The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific and
Cultural Organization

IAEA
International Atomic Energy Agency

2164-1

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Quantum versus Classical Dynamics of Strongly Nonlinear Resonators

Miles BLENCOWE
Department of Physics & Astronomy
Dartmouth College, Hanover
NH 03755
U.S.A.

Quantum versus Classical Dynamics of Strongly Nonlinear Resonators

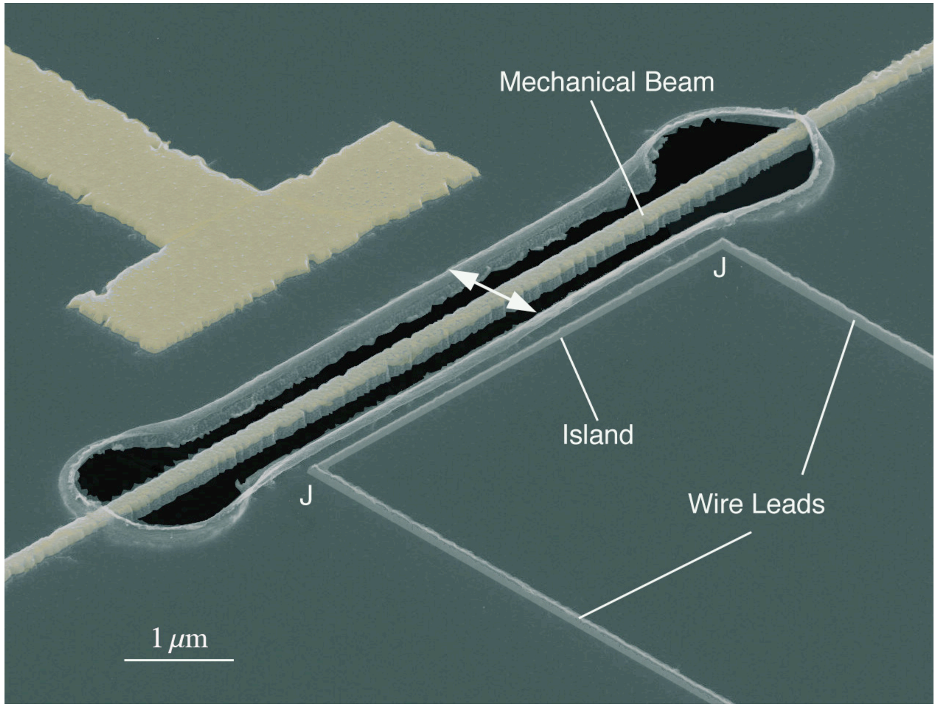
Miles Blencowe, Alex Rimberg (*Dartmouth College, USA)*
Andrew Armour (*Nottingham, UK*)

Wish to prepare a mesoscale mechanical resonator in a quantum superposition state and then watch it become classical.

Few tens of mK

$$\hbar\omega \lesssim k_B T$$

Few tens of MHz

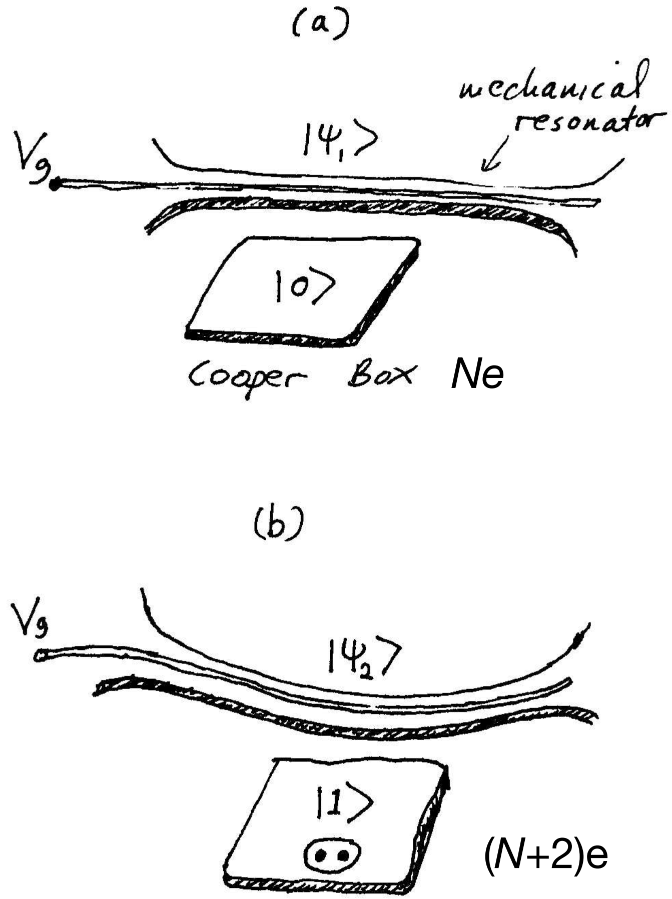


[M.D. LaHaye *et al.*, Science **304**, 74 (2004)]

Environmental decoherence at mK temperatures: two level systems, phonon radiation into supports,….

Use superconducting qubit to drive and probe mechanical resonator in a superposition state

[A.D. Armour, M.P.B. & K.C. Schwab, PRL **88**, 148301 (2002)]



$$(\alpha|0\rangle + \beta|1\rangle)\,|\Psi_1\rangle \rightarrow \alpha|0\rangle|\Psi_1\rangle + \beta|1\rangle|\Psi_2\rangle$$

But charge basis states strongly affected by environment noise.
Instead use charge degenerate basis states:
[E. Buks and M.P.B., PRB **74**,174504 (2006)]

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right)$$

Resulting (dispersive) qubit-resonator Hamiltonian:

$$H = \Delta\sigma_z + \hbar\omega_1\sigma_z a^\dagger a + \hbar\omega a^\dagger a$$

Observed in LaHaye *et al*., Nature '09

- Qubit basis states shift oscillator frequency

- Qubit-resonator interaction strength is second order in electro-mechanical coupling λ:

$$\omega_1 = \lambda^2/(\hbar\Delta) \qquad \lambda \approx \frac{\Delta x_{\rm zp}}{d}\frac{C_m}{2C_J}eV_{\rm gate}$$

Evolution of mechanical resonator assuming coherent state (for simplicity)

Initial qubit state:

$$\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

Apply a π-pulse

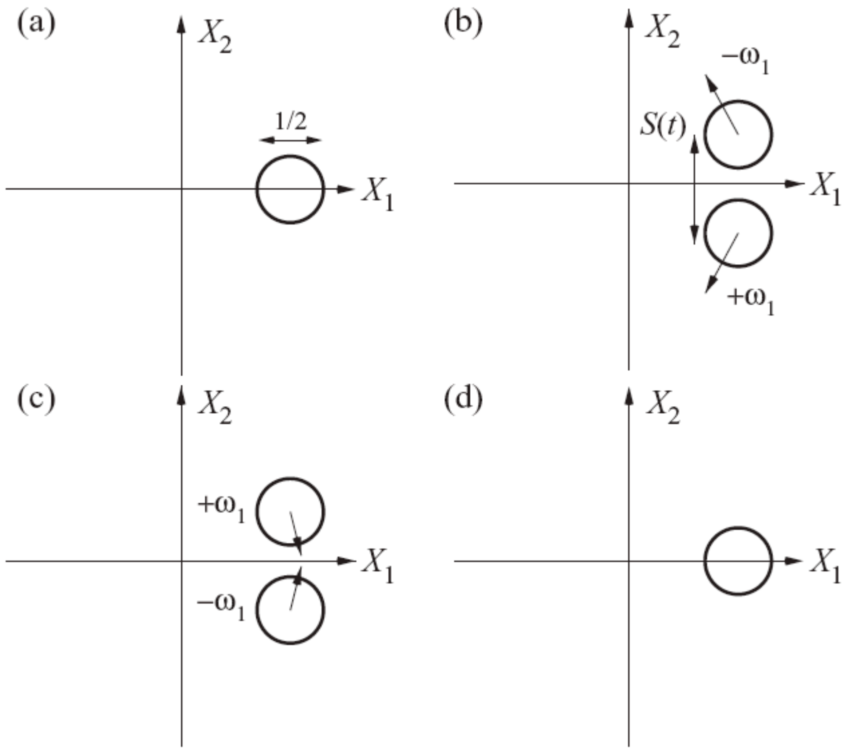


Figure 1. Schematic illustration of the evolution of the mechanical resonator in phase space during the echo sequence. Initially (a) the resonator is prepared in a coherent state and the qubit is prepared in a superposition of states. The two qubit states couple to the resonator leading to different effective frequencies $\omega \pm \omega_1$ so that in the frame rotating at the resonator frequency the two mechanical states start to pull apart (b). A π pulse inverts the qubit state and hence interchanges the relative frequencies of the two resonator states (c). When the periods of evolution before and after the inversion of the qubit are the same the resonator will return to its initial state (d) *in the absence of dissipation*.

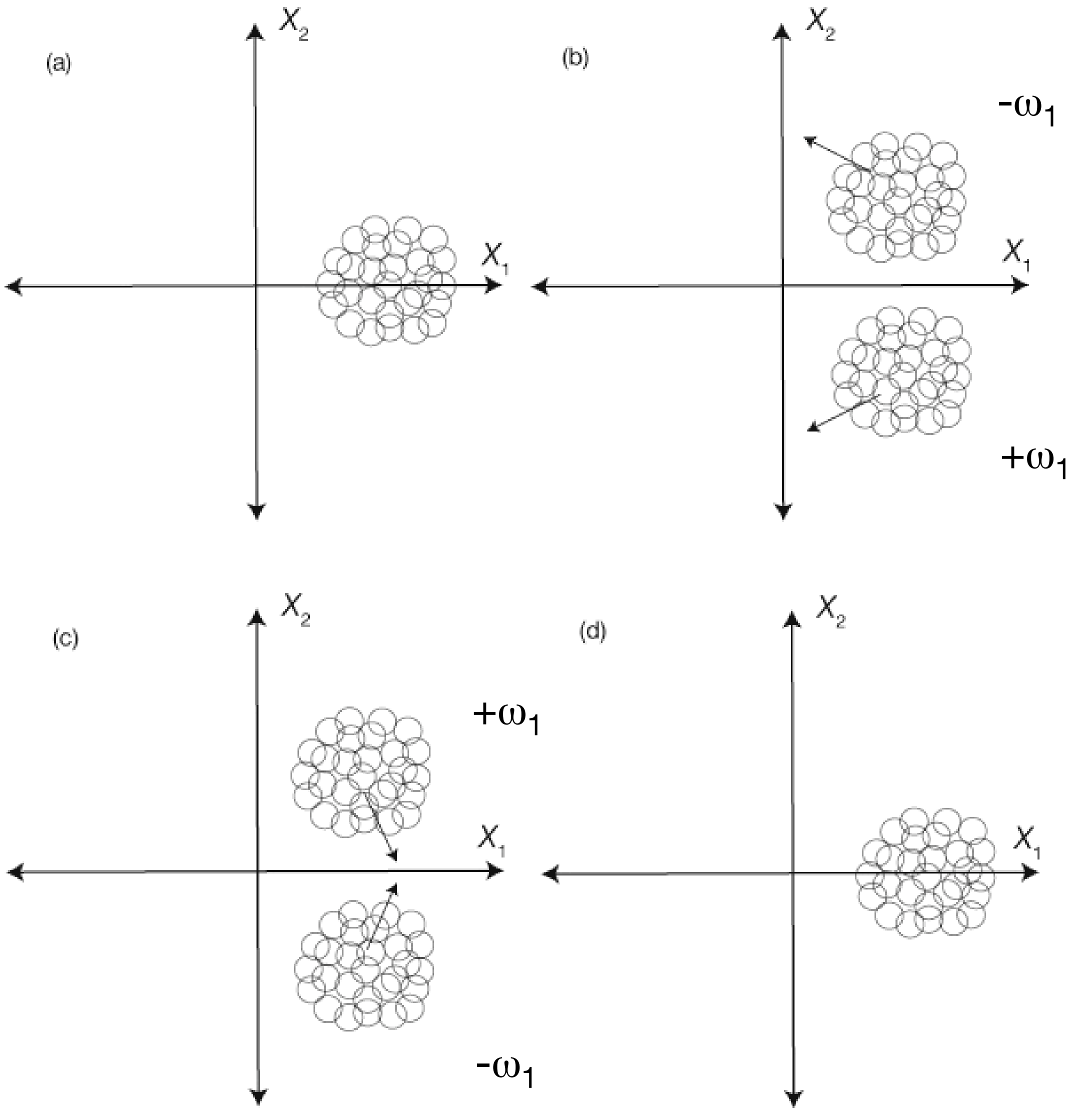
[A.D. Armour and M.P.B, New J. Phys. **10**, 095004 (2008); M.P.B. and A.D. Armour, New J. Phys. **10**, 095005 (2008)]

Evolution of mechanical resonator initially in displaced thermal state

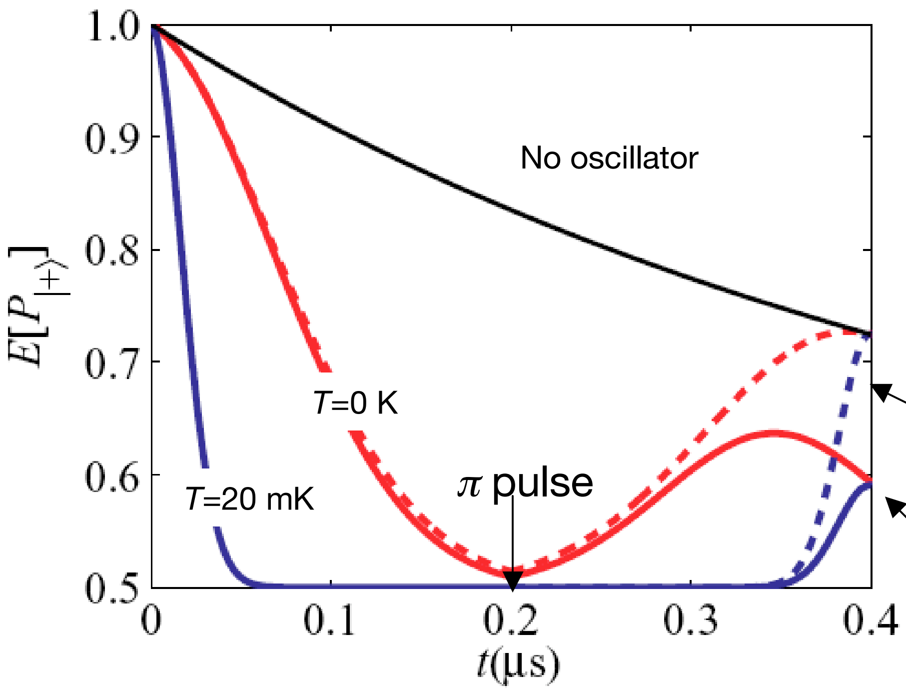
Initial qubit state:

$$\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

Apply a π-pulse



Probability of qubit to be in $|+\rangle$ state vs time



$$
\begin{aligned}
\omega/2\pi &= 50 \text{ MHz} \\
2\Delta/h &= 5 \text{ GHz} \\
T_2 &= 0.5\ \mu s \\
T &= 20 \text{ mK}
\end{aligned}
$$

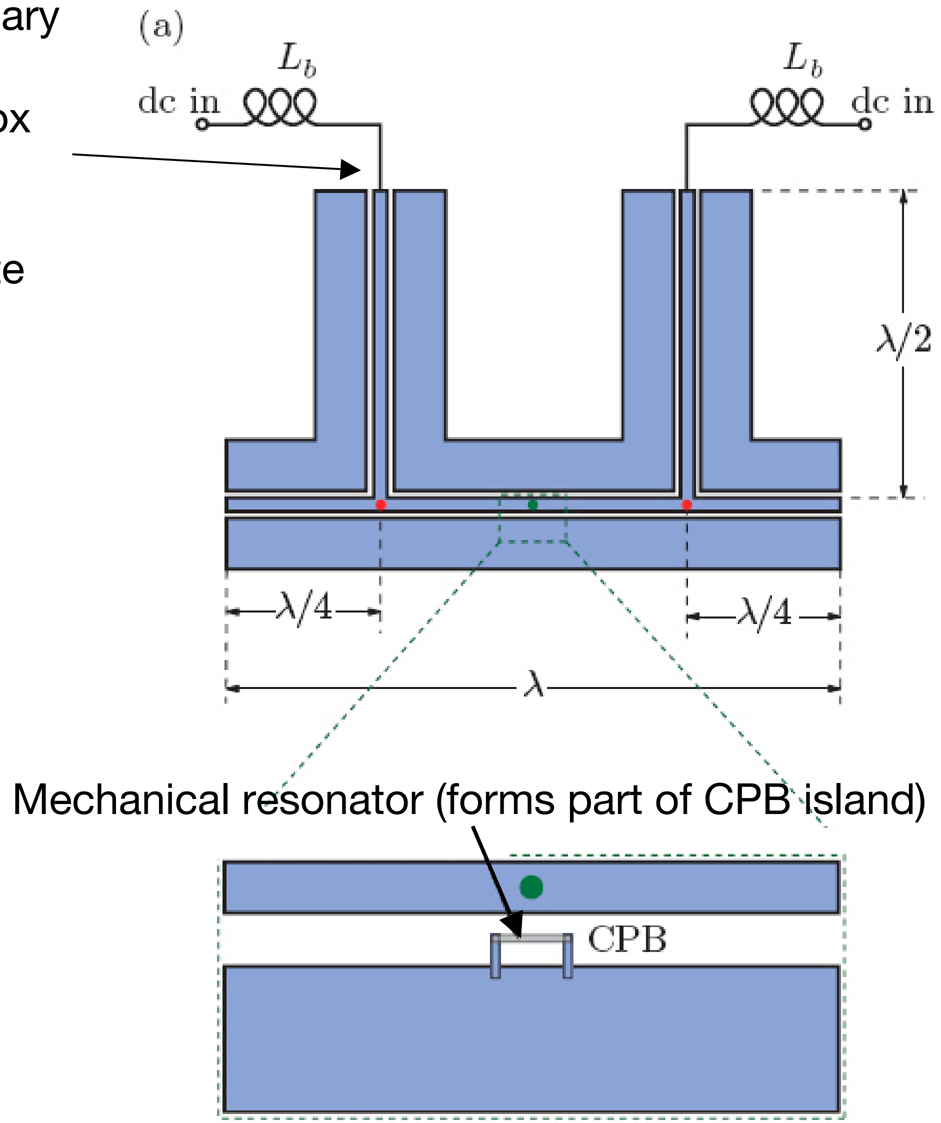
No oscillator dissipation (dashed)

With oscillator dissipation (solid)

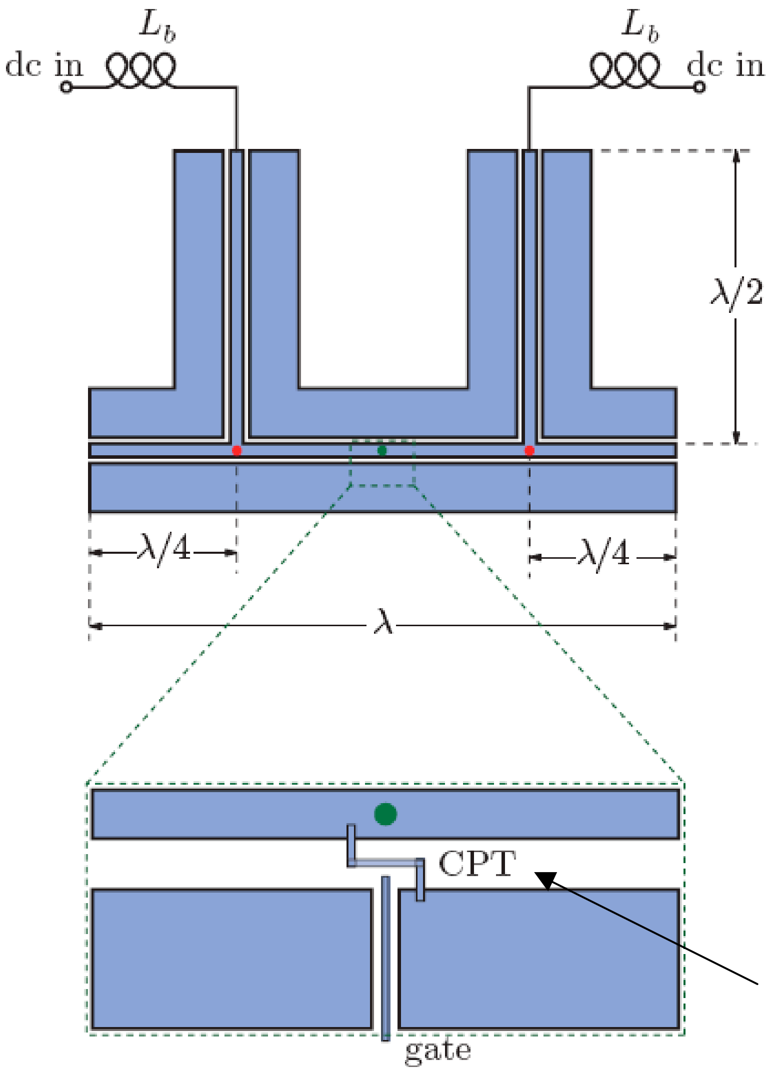
Envelope of oscillations in $P_{|+\rangle}$ in an echo experiment with a π pulse applied at $t(=t_1) = 0.2\,\mu s$. The blue curves are for coupling strength $\kappa = 0.2$, with the resonator starting in a displaced thermal state, with an initial displacement given by $\alpha_0 = 25$ and a width which is set by the temperature of its surroundings: $\bar{n} = \bar{m} = 10$. The red curves are for the same parameters, but now the initial state, though displaced from the origin by the same amount as before, is a pure coherent state with $\bar{m} = 0$. In each case, the full curve is for $Q = 3000$ and the dashed curve is for the case without any mechanical dissipation. The black curve is the result that would be obtained without any coupling to the mechanical resonator.

Coplanar waveguide-based realization of scheme

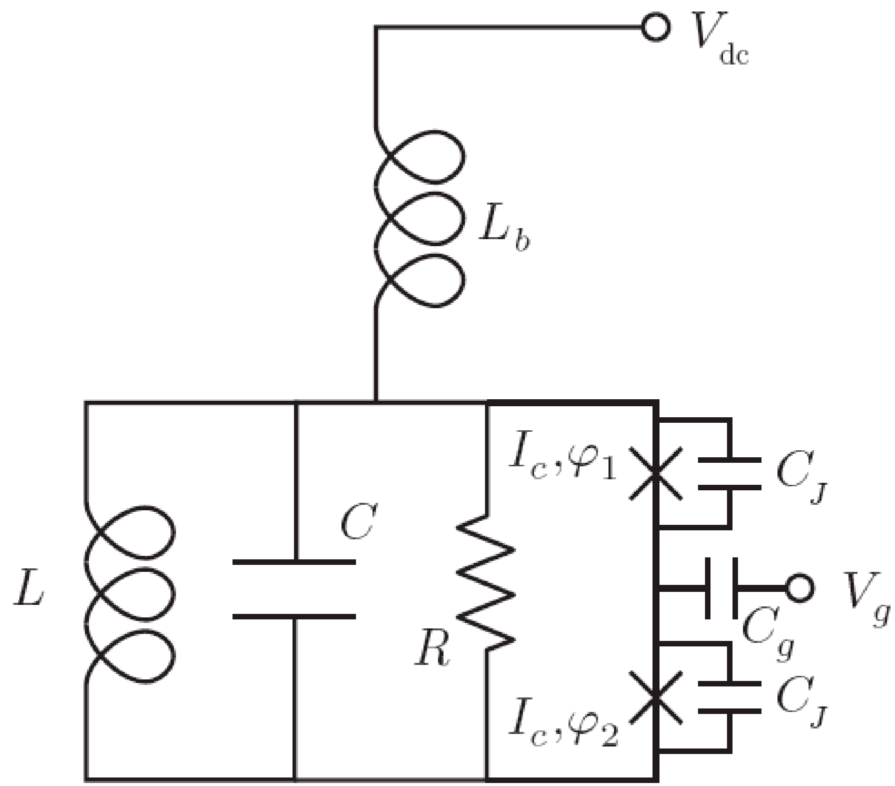
Centre conductor dc bias: necessary for
- Strongly coupling Cooper pair box (CPB) to mechanical resonator
- MHz ac driving of mechanical resonator→displaced thermal state



“Warm-up” investigation: quantum versus classical dynamics of a strongly nonlinear, low noise cavity-Cooper pair transistor system (no mechanical resonator)



Cooper pair transistor (CPT)



Single microwave mode equivalent circuit

Mechanical equivalent: oscillator-driven pendulum system

$$\gamma_\pm = (\varphi_1 \pm \varphi_2)/2$$

$$\mathcal{H} = \frac{p_+^2}{2M_+} + \frac{1}{2}M_+\omega_+^2\gamma_+^2 + \frac{p_-^2}{2M_-} + M_-\omega_-^2 \cos\gamma_- \cos(\gamma_+ + \omega_d t)$$

$$\omega_d = \frac{L}{L_b}\frac{eV_{dc}}{\hbar}$$

$$\frac{M_+}{M_-} = \frac{R_K}{Z}\frac{E_{C_J}}{\hbar\omega_0} = \frac{2C}{C_J} \gg 1 \qquad \frac{\omega_-}{\omega_+} = 2\frac{\sqrt{E_J E_{C_J}}}{\hbar\omega_0} \gg 1$$

Have slow, massive oscillator (cavity mode: phase coord. γ_+) interacting with fast, low mass pendulum (CPT: phase γ_-, charge $N = -p_-/\hbar$ coords.).

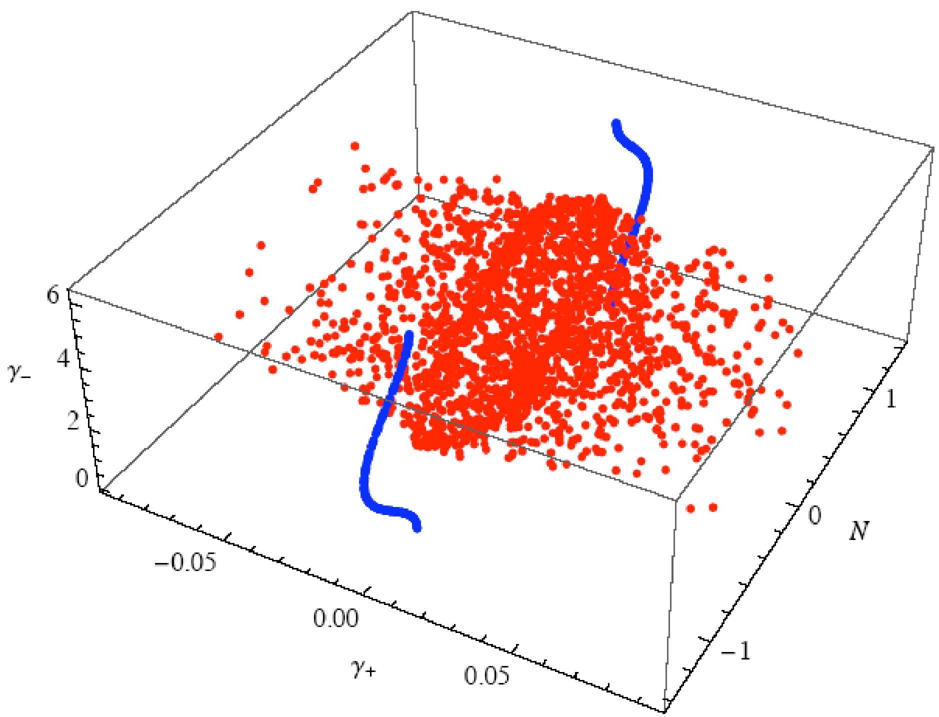
$$\omega_d = \frac{L}{L_b} \frac{eV_{dc}}{\hbar}$$

Drive frequency ω_d tuned by varying external dc bias V_{dc} . No external rf drive required (self-oscillating via ac Josephson effect)$\Rightarrow$low noise.

Example strobe plot

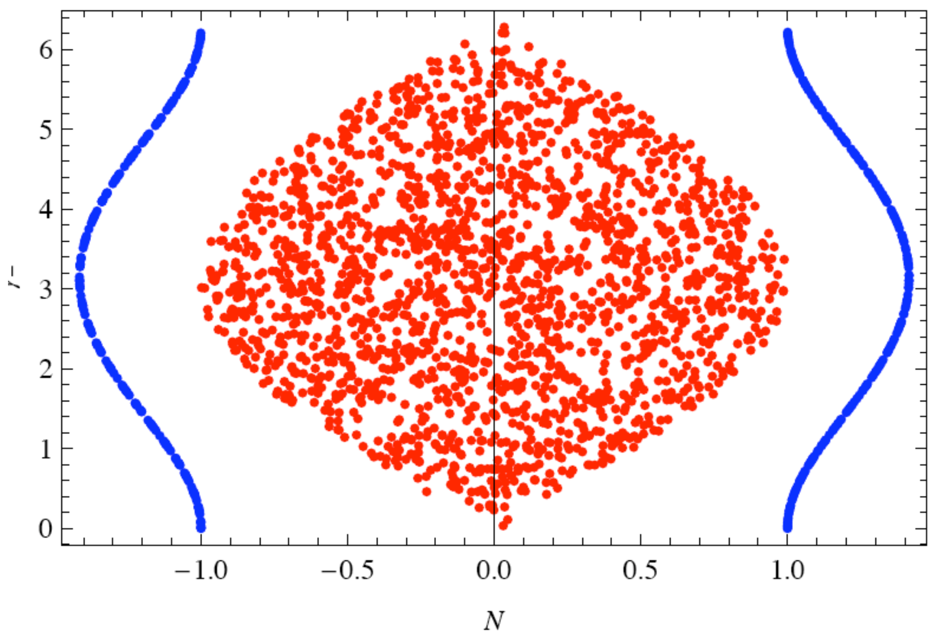
Damping/noise on oscillator (cavity mode) $\omega_d = \omega_+$

Find coexisting chaotic (red) and aperiodic (blue) attractors:



3D projection

2D projection



Quantum dynamics (in progress):

- Complementary viewpoints: cavity mode photons interact with Cooper pair current in transistor ↔ oscillator coupled to driven pendulum.

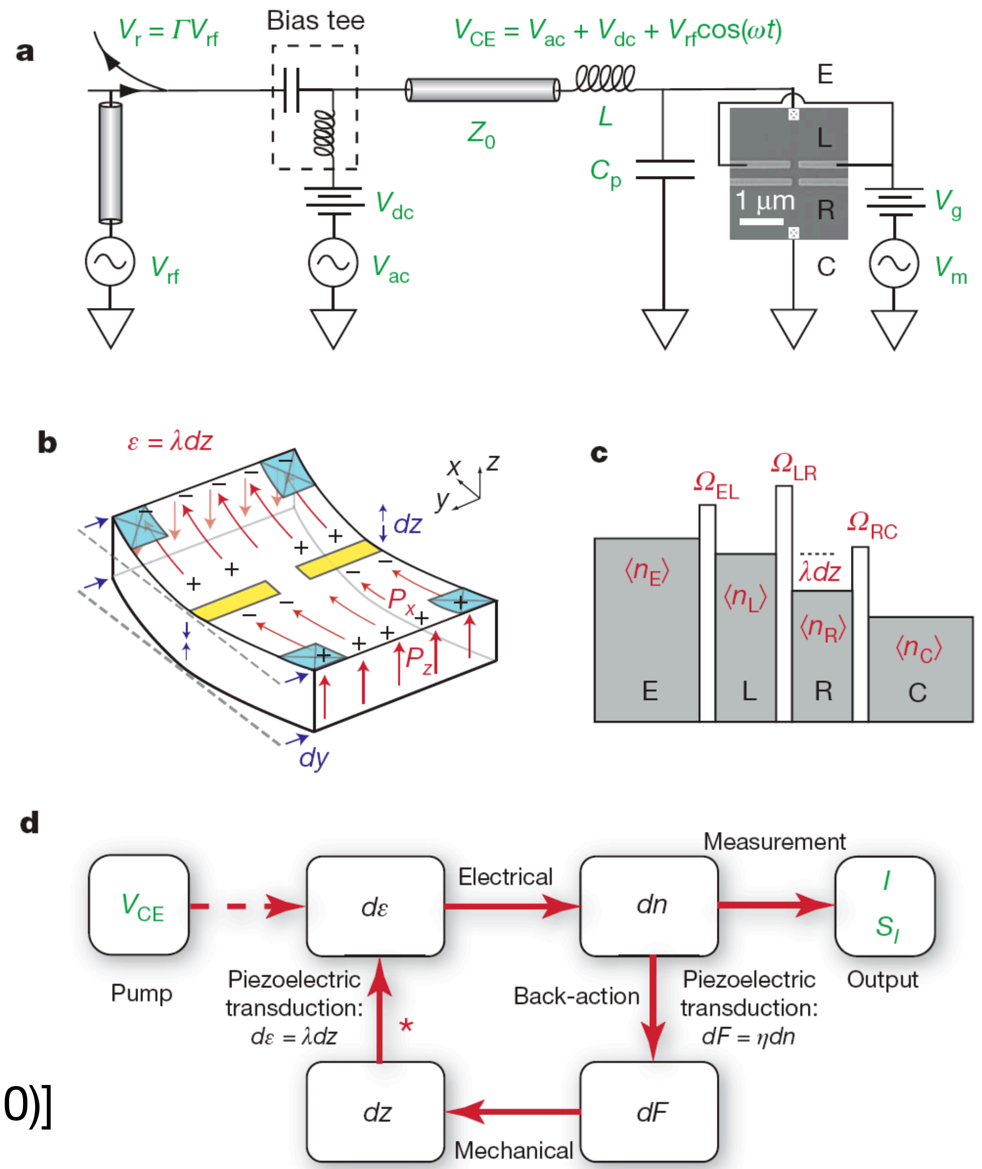
- Classical and quantum dynamics sensitive to type of environment (cavity loss, gate voltage noise on CPT etc).

- Quantum phase space description/visualization for pendulum coordinate (particle on a circle): appropriate Wigner/Husimi representation.

- Conditions for approximate emergence of classical dynamics from quantum dynamics [Katz et al., PRL ‘07].

- Possible quantum dynamics [quantum activation (Dykman, PRE ‘07), tunneling between aperiodic attractors through chaotic ‘sea’ etc].

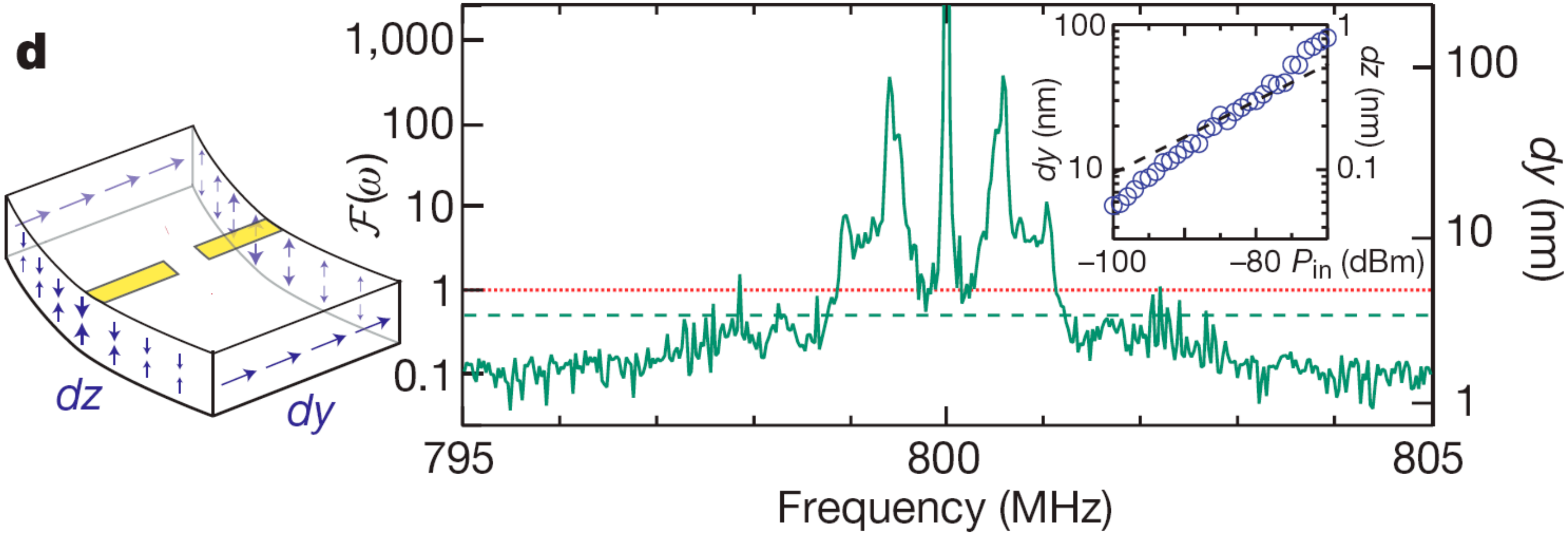
Another possible path towards demonstrating quantum signatures of macroscopic mechanical resonator?



rf-quantum point contact piezoelectrically coupled to mechanical resonator (host GaAs crystal)

[J. Stettenheim *et al*., Nature **466**, 86 (2010)]

Tunneling current shot noise found to be strongly frequency dependent



of the peak in the shot noise. **a**, Left axis: frequency-dependent Fano factor, $\mathcal{F}(\omega)$, for sample A for $P_{\text{in}} = -68$ dBm, showing drastically super-Poissonian ($\mathcal{F} \gtrsim 100$) and sub-Poissonian ($\mathcal{F} \approx 0.1$) noise as a function of frequency (on a logarithmic scale). The red dotted line indicates Poissonian noise and the green dashed line gives the Fano factor $\mathcal{F}(\omega) = \mathcal{F}_{\text{dc}} = 0.5$ expected for an uncoupled detector. Right axis: displacement *dy* versus frequency, showing the strongly non-thermal nature of the resonator dynamics. Inset, displacements *dy* and *dz* versus input power for sample A.

[J. Stettenheim *et al*., Nature **466**, 86 (2010)]

$\sim 10^4$ *e* dipolar charge fluctuations due to electrons tunneling through point contact barrier causes ~ 50 nm amplitude vibrations of mm^3 host crystal.

Mass equivalent to jumping flea causing Mt. Everest to sway by a few metres.

Model Hamiltonian:

$$H = H_{\text{sys}} + H_{\text{bath}} + H_{\text{int}}$$

$$H_{\text{sys}} = \hbar\omega_m a^\dagger a + \sum_L (\varepsilon_L + \lambda\hat{z}/2) b_L^\dagger b_L + \sum_R (\varepsilon_R - \lambda\hat{z}/2) b_R^\dagger b_R$$

$$H_{\text{bath}} = \sum_E \varepsilon_E b_E^\dagger b_E + \sum_C \varepsilon_C b_C^\dagger b_C$$

$$H_{\text{int}} = \sum_{E,L} \hbar\Omega_{EL}(b_E^\dagger b_L + b_L^\dagger b_E) + \sum_{C,R} \hbar\Omega_{CR}(b_C^\dagger b_R + b_R^\dagger b_C) + \sum_{L,R} \hbar\Omega_{LR}(b_L^\dagger b_R + b_R^\dagger b_L)$$

Born-Markov approximated classical master equation:

$$\frac{\partial\rho}{\partial t}=\left[\omega_m^2 z+\frac{\lambda}{2m}\left(\sum_L n_L-\sum_R n_R\right)\right]\frac{\partial\rho}{\partial v}-v\frac{\partial\rho}{\partial z}+\gamma_{\text{ext}}\frac{\partial}{\partial v}\left(v\rho+\frac{k_B T}{m}\frac{\partial\rho}{\partial v}\right)$$

$$-2\pi\hbar\sum_{E,L}\Omega_{EL}^2\delta\left(\varepsilon_L+\lambda z/2-\varepsilon_E\right)\begin{Bmatrix}\left[\left(1-n_L\right)\rho\left(n_L\right)-n_L\rho\left(n_L-1\right)\right]\left\langle n_E\right\rangle\\+\left[n_L\rho\left(n_L\right)-\left(1-n_L\right)\rho\left(n_L+1\right)\right]\left(1-\left\langle n_E\right\rangle\right)\end{Bmatrix}$$

$$-2\pi\hbar\sum_{C,R}\Omega_{CR}^2\delta\left(\varepsilon_R-\lambda z/2-\varepsilon_C\right)\begin{Bmatrix}\left[\left(1-n_R\right)\rho\left(n_R,N\right)-n_R\rho\left(n_R-1,N+1\right)\right]\left\langle n_C\right\rangle\\+\left[n_R\rho\left(n_R,N\right)-\left(1-n_R\right)\rho\left(n_R-1,N-1\right)\right]\left(1-\left\langle n_C\right\rangle\right)\end{Bmatrix}$$

$$-2\pi\hbar\sum_{L,R}\Omega_{LR}^2\delta\left(\varepsilon_L-\varepsilon_R+\lambda z\right)\begin{Bmatrix}n_L\left(1-n_R\right)\rho\left(n_L,n_R\right)-\left(1-n_L\right)n_R\rho\left(n_L+1,n_R-1\right)\\+\left(1-n_L\right)n_R\rho\left(n_L,n_R\right)-n_L\left(1-n_R\right)\rho\left(n_L-1,n_R+1\right)\end{Bmatrix}$$

Can a quantum flea induce quantum behaviour in Everest?

Smaller, higher *Q* resonators: quantum signatures in the QPC current noise?

[Bennett and Clerk, PRB ‘08]