



The Abdus Salam  
International Centre for Theoretical Physics



2164-5

**Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the  
Quantum Regime**

*6 - 10 September 2010*

**From Cavity Opto-Mechanics to Quantum Phase-Transitions**

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# From Cavity Opto-Mechanics to Quantum Phase-Transitions

Tilman Esslinger ETH Zürich

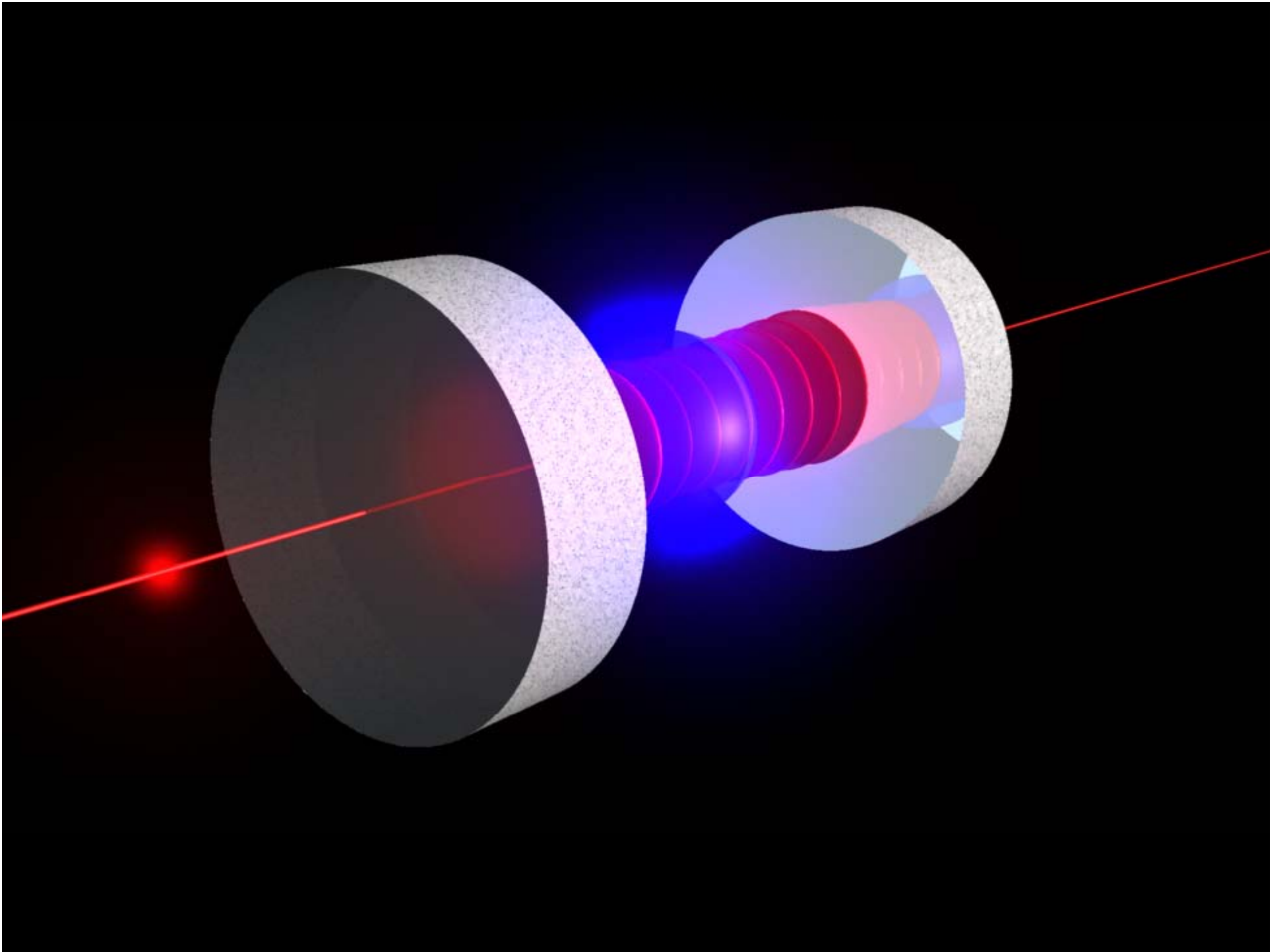
Funding: ETH, EU (ERC, NameQuam, Scala), QSIT, SNF

[www.quantumoptics.ethz.ch](http://www.quantumoptics.ethz.ch)



Quantum Gases ↔ Quantum Optics

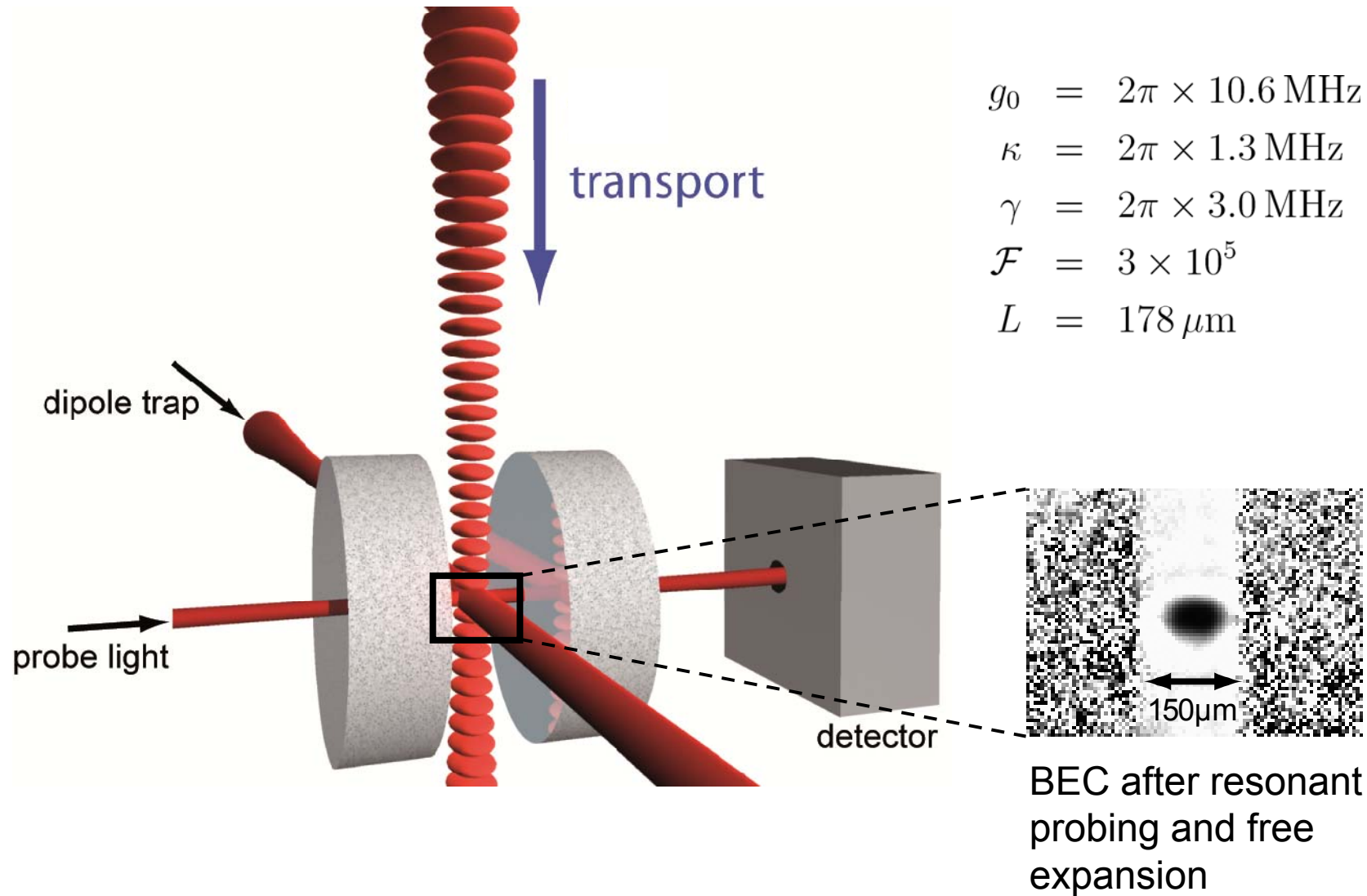




Cavity opto-mechanics

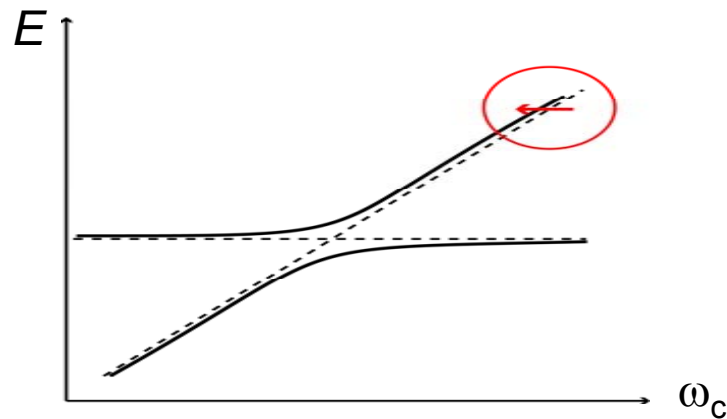
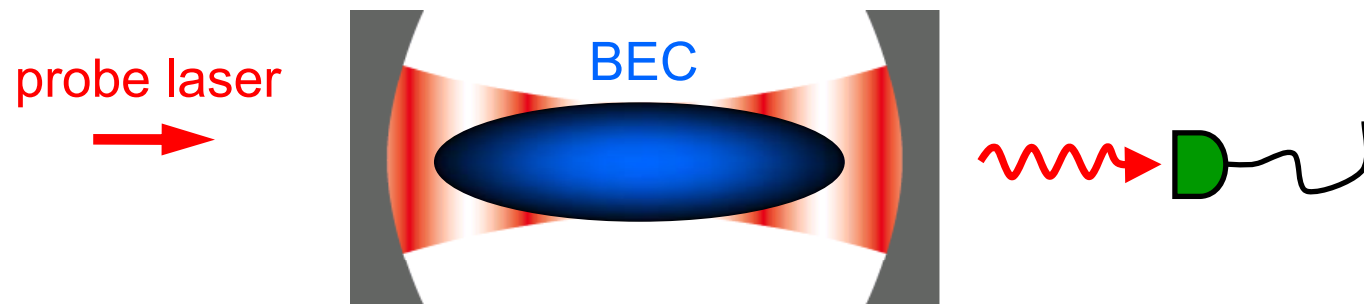
Dicke Quantum Phase Transition

# Experimental setup



See also: Zimmermann, Hemmerich, Stamper-Kurn, Reichel

# Spectroscopy

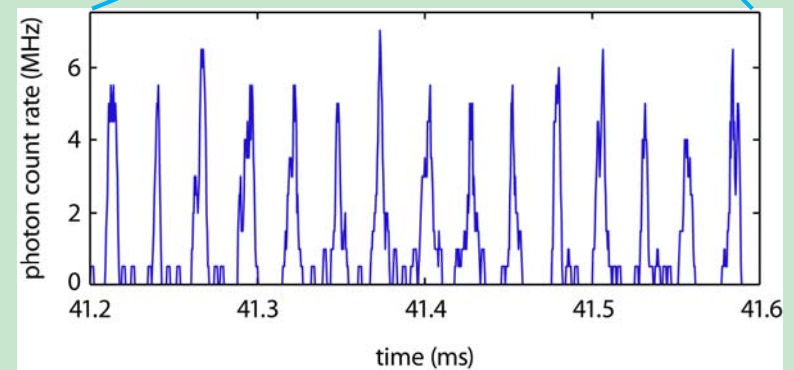
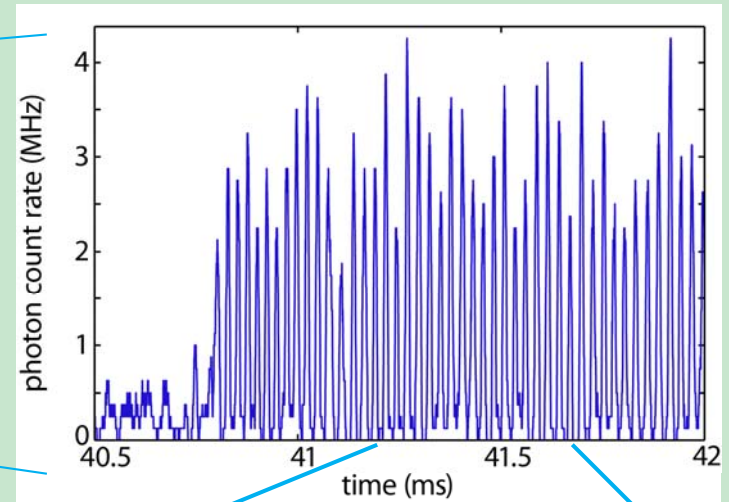
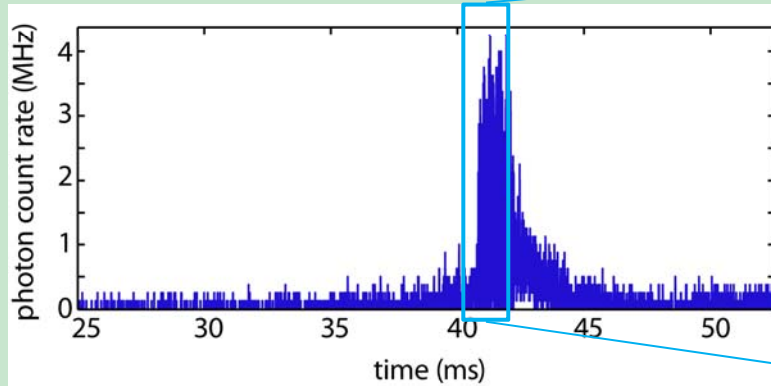


The bare cavity resonance frequency is refractively shifted by BEC

See also: D. Stamper-Kurn group: PRL 99, 213601 (2007), Nat. Phys. 4, 561 (2008)

# BEC Cavity Dynamics

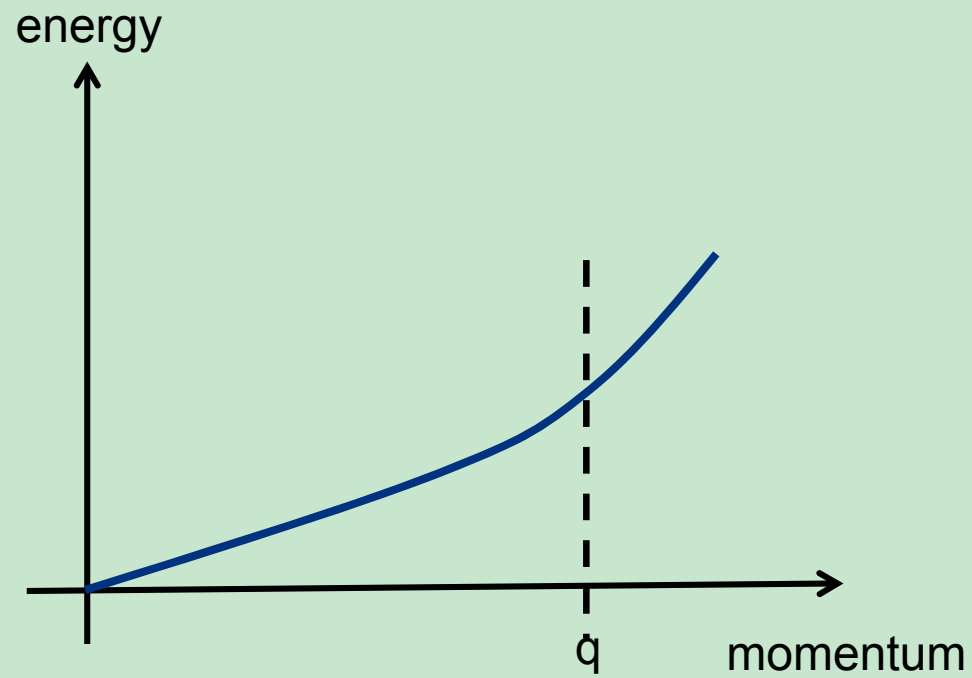
BEC



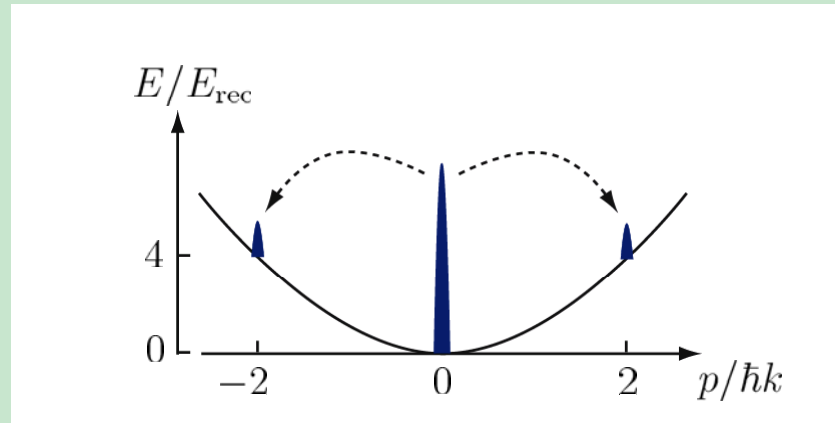
**35 kHz – 25 kHz**



# Coupling BEC-Cavity



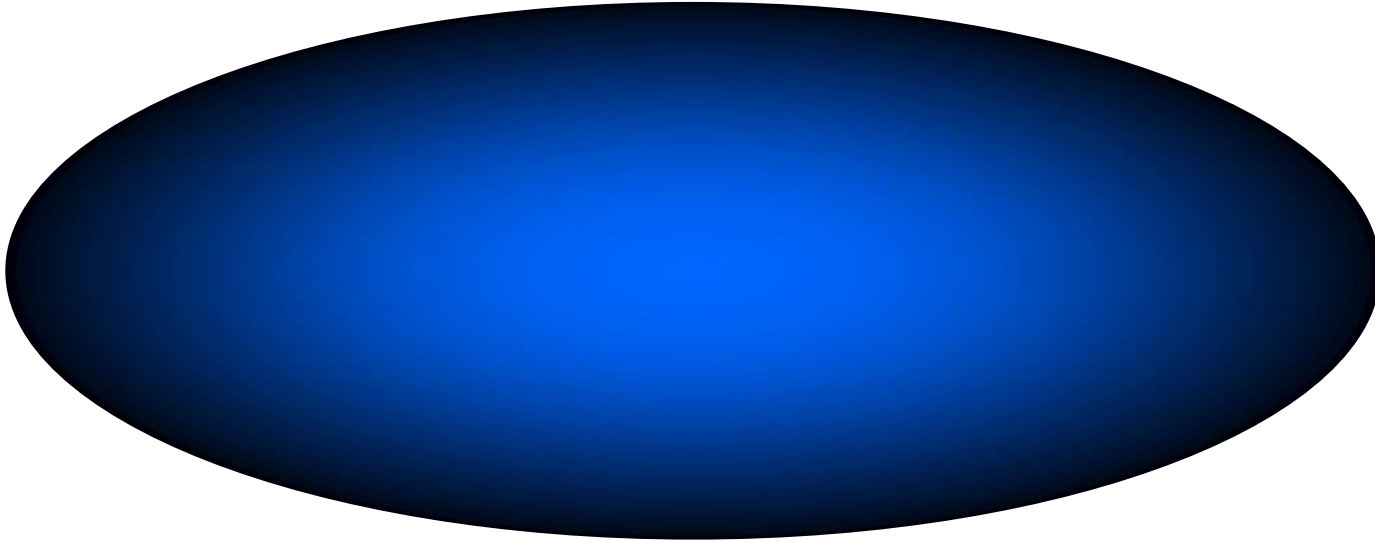
# BEC as a mechanical oscillator



$$\Psi(x, t) = c_0(t) |p = 0\rangle + c_2(t) |p = \pm 2\hbar k\rangle$$

$$|\Psi(x, t)|^2 \approx N + c_2 \sqrt{N} \cos(2kx) \cos(4\omega_r t)$$

# BEC as a mechanical oscillator



# Cavity opto-mechanics with a BEC

$$H = \int \widehat{\Psi}^\dagger(x) \left( \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \hbar U \cos^2(kx) \hat{a}^\dagger \hat{a} \right) \Psi(x) dx - \Delta \hbar \hat{a}^\dagger \hat{a} - H_{in/out}$$



$$\Psi(x) = \sqrt{N} |p=0\rangle + \hat{c}_2 |\pm 2\hbar k\rangle$$



$$H = 4\hbar\omega_{rec} \hat{c}^\dagger \hat{c} - \Delta \hbar \hat{a}^\dagger \hat{a} + \hbar g (\hat{c} + \hat{c}^\dagger) \hat{a}^\dagger \hat{a} - H_{in/out}$$

# Cavity opto-mechanics with a BEC


Full Hamiltonian





two mode description



$$H = 4\hbar\omega_{rec} \hat{c}^\dagger \hat{c} - \Delta\hbar \hat{a}^\dagger \hat{a} + \hbar g (\hat{c} + \hat{c}^\dagger) \hat{a}^\dagger \hat{a} - H_{in/out}$$

  
mechanical  
mode

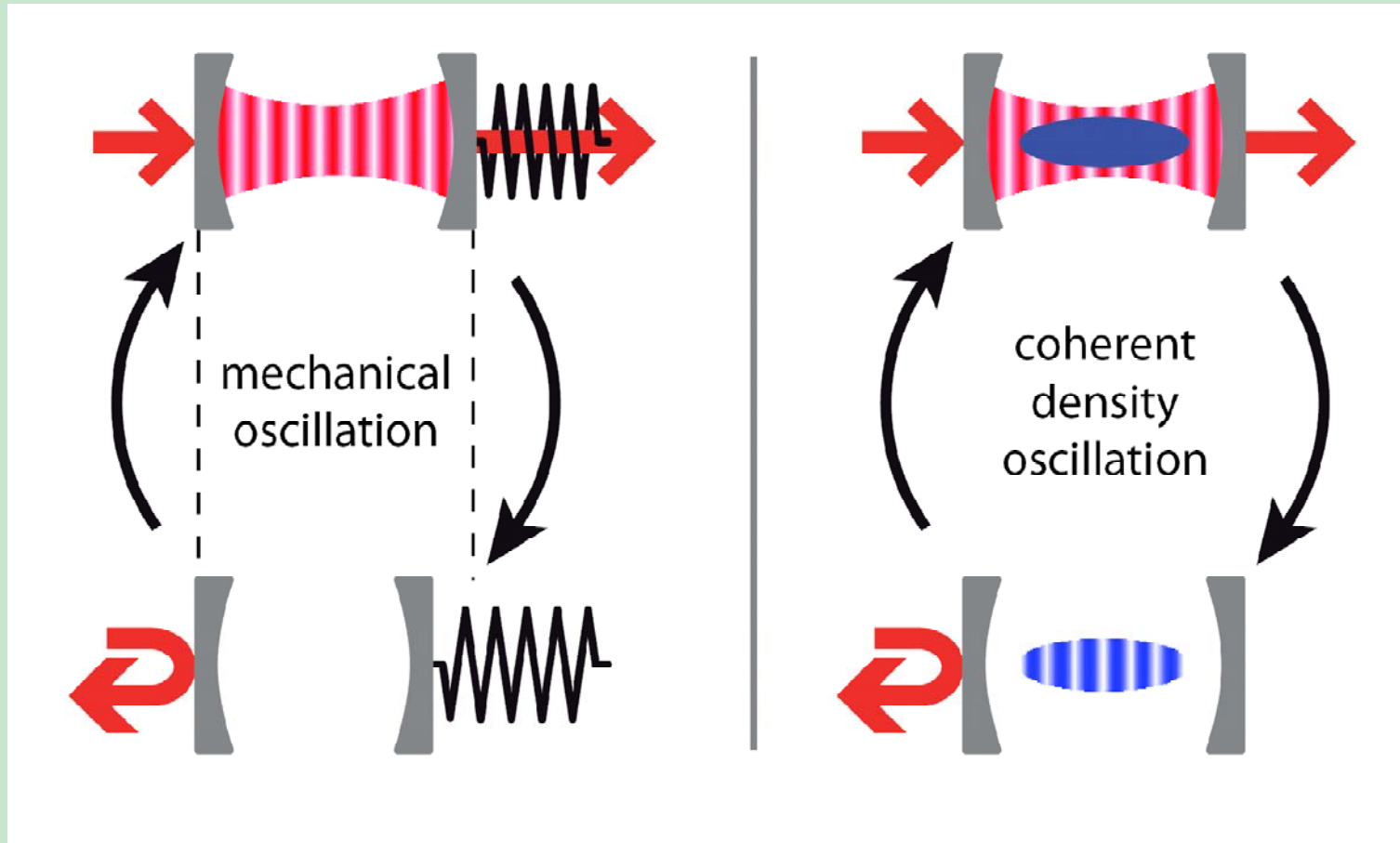
  
optical  
mode

  
strong coupling:  
 $g \approx 0.3 \kappa$

  
pump/decay



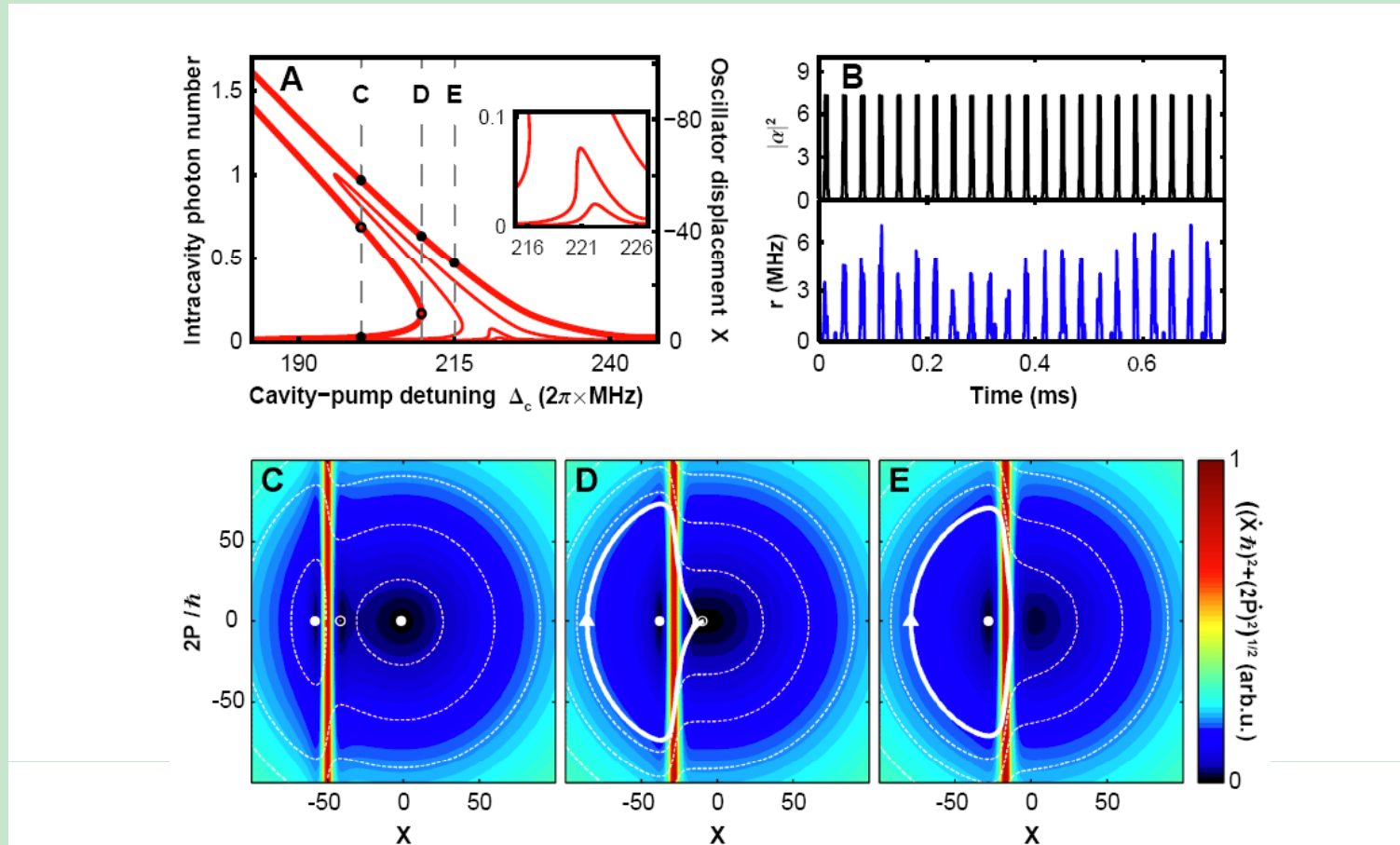
# Cavity opto-mechanics with a BEC



F. Brennecke, S. Ritter, T. Donner, T. Esslinger, Science 322, 235 (2008)

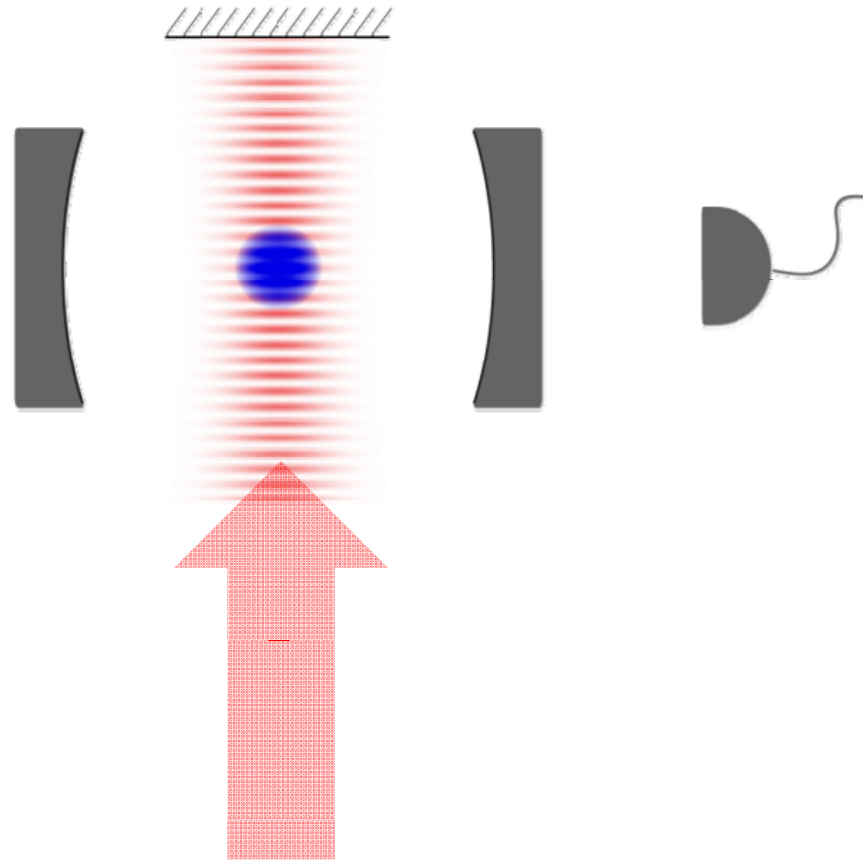
See also: Stamper-Kurn (Berkeley), Nature Physics 2008

# Dynamics of BEC cavity system



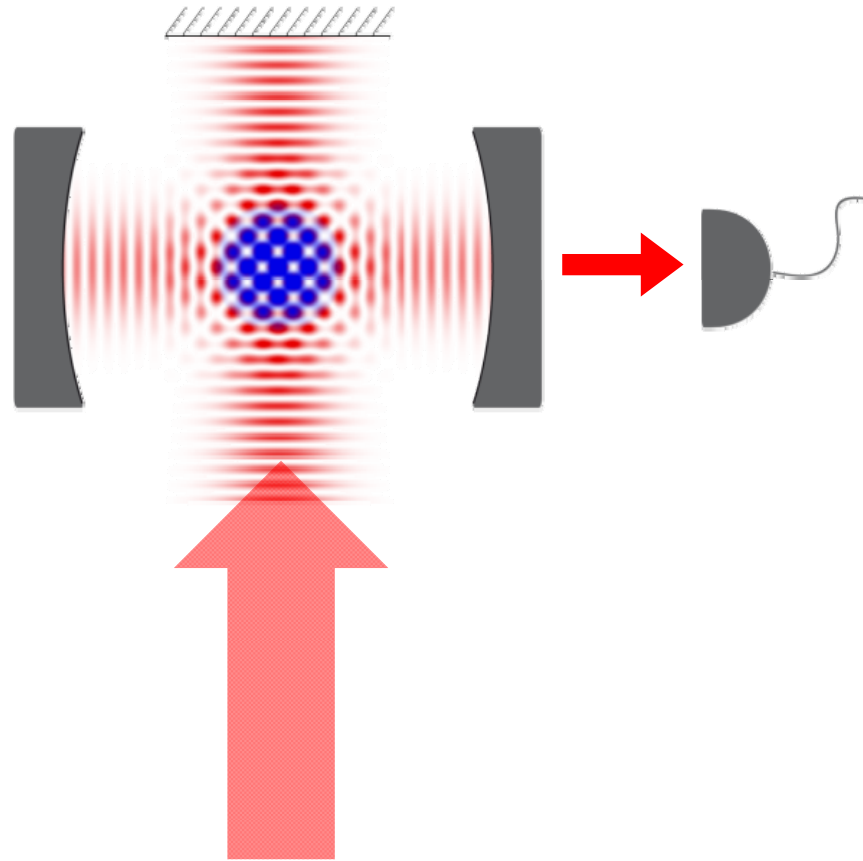
F. Brennecke, S. Ritter, T. Donner, T. Esslinger, Science 322, 235 (2008)

# Transverse Pumping



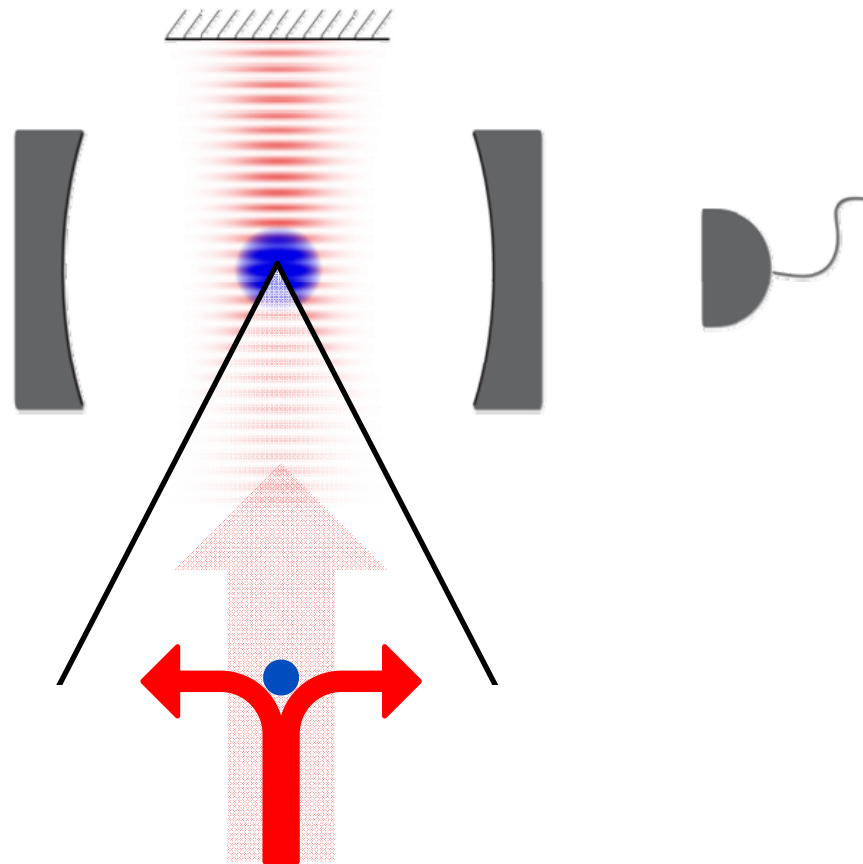
Theory: H. Ritsch, P. Domokos, Exp. with thermal atoms: V. Vuletic

# Phase Transition



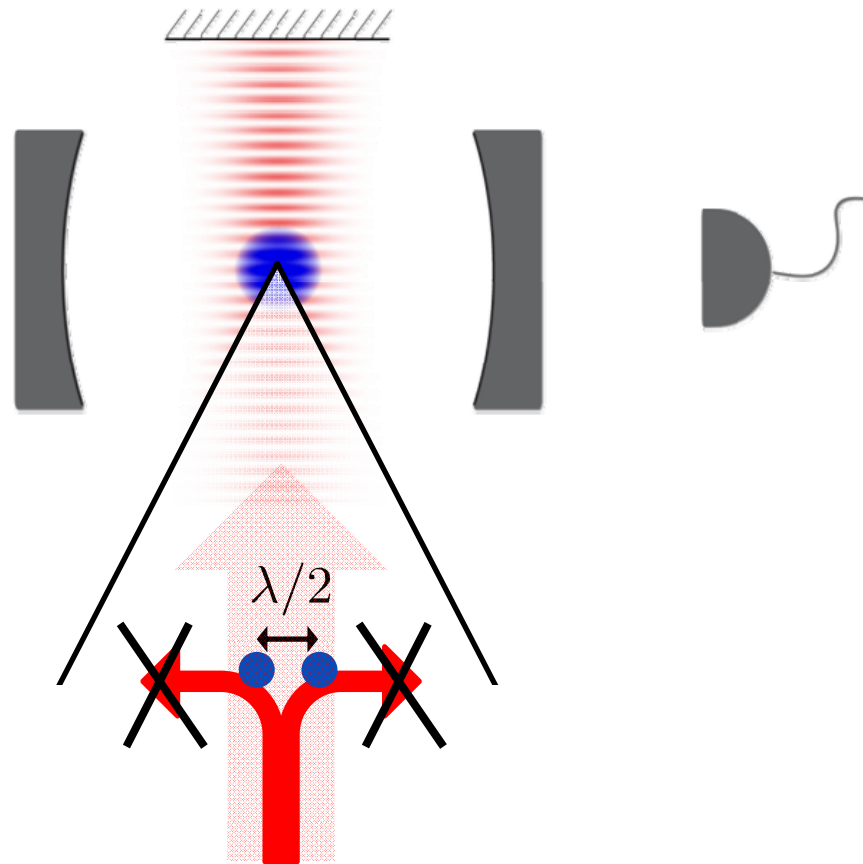
Theory: H. Ritsch, P. Domokos, Exp. with thermal atoms: V. Vuletic

# Scattering from a single atom

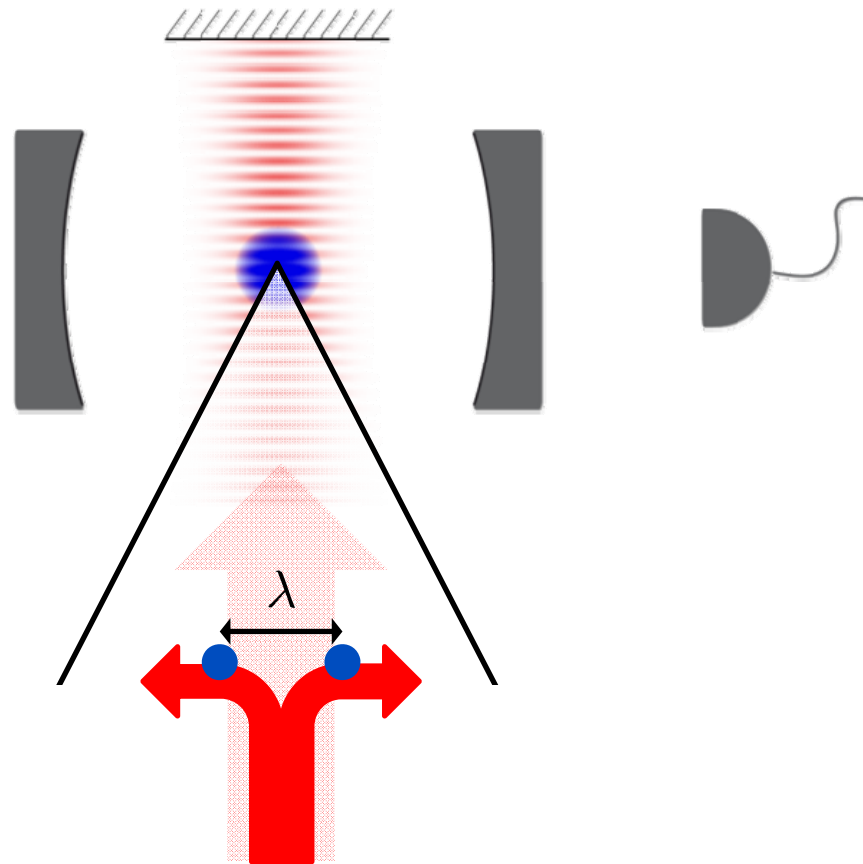




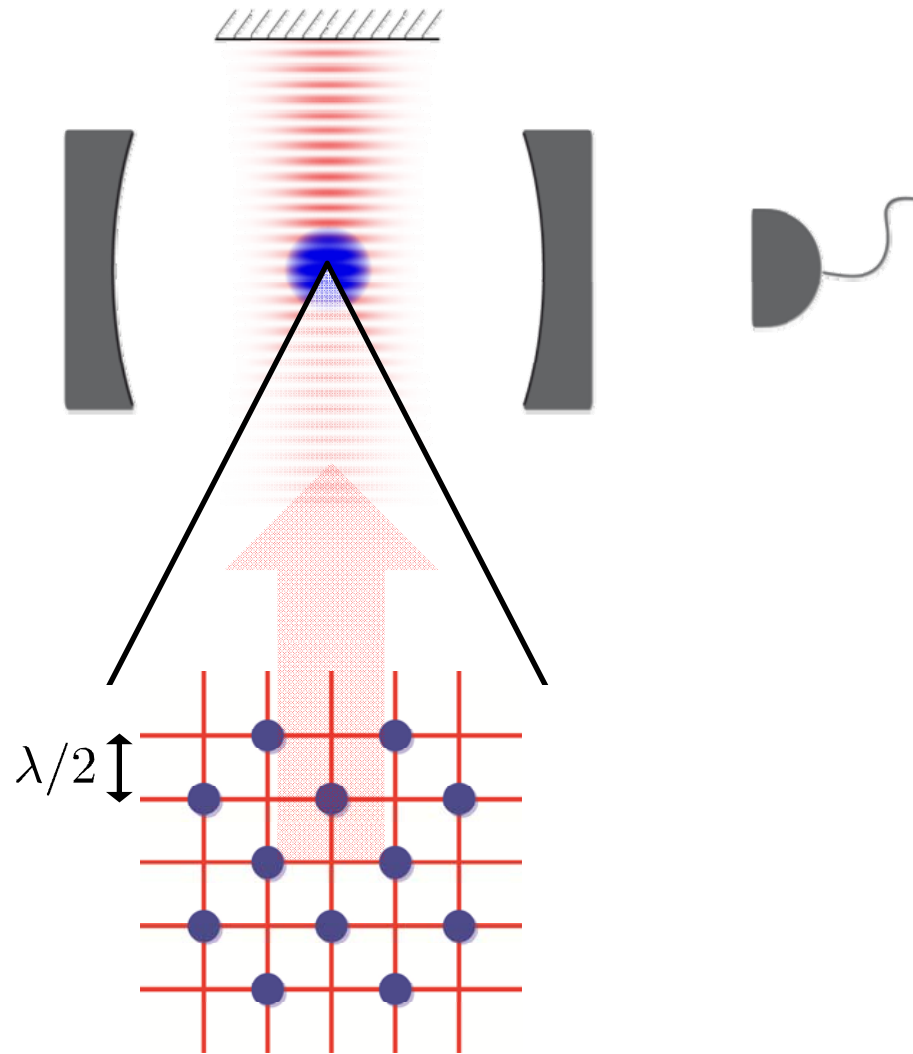
# Scattering from two atoms: Interference



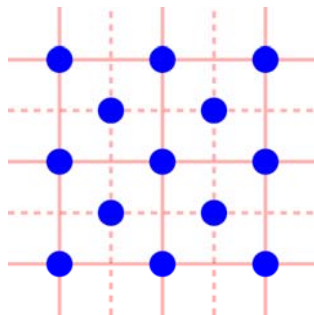
# Scattering from two atoms: Interference



# Self-organization

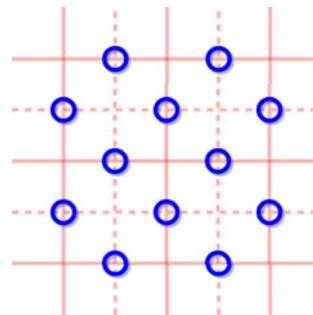


# Symmetry-breaking



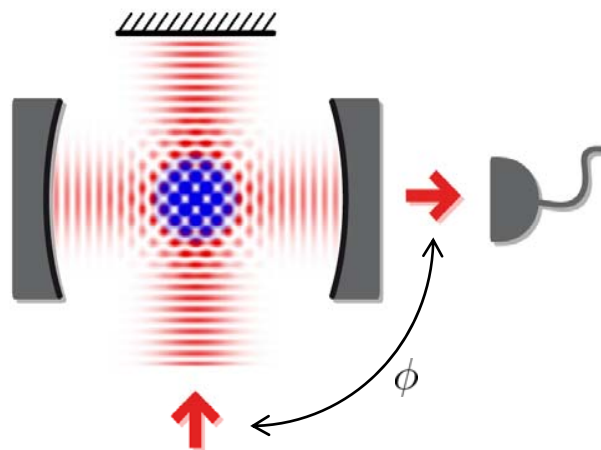
even

$$\phi = 0$$

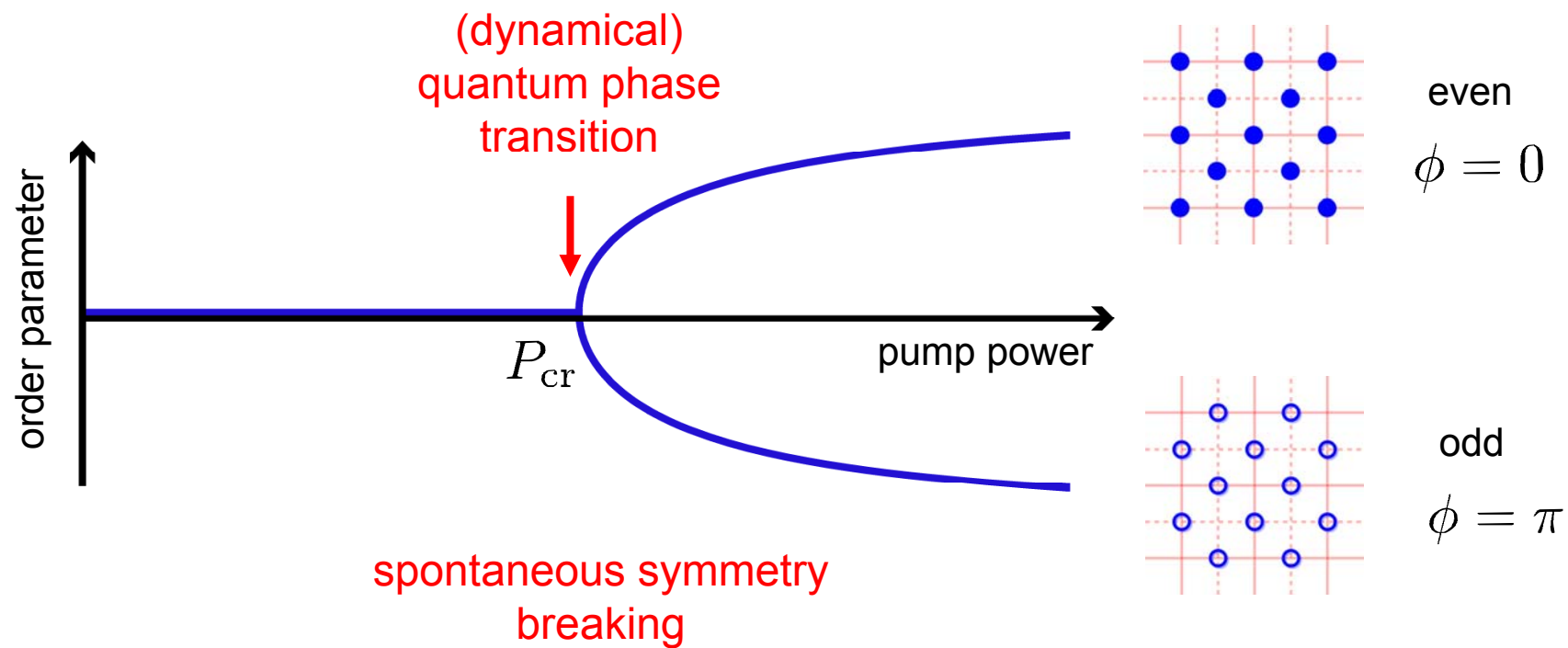


odd

$$\phi = \pi$$

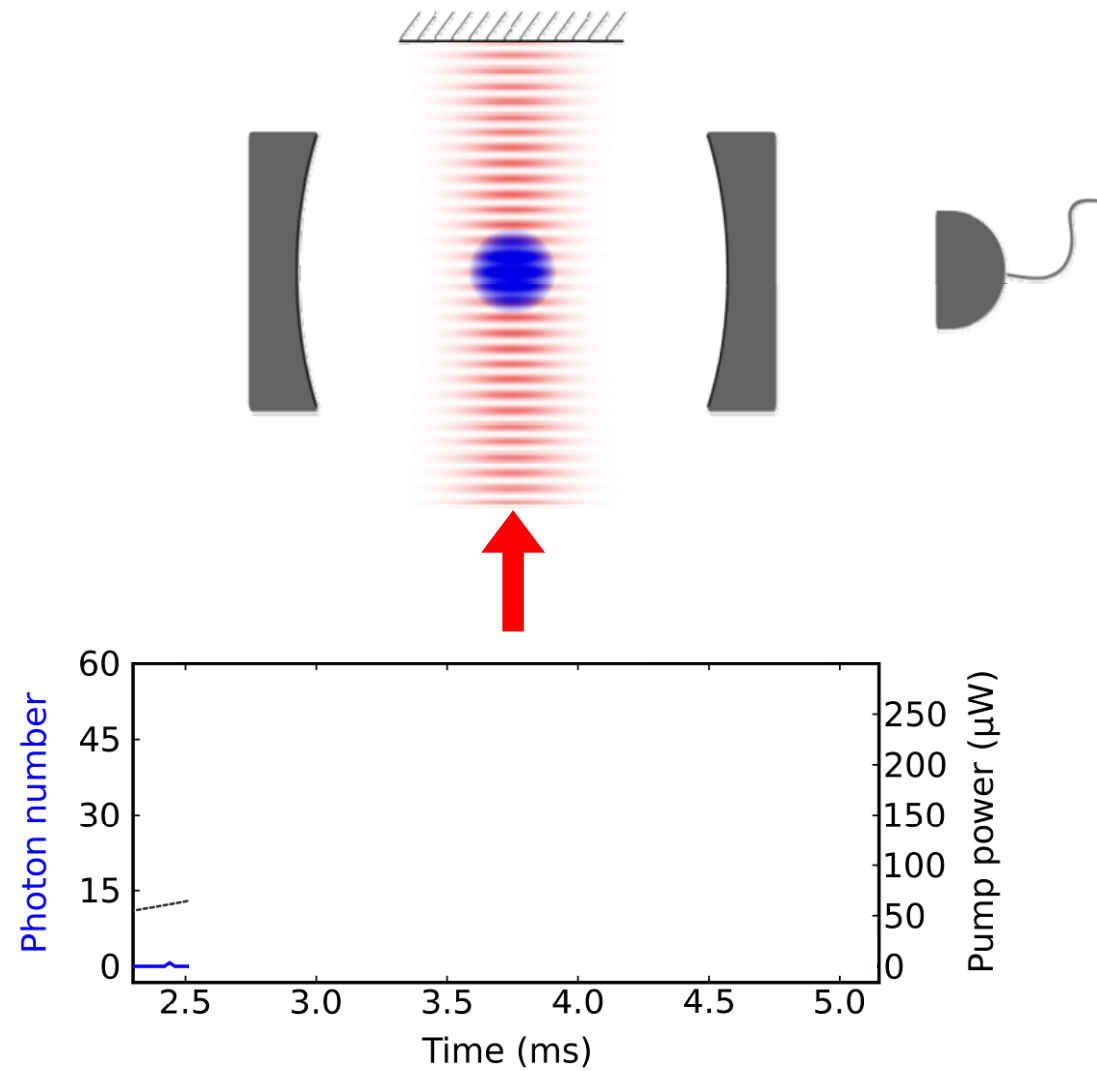


# Self-organization

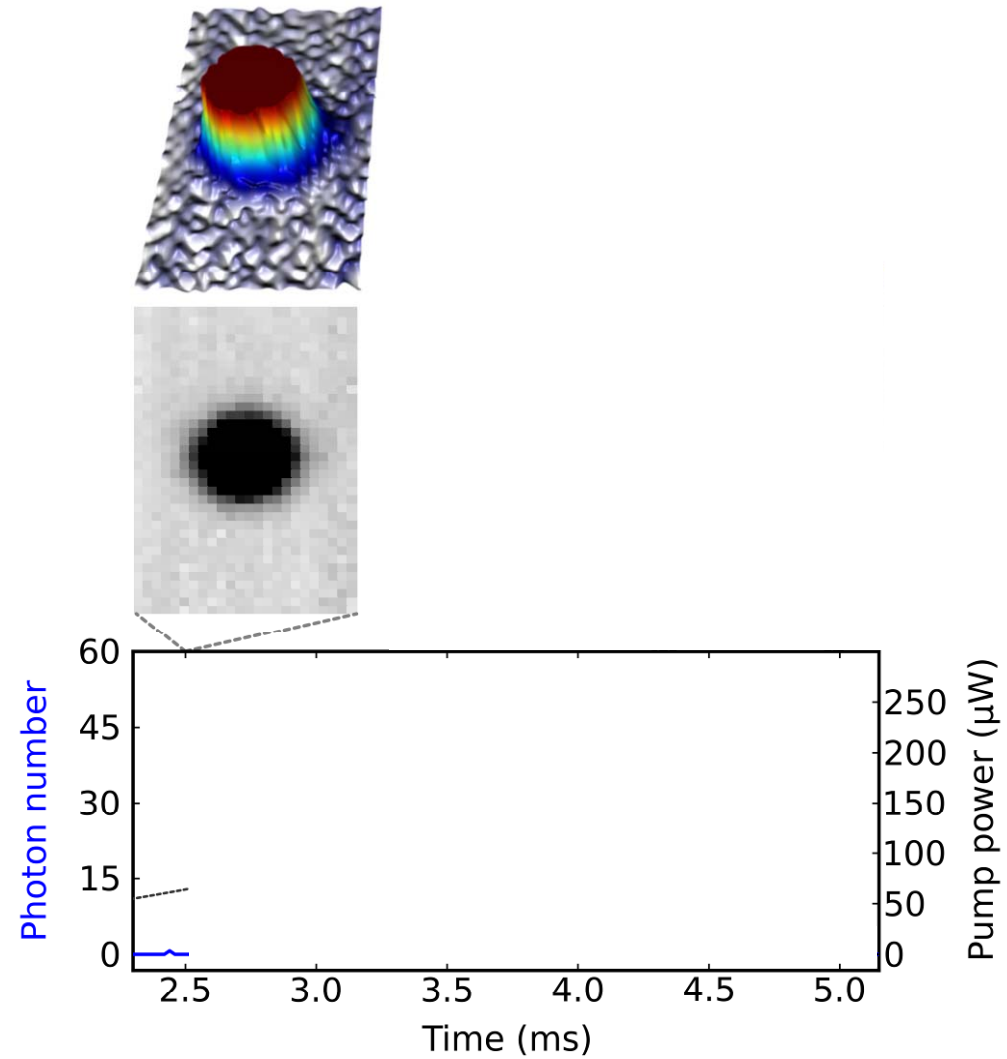




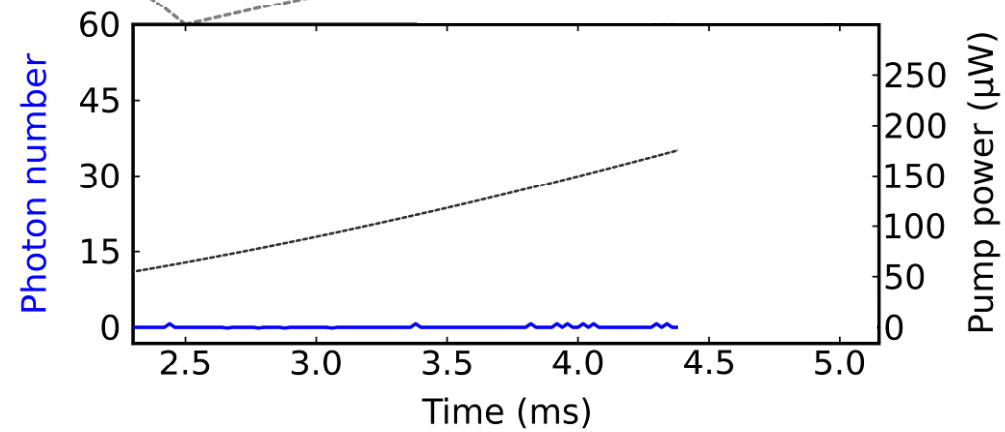
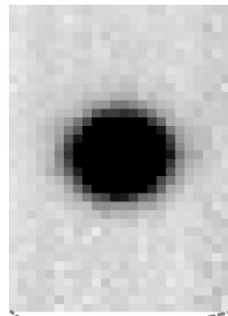
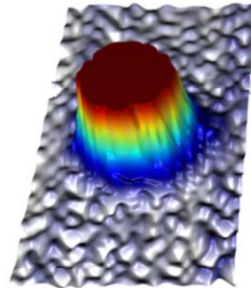
# Observing Self-Organization



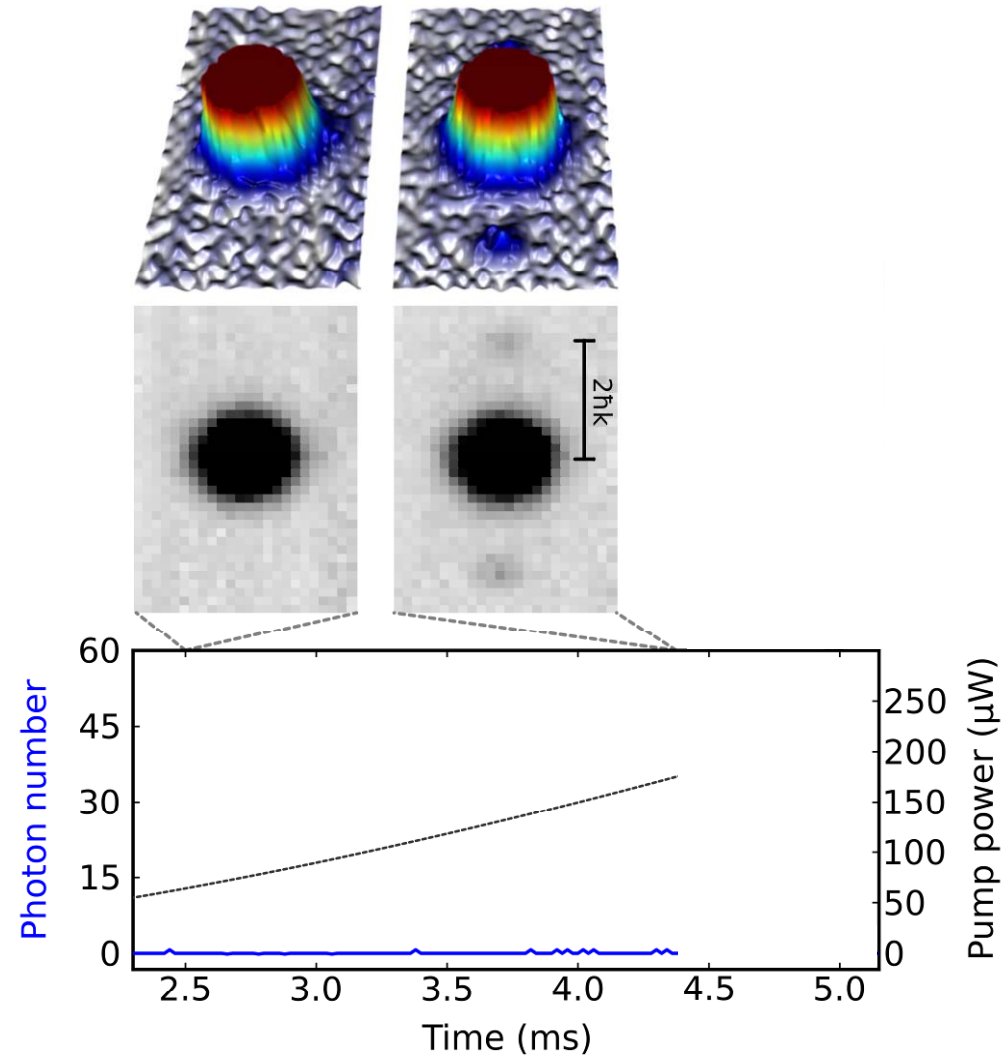
# Observing Self-Organization



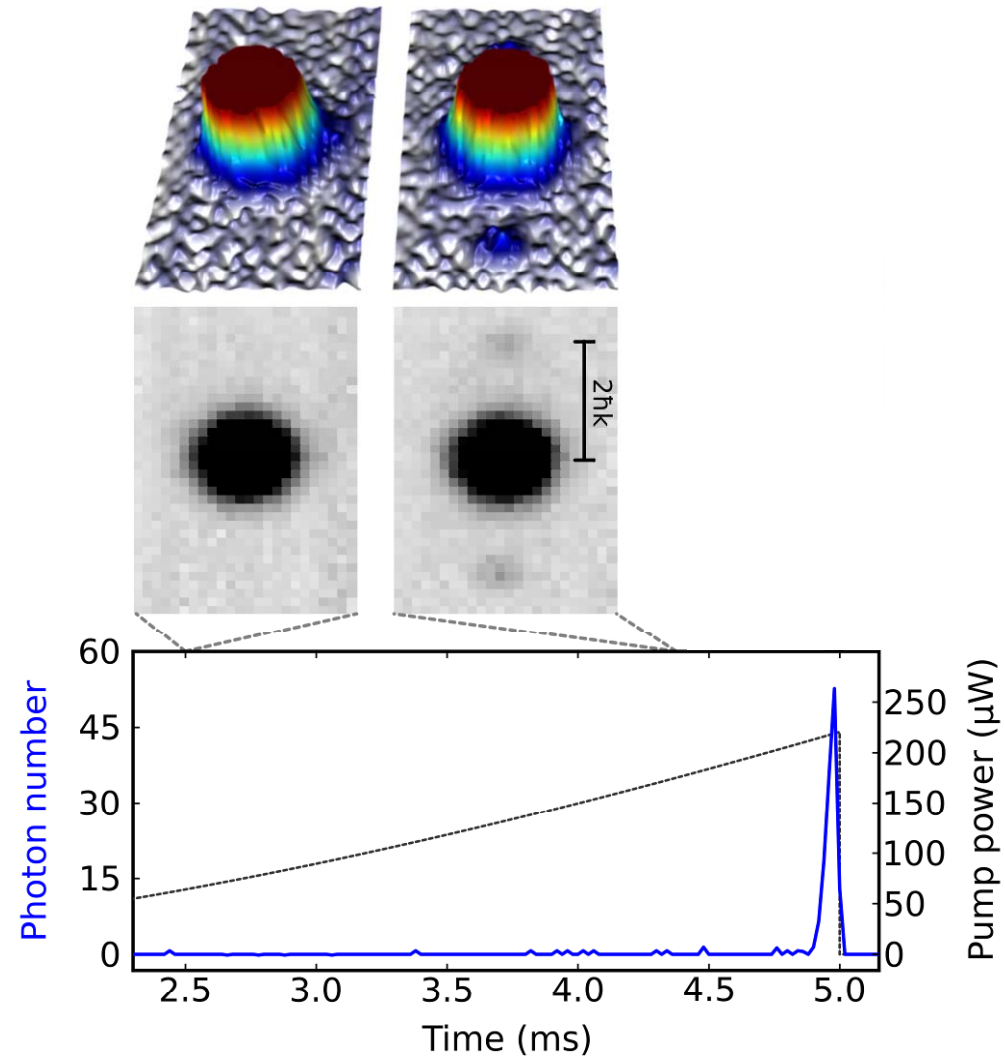
# Observing Self-Organization



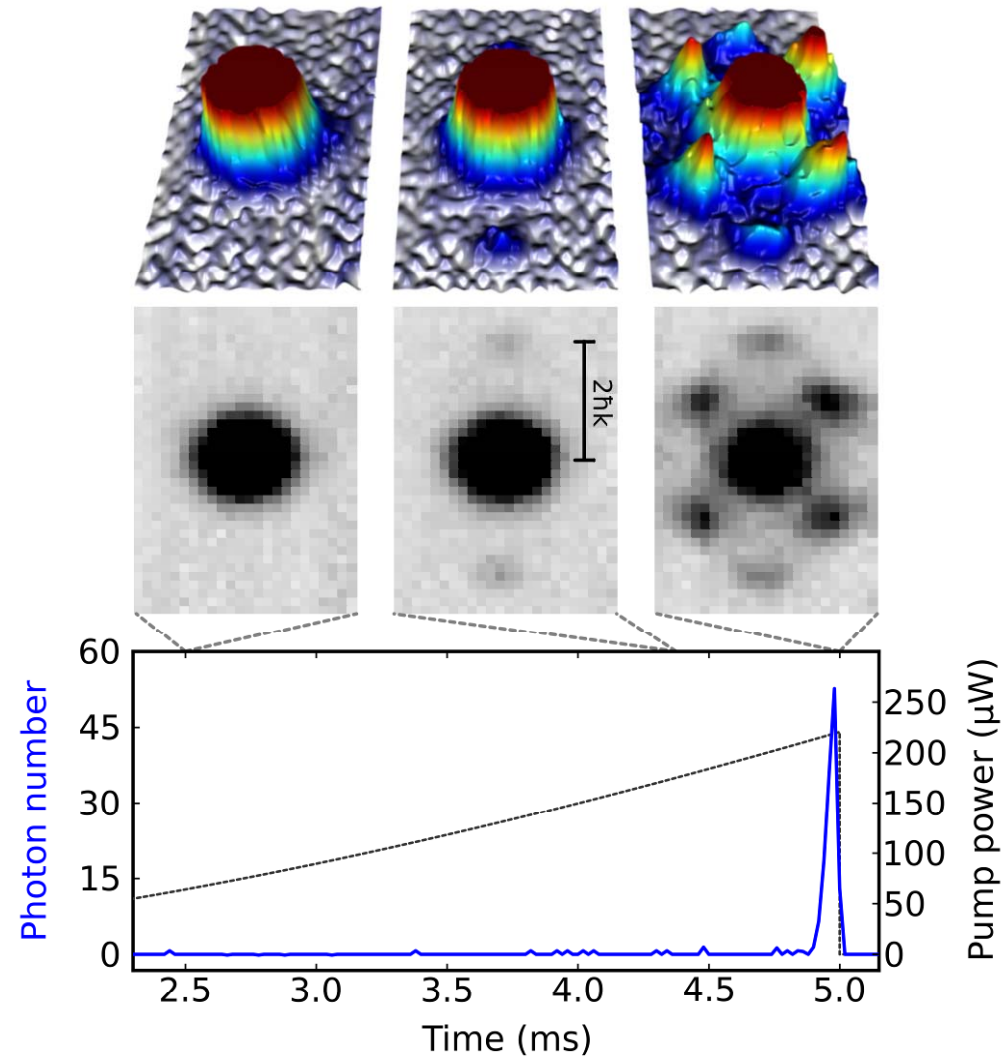
# Observing Self-Organization



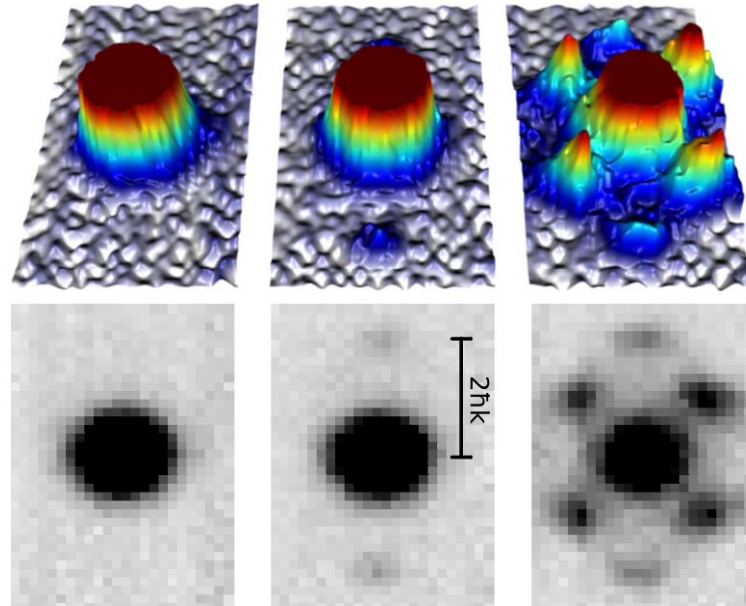
# Observing Self-Organization



# Observing Self-Organization



# Observing Self-Organization



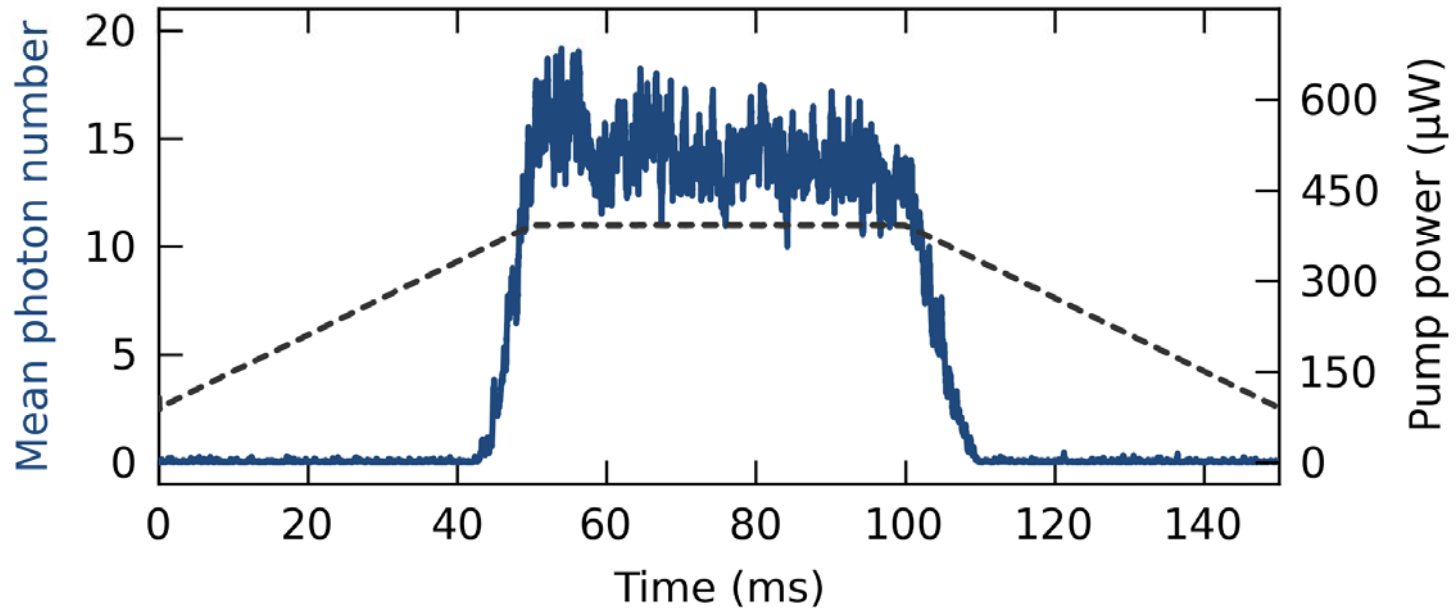
Coexistence of:

- non-trivial diagonal long-range order
- off-diagonal long-range order



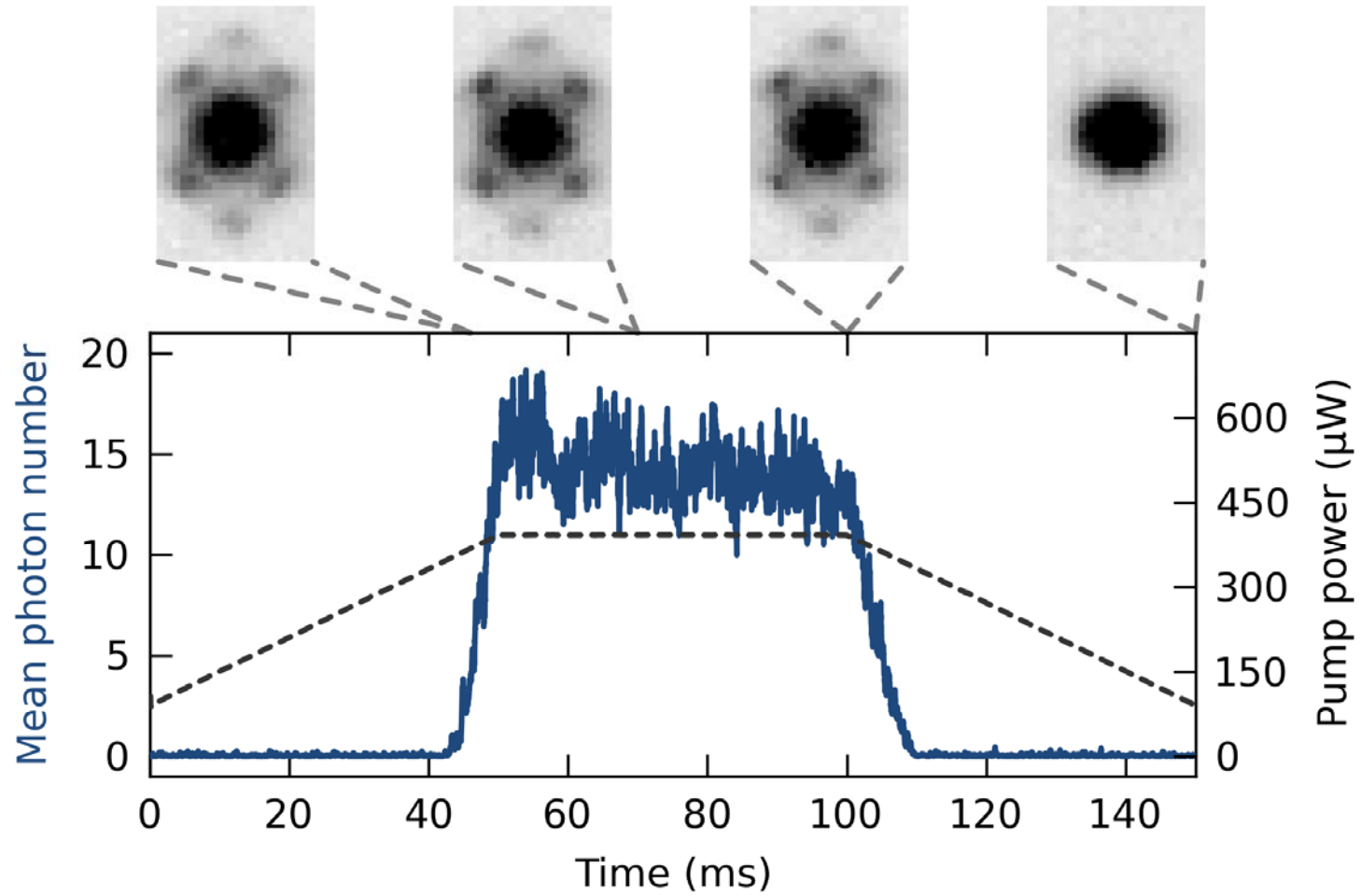
The atoms can be regarded as a Supersolid

# Stability





# Stability and Dephasing



PHYSICAL REVIEW A **75**, 013804 (2007)

**Proposed realization of the Dicke-model quantum phase transition  
in an optical cavity QED system**

F. Dimer,<sup>1</sup> B. Estienne,<sup>2</sup> A. S. Parkins,<sup>3,\*</sup> and H. J. Carmichael<sup>1</sup>

<sup>1</sup>*Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand*

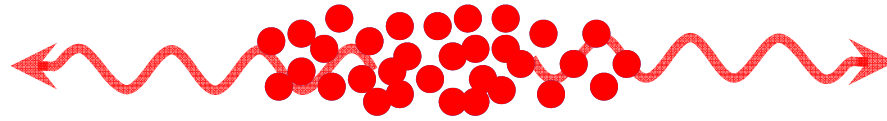
<sup>2</sup>*Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie, 4 place Jussieu,  
F-75252 Paris Cedex 05, France*

<sup>3</sup>*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA*  
(Received 18 July 2006; published 8 January 2007)

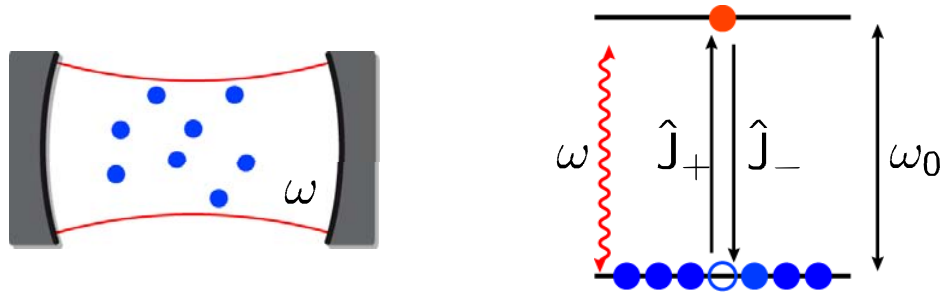


**“molecules interacting with a  
common radiation field cannot  
be treated as independent”  
*R.H. Dicke (1953)***

# Super-radiance



# Dicke Model



$$\hat{H}_{\text{Dicke}} = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{J}_z + \frac{\lambda}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) (\hat{J}_+ + \hat{J}_-)$$

On the Superradiant Phase Transition for  
Molecules in a Quantized Radiation Field:  
the Dicke Maser Model

KLAUS HEPP

Physics Department, ETH, Zürich, 8049 Switzerland

AND

ELLIOTT H. LIEB\*

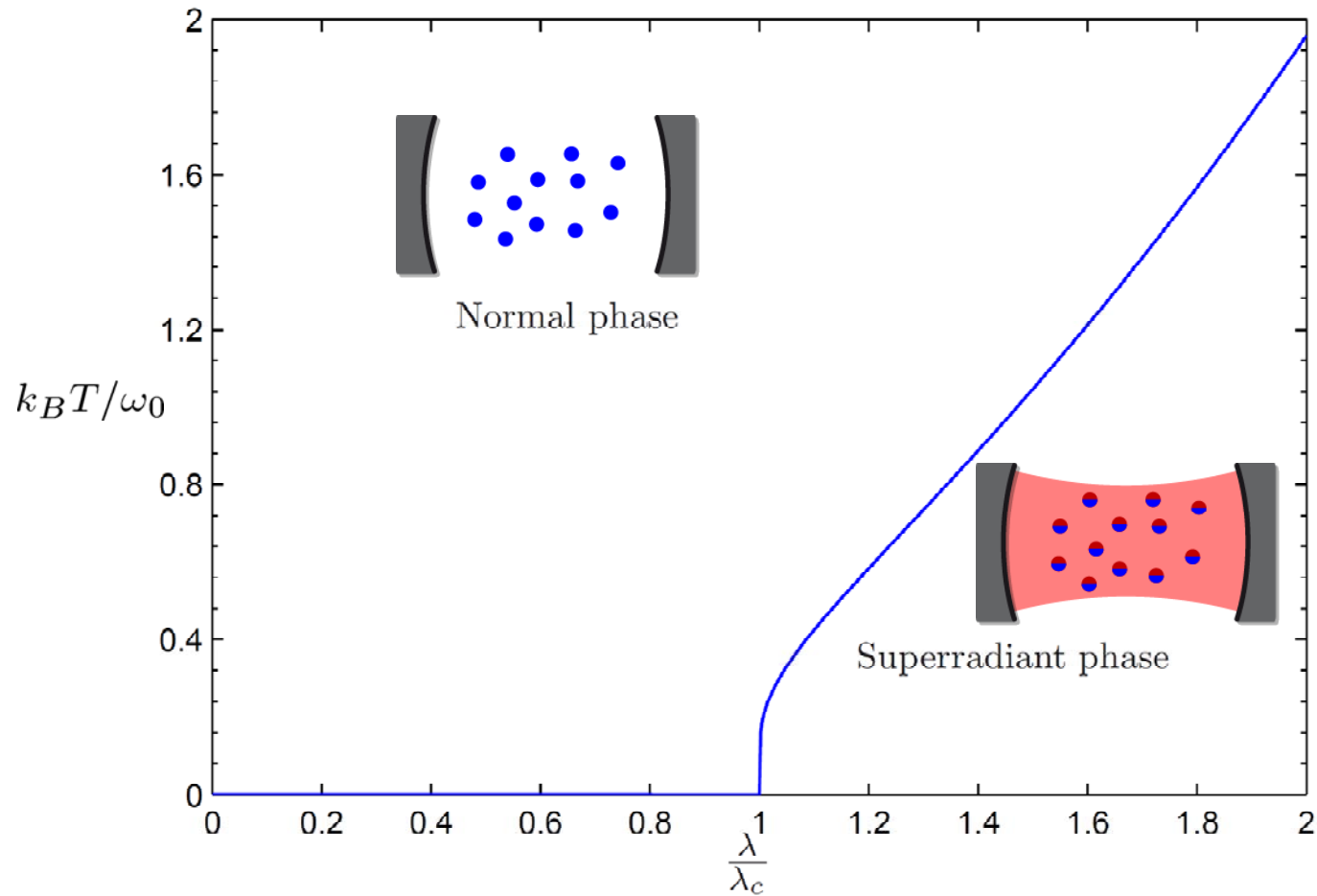
*Mathematics Department, MIT, Cambridge, Mass. 02139, USA*

A system of  $N$  two-level molecules coupled to finitely many modes of a quantized radiation field via a truncated dipolar interaction is investigated. The thermodynamic and correlation functions can be exactly computed in the limit  $N \rightarrow \infty$ . The system exhibits a second order phase transition from normal to superradiance. Different effective Hamiltonians with linear Heisenberg equations of motion become asymptotically exact in the limit  $N \rightarrow \infty$ .

ANNALS OF PHYSICS: 76, 360–404 (1973)

# Super-radiant phase transition

$$\hat{H}_{\text{Dicke}} = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{J}_z + \frac{\lambda}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) (\hat{J}_+ + \hat{J}_-)$$



$$\lambda_c = \sqrt{\omega \omega_0} / 2$$

Recent work: T. Brandes, ...

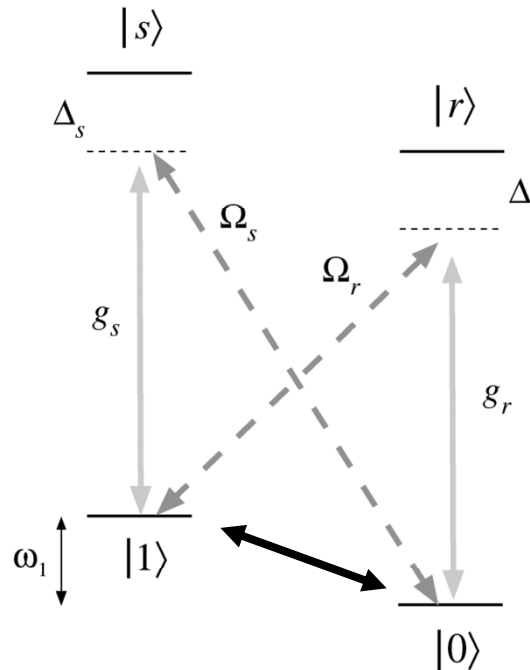
## Proposed realization of the Dicke-model quantum phase transition in an optical cavity QED system

F. Dimer,<sup>1</sup> B. Estienne,<sup>2</sup> A. S. Parkins,<sup>3,\*</sup> and H. J. Carmichael<sup>1</sup>

<sup>1</sup>*Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand*

<sup>2</sup>*Laboratoire de Physique Théorique et Hautes Energies, Université Pierre et Marie Curie, 4 place Jussieu, F-75252 Paris Cedex 05, France*

<sup>3</sup>*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125, USA*  
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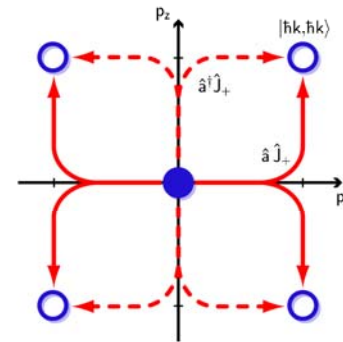
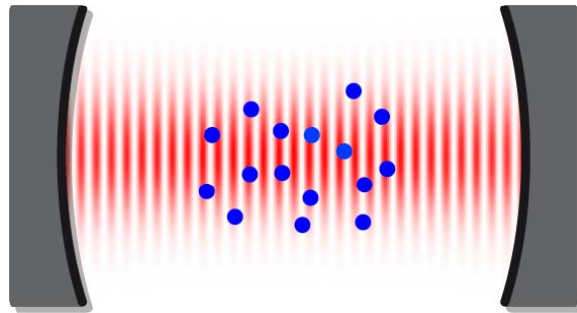


described by the  
Dicke Hamiltonian



# Dicke Model

$$\hat{H}_{\text{Dicke}} = \omega \hat{a}^\dagger \hat{a} + \omega_0 \hat{J}_z + \frac{\lambda}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) (\hat{J}_+ + \hat{J}_-)$$

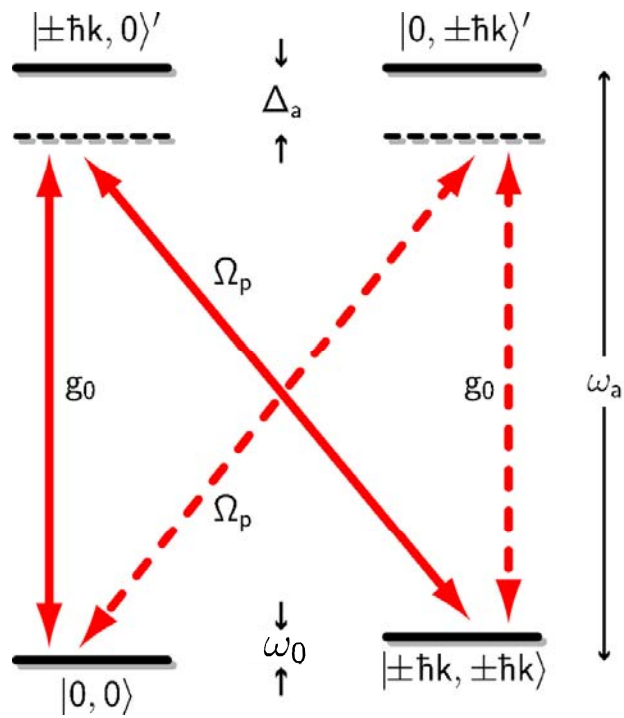


K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Nature 464, 1301 (2010)

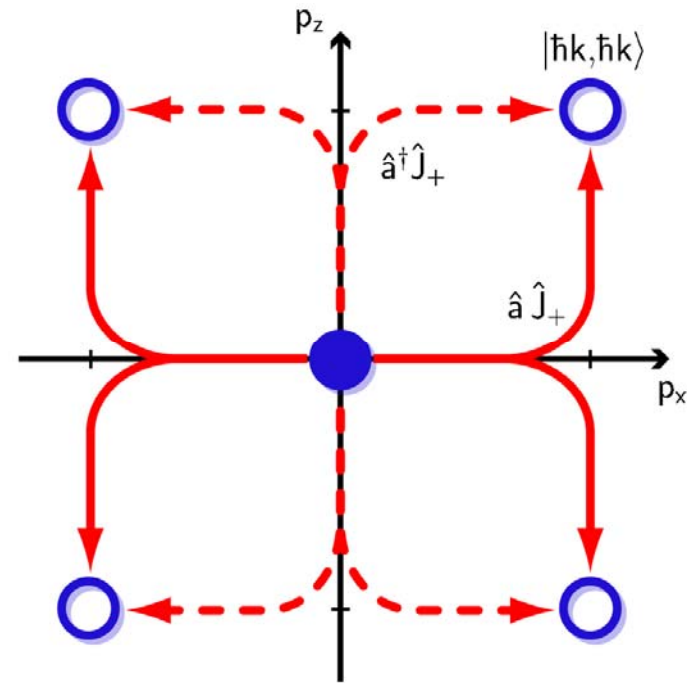
1D theory: D. Nagy, G. Kónya, G. Szirmai, P. Domokos, PRL 104, 130401 (2010).

# Two-Mode Description

energy diagram



momentum diagram

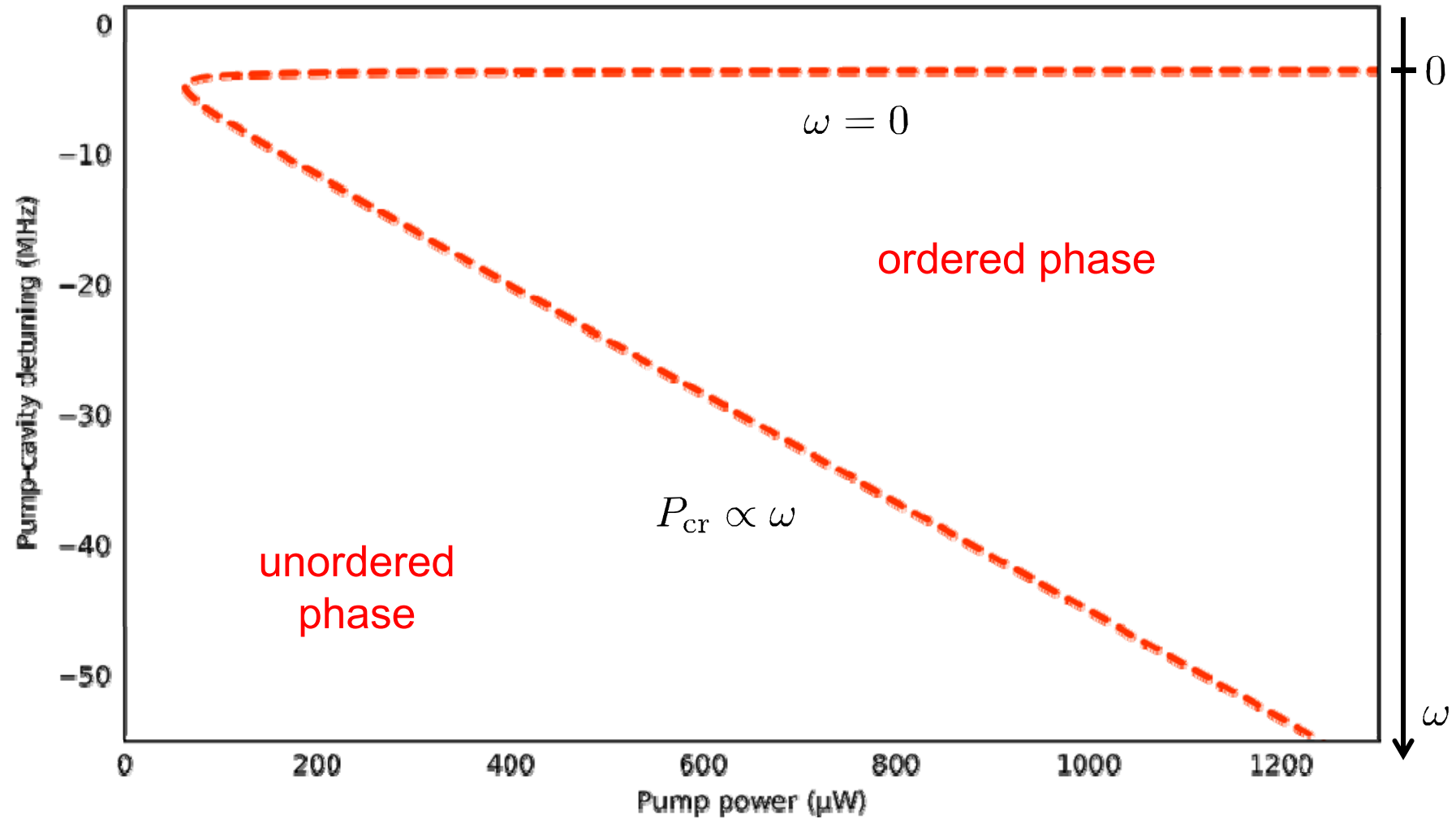


$$\hat{J}_+ = \sum_i |\pm\hbar k, \pm\hbar k\rangle_i \langle 0, 0| = \hat{J}_-^\dagger$$

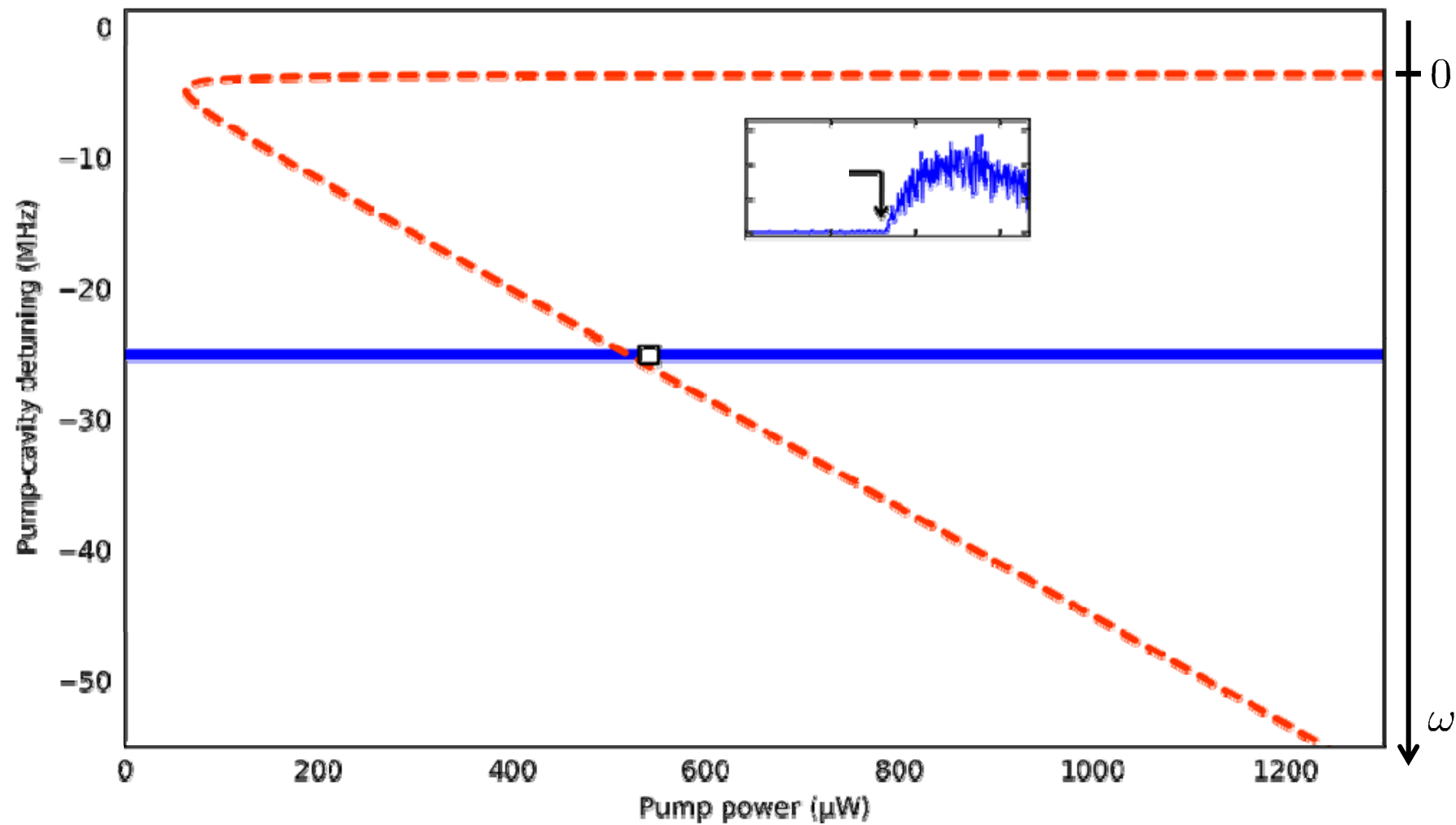
# Zero Temperature Phase Diagram

$$\lambda_{\text{cr}} = \sqrt{\omega\omega_0}/2$$

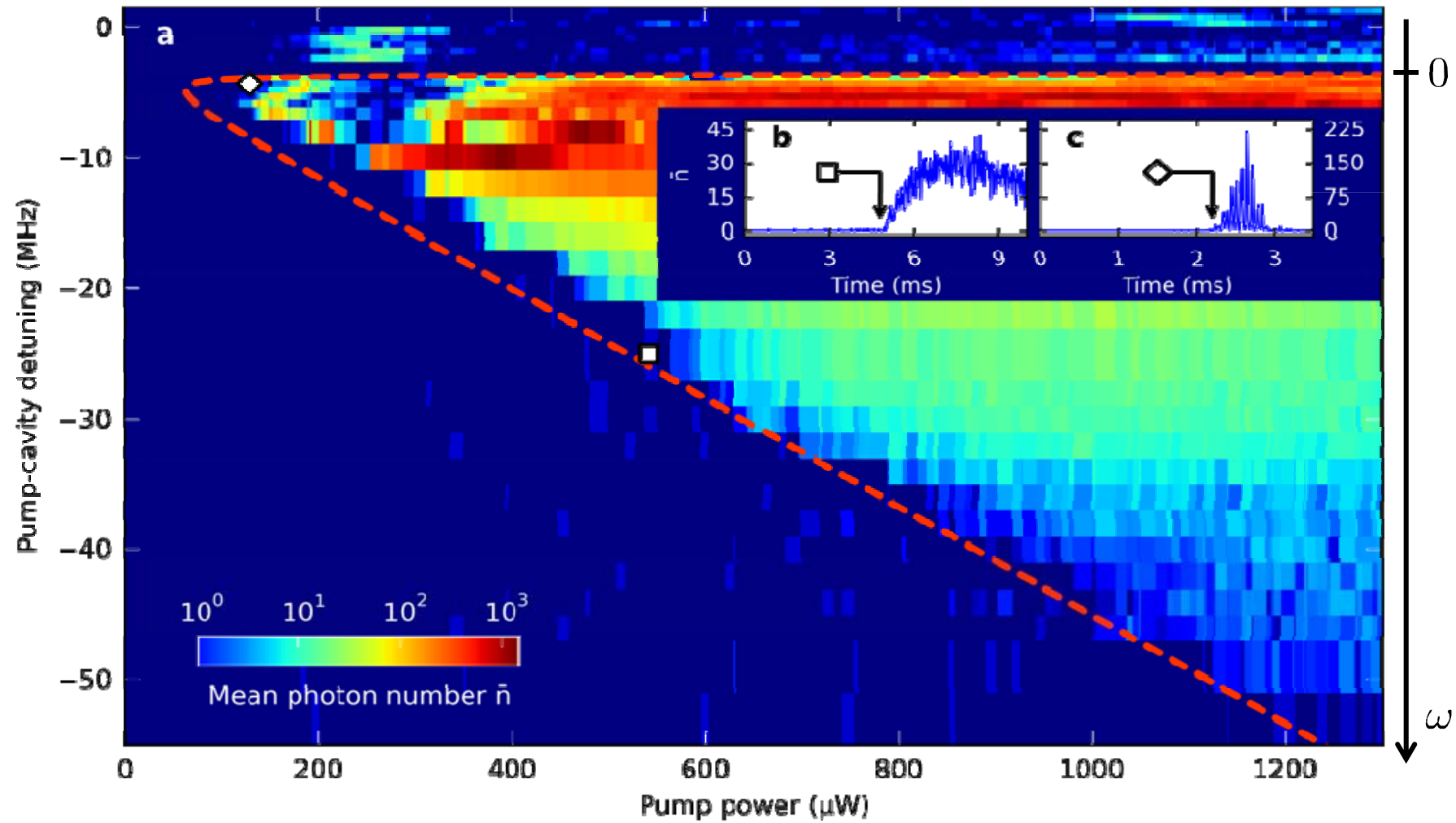
$$P_{\text{cr}} \propto \lambda_{\text{cr}}^2$$



# Zero Temperature Phase Diagram

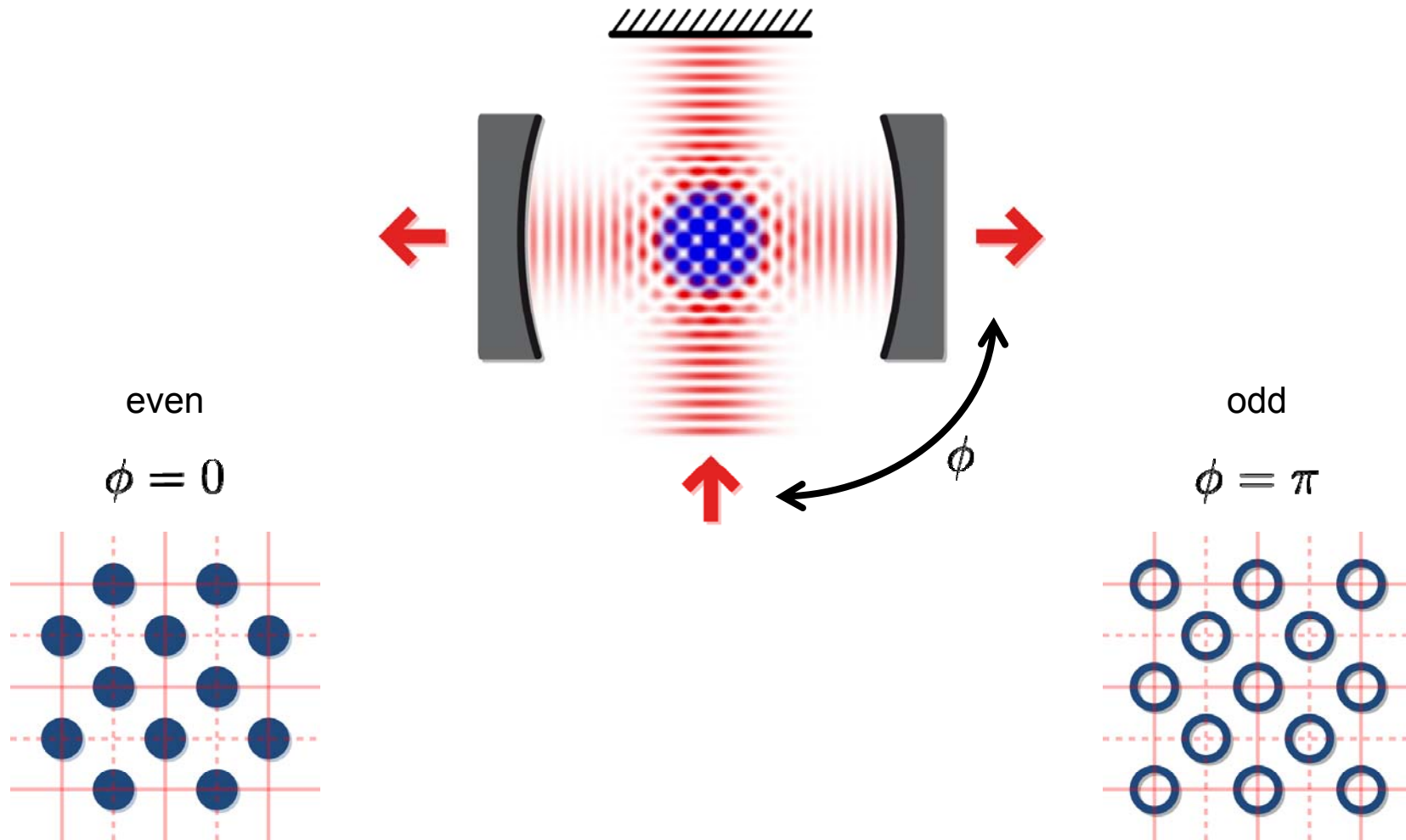


# Zero Temperature Phase Diagram

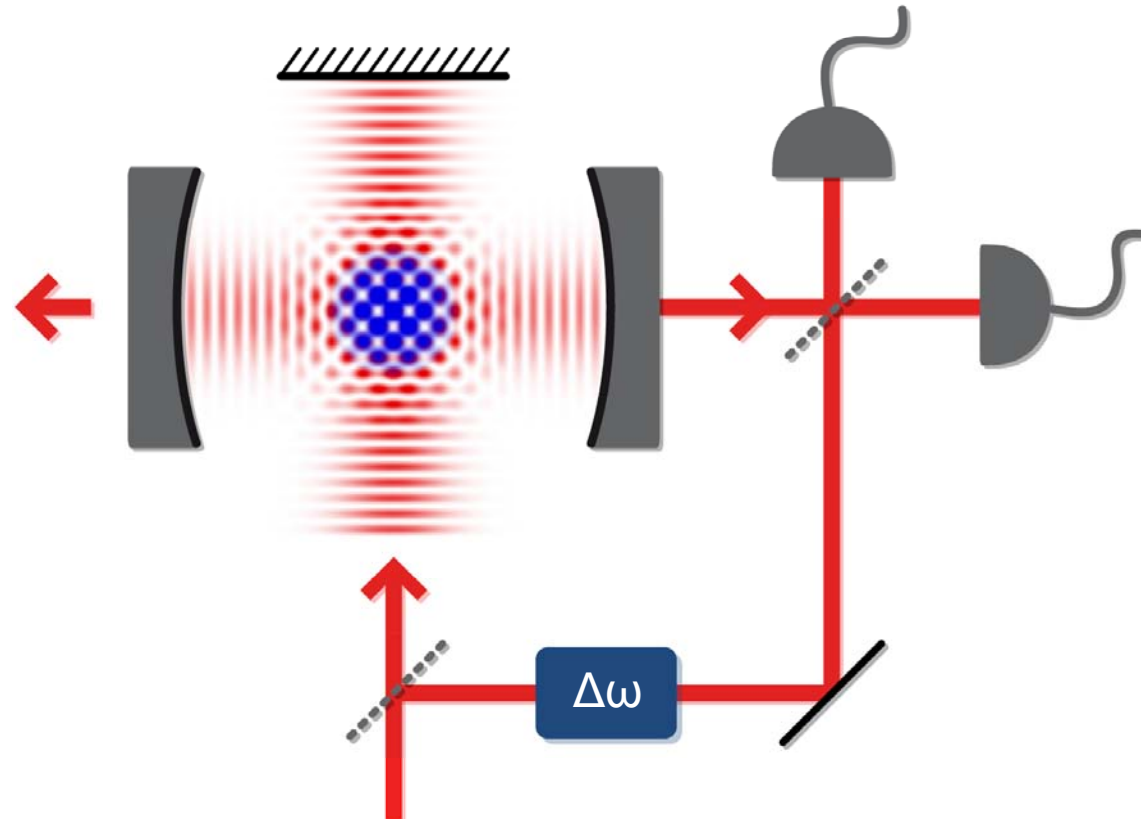


K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger. Nature 464, 1301 (2010)

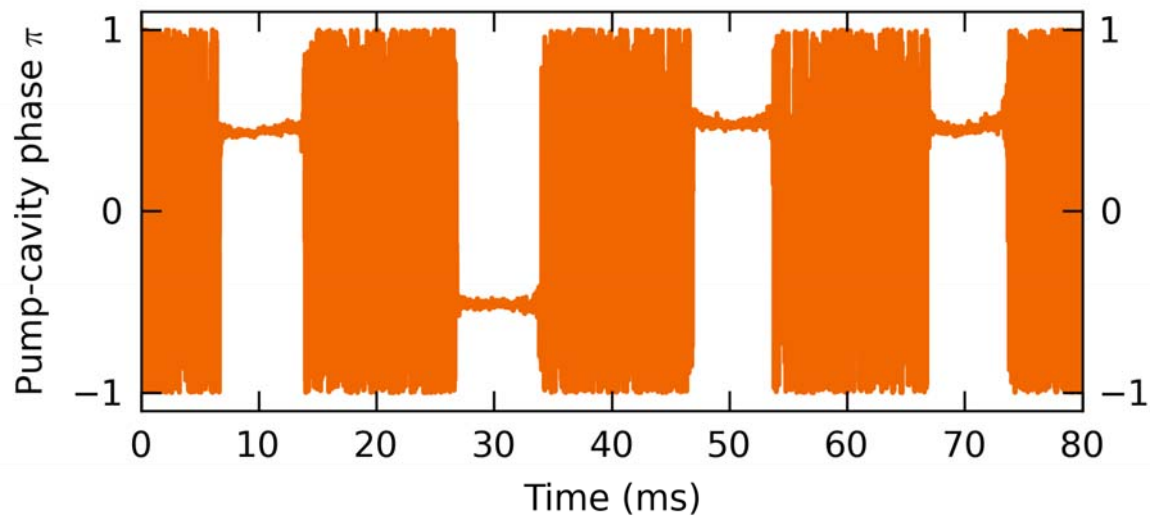
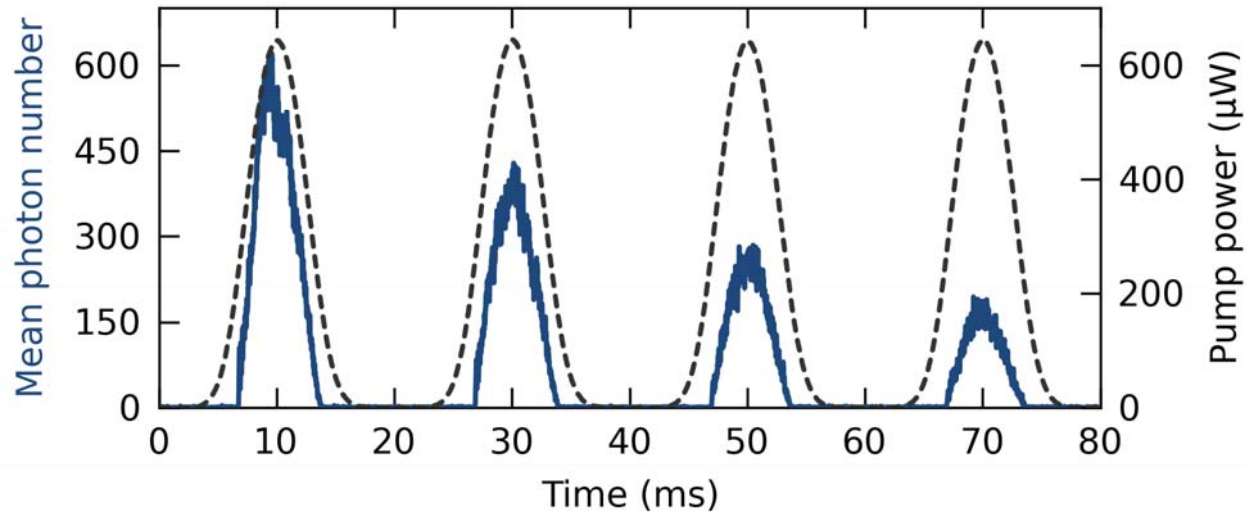
# Phase Sensitive Detection



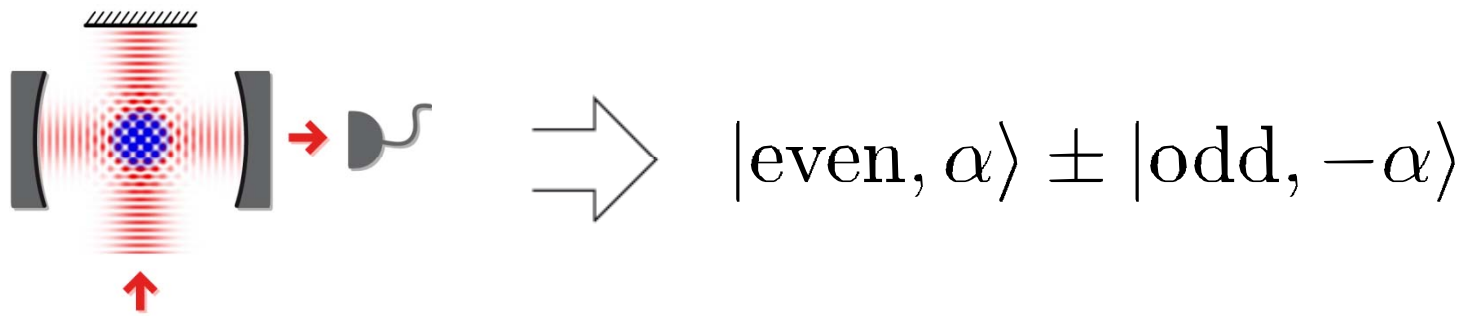
# Phase Sensitive Detection



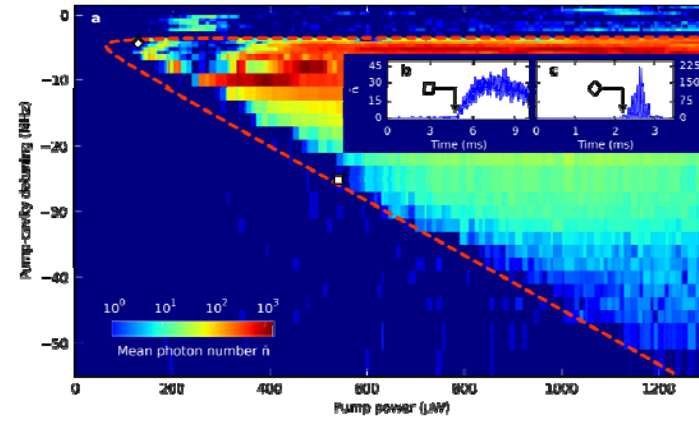
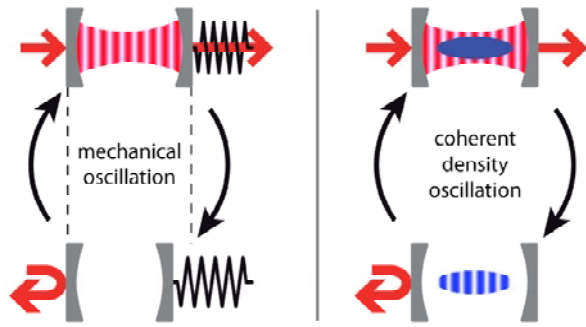
# Phase Sensitive Detection







# Summary



# Thanks !

**Funding: ETH, SNF, QSIT, EU (NameQuam, SCALA), ERC**

## Quantum Gases in Optical Lattices

Leticia Tarruell  
Daniel Greiff  
Thomas Uehlinger  
Robert Jördens



## BEC and Cavity

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Kristian Baumann  
Raphael Mottl  
Silvan Leinss

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Alexander Frank

## Administration

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Discussions: Eugene Demler, Lode Pollet, Vito Scarola, Sebastian Huber, Matthias Troyer, Hans-Peter Büchler, J. Blatter, E. Altman, ...