A phonon-tunneling approach to support-induced dissipation of nanomechanical resonators

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State of the art optomechanical and nanomechanical setups are close to allowing for the observation of quantum effects in a "macroscopic" mechanical system. A major challenge that remains to be addressed is understanding and controlling mechanical dissipation in these systems. Here we analyze the dissipation mechanism induced by the unavoidable coupling of the resonator to the substrate (known as clamping losses). We follow a Hamiltonian treatment and derive the Caldeira-Leggett model that determines the quantum Brownian motion of a given resonance starting from the elastic scattering eigenmodes of the entire structure including the substrate. Our "phonon tunneling" approach provides the leading contribution in the aspect ratio or, more generally, in $k_R d$, where $1/k_R$ is the characteristic length scale over which the resonator mode varies appreciably and d the characteristic dimension of the contact area from which the resonator is suspended. It leads to a "master formula" for the dissipation $1/Q$ that is applicable to a very wide range of high-Q resonators including planar structures (e.g. bridges), pedestal geometries (e.g. microdisks), and single-walled carbon nanotubes (CNT). The resulting limits for the Q-values have a strong geometric character.

Based on this master formula for the dissipation, we have developed an efficient FEM-enabled numerical method that can be used as an aid to design in order to minimize the clamping loss of complex geometries. We apply this concept to free-free micromirror structures relevant for Fabry-Perot based optomechanics. In addition, this design allows to isolate support-induced losses from other dissipation channels. Thus, we perform a rigorous test of the theory developed and demonstrate the strong geometric dependence of this loss mechanism. Furthermore, we analyze the case of high-stress nanomechanical resonators and test the theory on Si_3N_4 membranes with circular and square geometries. The Q-values of different harmonics present a striking non-monotonic behavior which is successfully explained. For the circular geometry we identify a class of modes for which destructive interference of the radiated waves leads to an exponential suppression of the damping rate as the harmonic index is increased, rendering these modes effectively clamping-loss free. This may provide a route towards ultra-high-Q nanomechanics and is directly relevant to dispersive optomechanical setups utilizing a thin membrane.

Beyond enabling reliable predictions for the design-limited Q, our approach allows to determine the spectrum of the environment. This is relevant to analyze the quantum regime of scenarios where the resonator is strongly coupled to an anharmonic system or exhibits strong nonlinearity. We find that a cantilever geometry coupled to a 3D support presents superohmic behavior for the torsional resonances and ohmic behavior otherwise, while coupling to a 2D support can induce $1/f$ noise. However, for more general geometries the environmental noise experienced by a given resonator mode may not be captured by a power law spectral density, presenting structure on a scale comparable to the resonant frequency.