$\psi_1\vec{\sigma}\hat{\epsilon}\psi_2\cdot\vec{S}$$

SU(2) invariant form

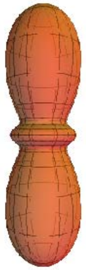


Angular momentum and Composite pairing



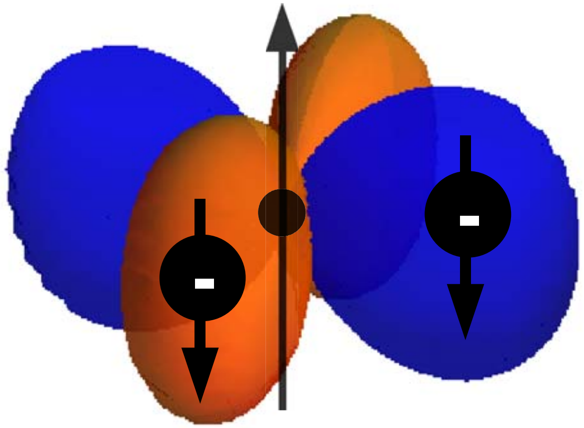


$$\Delta l = 2$$



$$\sim |\pm\frac{5}{2}\rangle$$

$$|\pm\frac{1}{2}\rangle$$



d-wave gap originates from the crystal fields

The two channel Kondo model in Symplectic-N

Now include both channels:

$$\sum_{\Gamma j} \frac{J_\Gamma}{N} \psi^\dagger_{j\Gamma} \vec{\sigma} \psi_{j\Gamma} \cdot \vec{S}_j = -\sum_{\Gamma} \frac{J_\Gamma}{N} \left[\left(\psi^\dagger_\Gamma f \right) \left(f^\dagger \psi_\Gamma \right) + \left(\psi^\dagger_\Gamma \hat{\epsilon}^\dagger f^\dagger \right) \left(f \hat{\epsilon} \psi_\Gamma \right) \right]$$

$$\longrightarrow \left[\left(f^\dagger_\alpha \psi_{1\alpha} \right) V_1 + \left(\tilde{\alpha} f_{-\alpha} \psi_{2\alpha} \right) \Delta_2 + \mathrm{h.c.} \right] + N \left(\frac{|V_1|^2}{J_1} + \frac{|\Delta_2|^2}{J_2} \right)$$

Solving the model:

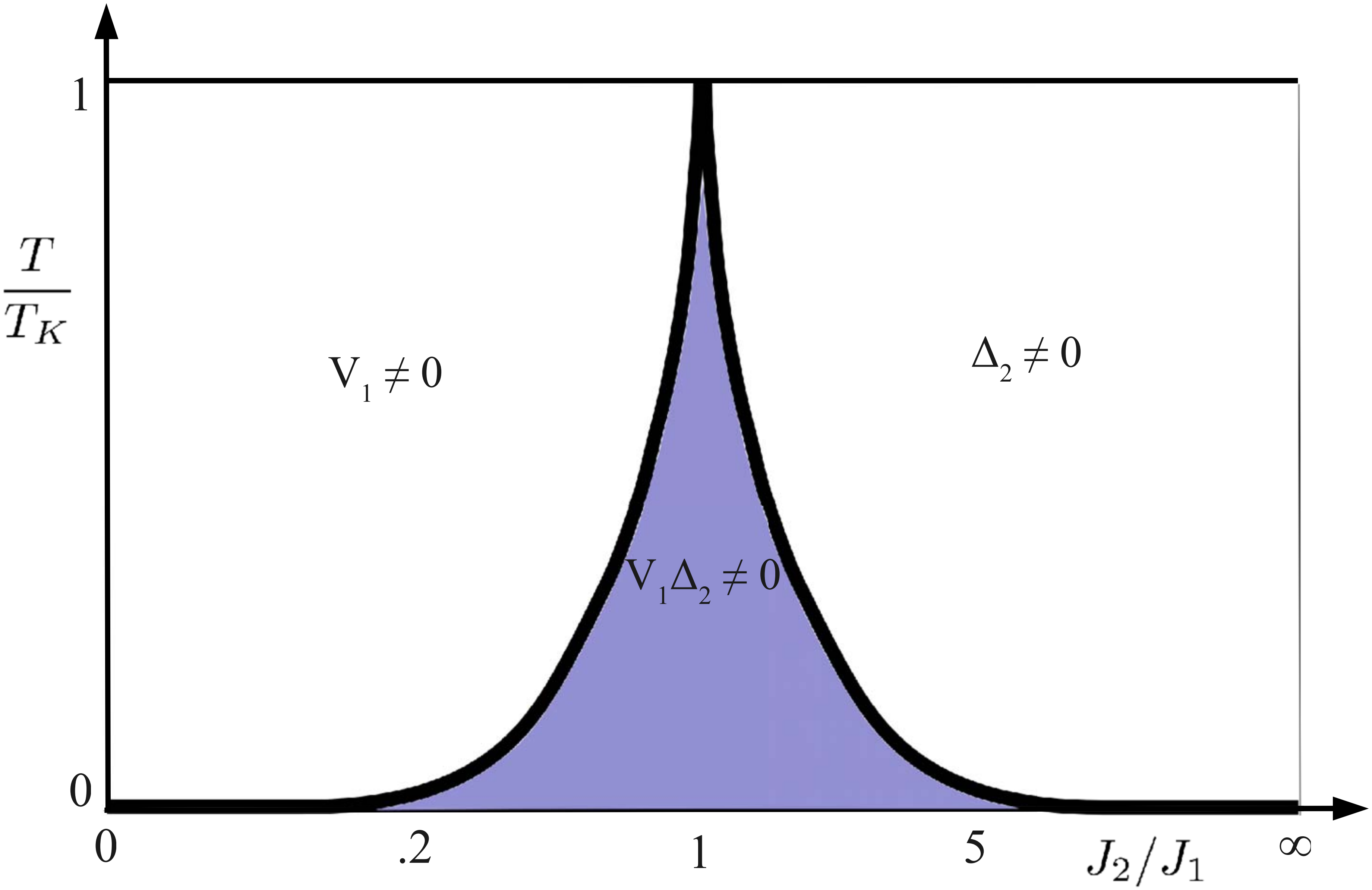
Diagonalize (analytically)

Minimize the free energy (numerically)

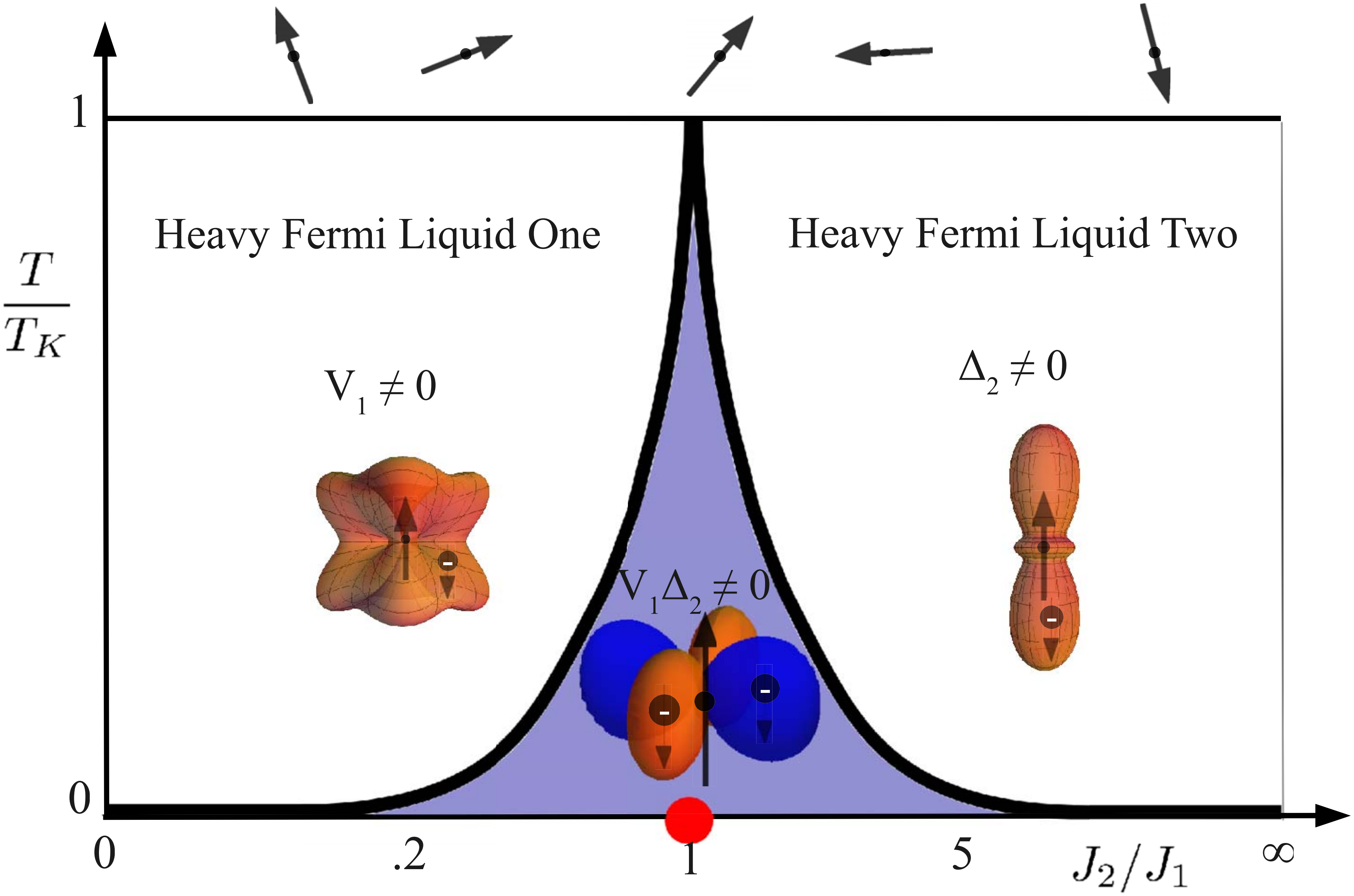
$V_1, \Delta_2 \quad \lambda$

Lagrange multiplier enforcing $n_f = 1$

The Phase Diagram



The Phase Diagram



Outline

- The materials: how are the 115 superconductors special?
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  - Composite pairing
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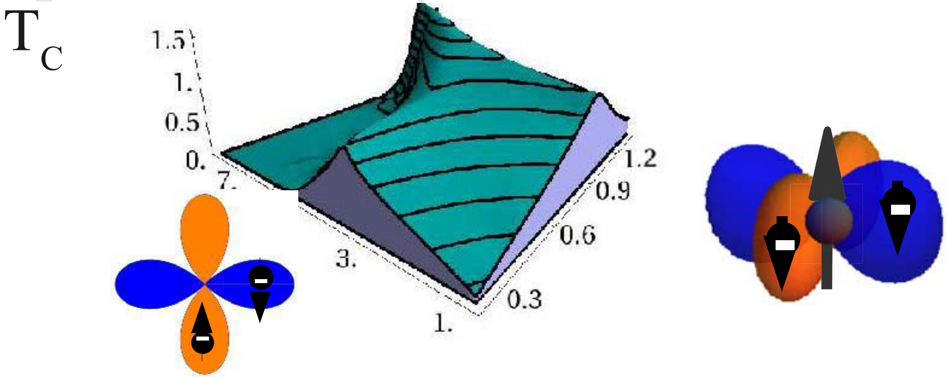


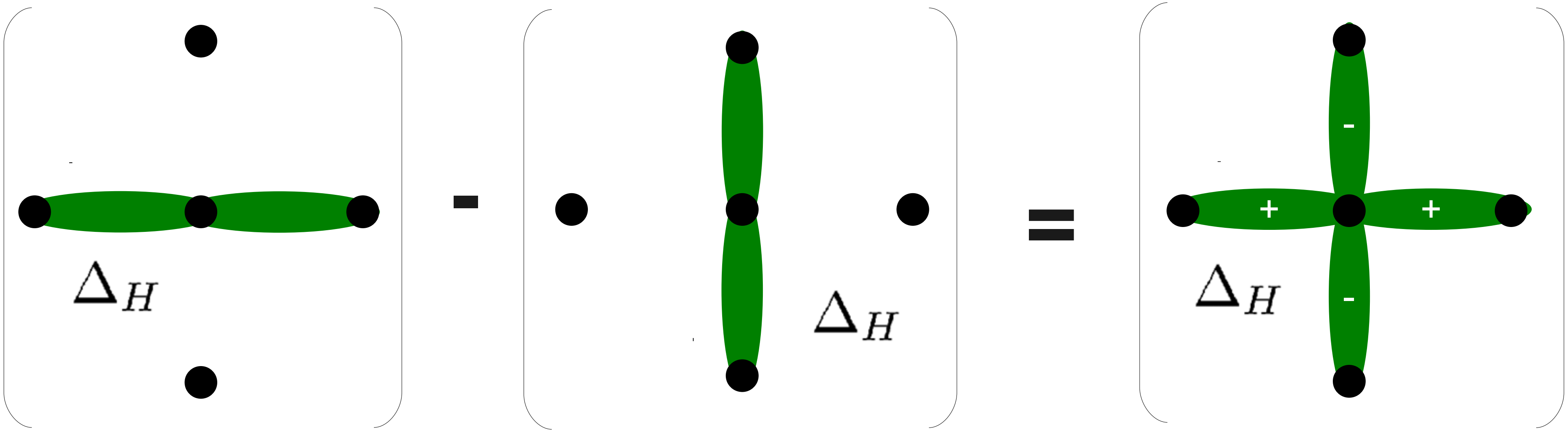
The Two channel Kondo-Heisenberg Model

$$H = \overbrace{\sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1j\beta}\cdot\vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2j\beta}\cdot\vec{S}_j}^{\text{Two channel Kondo Model}} + \underbrace{J_H \sum_{\langle ij\rangle} \vec{S}_i\cdot\vec{S}_j}_{\text{Heisenberg Model}}$$

The Two channel Kondo-Heisenberg Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1j\beta}\cdot\vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2j\beta}\cdot\vec{S}_j + J_H \sum_{\langle ij\rangle} \vec{S}_i\cdot\vec{S}_j$$

Form valence bonds $J_H \vec{S}_i\cdot\vec{S}_j \longrightarrow \Delta_H f_i^\dagger \cdot \hat{\epsilon} \cdot f_j^\dagger + \text{h.c.}$



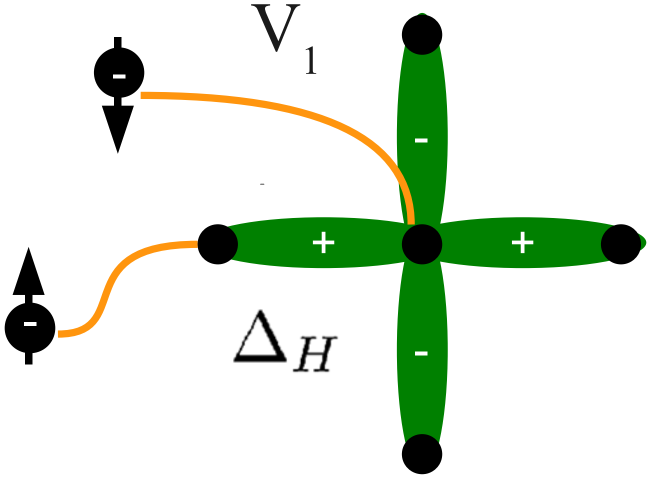
d-wave

Spin liquid state

Anderson 1973
Baskaran, Zou and Anderson 1987

The Two channel Kondo-Heisenberg Model

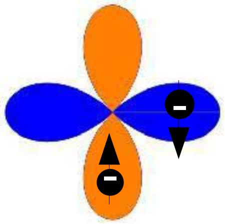
$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha} \vec{\sigma}_{\alpha\beta} \psi_{1j\beta} \cdot \vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha} \vec{\sigma}_{\alpha\beta} \psi_{2j\beta} \cdot \vec{S}_j + J_H \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Valence bonds become charged, mobile Cooper pairs

Inherit their d-wave nature from the antiferromagnetism

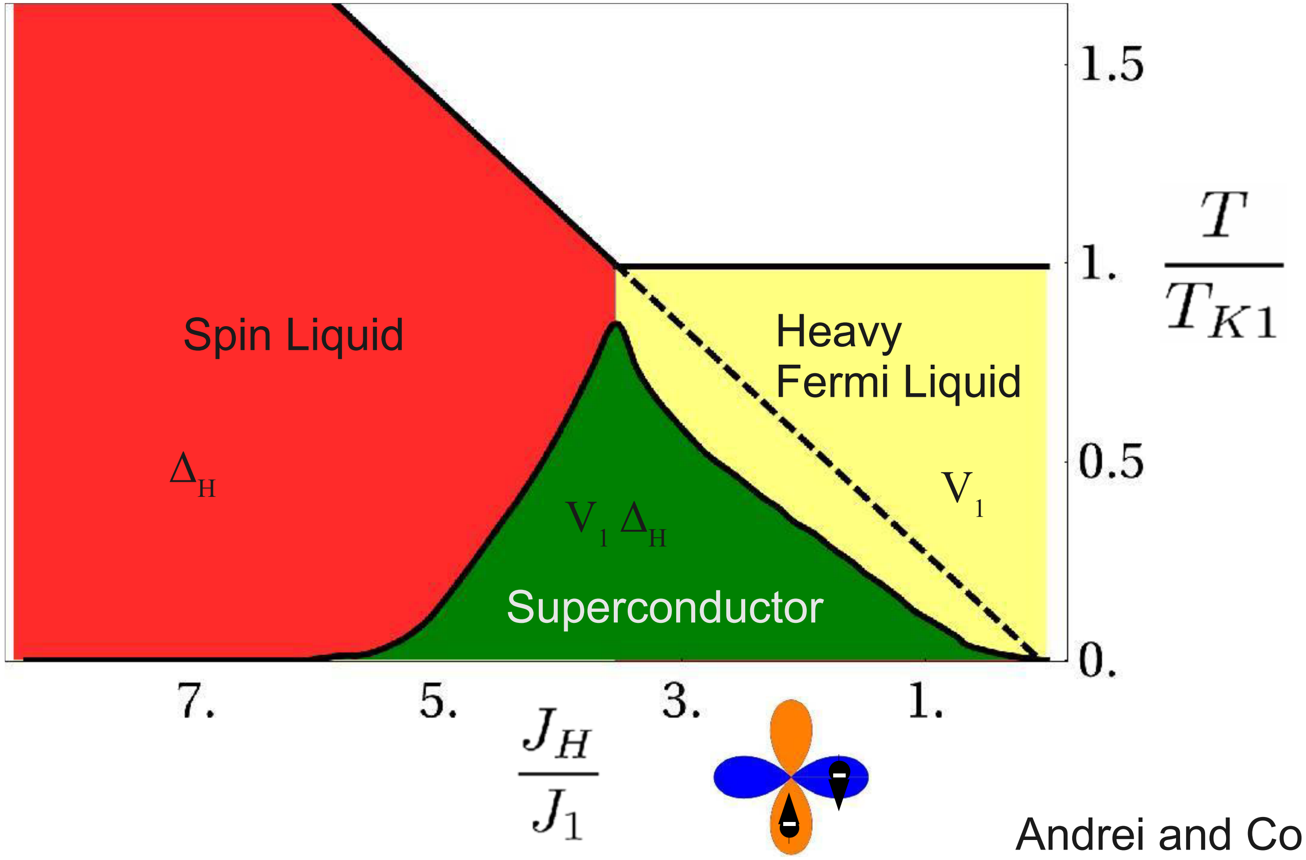
Two band version of RVB pairing



Anderson 1987
Andrei and Coleman 1989
Miyake, Schmitt-Rink, Varma 1986

The Two channel Kondo-Heisenberg Model

$$H = \sum_k \epsilon_k c_k^\dagger c_k + J_1 \sum_j \psi^\dagger_{1j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{1j\beta}\cdot\vec{S}_j + J_2 \sum_j \psi^\dagger_{2j\alpha}\vec{\sigma}_{\alpha\beta}\psi_{2j\beta}\cdot\vec{S}_j + J_H \sum_{\langle ij\rangle} \vec{S}_i\cdot\vec{S}_j$$

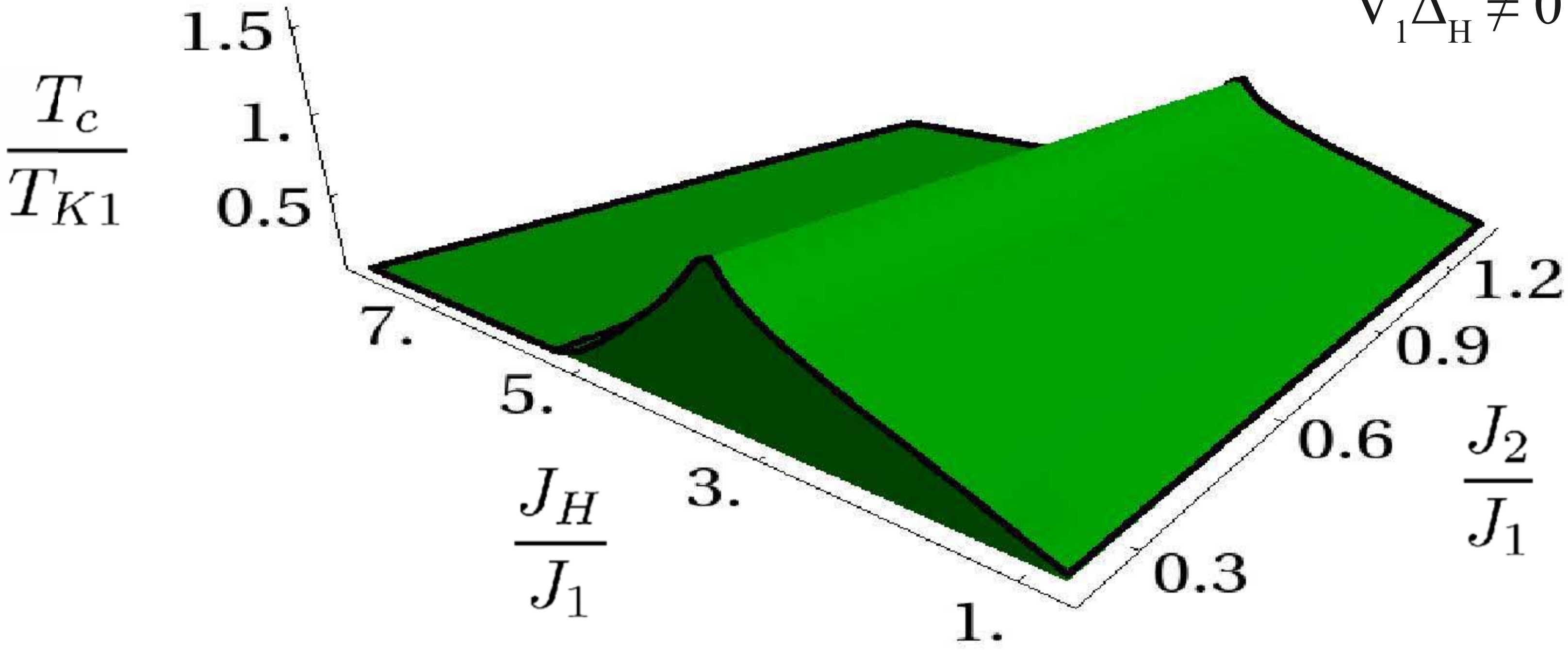


Andrei and Coleman 1989

The Phase Diagram

Magnetic pairing

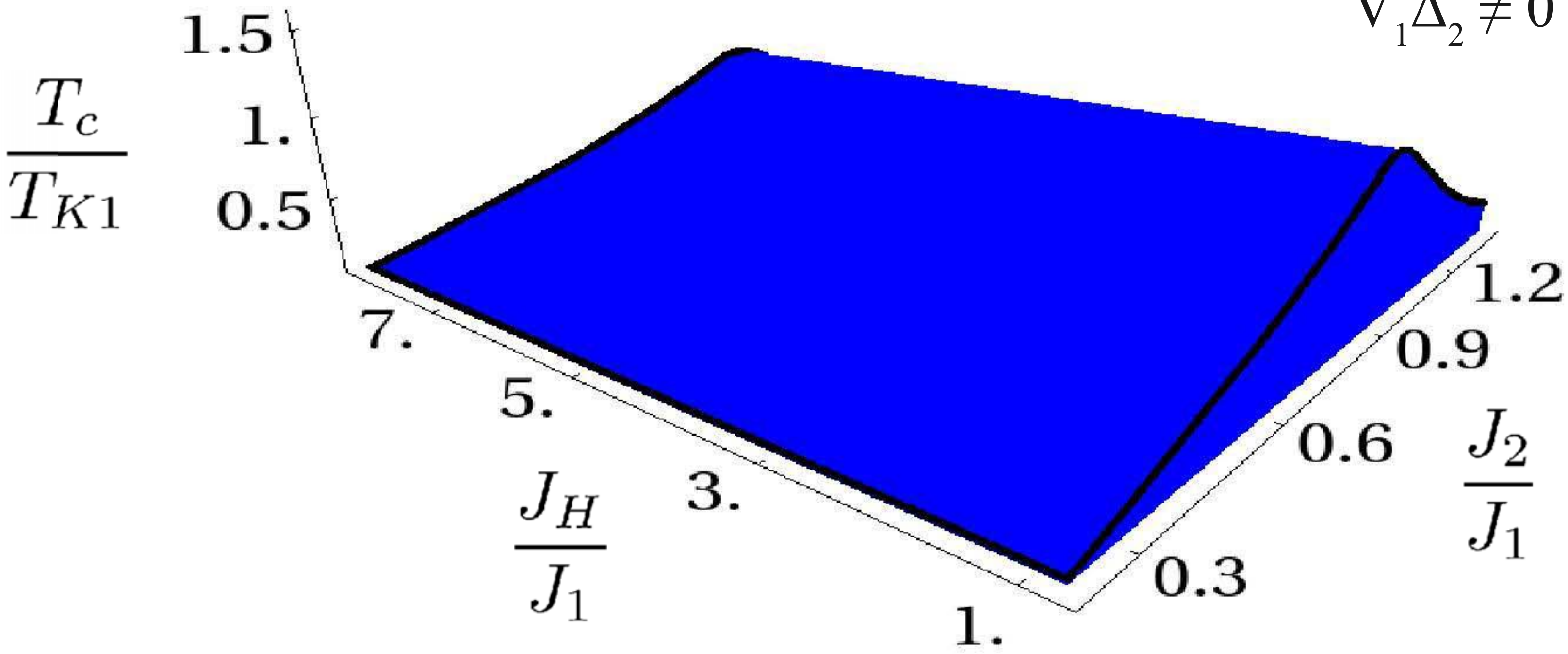
$V_1 \Delta_H \neq 0$



The Phase Diagram

Composite pairing

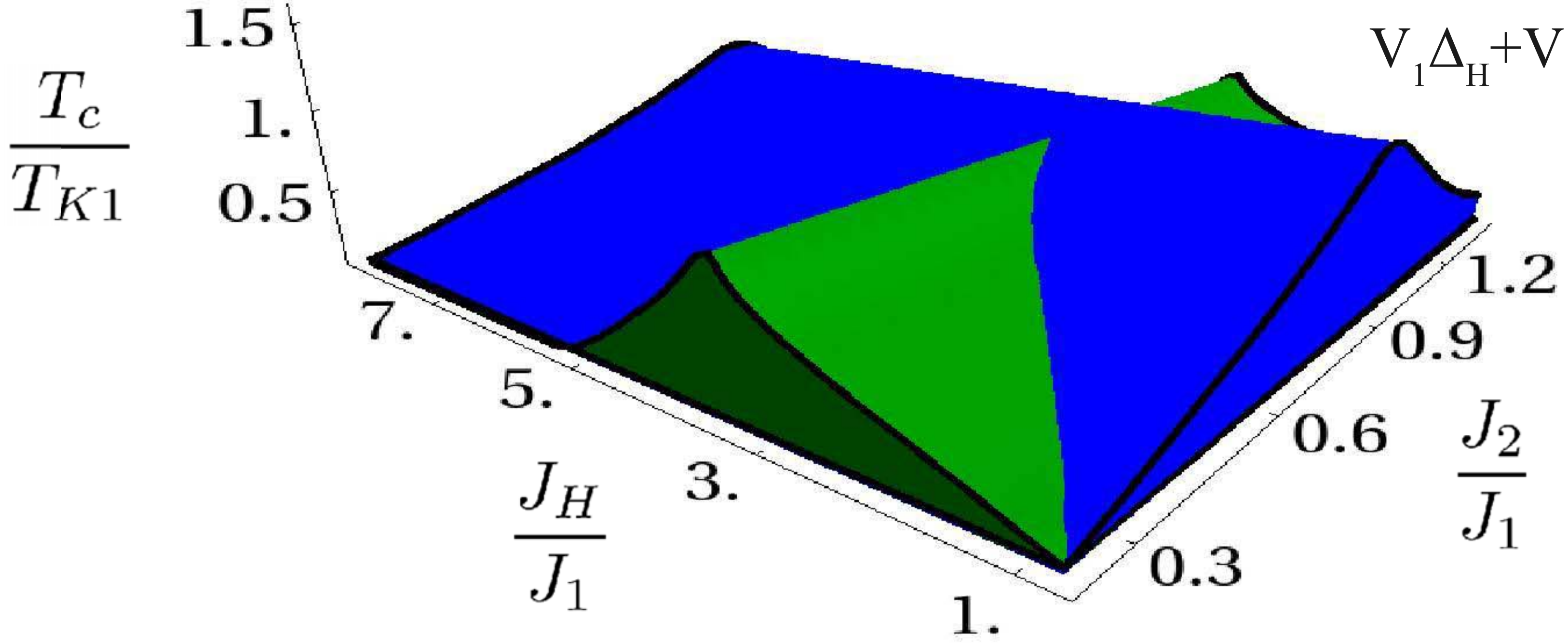
$V_1\Delta_2 \neq 0$



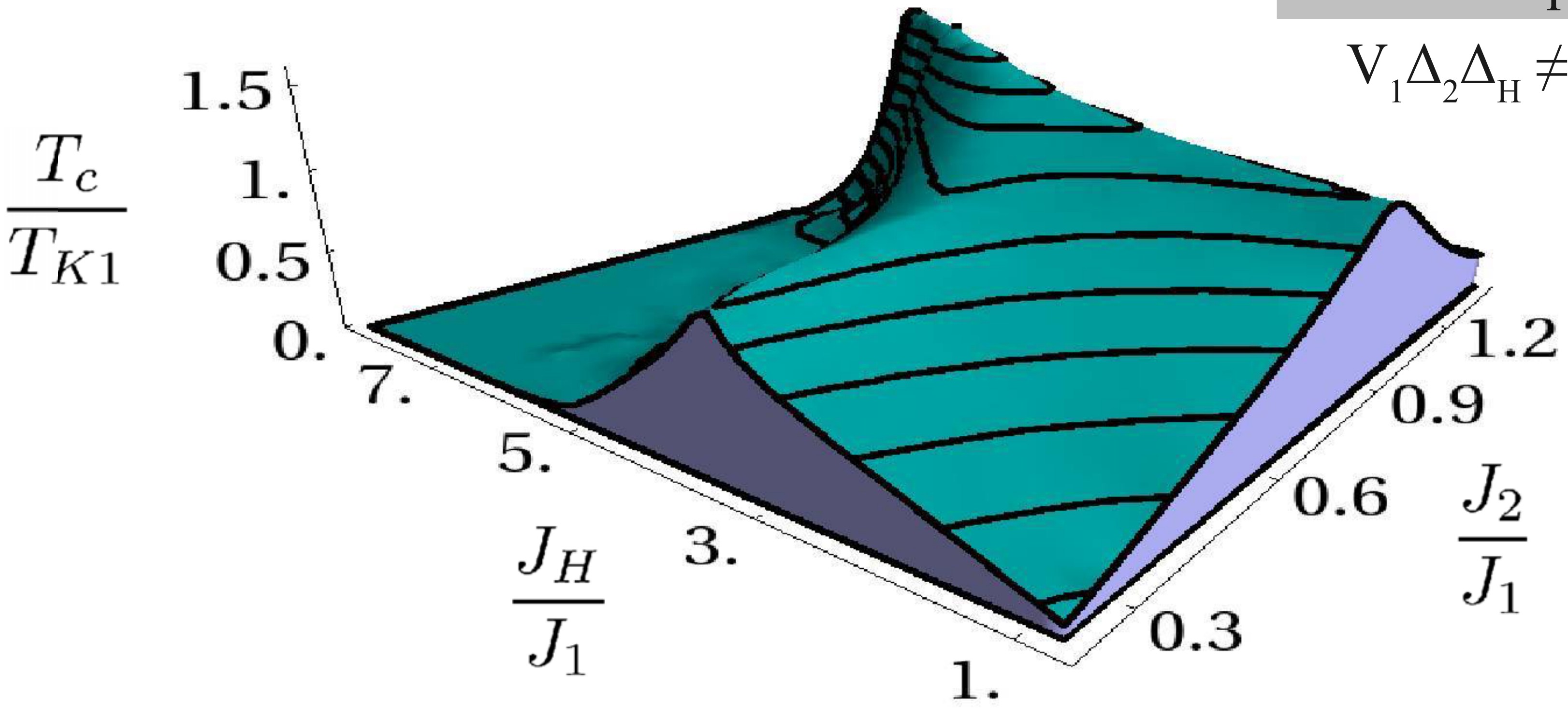
The Phase Diagram

Decoupled magnetic and composite pairing

$V_1\Delta_H + V_1\Delta_2 \neq 0$



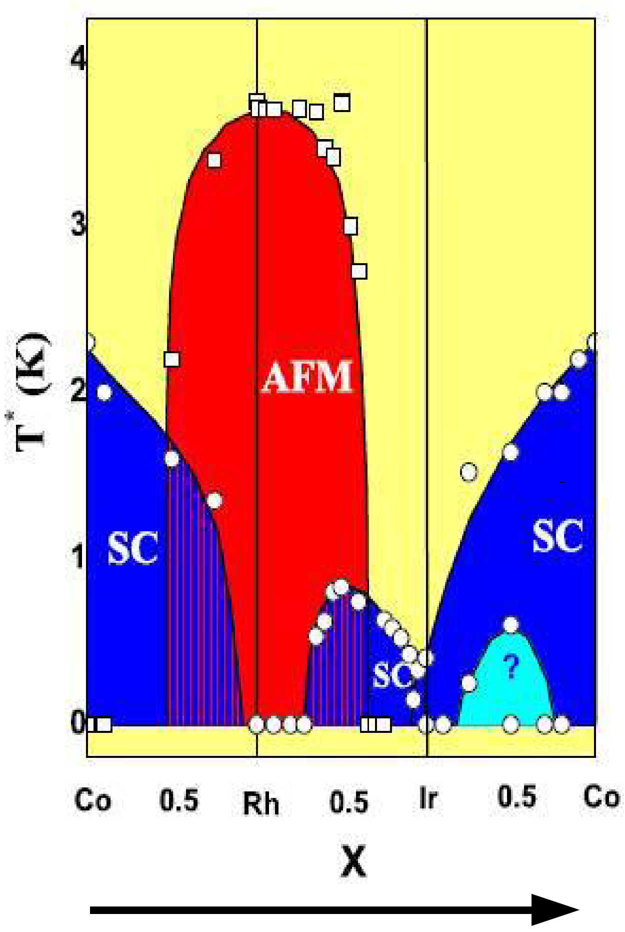
The Phase Diagram



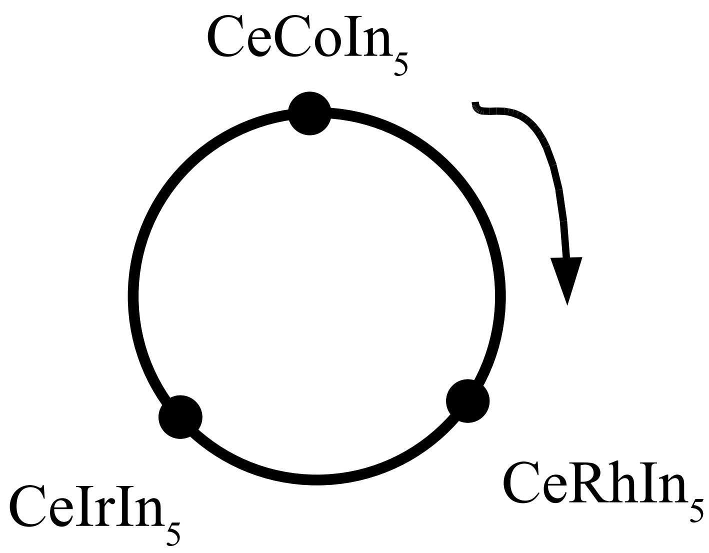
"Tandem" pairing

$V_1 \Delta_2 \Delta_H \neq 0$

Comparision to the Ce 115s



Chemical pressure traces
a path through phase space



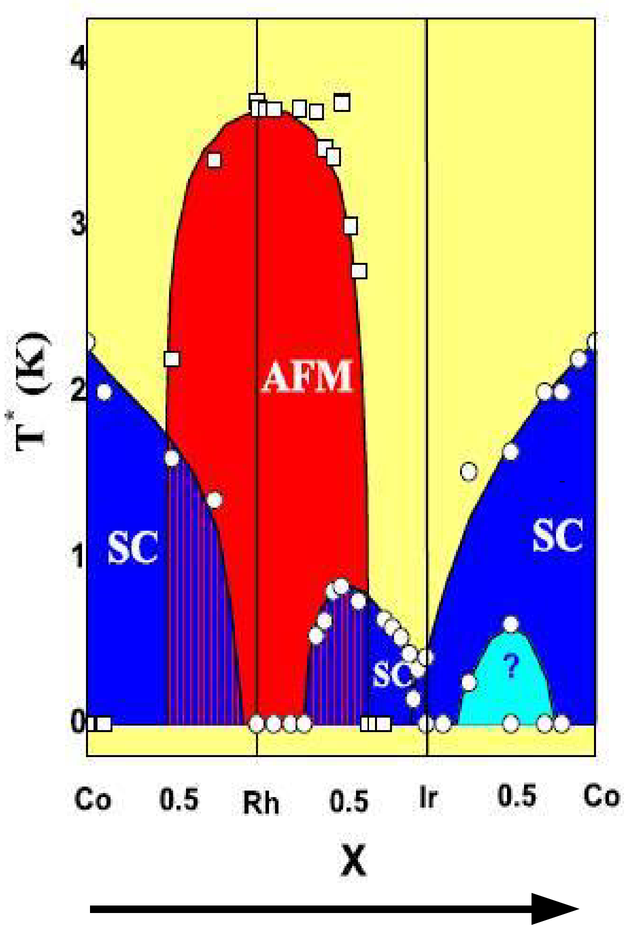
$CeCoIn_5$

$CeRhIn_5$

$CeIrIn_5$

Sarrao and Thompson 2007

Comparision to the Ce 115s



Chemical pressure traces
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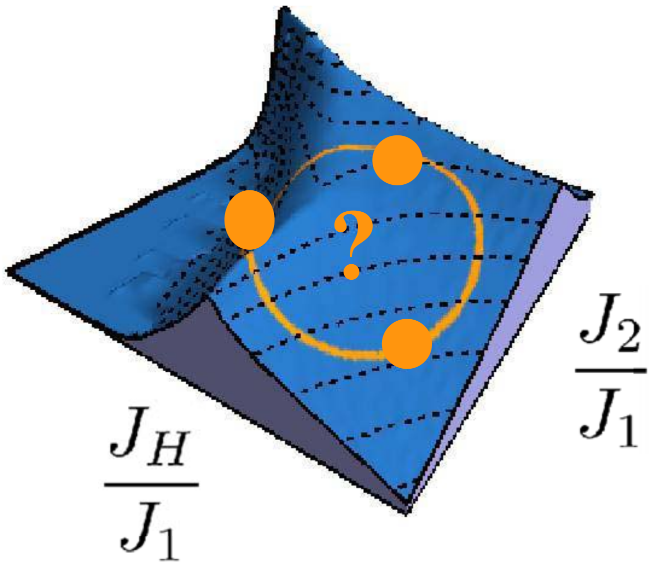

Sarrao and Thompson 2007

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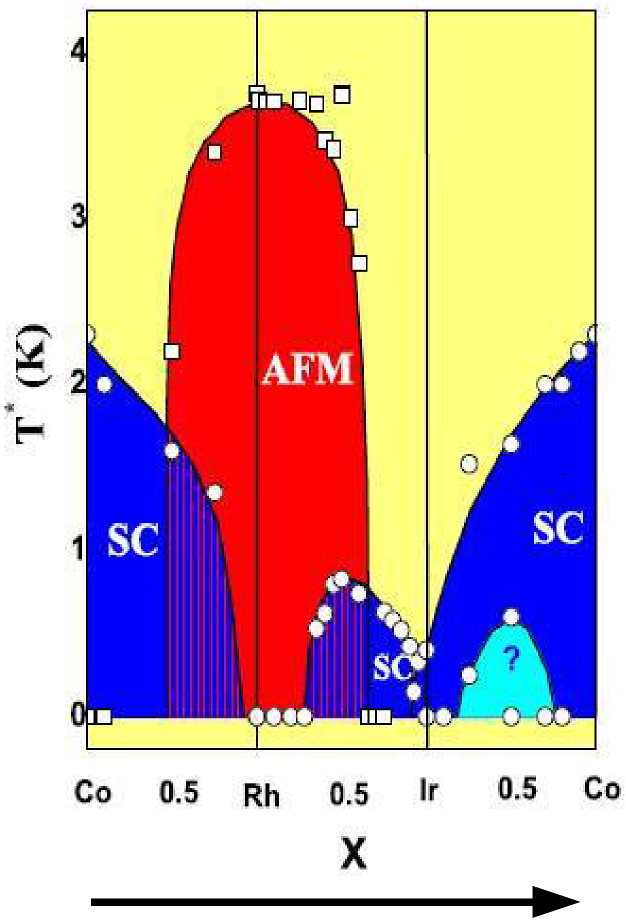

Sarrao and Thompson 2007

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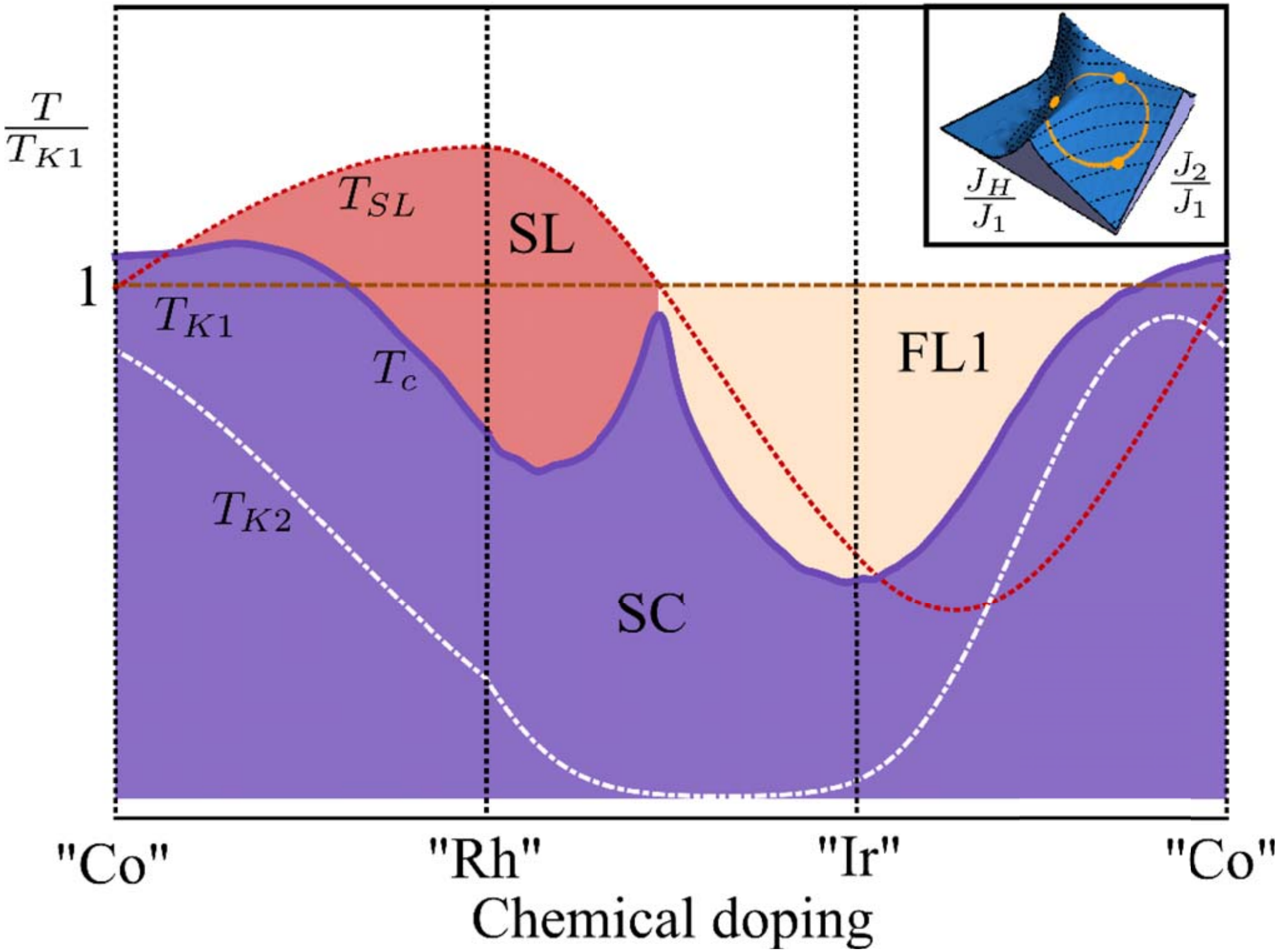

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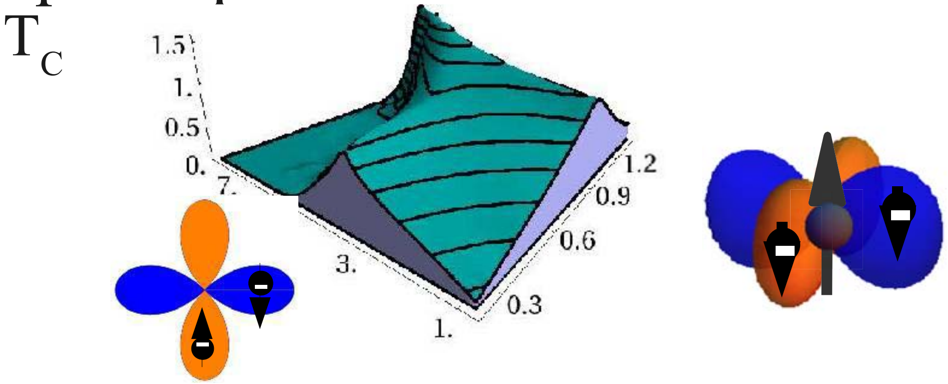
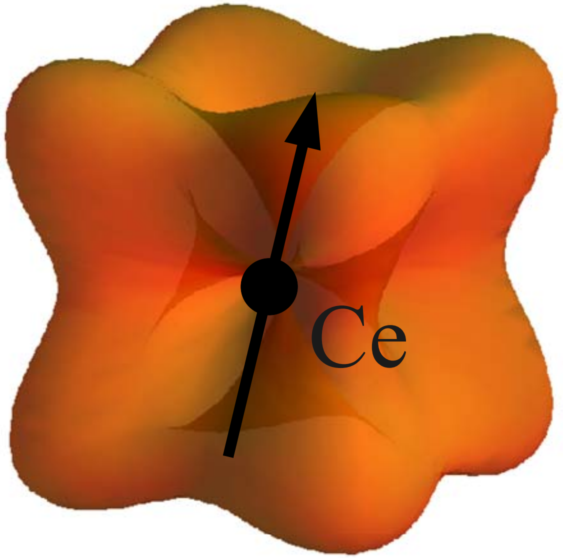
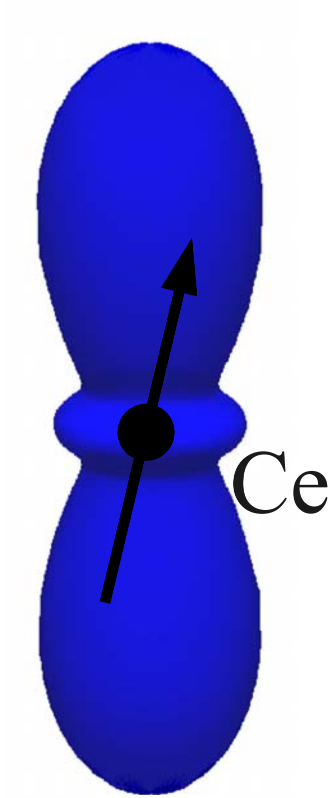


Electrostatically active tandem condensate



Γ_7^+

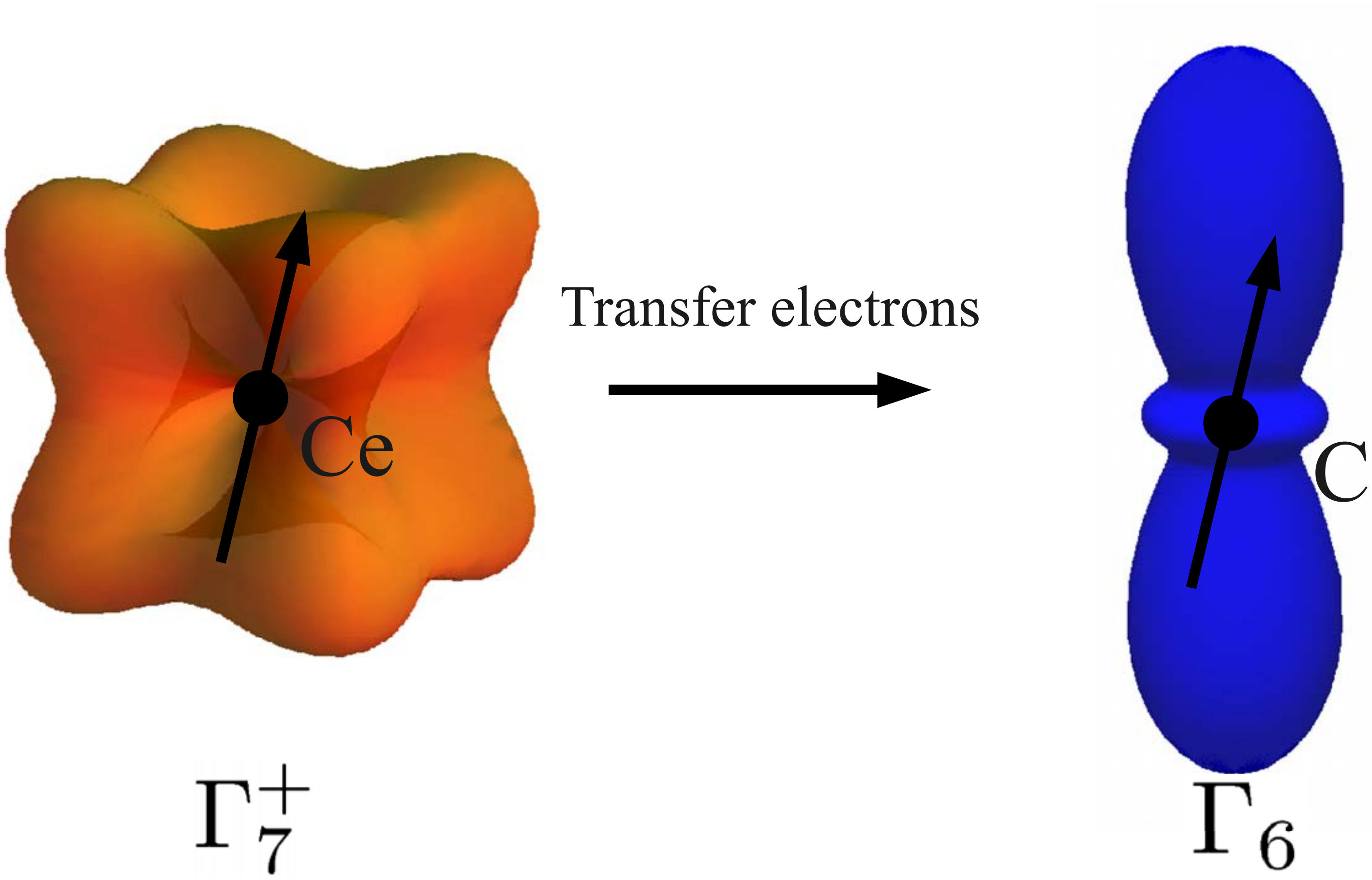
Hole Kondo effect



Γ_6

Electron Kondo effect

Electrostatically active tandem condensate

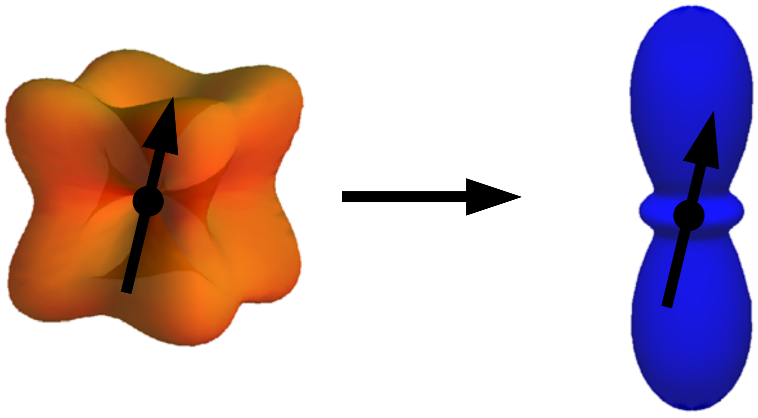


Composite pairing redistributes the f-electron charge
the condensate acquires a **quadrupole moment**

$$Q_{zz} \propto \Psi_C^2$$

$$\Delta\rho(\mathbf{x}) \propto V_1^2 \rho_{7+}(\mathbf{x}) - \Delta_2^2 \rho_6(\mathbf{x})$$

Electrostatically active tandem condensate



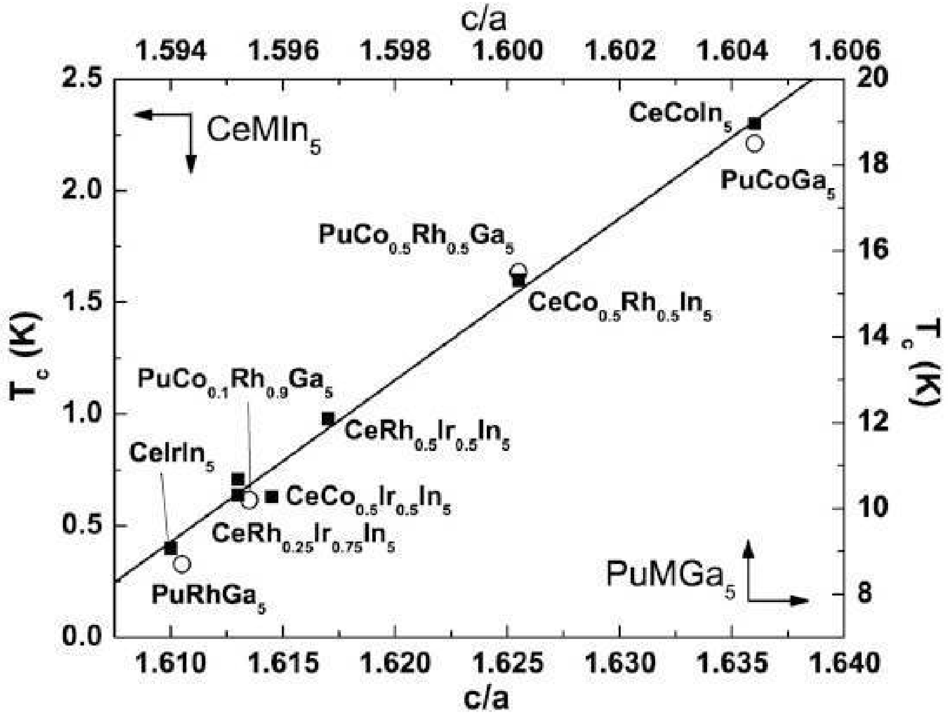
Γ_7^+ Γ_6

$$Q_{zz} \propto \Psi_C^2$$

The quadrupole moment couples to tetragonal strain ~ c/a

$$\Delta F \propto -Q_{zz}\frac{c}{a}$$

$$F = \alpha_2\left[T - (T_c + \lambda\frac{c}{a})\right]\Psi_C^2$$

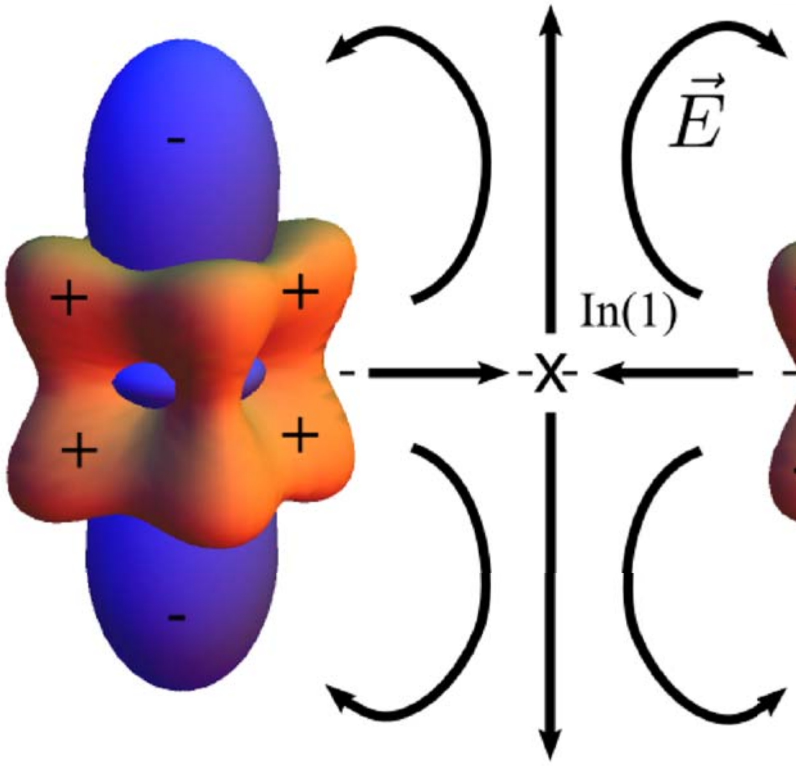


T_c increases linearly with c/a

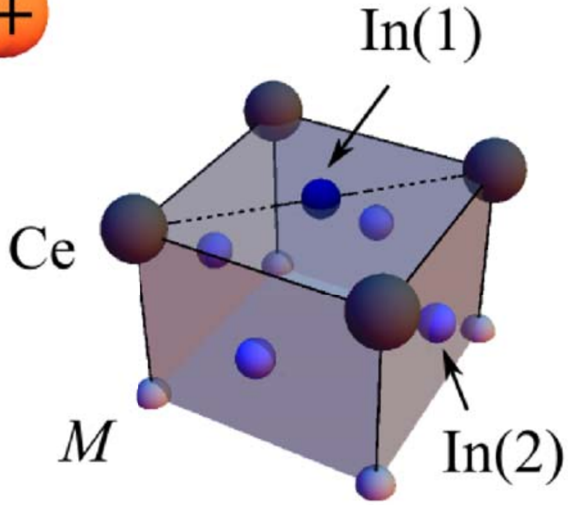
Bauer *et al.* PRL **93**, 147005 (2004)

Electrostatically active tandem condensate

$$\Delta\nu_{NQR} \approx 5\text{kHz/K} \times (T_c - T)$$



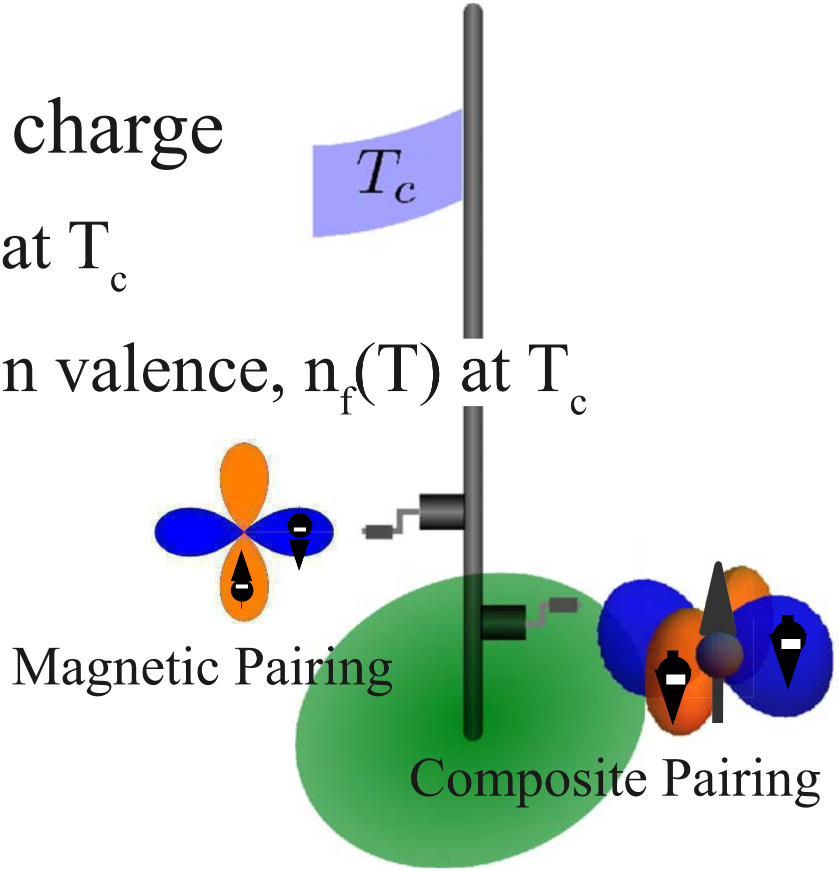
$$\frac{\partial E_z}{\partial z}\Big|_{In(1)} > 0$$



$$\Delta\rho(\mathbf{x}) \propto V_1^2\rho_{7+}(\mathbf{x}) - \Delta_2^2\rho_6(\mathbf{x})$$

Summary

- Spins quench as they pair in the 115s
  - Must be incorporated directly into the condensate
- Composite and magnetic pairing work in **tandem** to drive superconductivity
- Composite pairing redistributes charge
  - Observable as a sharp NQR shift at T_c
  - Or as a sharp shift in the f-electron valence, $n_f(T)$ at T_c



Open questions

- How does disorder affect different pairing mechanisms?
- Quantum criticality?
- Can the idea of tandem pairing be extended to other families of superconductors?