The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA

2164-11

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Euler Buckling Instability in Nanoelectromechanical Systems

Felix von OPPEN

Freie Univ. Berlin
Dahlem Center for Complex Quantum Sys.
Arnimallee 14, Dahlem, D-14195
Berlin
GERMANY

Freie Universität Berlin

Euler buckling instability in nanoelectromechanical systems

Felix von Oppen

FU Berlin

Projektträger für das Bundesministerium für Bildung und Forschung

DIP

Deutsche Forschungsgemeinschaft DFG

SPP 1285

Deutsche Forschungsgemeinschaft DFG

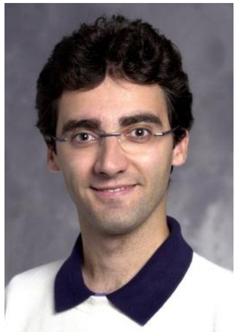
SPP 1243

Sfb 658

Collaborators

Guillaume Weick (Strasbourg)

Fabio Pistolesi (Bordeaux)



Eros Mariani (Exeter)

G. Weick. F. Pistolesi, E. Mariani, FvO, PRB **81**, 121409(R) (2010)

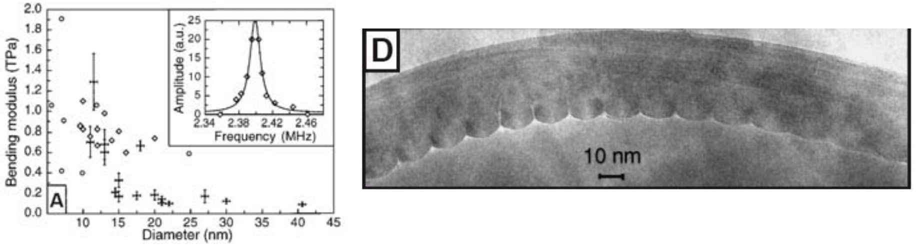
G. Weick, F. Pistolesi, FvO, in preparation

Nanomechanical instabilities

Freie Universität Berlin

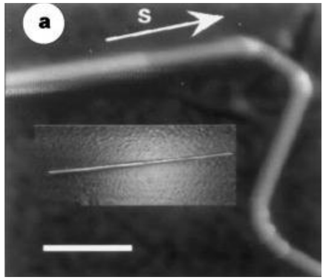
- bending & buckling of nanotubes

electrostatic deflection



de Heer group, Nature 1999

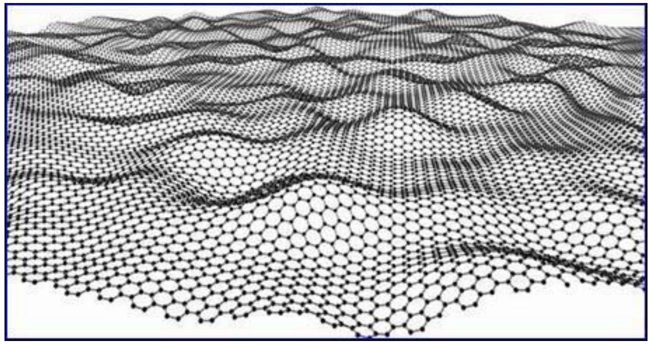
wrinkling under compression



Falvo et al., Nature 1997

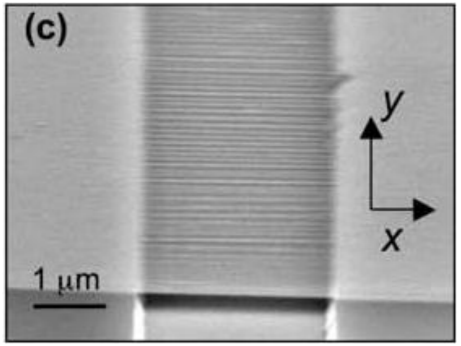
- rippling & wrinkling of suspended graphene

rippling



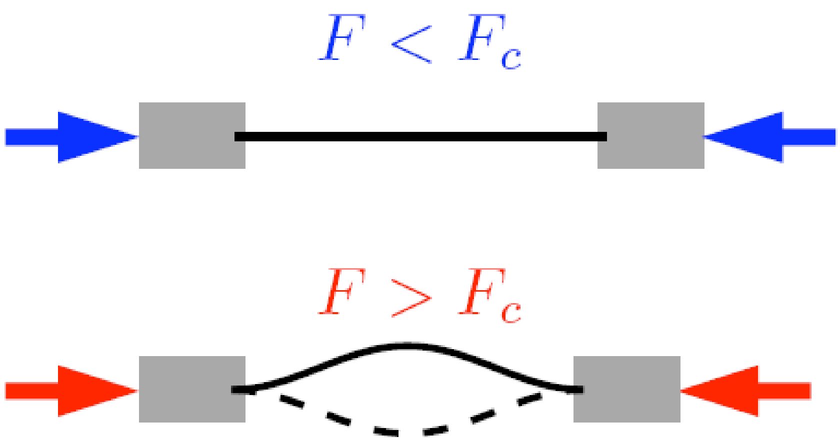
Meyer *et al.* Nature 2007

wrinkling



Lau group, Nature Nanotech (2009)

Euler buckling instability



Elastic rod buckles when compression exceeds critical force F_c

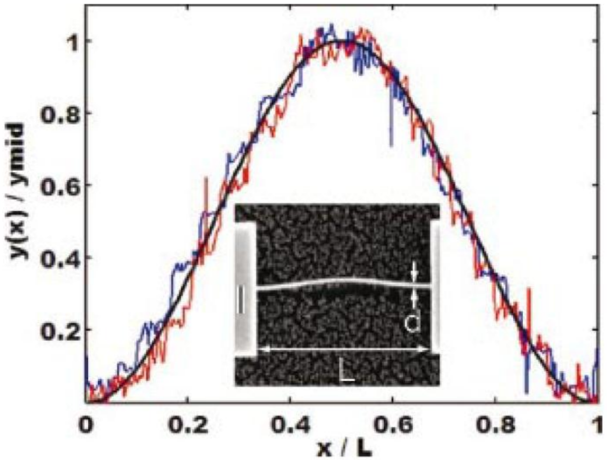
L. Euler (1744)

Euler buckling

Nanomechanical buckling

Freie Universität Berlin

- buckling of nanobeams



Euler buckling of SiO_2 nanobeams

- ➢ nanobeams released from Si substrate by reactive ion etching
- ➢ caused by SiO_2/Si strain

Carr et al., APL 2003

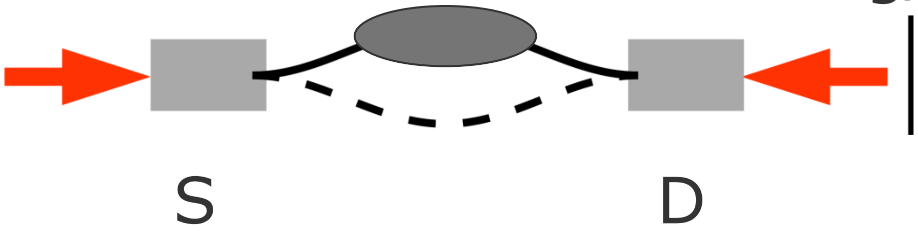
- probing nanomechanical quantum fluctuations

  - ➢ smearing of the Euler instability due to quantum flucutations
  - ➢ quantum coherence in nanobeams

e.g. Werner & Zwerger, EPL 2004

Euler instability in NEMS

quantum dot:
semiconducting/metallic island



e.g.
- nanobeam
- carbon nanotube

Question:

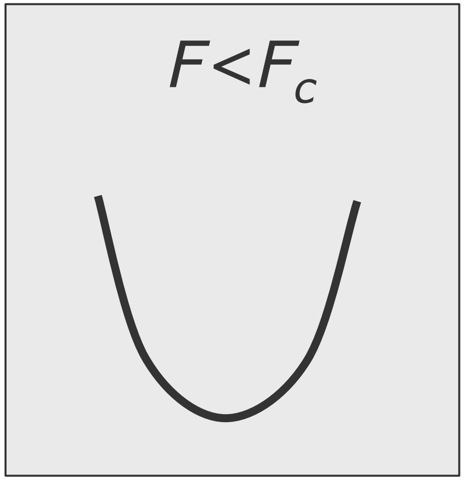
- interplay between mechanical and electrical degrees of freedom

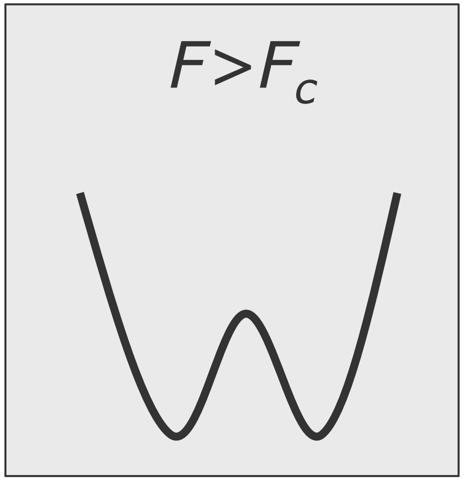
  - ➢ modification of Coulomb blockade by Euler instability?
  - ➢ backaction of Coulomb blockade on Euler?

Motivation:

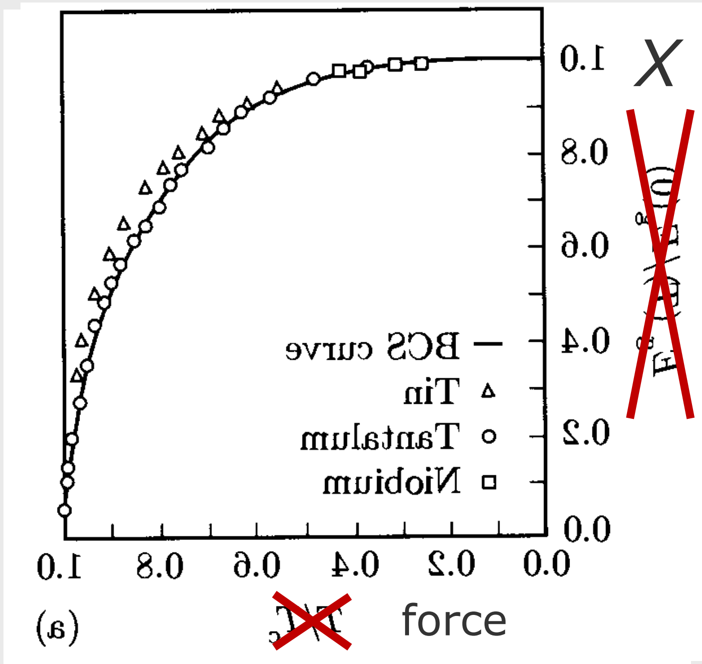
- pronounced mechanical nonlinearities
- strong electron-vibron coupling close to instability

Theory of Euler instability





$$H_{\mathrm{vib}} = \frac{P^2}{2m} + \frac{m\omega^2}{2}X^2 + \frac{\alpha}{4}X^4$$



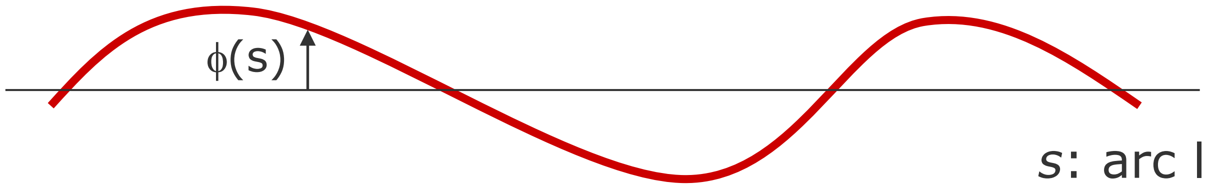
Superconductivity / Euler instability

- buckling is **continuous** instability
- buckled state for $F>F_c$: $X \sim \pm\sqrt{F-F_c}$

Theory of buckling instability

Freie Universität Berlin

template for Landau theory of
continuous phase transitions:



s: arc length

action:

$$S[\phi(s)] = \int dt \int ds \left\{ \frac{\sigma}{2}\dot{\phi}^2 - \frac{\kappa_{\text{eff}}}{2}\frac{(\phi'')^2}{1-(\phi')^2} - F\sqrt{1-(\phi')^2} \right\}$$

curvature energy

fixed distance between endpoints

harmonic approximation

$$S[\phi(s)] \simeq \frac{1}{2}\int dt \int ds \left\{ \sigma\dot{\phi}^2 - \kappa_{\text{eff}}(\phi'')^2 + F(\phi')^2 \right\}$$

Theory of buckling instability

Freie Universität Berlin

- restrict theory to unstable mode with mode amplitude *X*

- include anharmonic corrections

$$H_{\mathrm{vib}} = \frac{P^2}{2m} + \frac{m\omega^2}{2}X^2 + \frac{\alpha}{4}X^4$$

critical force $F_c = \kappa_{\mathrm{eff}}(2\pi/L)^2$

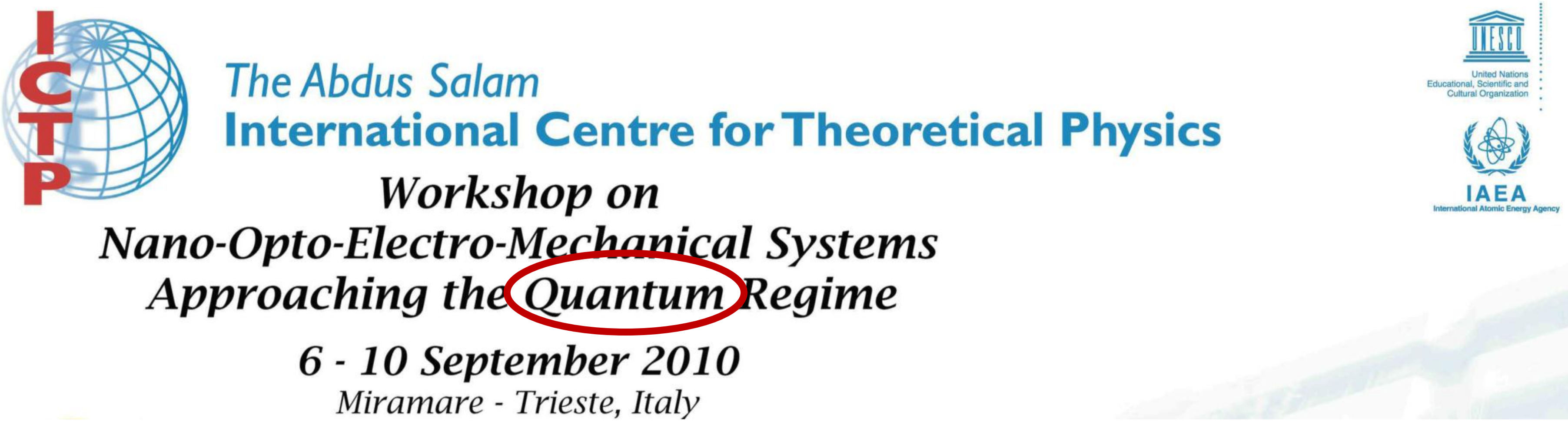
anharmonicity $\alpha = (\pi/2L)^4 F_c L$

with frequency $\omega^2 = (\kappa_{\mathrm{eff}}/\sigma)(2\pi/L)^4(1 - F/F_c)$

- Euler instability
- "critical" slowing down

Quantum vs classical

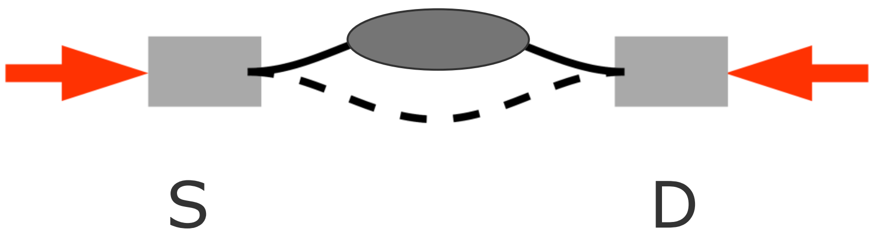
Freie Universität Berlin



critical slowing down:

$$I >> \omega$$

sufficiently close to the instability

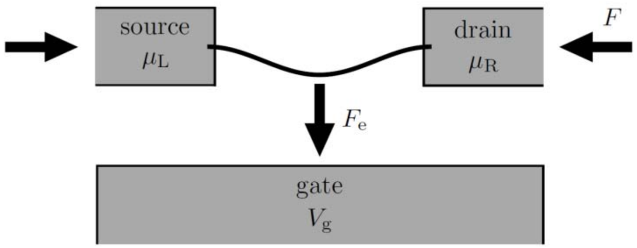


→ classical Langevin dynamics

EXACT

Electron-vibron coupling

Capacitive coupling



$$H_c = \lambda X \hat{n}$$

current-induced force:

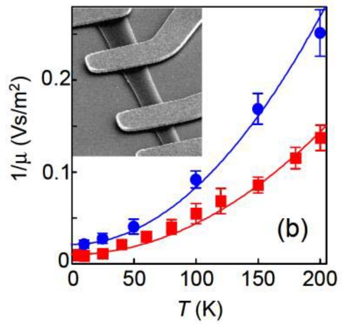
$$F_{curr} = -\lambda \langle \hat{n} \rangle_X$$

Intrinsic coupling

intrinsic electron-vibron coupling

$$H_c = \frac{g}{2} X^2 \hat{n}$$

Compare suspended graphene :



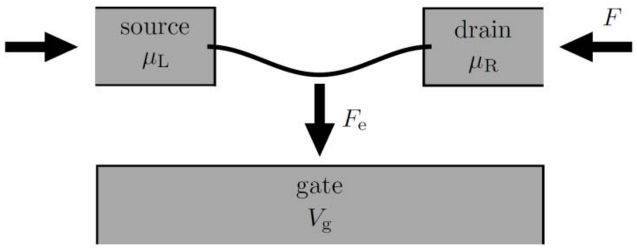
$$\rho \sim T^2$$

Geim group arXiv:1008.2522

See also:
E. Mariani, FvO, PRL (2008)
& arXiv:1008.1631

	capacitive electron-vibron coupling	**intrinsic electron-vibron coupling**
semiconductor quantum dot	• more easily realized experimentally • more pronounced effect on Coulomb blockade	
metallic quantum dot		• consistent w/ symmetry of Euler instability • more pronounced effect on Euler

Strong e-vib coupling



eff. gate voltage

$$H_c = \overbrace{\lambda X}\hat{n}$$

addition of a single electron $\Delta n = 1$

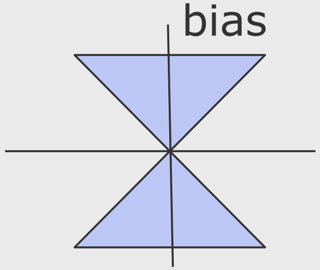
displacement

$\Delta X = \lambda / m\omega^2$

diverges at instability !

eff. shift in gate voltage

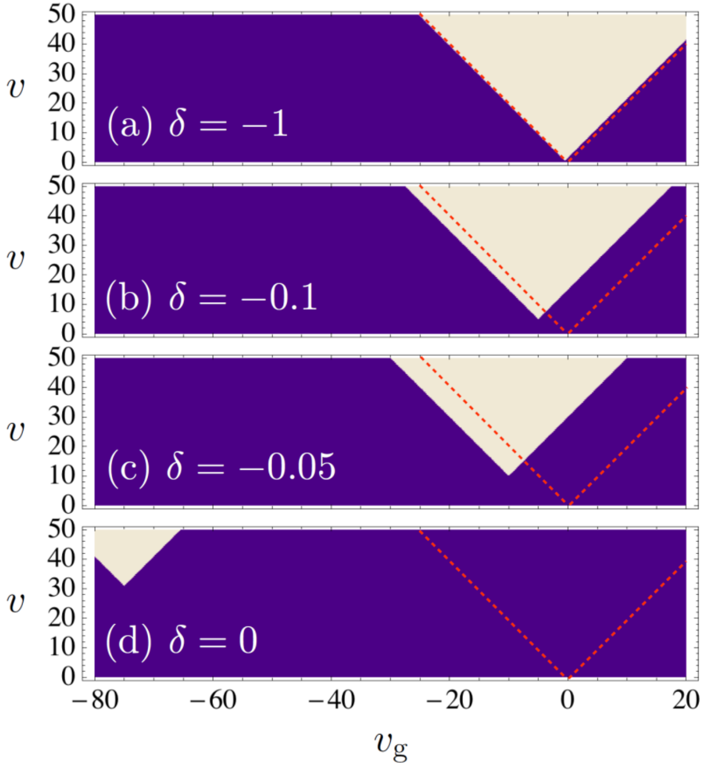
$$\Delta v_g = \lambda^2 / m\omega^2 \rightarrow \infty$$



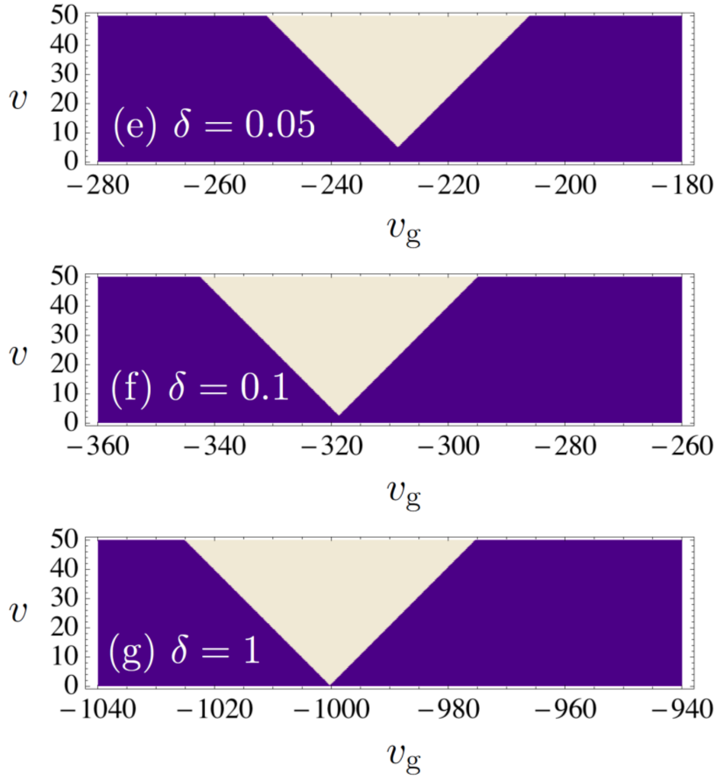
phonon blockade

Strong enhancement of phonon blockade

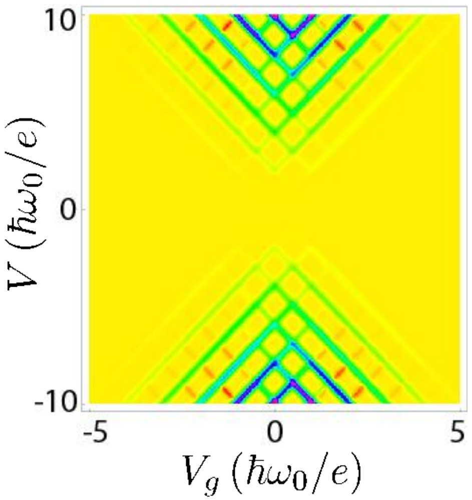
below the instability



above the instability

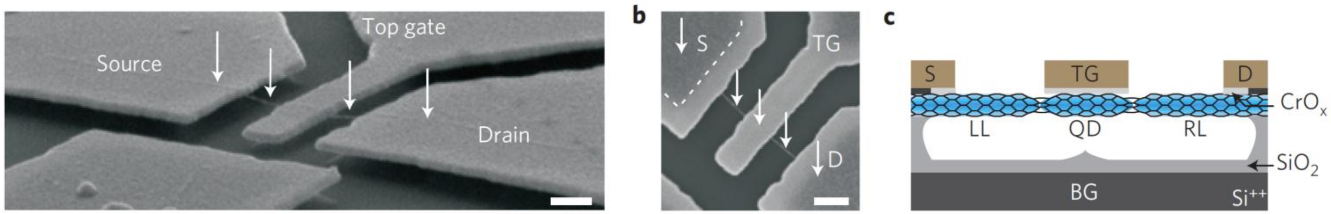


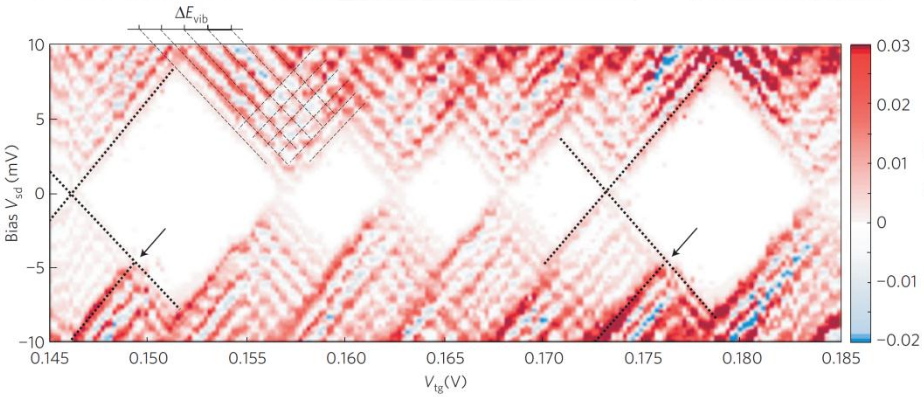
Quantum analog:
Franck-Condon blockade

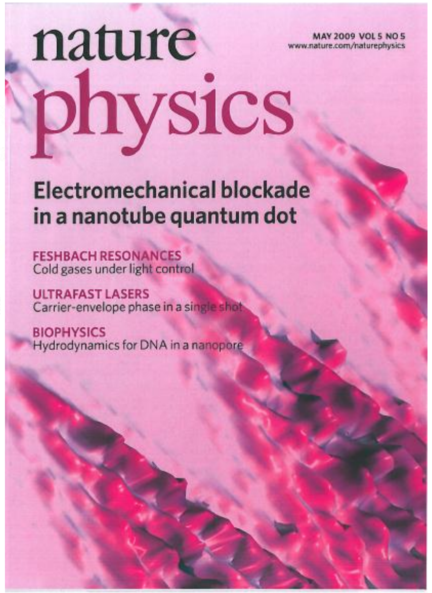


Theoretical prediction

Koch & FvO, PRL (2005)
Koch, Raikh, FvO, PRL (2005)
Koch, FvO, Andreev, PRB (2006)



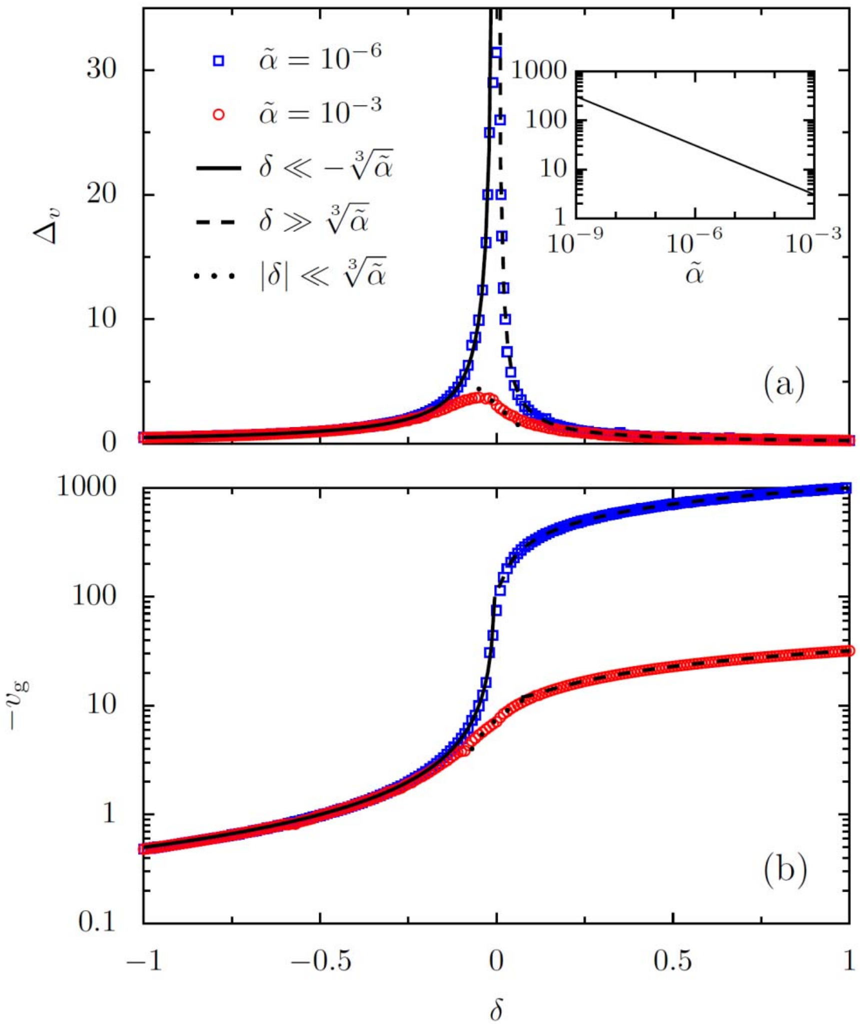




Experimental observation

R. Leturcq, C. Stampfer, K. Inderbitzin,
L. Durrer, C. Hierold, E. Mariani,
M.G. Schultz, FvO, K. Ensslin,
Nature Phys. **5**, 327 (2009)

Strong enhancement of phonon blockade

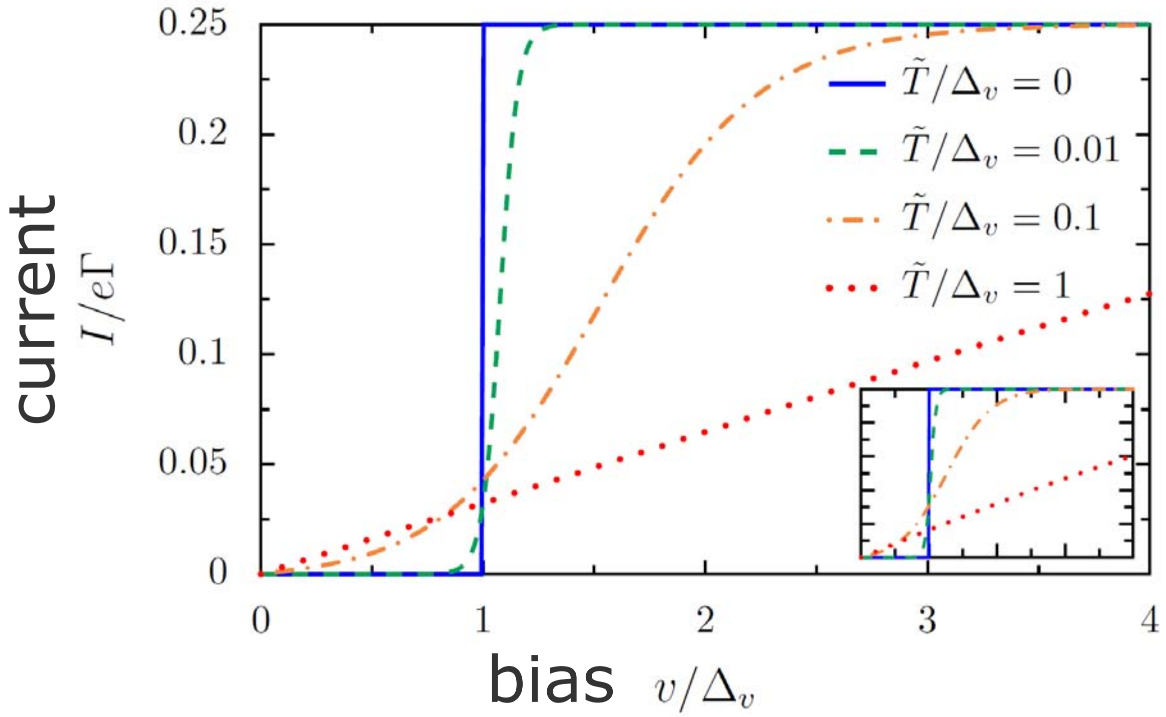


- gap increases sharply near F_c
- increase limited by quartic term
- relative increase stronger for weaker e-vib coupling

- Coulomb diamond shifts in gate voltage
- small shift below instability
- orders of magnitude larger shift on buckled side

Temperature effects

Freie Universität Berlin



gap observable as long as $T \ll \text{gap}$

Analogy with Landau theory

Freie Universität Berlin

Euler instability

displacement *X*

force $\delta \sim F\text{-}F_c$

$$\Delta_v = \begin{cases} -\dfrac{1}{2\delta}, & -1 \leqslant \delta \ll -\sqrt[3]{\tilde{\alpha}} \\ \dfrac{1}{4\delta}, & \delta \gg \sqrt[3]{\tilde{\alpha}}, \\ \dfrac{\sqrt[3]{2}-1}{\sqrt[3]{\tilde{\alpha}}}\left(\dfrac{3}{2^{4/3}} - \dfrac{\delta}{\sqrt[3]{\tilde{\alpha}}}\right), & |\delta| \ll \sqrt[3]{\tilde{\alpha}}. \end{cases}$$

Landau theory

order parameter *m*

red. temperature $(T_c\text{-}T)/T_c$

Curie law $m \sim B/(T-T_c)$

$m \sim (B/\alpha)^{1/3}$ at $T = T_c$

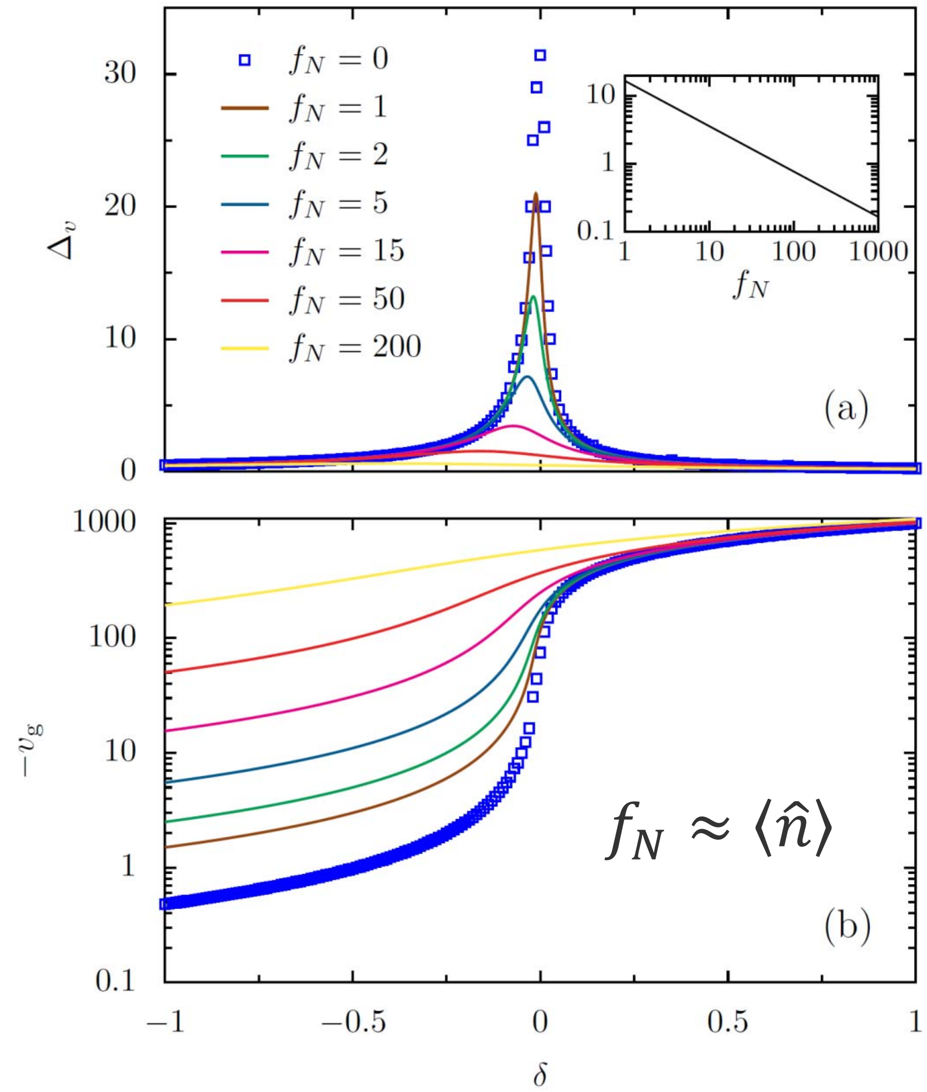
Effect of offset charges

so far: $\langle \hat{n} \rangle_X$ is of order unity

in general, $\langle \hat{n} \rangle_X$ can take on any value due to „offset“ charges:

$$V(X) = \lambda X \langle \hat{n} \rangle_X + \frac{m\omega^2}{2} X^2 + \frac{\alpha}{4} X^4$$

analogous to symmetry-breaking field in Landau theory



Full Langevin dynamics

Non-equilibrium Born-Oppenheimer approximation:

current-induced force $F_{curr} = \lambda \langle \hat{n} \rangle_X$

adiabatic limit	first corrections to adiabatic limit in ω_0/Γ	
compute $\langle \hat{n} \rangle_X$ at fixed X	retardation due to slow vibrational dynamics:	thermal & shot noise of $\langle \hat{n} \rangle_X$:
conservative average force	frictional force	fluctuating force

$$\ddot{X} + \gamma(X)\dot{X} = F_{\text{eff}}(X) + \delta F(X,t)$$

Langevin equation

Freie Universität Berlin

Occupation $\langle \hat{n} \rangle_X$ follows from Boltzmann-Langevin equation

$$\frac{dn}{dt} = \{n, H_{\text{vib}}\} + \Gamma_+(1-n) - \Gamma_- n + \delta J_+ - \delta J_-$$

tunneling rates into & out of quantum dot

$$\langle \delta J_-(t)\, \delta J_-(t') \rangle = \Gamma_- n\, \delta(t-t')$$

perturbative treatment of Poison bracket:

$$\gamma(X) = \frac{\lambda}{m\Gamma} \partial_X \langle \hat{n} \rangle_X$$

damping

$$\langle \delta F(t) \delta F(t') \rangle = \frac{2\lambda^2}{\Gamma} \langle \hat{n} \rangle_X (1 - \langle \hat{n} \rangle_X)$$

fluctuations

Fokker-Planck equation

Boltzmann equation for $\mathcal{P}_n(X,t)$:

$$\partial_t \mathcal{P}_n = \{\mathcal{H}_n, \mathcal{P}_n\} - (-1)^n \Gamma_{01}(X)\mathcal{P}_0 + (-1)^n \Gamma_{10}(X)\mathcal{P}_1$$

small:

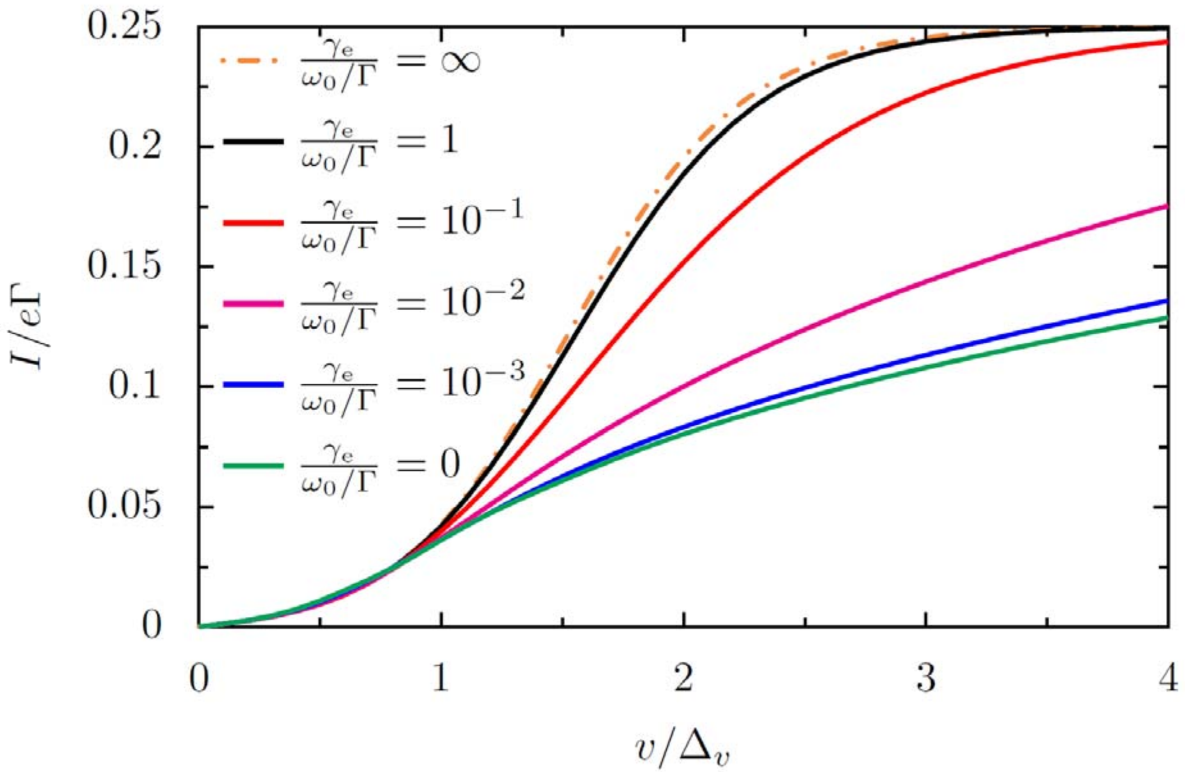
$$\mathcal{P}_0(X,P,t) = \frac{\Gamma_{10}(X)}{\Gamma(X)}\mathcal{P}(X,P,t) - \delta\mathcal{P}(X,P,t)$$

$$\mathcal{P}_1(X,P,t) = \frac{\Gamma_{01}(X)}{\Gamma(X)}\mathcal{P}(X,P,t) + \delta\mathcal{P}(X,P,t)$$

$$\partial_t \mathcal{P} = -\frac{P}{m}\partial_X \mathcal{P} - F_{\mathrm{eff}}(X)\partial_P \mathcal{P} + \frac{\eta(X)+\eta_{\mathrm{e}}}{m}\partial_P(P\mathcal{P}) + \left(\frac{D(X)}{2} + \eta_{\mathrm{e}} k_{\mathrm{B}} T\right)\partial_P^2 \mathcal{P}$$

η_e: intrisinc damping; accounts for *Q* of vibron

Consequences of fluctuations



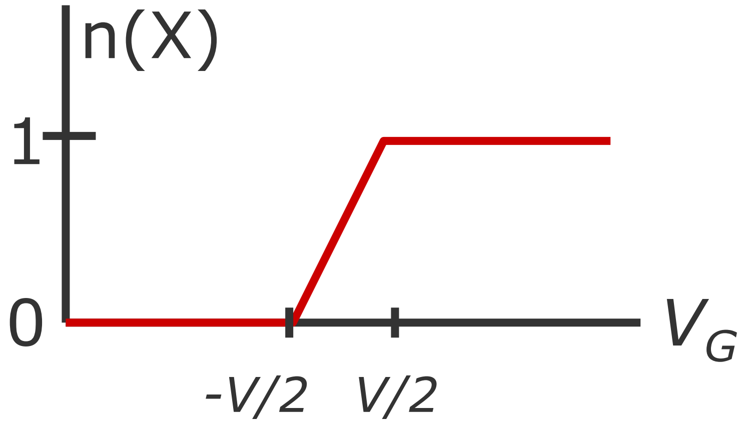
phonon blockade more pronounced
- for low *Q*
- slow oscillator

Intrinsic coupling

metallic quantum dot:

$$H_c = \frac{g}{2} X^2 \hat{n} \quad \Rightarrow \quad F_{\text{eff}}(X) = -gXn(X)$$

- effect of e-vib coupling: $V_G \rightarrow V_G - gX^2/2$



$$n_0(x) = \begin{cases} 1, & v_g(x) > v/2, \\ \frac{1}{2} + \frac{v_g(x)}{v}, & -v/2 \le v_g(x) \le v/2 \\ 0, & v_g(x) < -v/2 \end{cases}$$

Current-induced potential

Mean-field theory:

$$F_{\mathrm{curr}} = -gX\langle \hat{n} \rangle_X \quad \Rightarrow \quad V_{\mathrm{curr}}(x) = \frac{g}{2V}\left(V_+ X^2 - \frac{1}{4} g X^4\right)$$

more significant at small bias

stabilizes $X=0$

destabilizes $X=0$

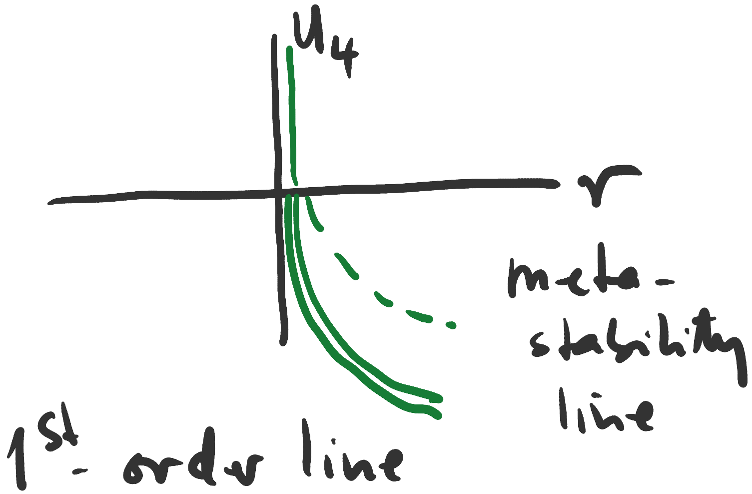
compare to bare vibron Hamiltonian $H_{\mathrm{vib}} = \frac{P^2}{2m} + \frac{m\omega^2}{2}X^2 + \frac{\alpha}{4}X^4$

unstable X^4 term (at small X) when $V<g^2/2\alpha$

Analogy with tricritical points

Tricritical point

$$f = \frac{1}{2} r\phi^2 + u_4\phi^4 + u_6\phi^6$$

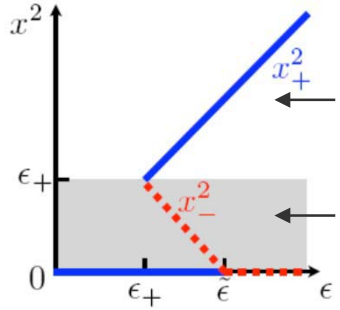


$$r_{1st} = \frac{u_4^2}{2u_6}$$

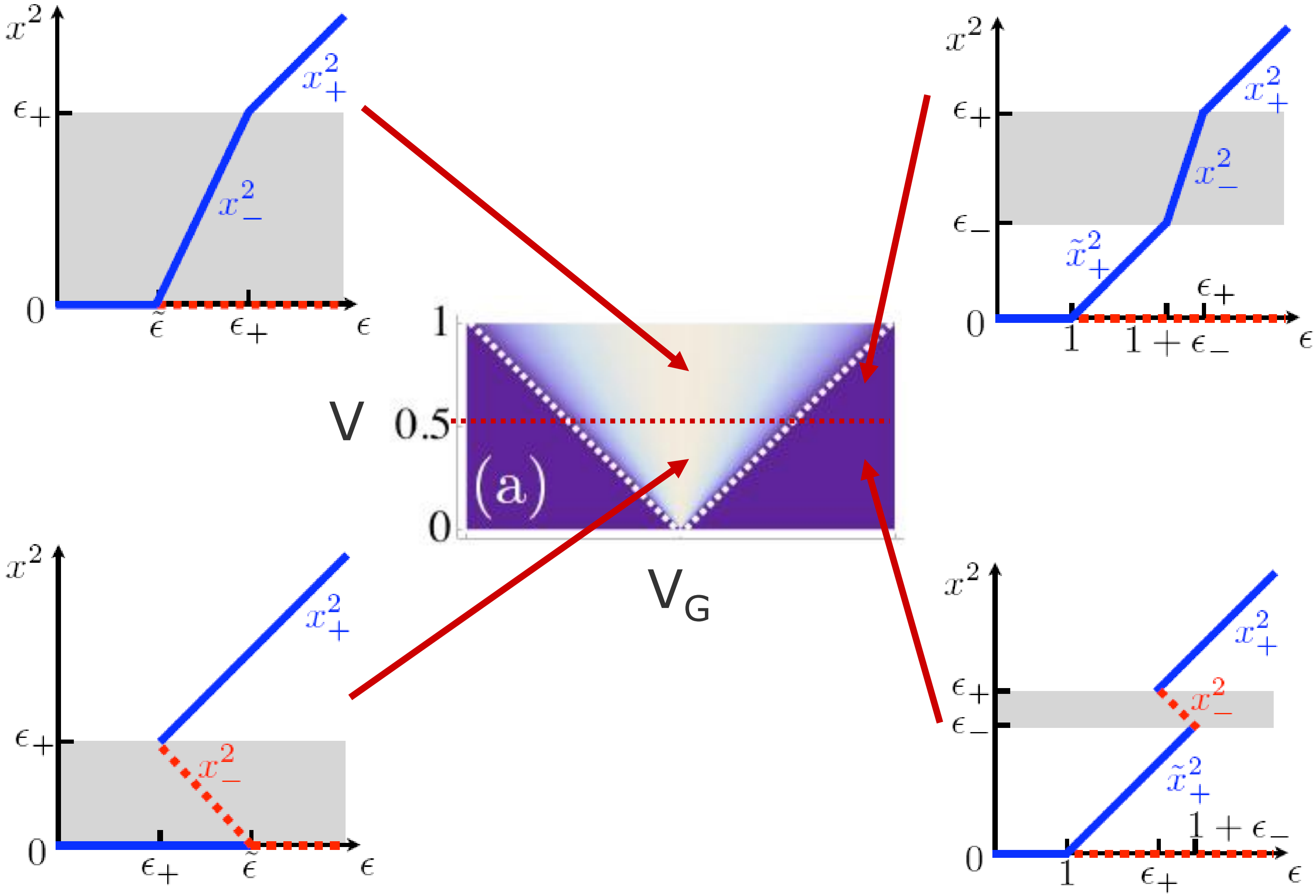
Euler instability

$$V_{\text{tot}} = \frac{1}{2}\left(m\omega^2 + \frac{gV_+}{2V}\right)^2 X + \left(\frac{\alpha}{4} - \frac{g^2}{8V}\right) X^4$$

- valid for "small" X^2
- V controls sign of X^4 term
- region of metastability



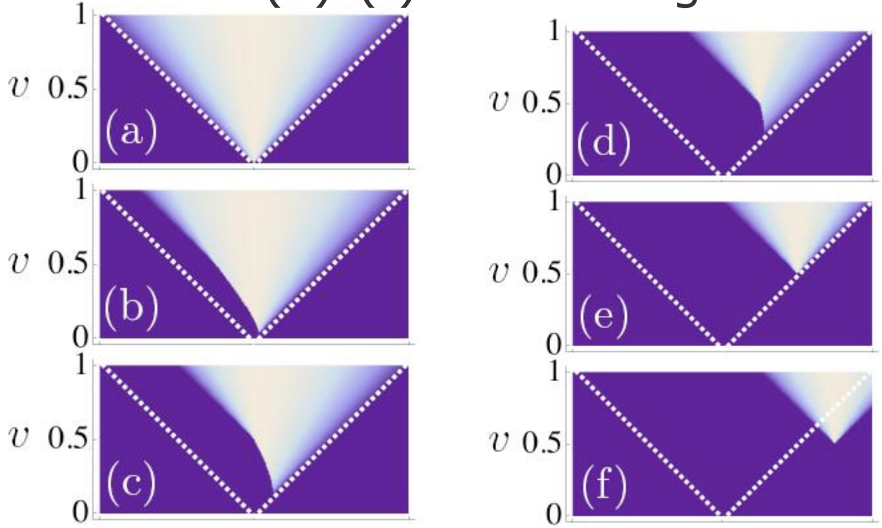
Discontinuous Euler instability



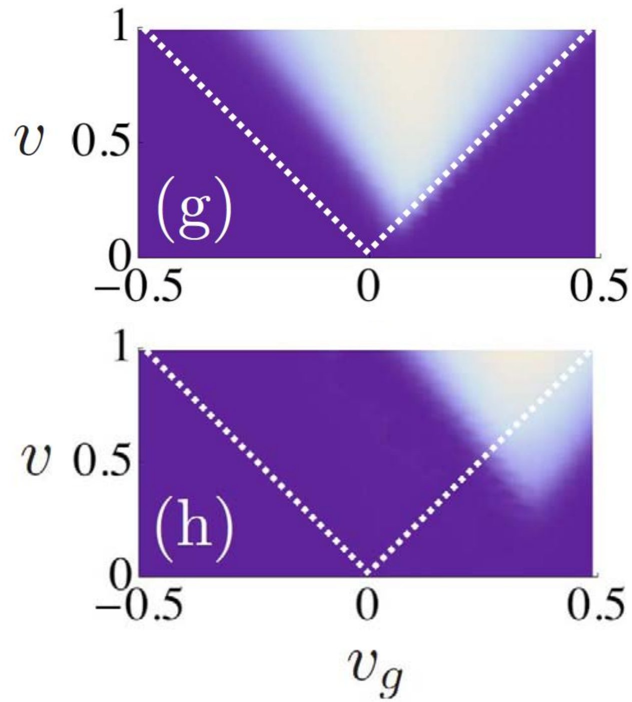
Transport

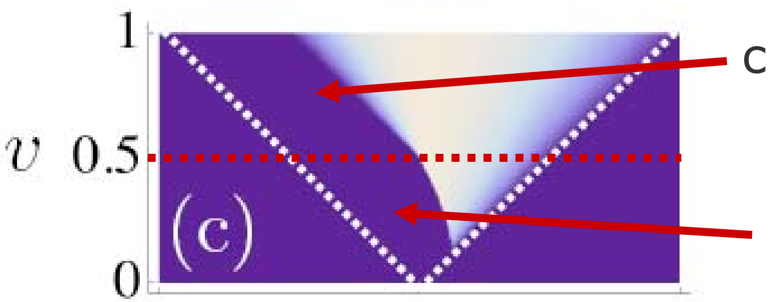
mean-field theory

(a)-(f): increasing *F*



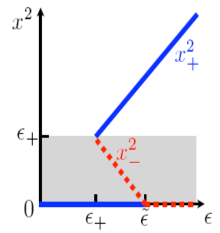
full Langevin dynamics





classical phonon blockade

“tricritical” blockade



Conclusions

Nanoelectromechanics near mechanical instabilities

- Euler instability as paradigm of mechanical instability
- "critical slowing down" makes problem inherently classical, and allows for asymptotically exact solution
- capacitive coupling/semiconductor dot: strong enhancement of phonon blockade
- intrinsic coupling/metallic dot: tricritical Euler instability