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Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

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Feedback Control of Quantum Transport

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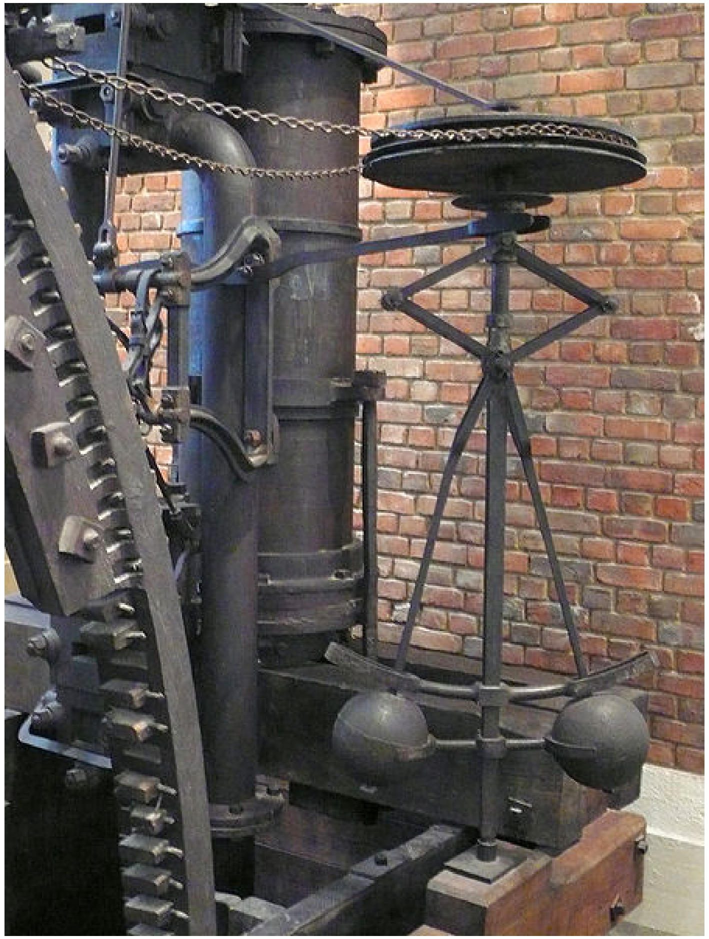
Feedback Control of Quantum Transport

Tobias Brandes (Institut für Theoretische Physik, TU Berlin)

- Examples, basic idea.
- Quantum transport.
- Open questions.

Feedback Control: Examples

J. C. Maxwell (1868)



Centrifugal Governor

- Stochastic input is **stabilized**.

Feedback Control: Examples

S. van der Meer (1972), Nobel Prize (1984) - discovery of W and Z bosons

Stochastic cooling of particle collider beam

- Transverse kicks correct trajectory.

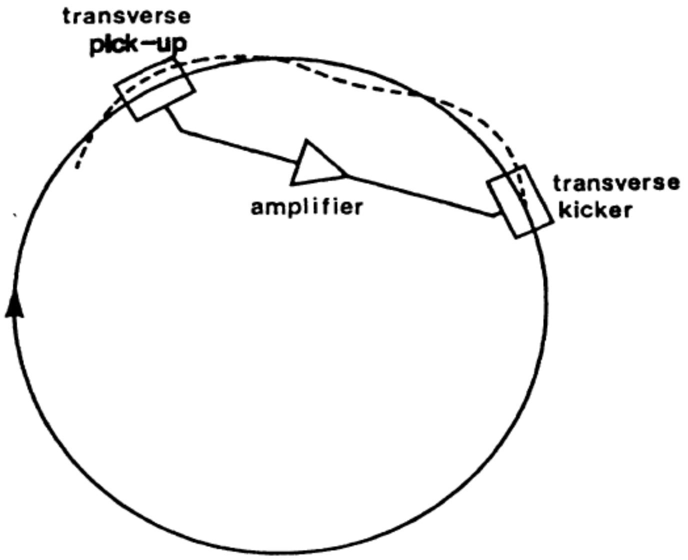


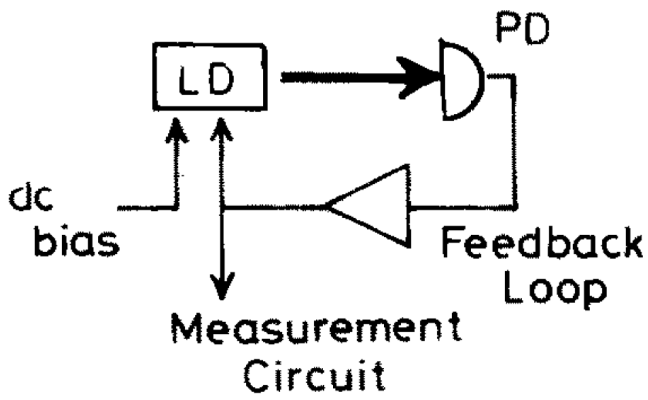
FIG. 2. Cooling of the horizontal betatron oscillation of a single particle.

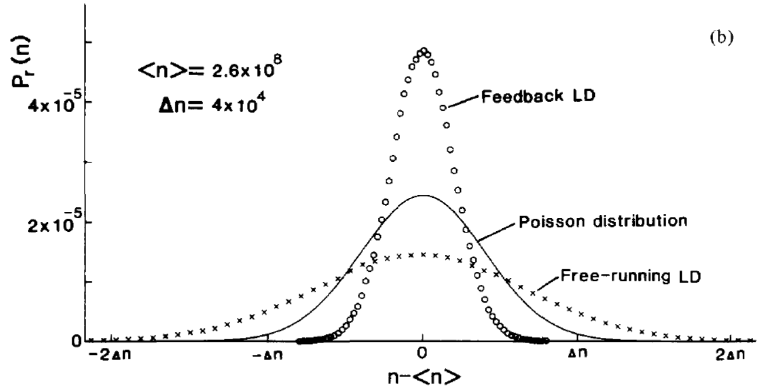
Feedback Control: Examples

S. Machida and Y. Yamamoto (1986)

Negative Feedback Semiconductor Laser

- Photodetector signal corrects laser diode pump current.
- Photon statistics changed into sub-Poissonian.



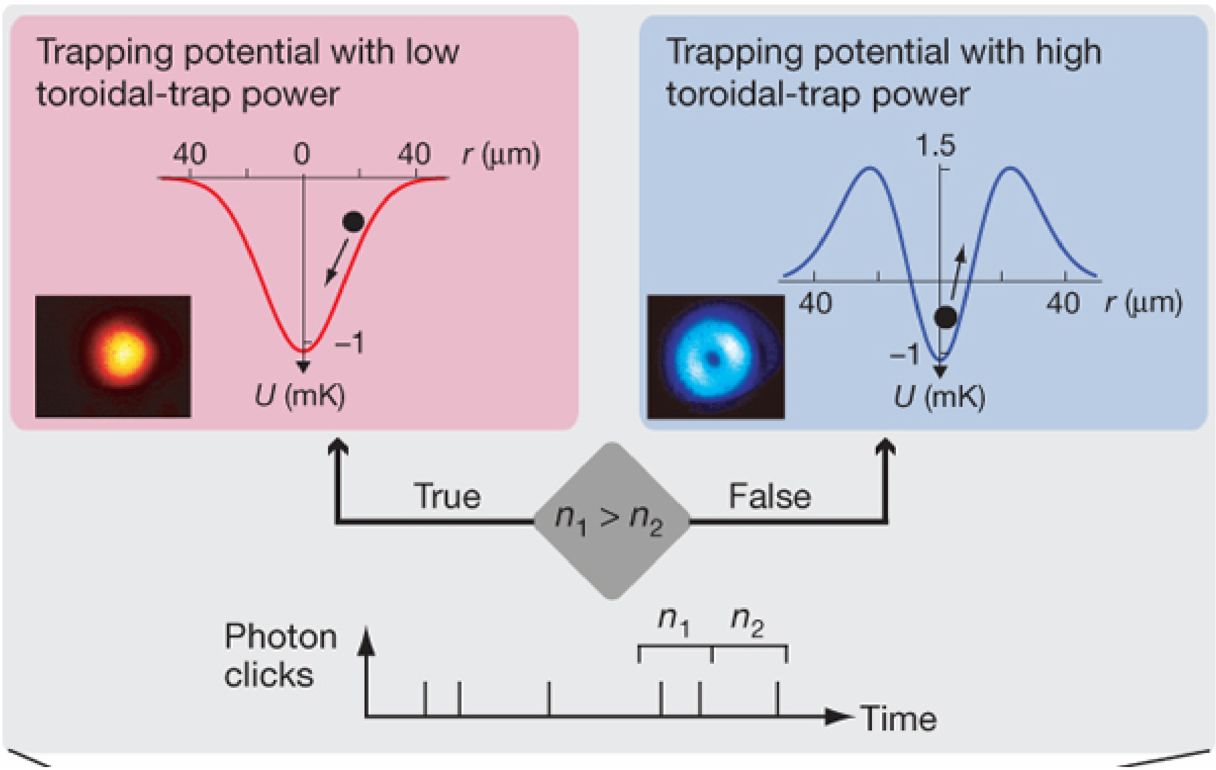


Feedback Control: Examples

A. Kubanek, M. Koch, C. Sames, A. Ourjoumtsev, P. W. H. Pinkse, K. Murr and G. Rempe (2009)

Feedback control of a single-atom trajectory

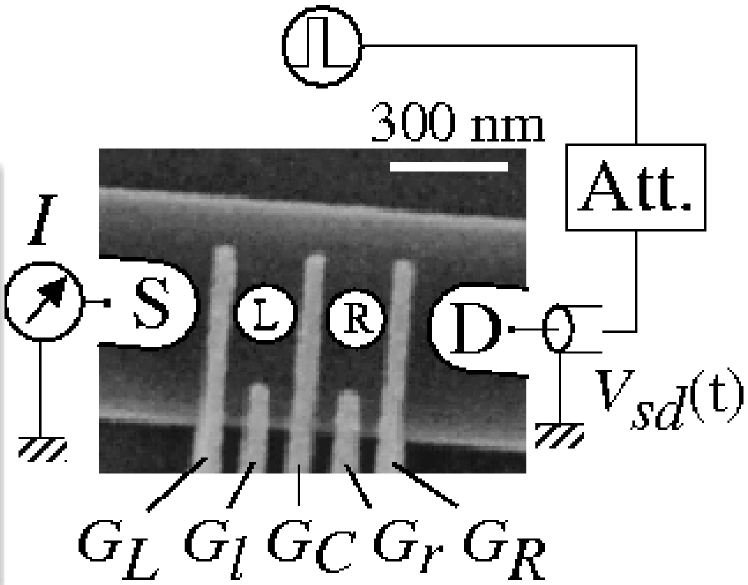
- Potential switch conditioned on photon count.



Quantum Feedback Control

Basic Goals

- Feedback control at nanoscales.
- Feedback control of quantum dynamics.
- Microscopically justify classical feedback schemes.



Double quantum dot.

Challenge for Quantum Systems

- Include feedback control into Schrödinger equations, Liouville-von-Neumann equations.

Quantum Feedback Control

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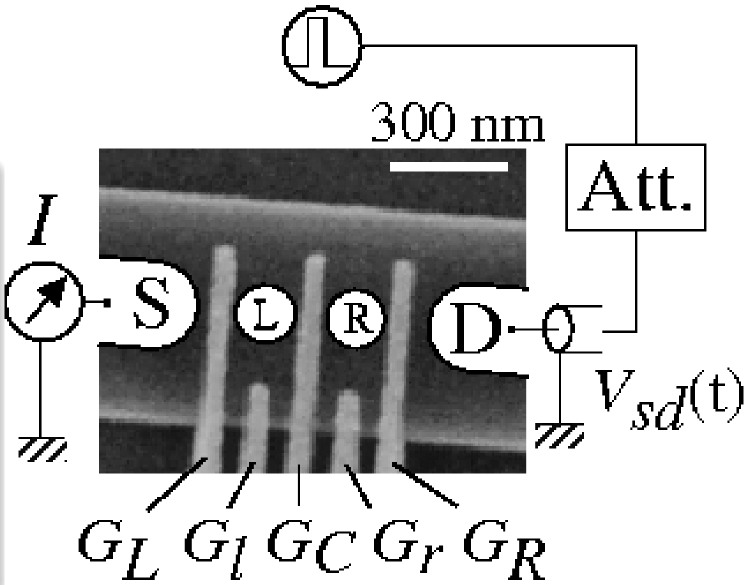

Double quantum dot.

Challenge for Quantum Systems

- Measurement process.
- Quantum noise.

Quantum Feedback Control

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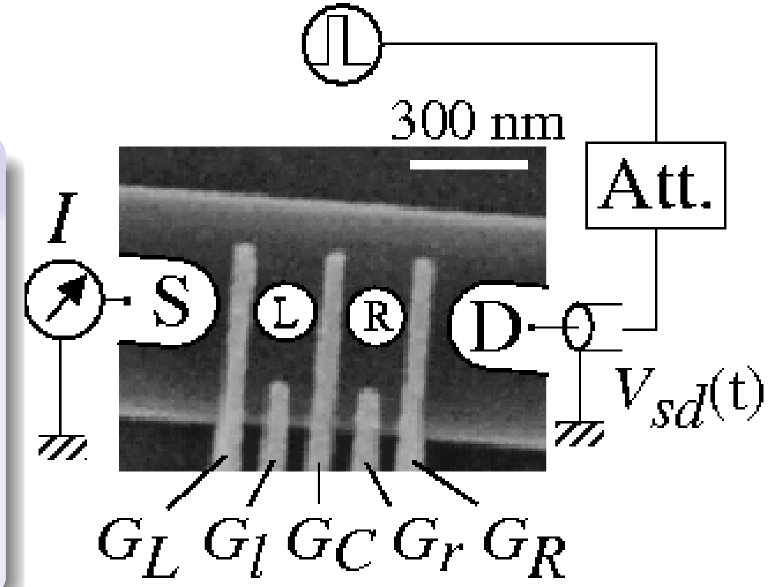

Double quantum dot.

Quantum feedback control.
Belavkin
Milburn
Wiseman
Doherty
Korotkov
Mabuchi

...

Quantum Feedback Control

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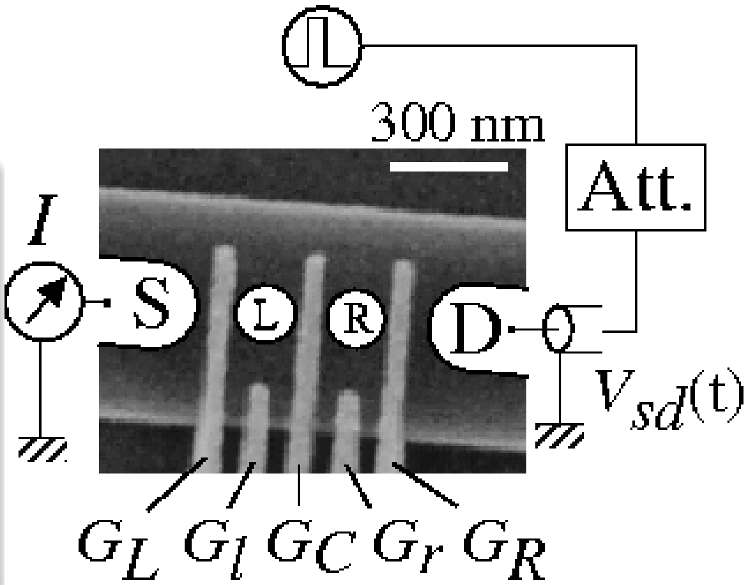
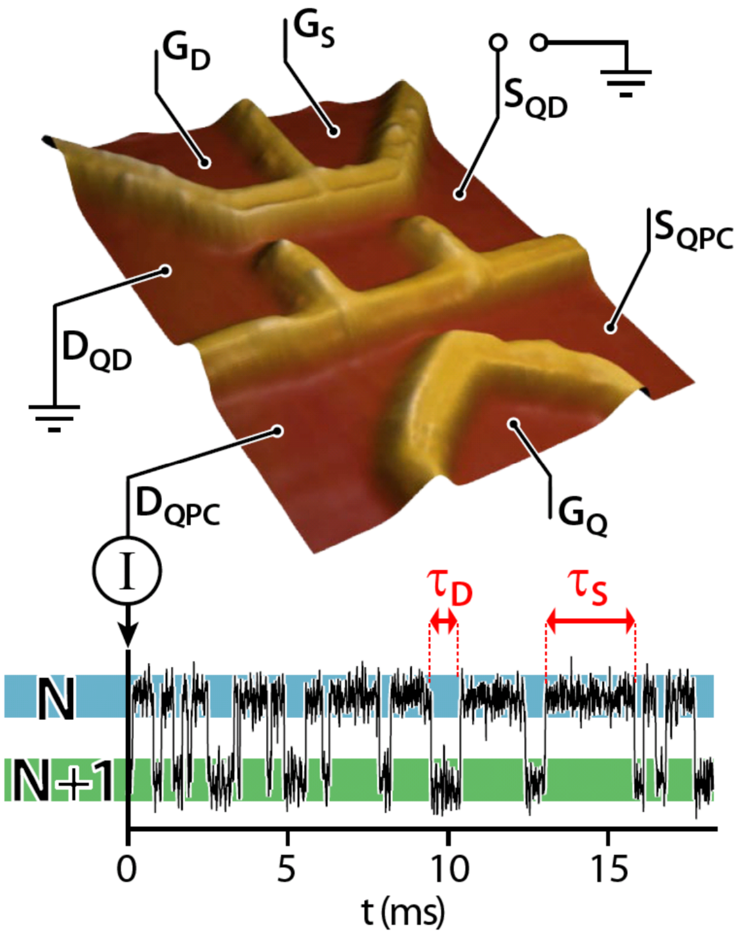


Double quantum dot.

This work: feedback control for quantum transport.

Quantum Transport

Electron Statistics in Quantum Dots

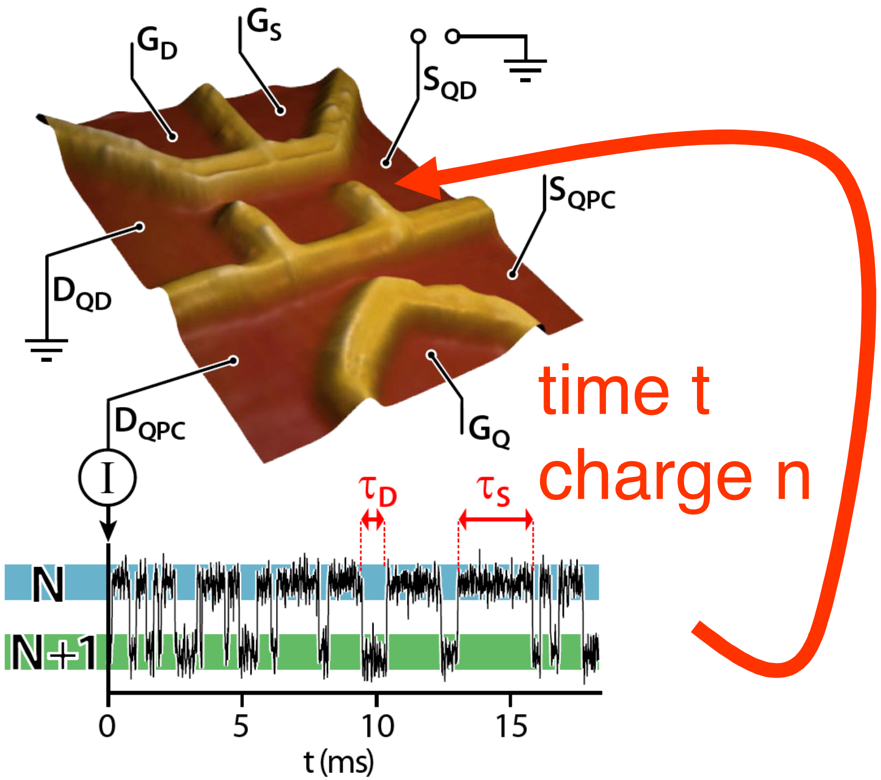


Full Counting Statistics

- Probability $p(n,t)$ of n electrons after time t.

C. Flindt, C. Fricke, F. Hohls, T. Novotný, K. Netocný, T. Brandes, and R. J. Haug; PNAS **106**, 10116 (2009).

Feedback Control of Electron Statistics



Aim: to suppress current fluctuations

- n electrons measured after time t, target current I_0.
- Charge error $\delta q_n(t) \equiv I_0 t - n$.
- Speed up ($\delta q_n(t) > 0$) or slow down ($\delta q_n(t) < 0$) tunneling.

Quantum Transport Model

No-Feedback Master Equation

- Open system Hamiltonian.

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{\rm res} + \mathcal{H}_T.$$

  - $\mathcal{H}_S$ system.
  - $\mathcal{H}_{\rm res}$ reservoir.
  - $\mathcal{H}_T$ system-reservoir coupling.

- Reduced density matrix $\rho(t)$, Liouvillian $\mathcal{L}$, Born-Markov approximation

$$\dot{\rho}(t) = \mathcal{L}\rho(t).$$

Quantum Transport Model

Markovian Master equation $\dot{\rho}(t) = \mathcal{L}\rho(t)$: unravelling, quantum jumps

$\mathcal{L} = \mathcal{L}_0 + \mathcal{J}$, $\mathcal{J}$: jump super-operator

$$\rho(t) = \sum_{n=0}^{\infty} \rho^n(t) \equiv \sum_{n=0}^{\infty} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho^c(t; t_n, ..., t_1)$$

$$\rho^c(t; t_n, ..., t_1) \equiv e^{\mathcal{L}_0 \cdot (t - t_n)} \mathcal{J} e^{\mathcal{L}_0 \cdot (t_n - t_{n-1})} \mathcal{J} ... \mathcal{J} e^{\mathcal{L}_0 \cdot t_1} \rho_0$$

- Non-unitary 'free' time-evolution, interrupted by n quantum jumps at times t_i (Carmichael; Zoller; Moelmer; Hegerfeldt;... 1980s).
- Full counting statistics (FCS)

$$p(n, t) \equiv \mathrm{Tr} \rho^n(t).$$

Feedback Master Equation

Conditional density matrix $\rho^n(t) \equiv \mathrm{Tr}_{\mathrm{res}} P_n \rho_{\mathrm{total}}(t) P_n$

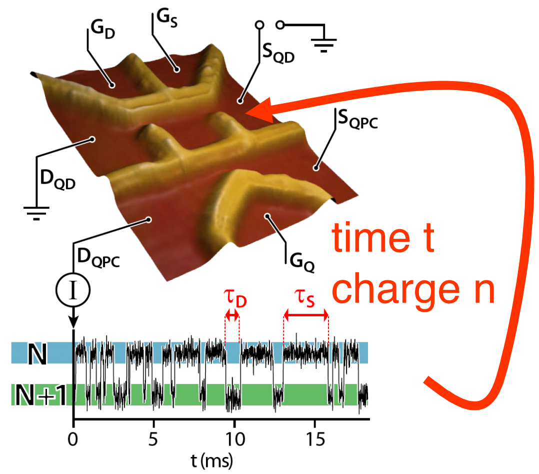
- Partial trace, keeps track of reservoir charge n.

$$\dot{\rho}^n(t) = \mathcal{L}_0(t,n)\rho^n(t) + \mathcal{J}(t,n)\rho^{n-1}(t).$$

- Example: junction with bare tunnel rate Γ,

$$-\mathcal{L}_0(t,n) = \mathcal{J}(t,n) = \Gamma\left(1 + g\delta q_n(t)\right).$$

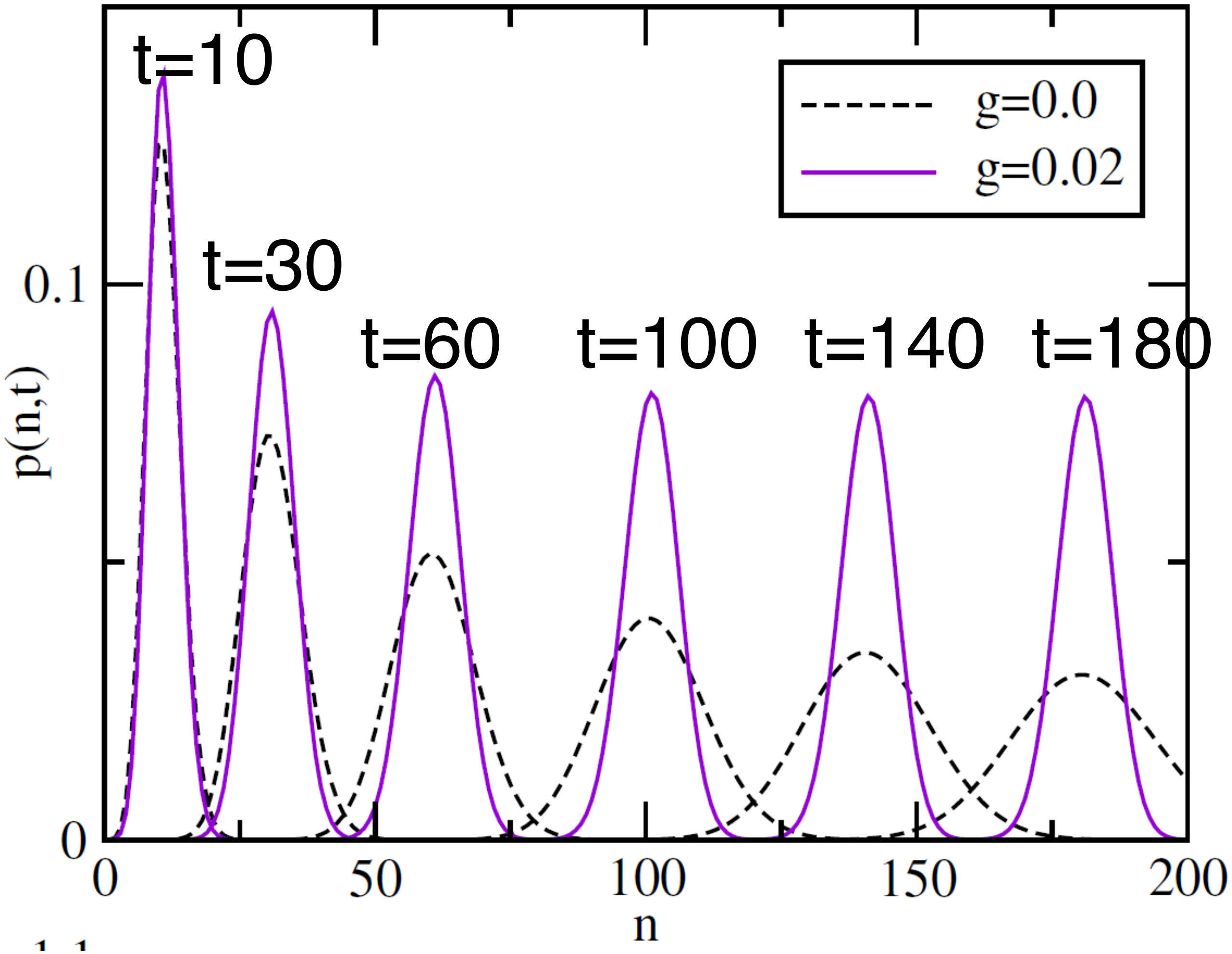
- $g \geq 0$: feedback strength.
- $\delta q_n(t) \equiv I_0 t - n$ error charge.
- I_0: target current, t: time.



Tunnel Junction Model

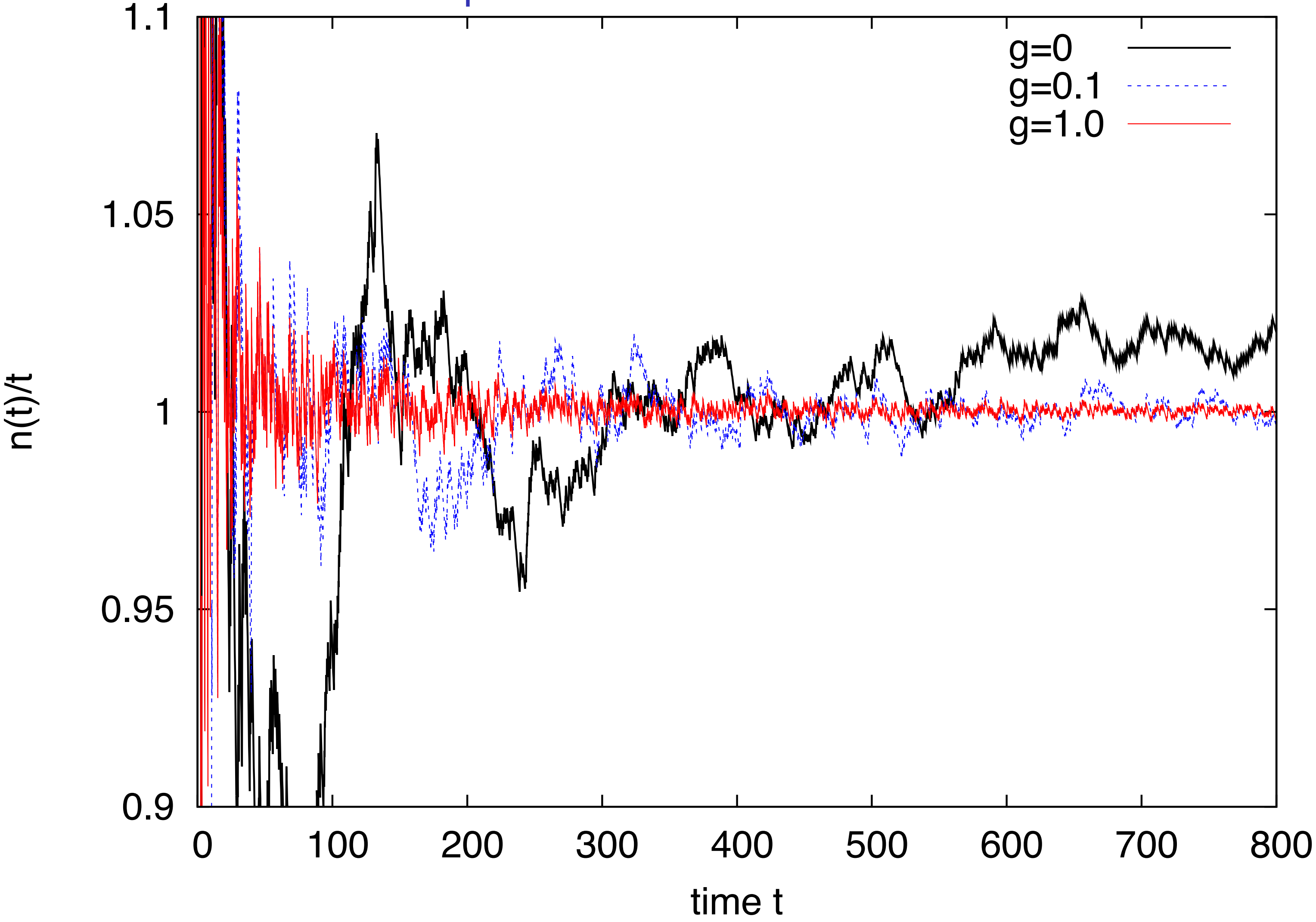
Full counting statistics $p(n,t)$: numerical results

- Feedback freezes in the counting statistics!

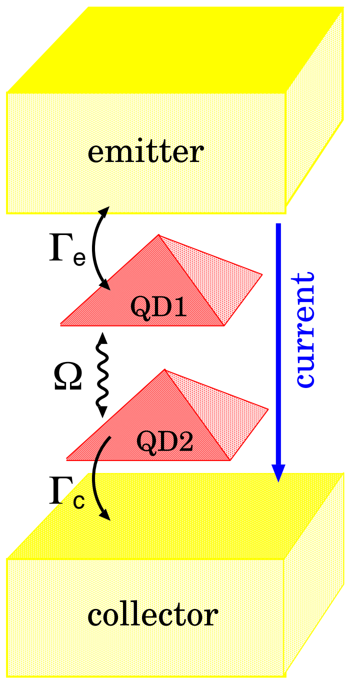


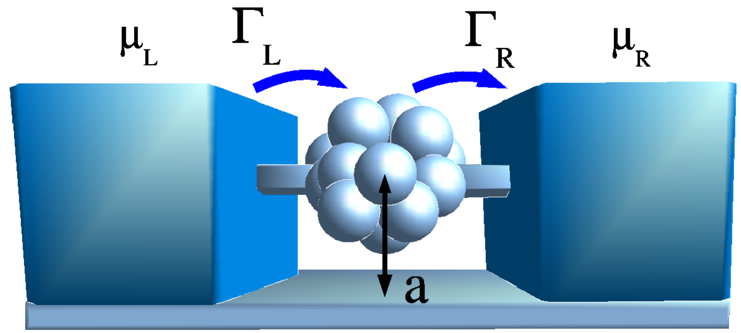
Tunnel Junction Model

Simulation of *n*-resolved Master equation



Feedback in Coupled Quantum System





- Charge qubits, electron-phonon dissipation.

G. Kießlich, E. Schöll, T. Brandes, F. Hohls, and R.J. Haug; Phys. Rev. Lett. **99**, 206602 (2007).

- Molecular transport.

H. Hübener and T. Brandes, Phys. Rev. Lett. **99**, 247206 (2007).

Mathematical Challenges

Numerical stability

- Large ODE systems.

$$\dot{\rho}^n(t) = \mathcal{L}_0(t,n)\rho^n(t) + \mathcal{J}_-(t,n)\rho^{n-1}(t) + \mathcal{J}_+(t,n)\rho^{n+1}(t).$$

- Conditional density matrix $\rho^n(t)$.
- Partial trace, keeps track of reservoir charge n.

Mathematical Challenges

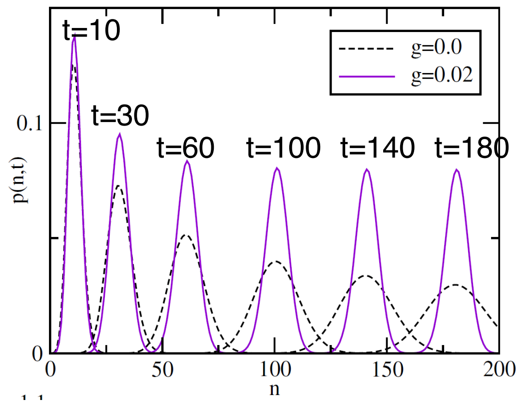
Analytical results

- Cumulants of feedback-frozen $p(n, t \to \infty)$.
- PDE systems.

Example: tunnel junction,

- Generating Function $\rho(\chi, t) \equiv \sum_n e^{in\chi} p(n, t)$.

$$\dot{\rho}(\chi, t) = (e^{i\chi} - 1)\left(1 + g\left(I_0 t - \frac{\partial}{\partial i\chi}\right)\right)\rho(\chi, t).$$



- Cumulants $C_1(t) = \Gamma t$, $C_{n \geq 2}(t \to \infty) = -\frac{1}{g} \times$ Bernoulli-Seki number.

T. Brandes, Phys. Rev. Lett. **105**, 060602 (2010).

Relation between feedback and no-feedback distribution

	no Feedback	**homogeneous Feedback**
$p(n,t)$	diffusive decay	frozen
$\frac{\partial}{\partial t}\rho(\chi,t) =$	$\mathcal{L}(\chi)\rho(\chi,t)$	$\mathcal{L}(\chi) f\left(I_0 t - \frac{\partial}{\partial i\chi}\right)\rho(\chi,t)$
type	ODE	PDE
CGF	$\lambda_0(\chi)\times t$	$h(\chi) \equiv \ln \mathrm{Tr}\, \rho(\chi,t) - i\chi I_0 t$
cumulants	$\langle\langle I^n \rangle\rangle \times t \equiv (-i)^n \lambda_0^{(n)}(\chi)\times t$	$(-i)^n h^{(n)}(\chi)$

CGF $h(\chi)$ for homogeneous feedback $(t\to\infty)$

$$\frac{i\chi}{\lambda_0(\chi)} I_0 = e^{-h(\chi)} f\left(-\frac{\partial}{\partial i\chi}\right) e^{h(\chi)}.$$

Relation between feedback and no-feedback distribution

Explicit Formulas: First three cumulants

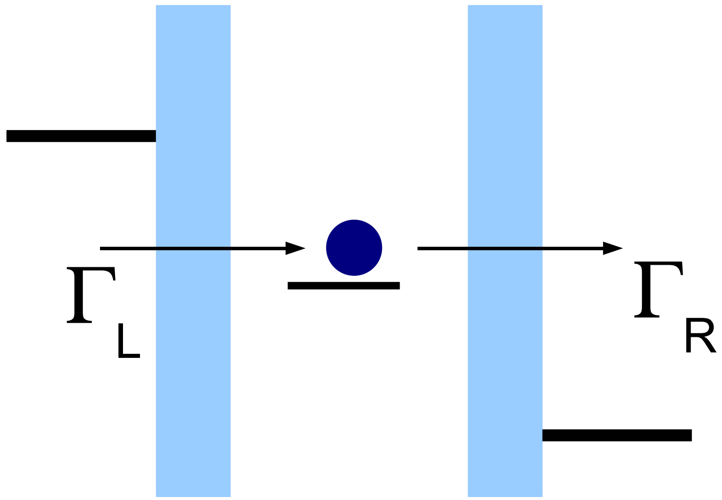
no FB	linear Feedback	exponential Feedback
$\langle\langle I^1\rangle\rangle =$	I_0	I_0
$\langle\langle I^2\rangle\rangle =$	$I_0 \times 2gC_2$	$I_0 \times 2gC_2 + O(g^2)$
$\langle\langle I^3\rangle\rangle =$	$I_0 \times \left(6g^2 C_2^2 + 3gC_3\right)$	$I_0 \times \left(3g^2 C_2^2 + 3gC_3 - \frac{3}{2}g^2 C_4\right) + O(g^3)$

Fano Factor $F(g=0)$ from second frozen cumulant $C_2(g>0)$

$$F = 2g\,C_2 + O(g^2), \quad F \equiv \frac{\langle\langle I^2\rangle\rangle}{\langle\langle I^1\rangle\rangle}$$

Relation between feedback and no-feedback distribution

Example: Single level dot



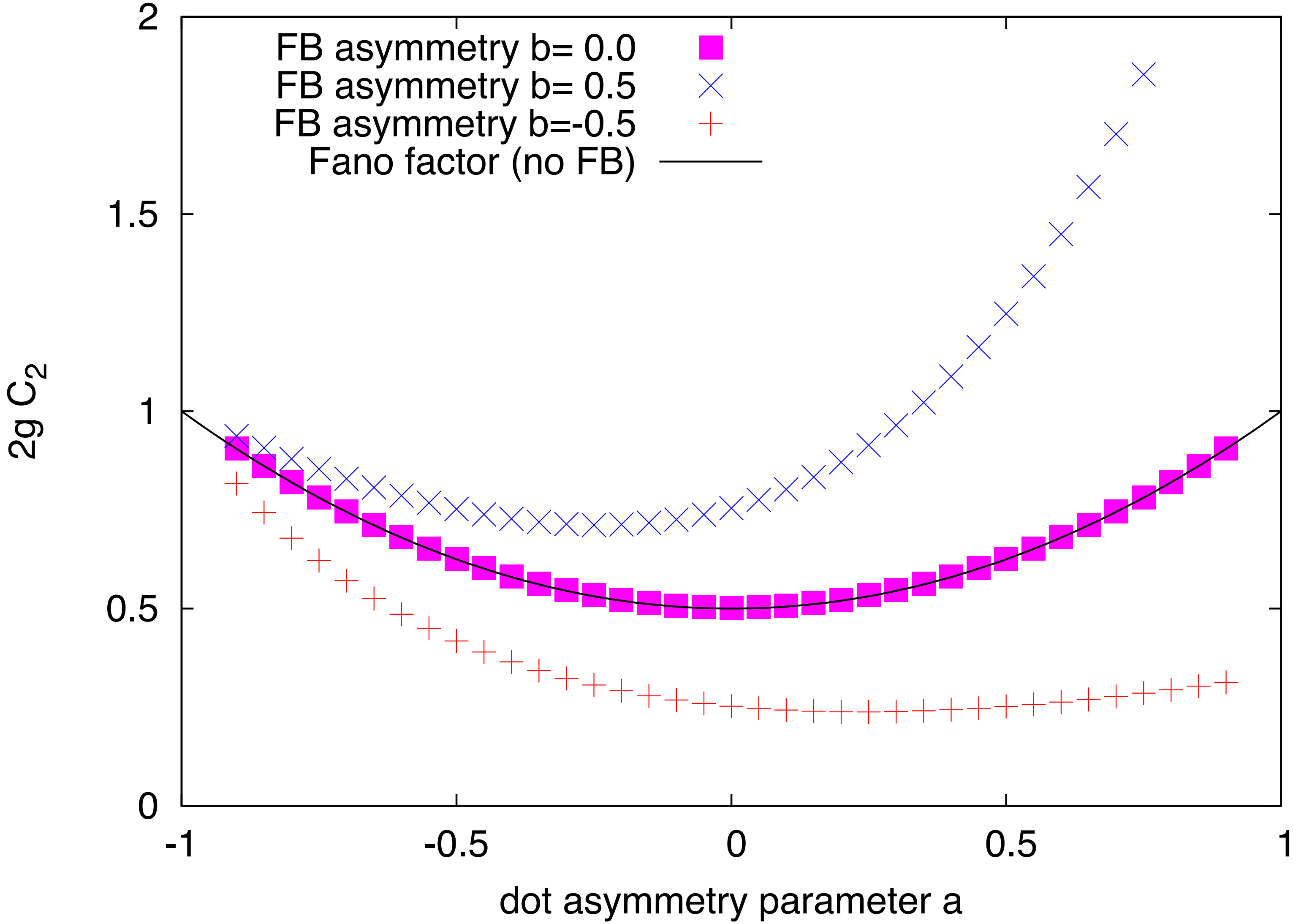
Dot asymmetry

$$\Gamma_R = \Gamma_L \frac{1-a}{1+a}, \quad -1 \leq a \leq 1.$$

Feedback asymmetry

$$g_R = g_L \frac{1-b}{1+b}, \quad -1 \leq b \leq 1.$$

Relation between feedback and no-feedback distribution

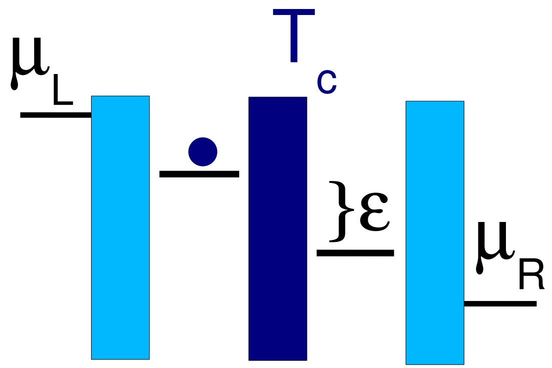


- Homogeneous FB $b = 0$ recovers Fano factor $F = 2gC_2 = \frac{1}{2}(1 + a^2)$.

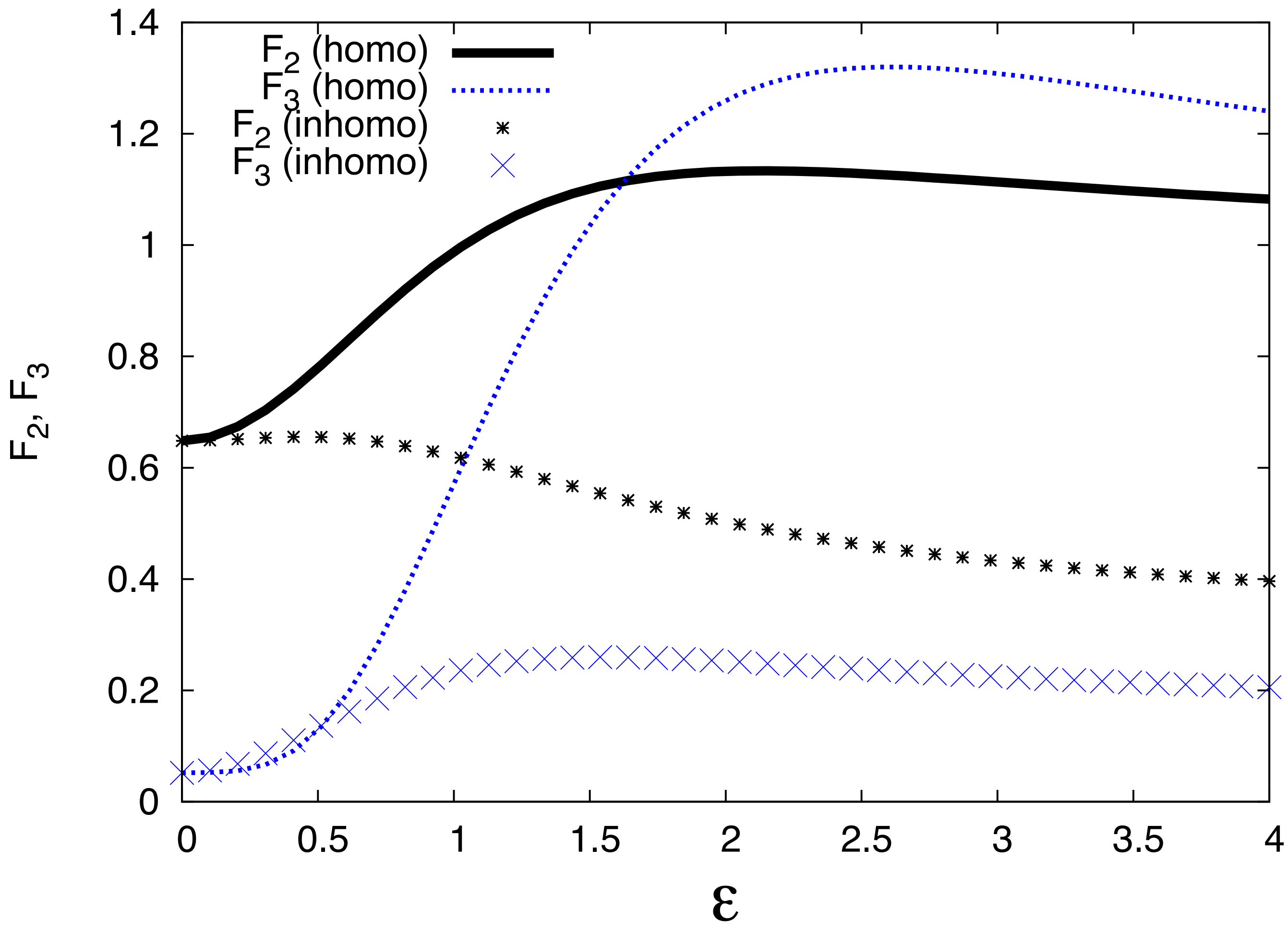
$(g = 0.02$ here)

Relation between feedback and no-feedback distribution

Example: double quantum dot



$\Gamma_L = 10\Gamma_R, \; T_c = \Gamma_R$



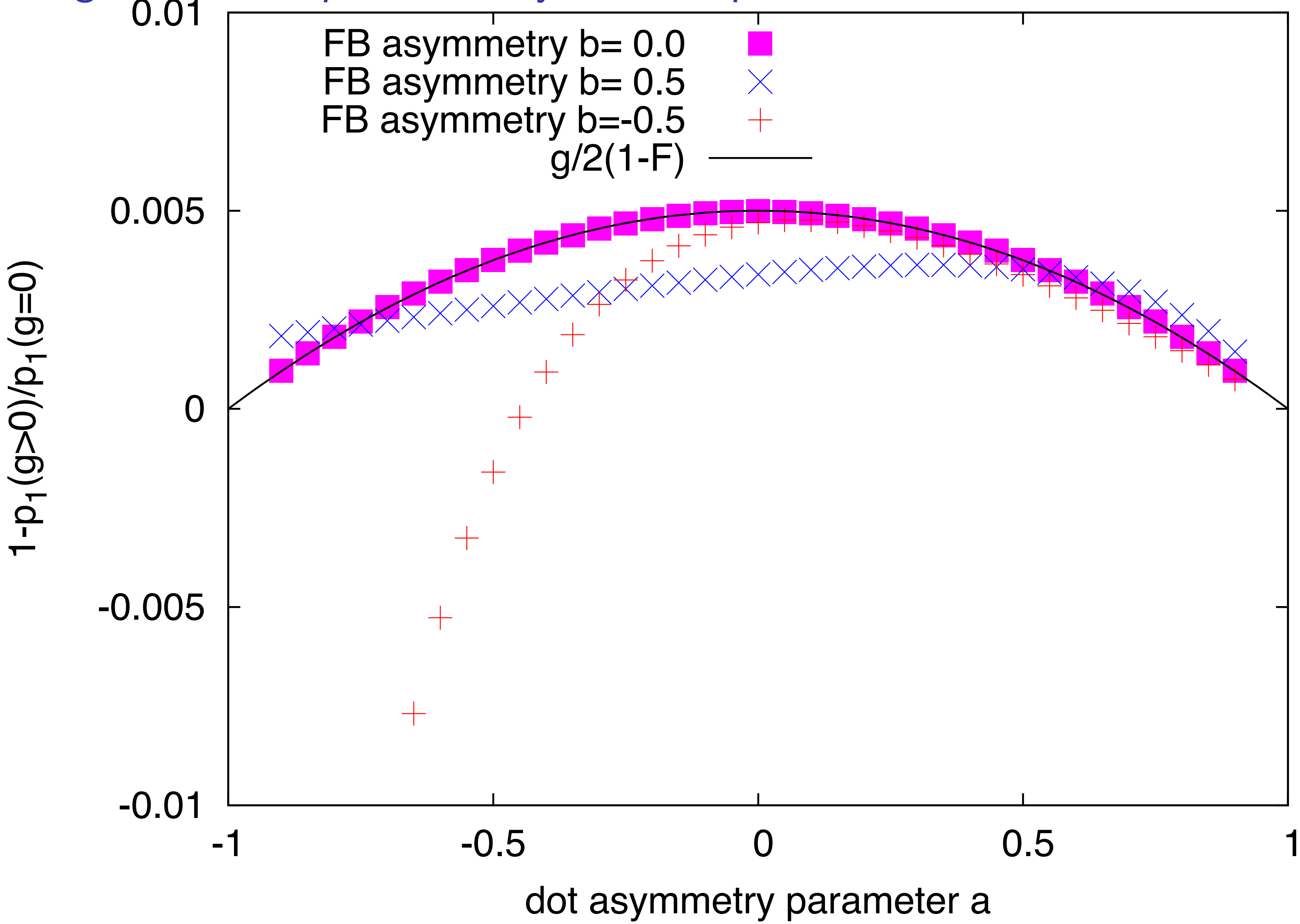
- $F_{2/3}$ reconstructed Fano factor/ skewness.
- inhomogenous FB: ε not changed.

Feedback control and internal states

- How does feedback affect internal system state?

Feedback control and internal states

Example: Single level dot, p_1 stationary dot occupation.

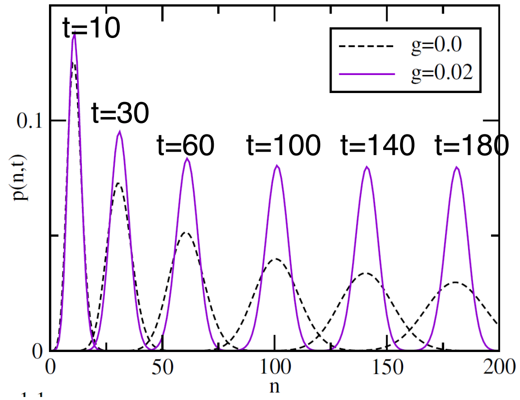


- $b = 0$: relation $1 - p_1(g > 0)/p_1(g = 0) = \frac{g}{2}(1 - F)$.

Open Questions

Open Questions

- Thermodynamic interpretation of frozen FCS: information gain.



Related work: "Generalized Jarzynski Equality under Nonequilibrium Feedback Control"

$$\langle e^{-\beta(W-\Delta F)-I} \rangle = 1$$

T. Sagawa and M. Ueda, Phys. Rev. Lett. **104**, 090602 (2010).

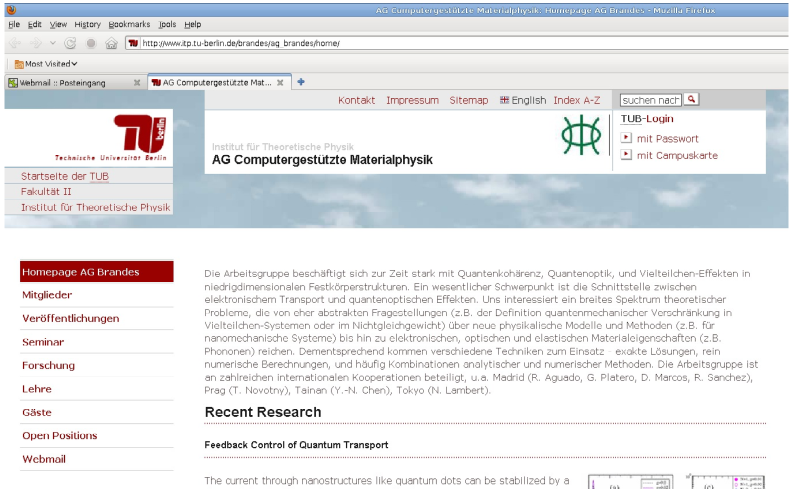
Open Questions

- Efficiency as an accurate charge transfer device.
- To go beyond Markovian master equation.
- Semiclassical limits.
- Fully quantum feedback loops.

Summary

- Instantaneous feedback of $I_0 t - n \rightsquigarrow$ frozen FCS at large times.

PhD / PostDoc positions available



Feedback efficiency

Comparison with pump/turnstile

Single electron turnstile model

- Single level dot, rates $\Gamma_{L/R}(t) = \gamma T \sum_{j=1}^{\infty} \delta(t - t_{j,L/R})$.
- Strong bias from left to right, cycles electrons in - out -in -out ...

- Transfers on average $\langle n \rangle = j \tanh(\gamma T/2)$ electrons after j cycles.
- Fluctuations $c_2 \equiv \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle / (2 \cosh^2(\gamma T/2))$
  - grow with time.
  - surpass corresponding feedback system fluctuations after time

$$t^* = \frac{4}{\Gamma} C_2^{\mathrm{FB}} \cosh^2(\gamma T/2) = \frac{1}{2gI} \cosh^2(\gamma T/2).$$

($I = \Gamma/2$ current for symmetric dot, g FB coupling strength).

Feedback efficiency

Comparison with pump/turnstile

- Continuous operation for long times: feedback always wins!

Feedback efficiency

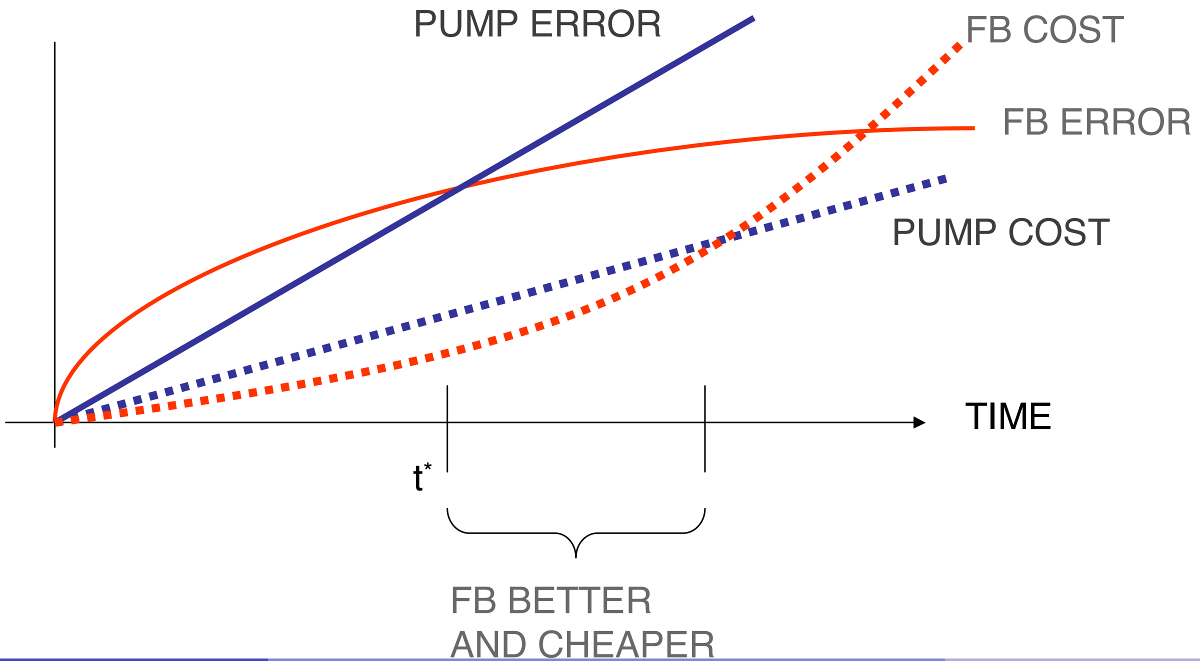
Comparison with pump/turnstile

However: the costs ....

Feedback efficiency

Comparison with pump/turnstile

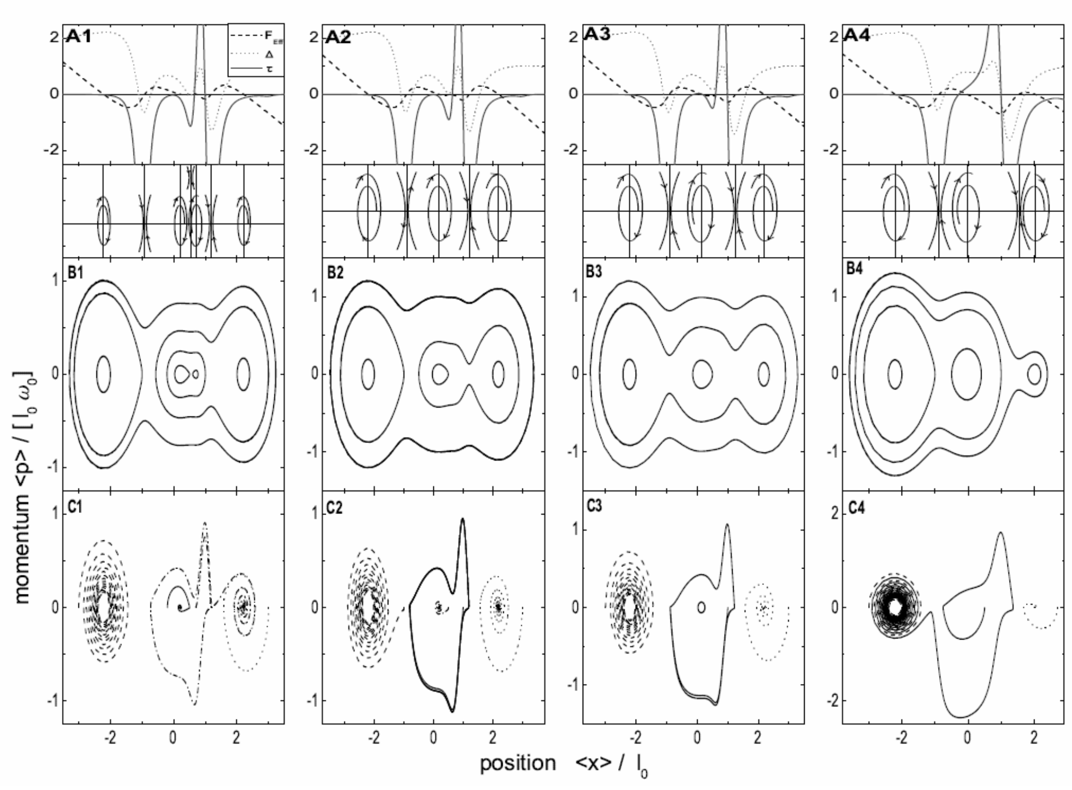
- FB requires continuous monitoring of bath.
- Cost of high-precision *multiplication* in feedback function $I_0 t - n$ is super-linear in t.



Semiclassical Limit

- Double quantum dot + single resonator mode.
- Expansions around classical trajectories: limit cycles.

R. Hussein, A. Metelmann, P. Zedler, T. Brandes; arXiv:1006.2076 (2010).

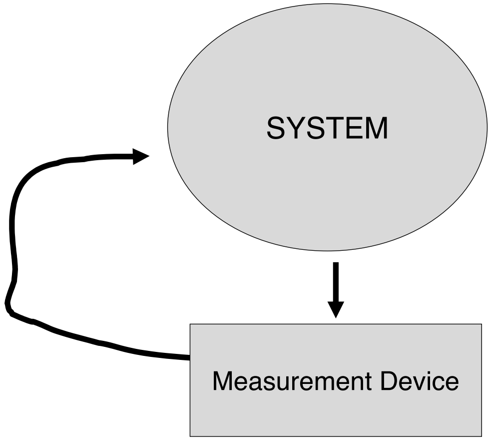


n-dependent rates: Milburn's detector feedback model

G. J. Milburn, J. Mod. Opt **38** (10), 1973 (1991)

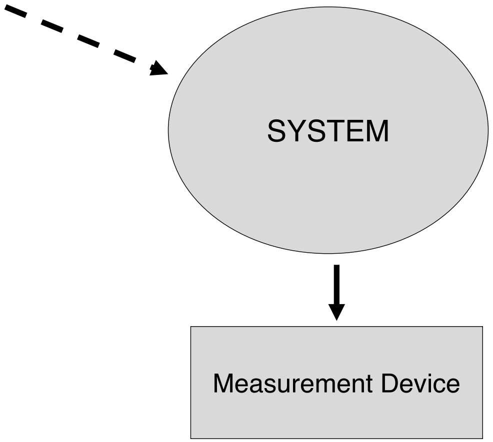
- Random classical variable $x = 0, 1$: '(not) transmitted'.
- Generating function $\langle e^{-i\lambda x}\rangle = 1 + p(e^{-i\lambda} - 1)$ with $p = \langle x \rangle$.
- Now N independent transmissions: variables x_l with $\langle x_n \rangle \equiv p_n$.
- Variable $X_N \equiv \sum_{l=1}^{N} x_l$, $\langle e^{-i\lambda X_N}\rangle = \Pi_{n=1}^{N} \left(1 + p_n(e^{-i\lambda} - 1)\right)$.
- Specific choice of the p_n (e.g. saturation at large n) $\rightsquigarrow$ some control of $\langle X_N \rangle$, $\langle X_N^2 \rangle$.

Closed versus Open Loop Control



CLOSED LOOP (FEEDBACK)

- System parameters are permanently changed, conditioned on measurement result.



Open loop (no feedback)

- 'Design' of system parameters.

Quantum Feedback Control

Some key players: Belavkin; Wiseman, Milburn (textbook 2010!); Doherty, Jacobs; Korotkov; Mabuchi;....

- Measurement vs. coherent feedback control.
- Markovian measurement feedback $\rightsquigarrow$ Lindblad Master equation.
  - Avoiding decoherence of cat states.
  - Purification of otherwise mixed qubit states: resonance fluorescent atoms.
- Solid state context: qubit coupled to detector, n-resolved master equation.