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Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Nano-Electro-Mechanics of Superconducting Weak Links

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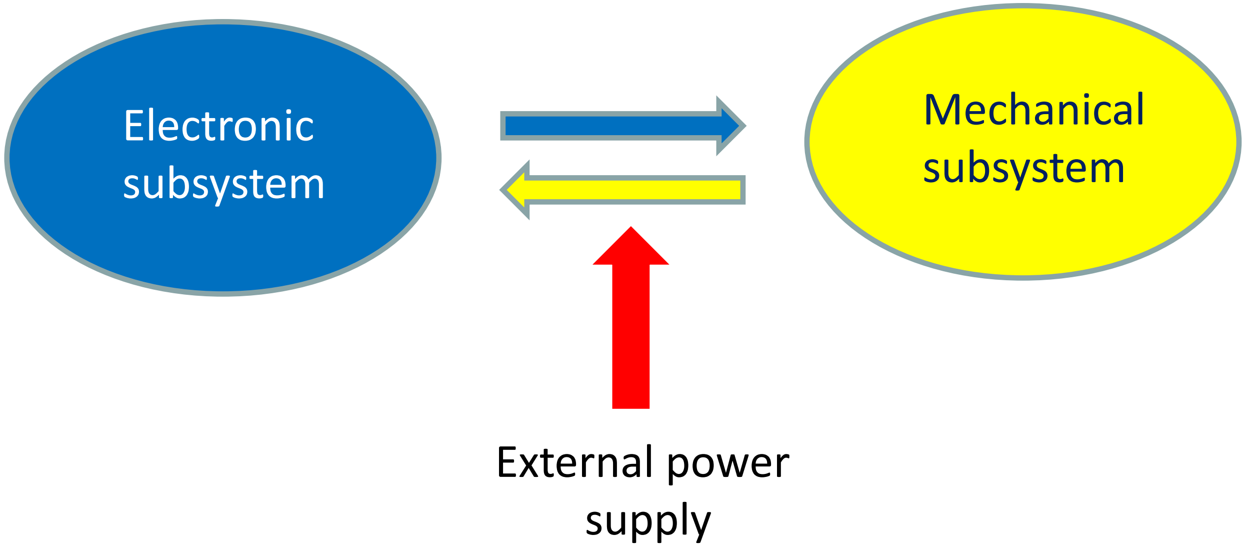
ICTP, Trieste, Sept. 6-10, 2010

Energy Transfer in NEM Systems



Two Examples of Controllable NEM Energy Transfer

I. Single Electronic Shuttle Devices

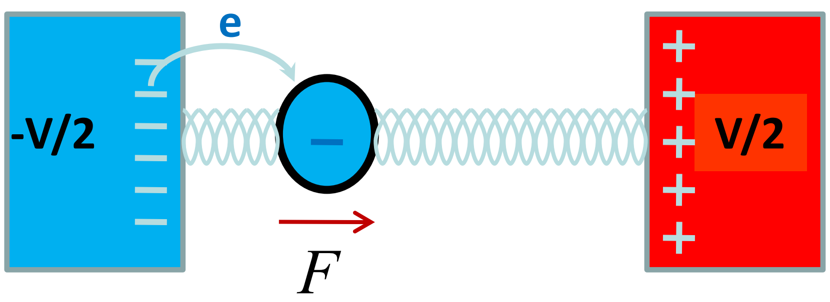
II. NEM Superconducting Weak Links

I. F. Santandrea et al.: Cooling of the vibrations of a dc-biased nanomechanical resonator determined by quantum interference effects.

II. G.Sonne et al.: Cooling of a suspended carbon nanotube by an ac Josephson current flow.

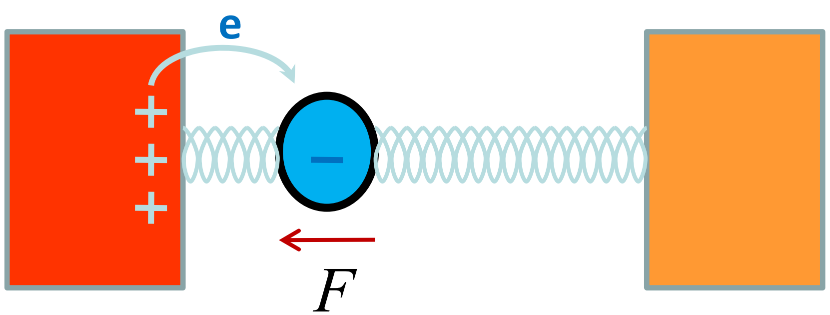
Shuttling in Coulomb Blockade Device

$|V| > V_c$



$|V| < V_c$

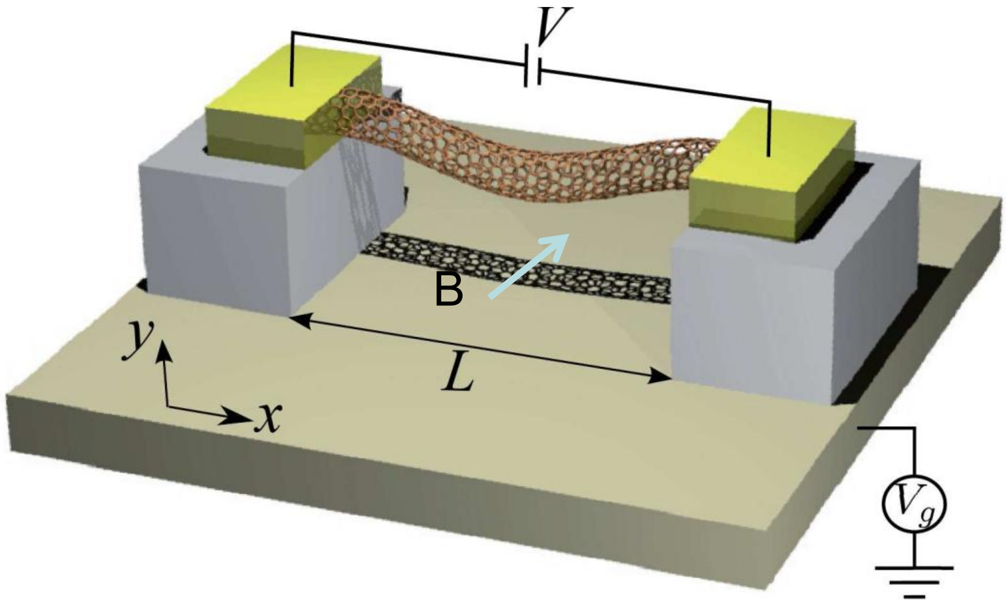
Thermally activated transport



Nano-Electro-Mechanics of Superconducting Weak Links

- Superconducting weak link as NEM device
- Superconductive pumping of nanovibrations
- NEM-induced superconductive cooling of mechanical resonator
- Conclusions

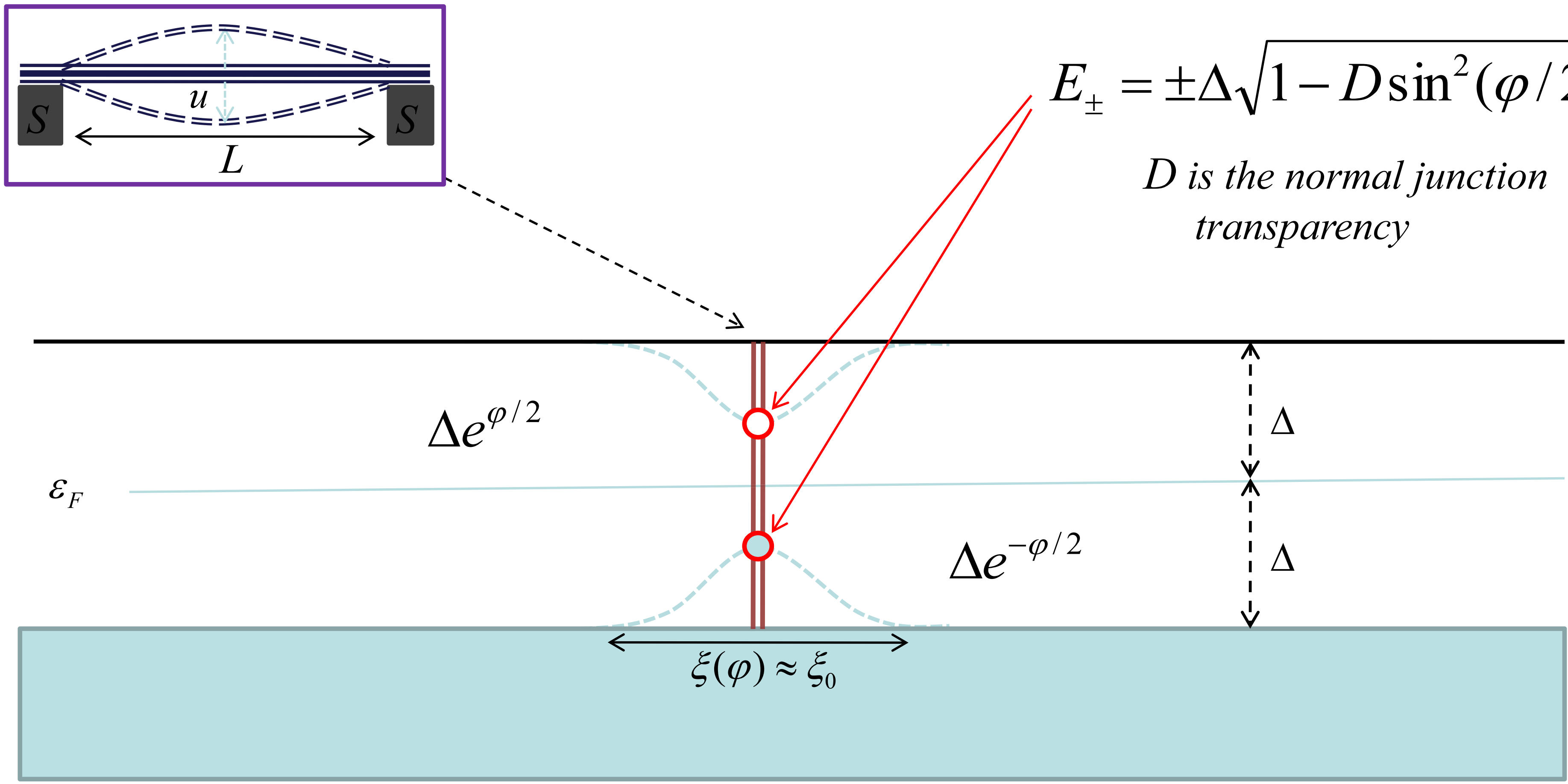
Suspended Nanowire-Based Superconducting Weak Link



Andreev States

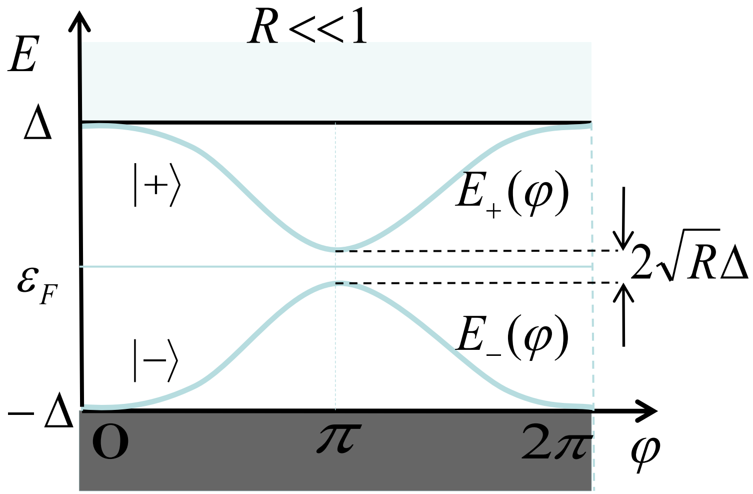
$$E_{\pm} = \pm\Delta\sqrt{1 - D\sin^2(\varphi/2)}$$

D is the normal junction transparency

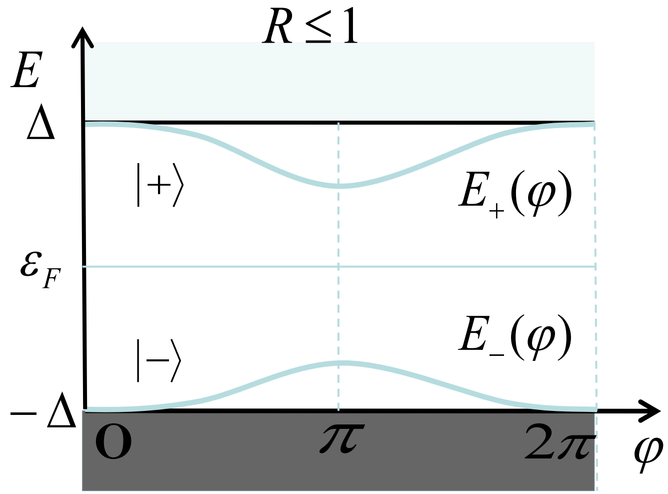


$$\xi_0 = \frac{\hbar v_F}{\Delta}$$

Effect of Backscattering

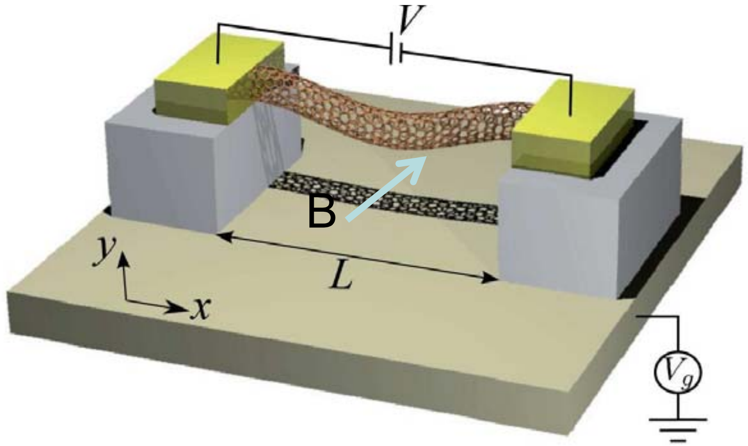


Nearly ballistic weak link:
(cooling of nanovibrations)



Josephson weak links:
(pumping of nanovibrations)

Josephson Weak Links (D<<1)

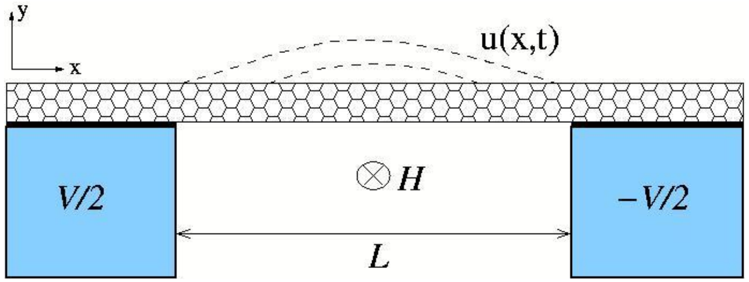


Superconductive Pumping of Nanovibrations

G.Sonne et al. Phys.Rev. B **78**, (2008)

Supercurrent-Driven Nanomechanics

Model: Driven, damped nonlinear oscillator
G. Sonne et al. PR B **78**, 144501 (2008)



$$m\ddot{u} + \gamma\dot{u} + ku = \underline{HLJ_c \sin(\varphi)}$$

Driving Lorentz force

$$\dot{\varphi} = (2eV/\hbar) + (2e[\underline{-2HL\dot{u}}]/\hbar)$$

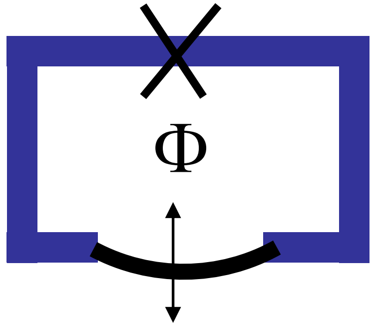
Induced el.motive force

$$\underline{j_{dc}V = \gamma\langle \dot{u}^2(t)\rangle}$$

Energy balance in stationary regime
determines time-averaged dc supercurrent

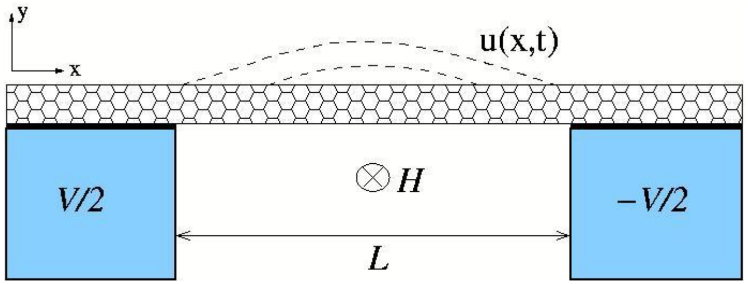
Compare:

NEM resonator as part of a SQUID



Buks, Blencowe PRB 2006
Zhou, Mizel PRL 2006
Blencowe, Buks PRB 2007
Buks et al. EPL 2008

Giant Magnetoresistance



V ⟶
Alternating Josephson current ⟶
Alternating Lorentz force, F_L ⟶
Mechanical resonances

$$F_L = HLI_c \sin(2eVt/\hbar - 4eHLu(t)/\hbar) \approx$$

$$\underbrace{HLI_c \sin(2eVt/\hbar)}_{(I)} - \underbrace{[4eH^2L^2I_c/\hbar]\,u(t)\cos(2eVt/\hbar)}_{(II)}$$

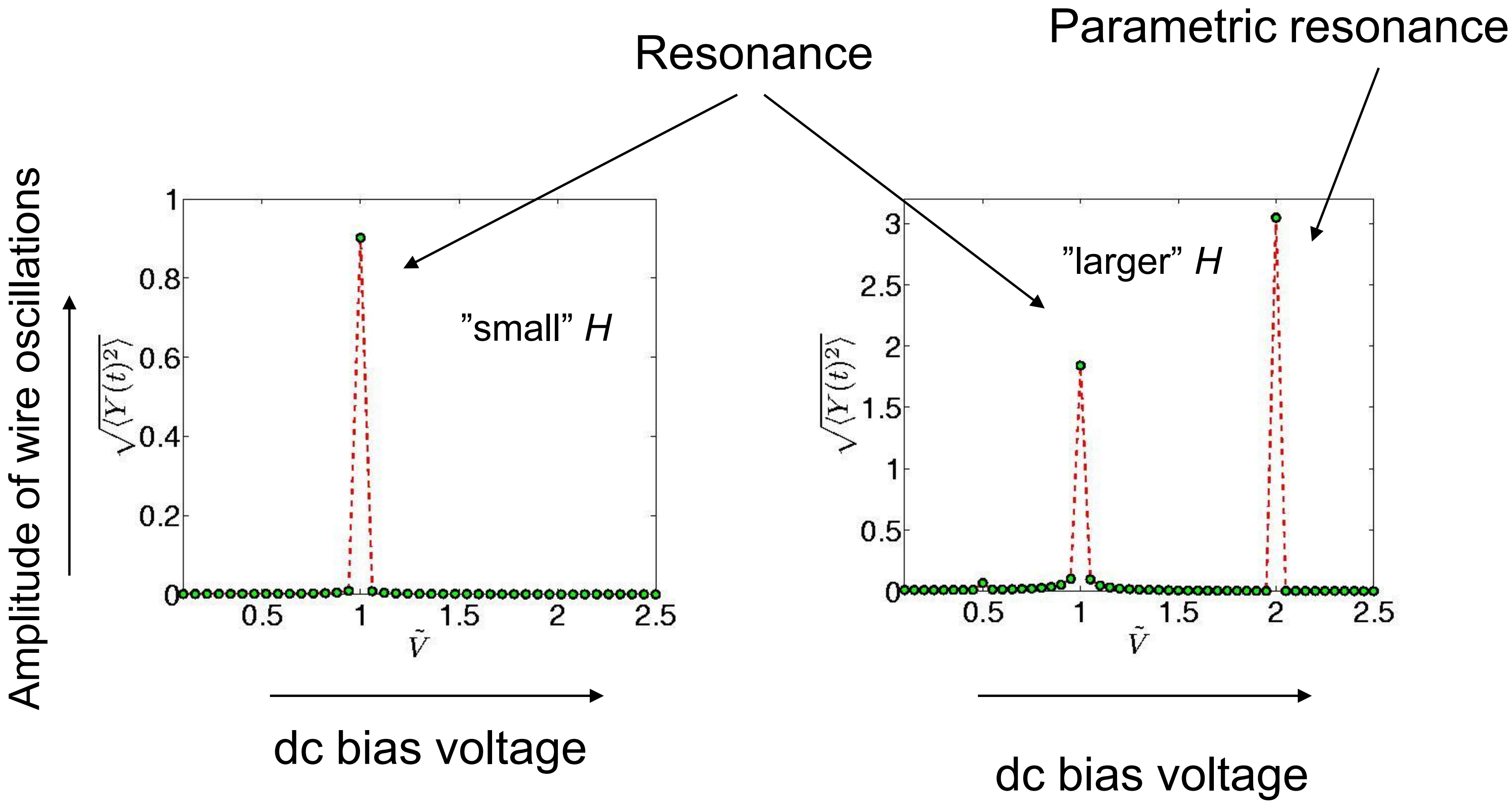
Force (I) leads to resonance at $2eV/\hbar = \omega$

Force (II) leads to parametric resonance at $2eV/\hbar = 2\omega$

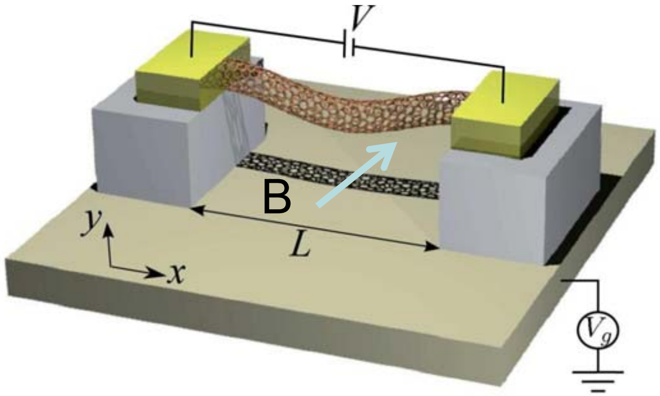
Accumulation and dissipation of a finite amount of energy during each nanowire oscillation period means that $W = V\langle I(t)\rangle \neq 0$ and therefore a nonzero average (dc) supercurrent on resonance

Giant Magnetoresistance

The onset of the parametric resonance depends on magnetic field *H*. By increasing H the resistance $R = V / \langle j(t) \rangle$ jumps from $R = \infty$ to a finite value.



Nearly Ballistic Superconducting NEM Weak Link



Superconductive Cooling of Nanovibrations

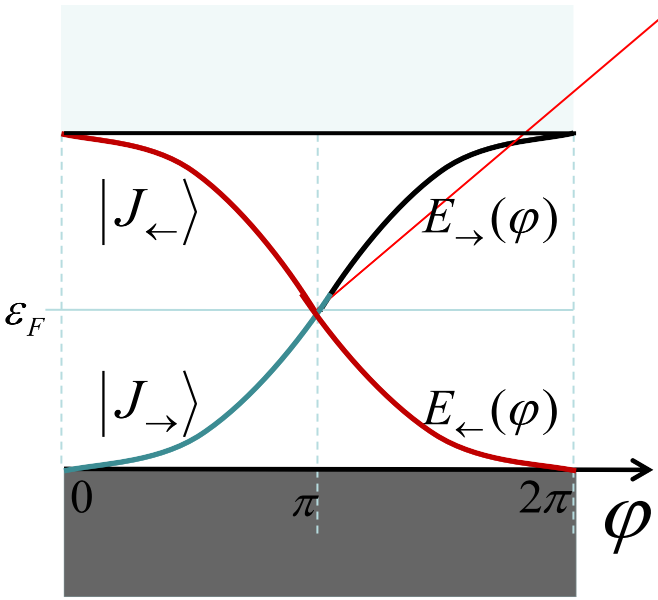
*A **refrigerator** is a cooling appliance comprising a **thermally insulated compartment** and a **heat pump** to transfer heat from it to the **external enviroment**. (Wikipedia)*

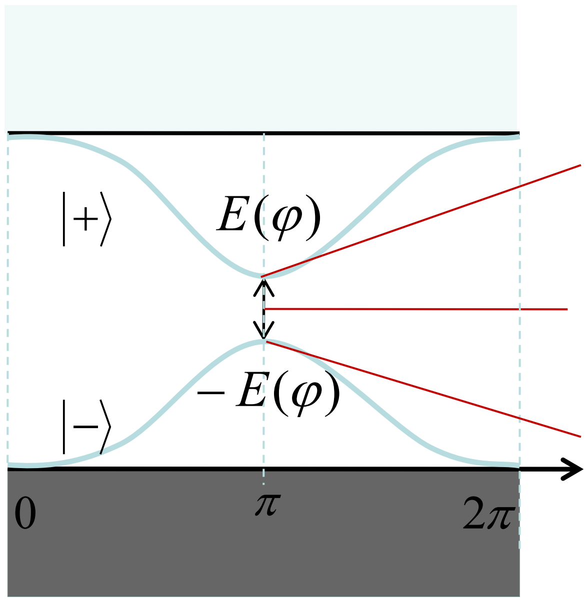
G.Sonne et al. PRL ,**104**, 226802 (2010)

Andreev Levels

Completely transparent junction: normal reflection probability R=0

Finite small reflection R<<1 removes degeneracy at φ=π





$$|+\rangle = \frac{1}{\sqrt{2}}\left(|J_{\rightarrow}\rangle + |J_{\leftarrow}\rangle\right)$$

$$\delta E = 2\Delta\sqrt{R}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|J_{\rightarrow}\rangle - |J_{\leftarrow}\rangle\right)$$

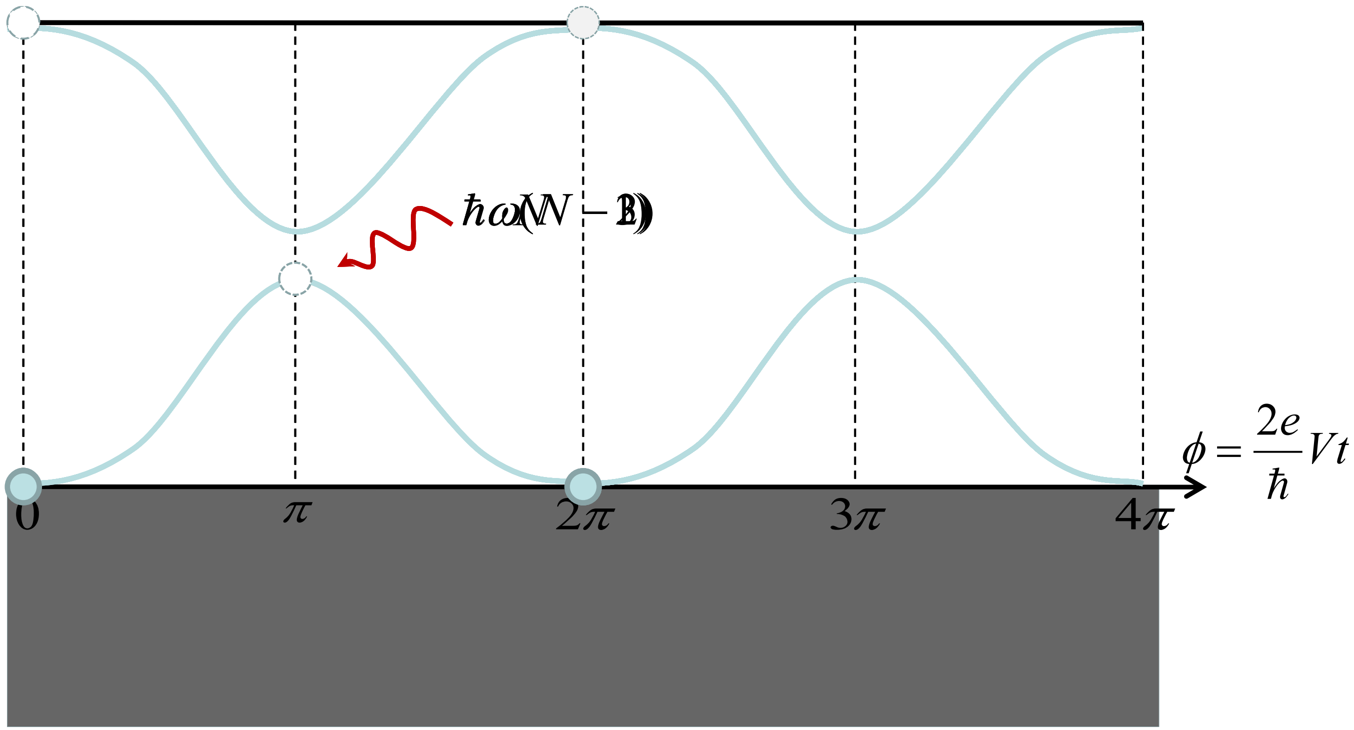
$$E\;(\varphi) = \pm\Delta\cos(\varphi/2)$$

$$\hat{H}_{eff} = \Delta\cos(\varphi/2)\hat{\sigma}_z$$

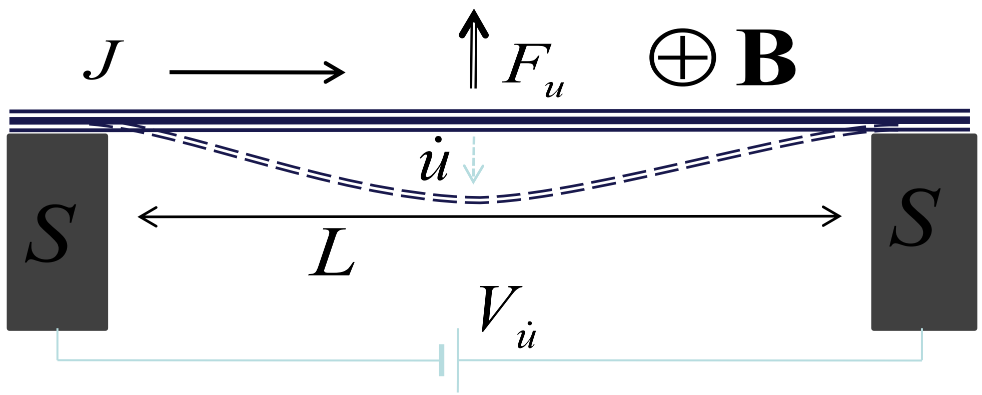
$$E(\varphi) = \Delta\sqrt{R + D\cos^2(\varphi/2)};\quad D = 1 - R$$

$$\hat{H}_{eff} = \Delta\sqrt{D}\cos(\varphi/2)\hat{\sigma}_z + \sqrt{R}\Delta\hat{\sigma}_x$$

Quantized Pumping out the Heat from Nanovibrations



Magneto-motive NEM Coupling



$$F_u = LBJ$$

$$V_{\dot{u}} = -\dot{u}BL$$

$$\hat{J} \equiv \frac{1}{L}\int_{-L/2}^{L/2} \hat{j}(x)dx \qquad \langle J_{\rightarrow} | \hat{J} \,| J_{\leftarrow} \rangle \cong 0$$

$$\hat{J} \cong -e\hbar^{-1}\Delta\sin(\varphi/2)\hat{\sigma}_z$$

$$\dot{\varphi} = 2e\hbar^{-1}V_{\dot{u}} = -2e\hbar^{-1}BL\dot{u}$$

$$\varphi = \varphi_0 - 2e\hbar^{-1}BLu$$

$$\hat{H}_{eff} = \Delta\sqrt{D}\cos\left(\frac{\varphi}{2} - \frac{eBL\hat{u}}{\hbar}\right)\hat{\sigma}_z + \Delta\sqrt{R}\hat{\sigma}_x + \frac{\hat{P}_u^2}{2M} + k\frac{\hat{u}^2}{2}$$

Voltage Biased NEM Weak Link

$$\hat{\mathrm{H}}_{\mathrm{eff}} = E(\varphi)\hat{\sigma}_z + \hbar\omega_0 \hat{a}^+\hat{a} - \tilde{B}\alpha(\varphi)\Delta\hat{\sigma}_x(\hat{a}^+ + \hat{a})$$

$$E(\varphi) = \Delta\sqrt{R + D\cos^2(\varphi/2)}$$

$$\alpha(\varphi) \equiv \frac{\sqrt{R}\sin(\varphi/2)}{\sqrt{R + D\cos^2(\varphi/2)}} = \begin{cases} 0 \ \ if \ \varphi = 2\pi n \\ 1 \ \ if \ \ \varphi = \pi(2n+1) \end{cases}$$

$$V_{ext} \Rightarrow \varphi(t) = \frac{2e}{\hbar} V_{ext} t$$

Under adiabatic conditions

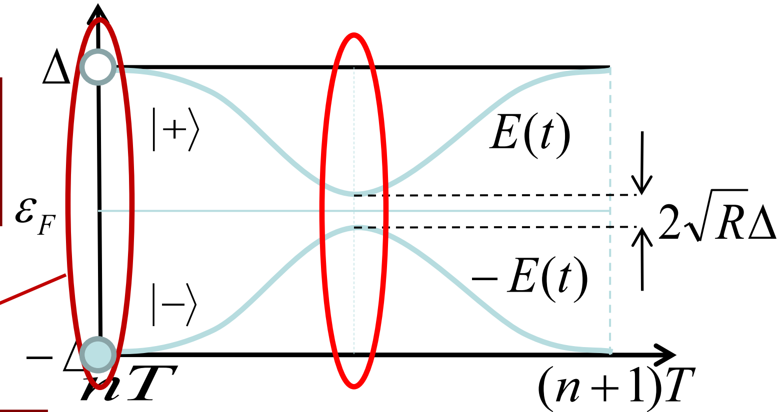
$V_{ext} << R\Delta,$

$\mathrm{H}_{\mathrm{eff}} \Longrightarrow H_V(t)$

$$\hat{H}_V(t) = E(\varphi(t))\hat{\sigma}_z + \hbar\omega_0 \hat{a}^+ a - \tilde{B}\alpha(\varphi(t))\Delta\hat{\sigma}_x(\hat{a}^+ + \hat{a})$$

Liouville- von Neumann Equation

$$\frac{\partial}{\partial t}\hat{\rho}(t)=\frac{1}{i\hbar}\left[H_V(t),\hat{\rho}(t)\right]+\gamma L(\hat{\rho}), \qquad t\in(nT,nT+T)$$



Boundary conditions

$$\hat{\rho}(t=nT+\delta)=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\otimes\rho_{in}(nT), \qquad \rho_{in}(nT)=\underset{e}{Tr}\,\hat{\rho}(nT-\delta)$$

$$e^{-\frac{i}{\hbar}\int_0^T H_0(t)dt}\,\hat{\rho}(nT+T-\delta)\,e^{+\frac{i}{\hbar}\int_0^T H_0(t)dt}=\hat{\rho}(nT+\delta)+\frac{B}{B_0}\int_0^T dt\left[\tilde{H}_{\mathrm{int}}(t'),\hat{\rho}(nT+\delta)\right]+\left(\frac{B}{B_0}\right)^2\ldots$$

$$\hat{H}_0(t)=E(\varphi(t))\hat{\sigma}_z+\hbar\omega_0\hat{a}^+a;\quad \tilde{H}_{\mathrm{int}}(t)=\Delta\alpha(\varphi(t))\,e^{\sigma_z\frac{i}{\hbar}\int_0^t E(t)dt}\,\hat{\sigma}_x\,e^{-\sigma_z\frac{i}{\hbar}\int_0^t E(t)dt}\left(\hat{a}^+e^{i\omega t}+\hat{a}e^{-i\omega t}\right)$$

The main contributions are given by the points $t = (n+1/2)T$ where electromechanical coupling has a maximal value

$$E\left(\frac{T}{2}+\delta t\right)\cong\Delta\sqrt{R}+\frac{\Delta}{\sqrt{R}}\left(\frac{eV\delta t}{\hbar}\right)^2$$

Evolution During Single Period of Josephson Oscillations

Initial state $|-\rangle \otimes |n\rangle$

$\xrightarrow{P_-(n)}$ $|+\rangle \otimes |n-1\rangle$ *Final state*

$\xrightarrow{P_0(n)}$ $|-\rangle \otimes |n\rangle$ *Final state*

$\xrightarrow{P_+(n)}$ $|+\rangle \otimes |n+1\rangle$ *Final state*

$$P_{\mp}(n) = n\frac{B^2}{B_0^2}\left(\frac{\Delta}{eV}\right)^2 \left|\int_{-\infty}^{\infty} d\tau \cos\left(\frac{1}{3\sqrt{R}}\frac{\Delta}{eV}\tau^3 + \frac{2\Delta\sqrt{R} \mp \hbar\omega}{eV}\tau\right)\right|^2 \qquad P_+ + P_- + P_0 = 1$$

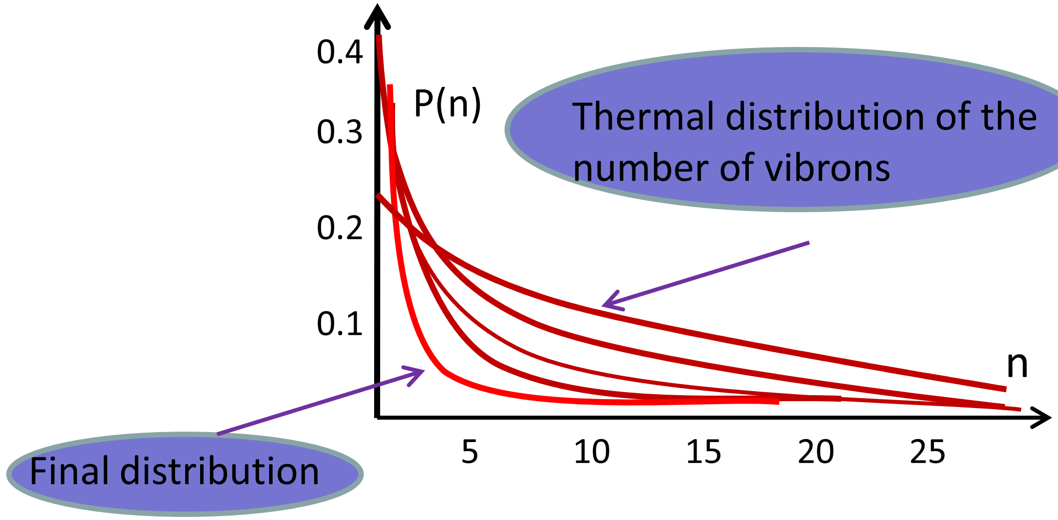
Under resonant conditions $2\Delta\sqrt{R} = \hbar\omega$

$$P_-(n) = n\frac{\pi}{2}\frac{B^2}{B_0^2}\left(\frac{\hbar\omega\cdot\Delta}{e^2V^2}\right)^{2/3}; \quad P_+(n) = 0{,}02P_-(n)$$

Numerical Estimations

$$\hbar\omega = 10^{-6}\,\mathrm{eV}$$
$$k_B T = 5\hbar\omega$$
$$\Delta = 10\hbar\omega$$
$$L = 200\,\mathrm{nm}$$
$$u_0 = 20\,\mathrm{pm}$$
$$B = 1\,\mathrm{T}$$
$$V = 10^{-7}\,\mathrm{V}$$
$$\mathrm{Q} = 10^5$$

Voltage applied between superconductors generates cooling of the nanowire vibrations



$$\langle n \rangle_{final} = 0.1$$

Superconducting "Nano-Thermometer"

$$\bar{j} = \langle n \rangle \frac{2e}{\hbar} \Gamma \Delta, \quad \Gamma \equiv \pi \frac{B^2}{B_0^2} \left(\frac{\hbar\omega \cdot \Delta}{e^2 V^2} \right)^{2/3}$$

$$\Gamma Q << \hbar\omega / 2eV \Rightarrow \langle n \rangle = n_{eq} \cong \frac{\mathrm{k_B T}}{\hbar\omega}, \qquad \bar{j}_{\mathrm{eq}} = n_{eq} \frac{2e}{\hbar} \Gamma \Delta$$

$$\frac{\bar{j}}{\bar{j}_{\mathrm{eq}}} = \frac{\hbar\omega}{kT} \langle n \rangle$$

Conclusions

1. Voltage-biased superconducting weak link drives vibrational motion when an external magnetic field is switched on.

2. Both resonant pumping and an efficient cooling of nanovibrations can be achieved depending on the amount of electronic backscattering.

3. The refrigerating effect corresponding to the average occupation number of vibrations <n>=0.1 can be achieved.