Seasonal forecast calibration and combination

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Plan of lectures
Part 1: Concepts and methods (Niño-3.4)
Part 2. South American precipitation forecasts

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Seasonal forecast calibration and combination

Part 1: Concepts and methods (Niño-3.4)

Motivating questions

Calibration

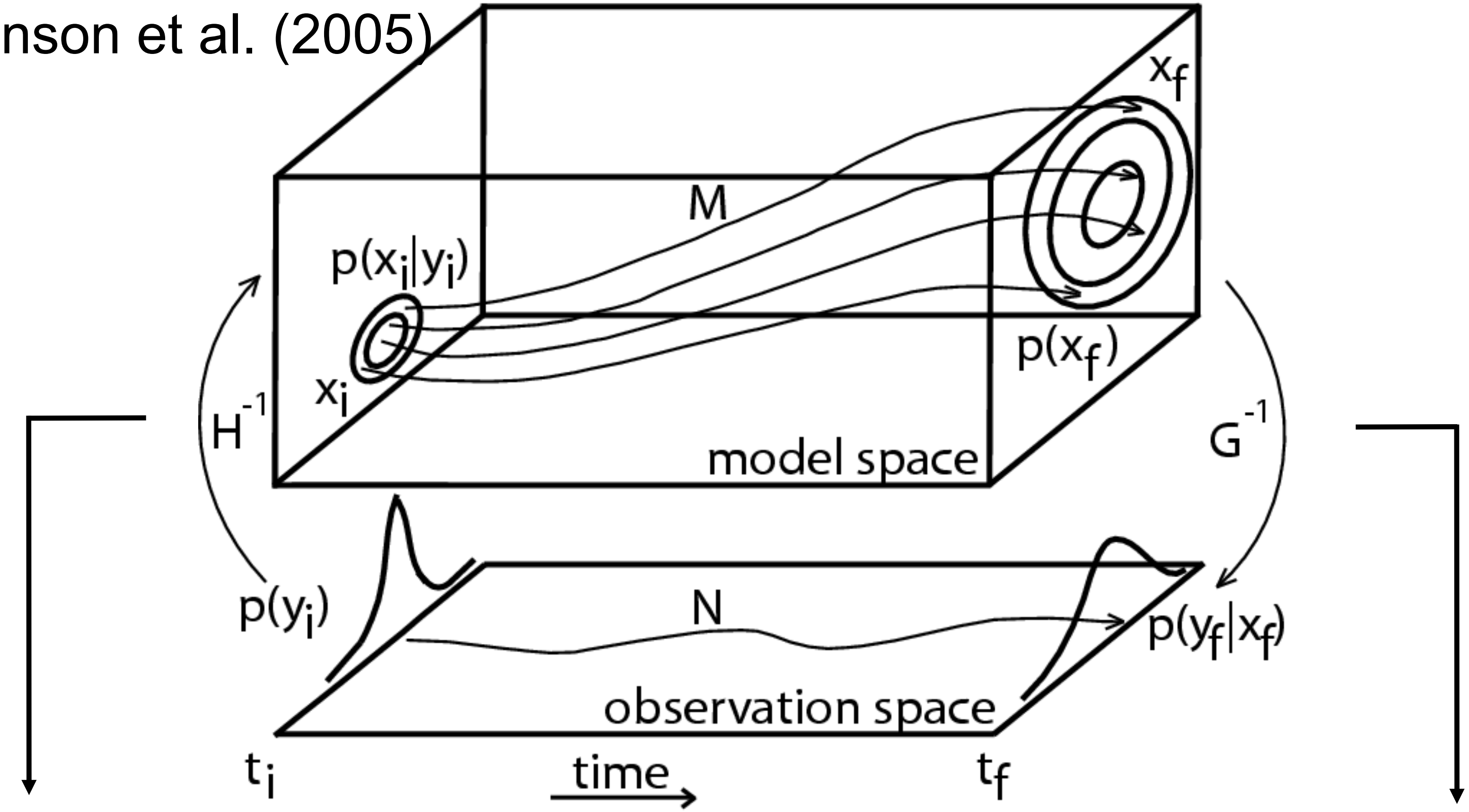
- Why do forecasts need it?
- Which are the best ways to calibrate?
- How to get good probability estimates?
- Who should do it?

Combination

- Why combine forecasts?
- Should model predictions be weighted or selected?
- How best to combine?
- Who should do it?

Conceptual framework for the calibration and combination of forecasts

Stephenson et al. (2005)



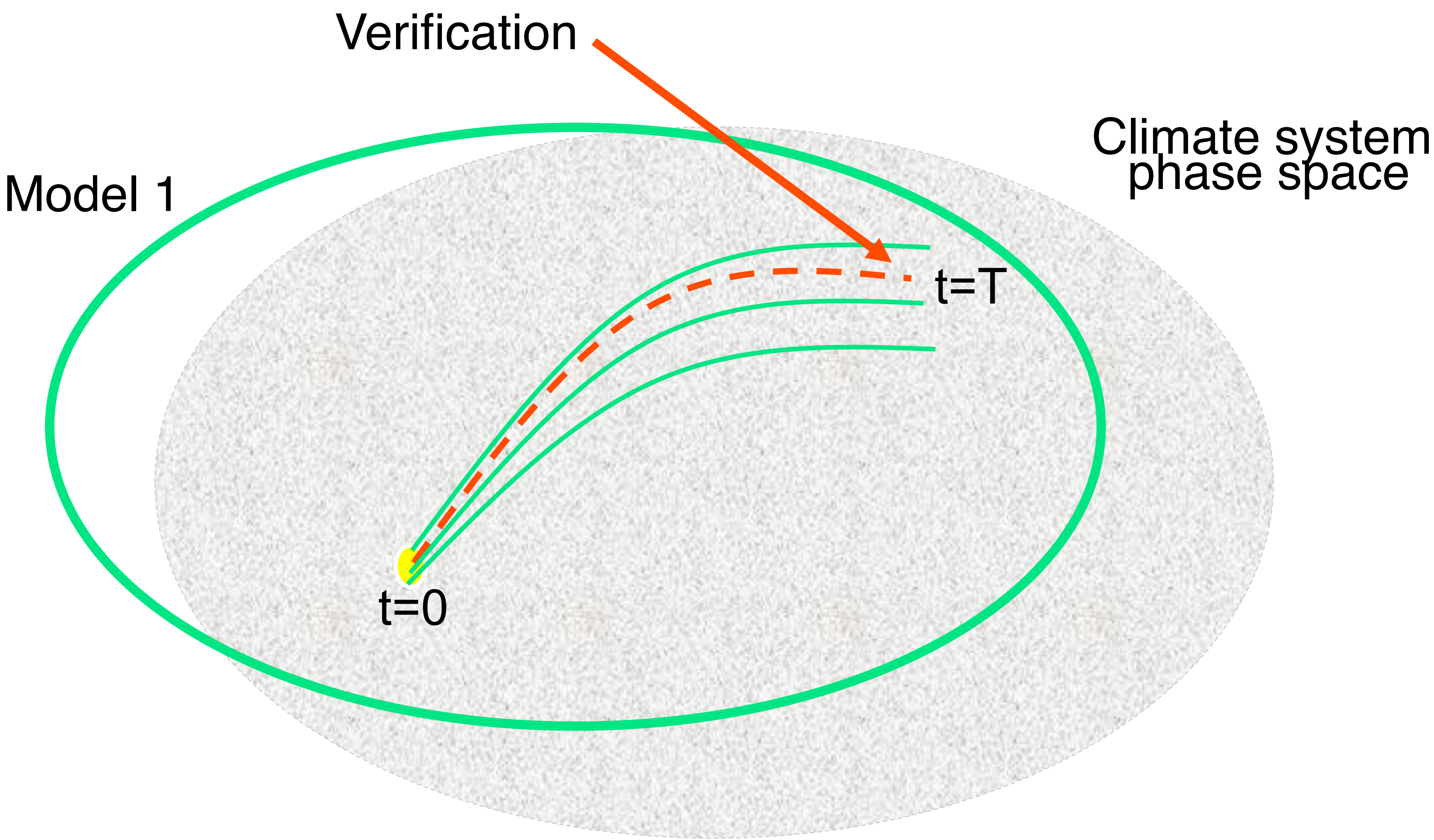
Data assimilation

$$p(x_i \mid y_i) = \frac{p(y_i \mid x_i)p(x_i)}{p(y_i)}$$

"*Forecast assimilation*

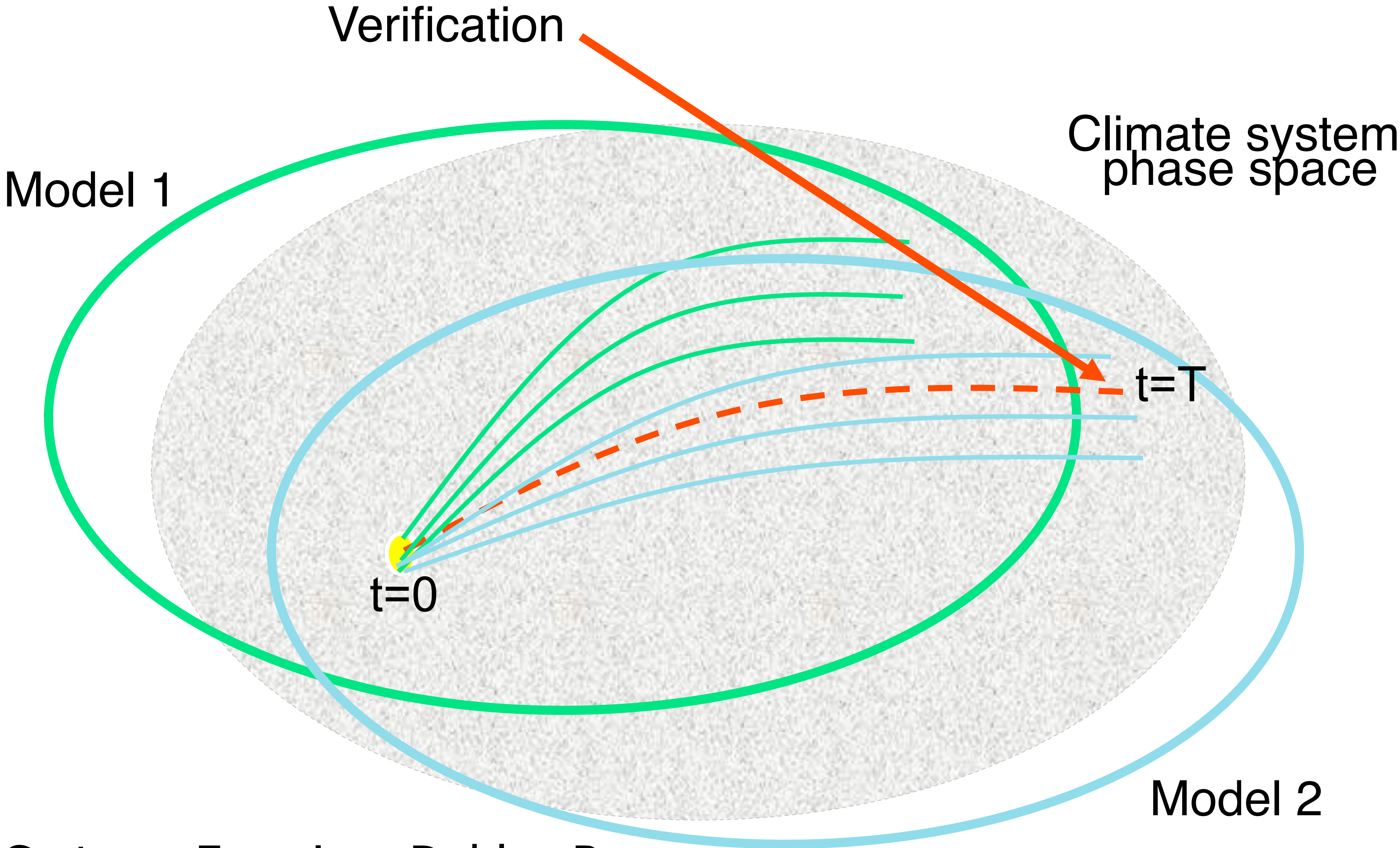
$$p(y_f \mid x_f) = \frac{p(x_f \mid y_f)p(y_f)}{p(x_f)}$$

Motivation for combination: Multi-model ensemble forecasts



Cortesy: Francisco Doblas-Reyes

Motivation for combination: Multi-model ensemble forecasts



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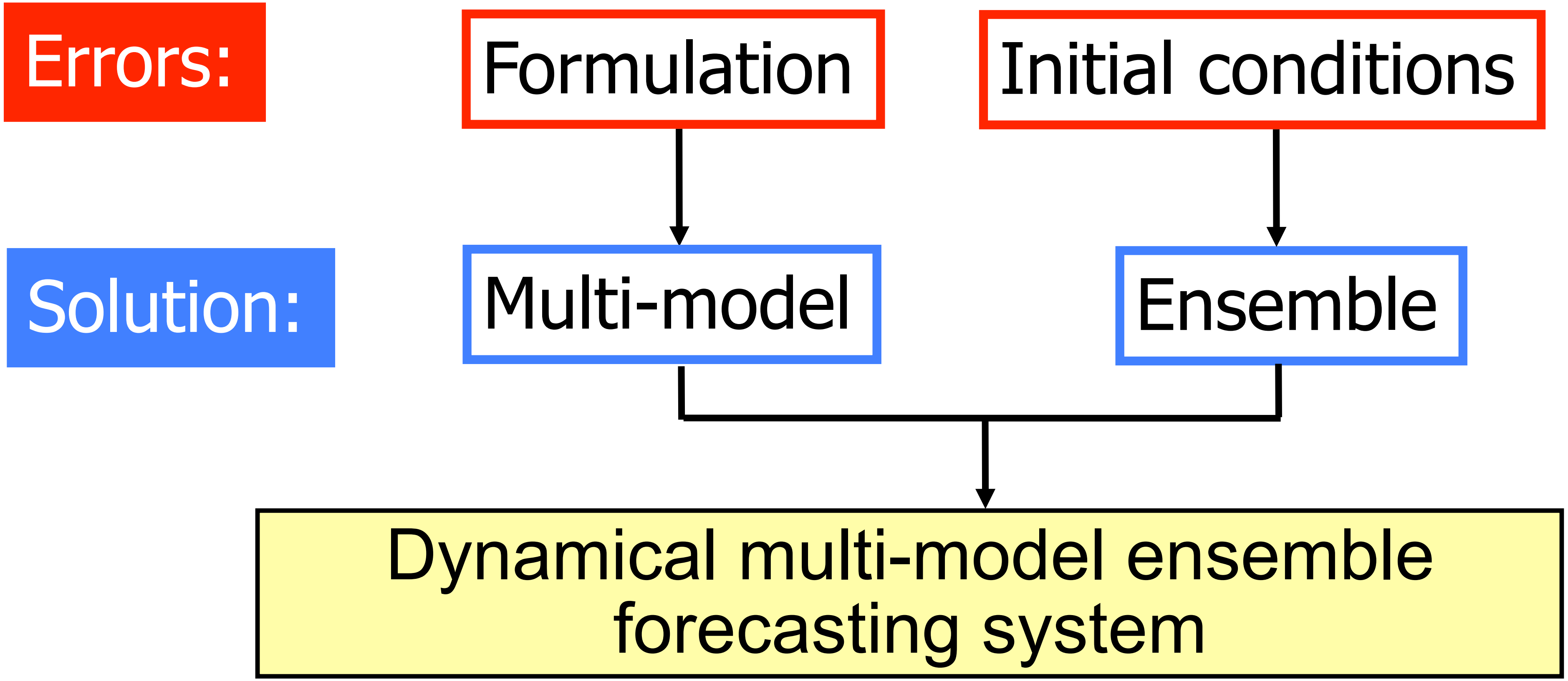
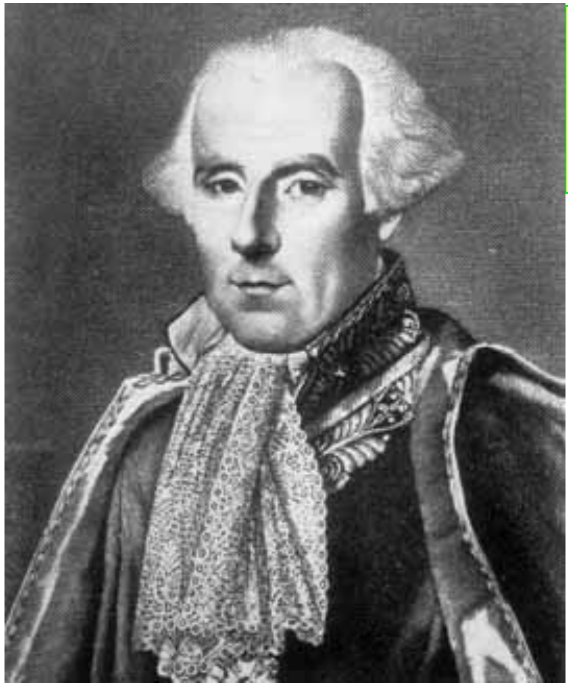
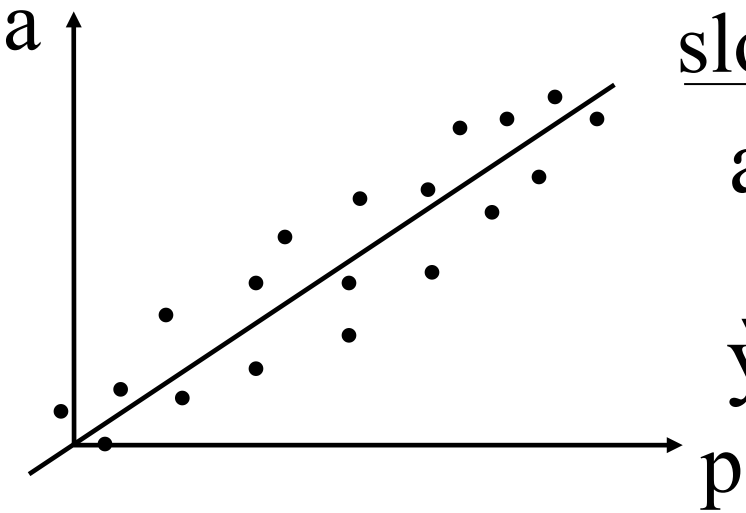
Multi-model ensemble forecasts



The Pioneer of combination



Pierre-Simon Laplace
(1749-1827)



slope residual

$$a = y\,p + x$$

$$y = \alpha x + \varepsilon$$

Laplace (1818): Combination of two estimators

y_{LS} : Least square $\rightarrow \min(x^2)$

y_{MS} : Method of Situation (Boscovich, 1757) $\rightarrow \min(|x|)$

$$\hat{y} = y_{LS} - (y_{LS} - y_{MS})C \qquad \text{where } C = C(x, \phi(x))$$

In modern times…

+ numerical = methods

Forecasts from

European Centre for Medium-Range Weather Forecasts

METEO FRANCE

Max-Planck-Institut für Meteorologie
MaxPlanck Institute for Meteorology

⋮

Have combined forecasts better skill than individual forecasts?

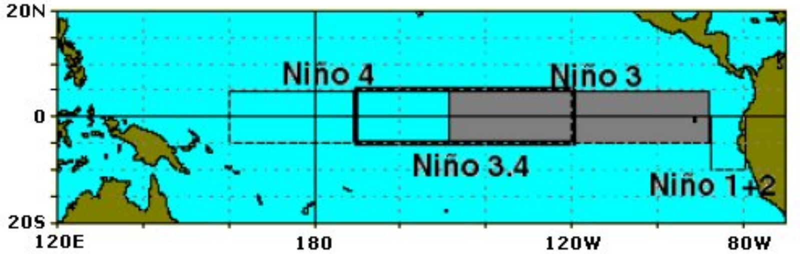
Forecast combination literature

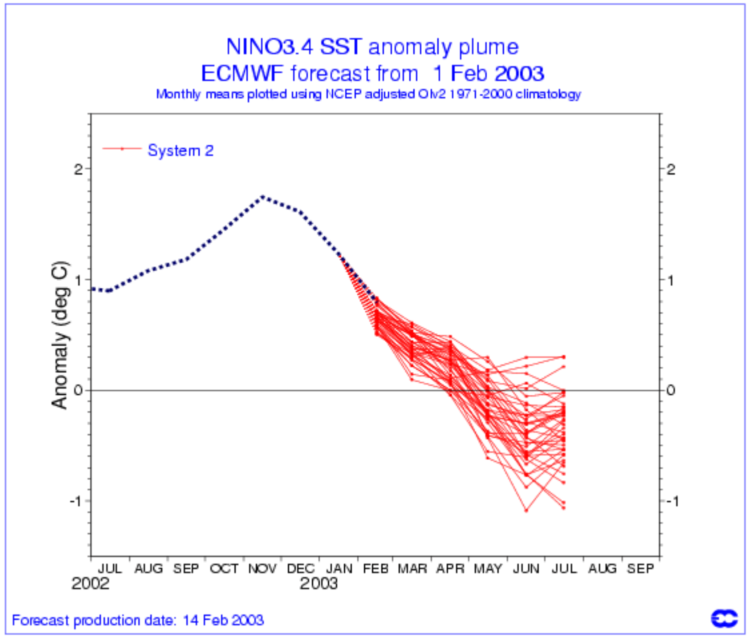
- Trenkler and Gotu (2000): ~600 publications (1970-2000)
- Widely applied in Economics and Meteorology
- Overlap of methods in these areas
- Combined forecasts are better than individual forecasts

Some issues

- What is the best method for combining?
- Is it worth combining all available forecasts?

DEMETER: coupled multi-model ensemble forecasts





Nino-3.4 index (Y)

Period: 1987-99

Ensemble: 9 members

Aug -> Dec

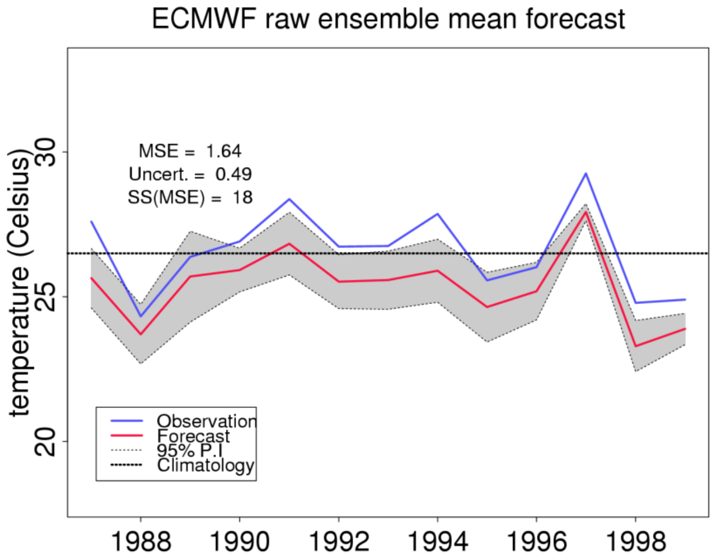
5 months lead

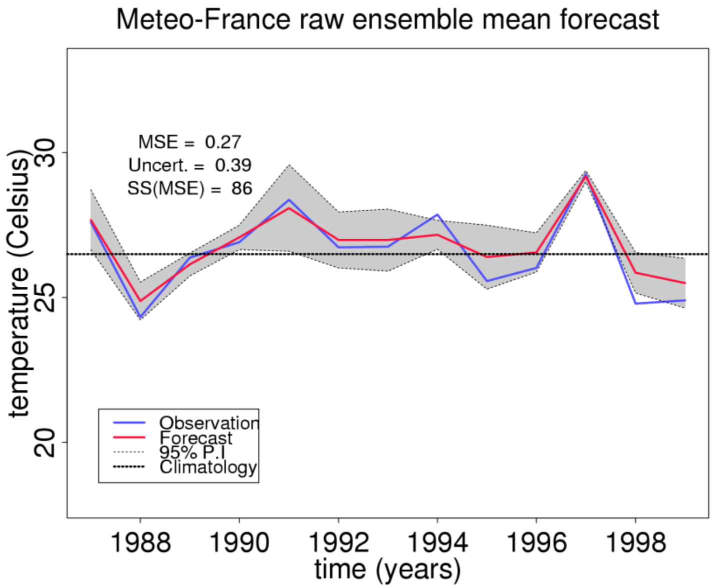
$$Y \sim N(\mu_t, \sigma_t^2)$$

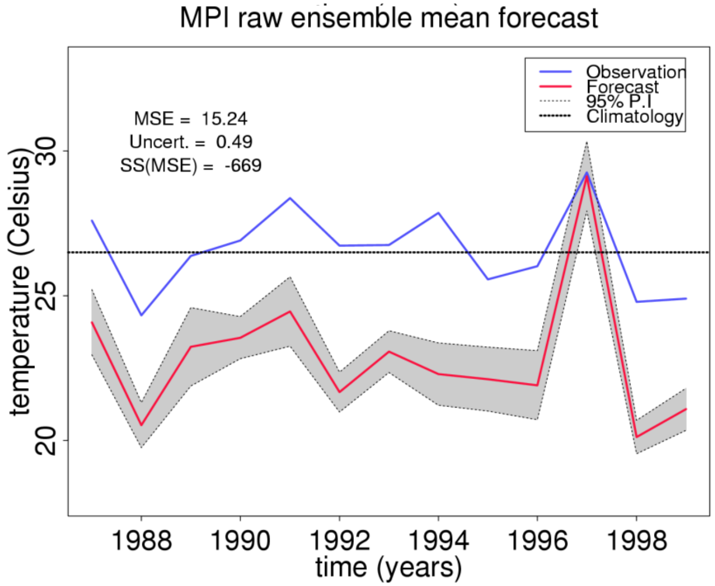
ECMWF
Meteo-France (MF)
Max Planck Institut (MPI)

DEMETER web page: http://www.ecmwf.int/research/demeter

December Nino-3.4 raw forecasts







$$Y \sim N(\mu_t, \sigma_t^2)$$

$$\hat{\mu}_t = \overline{X}_t$$

$$\hat{\sigma}_t = s_X$$

$$95\%\ \text{P.I.:}\ [\hat{\mu}_t - 1.96\hat{\sigma}_t, \hat{\mu}_t + 1.96\hat{\sigma}_t]$$

Forecast calibration and combination

$$F(t) = w_o + \sum_{i=1}^{M} w_i \overline{X}_i(t)$$

$\overline{X}$: ensemble mean

M : models

F : combined forecast

w_o and w_i : constants

Linear combination of M ensemble mean forecasts $\overline{X}$

Calibration and combination methods

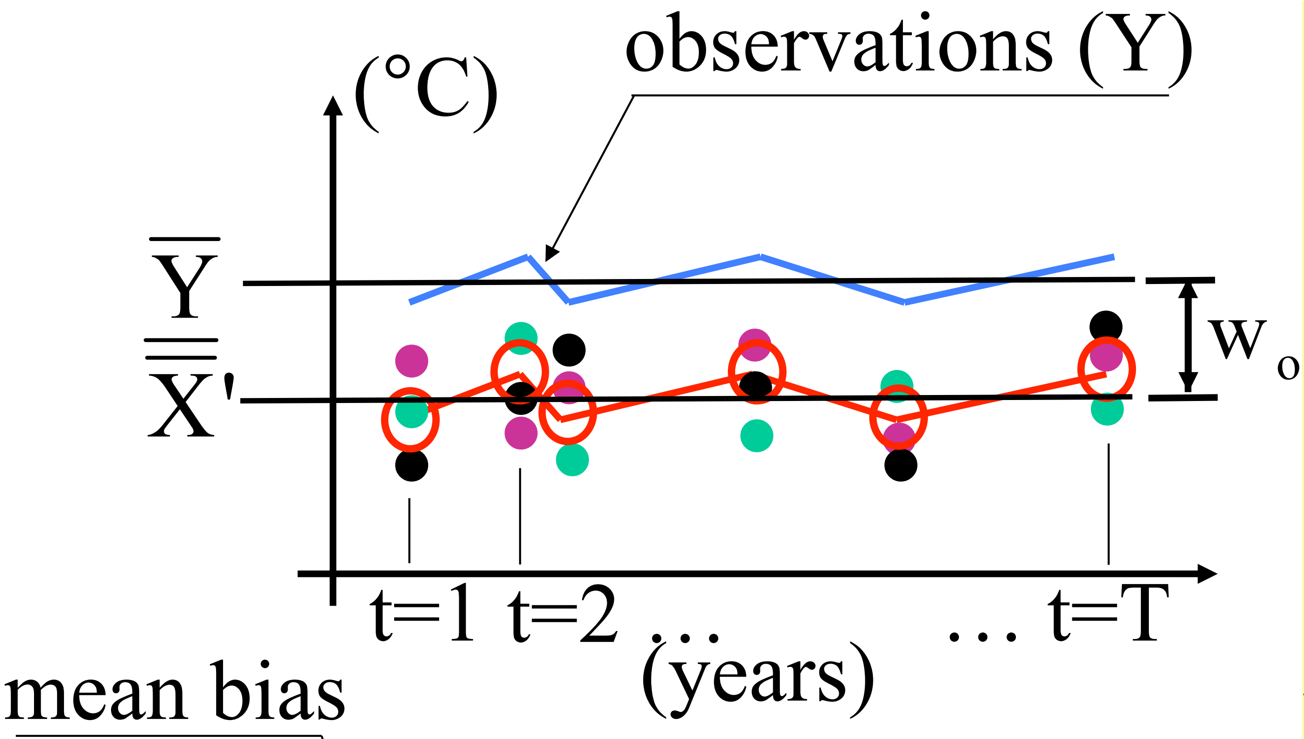
$$F(t) = w_o + \sum_{i=1}^{M} w_i \overline{X}_i(t)$$

How to estimate w_o and w_i ?

Kharin and Zwiers (2002), J Climate

- Bias-removed multimodel ensemble mean forecast (Uem)
- Regression-improved multimodel ensemble mean forecast (Rem)
- Regression-improved multimodel forecast (Rall)

Bias-removed multimodel ensemble mean forecast (Uem)



Raw forecasts

- Model 1 ($\overline{X}_1$)
- Model 2 ($\overline{X}_2$)
- Model 3 ($\overline{X}_3$)
- Multimodel ensemble mean (X′)

$$\overline{X}' = (\overline{X}_1 + \overline{X}_2 + \overline{X}_3)/3$$

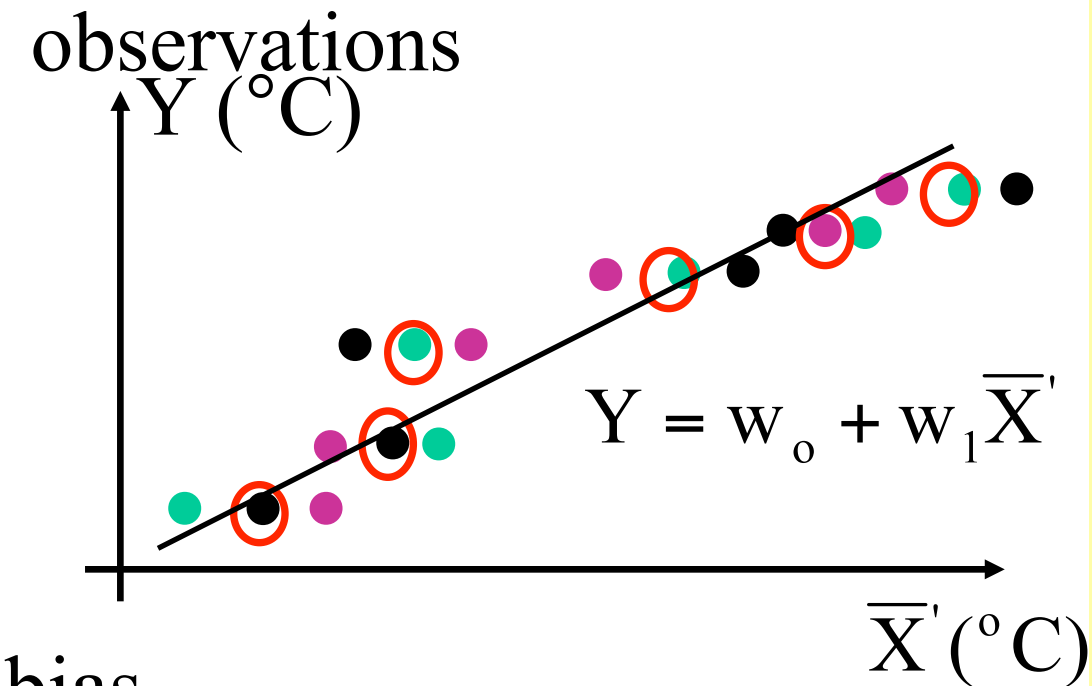
$$F(t) = w_o + \sum_{i=1}^{M=3} w_i \overline{X}_i(t)$$

$$w_o = \frac{1}{T}\sum_{t=1}^{T} Y_t - \overline{X}'_t = \overline{Y} - \overline{\overline{X}}'$$

$$Uem = w_o + \overline{X}'$$

$$w_i = \frac{1}{M}$$

Regression-improved multimodel ensemble mean forecast (Rem)



Raw forecasts

- Model 1 ($\bar{X}_1$)
- Model 2 ($\bar{X}_2$)
- Model 3 ($\bar{X}_3$)
- Multimodel ensemble mean ($\bar{X}'$)

$$\bar{X}' = (\bar{X}_1 + \bar{X}_2 + \bar{X}_3)/3$$

mean bias

$$F(t) = w_o + \sum_{i=1}^{M=3} w_i \bar{X}_i(t) \qquad \hat{w}_o = \bar{Y} - \hat{w}_1 \bar{\bar{X}}' \qquad \hat{w}_1 = \frac{s_Y}{s_{\bar{X}'}} r$$

$$\mathrm{Rem} = w_o + w_1 \bar{X}' \qquad w_i = \frac{1}{M} w_1$$

Regression-improved multimodel forecast (Rall)

Multiple linear regression in matricial notation :
Y: 1 x n (observations)
X: M x n (forecasts)

Raw forecasts
- Model 1 ($\overline{X}_1$)
- Model 2 ($\overline{X}_2$)
- Model 3 ($\overline{X}_3$)

$$X = [\overline{X}_1 \ \overline{X}_2 \ \overline{X}_3]^T$$

$$Y = \xi + HX$$

$$\hat{H} = S_{YX} S_{XX}^{-1}$$

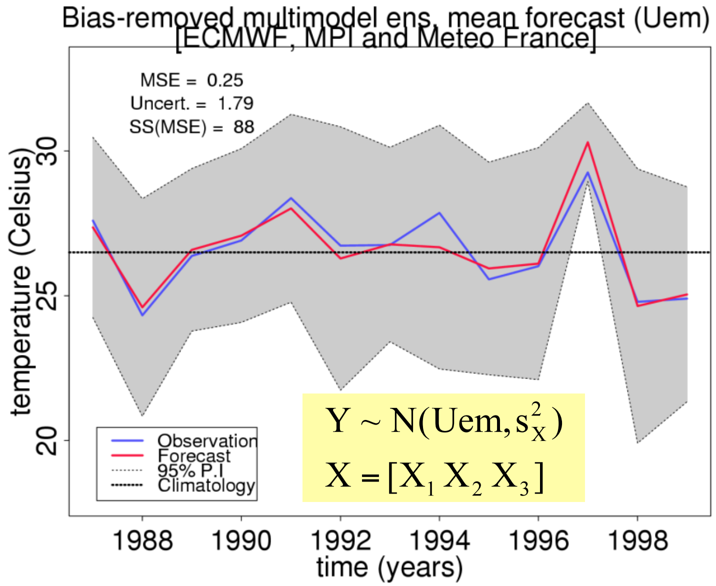
$$\hat{\xi} = \overline{Y} - \hat{H}\overline{X}$$

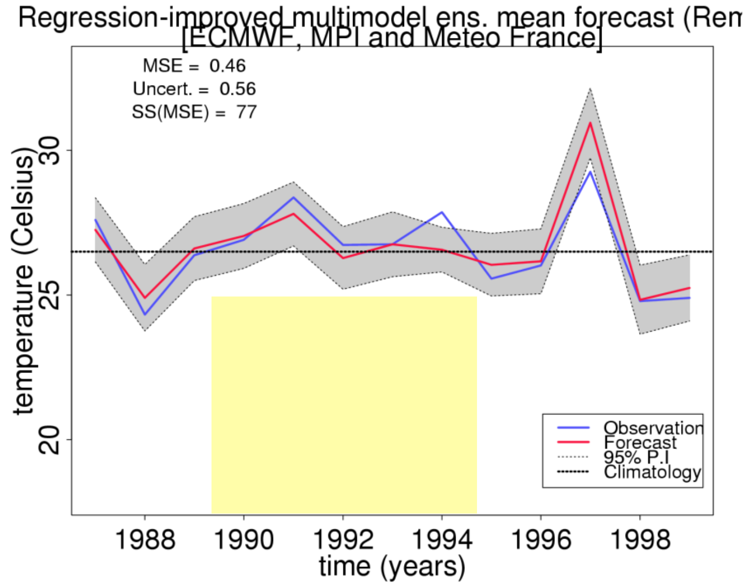
$$Rall = w_o + \sum_{i=1}^{M=3} w_i \overline{X}_i(t)$$

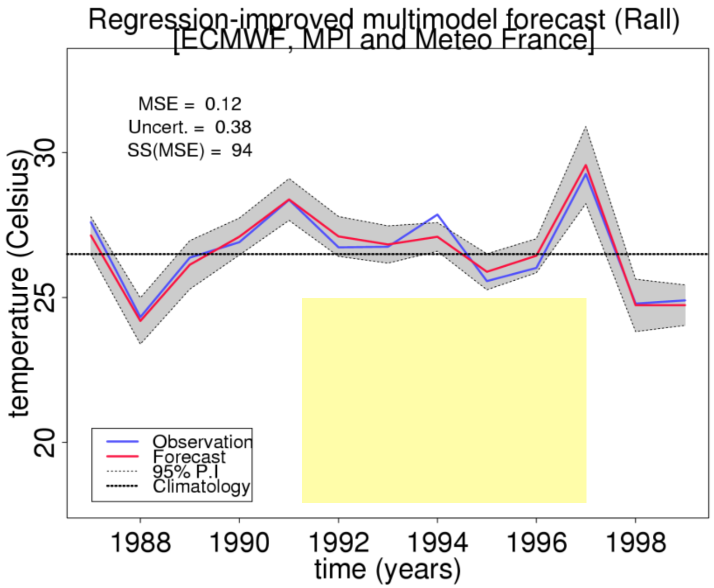
$$w_o = \hat{\xi}$$

$$\sum_{i=1}^{M=3} w_i \overline{X}_i(t) = \hat{H}X$$

Combined forecasts







The Bayesian approach

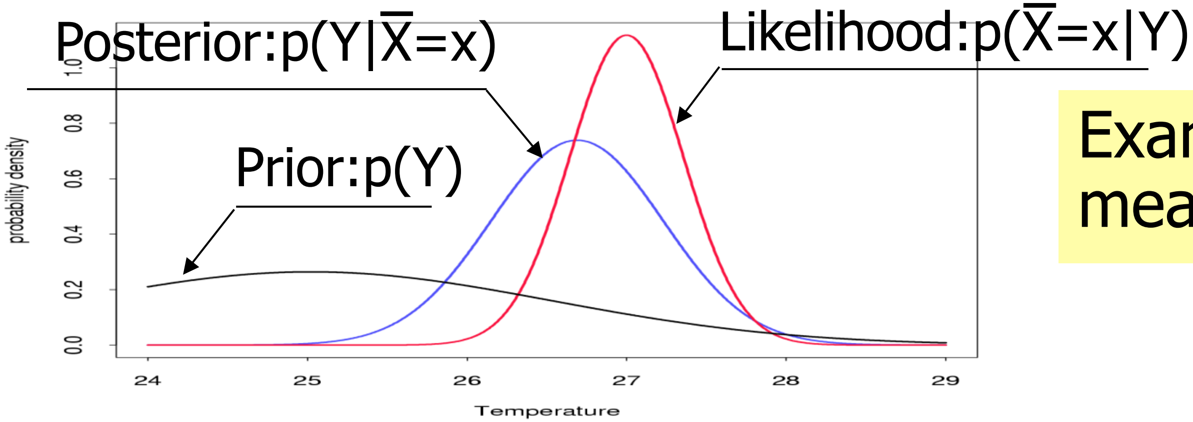
Thomas Bayes (1701-1761)

The process of belief revision on any event Y consists in updating the probability of Y when new information $\bar{X}$ becomes available

Y: Observed December Nino-3.4 index

$\bar{X}$: Ensemble mean forecast of Y for December

$$p(Y \mid \bar{X} = x) = \frac{p(\bar{X} = x \mid Y)\, p(Y)}{p(\bar{X} = x)}$$



Example: Ensemble mean ($\bar{X}$=x=27°C)

Bayesian multi-model forecast (B): Forecast assimilation

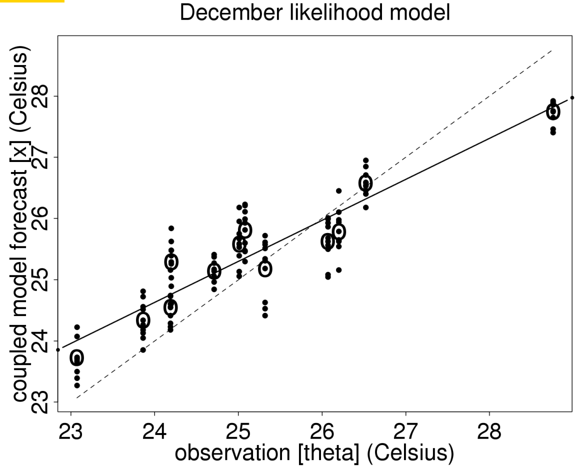
Prior: $Y \sim N(Y_b, C)$

bias

Likelihood:

calibration

$$G = S_{XY} S_{YY}^{-1}$$

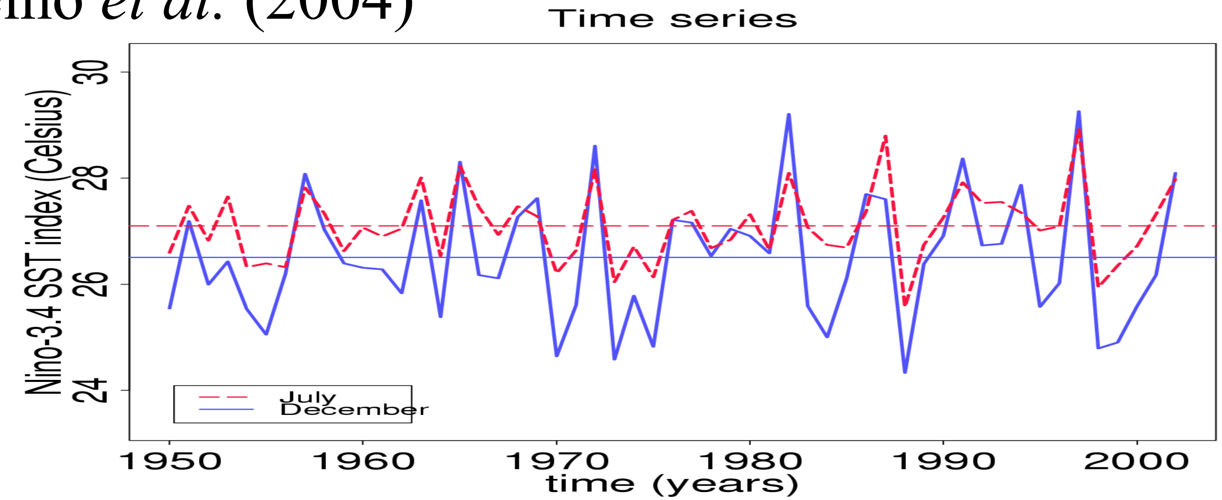


Posterior:

Coelho (2005)
Stephenson *et al.* (2005)

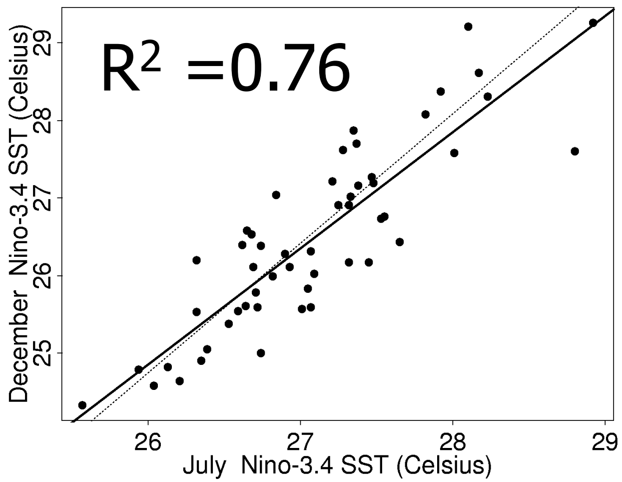
Nino-3.4 index observational data

Coelho *et al.* (2004)



Nino-3.4 index mean values:
Jul: 27.1°C
Dec: 26.5°C
r: 0.87

July and December Reynolds OI V2 SST (1950-2001)



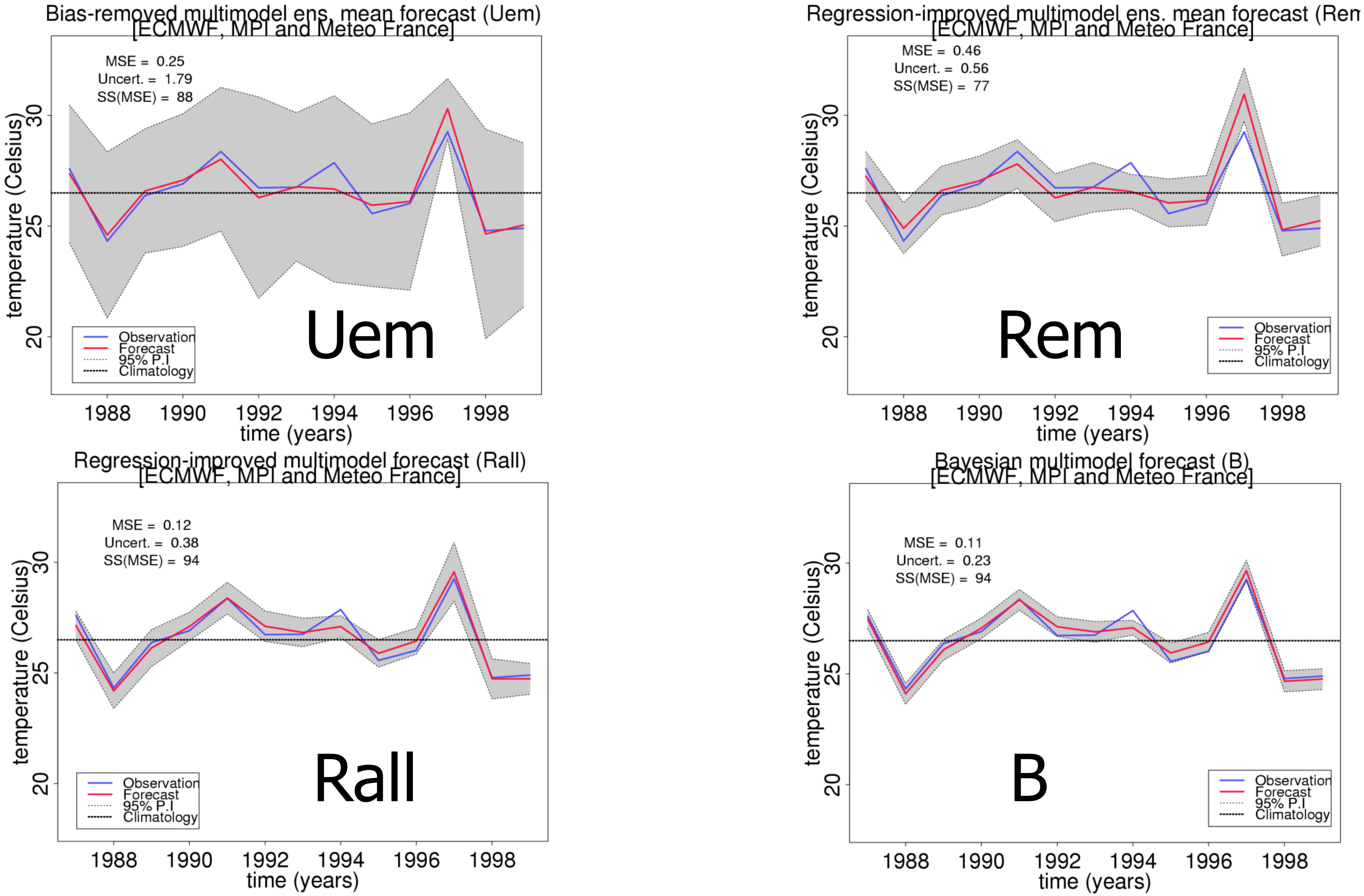
$$Y_t \mid \psi_t \sim N(\beta_o + \beta_1 \psi_t, \sigma_{0t}^2) \qquad \beta_o = -14.14°C \quad \beta_1 = 1.50$$

Y_t = December Nino − 3.4 index for the year t

ψ_t = July Nino − 3.4 index for the year t

$$\therefore \text{Prior}: \; Y \sim N(\mu_{ot}, \sigma_{ot}^2) \qquad \mu_{ot} = \beta_o + \beta_1 \psi_t$$

All combined forecasts



Why are Rall and B so similar?

Combined forecasts in Bayesian notation

Forecast	Likelihood	Prior	Posterior
Uem			
Rem		$Y \sim N(\overline{Y}, S_{YY})$	
Rall		$Y \sim N(\overline{Y}, S_{YY})$	
B		$Y \sim N(\mu_{ot}, \sigma^2_{ot})$	

Bayesian combination

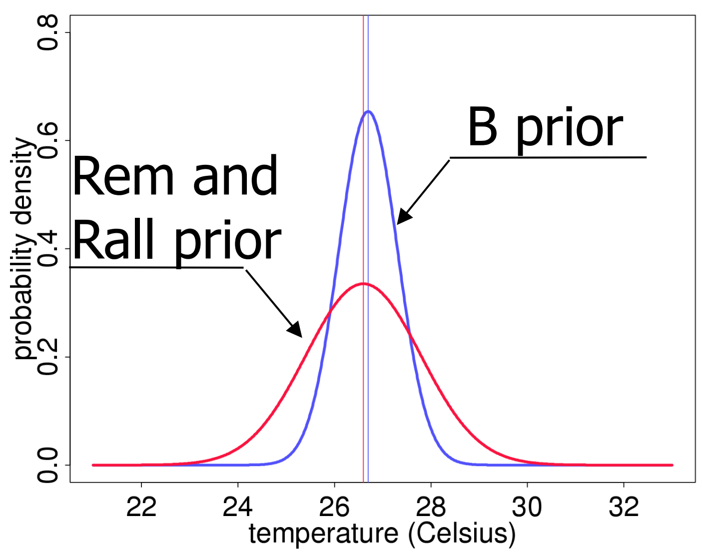
Likelihood:

Prior: $Y \sim N(Y_b, C)$

Regression of Y on X

$$Y = HX + \xi$$

$$H = S_{YX} S_{XX}^{-1}$$



Skill and uncertainty measures

Forecast	MSE $(°C)^2$	Skill Score (%)	Uncert. $(°C)^2$
Climatology	1.98	0	1.20
ECMWF (raw)	1.64	18	0.49
ECMWF (bias-removed)	0.18	91	0.49
ECMWF (regression-improved)	0.22	89	0.47
MF (raw)	0.27	86	0.39
MF (bias-removed)	0.22	89	0.39
MF (regression-improved)	0.20	90	0.46
MPI (raw)	15.24	-669	0.49
MPI (bias-removed)	1.52	23	0.49
MPI (regression-improved)	1.06	46	0.86
Uem	0.25	88	1.79
Rem	0.46	77	0.56
Rall	0.12	94	0.38
B	0.11	94	0.23

Skill Score = [1- MSE/MSE(climatology)]*100%

Summary

- Statistical calibration and combination is analogous to data assimilation and is a fundamental and essential part of the forecasting process: *forecast assimilation* (see Stephenson et al. 2005 for more details)

- Forecast can be combined and calibrated in several different ways

- Combined and calibrated forecasts have better skill than individual forecasts

- Rall and B had best skill for Nino-3.4 example

- Inclusion of very biased forecast has not deteriorated combination

Additional information

•**Coelho C.A.S.**, 2005: "*Forecast Calibration and Combination: Bayesian Assimilation of Seasonal Climate Predictions*". PhD Thesis. University of Reading, 178 pp.

•Stephenson, D. B., **C.A.S. Coelho**, F. J. Doblas-Reyes, and M. Balmaseda, 2005: *"Forecast Assimilation: A Unified Framework for the Combination of Multi-Model Weather and Climate Predictions." Tellus A,* Vol. **57**, 253-264.

• **Coelho C.A.S.**, S. Pezzulli, M. Balmaseda, F. J. Doblas-Reyes and D. B. Stephenson, 2004: "*Forecast Calibration and Combination: A Simple Bayesian Approach for ENSO". Journal of Climate*. Vol. **17**, No. 7, 1504-1516.

• **Coelho C.A.S**., S. Pezzulli, M. Balmaseda, F. J. Doblas-Reyes and D. B. Stephenson, 2003: "*Skill of Coupled Model Seasonal Forecasts: A Bayesian Assessment of ECMWF ENSO Forecasts". ECMWF Technical Memorandum* No. 426, 16pp.

Available at http://www.cptec.inpe.br/~caio