The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA

2164-10

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Quantum Signatures of the Dynamics of a Vibrational Mode of a Thin Membrane within an Optical Cavity

David VITALI

School of Science & Tech.
University of Camerino
via Madonna delle Carceri, 9b, Camerino 62032 MC
ITALY

Quantum signatures of the dynamics of a vibrational mode of a thin membrane within an optical cavity

David Vitali
M. Karuza, C. Biancofiore, G. Di Giuseppe, R. Natali, M. Galassi, P. Tombesi
School of Science and Technology, Physics Division, University of Camerino, Italy

ICTP Workshop on "Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime", Sept. 6-10, 2010, Trieste

Outline of the talk

1. Optomechanical systems: the case of a thin membrane within a Fabry-Perot cavity (also with some experimental results)

2. Theory predictions on quantum phenomena: entanglement, ground-state cooling (with one or two mechanical modes), ponderomotive squeezing of the light mode

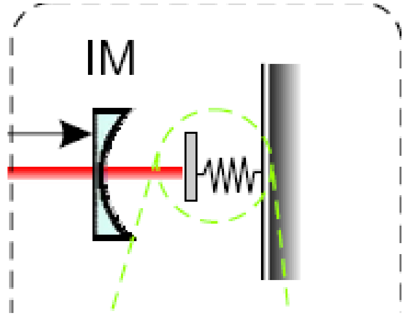
Why entering the quantum regime for opto- and electro-mechanical systems ?

• **quantum-limited sensors**, i.e., working at the sensitivity limits imposed by Heisenberg uncertainty principle

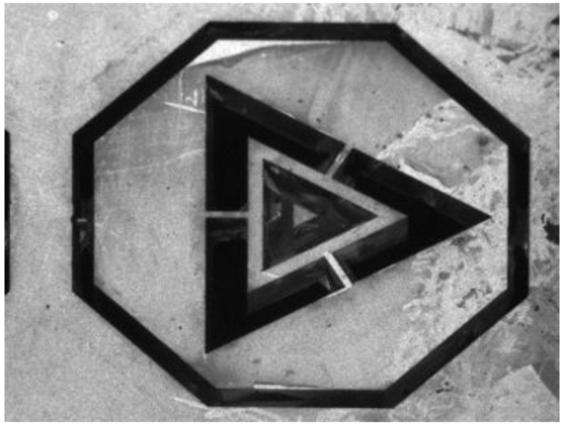
• exploring the **boundary between the classical macroscopic world and the quantum microworld** (how far can we go in the demostration of macroscopic quantum phenomena ?)

• **quantum information applications** (optomechanical and electromechanichal devices as **light-matter interfaces** and **quantum memories), or transducers for quantum computing architectures**

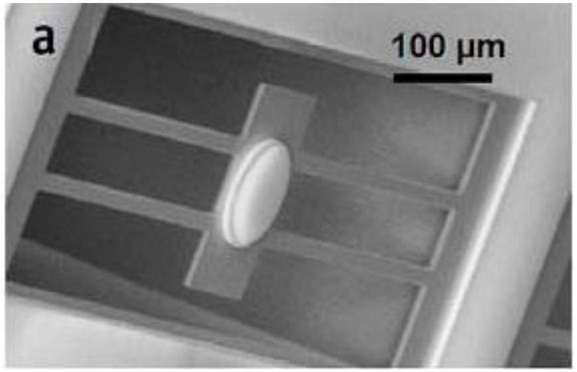
We focus on cavity optomechanics



1. Fabry-Perot cavity with a moving micromirror



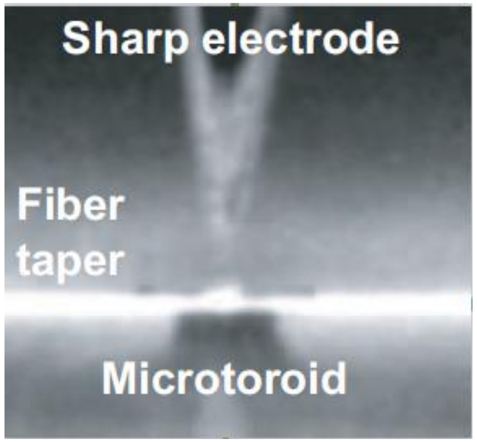
micropillar mirror (LKB, Paris)



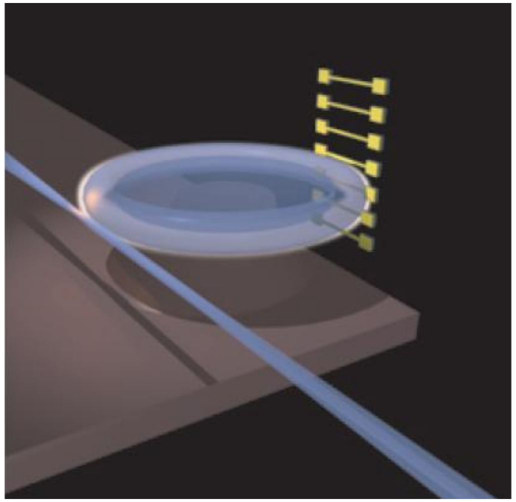
Monocrystalline Si cantilever, (Vienna)

2. Silica toroidal optical microcavities

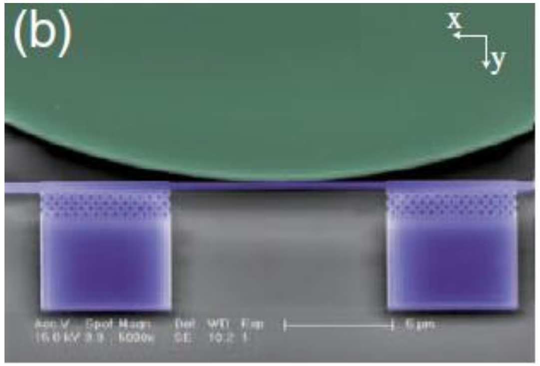
spoke-supported microresonator (Munich, Lausanne)



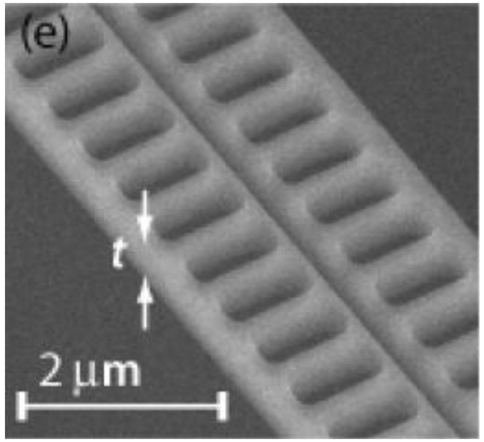
With electronic actuation, (Brisbane)



Evanescent coupling of a SiN nanowire to a toroidal microcavity (Munich, Lausanne)

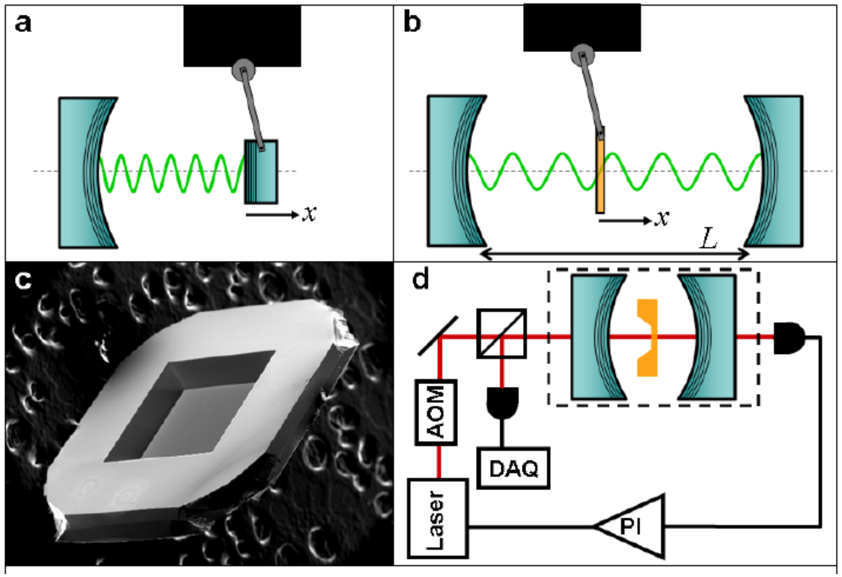


microdisk and a vibrating nanomechanical beam waveguide (Yale)



Photonic crystal "zipper" cavity (Caltech)

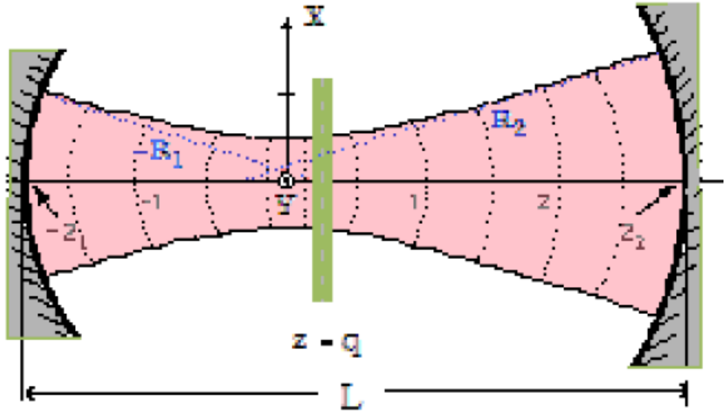
"membrane in the middle" scheme: Fabry-Perot cavity with a thin SiN membrane inside (Yale, and more recently Caltech, Camerino)



We focus here on the cavity-membrane system

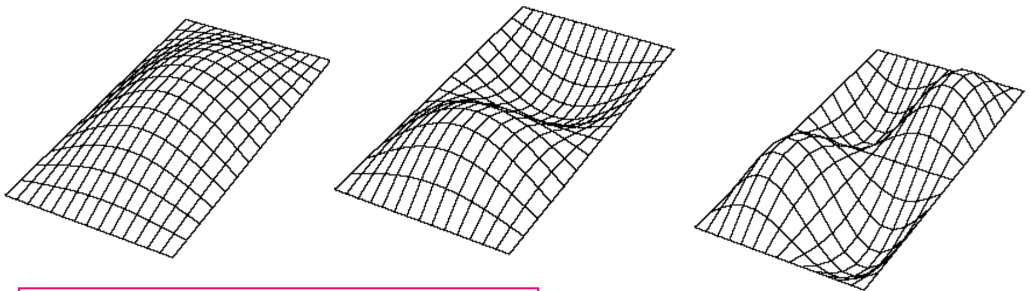
Many cavity modes (still Gaussian TEM_{mn} for an aligned membrane close to the waist)

$$H_{cav} = \sum_k \hbar \omega_k a_k^+ a_k$$



Many vibrational modes $u_{mn}(x,y)$ of the membrane

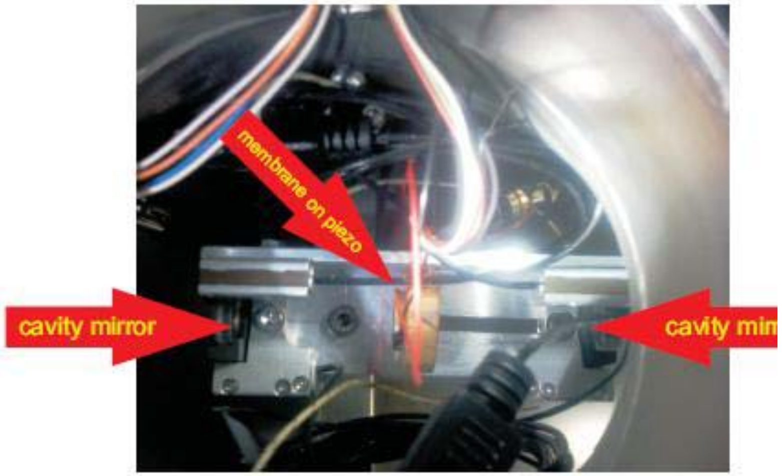
$$u_{mn}(x,y) = \sin \frac{n\pi x}{d} \sin \frac{m\pi y}{d}$$



$$\Omega_{nm} = \sqrt{\frac{\pi T}{\rho t d^2}\left(m^2 + n^2\right)}$$

Vibrational frequencies

T = surface tension
ρ= SiN density,
t = membrane thickness
d = membrane side length
m,n = 1,2…



$$z(x,y)=\sum_{n,m}\sqrt{\frac{\hbar}{M\Omega_{nm}}}q_{nm}u_{nm}(x,y)$$

Membrane axial deformation field

$$M=\frac{\rho t d^2}{4}$$ Mode mass

$$[q_{nm},p_{lk}]=i\delta_{nl}\delta_{mk}$$

Dimensionless position and momentum of vibrational modes

$$H_M=\sum_{n,m}\frac{\hbar\Omega_{nm}}{2}\left(p_{nm}^2+q_{nm}^2\right)$$

Mechanical Hamiltonian

Optomechanical interaction due to radiation pressure

$$H_{\text{int}}=-\int dxdy\,P_{rad}(x,y)\,z(x,y)$$

(at first order in z)

$$P_{rad}(x,y)=\varepsilon_0\left(n_M^2-1\right)\int_{-t/2}^{t/2}dz\left(\dot{\vec{E}}(x,y,z)\times\vec{B}(x,y,z)\right)_z$$

Radiation pressure field

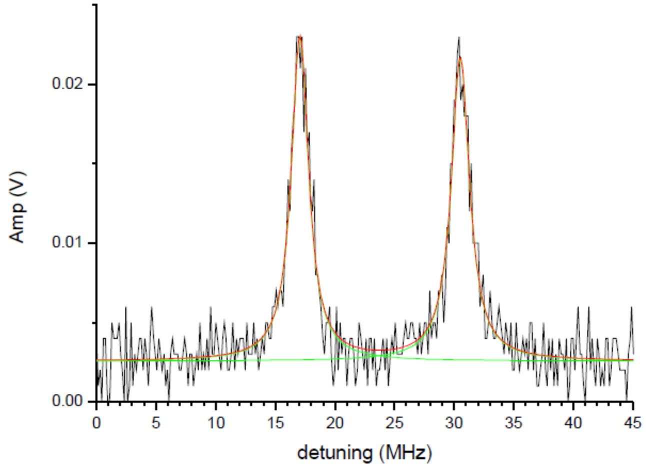
$$\hat{H}_{int} = -\hbar \sum_{l,k,n,m} c_{nmlk} a_l^+ a_k q_{nm}$$

Trilinear coupling describing photon scattering between cavity modes mediated by the vibrating membrane

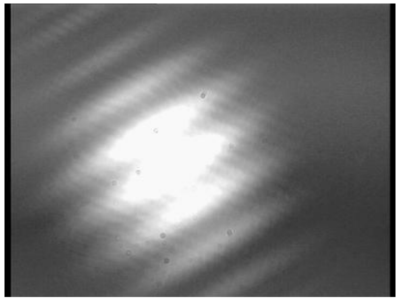
$$c_{nmlk} = \frac{\sqrt{\omega_l \omega_k}}{L} \sqrt{\frac{\hbar}{M\Omega_{nm}}} \beta_{nmlk}$$

$\boldsymbol{\beta_{nmlk}}$ = dimensionless coupling constants depending upon membrane position, thickness, transverse spatial overlap between optical and vibrational modes…..

Some first experimental data in Camerino



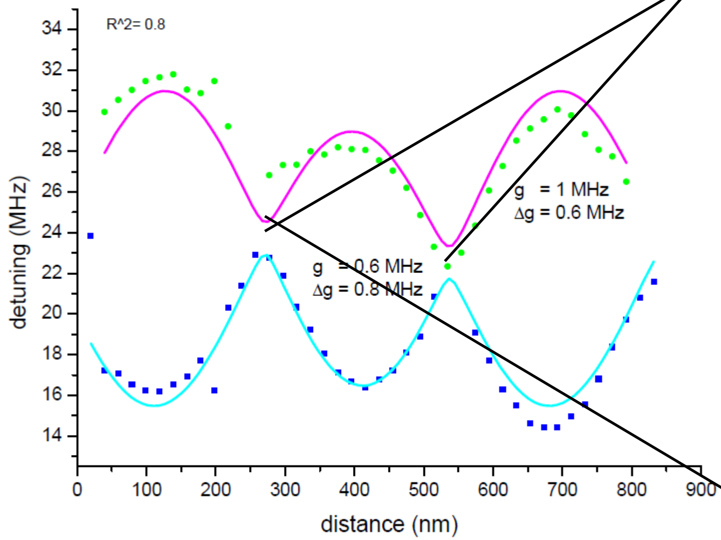
We have observed scattering between modes: simultaneous presence of a TEM00 mode (driven by the laser) and TEM0n (n ≥6) mode (scattered by the membrane)



CCD camera picture of the transverse patterns of the intracavity mode, showing the simultaneous presence of a TEM00 and TEM0n (n ≥6) mode

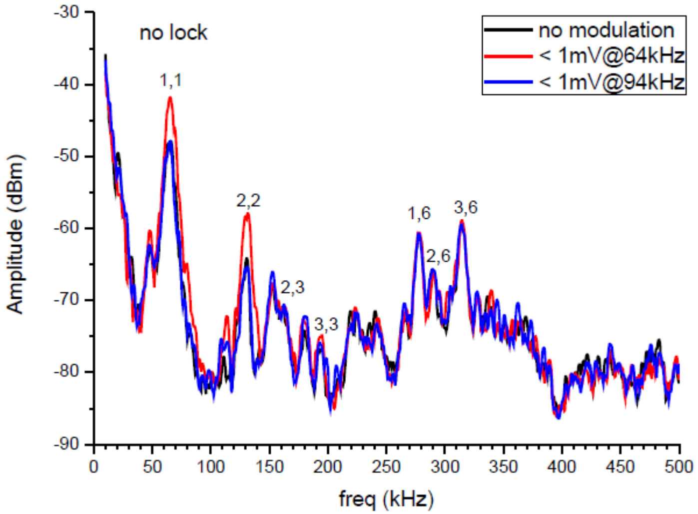
Mode coupling and the corresponding frequency shifts can be **tuned by adjusting the position and orientation of the membrane**

Avoided crossing



Relative frequency of the two modes TEM00 and TEM0n versus the membrane displacement. The data are consistent with a splitting of about 1 MHz (see also J. Sankey et al., Nat. Phys, July 2010, for a much more detailed study of mode coupling)

Coupling quadratic in q



Spectrum of the transmitted signal

Excitation spectrum of the vibrational modes of the SiN membranes, both in the presence and in absence of electromechanical driving (room temperature, low mechanical Q -> well in the classical regime)

Let us now focus on a simpler situation: single mechanical oscillator, nonlinearly coupled by radiation pressure, to a single optical oscillator

This is possible when:

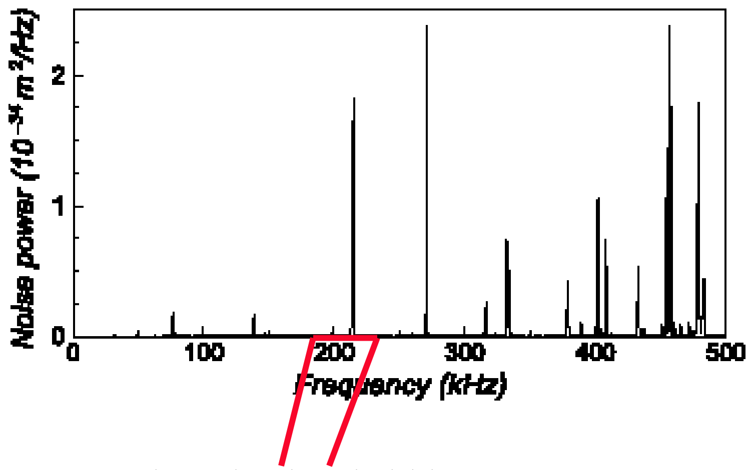
• The external laser (with frequency $\omega_L \approx \omega_a$) **drives only a single cavity mode *a*** and scattering into the other cavity modes is negligible (no frequency close mode)

• a **bandpass filter** in the detection scheme can be used, isolating a single mechanical resonance

$$\hat{H}_{\text{int}} \approx -\hbar G_0 a^+ a q$$

$$\hat{H}_{drive} = i\hbar\left(E e^{-i\omega_L t} a^+ - E^* e^{i\omega_L t} a\right)$$

$E = \sqrt{\dfrac{2\kappa P_L}{\hbar \omega_L}}$ amplitude of the driving laser with input power P_L



detection bandwidth

Also damping and noise act on the system…..

- **The membrane** is in contact with an **ohmic environment at temperature *T***;

Fluctuation-dissipation theorem ⇒ presence of a **quantum Langevin force ξ** with correlation functions

$$\left\langle \xi(t)\xi(t')\right\rangle = \frac{\gamma_m}{\omega_m}\int \frac{d\omega}{2\pi} e^{i\omega(t-t')}\omega\left[\coth\left(\frac{\hbar\omega}{kT}\right)+1\right]$$

Gaussian, generally non-Markovian

- **The cavity mode** is damped by **two** independent processes:

1. photon leakage through the mirrors, with decay rate κ_1
2. **absorption by the membrane**, with decay rate κ_2(q)**, non-standard because of membrane position dependence --> further nonlinearity**

Each decay is associated with a **vacuum input Langevin noise $a_{in}^{j}(t)$** with correlation functions

$$\left\langle a_{in}^{j}(t)a_{in}^{k}(t')\right\rangle = \left\langle a_{in}^{j}(t)^{+} a_{in}^{k}(t')\right\rangle = 0 \qquad \left\langle a_{in}^{j}(t)a_{in}^{k}(t')^{+}\right\rangle = \delta_{jk}\delta(t-t')$$

Gaussian, Markovian

Description in terms of Heisenberg-Langevin equations

(in the frame rotating at ω_L)

$$\dot{a} = -i[\omega_a - \omega_L - G_0 q]a - [\kappa_1 + \kappa_2(q)]a + E + \sqrt{2\kappa_1}a^{(1)}{}_{in} + \sqrt{2\kappa_2(q)}a^{(2)}{}_{in}$$

$$\dot{q} = \omega_m p$$

$$\dot{p} = -\omega_m q + G_0 a^+ a - \gamma_m p + \xi + \frac{\partial_q \kappa_2(q)}{\sqrt{2\kappa_2(q)}}\left[a a^{(2)}{}_{in}{}^+ + a^{(2)}{}_{in} a^+\right]$$

Nonlinear cavity decay

Nonlinear noise

Additional non-standard terms due to membrane absorption;

how much do they affect quantum effects ?

Classical steady state and linearization around it

Strong driving E and high-finesse cavity $\Rightarrow$ **steady-state with an intense intracavity field (amplitude α_s)** and deformed membrane.

We focus on the linearized dynamics of the **quantum fluctuations around this steady state** (only cavity mode is linearized $\Rightarrow$ **exact for $|\alpha_s| >> 1$**)

$$a \to \alpha_s + \delta a \qquad q \to q^s + \delta q$$

$$\kappa = \kappa_1 + \kappa_2\left(q^s\right)$$

$$\alpha_s = \frac{E}{\kappa + i\Delta(\alpha_s)}$$

$$\Delta(\alpha_s) = \omega_c - \omega_L - \frac{G_0^2|\alpha_s|^2}{\omega_m}$$

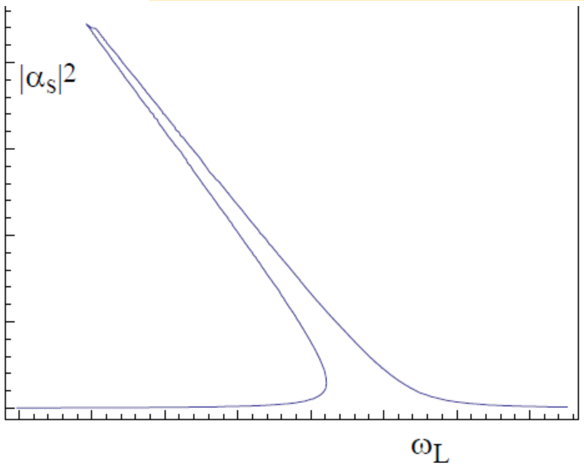
steady-state radiation pressure shift

Nonlinear eqn. for the intracavity steady-state amplitude

Effective cavity detuning

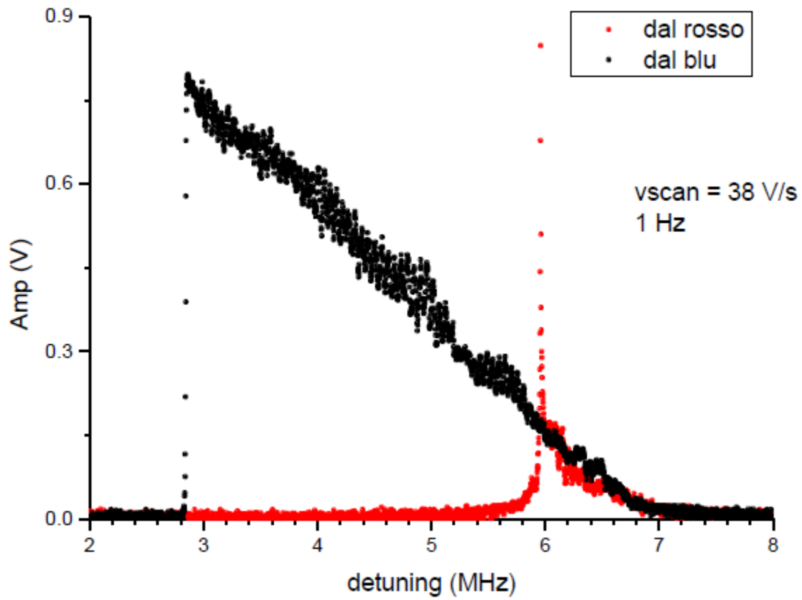
$\Rightarrow$ **Radiation pressure optical bistability** (Dorsel et al., 1983, more recently in cavity-BEC systems, (see Esslinger talk)

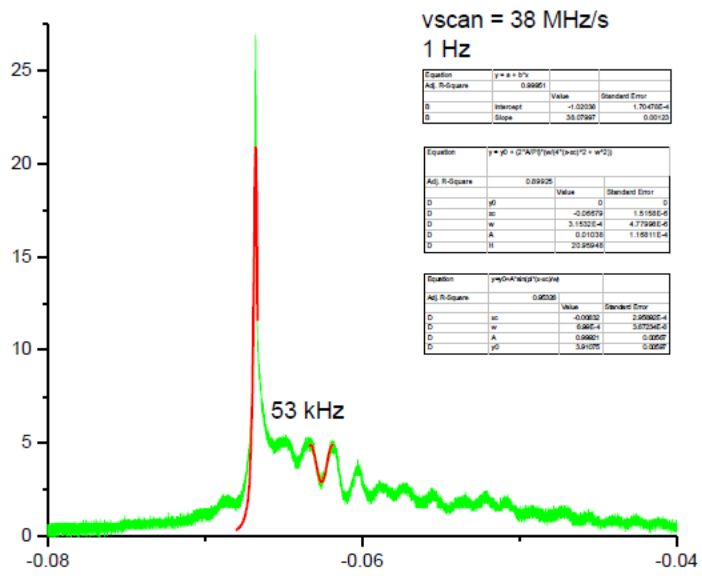
Optical bistability by radiation pressure observed also in our cavity-membrane system



$$|\alpha_s|^2 = \frac{|E|^2}{\kappa^2 + \Delta^2(\alpha_s)}$$

Experimental data





Dynamical transition to the new steady state at mechanical frequencies

Back to theory: Quantum dynamics of the fluctuations: Linearized quantum Langevin equations

$$\delta\dot{q} = \omega_m \delta p$$

$$\delta\dot{p} = -\omega_m \delta q - \gamma_m \delta p + G\delta X + \xi + \frac{\partial_q \kappa_2(q^s)\alpha_s}{\sqrt{\kappa_2(q)}} Y^{(2)}{}_{in}$$

$$\delta\dot{X} = -\kappa\delta X + \Delta\delta Y - \sqrt{2}\alpha_s \partial_q \kappa_2(q^s)\delta q + \sqrt{2\kappa_1} X^{(1)}{}_{in} + \sqrt{2\kappa_2(q^s)} X^{(2)}{}_{in}$$

$$\delta\dot{Y} = -\kappa\delta Y - \Delta\delta X + G\delta q + \sqrt{2\kappa_1} Y^{(1)}{}_{in} + \sqrt{2\kappa_2(q^s)} Y^{(2)}{}_{in}$$

$$\delta X = \frac{\delta a + \delta a^+}{\sqrt{2}}$$ Amplitude quadrature

$$\delta Y = \frac{\delta a - \delta a^+}{i\sqrt{2}}$$ Phase quadrature

$$X_{in}^{(j)} = \frac{\delta a^{(j)}{}_{in} + \delta a^{(j)}{}_{in}{}^+}{\sqrt{2}}$$ Amplitude noise

$$Y_{in}^{(j)} = \frac{\delta a^{(j)}{}_{in} - \delta a^{(j)}{}_{in}{}^+}{i\sqrt{2}}$$ Phase noise

Additional terms due to membrane absorption

$$G = 2G_0 \sqrt{\frac{P_L \kappa}{\hbar\omega_L(\Delta^2 + \kappa^2)}}$$

Effective radiation pressure coupling

1. STEADY STATE ENTANGLEMENT

When the system is **stable**, it reaches for t → ∞ a **Gaussian steady state**, due to:

1. Linearized dynamics
2. Gaussian quantum noises

ρ Gaussian ⇔ Gaussian characteristic function

$$\Phi(\vec{\lambda}) = \mathrm{Tr}\left[\rho e^{-i\vec{\lambda}^T\vec{\xi}}\right] = \exp\left[-\frac{\vec{\lambda}^T V \vec{\lambda}}{2} + i\vec{d}^T\vec{\lambda}\right] \qquad \vec{\xi}^T = (\delta q, \delta p, \delta X, \delta Y)$$

$$V_{ij} = \frac{\langle \xi_i \xi_j + \xi_j \xi_i \rangle}{2} - \langle \xi_i \rangle \langle \xi_j \rangle$$

correlation matrix (CM) fully characterizing the steady state and its entanglement properties (we use log-negativity)

Review paper: C. Genes, A. Mari, D. Vitali and P. Tombesi, *Quantum Effects in Optomechanical Systems*, Advances in Atomic, Molecular, and Optical Physics, Vol. 57, Academic Press, 2009, pp. 33-86.

2. GROUND STATE COOLING OF THE MEMBRANE MODES

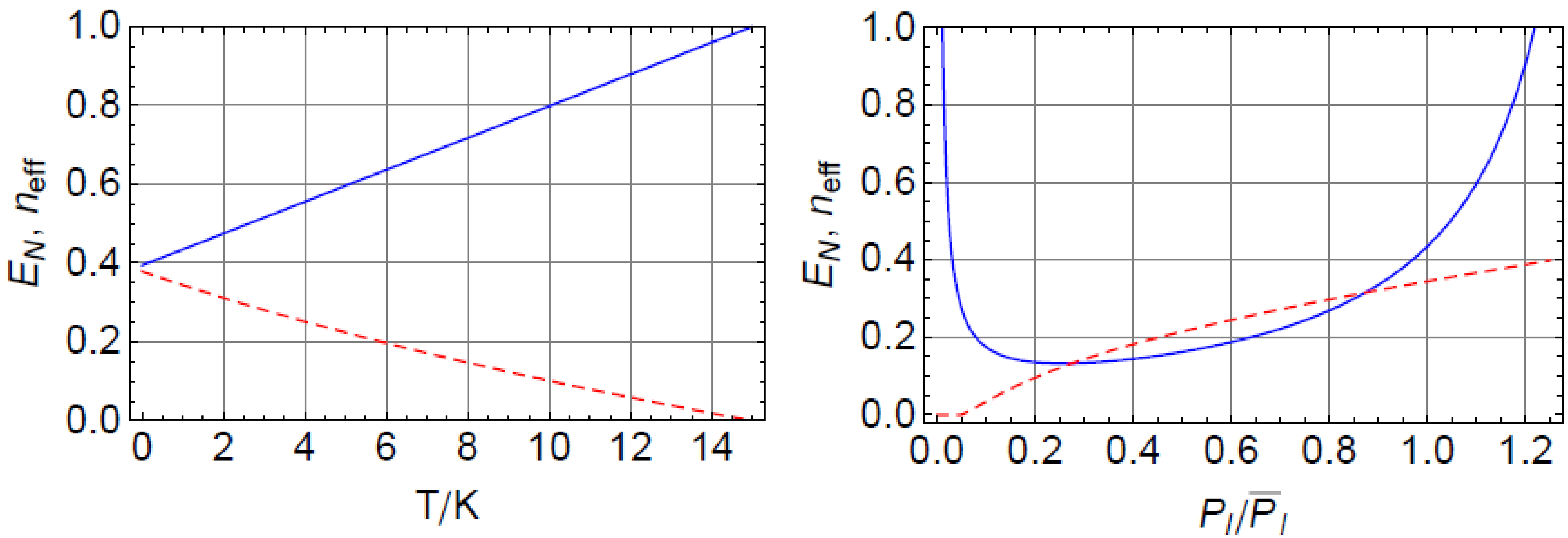
The steady state CM, *V*, contains also the info about the stationary energy of the membrane mode, *U*

$$V_{11} = \langle \delta q^2 \rangle \qquad V_{22} = \langle \delta p^2 \rangle$$

$$U = \frac{\hbar\omega_m}{2}\left[\langle \delta q^2 \rangle + \langle \delta p^2 \rangle\right] \equiv \hbar\omega_m \left(n_{eff} + \frac{1}{2}\right)$$

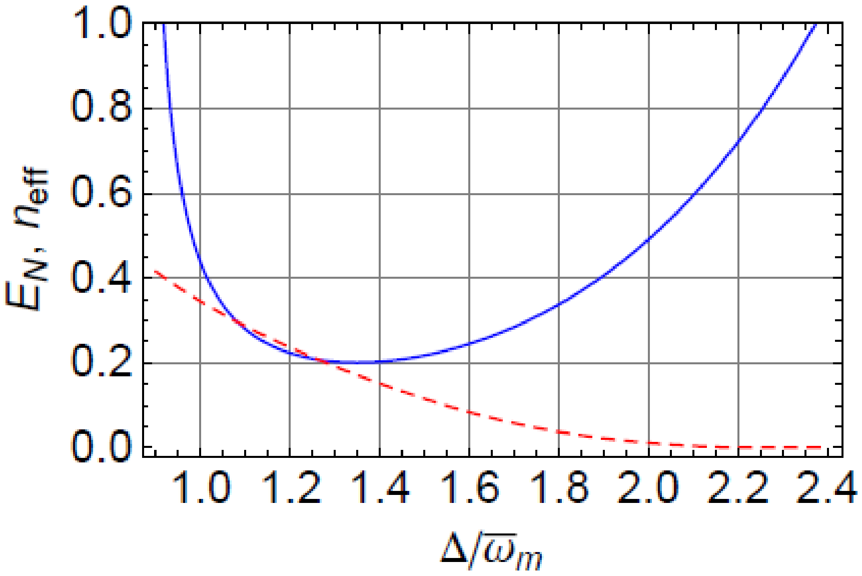
Is it possible to get **simultaneous optomechanical steady-state entanglement and ground state cooling** ($\delta q^2 = \delta p^2 = ½$) of a membrane mode with state of the art parameters**, despite membrane absorption (Im n ~ 10^{-4})**

For parameters similar to those of our current experiment: M = 35 ng, $\omega_m/2\pi$ = 250 KHz, $Q_m = 10^6$, P_L = 650 μW, L = 7 cm, F_0 = 20000, T = 4 K, t = 50 nm, $\Delta \sim \omega_m$, $n_M = 2.2 + i\,10^{-4}$

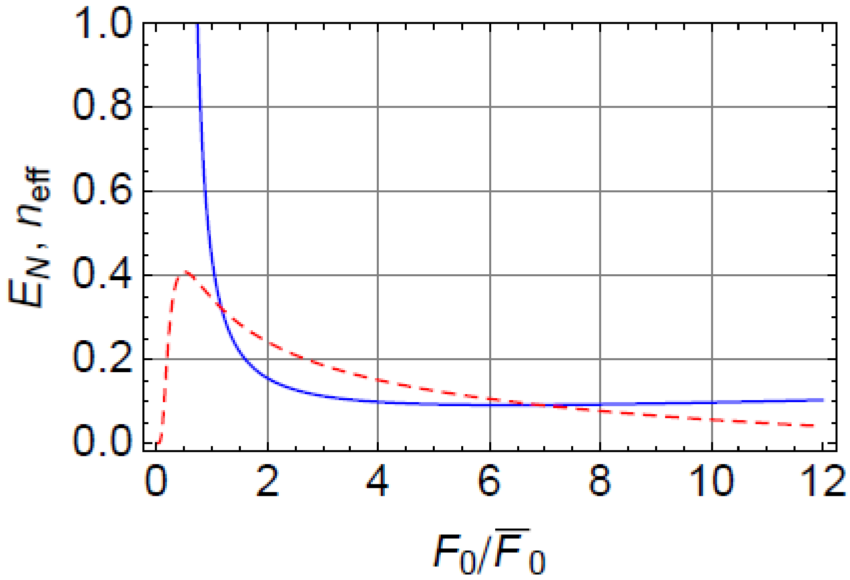


Blue: n_{eff} = ground state occupancy

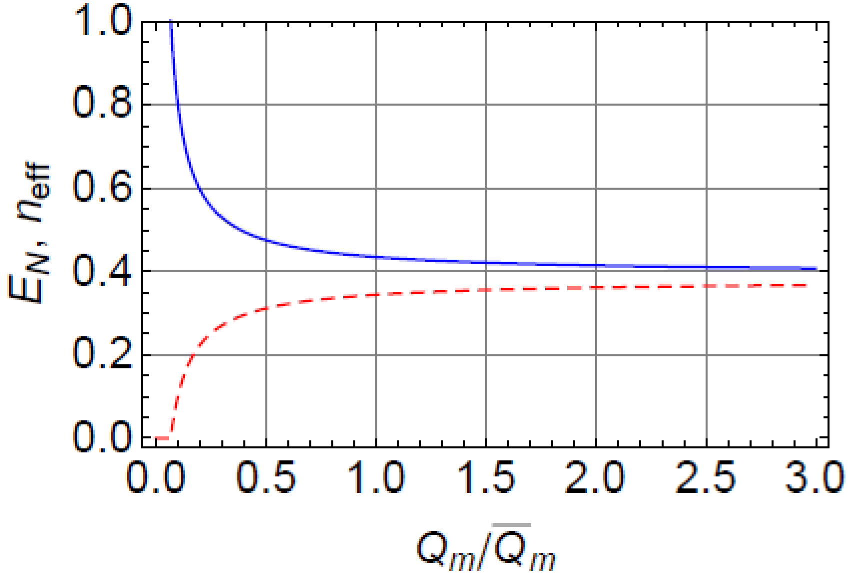
Red: E_N, Log-negativity

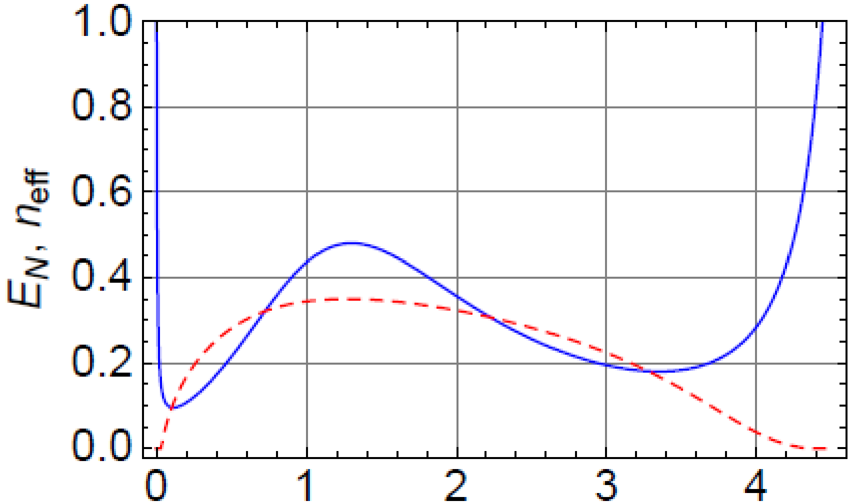


Cavity resonant with the laser blue sideband



(e) Finesse, $\bar{F}_0 = 22\,000$



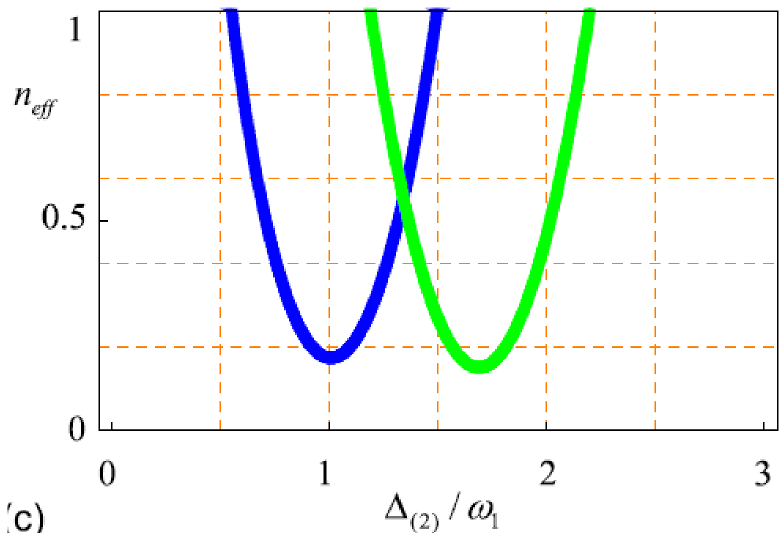


t (50 nm) membrane thickness

Relaxing the single mechanical mode description:
What if a nearby mechanical mode is present ?

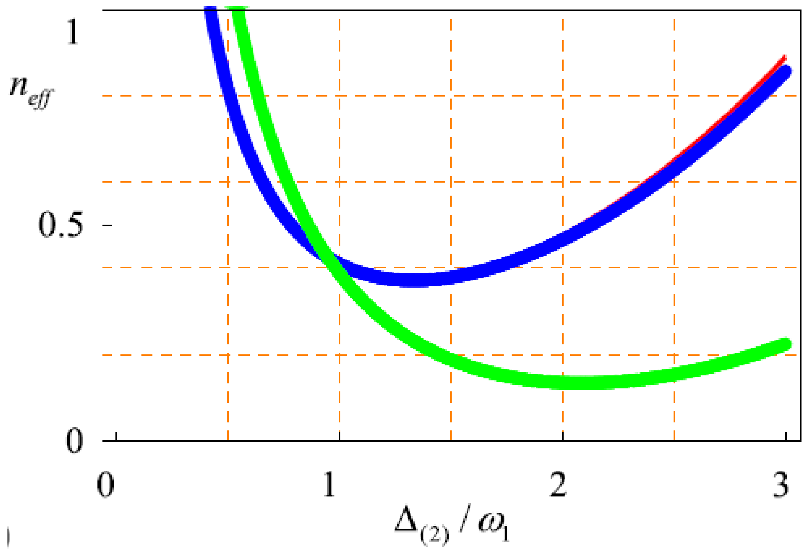
Everything depends upon the **frequency mismatch between the two modes $\delta\omega_{21} = \omega_2 - \omega_1$**

Cooling is not disturbed if the two modes are not too close: the two modes are even simultaneously cooled



$F = 1.5 \cdot 10^5$, $\kappa \approx 0.2\omega_m$

$\omega_2 = 1.7\omega_1$

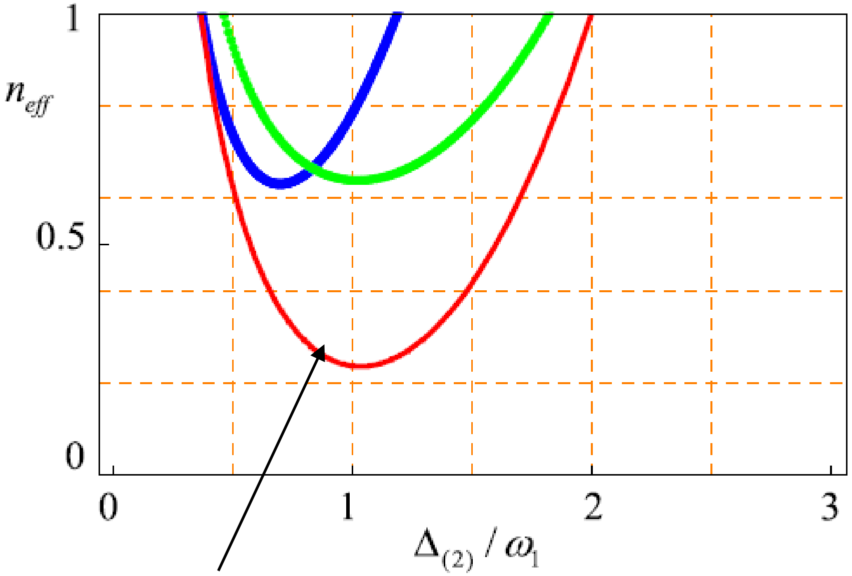


$F = 3 \cdot 10^4$, $\kappa \approx \omega_m$

Cooling is inhibited when the frequencies are close!

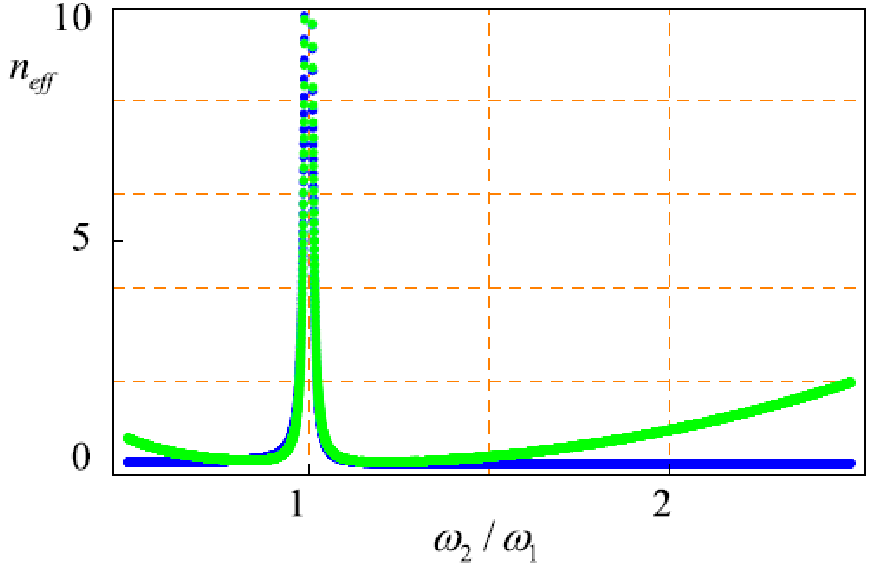
It happens when the modes are separated by less than the effective mechanical width, $\delta\omega_{21} < \Gamma_2$ (net laser cooling rate)

$\omega_2 = 0.95\omega_1$

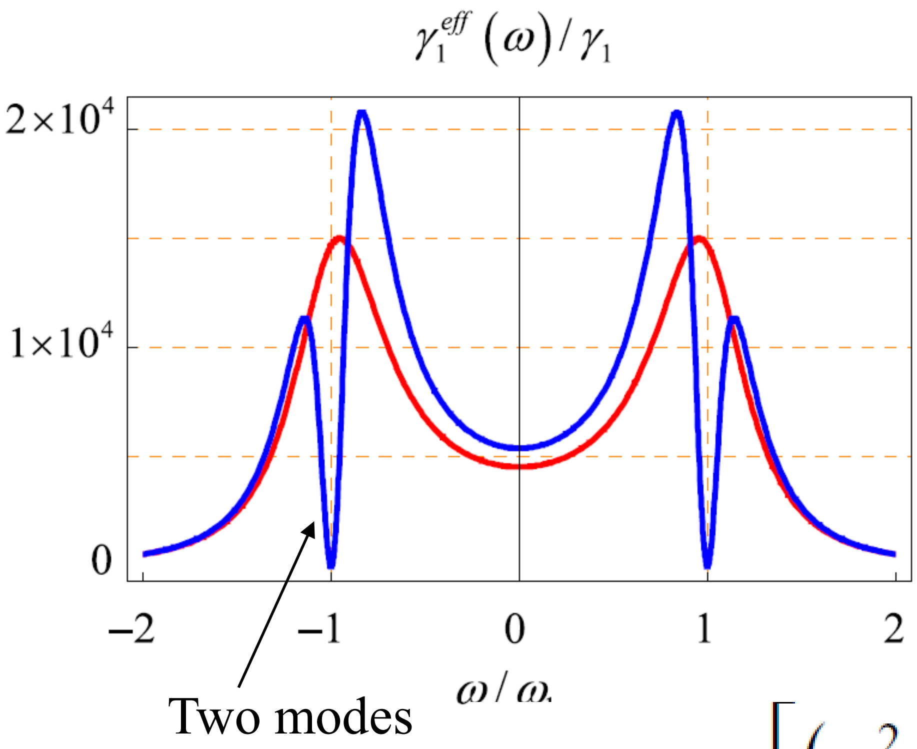


one mode only

$\Delta_2 = \omega_1$



C. Genes et al., New J. Phys. 10 (2008) 095009



This inhibition is due to a **classical destructive interference phenomenon**, similar to a classical analogue of electromagnetically induced transparency (EIT)

$$\gamma_i^{eff}(\omega) \simeq \gamma_1 + \Gamma_1 \frac{\left[\left(\omega_2^2 - \omega^2\right)^2 + \omega^2 \gamma_2 \Gamma_2\right]}{\left(\omega_2^2 - \omega^2\right)^2 + \omega^2 \Gamma_2^2}$$

⇓

$$\boxed{\gamma_i^{eff}(\omega_1) \simeq \gamma_1 + \Gamma_1(\gamma_2/\Gamma_2) \simeq \gamma_1}$$

when $\boldsymbol{\delta\omega_{21} \approx 0}$

Alternative explanation: when $\delta\omega_{21} = 0$, **radiation pressure couples the cavity mode only with the effective "center-of-mass" of the two mechanical modes**

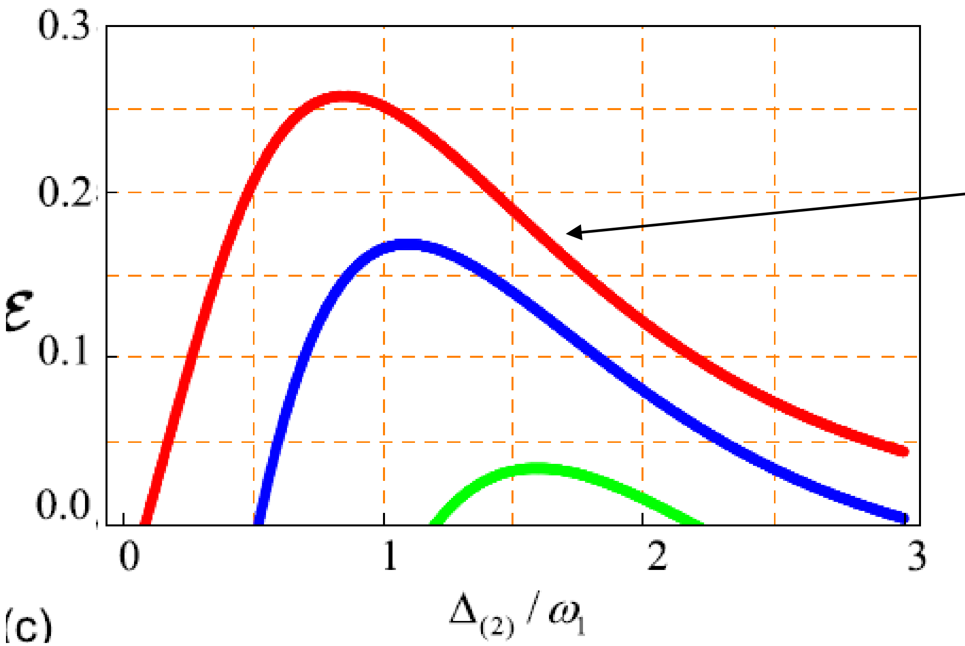
$$q_{\rm cm} = \frac{G_0^1 q_1 + G_0^2 q_2}{[G_0^1]^2 + [G_0^2]^2} \qquad q_{\rm r} = \frac{G_0^1 q_2 - G_0^2 q_1}{[G_0^1]^2 + [G_0^2]^2}$$

$$H_{mech} = \frac{\hbar\omega_{cm}}{2}\left(q_{cm}^2 + p_{cm}^2\right) + \frac{\hbar\omega_r}{2}\left(q_r^2 + p_r^2\right) + \frac{\hbar(\omega_2 - \omega_1) G_0^1 G_0^2}{[G_0^1]^2 + [G_0^2]^2}\left(q_{cm} q_r + p_{cm} p_r\right)$$

When $\delta\omega_{21} = 0$, the "relative motion" is decoupled from the center-of-mass and the cavity mode ⇒ is uncooled and therefore also the two modes are uncooled.

EFFECT OF NEARBY MODE ON ENTANGLEMENT

Similar to cooling: the two modes are simultaneously entangled with the cavity mode if the are not too close $\delta\omega_{21} > \Gamma_2$



$\omega_2 = 1.5\omega_1$

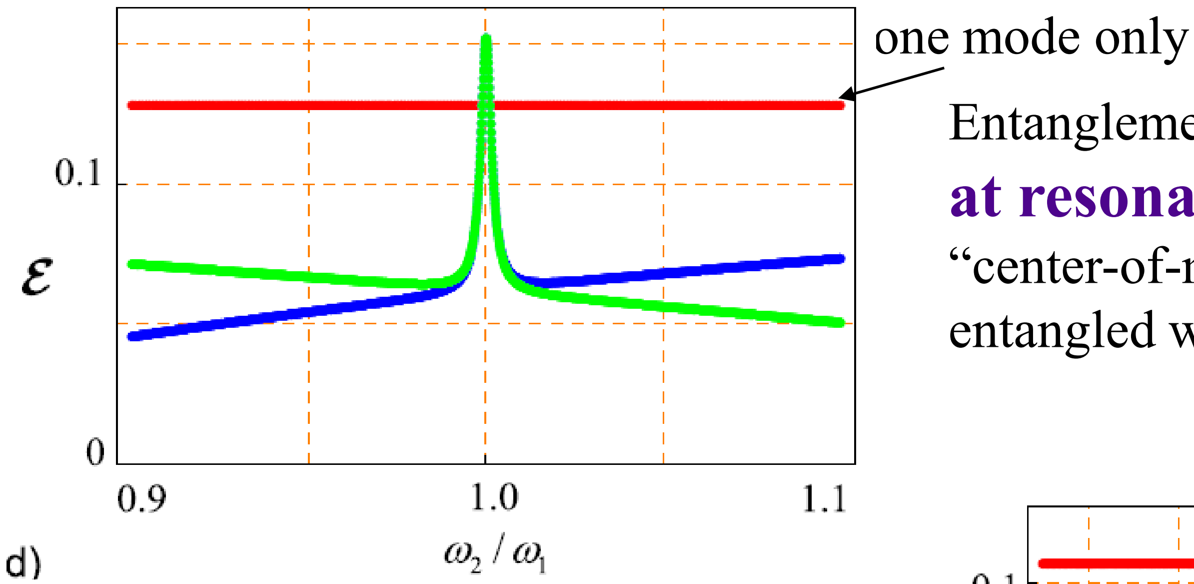
one mode only

Entanglement is more fragile and more affected than cooling

EFFECT OF NEARBY MODE ON ENTANGLEMENT

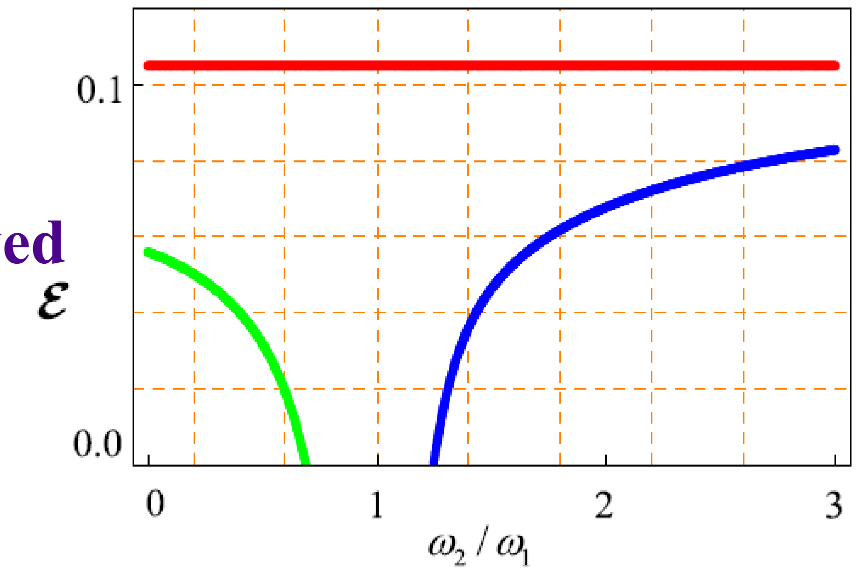
The situation is more involved when the modes are close $\delta\omega_{21} < \Gamma_2$

T = 0



Entanglement at $T = 0$ **increases at resonance** because the "center-of-mass" is strongly entangled with the cavity

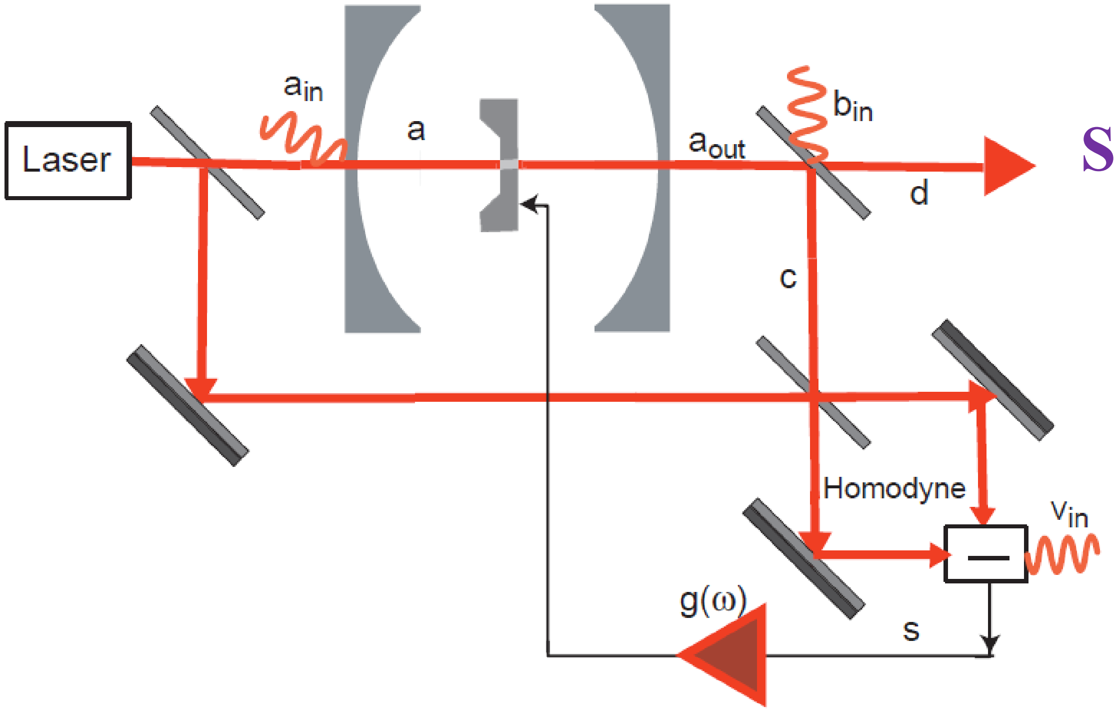
T = 0.4 K



But **entanglement at resonance is soon destroyed by temperature** due to the uncooled "relative motion"

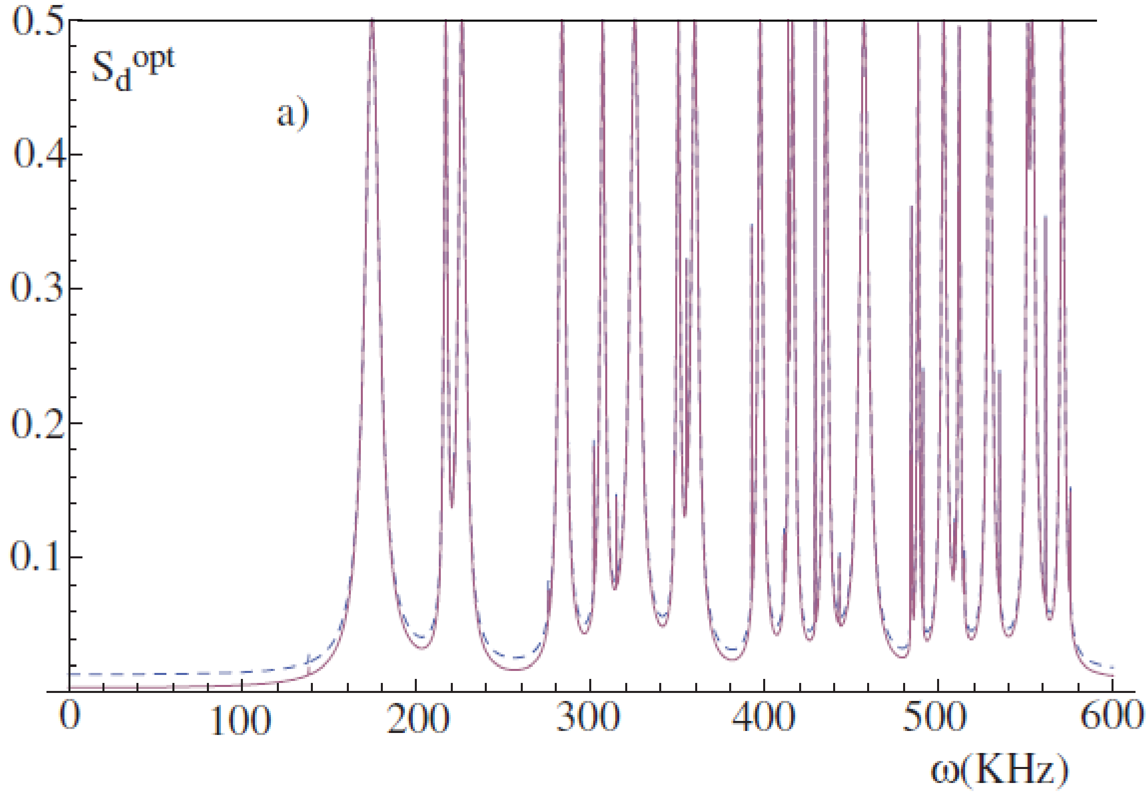
FURTHER POSSIBLE QUANTUM EFFECT:
GENERATION OF SQUEEZED LIGHT AT THE CAVITY OUTPUT

Predicted by Mancini-Tombesi, and Fabre et al. in 1994



Squeezed light

Feedback-assisted generation of squeezing ?



Shot noise

Optimized homodyne spectrum of the output light, cavity-membrane system; **feedback (full) yields little improvement** over no feedback (dashed)

Feedback does not help, **but squeezing is possible** with state-of-the art devices (main problem: low-frequency phase noise)

D. Vitali & P. Tombesi, CR Physique, to appear

CONCLUSIONS

1. Some preliminary **experimental** results with a cavity-membrane-in-the-middle system

2. **Membrane absorption** does not seriously affects ground state cooling and entanglement

3. **Simultaneous cooling and entanglement of two mechanical modes** is possible only if they are not too close in frequency

4. **Quadrature squeezing of the cavity output** is feasible with state-of-the art systems