



The Abdus Salam
International Centre for Theoretical Physics



2164-3

**Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the
Quantum Regime**

6 - 10 September 2010

Measuring the Quantum Harmonic Oscillator

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Measuring the quantum harmonic oscillator



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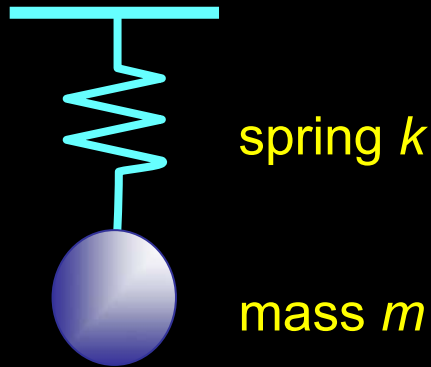


**NOEMS in the
quantum regime**

Abdus-Salam ICTP
Trieste Italy
September 2010

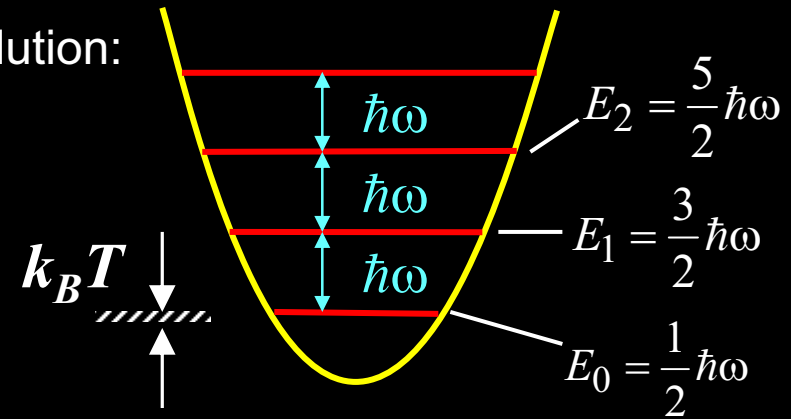
Monday September 6 9:40-10:20

Harmonic oscillator in the quantum limit



Resonance frequency: $\omega = \sqrt{\frac{k}{m}}$

Quantum solution:



Cooling to quantum ground state:

$$\hbar\omega \gg k_B T$$

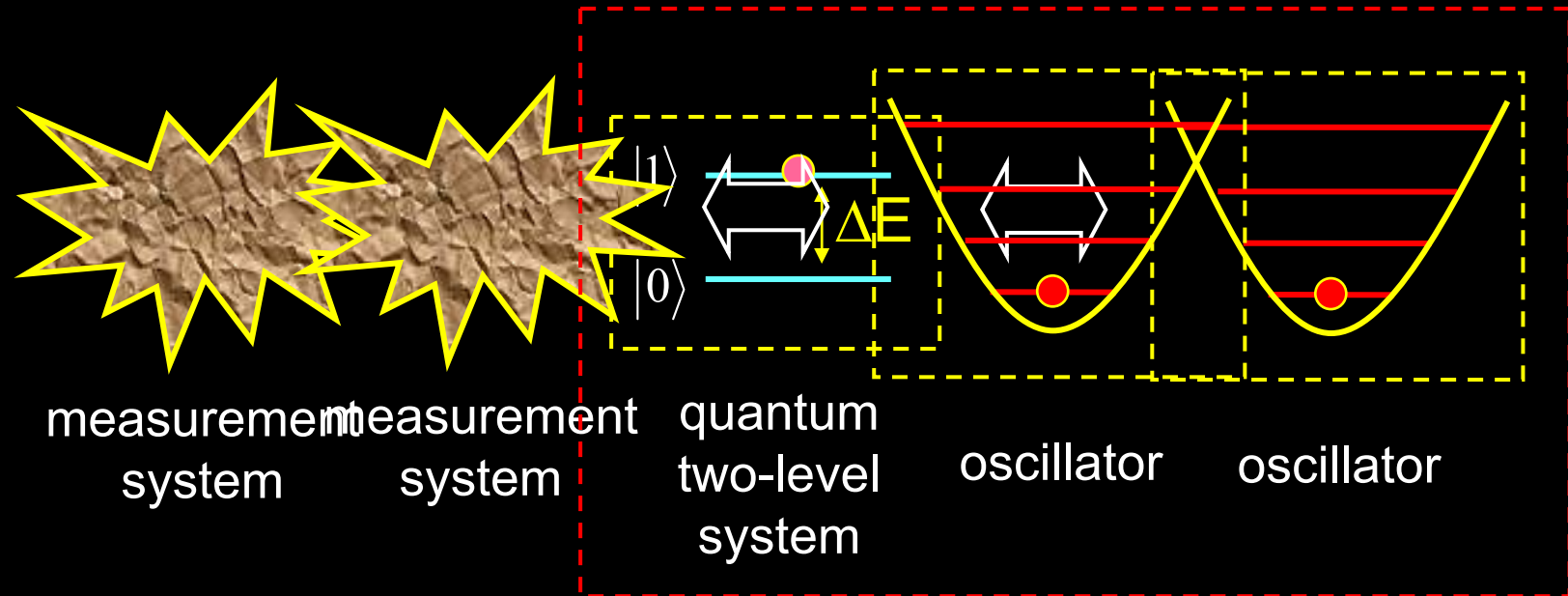
Conventional refrigeration: $T_{\min} = 20 \text{ mK} \implies \omega/2\pi > 1 \text{ GHz}$

Control and measurement present significant challenges

- Detection:
 $\langle x(t) \rangle$ identical in classical & quantum systems (correspondence limit)
- Detecting noise:
Amplifier noise & back-action noise dominate (Heisenberg uncertainty)
- Excitations:
A classical signal at $\omega/2\pi$ superposes many states (coherent state)

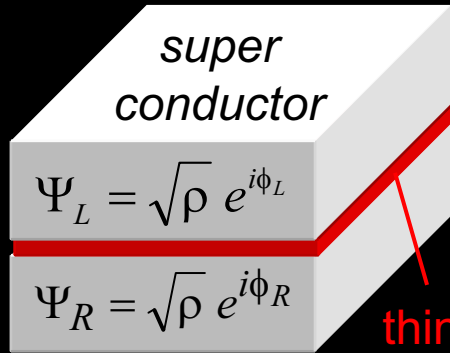
Measuring a harmonic oscillator in the quantum limit

- Measurement system destroys oscillator quantum state
- Interpose quantum two-level system



Coupled system quantum coherent
Allows complete quantum measurement & control

Josephson phase qubit: An electronic two-level system

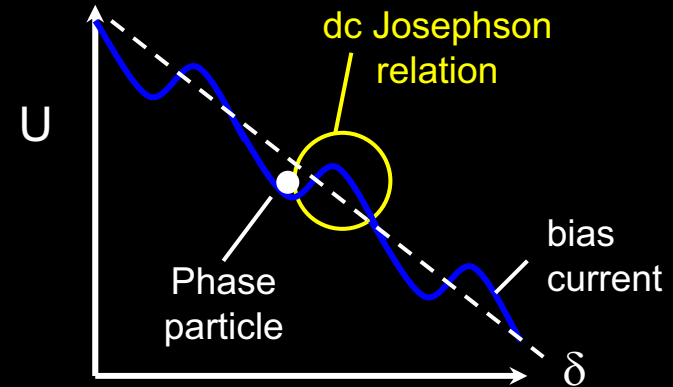


thin insulator (~1 nm)

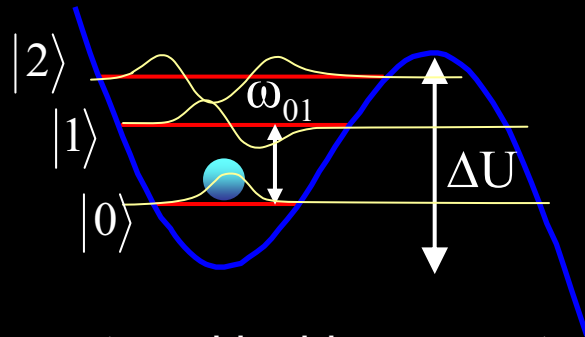
Phase difference

$$\delta = \phi_L - \phi_R$$

ac & dc
Josephson relations

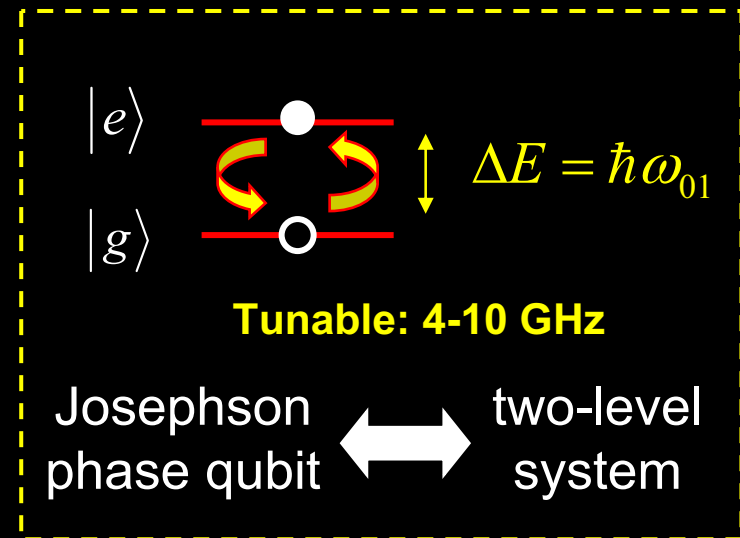


At 20 mK δ is a quantum variable:



- ω_{01} tuned by bias current: 4-10 GHz
- Nonlinearity makes $\omega_{12} \sim 0.95 \omega_{01}$

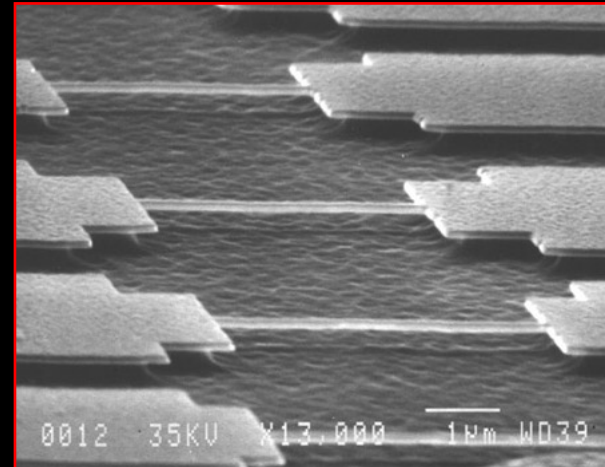
Phase qubit for quantum computation



Candidates for quantum harmonic oscillators

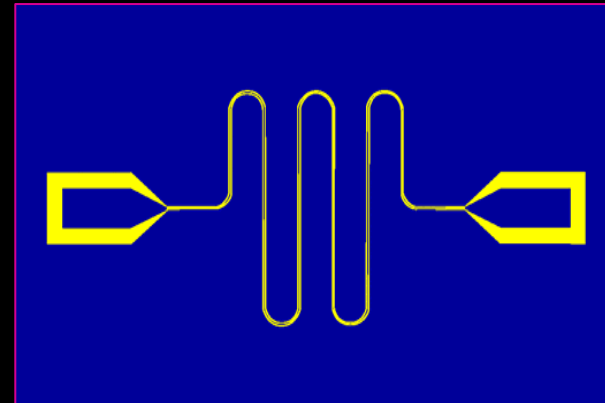
- Nanomechanical resonators

- Resonance frequencies up to ~ 10 GHz
- Integrable with phase qubit
- Quanta are *phonons*
- **Quality factors $\sim 10^3$**



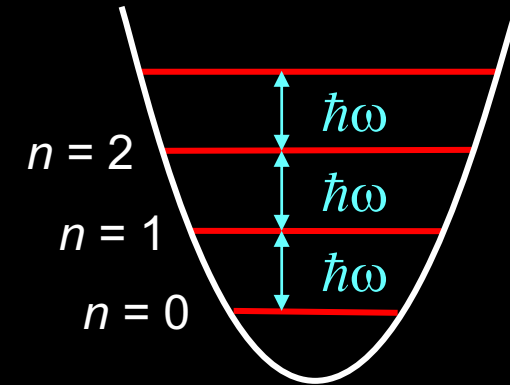
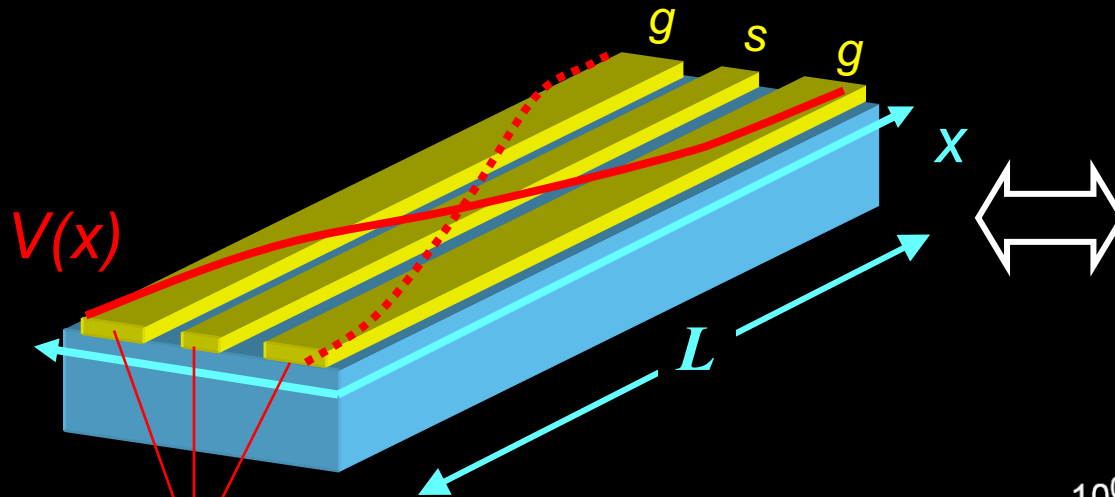
- Electromagnetic resonators

- Resonance frequencies up to ~ 100 GHz
- Integrable with phase qubit
- Quanta are *photons*
- **Quality factors $\sim 10^5$ - 10^6**



Max Hofheinz & Haohua Wang

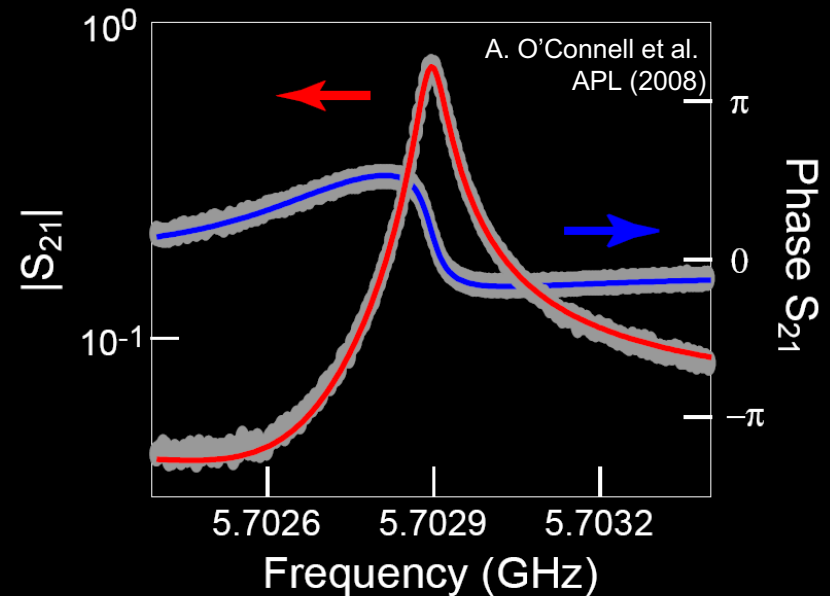
Half-wave coplanar stripline resonator



Photons in half-wave mode

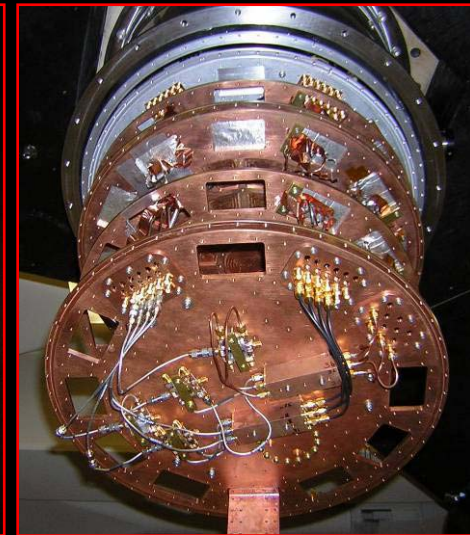
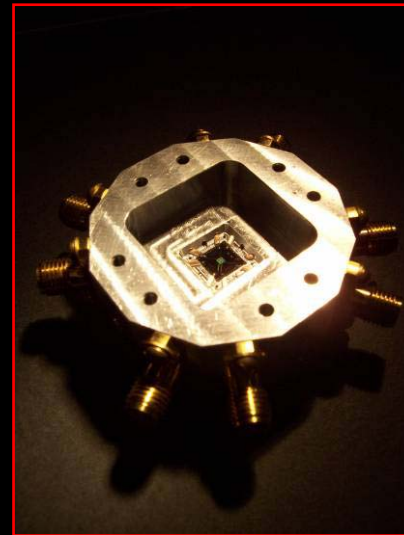
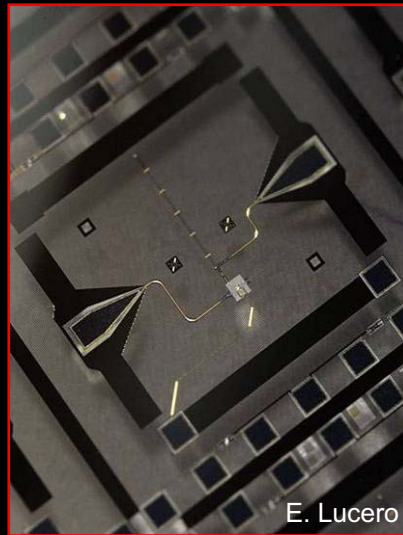
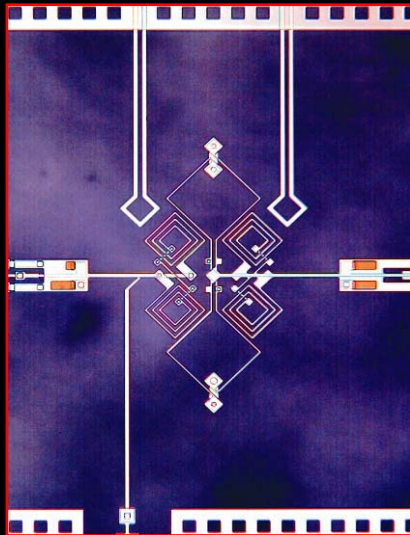
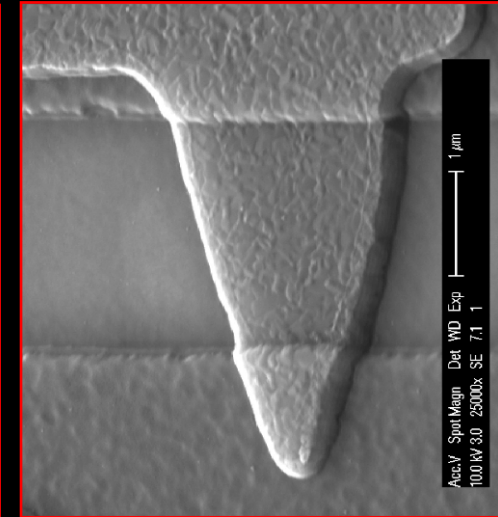
open terminations yield
voltage antinodes

- Wavelength $\lambda = 2L \sim 10\text{-}15$ mm
- Resonance frequency $\omega/2\pi \sim 5\text{-}10$ GHz
- Quality factor $Q \sim 10^5\text{-}10^6$
(determined by dielectric loss)
- Excitation lifetime \sim few μs

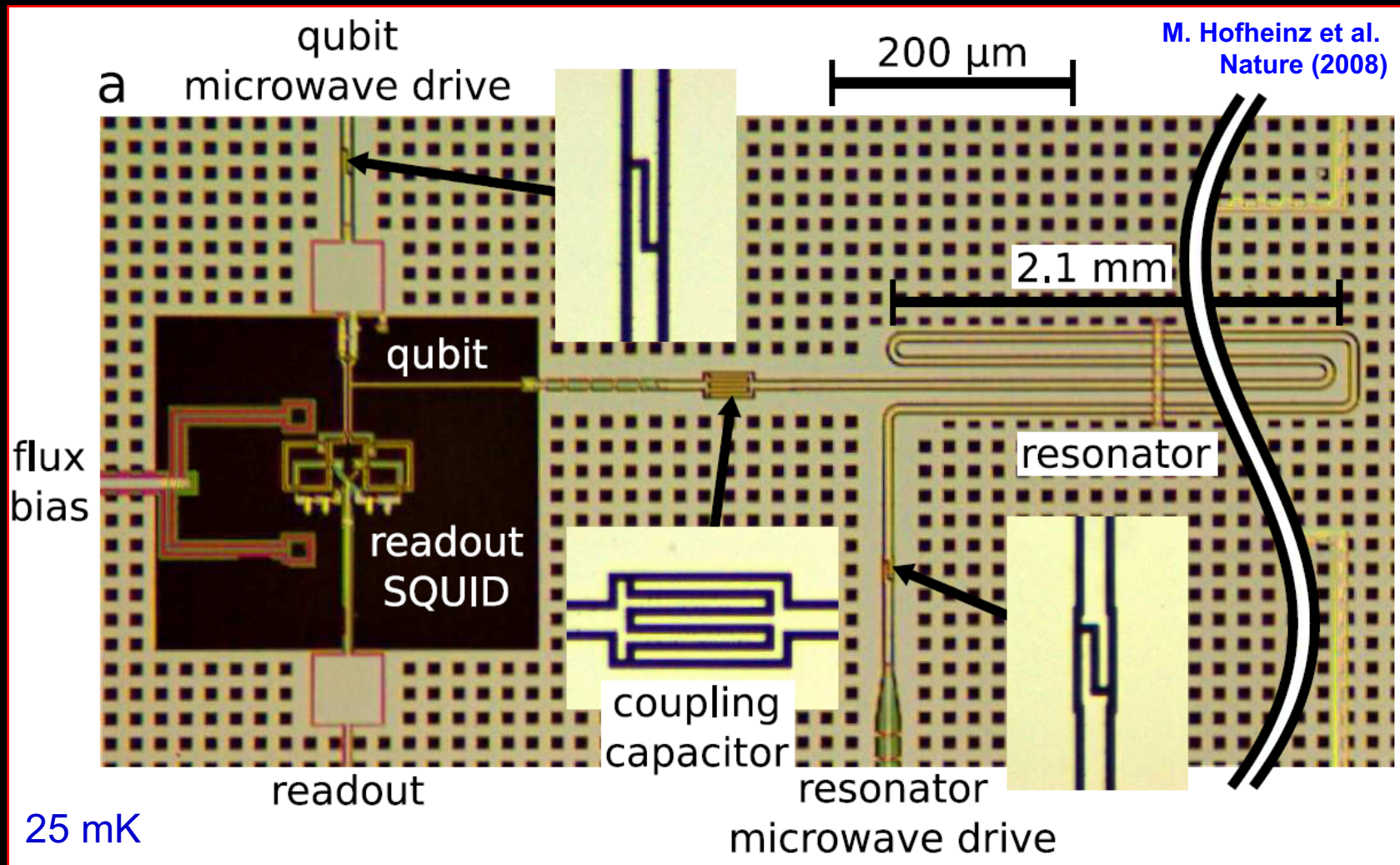


Making & measuring quantum devices

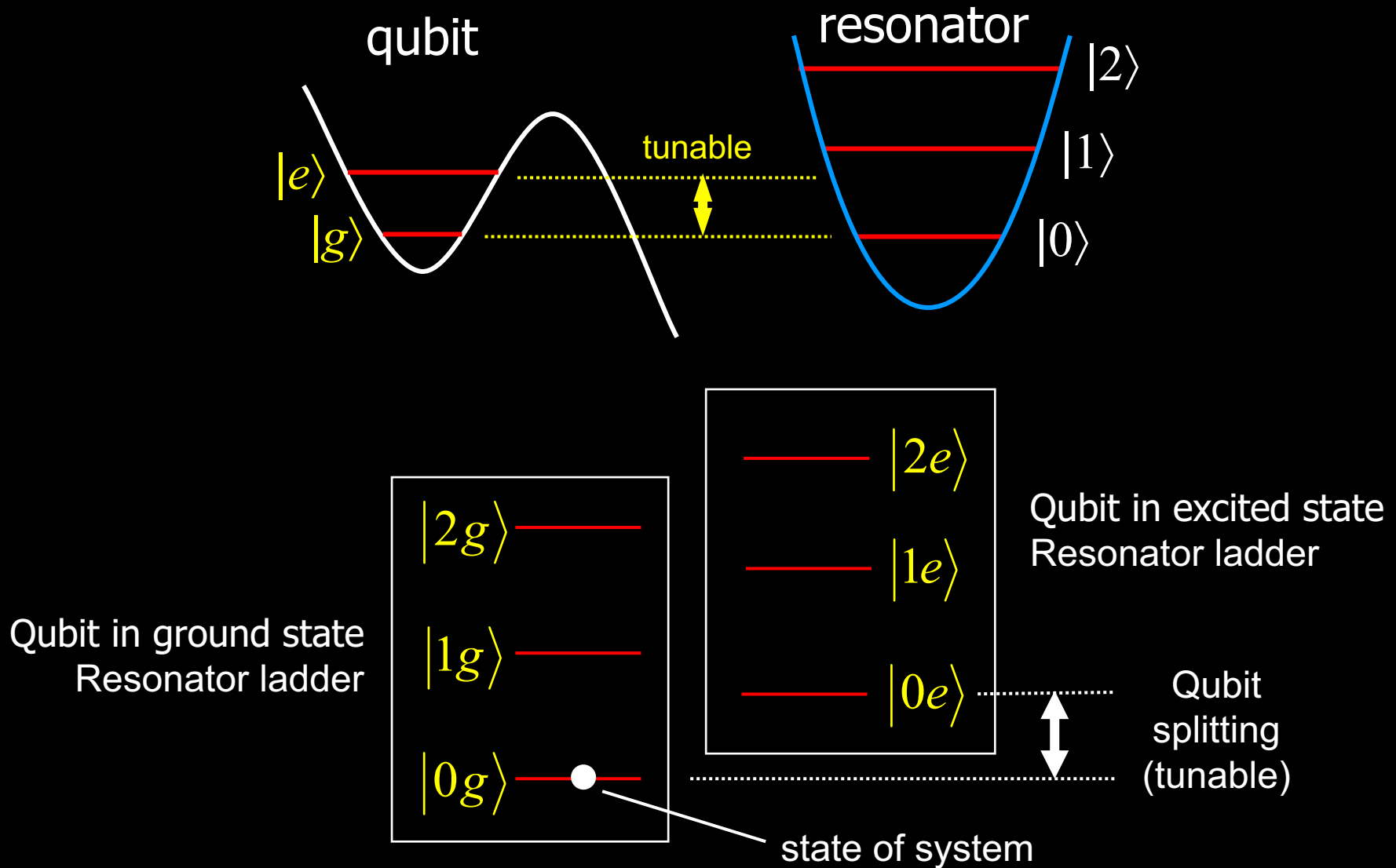
- 8 layers optical lithography, deposition, etching (UCSB Nanofabrication Facility)
- 3" sapphire wafers
- Diced into 1/4" x 1/4" chips
- Mount on dilution refrigerator (25 mK)
- Control with custom electronics



Coupled electromagnetic resonator & Josephson qubit

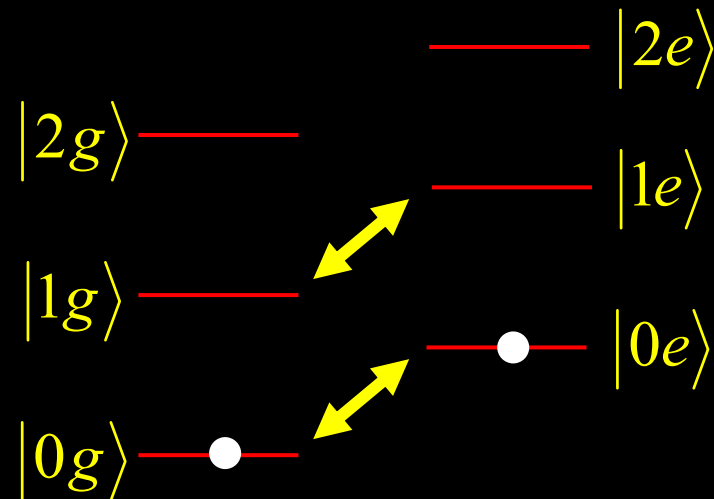


Coupled resonator-qubit energy levels



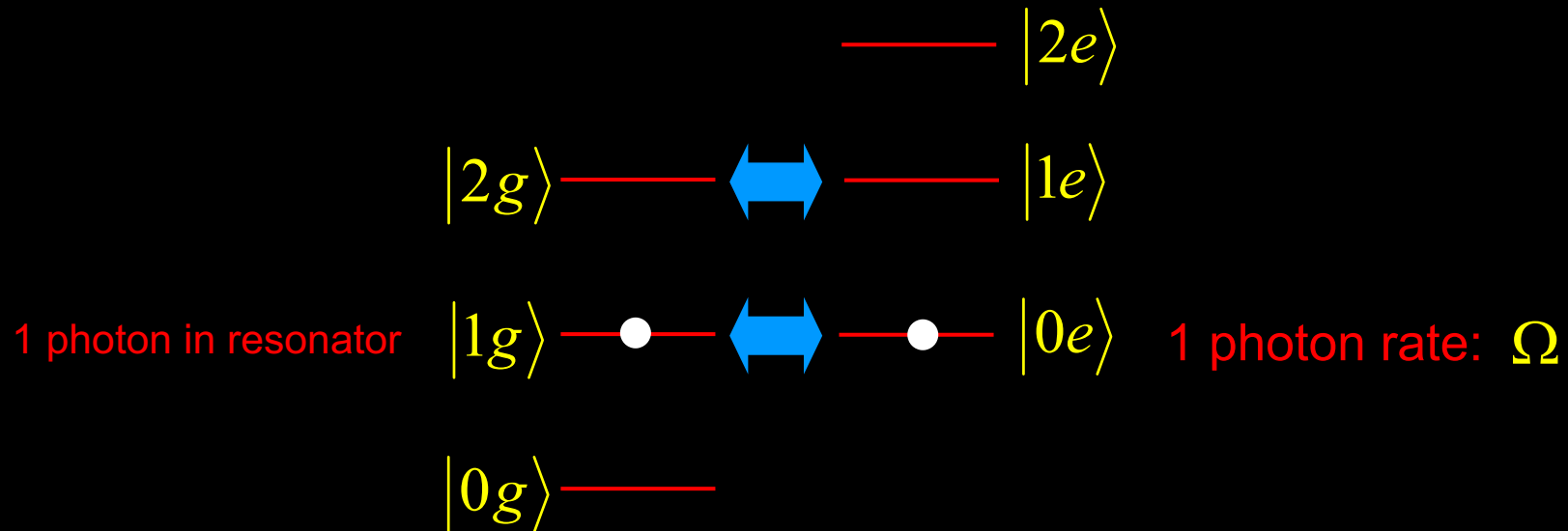
Resonator-qubit time-domain control

- Qubit off resonance (system in $|0g\rangle$ state)
- Apply microwave π pulse to qubit (goes to $|0e\rangle$ state)



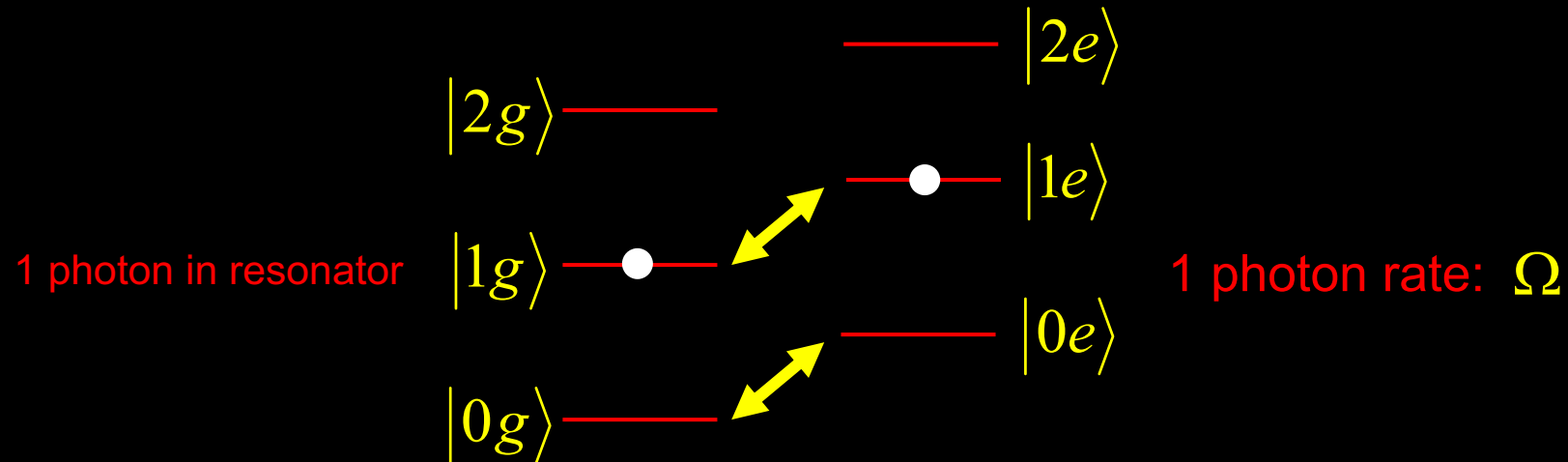
Resonator-qubit time-domain control

- Qubit off resonance (system in $|0g\rangle$ state)
- Apply microwave π pulse to qubit (goes to $|0e\rangle$ state)
- Tune qubit to resonator frequency
- Rabi oscillation: Transfer photon from qubit to resonator



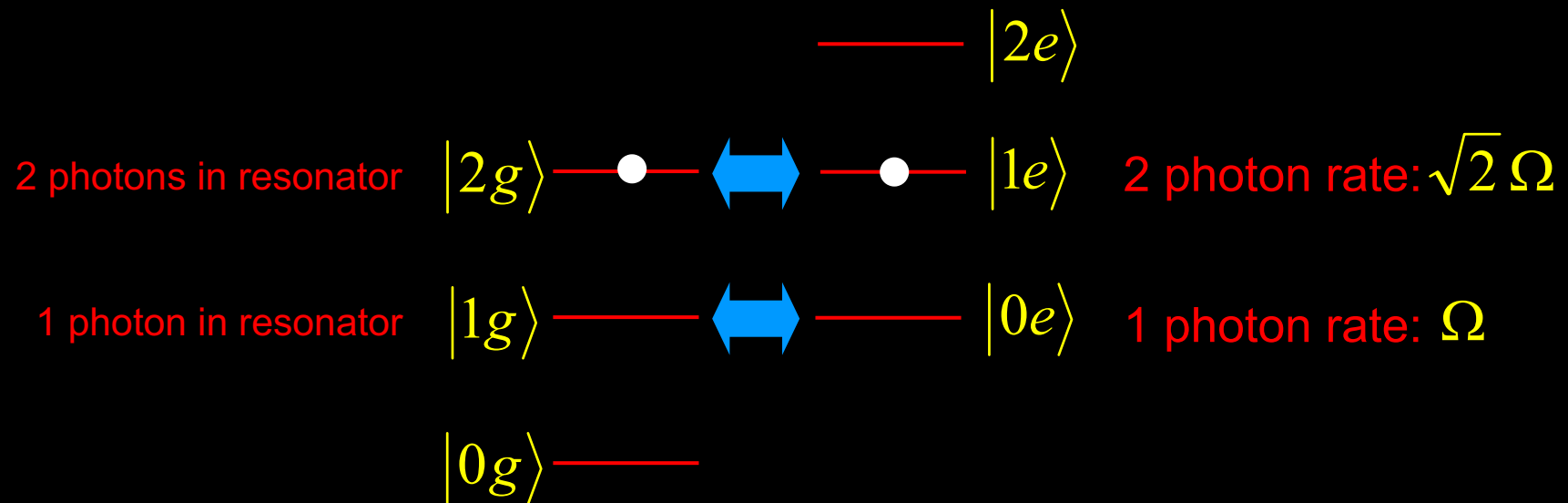
Adding more photons

- Detune qubit (system in $|1g\rangle$ state)
- Apply microwave π pulse to qubit (goes to $|1e\rangle$ state)



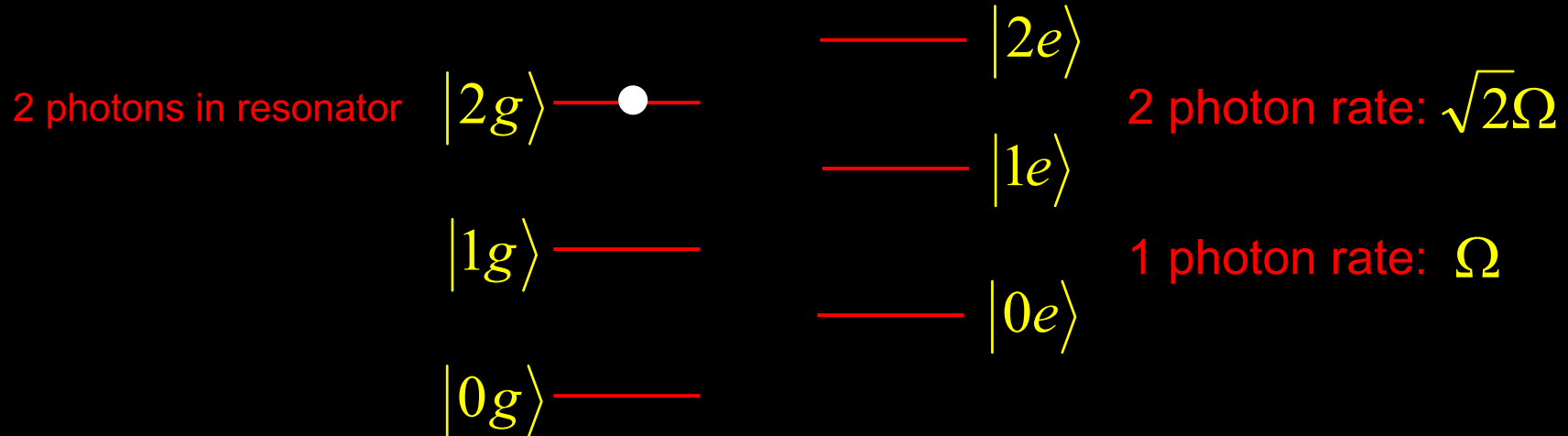
Adding more photons

- Detune qubit (system in $|1g\rangle$ state)
- Apply microwave π pulse to qubit (goes to $|1e\rangle$ state)
- Tune qubit to resonator, Rabi (goes to $|2g\rangle$ state)



Adding more photons

- Detune qubit (system in $|1g\rangle$ state)
- Apply microwave π pulse to qubit (goes to $|1e\rangle$ state)
- Tune qubit to resonator, Rabi (goes to $|2g\rangle$ state)
- Repeat for n photons: Each transfer \sqrt{n} faster

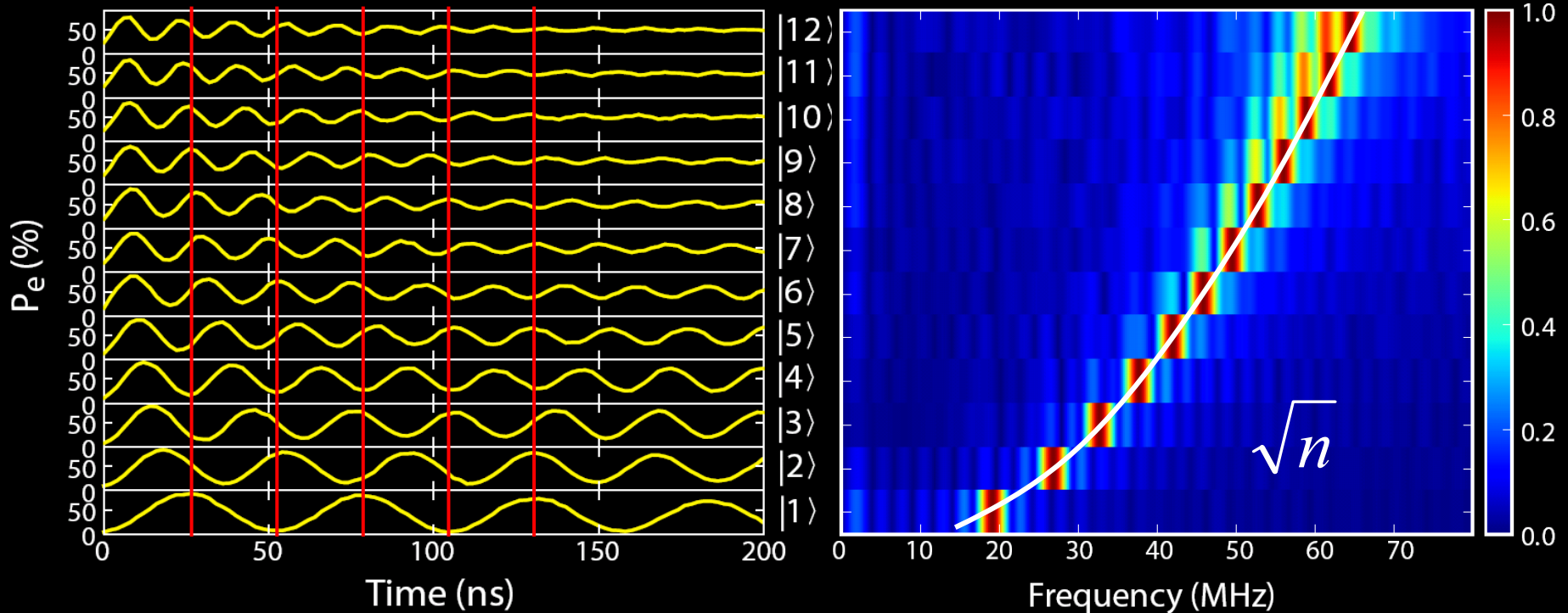
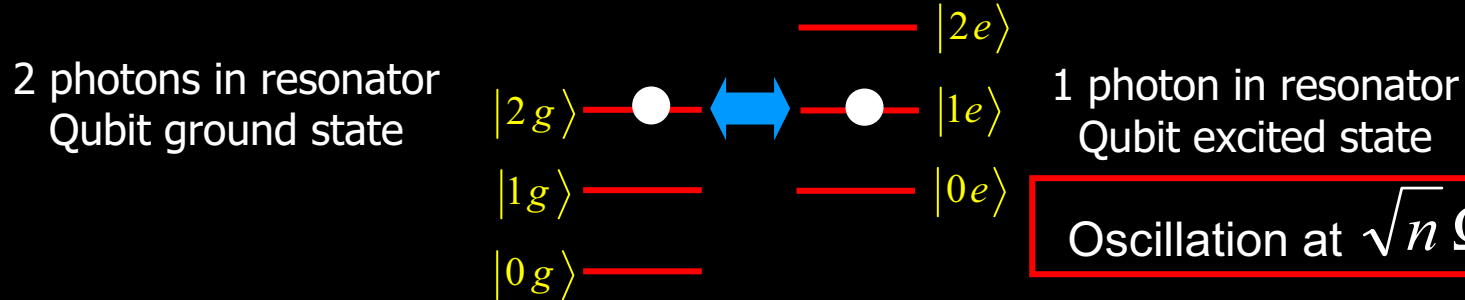


Pumping photons: $n = 1$ to 12 Fock states

Measurement of resonator state:

M. Hofheinz et al.
Nature (2008)

Bring ground-state qubit into resonance with resonator

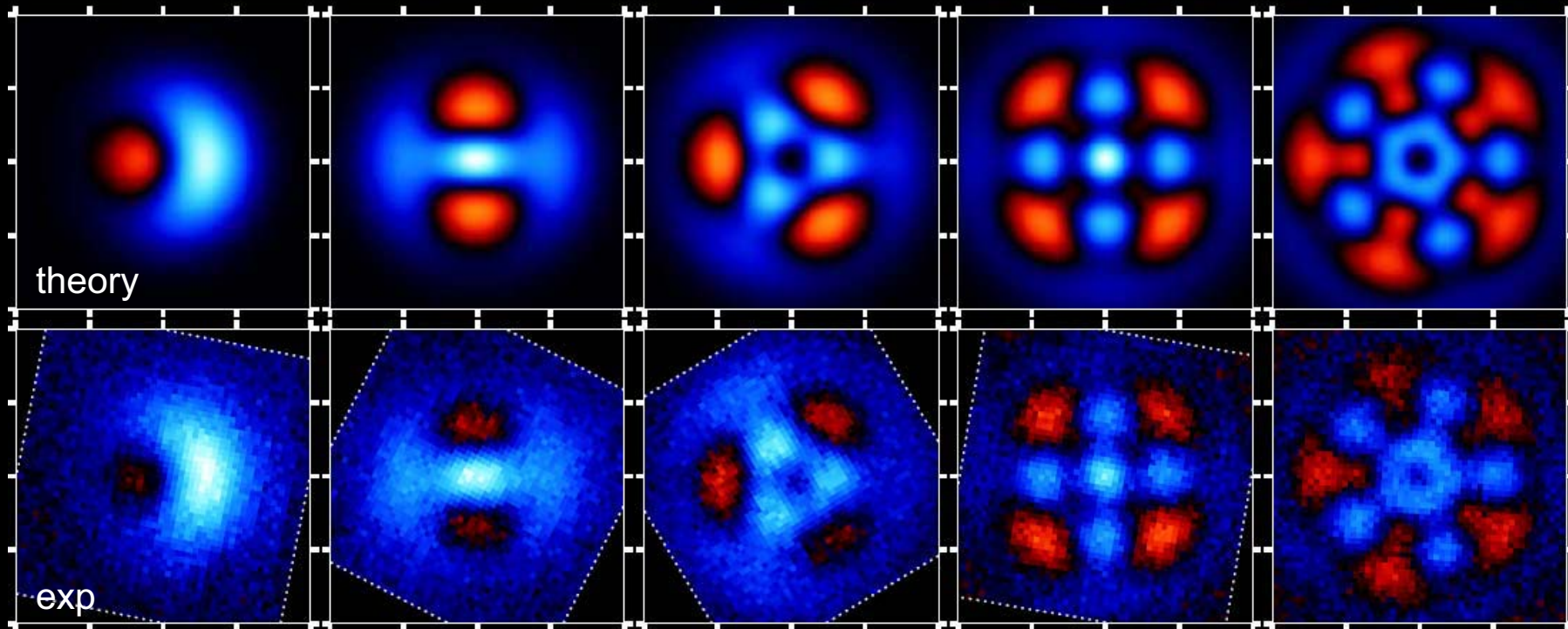


Superpositions by Wigner tomography

Prepare and measure $|0\rangle + |n\rangle$ states in resonator

M. Hofheinz et al.
Nature (2009)

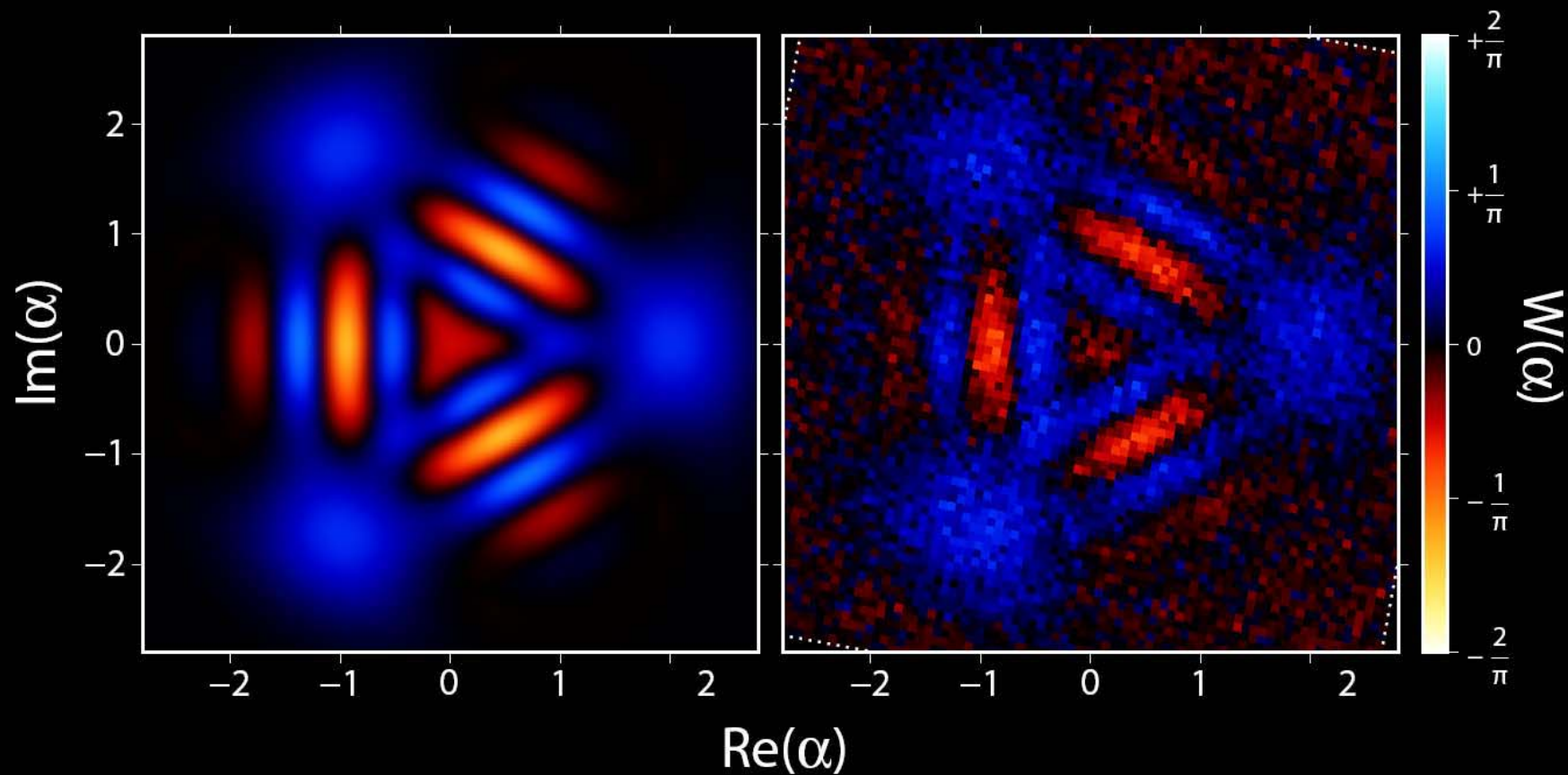
$$|\psi\rangle = |0\rangle + |1\rangle \quad |\psi\rangle = |0\rangle + |2\rangle \quad |\psi\rangle = |0\rangle + |3\rangle \quad |\psi\rangle = |0\rangle + |4\rangle \quad |\psi\rangle = |0\rangle + |5\rangle$$



"Voodoo" cat state: Alive + dead + zombie

alive dead zombie

$$|\psi\rangle = |\alpha = 2\rangle + |\alpha = 2e^{2\pi i/3}\rangle + |\alpha = 2e^{4\pi i/3}\rangle$$

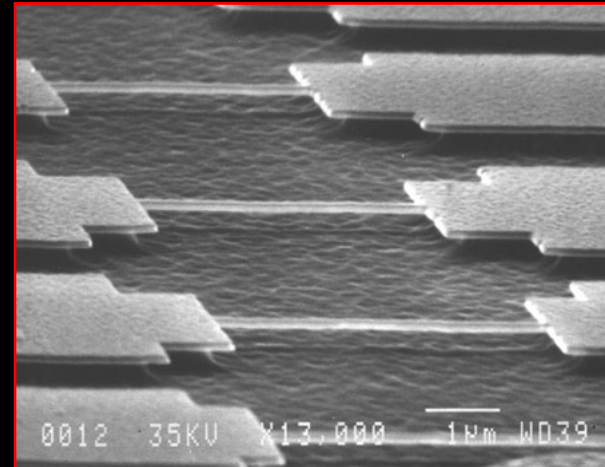


Candidates for quantum harmonic oscillators

- Nanomechanical resonators

- Resonance frequencies up to ~ 10 GHz
- Integrable with phase qubit
- Quanta are *phonons*
- **Quality factors $\sim 10^3$**

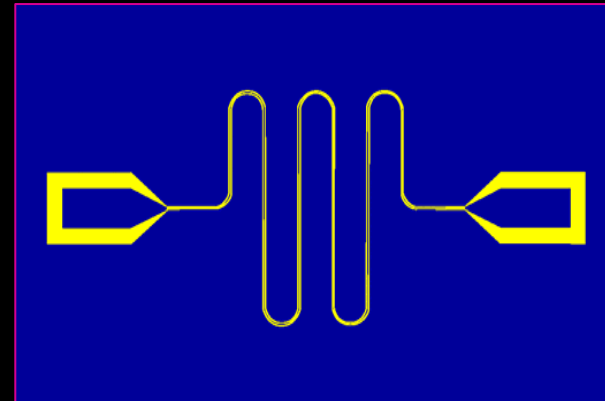
Aaron O'Connell



- Electromagnetic resonators

- Resonance frequencies up to ~ 100 GHz
- Integrable with phase qubit
- Quanta are *photons*
- **Quality factors $\sim 10^5$ - 10^6**

Max Hofheinz



Mechanical quantum oscillator

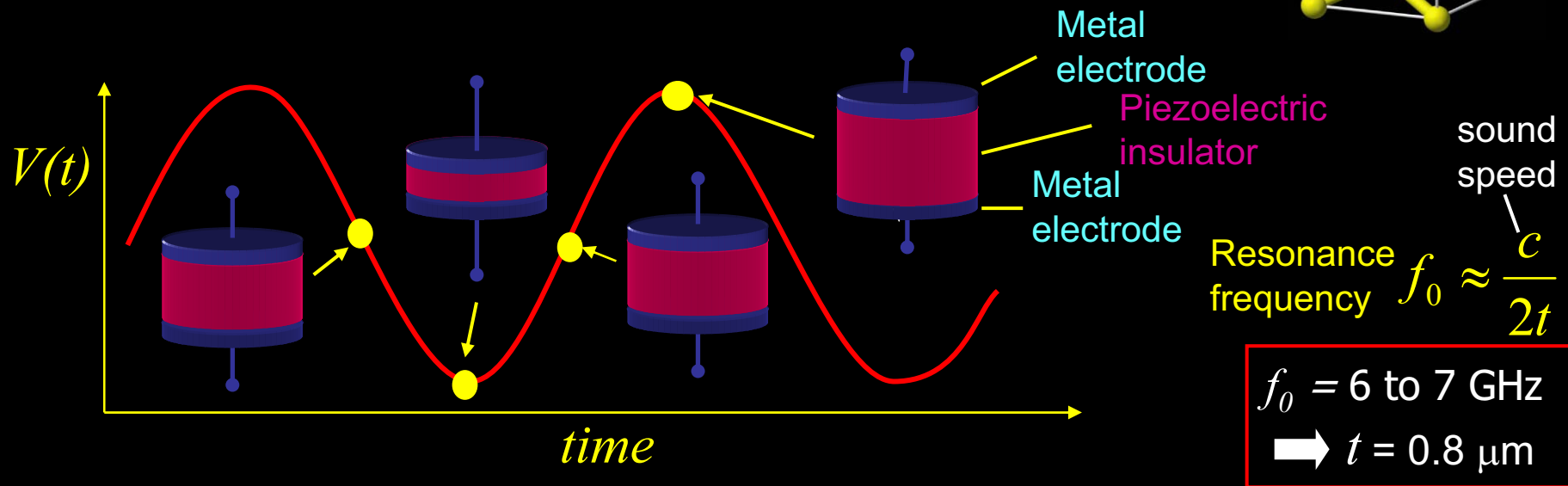
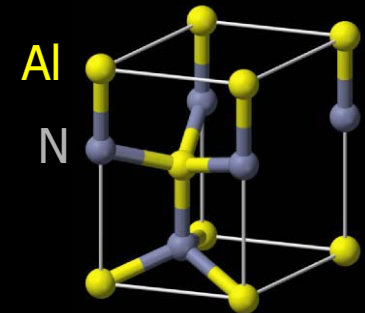
Requirements for mechanical oscillator:

- Resonance frequency $\gg 1$ GHz (want 6-7 GHz)
- Strong coupling of mechanics to electronics
- Testable in classical regime



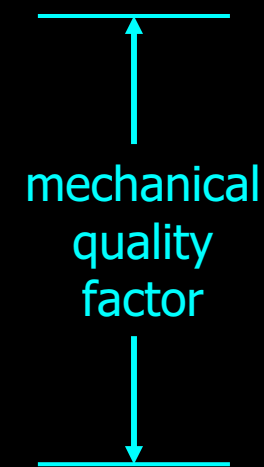
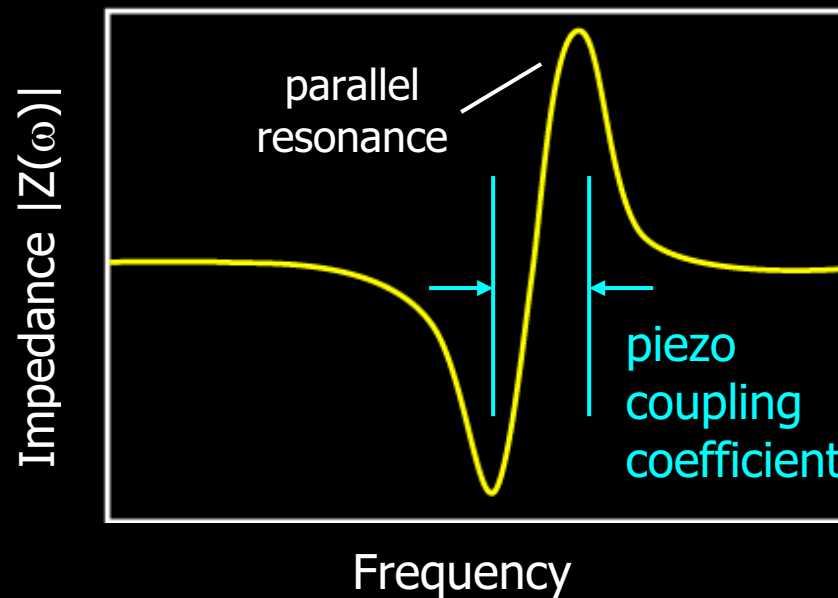
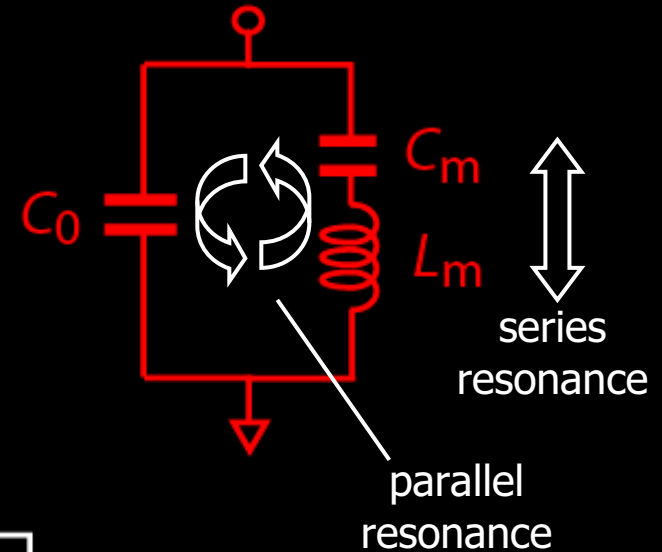
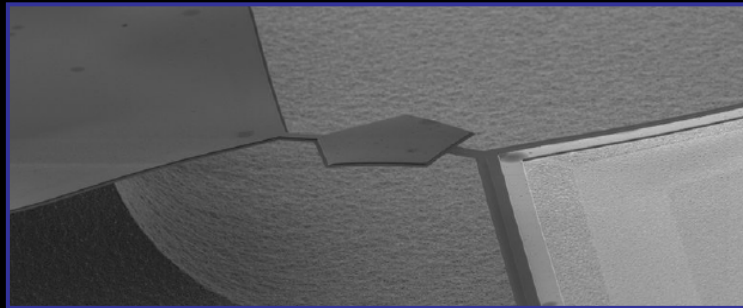
Our approach:

- Piezoelectric coupling: aluminum nitride
- Dilatational acoustic resonator (FBAR)



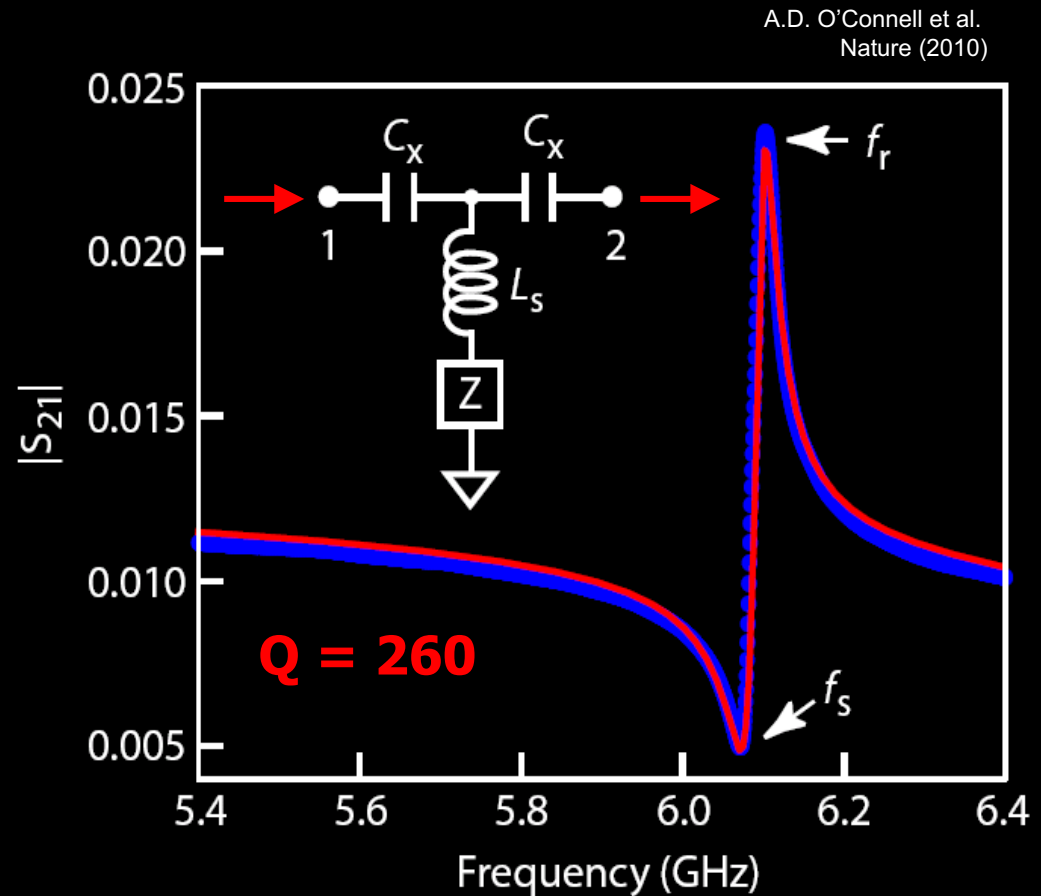
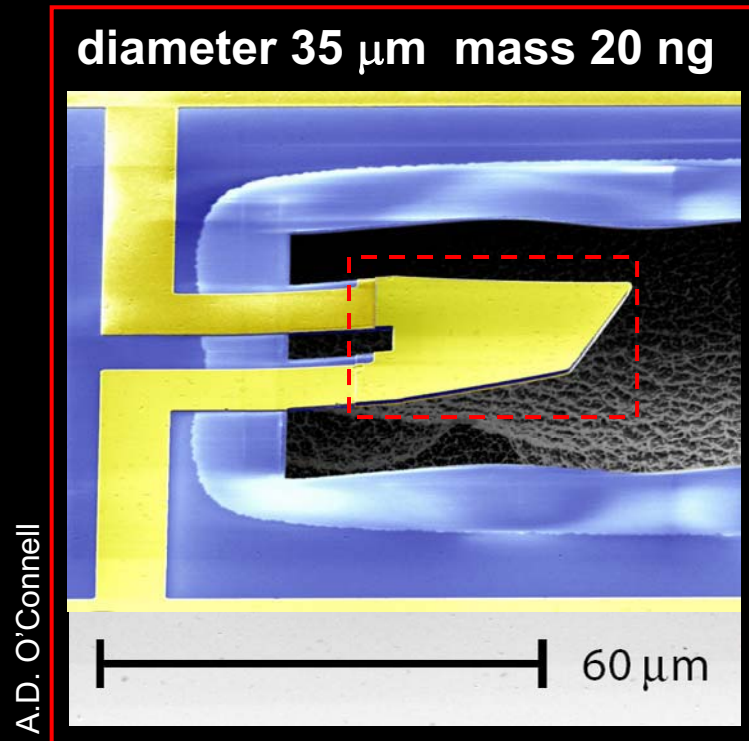
Resonator equivalent circuit

Electrical equivalent circuit:



Mechanical quantum oscillator

- Al top & bottom electrodes
- Sputtered AlN piezoelectric
- XeF_2 substrate release
- 4-7 GHz fundamental resonance
- Integrable with qubit fabrication



Six billionths of a second

Electromagnetic resonator: $Q \sim 10^5$

Energy lifetime $T_1 = \frac{Q}{2\pi f_r} = 2 \mu\text{s}$

Qubit-resonator photon transfer time ~ 25 ns (20 MHz coupling)

Lots of time for complex experiments (~ 40 gate operations)

Mechanical resonator: $Q \sim 260$

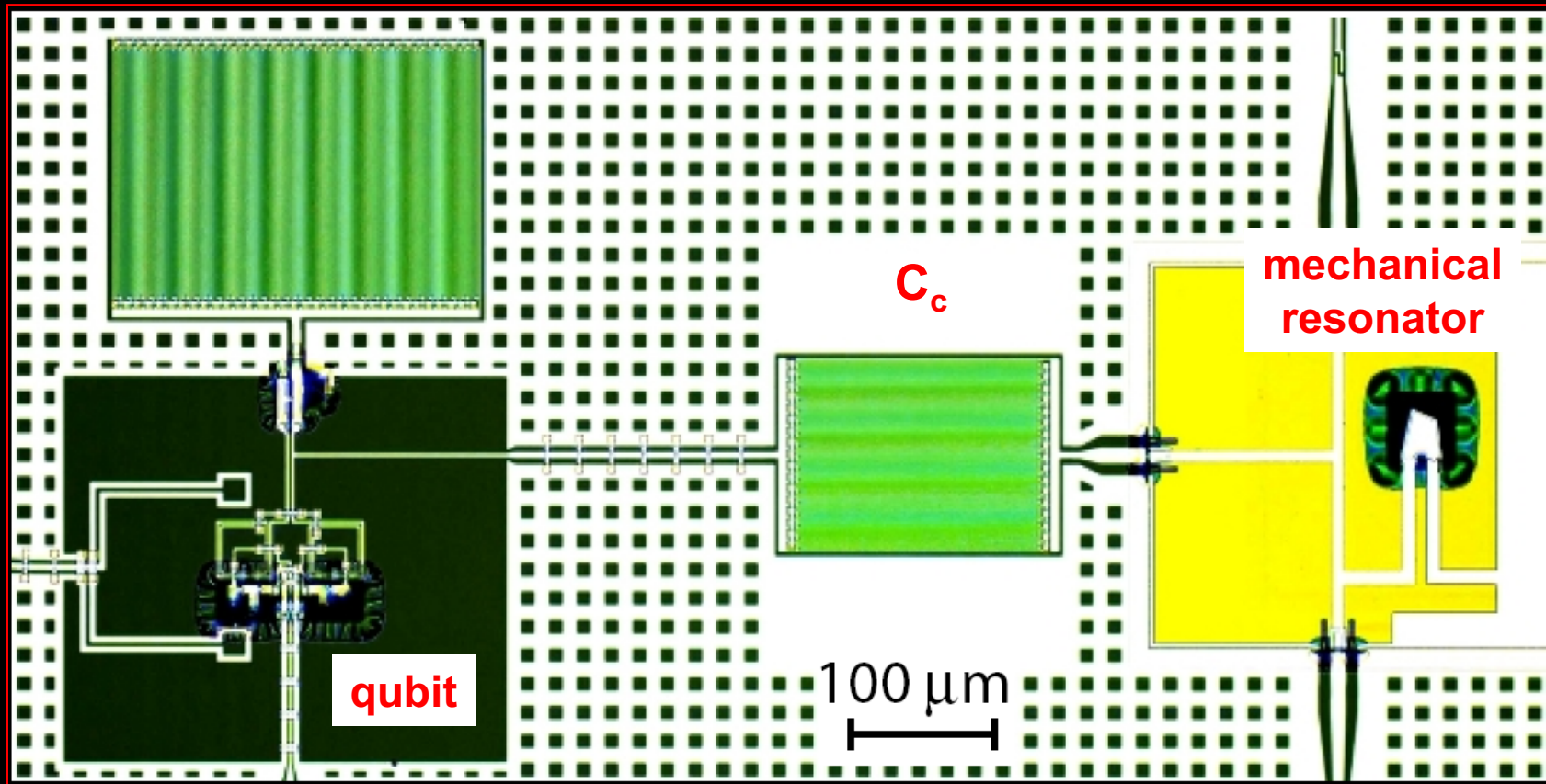
Energy lifetime $T_1 = \frac{Q}{2\pi f_r} = 6.7$ ns

- Fast energy transfer
- Fast measurement
- Strong qubit-resonator coupling

➤ Target gate time: ~ 5 ns to transfer excitation (~ 100 MHz coupling)

Integrated resonator & qubit

A.D. O'Connell et al. Nature (2010)

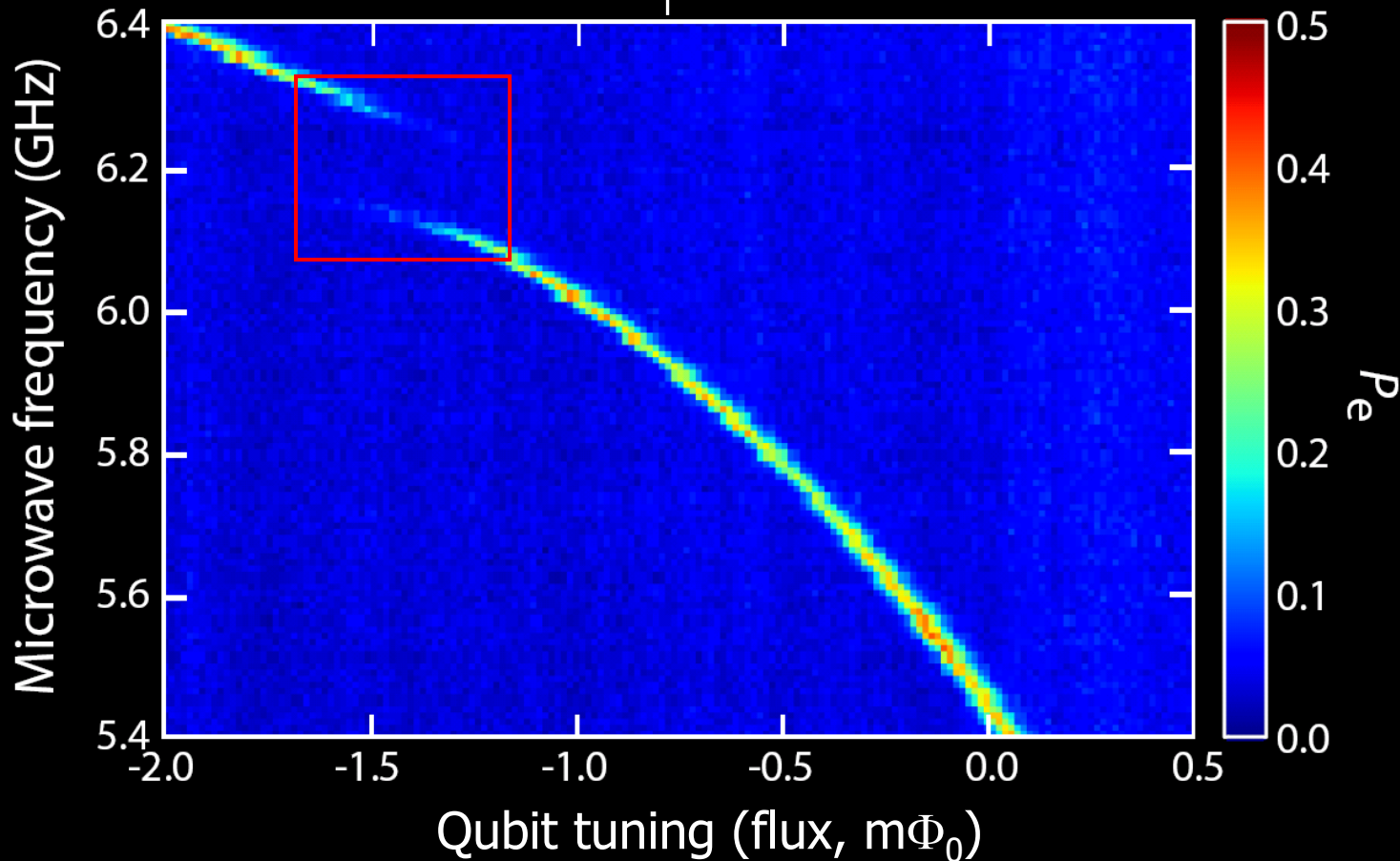
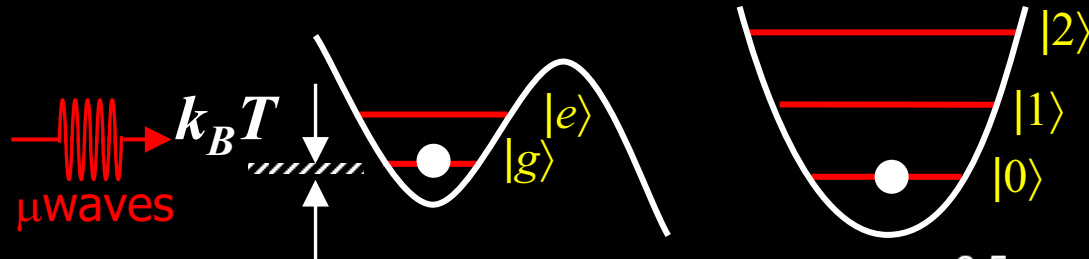


- Fabricate resonator, then qubit (13 layers)
- Suspend resonator at end
- Qubit & SQUID also mechanically suspended

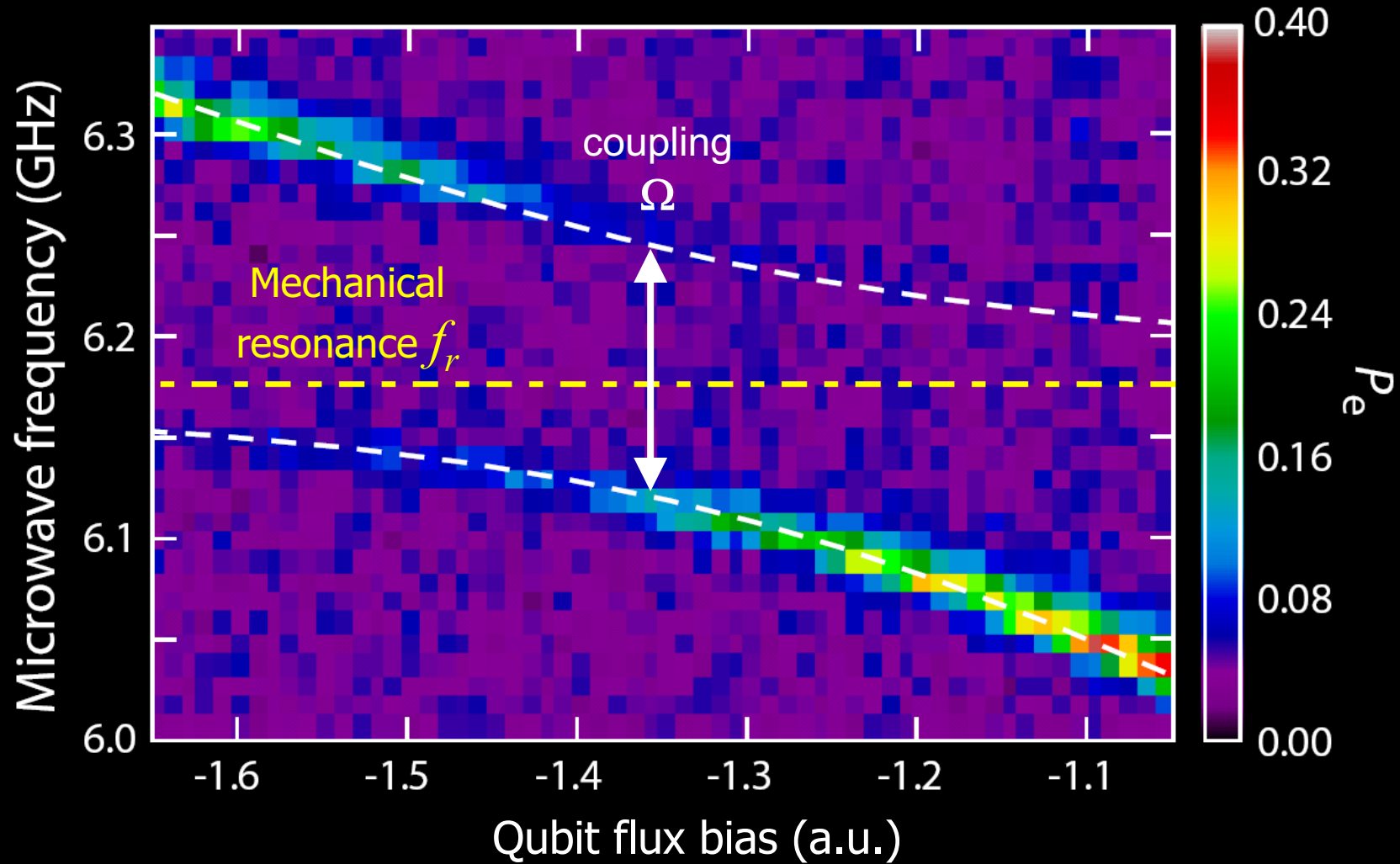
Design parameters:
Resonator 6.1 GHz
Coupling 110 MHz

Spectroscopy of coupled system

- Inject microwaves
- Measure qubit $P(e)$
- Sweep microwaves & qubit frequency



Spectroscopy of coupled system

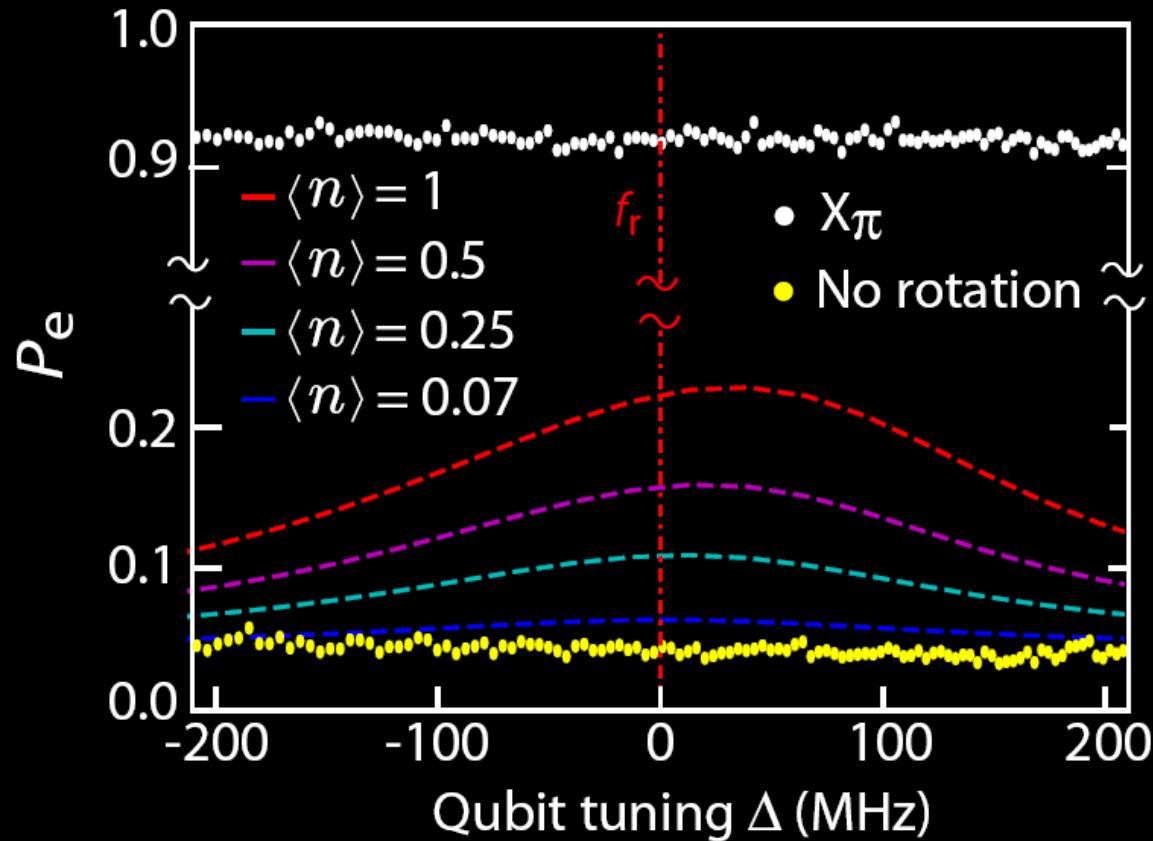
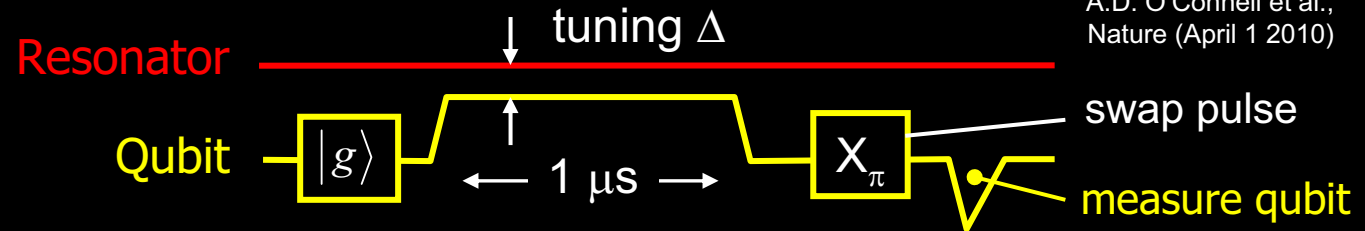


Mechanical resonance $f_r = 6.175$ GHz Coupling strength $\Omega = 124$ MHz

Quantum thermometry of mechanical resonator

A.D. O'Connell et al.,
Nature (April 1 2010)

Pulse
sequence:



With no X_π swap:

- Qubit always in $|g\rangle$ state

With X_π swap:

- Qubit always in $|e\rangle$ state

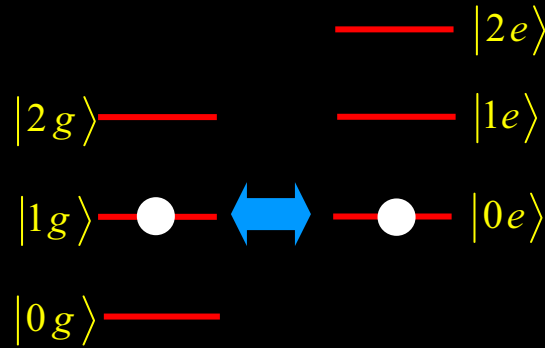
Thermal occupation:

- Less than 0.07 phonons
- Maximum ~ 0.01 phonons

Mechanical resonator in
quantum ground state

Electromechanical Rabi oscillations

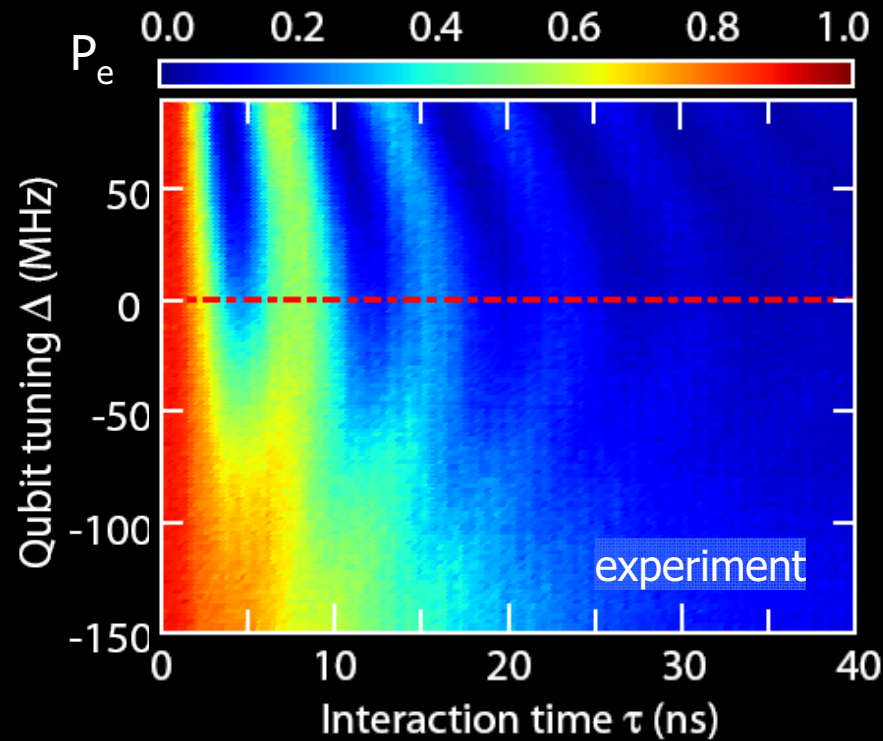
- Qubit in $|g\rangle$
- Tune to Δ of resonator
- Wait & measure:



A.D. O'Connell et al.,
Nature (April 1 2010)

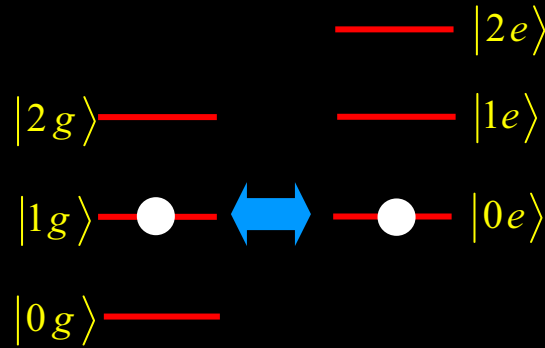
Oscillation
period:

$$T = \frac{1}{\sqrt{\Omega^2 + \Delta^2}}$$



Electromechanical Rabi oscillations

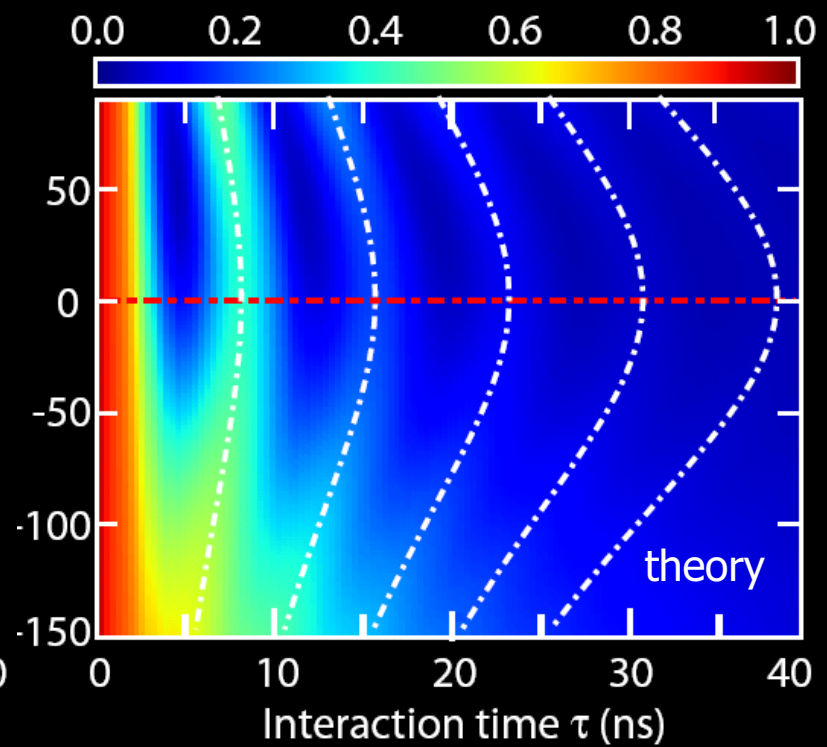
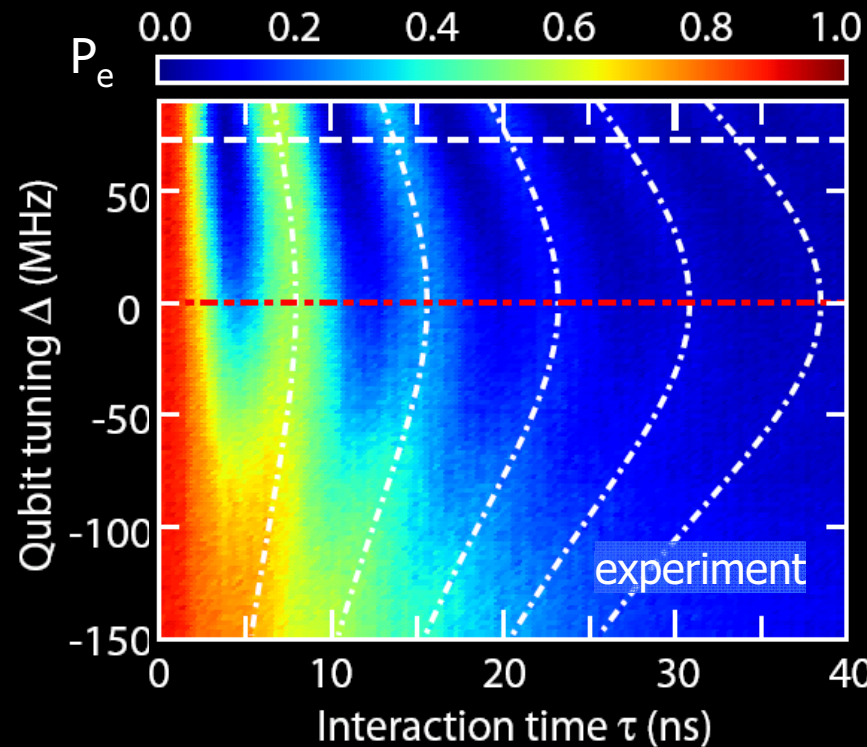
- Qubit in $|g\rangle$
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A.D. O'Connell et al.,
Nature (2010)

Oscillation
period:

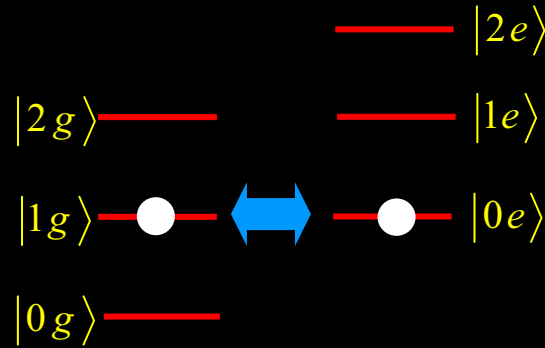
$$T = \frac{1}{\sqrt{\Omega^2 + \Delta^2}}$$



Electromechanical Rabi oscillations

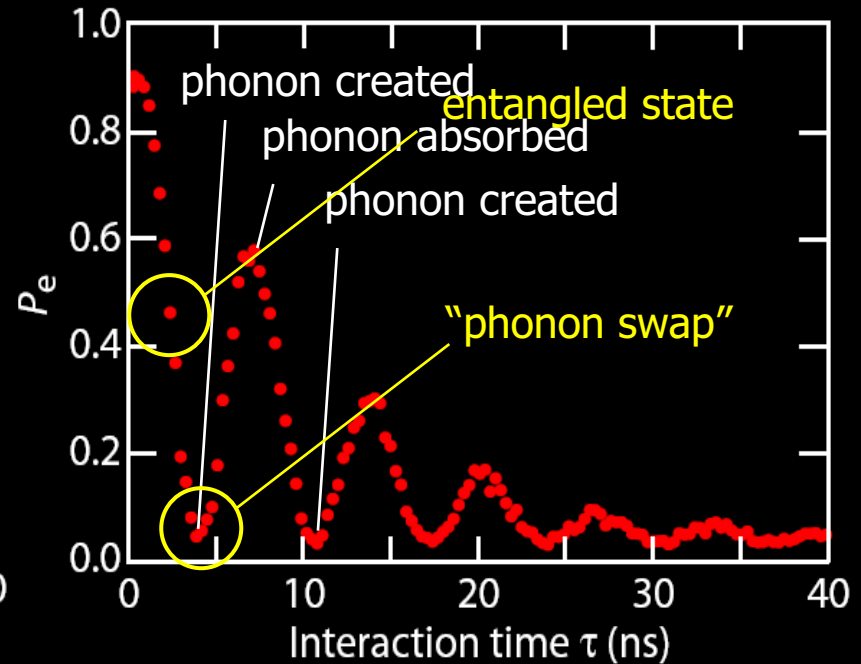
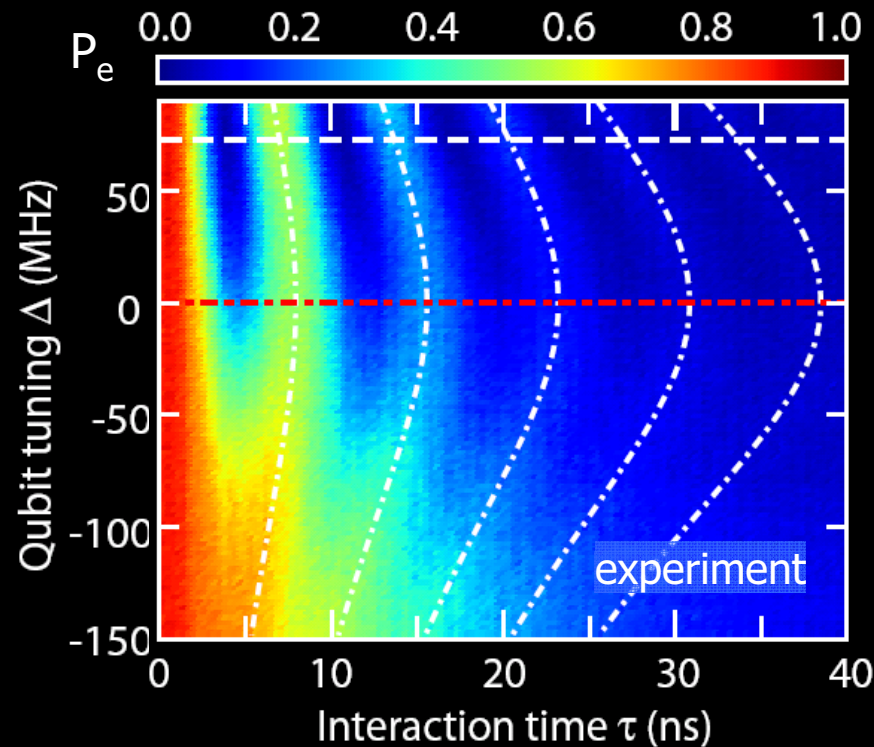
A.D. O'Connell et al.,
accepted, Nature (2010)

- Qubit in $|g\rangle$
- Tune to Δ of resonator
- Wait & measure:



Oscillation
period:

$$T = \frac{1}{\sqrt{\Omega^2 + \Delta^2}}$$



Single phonon lifetime & phase coherence time

- Create a single phonon
- Watch as it decays
- Extract single phonon T_1

$$T_1 = 6.1 \text{ ns}$$

(agrees with $Q = 260$)

$\Rightarrow T_1 = 6.7 \text{ ns}$

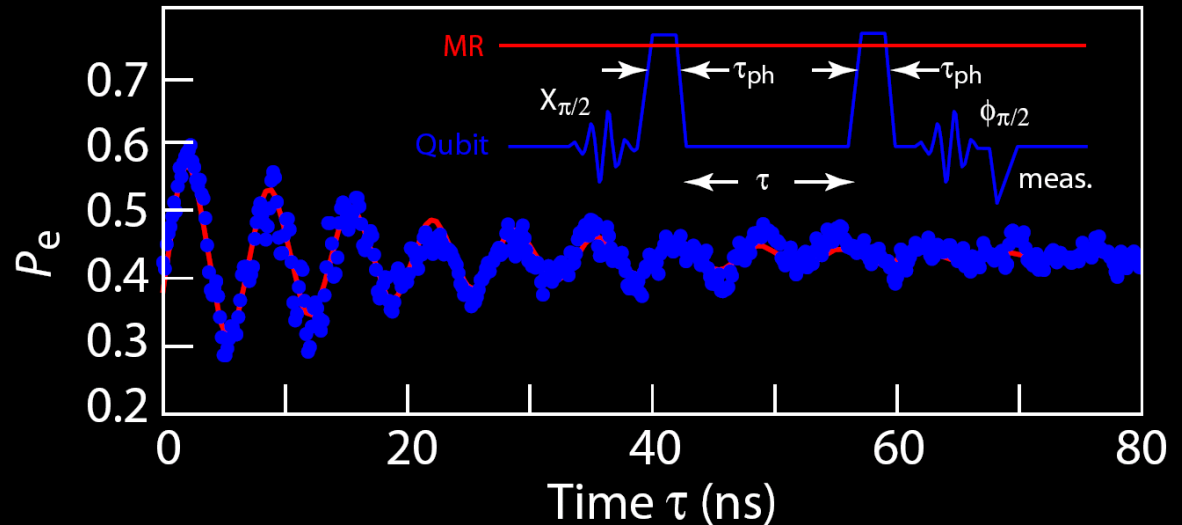
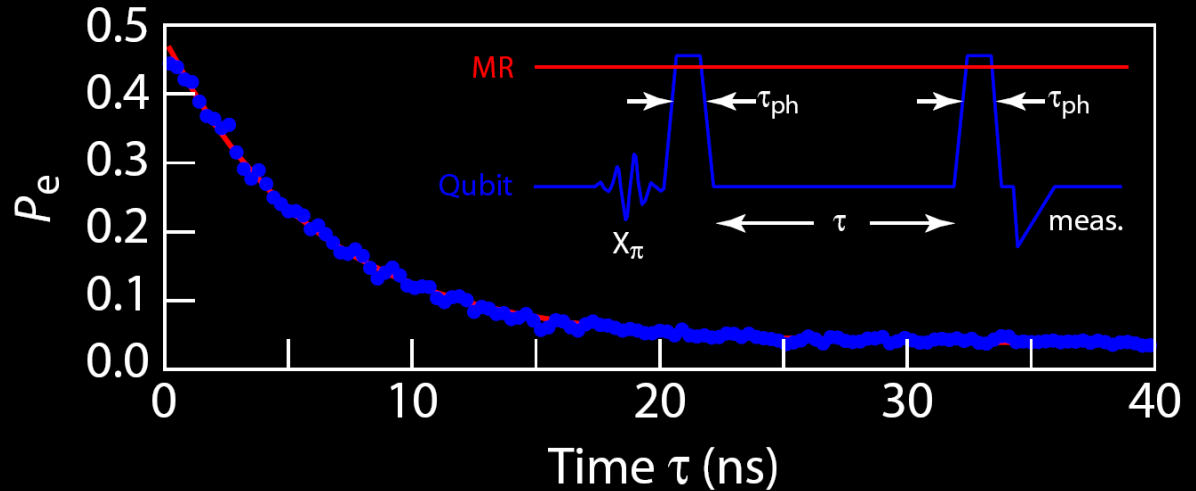
- Create superposition

$$|0\rangle + |1\rangle$$

- Watch as it decays
(Ramsey fringe)

- Extract single phonon T_2

$$T_2 \sim 2 T_1$$

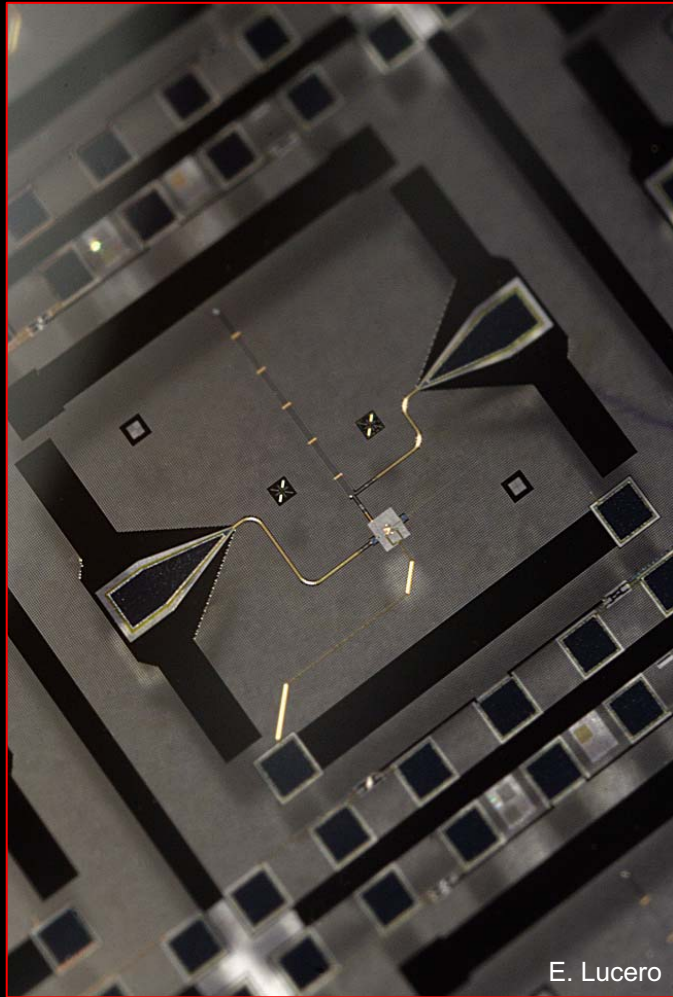


Quantum control of harmonic oscillators

Summary:

- Photon Fock $|n\rangle$ states
- Arbitrary superpositions of photon Fock states
- Cooling a mechanical resonator to its quantum ground state
- Creating and measuring a single mechanical phonon

Measuring the quantum harmonic oscillator



Cleland & Martinis groups:

Andrew N. Cleland
John M. Martinis

(Max Hofheinz)

Matteo Mariantoni

Haohua Wang

Martin Weides
(Eva Weig)

Yi Yin

Radek Bialczak
Michael Lenander
Erik Lucero
Matthew Neeley
Daniel Sank
James Wenner

Jean-Luc Fraikin
Chris McKenney
Aaron O'Connell
Michael Stanton
Amit Vainsencher

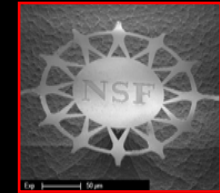
postdocs

graduate students

Support:

NSF

iARPA



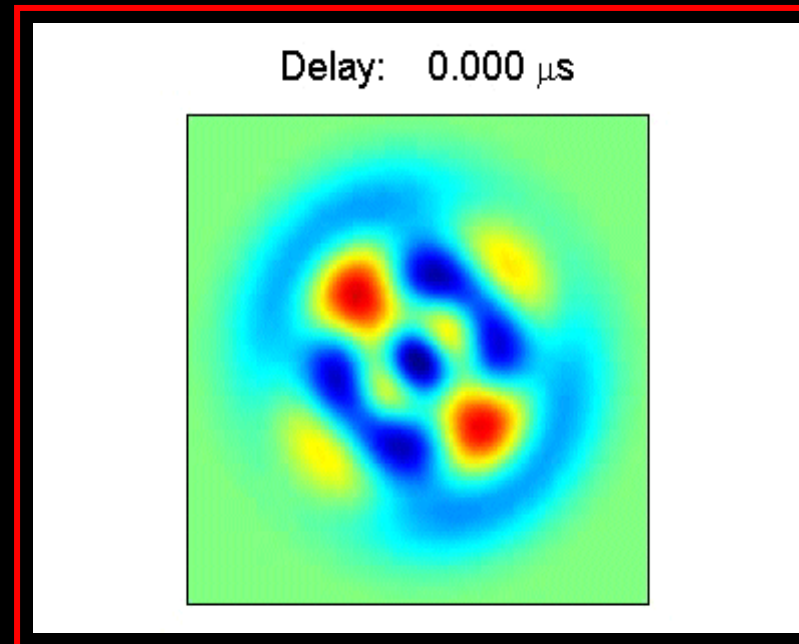
Decoherence as a movie

- Create superposed state using Law & Eberly protocol

$$|\psi\rangle = |0\rangle + i|2\rangle + |4\rangle$$

- Measure Wigner function using limited set of points
- Reconstruct full Wigner function using intrinsic symmetries

as a function
of time

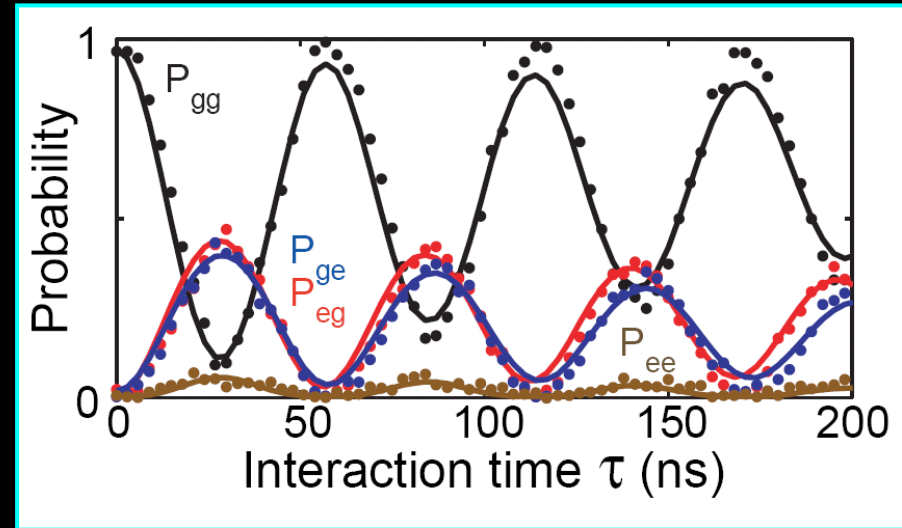


H. Wang et al.
PRL (2009)

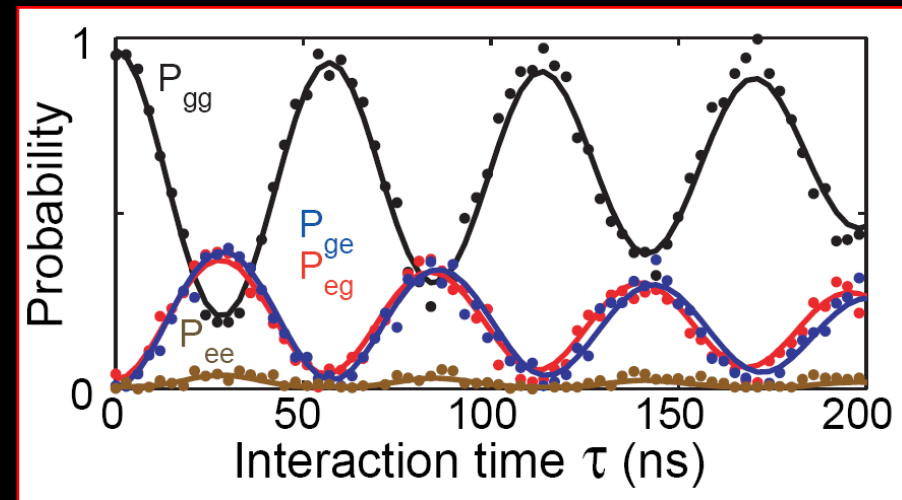
Quantitative
agreement with
Markovian master
equation

Entangling photons in two separate resonators

$N=1$ mixed state
Coincidence measurement
50% $|10\rangle$ and 50% $|01\rangle$



$N=1$ NOON state
 $|10\rangle + |01\rangle$



Entangling photons in two separate resonators

$N=1$ mixed state
Wigner tomography

50% $|10\rangle$ and 50% $|01\rangle$

$N=1$ NOON state

$|10\rangle + |01\rangle$

