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Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

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A Phonon-Tunneling Approach to Support-Induced Dissipation of Nanomechanical Resonators

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A phonon-tunneling approach to support-induced dissipation of nanomechanical resonators

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Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime, Trieste, September 2010

TECHNISCHE UNIVERSITÄT MÜNCHEN

T-34 Prof. W. Zwerger

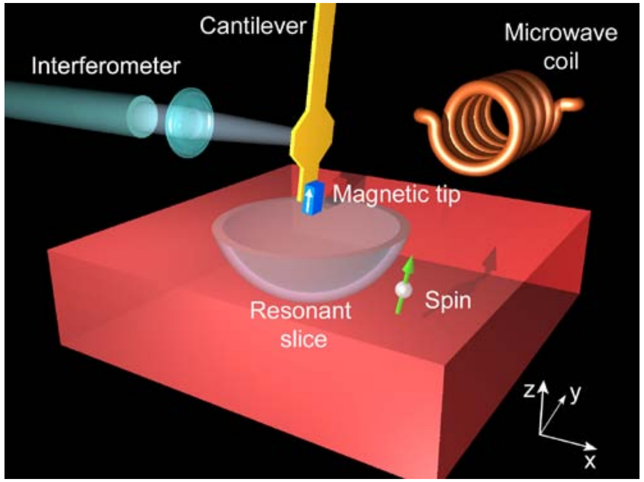
Outline

1. Motivation
2. Theory — Master formula for Q
3. “Free-free” resonators (Aspelmeyer group, Vienna)
4. Stressed membranes (Parpia-Craighead coll., Cornell)
5. Conclusions

Motivation: high-Q nanomechanics

Technological applications:

- signal-processing (on-chip UHF narrow-band filters)
- ultra-sensitive sensors at the molecular scale
- high resolution scanning-probe force microscopy



D. Rugar et al, Nature 2004

Quantum limited control of a single macroscopic mechanical degree of freedom

$\Downarrow$

- Initialize oscillator with long phonon lifetime in its quantum ground state.
- Quantum signatures: phonon jumps, non-classical states, Schrödinger cats . . .

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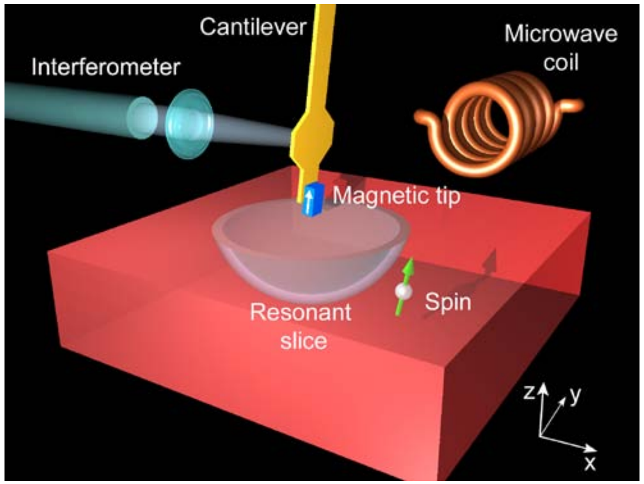

D. Rugar et al, Nature 2004

Quantum limited control of a single macroscopic mechanical degree of freedom

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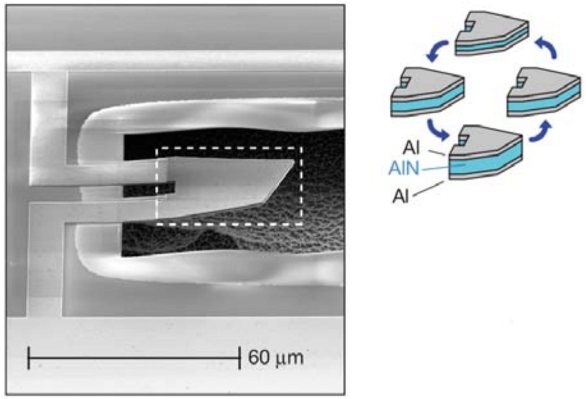
- "Quantum optics analogue": nanomechanical J-C model, TL "atom" $\rightarrow$ superconducting qubit, quantum dot (QD) or NV center.



O'Connell et al., Nature 2010

Motivation: high-Q nanomechanics

Room temperature cavity-assisted sideband-cooling to the ground-state:

$$\frac{k_B T}{\hbar \kappa Q} < 1 \text{ with } \kappa < \omega_m \text{ or } n_{\text{opt}} \sim \left(\frac{k_B T}{\hbar \omega_m Q}\right)^{2/3} \rightarrow fQ \sim 10^{13}\text{Hz}.$$

Importance of fQ

Predicting the mechanical dissipation remains a challenge.

Relevant mechanisms at low T and high-vacuum:

- two-level fluctuators
- anharmonic effects / phonon-phonon interactions (?)
- radiation of phonons (elastic-waves) into the supports

Motivation: high-Q nanomechanics

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$\Downarrow$

Which is the fundamental limit set by the design?

Temperature and scale independent

Other mechanisms add incoherently $1/Q_{\mathrm{tot}} = \sum_i 1/Q_{\mathrm{i}}$

Quantum dissipation of mechanical motion

Caldeira-Leggett model $\longrightarrow$ Generalized equation of motion

$$\ddot{\hat{X}}_R(t) + \int_0^t \mathrm{d}t' \, \gamma(t-t')\dot{\hat{X}}_R(t') + \omega_R^2 \hat{X}_R(t) = \hat{\xi}(t)$$

determined by environmental force spectral density $I(\omega)$.

$$I(\omega) = \omega \int_{-\infty}^{\infty} \mathrm{d}t \, \gamma(t) \frac{e^{i\omega t}}{2\omega_R}, \qquad \frac{1}{Q} = \frac{I(\omega_R)}{\omega_R},$$

$\gamma(t)$ is the symmetric dissipation-kernel [$\gamma(t) \sim \delta(t)$ in Ohmic case or in weak coupling limit] and $\hat{\xi}(t)$ the environmental noise.

Markov approximation may not suffice for "quantum nanomechanics" $\rightarrow$ e.g. structured spin-boson model for detector-resonator system, strongly non-linear resonators.

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Dissipation induced by unavoidable coupling to vibrations of the substrate $\rightarrow$ elastic wave radiation $\rightarrow$ clamping loss.

(M.C. Cross and R. Lifshitz, PRB 2001; D.M. Photiadis and J.A. Judge APL 2004)

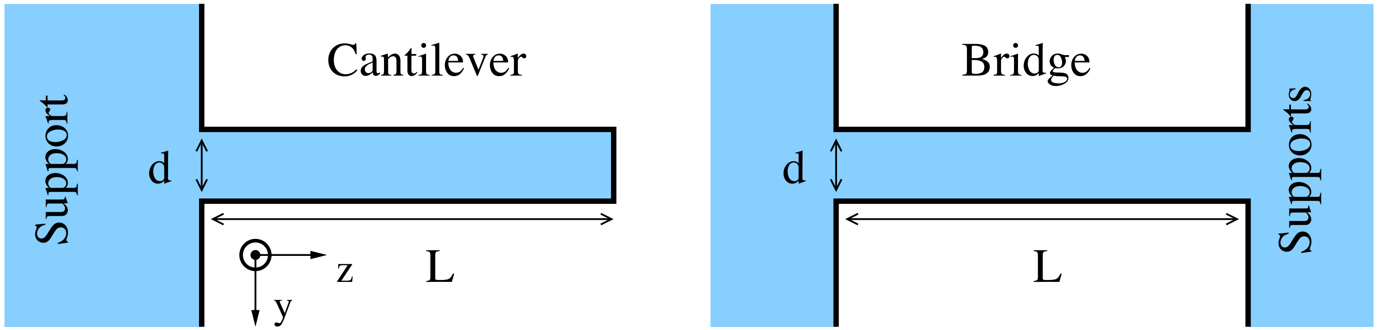
Objectives:

- General understanding of limit imposed on Q by the design of a small mechanical resonator.
- Microscopic derivation of quantum dissipative dynamics of a “macroscopic” mechanical degree of freedom.

IWR, Phys. Rev. B **77**, 245418 (2008).

Theory

Abrupt junctions with supports

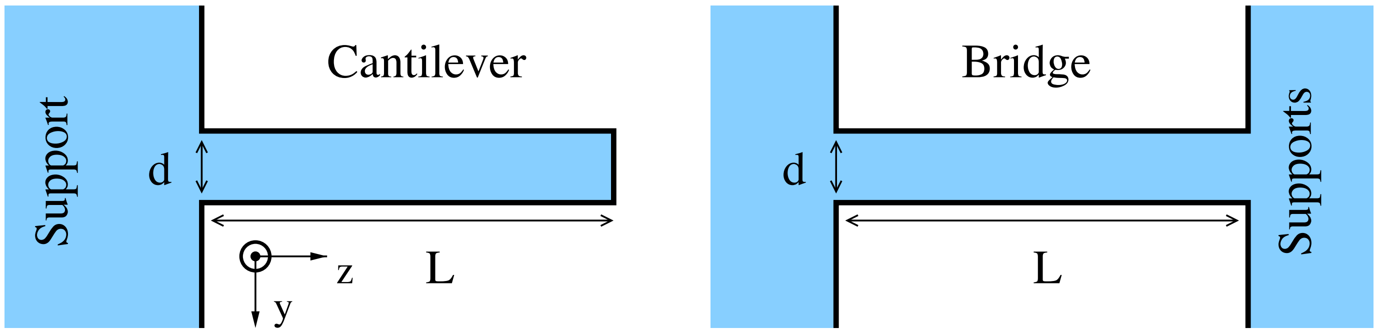


Ideal limit: phonon tunneling between the beam and the supports is the only source of dissipation $\longrightarrow$ upper bound for Q-values.

- Derivation from elasticity theory ($\lambda \gg a$) of phononic modes of the beam coupled to its substrate $\rightarrow$ scattering modes.
- d/L provides a natural small parameter.

Theory

Abrupt junctions with supports



- Derivation from elasticity theory ($\lambda \gg a$) of phononic modes of the beam coupled to its substrate $\rightarrow$ scattering modes.
- d/L provides a natural small parameter.
- Closed system $\leftrightarrow$ open system representations related by canonical transformation $\rightarrow$ reduction of $I(\omega)$ to overlaps between scattering modes and resonator mode.
- General expressions for $I(\omega)$ in terms of: (*i*) support DOS or (*ii*) transmission at beam-support junction.

Spectral densities and Q-values for beams

	General Relations (3D)		Monolithic structure ($\sigma = \frac{1}{3}$)	
	$I_n(\omega)$	Q_n	$Q_n(L, \mathrm{w}, \mathrm{t})$	Typical value for 150MHz
Compression	$\frac{\omega}{Q_n}$	$\frac{\rho_s c_t^3 L}{2\delta \mu_b \tilde{c}_c^3 \tilde{u}_c(\alpha) k_n}$	$\frac{0.88}{\delta} \frac{L^2}{\pi \mathrm{wt}} \frac{1}{n + \frac{\delta}{2}}$	3.2×10^4
Torsion	$\frac{\omega^3}{Q_n \omega_n^2}$	$\frac{\rho_s c_t^5 L}{2\delta \mu_b \langle r^2 \rangle \tilde{c}_t^5 \tilde{u}_t(\alpha, \gamma_z) k_n^3}$	$\frac{4.1}{\delta} \frac{\mathrm{w}^2 L^4}{\pi^3 \mathrm{t}^6} \frac{1}{(n + \frac{\delta}{2})^3}$	7.6×10^9
Vertical bending	$\frac{\omega}{Q_n}$	$\frac{\rho_s c_t^3 L}{4\delta C_n \mu_b \tilde{c}_v^3 \tilde{u}_v(\alpha) k_n^4}$	$\frac{3.9}{\delta C_n} \frac{L^5}{\pi^4 \mathrm{wt}^4} \left(\frac{3\pi}{2k_n L}\right)^4$	9.6×10^5
Horizontal bending	$\frac{\omega}{Q_n}$	$\frac{\rho_s c_t^3 L}{4\delta C_n \mu_b \tilde{c}_h^3 \tilde{u}_h(\alpha) k_n^4}$	$\frac{3.9}{\delta C_n} \frac{L^5}{\pi^4 \mathrm{tw}^4} \left(\frac{3\pi}{2k_n L}\right)^4$	3.9×10^5

- 2D slab supports $\longrightarrow$ $1/f$ noise: $I_{n,\beta \neq v} = \frac{\omega_n}{Q_n}, \; I_{n,v} = \frac{\omega_n^2}{Q_n \omega}$.
- Bridge with single support $\rightarrow$ interference effects can modify the scalings of Q and frequency dependence of $I(\omega)$.

Q-solver for generic geometry

Spectral density:

$$I(\omega) \approx \frac{\pi}{2\rho_s \rho_R \omega_R \omega} \int_q \left| \int_S \mathrm{d}\bar{S} \cdot \left(\boldsymbol{\sigma}_q \cdot \bar{u}'_R - \boldsymbol{\sigma}'_R \cdot \bar{u}_q \right) \right|^2 \delta[\omega - \omega(q)]$$

Master formula for dissipation:

$$\frac{1}{Q} \approx \frac{\pi}{2\rho_s \rho_R \omega_R^3} \int_q \left| \int_S \mathrm{d}\bar{S} \cdot \left(\boldsymbol{\sigma}_q^{(0)} \cdot \bar{u}'_R - \boldsymbol{\sigma}'_R \cdot \bar{u}_q^{(0)} \right) \right|^2 \delta[\omega_R - \omega(q)]$$

valid for isolated resonance and $kd \ll 1$,
resonator mode satisfies:

- non-singular $S \rightarrow 0 \implies$ free BC
- singular $S \rightarrow 0 \implies$ clamped BC

and support modes the converse.

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(i) Pedestal geometries: contact area $\parallel$ substrate, e.g. microspheres, microdisks, microtoroids, micropillars.

(ii) Planar geometries: contact area $\perp$ substrate, e.g. beams, in-plane modes of plates, flexural modes of plates.

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Fermi golden rule:

- free BC $\rightarrow H_{\mathrm{int}} = \int_S \mathrm{d}\bar{S}\cdot\boldsymbol{\sigma}_>\cdot\bar{u}_<$
- clamped BC $\rightarrow H_{\mathrm{int}} = -\int_S \mathrm{d}\bar{S}\cdot\boldsymbol{\sigma}_<\cdot\bar{u}_>$

- More general than "small contact area" $\rightarrow$ valid for generic high-Q resonance provided $|\Delta\omega_R|/\omega_R \ll 1$.
- Reduction to decoupled resonator mode + *free* wave propagation in the substrate.

Q-solver for generic geometry

Master formula for dissipation:

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$\downarrow$

Flexural modes of symmetric plate geometry inscribed in circle.

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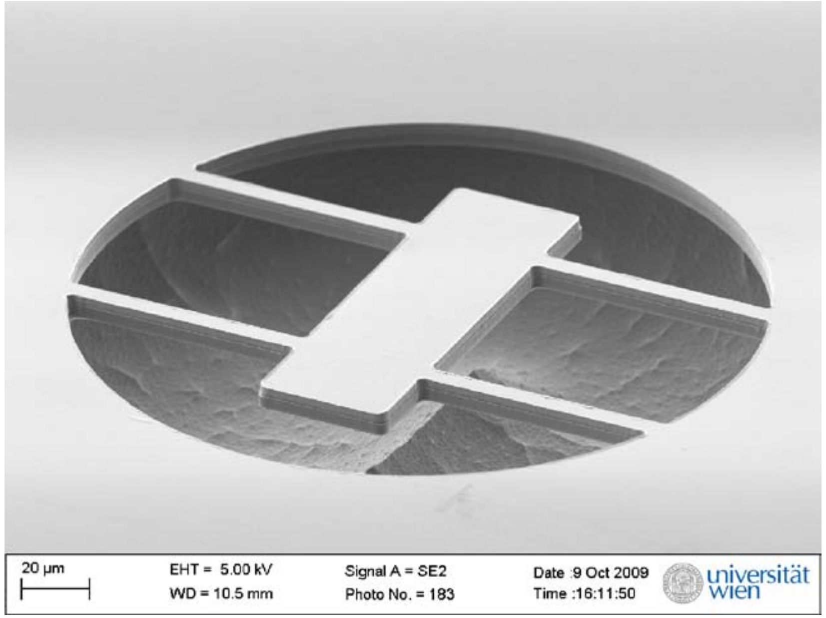
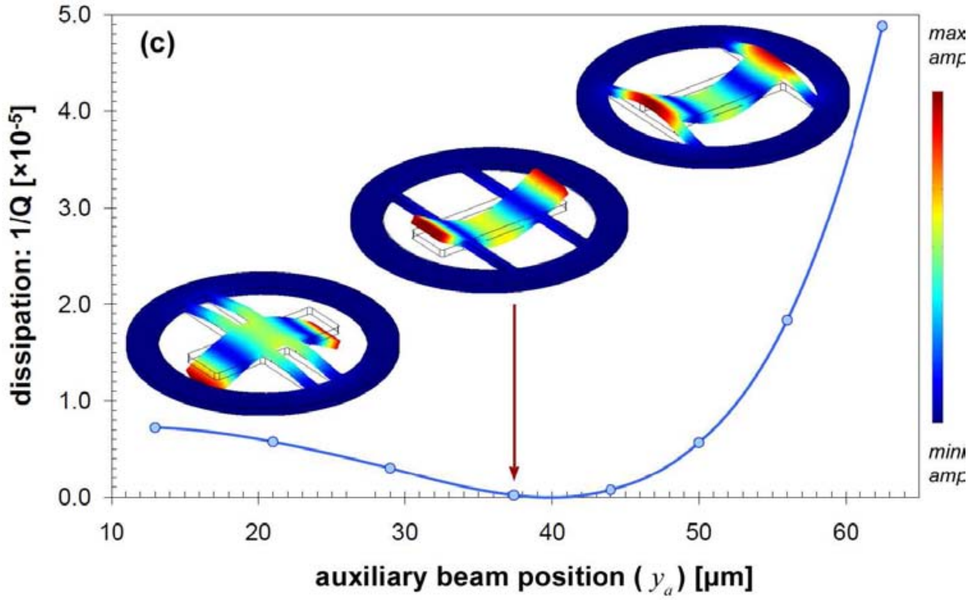
Q-solver: use FEM for resonator mode and cylindrical modes for support $\sim$ substrate $\sim$ $1/2$-space (with Garrett Cole).

"Free-free" DBR-micromirror structures

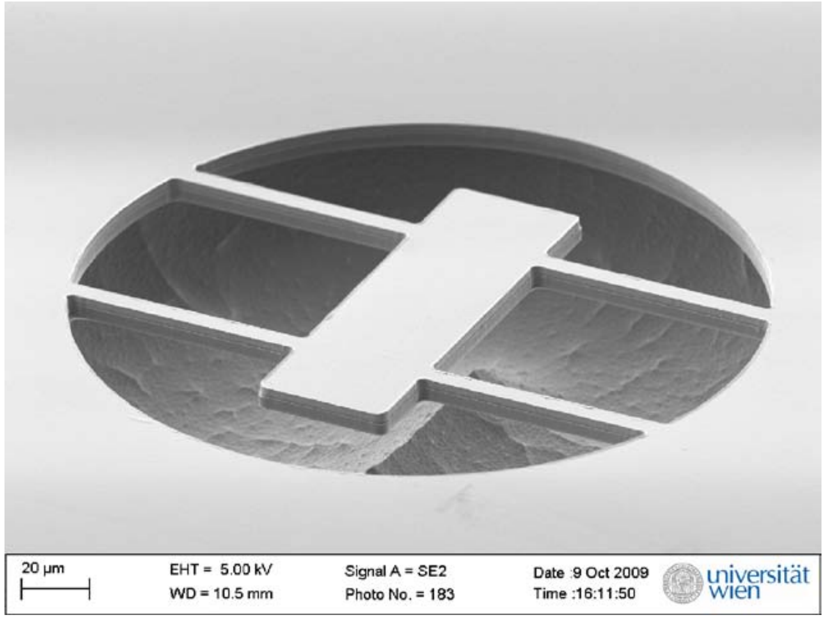


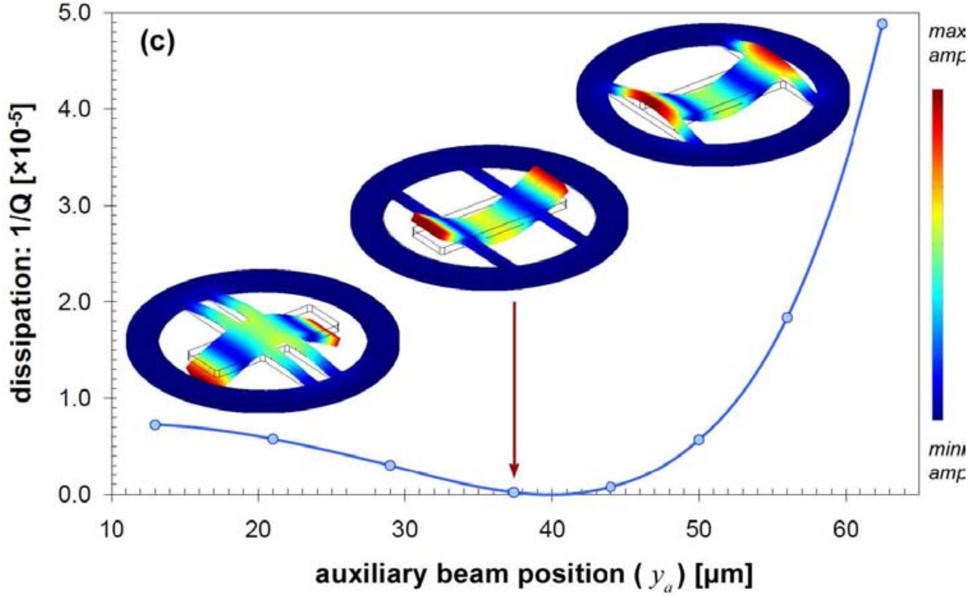


High-reflectivity DBR-micromirrors as used in optomechanical Fabry-Perot setups ($Q_{\mathrm{c-c}} \sim 10^3$).

Auxiliary beams act as noise filters provided their resonances are avoided — for generic positioning $Q_{\mathrm{f-f}}/Q_{\mathrm{c-c}} \sim (\mathrm{w}/\mathrm{w_s})^2$.

“Free-free” DBR-micromirror structures

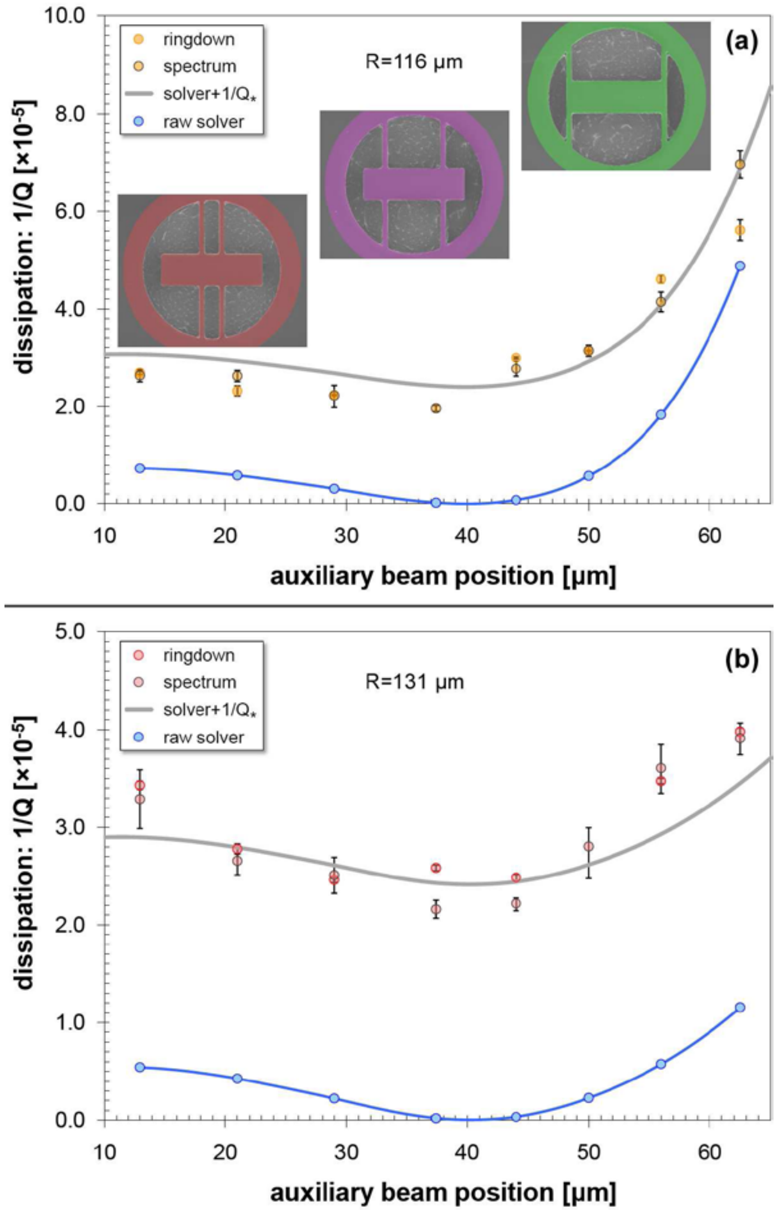




High-reflectivity DBR-micromirrors as used in optomechanical Fabry-Perot setups ($Q_{\mathrm{c-c}} \sim 10^3$).

Approximately preserves ω_0, S_{eff} and V_{eff}
$\longrightarrow$ “Free-free” design provides
a tool to study impact of geometry on Q.

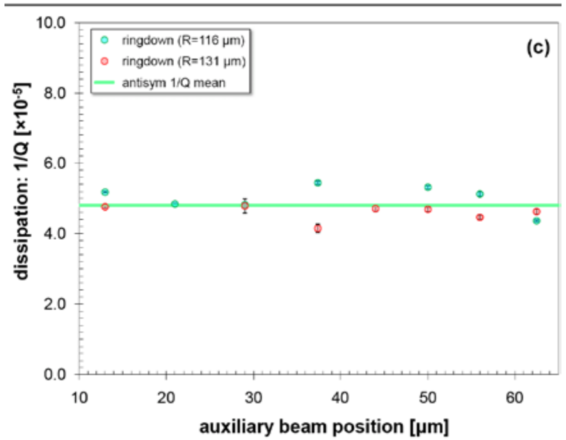
“Free-free” DBR-micromirror structures



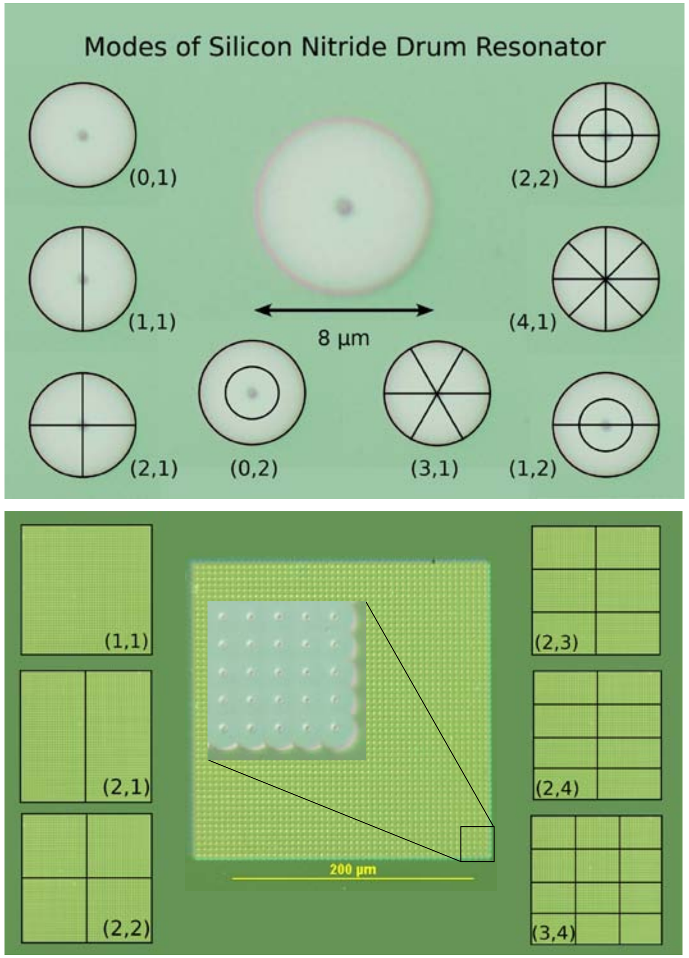
Single fit parameter:
“internal” dissipation offset

- $\Delta Q_{\mathrm{exp}}/Q_{\mathrm{exp}} \sim 260\%$ ($\sim 80\%$) vs. $\Delta f/f \sim 20\%$ ($\sim 10\%$) ($f \approx 2$MHz).
- At nodal positioning $Q_{\mathrm{th}} \approx 5 \times 10^6$ vs. $Q_{\mathrm{c-c}} \sim 10^3$.

Antisymmetric mode:



Stressed membranes



- Dispersive optomechanics
- Stress reduces internal dissipation (e.g. two-level systems)

$$\Downarrow$$

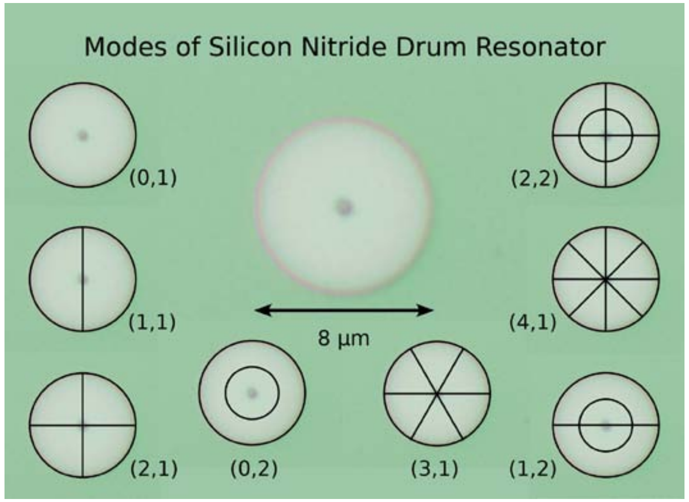
What is the effect of stress on clamping losses?

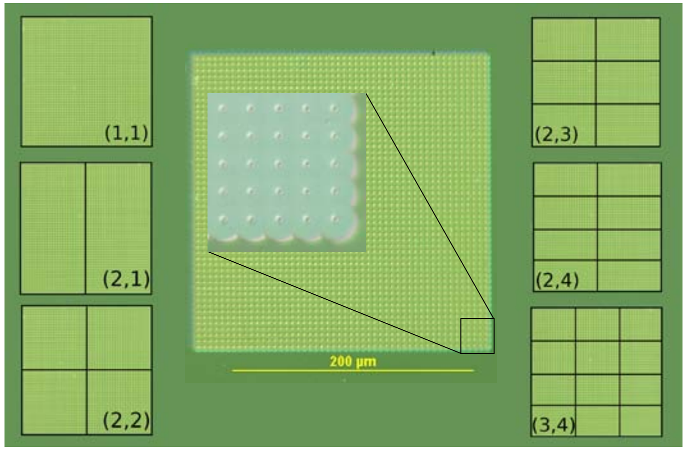
Assume "high stress" regime
$t^2/D^2 \ll \sigma/E_R \ll 1$

Drums:

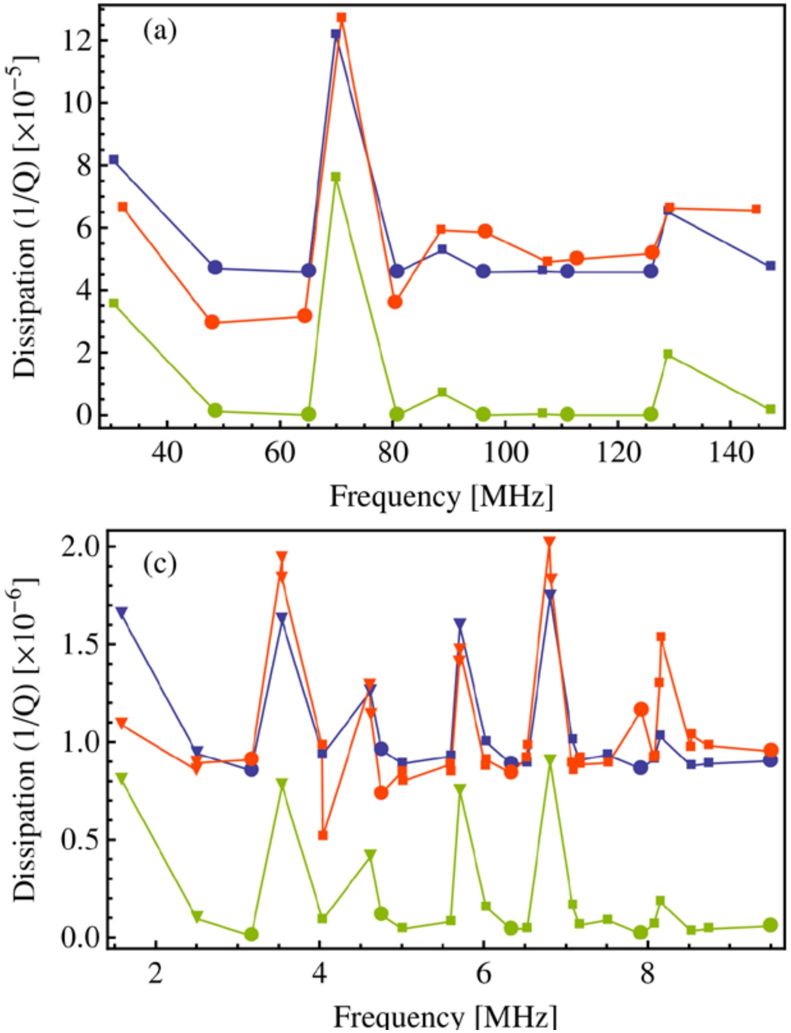
$$\frac{1}{Q_{nm}} = \frac{4\pi^2 \zeta_{nm} \rho_R t}{\rho_s D} \sum_{\gamma} \eta_\gamma^3 \tilde{u}_{n,\gamma}$$

Stressed membranes





Dissipation for different harmonics of a given resonator.



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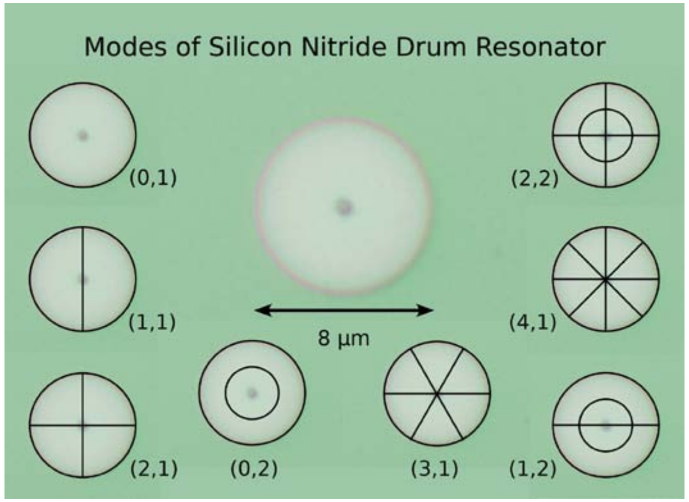
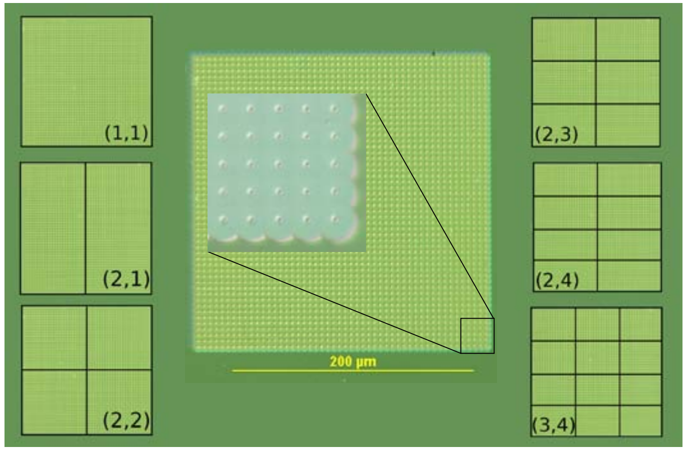
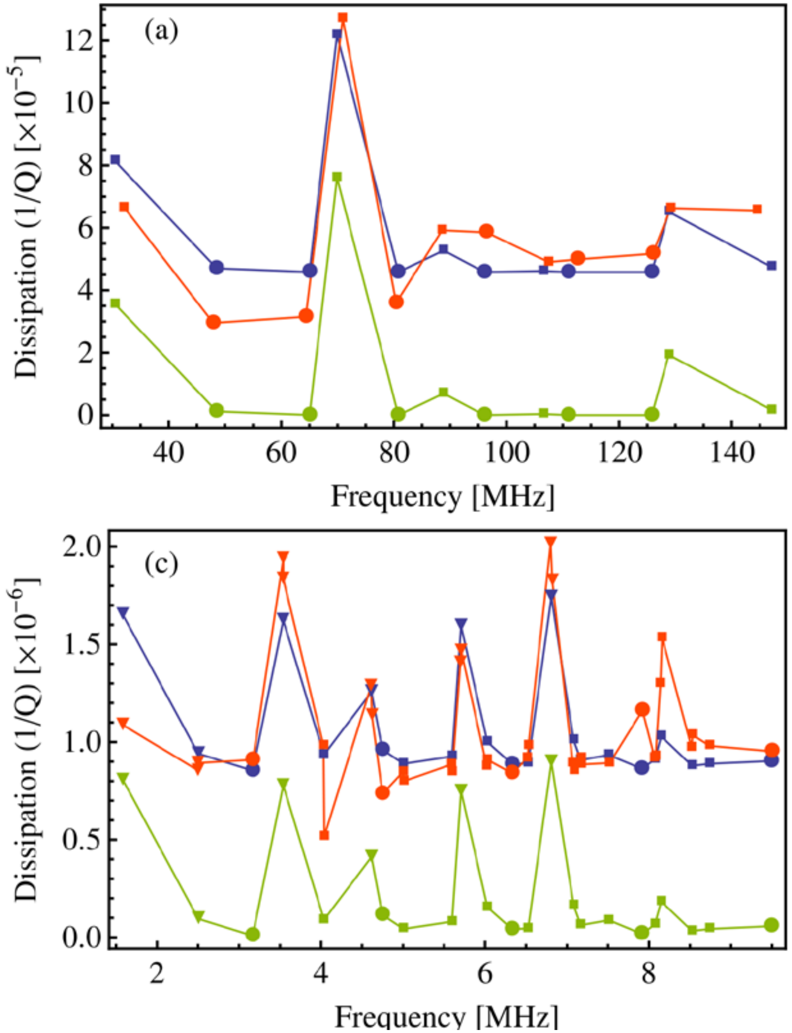


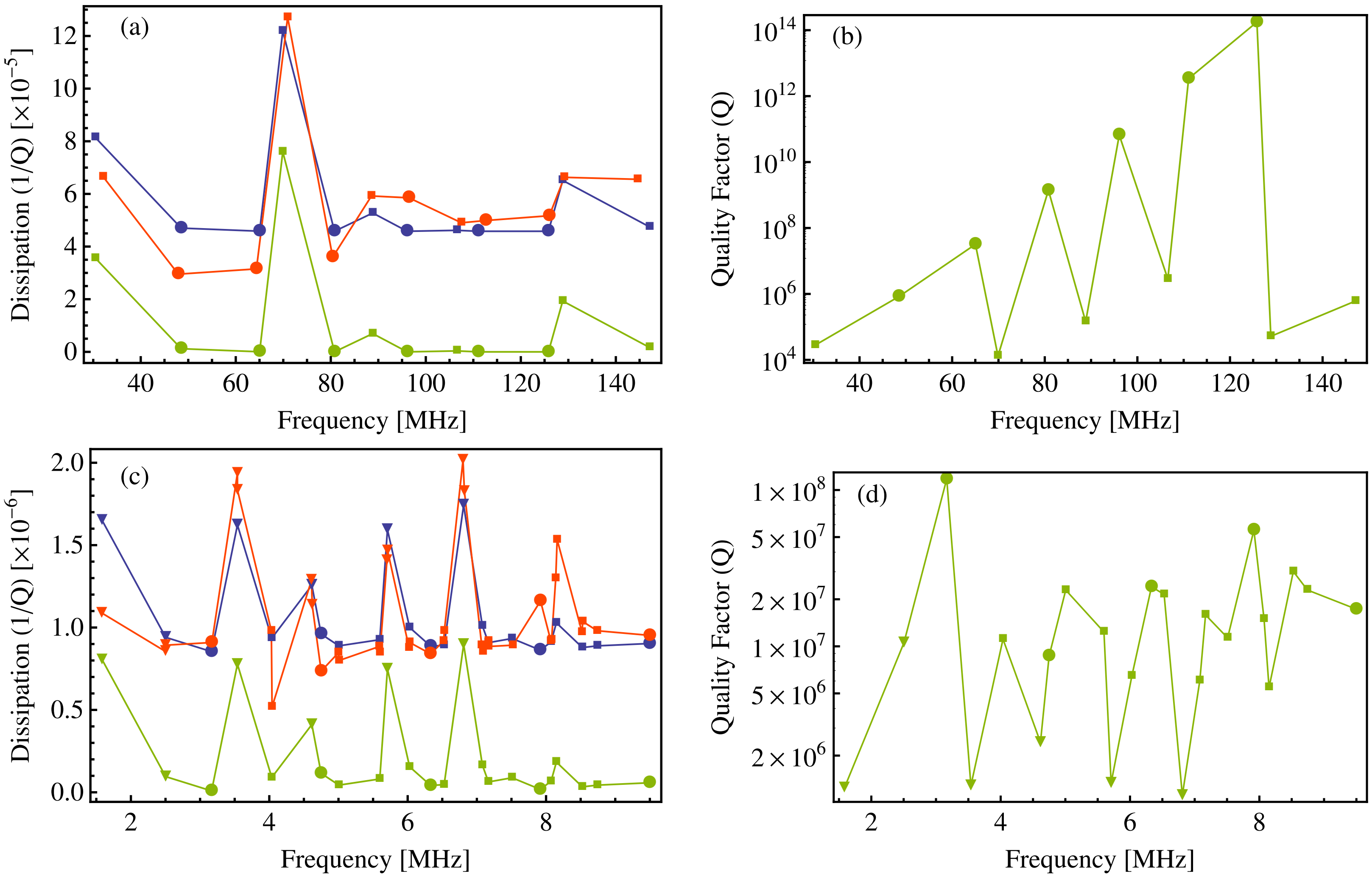




Square ($D = 252.3\mu$ t $= 12.5$nm): $E_s = 148$ GPa, $\rho_s = 3.75$gcm^{-3}, $1/Q_{\text{int}} = 8.5{\times}10^{-7}$.

Drum ($D = 14.5\mu$ t $= 110$nm): $E_s = 323$ GPa, $1/Q_{\text{int}} = 4.6 \times 10^{-5}$.

Stressed membranes



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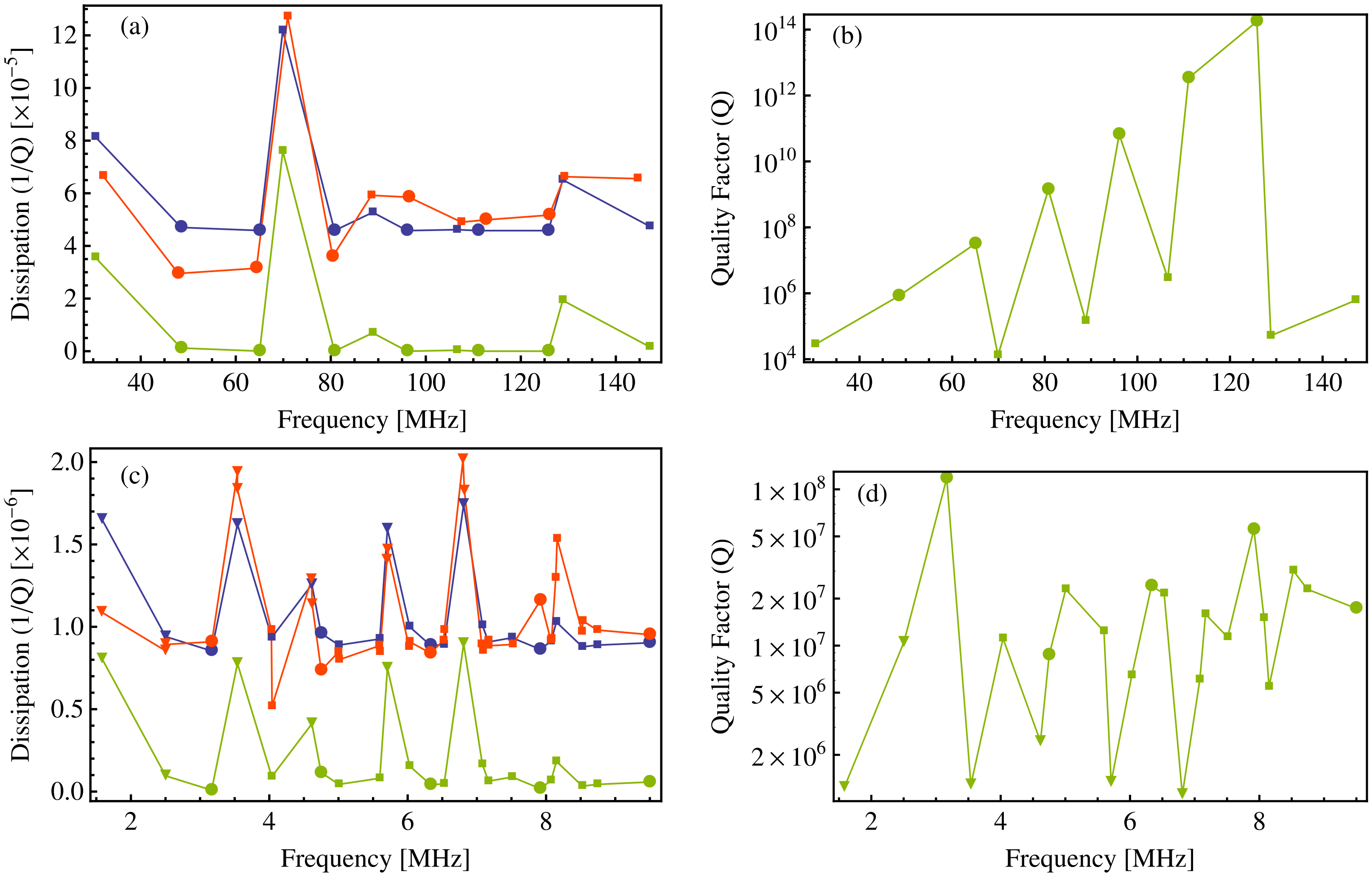
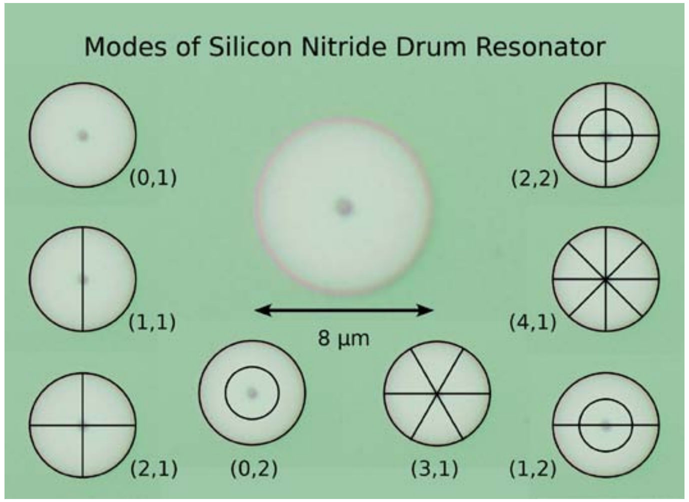
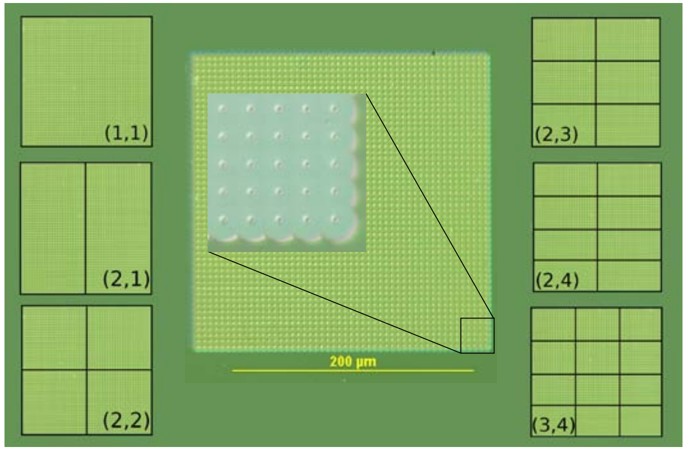
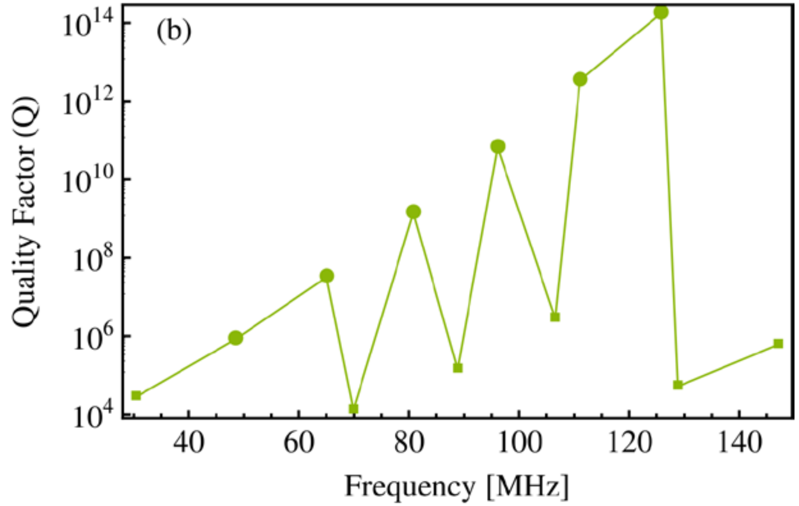


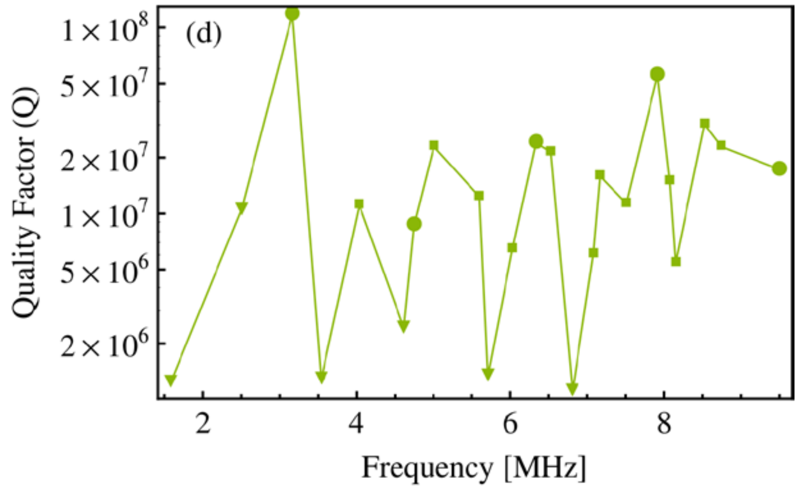
"Special" classes of high-Q harmonics $(n, 1)|_{n>0}$ for drum and $(n, n)|_{n>1}$ for square (circles) $\longrightarrow$ nodal lines intersect periphery at evenly spaced points [$fQ = 1.0 \times 10^{13}$Hz for $(6, 6)$].

Destructive interference effects









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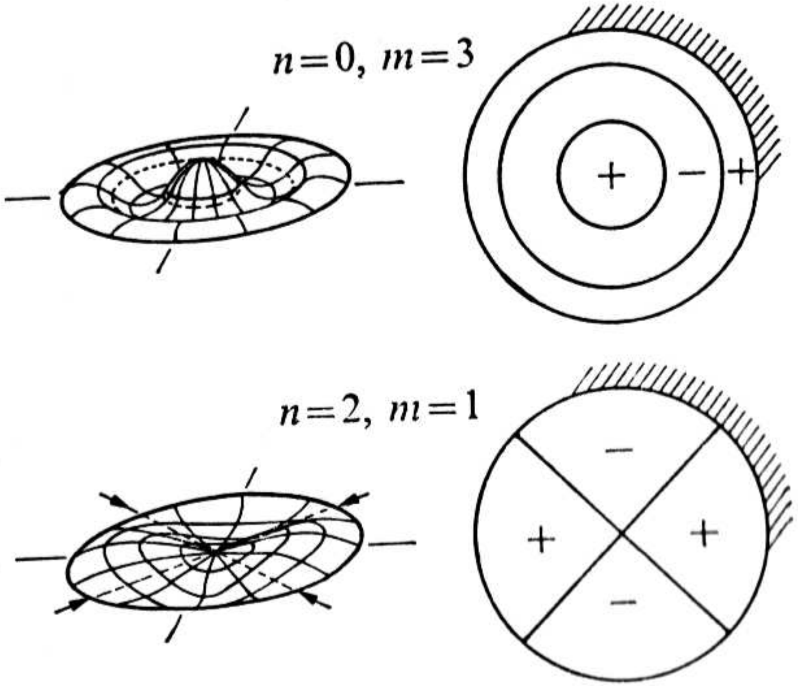
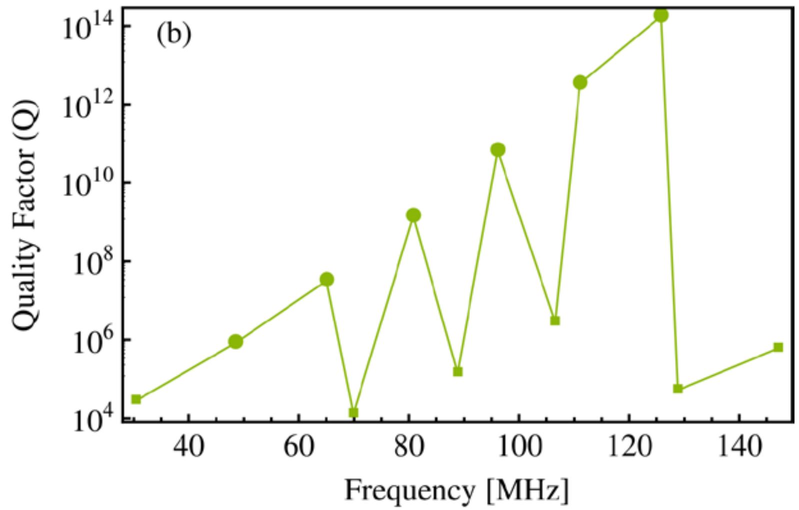
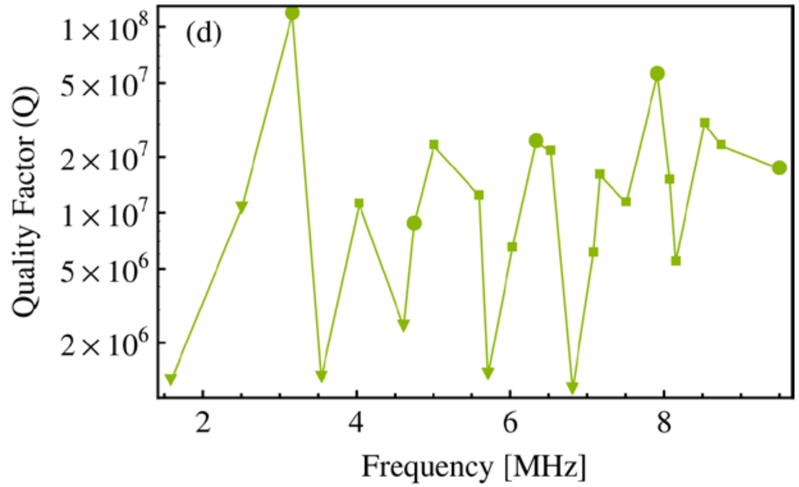






$c_R/c_\gamma \sim \sqrt{\sigma \rho_s / E_s \rho_R} \ll 1 \rightarrow$ resonant λ in substrate $\gg D$.

Destructive interference effects

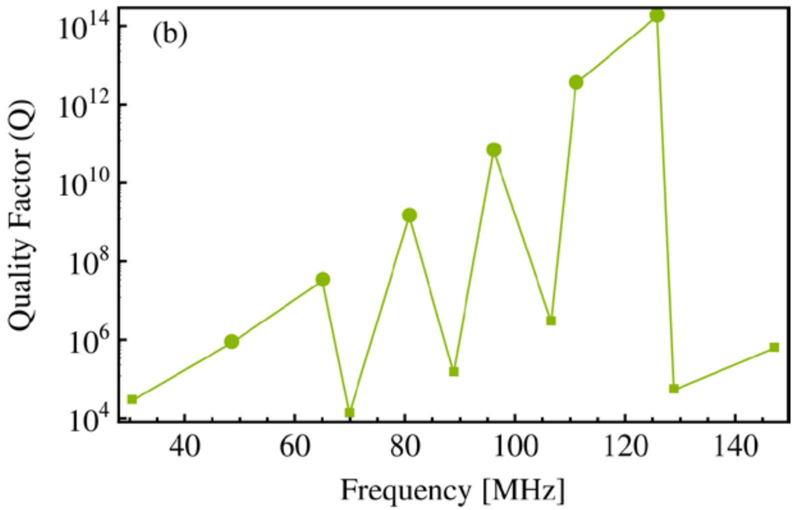
Analytical approximations for drum:

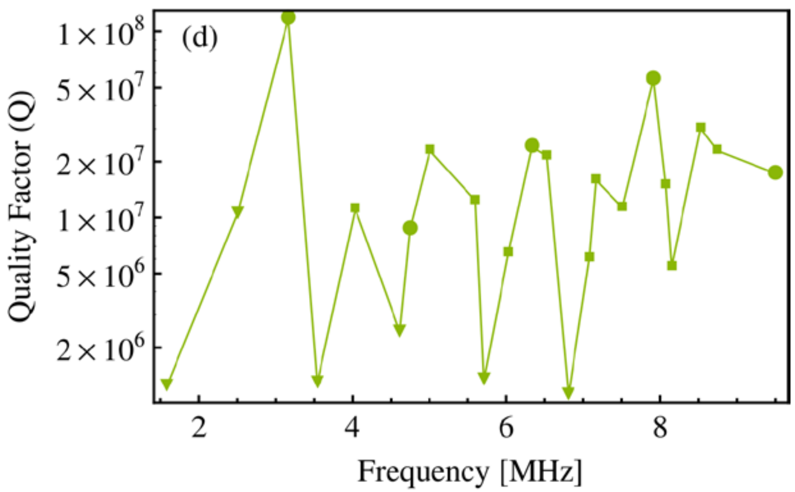
$$Q_{01} \approx \left. \frac{\rho_s c_t^3}{2\pi^2 \sigma_R c_R^2 \omega_{01} \tilde{u}_0(\nu_s)} \right|_{\nu_s = 1/3}$$

$$= 0.029 \sqrt{\frac{\rho_R}{\rho_s} \left(\frac{E_s}{\sigma} \right)^3} \frac{D}{t}$$

$$\frac{Q_{n1}}{Q_{01}} \sim n^{\frac{2n+1}{16}} \left(0.517 \frac{c_s}{c_R} \right)^{2n},$$

$$\frac{Q_{nm}}{Q_{n1}} \approx \left(\frac{\zeta_{n1}}{\zeta_{nm}} \right)^{2n+1}.$$





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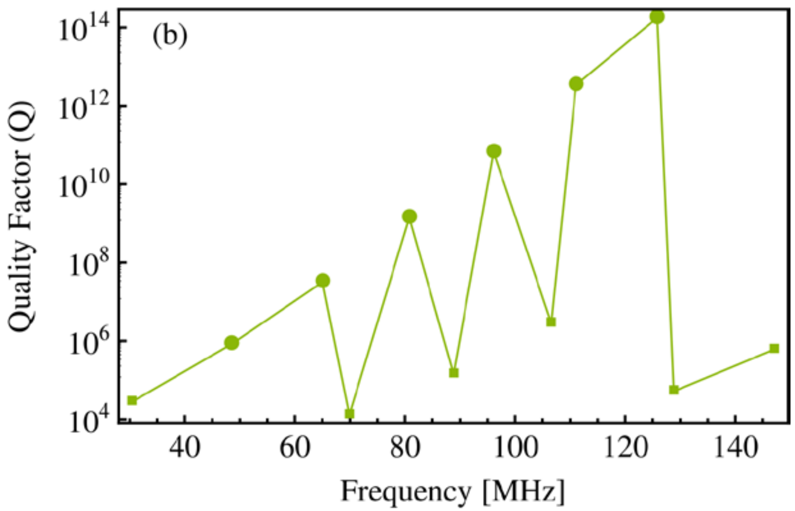

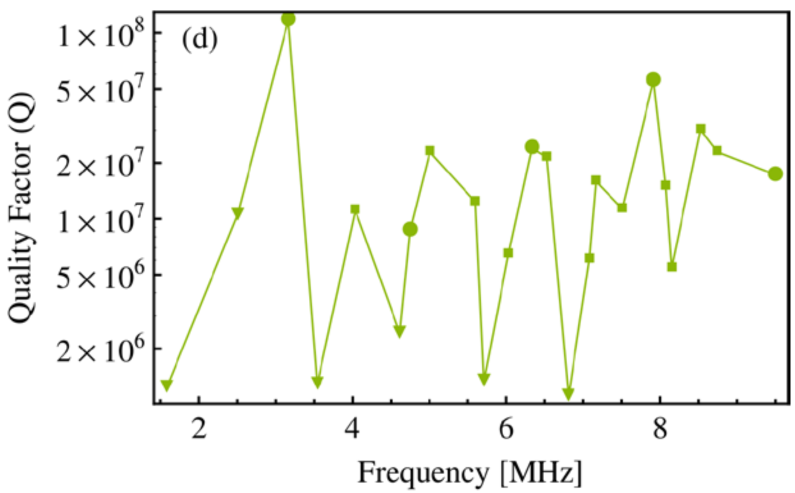

For square: damping rate → constant as n, m are increased with fixed ratio.

Destructive interference effects

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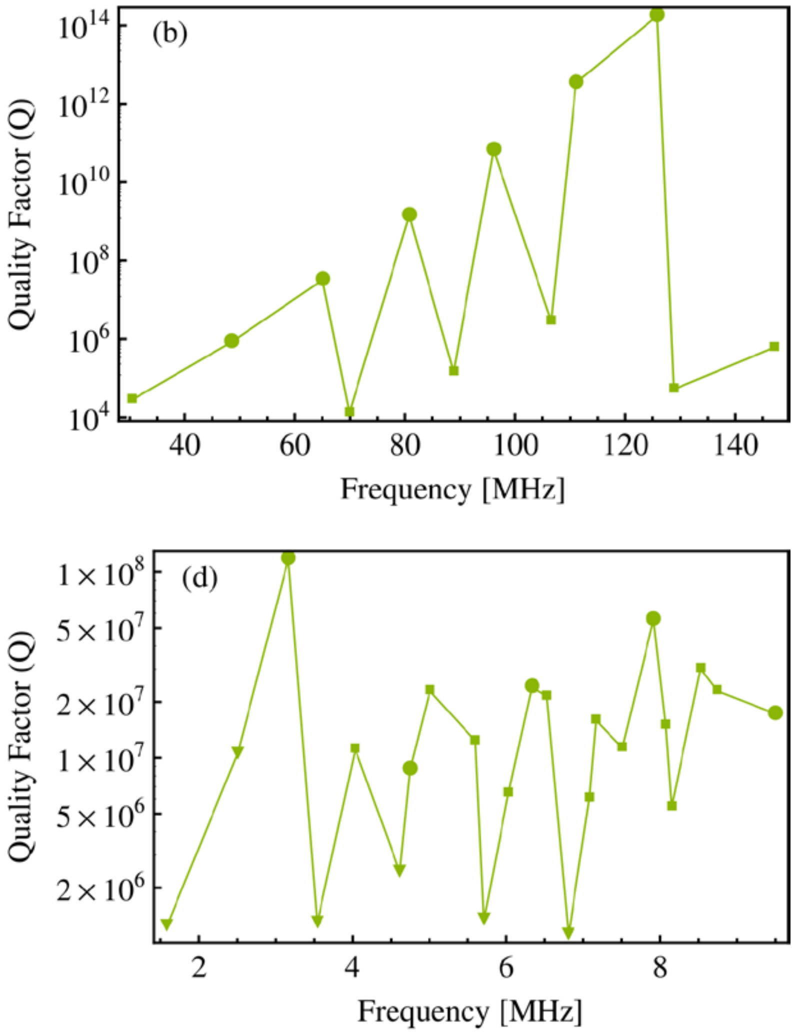
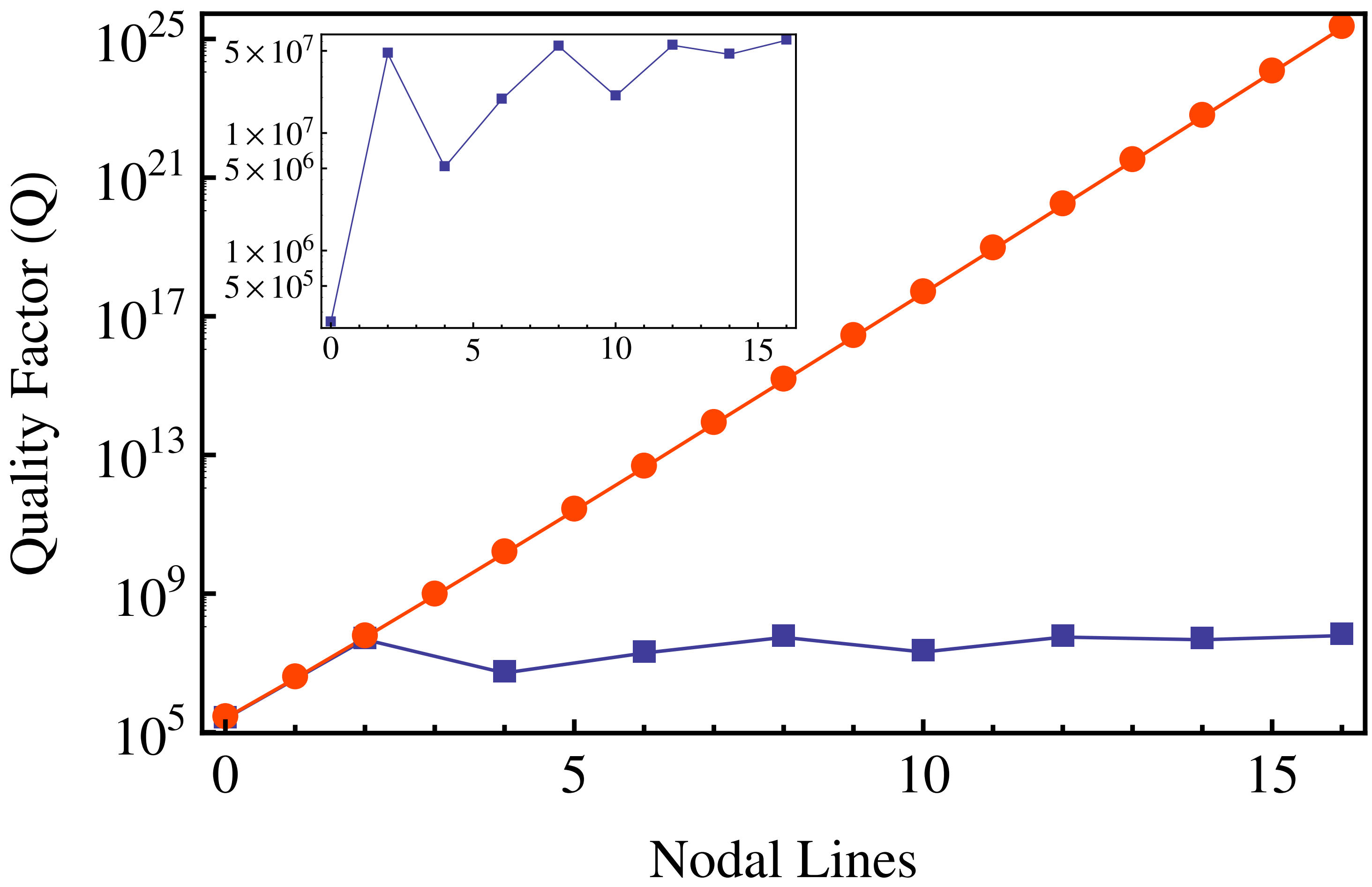
$$\frac{Q_{n1}}{Q_{01}} \sim n^{\frac{2n+1}{16}} \left(0.517 \frac{c_s}{c_R}\right)^{2n},$$

$$\frac{Q_{nm}}{Q_{n1}} \approx \left(\frac{\zeta_{n1}}{\zeta_{nm}}\right)^{2n+1}.$$



Azimuthal harmonics exhibit exponential suppression of phonon radiation → "asymptotically mute", e.g. $fQ_{\mathrm{clamp}} \gtrsim 10^{18}$Hz for $n \geq 6$ and thickness $t < 250$ nm.

Circle vs. square



Q-value of the nth harmonic in the series $(n, 1)|_{n\geq 0}$ $[(n, n)|_{n\geq 1}]$ for dimensions $D_n^{(c)} = (\zeta_{n1}/\zeta_{01})D_0^{(c)}$ $[D_n^{(s)} = nD_0^{(s)}]$, $D_0^{(c)} = \sqrt{2}\zeta_{01}D_0^{(s)}/\pi$ with $D_0^{(s)} = 50\,\mu\text{m} \implies f_n = 7.99\,\text{MHz}$.

Collaborators

“Free-free” geometries:

Garrett Cole, Katharina Werbach, Michael R. Vanner, and Markus Aspelmeyer (IQOQI and U. Vienna).

arXiv:0083230

Stressed membranes:

Rob Barton, Darren Southworth, Scott Verbridge, Rob Ilic, Harold Craighead, and Jeevak Parpia (Cornell).

Conclusions

- Clamping-loss limited Q-values have a strong geometric character.
- Generic Q-solver for guiding design optimization — quantitative understanding of support engineering.
- Mapping out of mechanical mode shape provides neat experimental test of the theory.

arXiv:0083230

- Striking non-monotonic behavior of membrane harmonics successfully explained.
- Destructive interference effects can lead to effectively clamping-loss free harmonics (requires 2D geometry).
- Stress increases the clamping loss.
- Typical optomechanical setups are limited by clamping loss.