The Abdus Salam International Centre for Theoretical Physics

2164-13

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Qubit-Coupled Nanomechanics

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Qubit-Coupled Nanomechanics

Matt LaHaye – Syracuse University

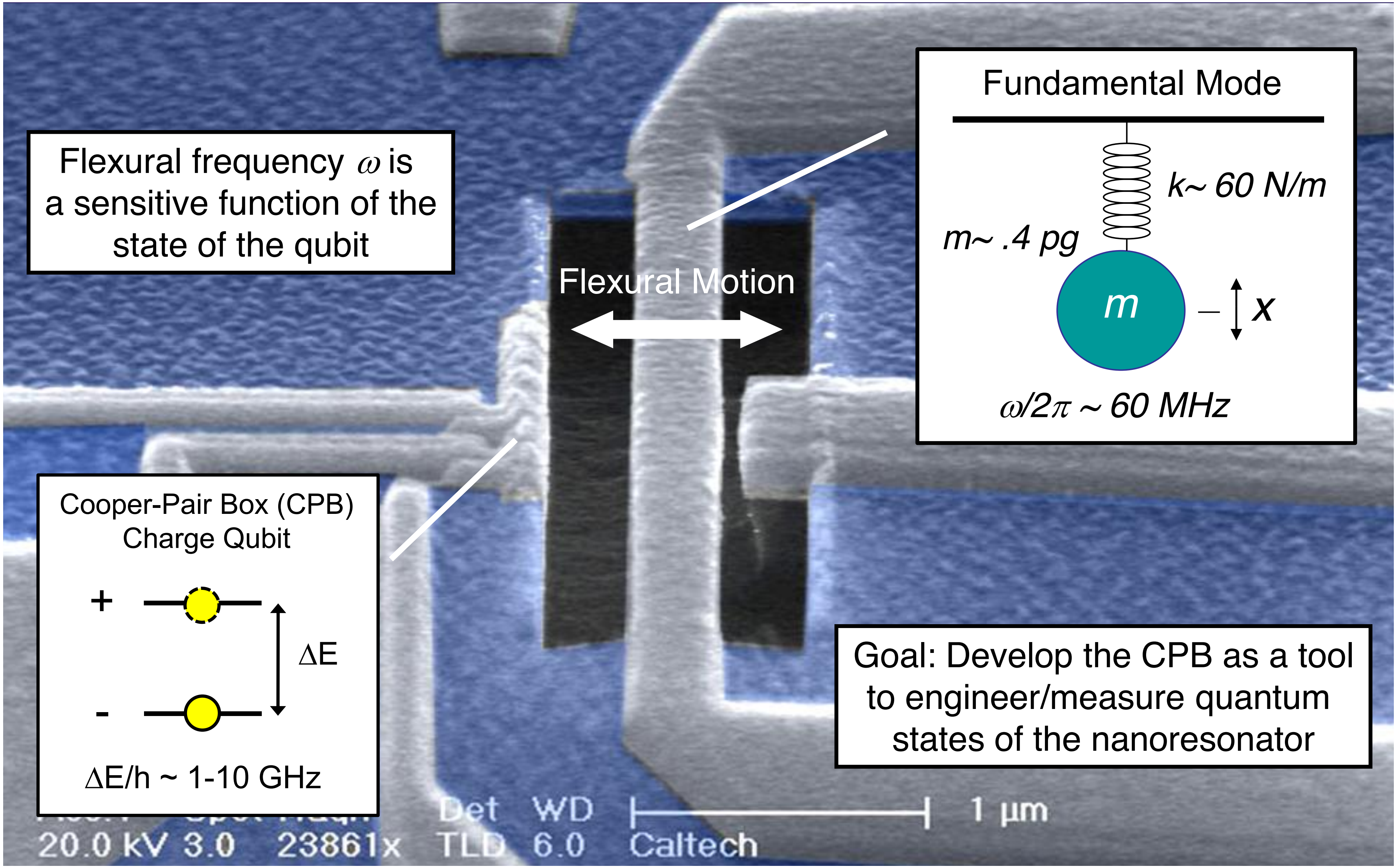
experiments performed at caltech with:

junho suh, michael roukes - caltech

keith schwab - caltech

pierre echternach - j p l

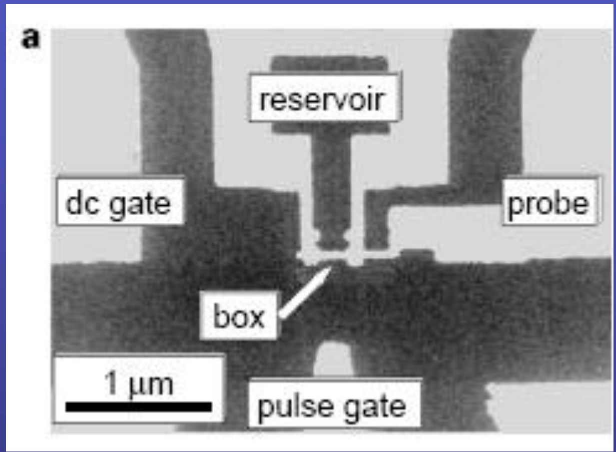
Qubit-coupled nanomechanical resonator



qubit-coupled nanomechanics

First proposed by A. Armour, M. Blencowe & K. Schwab: *PRL* **88** (2002) & *Physica B* **316** (2002).

Cooper-pair box (CPB) charge qubit



Nakamura et al., Nature, **398** 29 Apr. 1999

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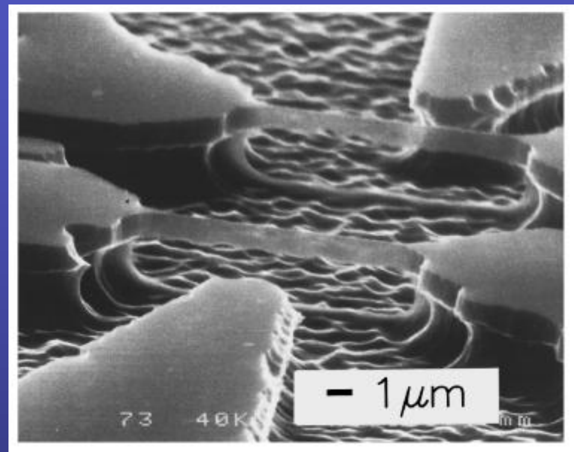
Nano-electromechanical resonator



Cleland & Roukes, APL **69** 28 Oct. 1996

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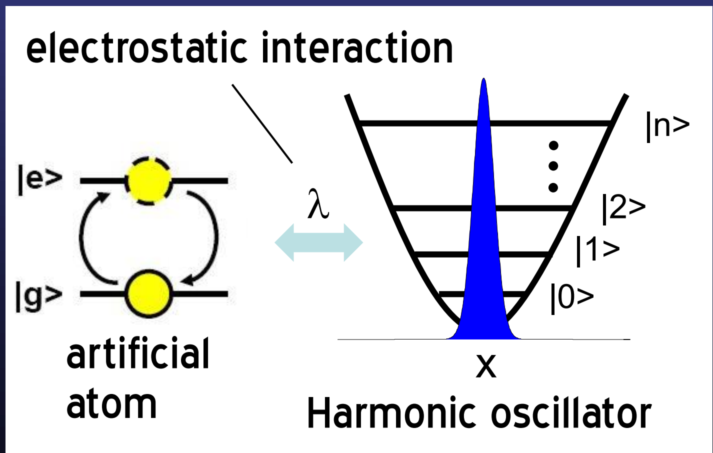


Qubit- coupled resonator analogous to atom-coupled photon cavity

outline

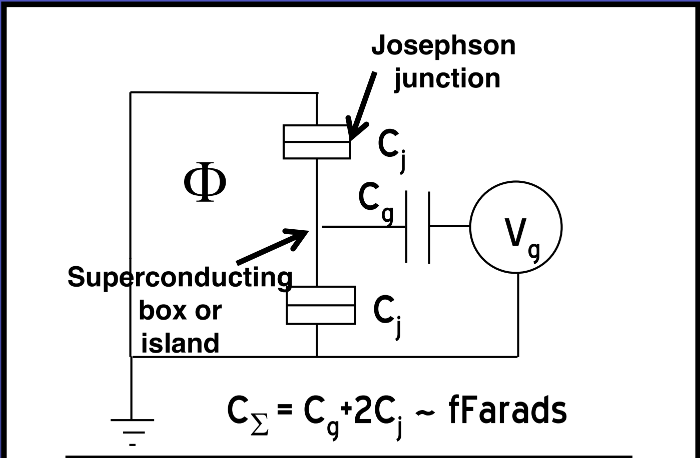
Brief review of the Cooper-pair box (CPB) charge qubit, how we couple the CPB and nanoresonator, dispersive interaction

Experiments: observe the dispersive interaction between CPB and nanoresonator and use it to perform spectroscopy of CPB and measurement of LZS-interference effects . CPB-based parametric amplification of mechanics.

Demonstrated coupling should be large enough to pursue more advanced quantum measurement proposals, e.g. 'lasing' and squeezing of nanoresonator.

Significant room for improvement to interaction strength. CPB/nanoresonator entanglement experiment looks within reach. Should also be able to approach strong coupling limit, a prerequisite for nanoresonator number-state detection.

Cooper-pair box (CPB) Charge Qubit



Schematic of CPB

- The CPB qubit is a two-level quantum system with a tunable energy gap

Hamiltonian $\hat{H} = [2E_C(1-2n-2n_g)]\hat{\sigma}_Z - [\frac{E_{J0}}{2}\cos(\pi\Phi/\Phi_0)]\hat{\sigma}_X$

Electrostatic Energy — Josephson Energy

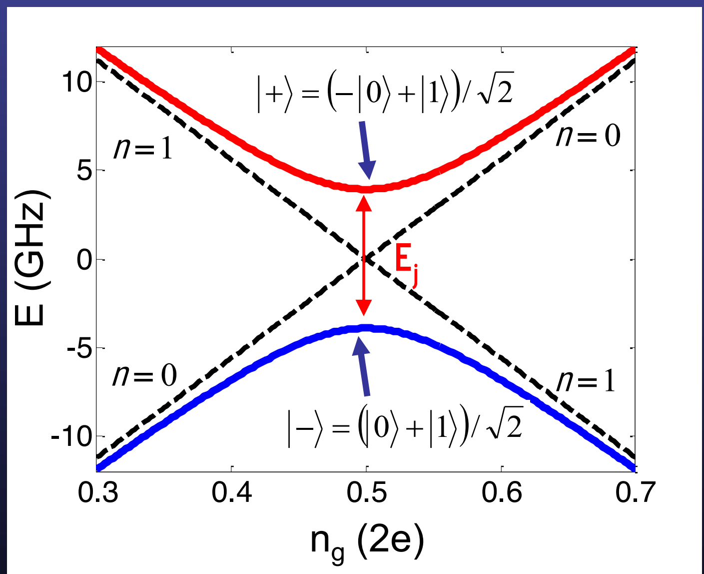
Charging Energy $E_C = \frac{e^2}{2C_\Sigma} \sim 1\ K$

Josephson Energy $E_{J0} \leq E_C$

Gate Charge $n_g = \frac{C_g V_g}{2e}$

Flux Quantum - Φ_0

Cooper-pairs on box - n



Energy Bands of CPB

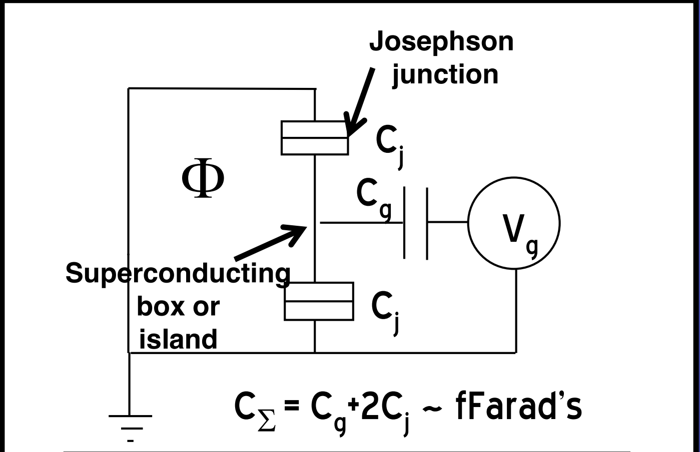
- The two states ($\pm$) are superpositions of n & n+1 Cooper-pairs on the CPB

Energy Gap $\Delta E = \sqrt{[4E_C(1-2n-2n_g)]^2 + [E_{J0}\cos(\pi\Phi/\Phi_0)]^2}$

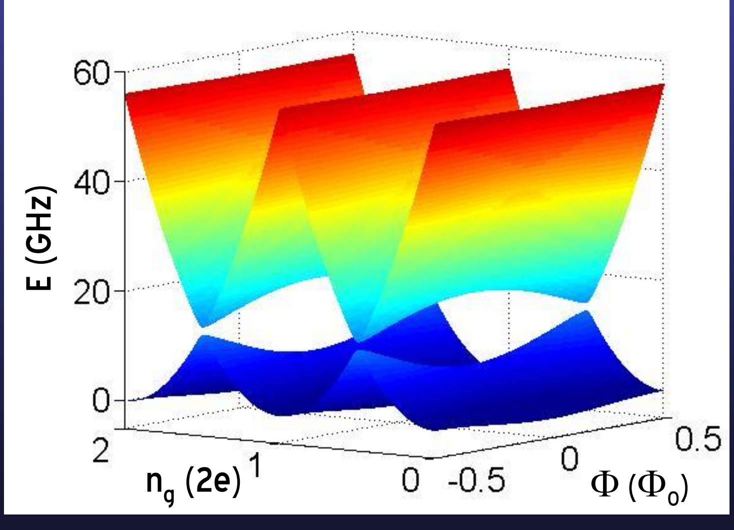
Tunable by adjusting gate voltage V_g

Tunable by adjusting applied flux Φ

Cooper-pair box (CPB) Charge Qubit



Schematic of CPB



Energy Bands Are Periodic in n_g and Φ

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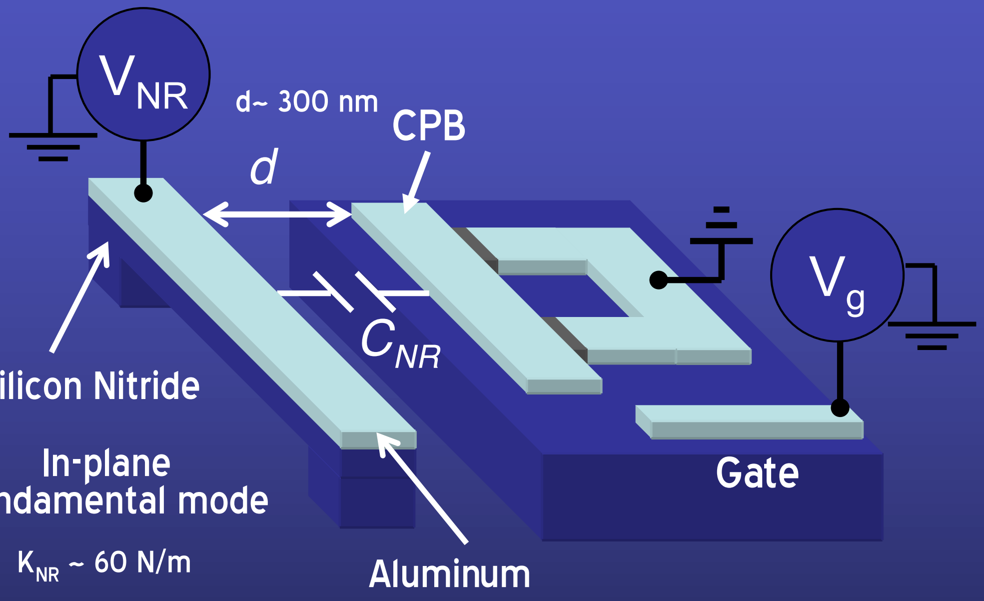
Tunable by adjusting gate voltage V_g

Tunable by adjusting applied flux Φ

CPB capacitively coupled to NEMS

Flexural motion of resonator couples to charge on the CPB island



In-plane
Fundamental mode

K_{NR} ~ 60 N/m

$\omega_{NR}/2\pi$ ~ 60 MHz

CPB-NEMS Interaction

$$\hat{H}_{Int} = \hbar\lambda\left(\hat{a}^{\dagger} + \hat{a}\right)\hat{\sigma}_Z$$

- λ: Electrostatic Coupling Constant
- $\left(\hat{a}^{\dagger} + \hat{a}\right)$: NEMS Position Operator
- $\hat{\sigma}_Z$: CPB Charge

$$\lambda = 2\frac{E_C}{\hbar}\frac{C_{NR}V_{NR}}{e}\sqrt{\frac{\hbar\omega_{NR}}{2K_{NR}d^2}}$$

Total Hamiltonian of the Coupled CPB & Nanoresonator

$$\hat{H}_T = \hbar\omega_{NR}\left(\hat{N}+1/2\right) + \frac{4E_C(1-2n_g)}{2}\hat{\sigma}_Z + \frac{E_J}{2}\hat{\sigma}_X + \hbar\lambda\left(\hat{a}^{\dagger}+\hat{a}\right)\hat{\sigma}_Z$$

- Mechanical quanta $\hat{N} = \hat{a}^{\dagger}\hat{a}$
- NEMS
- CPB electrostatic Energy
- CPB Josephson Energy
- Interaction

Reduces to
Jaynes-Cummings
Hamiltonian
at Charge Degenarcy

Dispersive limit of CPB-NEMS Hamiltonian

See E.K. Irish & Schwab, PRB 2003

CPB and NEMS far-detuned for our parameters

$$\Delta = \Delta E - \hbar\omega_{NR} >> \lambda\hbar$$

10 GHz — 60 MHz — MHz

Interaction Leads to Shift in Energy of Coupled CPB and NEMS (analogous to dispersive CQED)

Two Dispersive Effects Occur

CPB-state-dependent Frequency Shift in NEMS

Analogous to single-atom refractive shift in CQED

$$\Delta\omega_{NR} \approx \pm\chi$$

NEMS-Dependent shift in CPB transition

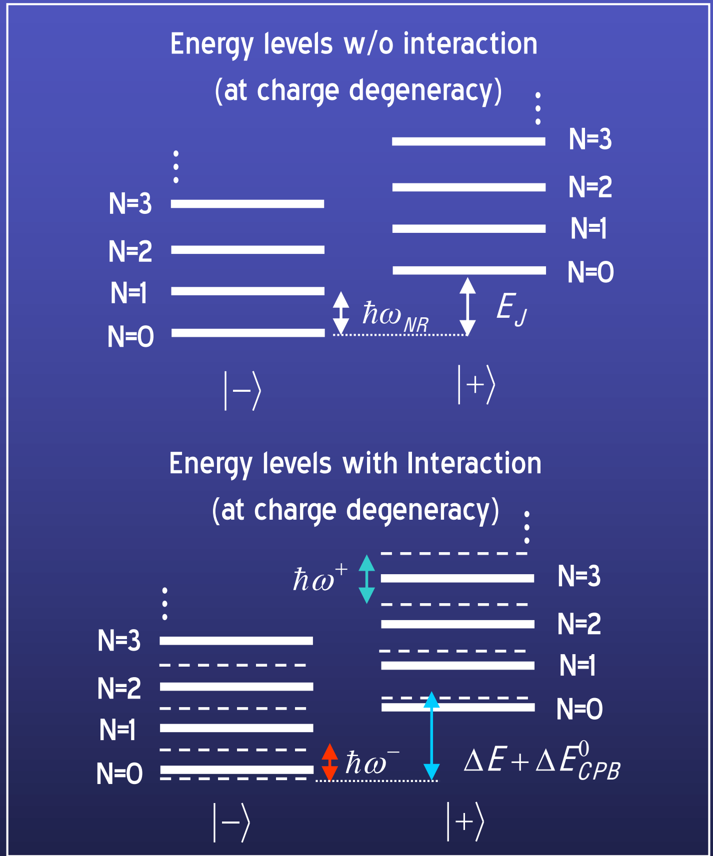
$$\Delta E_{CPB}^{(N)} \approx (2N+1)\cdot\hbar\chi$$

Analogous to Lamb Shift

Analogous to Stark Shift

$N \equiv \#$ of Quanta in Nanoresonator

Dispersive Shift of CPB/NEMS Energy



$$\omega^+ = \omega_{NR} + \chi \qquad \omega^- = \omega_{NR} - \chi$$

The shift χ is a function of CPB bias point and (in our experiments) is mainly due to CPB quantum capacitance.

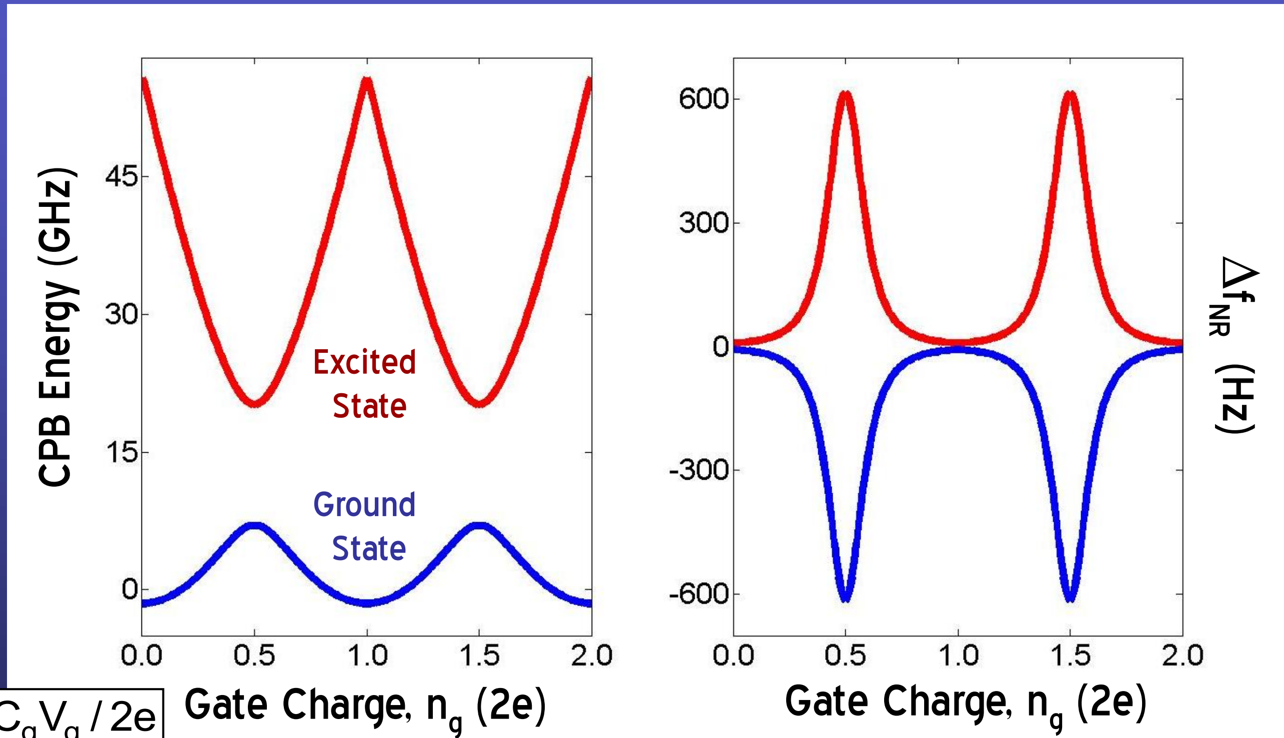
estimates for the nanomechanical frequency shift

Parameters

C_{NR} ~ 50 aF
d ~ 300 nm
V_{NR} ~ 10 V
$f_{NR} = \omega_o / 2\pi$ ~ 60 MHz

K ~ 60 N/m
$\lambda/2\pi$ ~ 2.0 MHz
E_C ~ 14 GHz
E_J ~ 13 GHz

CPB Energy & Nanoresonator Frequency Shift vs n_g



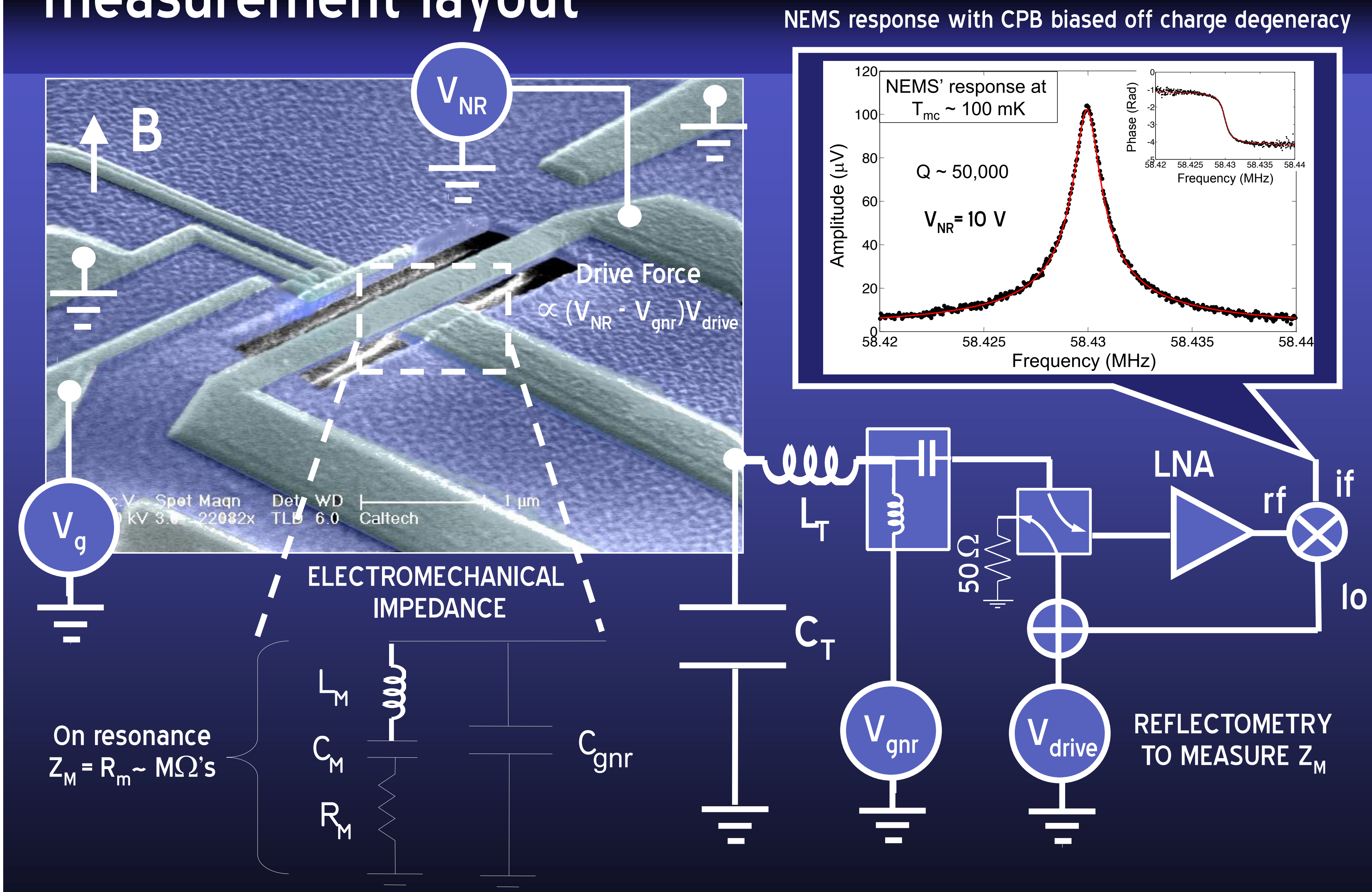
$n_g = C_g V_g / 2e$

Frequency shift is greatest at CPB avoided-level crossings*:

$$\Delta f_{NR} = \frac{\chi}{2\pi} \approx \pm \frac{\hbar \lambda^2}{\pi} \frac{E_{J0}^2 \cos^2(\pi\Phi/\Phi_0)}{\left(((4E_C(1-2n_g))^2 + E_{J0}^2 \cos^2(\pi\Phi/\Phi_0)\right)^{3/2}}$$

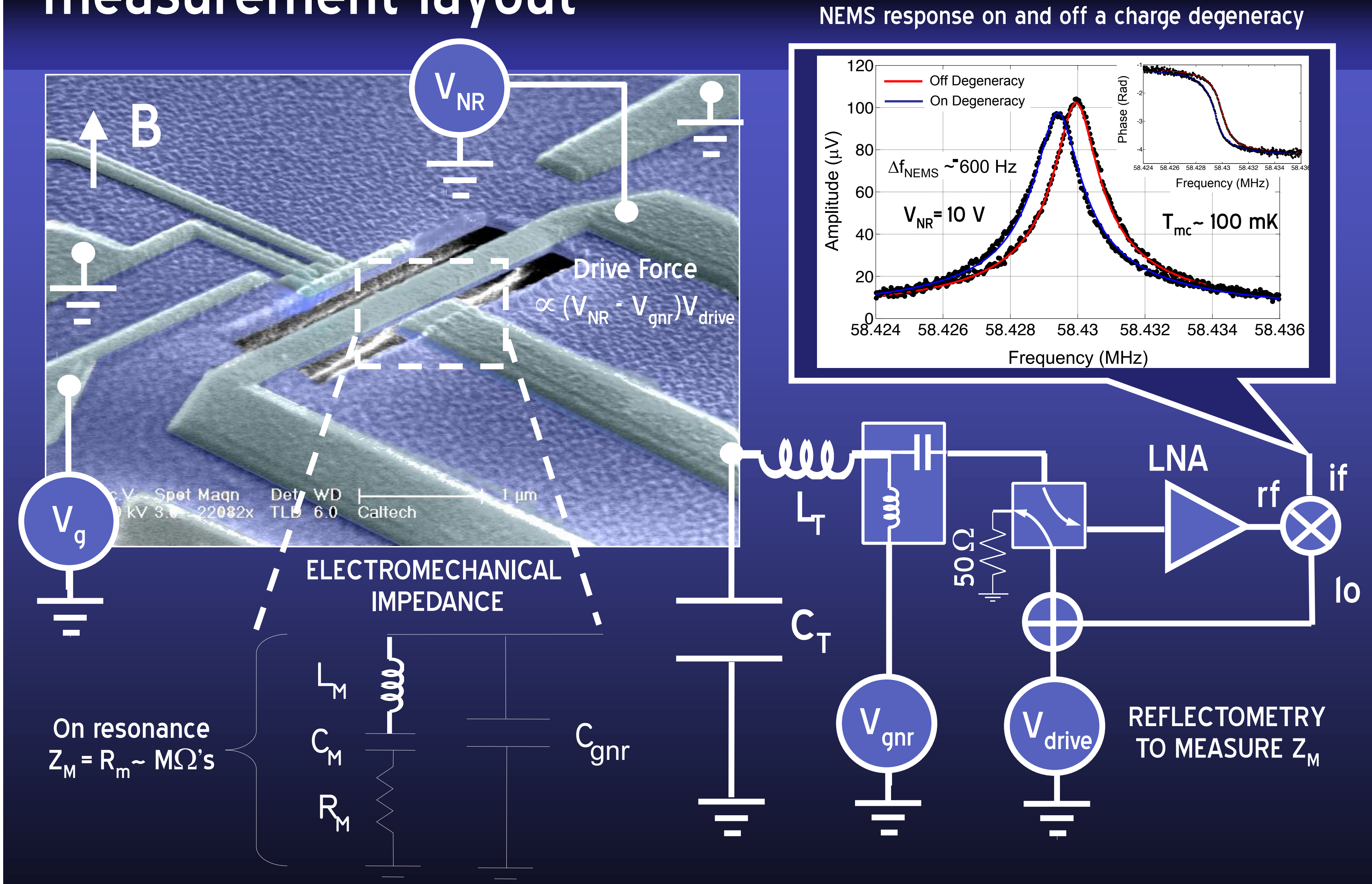
*This is the quantum capacitance effect measured via LC resonator in Sillanpaa et al., PRL 95 206806 (2005) and Duty et al., PRL 95 206807 (2005)

measurement layout



For details of capacitive detection scheme see: P. Truitt et al. Nano Letters 7, 120 (2007)

measurement layout

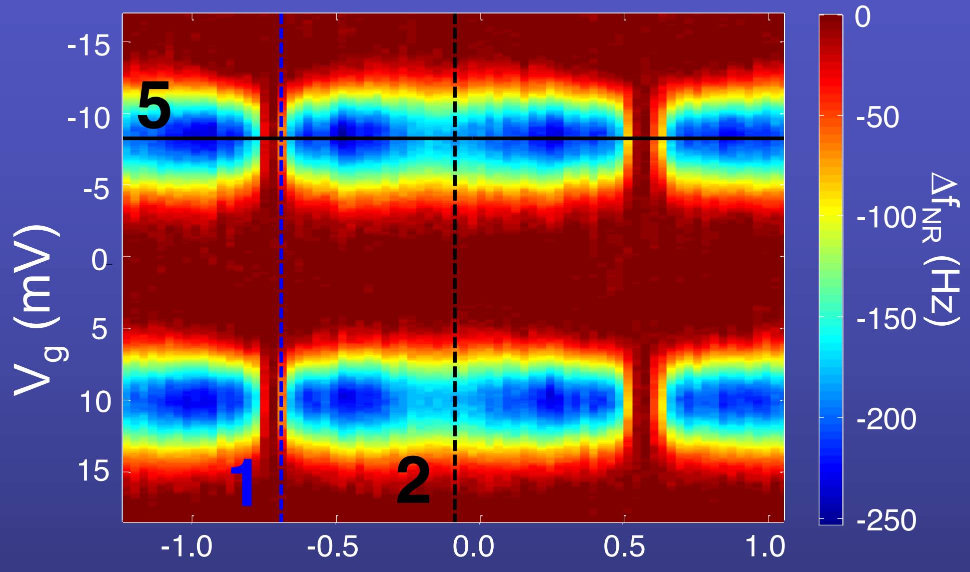


For details of capacitive detection scheme see: P. Truitt et al. Nano Letters 7, 120 (2007)

dispersive shift of NEMS: measurement vs. model

From M.D. LaHaye et al., Nature 459 , 960 (2009).

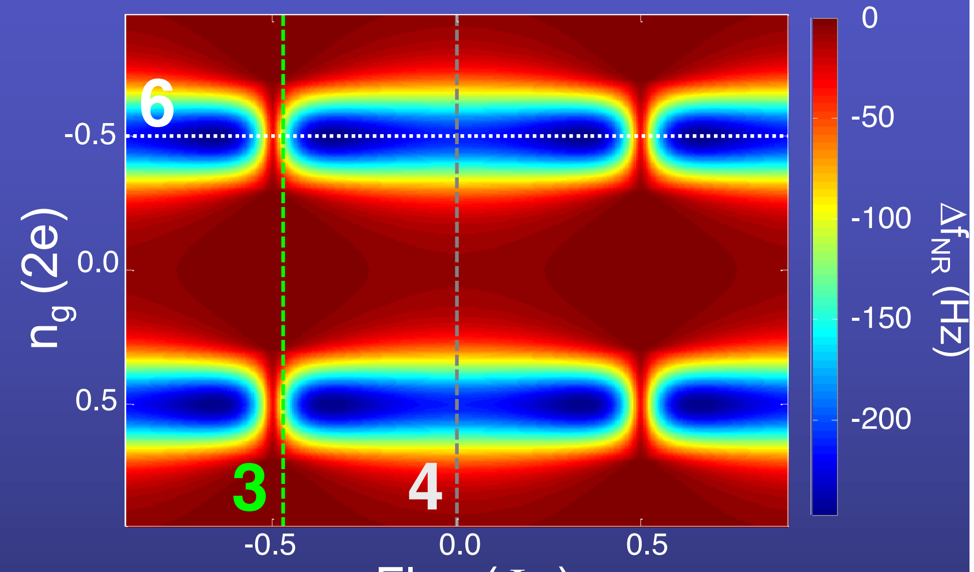
Measurement: V_{NR}= 7.0 V, $\omega_{NR}/2\pi$~60 MHz, T_{mc}~ 100 mK



Applied Magnetic Field (A.U.)

Note: Magnetic field applied on top of ~ 100 G

Model: $\lambda/2\pi$= 1.40 MHz, T =100mK
$E_{J,max}$ /h= 13.2 GHz, E_C /h= 14.0 GHz

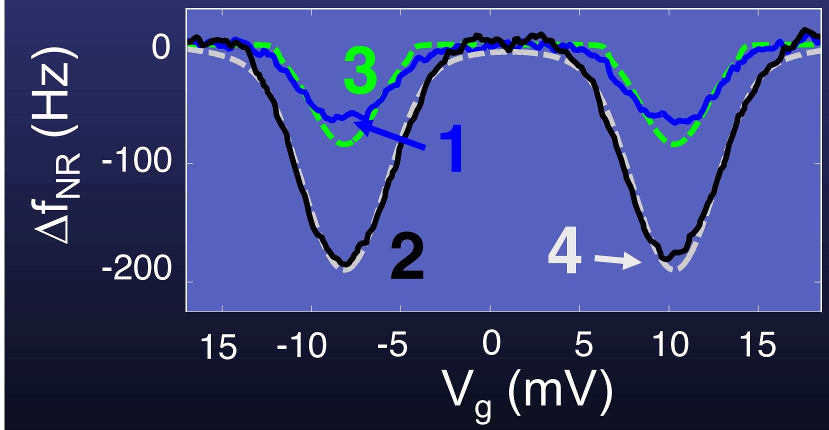


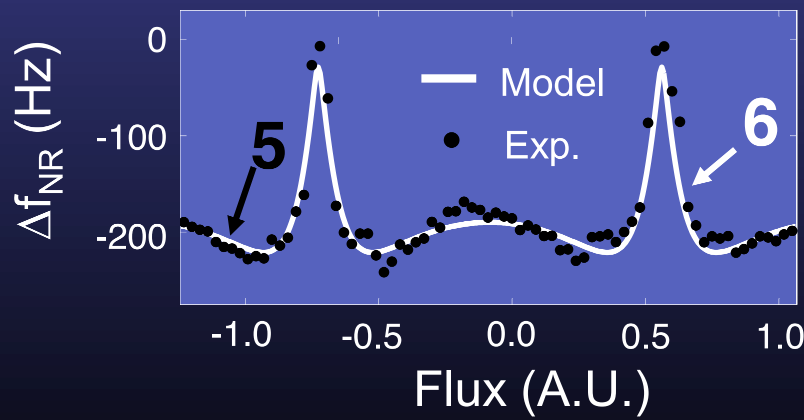
Flux (Φ_o)

Flux Periodicity:

$$E_J = E_{J,\max}\left|\cos\left(\pi\Phi/\Phi_o\right)\right|$$

Notes: Model convolved with 0.1 CP_{rms} charge noise, and includes thermal population of CPB excited state





dispersive shift of NEMS: measurement vs. model

With this coupling strength, proposals suggest it should be possible to implement qubit "lasing", squeezing of NEMS, and other techniques

(Kerr Nonlinearity) F.L. Semiao, K. Furuya, G. Milburn Phys. Rev. A **79**, 063811 (2009).

(Lasing) J. Hauss,, A. Federov, C. Hutter, A. Shnirman, G. Schon, PRL. **100**, 037003 (2008)

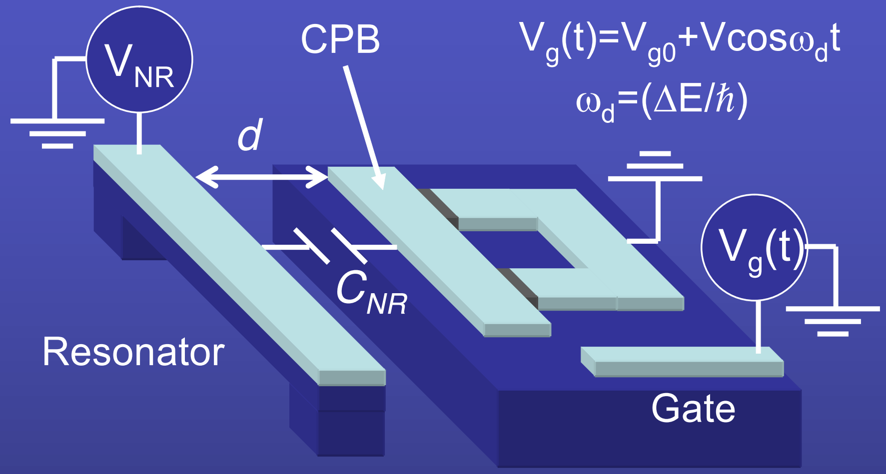
(Squeezing) P. Rabl,, A. Shnirman, P. Zoller. Phys. Rev. B **70**, 205304 (2004).

With realistic improvements to coupling strength, should be able to implement techniques to generate coherent superposition states

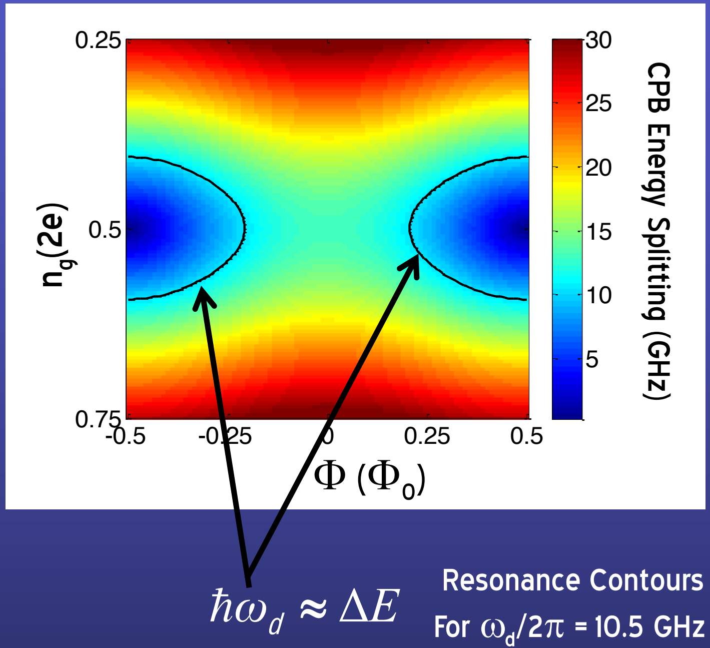
A. Armour, M. Blencowe, New J. Phys. **10** 095004 (2008).

NEMS-based spectroscopy of CPB

DEVICE SCHEMATIC



CPB ENERGY SPLITTING ΔE



- APPLY CW MICROWAVES RESONANT WITH CPB SPLITTING ΔE

$$\hbar\omega_d \approx \Delta E = \sqrt{(4E_C(1-2n_g))^2 + E_{J0}^2\cos^2\pi\frac{\Phi}{\Phi_0}}$$

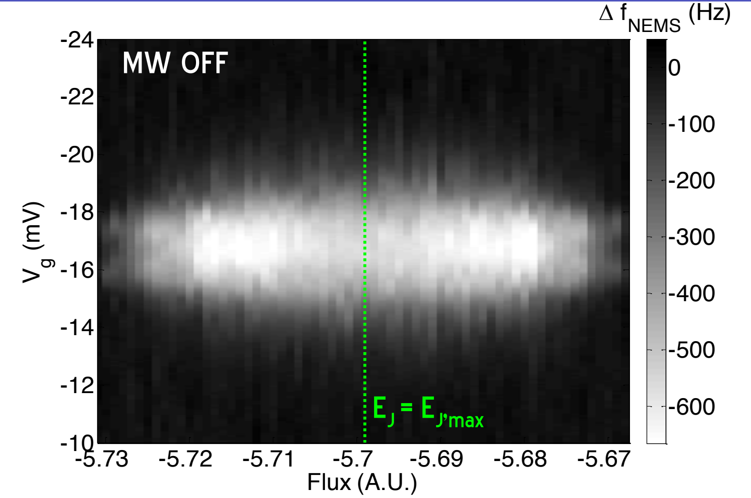
- FOR LARGE AMPLITUDE MW, CPB IS SATURATED (i.e. $P_- = P_+$, GIVEN BY BLOCH EQN.) ON RESONANCE CONTOURS

- AVERAGE NEMS FREQUENCY SHIFT GOES TO ZERO ALONG RESONANCE CONTOURS

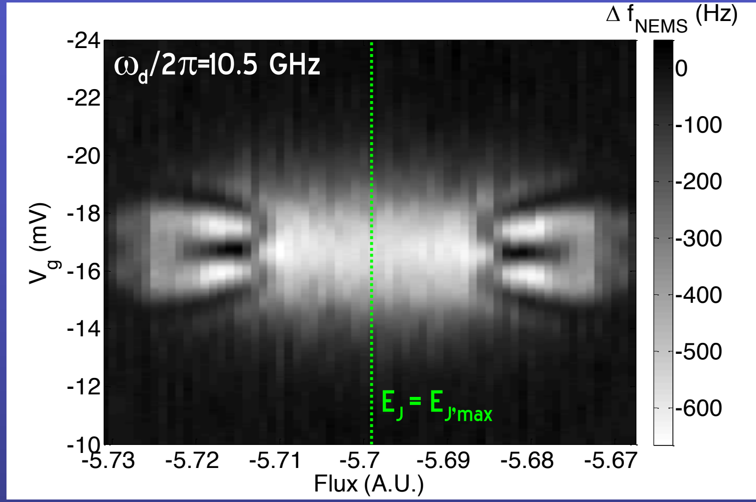
$$\Delta f_{NEMS}^- = -\Delta f_{NEMS}^+ \quad \Longrightarrow \quad \overline{\Delta f}_{NEMS} = P_-\Delta f_{NEMS}^- + P_+\Delta f_{NEMS}^+ = 0$$

NEMS-based spectroscopy of CPB

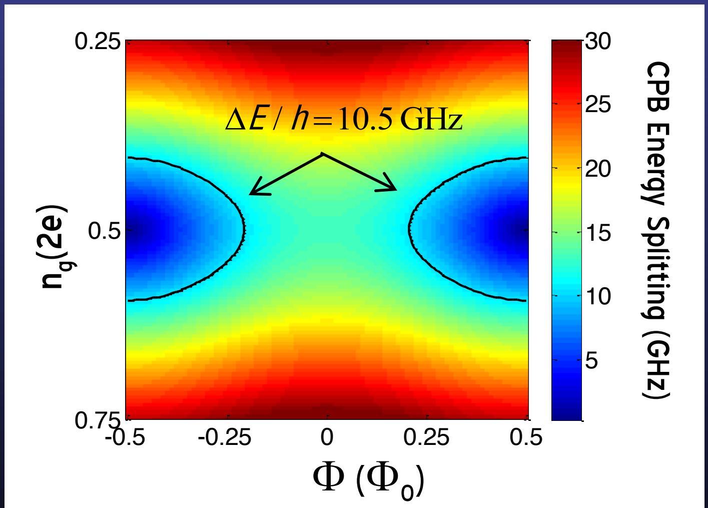
Measured NEMS Frequency Shift (MW OFF)



Measured NEMS Frequency Shift (MW ON)



CPB ENERGY SPLITTING ΔE

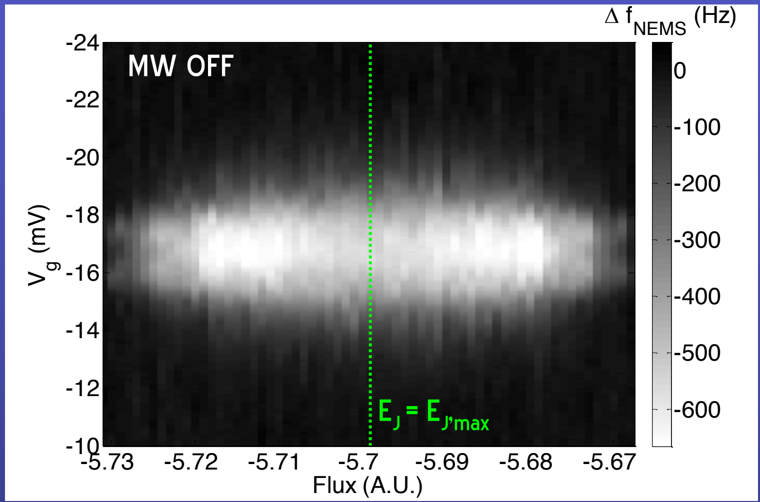


- Apply MW's to CPB gate, measure NEMS' frequency shift

- NEMS' frequency shift goes to zero at CPB bias points where we expect MWs to be resonant with CPB

NEMS-based spectroscopy of CPB

Measured NEMS Frequency Shift (MW OFF)



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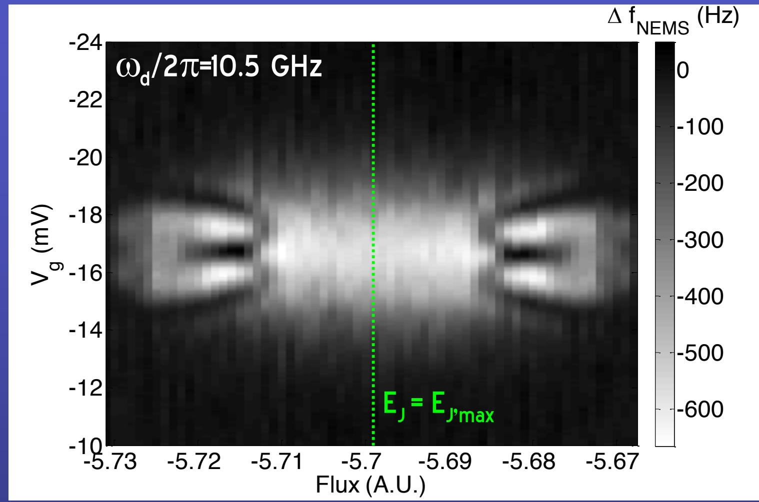
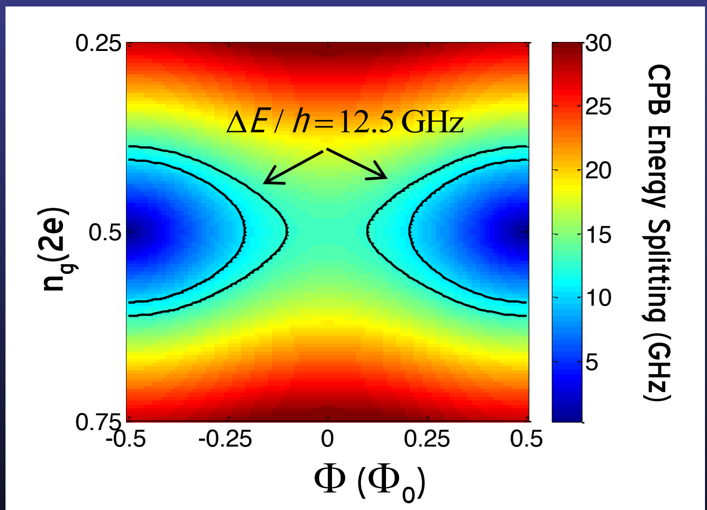


CPB ENERGY SPLITTING ΔE



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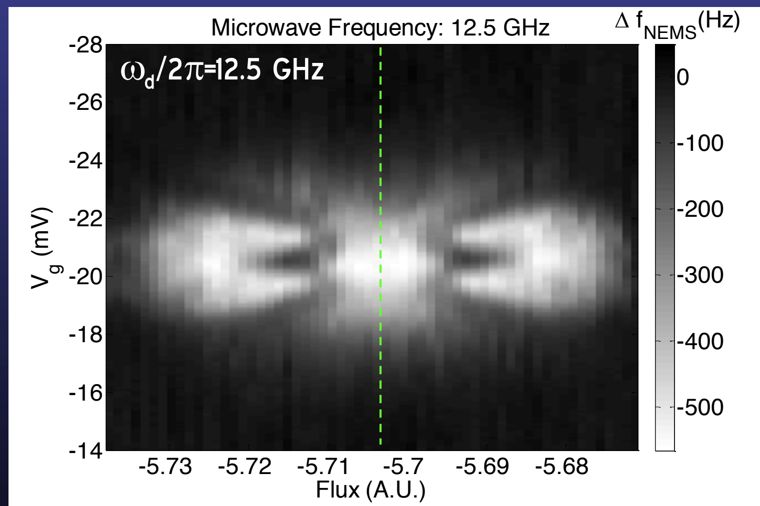
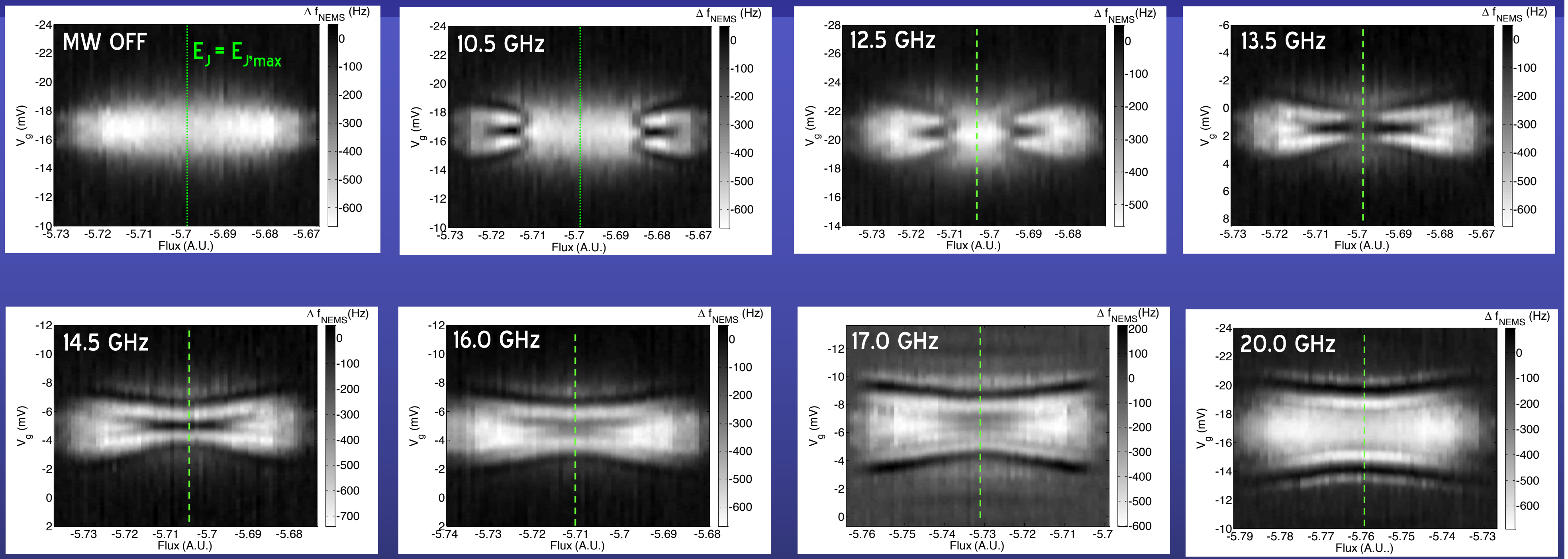
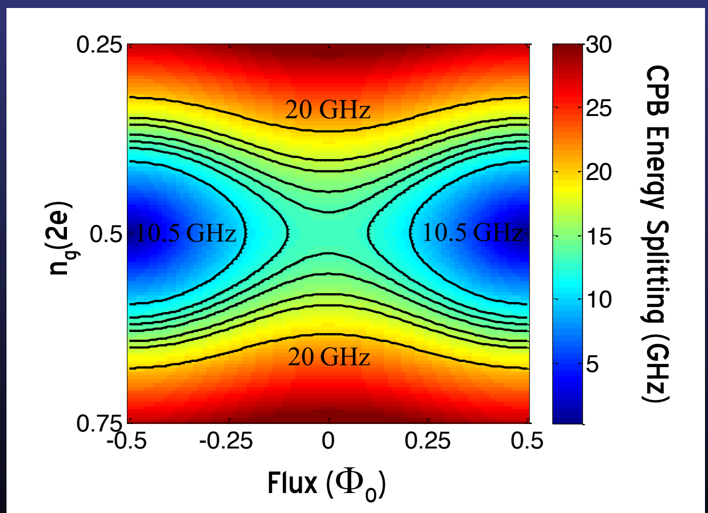


NEMS-based spectroscopy of CPB - measurements





- From spectroscopy and resonance condition can determine CPB parameters E_C and E_{J0}

Resonance condition

$$\hbar\omega_d = \Delta E = \sqrt{(4E_C(1-2n_g))^2 + E_{J0}^2 \cos^2 \pi \frac{\Phi}{\Phi_0}}$$

- Fitting data, find E_C ~13-14 GHz and E_{J0} ~13 GHz, agrees well with modeling data when no MW applied

Nanomechanical probe of LZS-interference in a CPB qubit

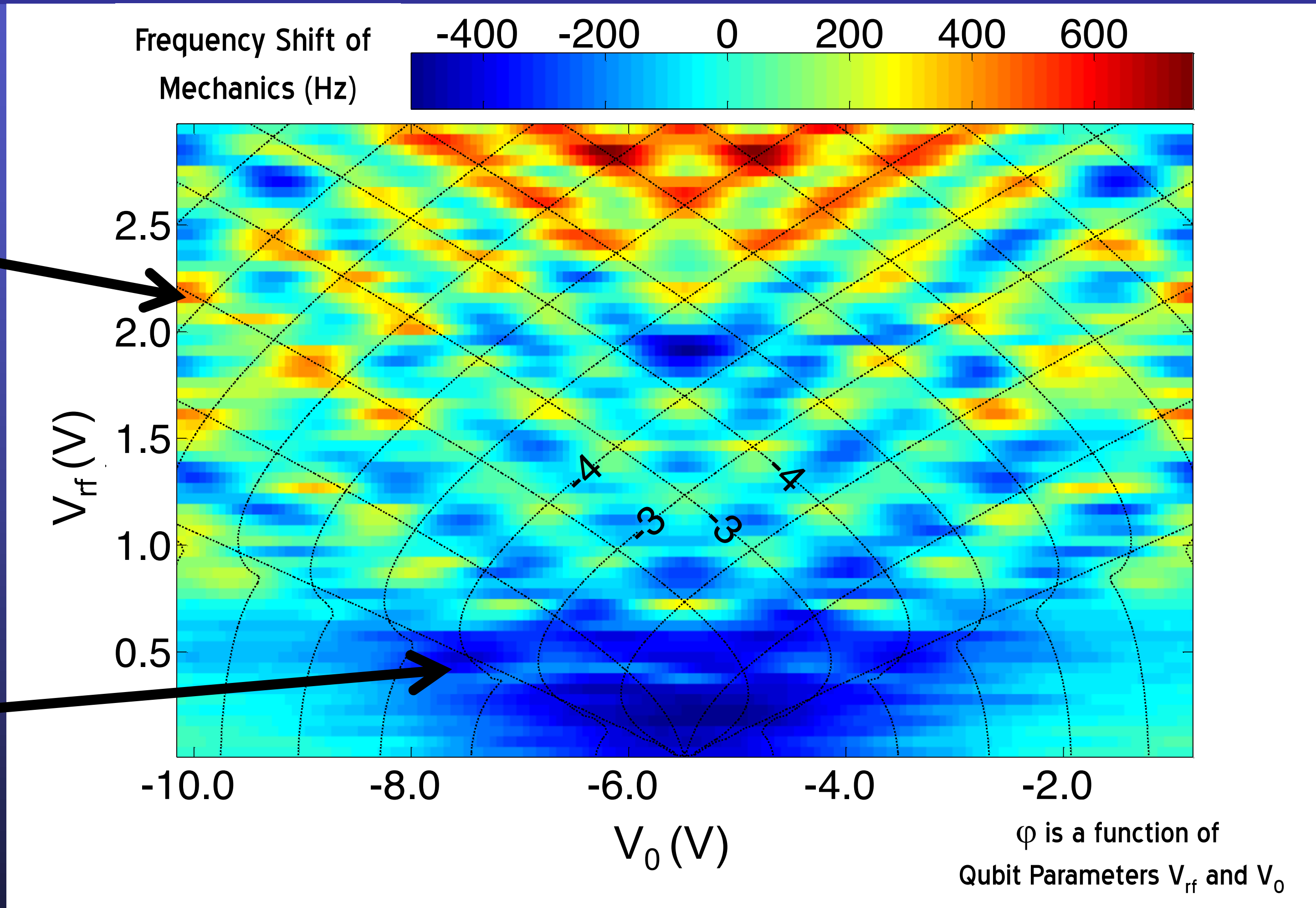
From M.D. LaHaye et al., Nature 459 , 960 (2009).

Positive Frequency Shift of Mechanics

Qubit in Excited State on Average

Negative Frequency Shift of Mechanics

Qubit in Ground State On Average



Qubit's State Vector

$$|\Psi(t)\rangle = \Psi^{-}|-\rangle + \Psi^{+}|+\rangle \quad \begin{cases} \Psi^{-} \propto \sin(\varphi/2) \\ \Psi^{+} \propto \cos(\varphi/2) \end{cases}$$

Landau-Zener-Stuckelberg (LZS) Interference Review: Shevchenko et al. Physics Reports 2010

prospects for number state detection

If you want to observe nanoresonator number state (N) statistics:

Could Measure
CPB 'Stark Shifts'

$$\Delta E_{CPB}^{(N)} \approx (2N+1) \cdot \hbar\chi$$

We haven't measured the Stark shifts, but we have measured χ (it's just the dispersive shift Δf_{NR}):

In these experiments

$$\chi / 2\pi \sim kHz$$

Would expect kHz shift in CPB transition frequency for a change of one phonon in nanoresonator.

Could we measure this? How does it compare to the damping?

prospects for strong dispersive coupling limit

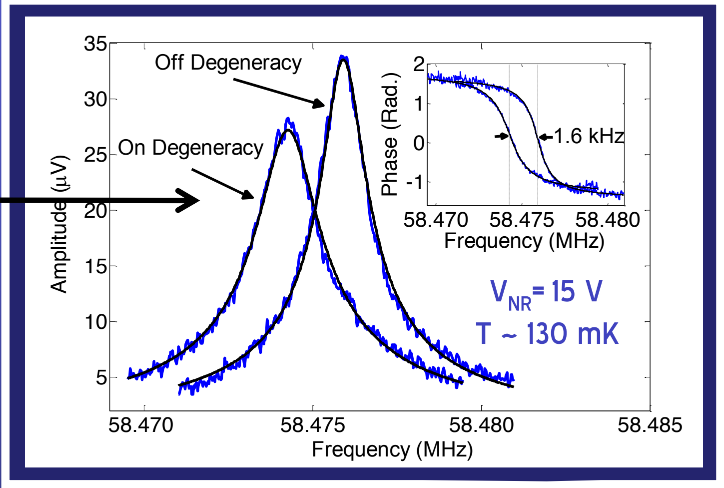
Definition of strong coupling limit: Dispersive interaction exceeds qubit and NEMS linewidth

$$\frac{\chi}{2\pi} > [\frac{\gamma_{NEMS}}{2\pi}, \frac{\gamma_{CPB}}{2\pi}]$$

$\frac{\chi}{2\pi} \approx 1.6$ kHz

$\frac{\gamma_{NEMS}}{2\pi} \approx 1-2$ kHz

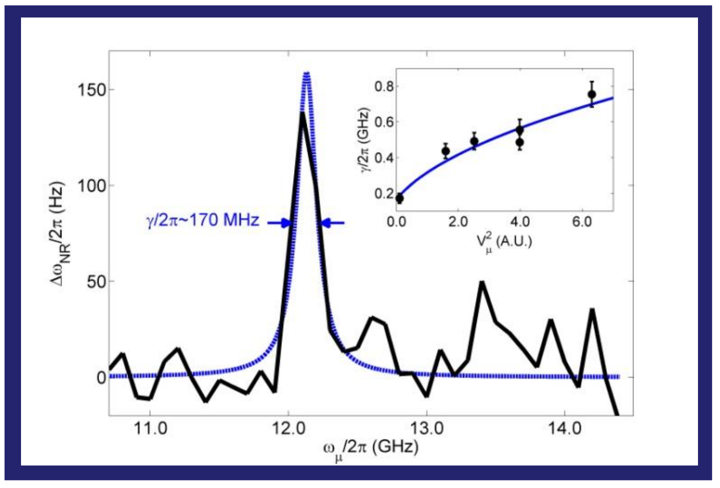
Present Sample: NEMS Linewidth



Demonstrated $\chi/2\pi > \gamma_{NEMS}/2\pi$

With conservative improvements to sample geometry, should achieve χ ~ 100's kHz

Present Sample: CPB Linewidth



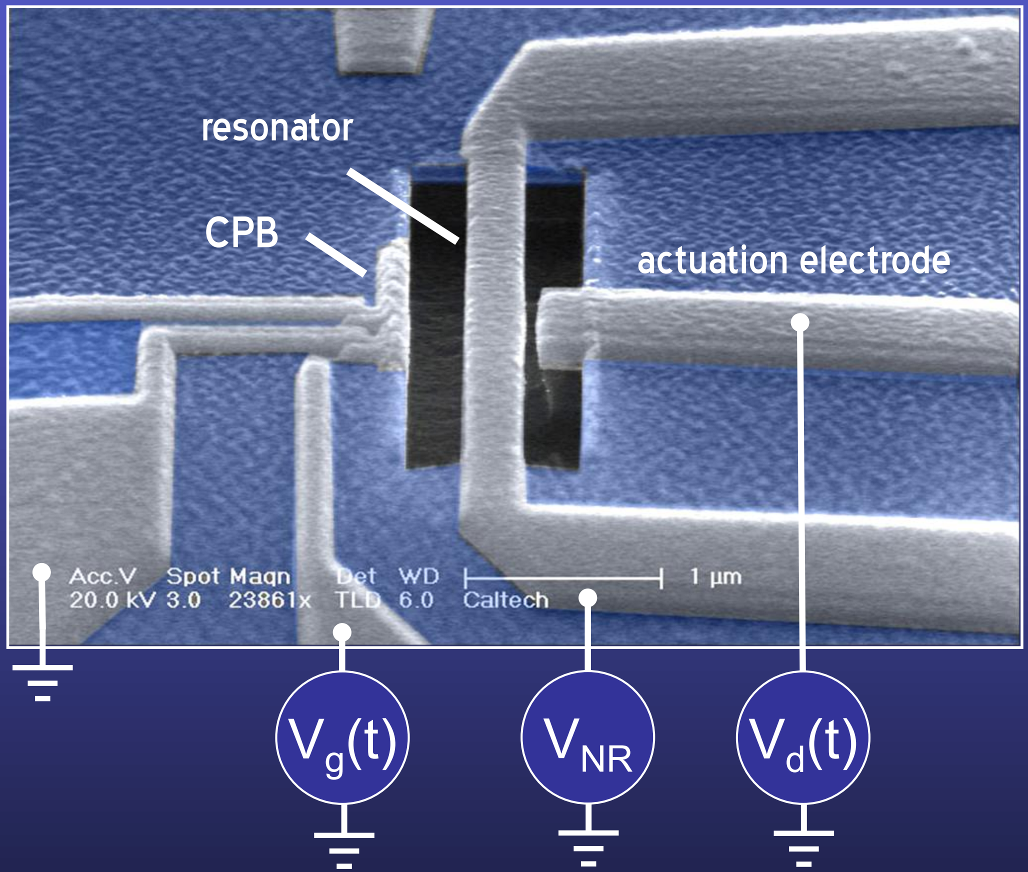
Present sample: $\gamma_{CPB}/2\pi >> \chi/2\pi$

$\frac{\gamma_{CPB}}{2\pi} \approx 170$ MHz

However, there is significant room to improve, e.g. in circuit QED, $\gamma_{CPB}/2\pi < 1$ MHz

e.g. see Wallraff et al., Nature 431 (2004)

parametric amplification of nanomechanical motion



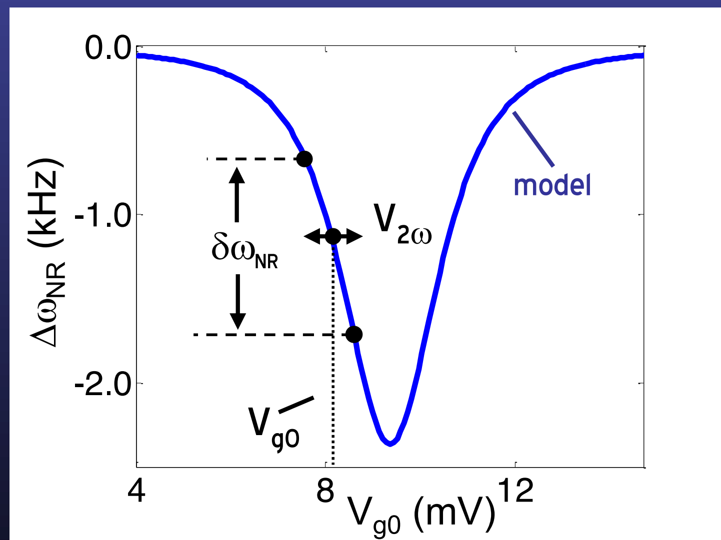
Drive NEMS via actuation electrode

$$V_d(t) = V_{d0} + V_d \sin(\omega_{NR} t + \varphi)$$

Pump CPB gate at $2\omega_{NR}$

$$V_g(t) = V_{g0} + V_{2\omega} \sin(2\omega_{NR} t)$$

Parametrically Pump NEMS



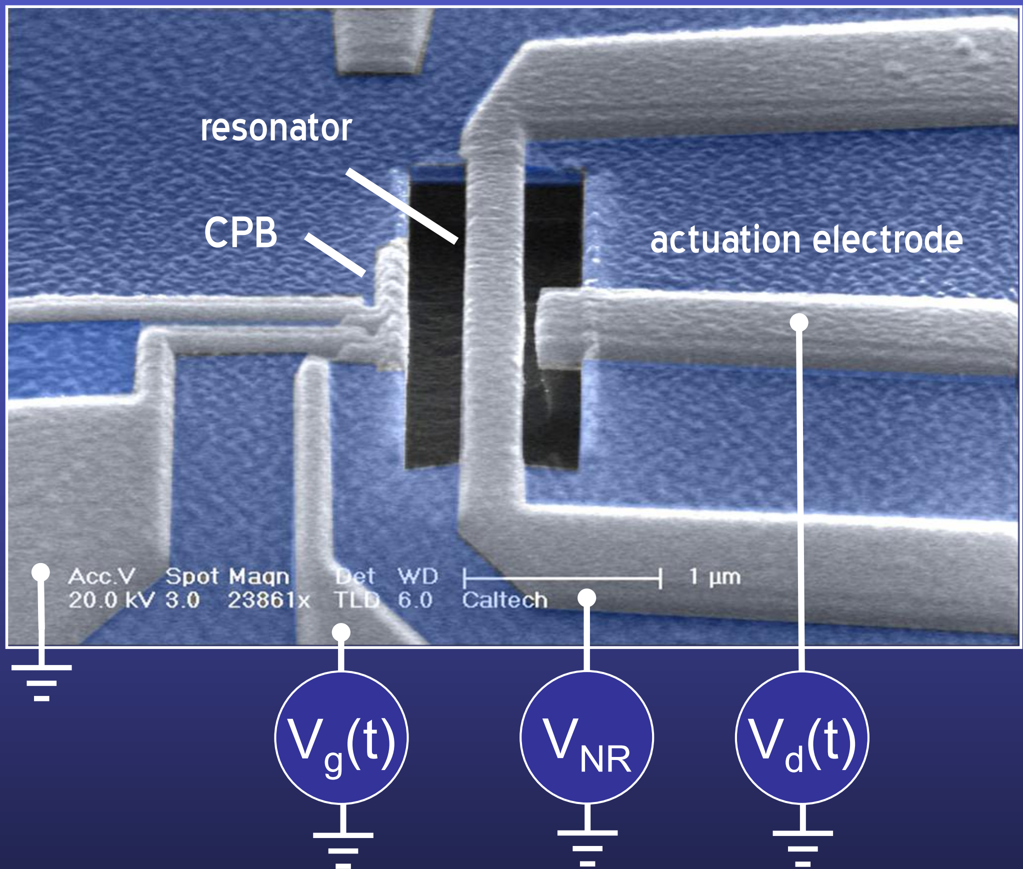
Phase-Sensitive Mechanical Gain

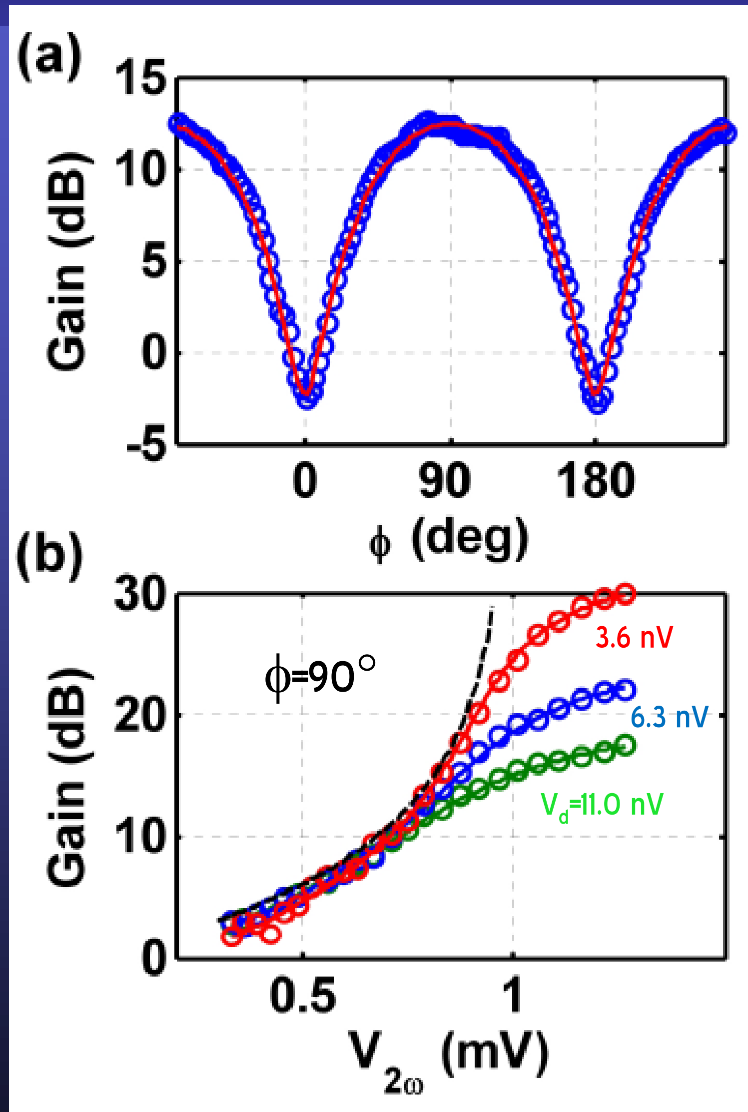
$$\text{Gain} = \left[\frac{\cos^2\varphi}{1 + \delta\omega_{NR} Q / \omega_{NR}} + \frac{\sin^2\varphi}{1 - \delta\omega_{NR} Q / \omega_{NR}} \right]$$

$$\delta\omega_{NR} \approx \frac{\partial}{\partial V_g}(\Delta\omega_{NR}) \cdot V_{2\omega}$$

parametric amplification of nanomechanical motion

Measurements taken by Junho Suh





Phase-Sensitive Mechanical Gain

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parametric amplification of nanomechanical motion

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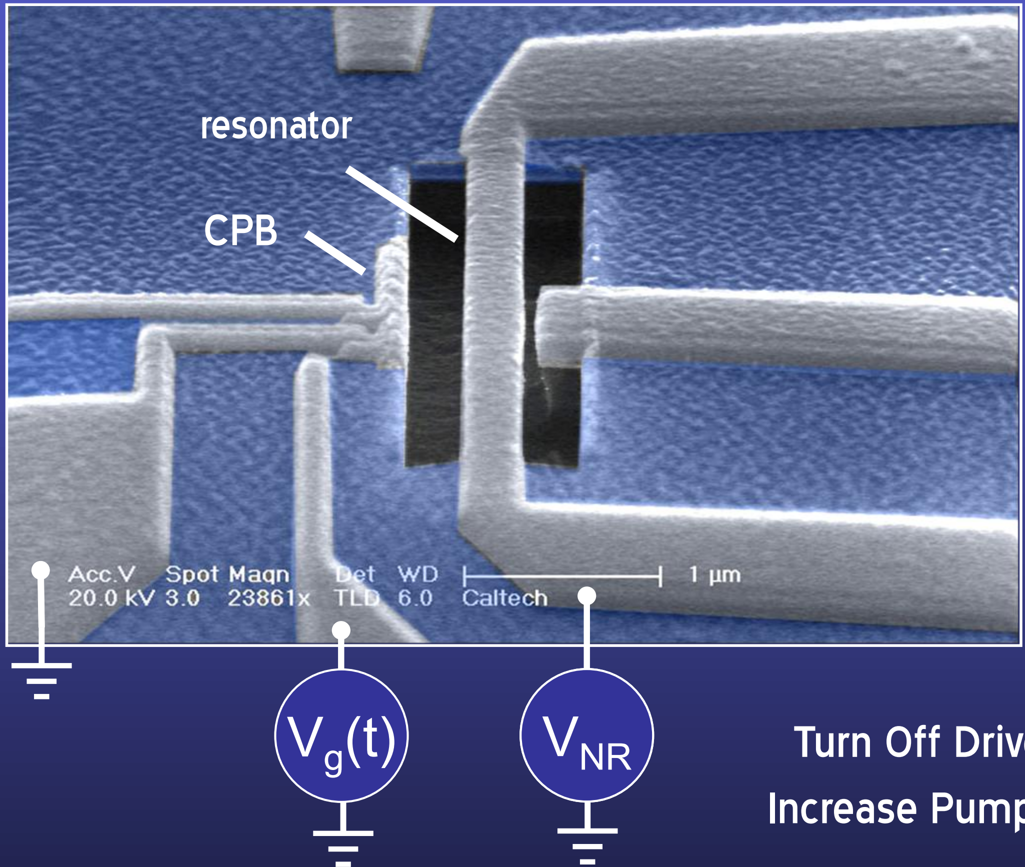
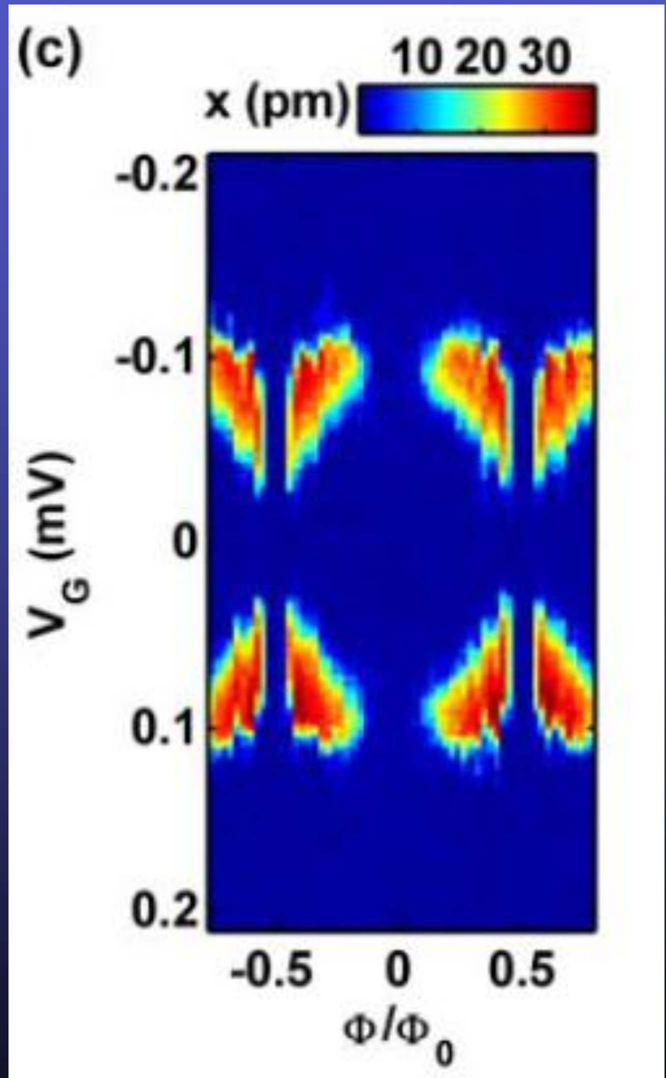


Turn Off Drive &
Increase Pump $V_{2\omega}$

Parametric-Oscillation of the Mechanics



Phase-Sensitive Mechanical Gain

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parametric amplification of nanomechanical motion

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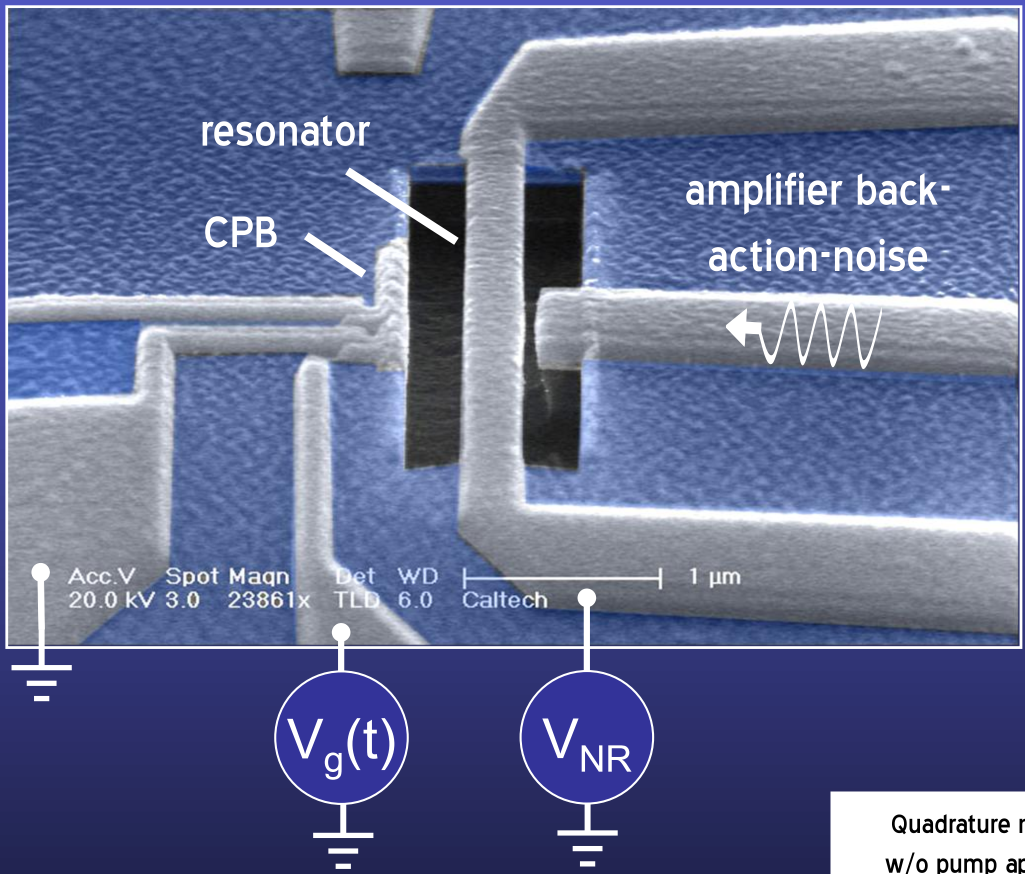
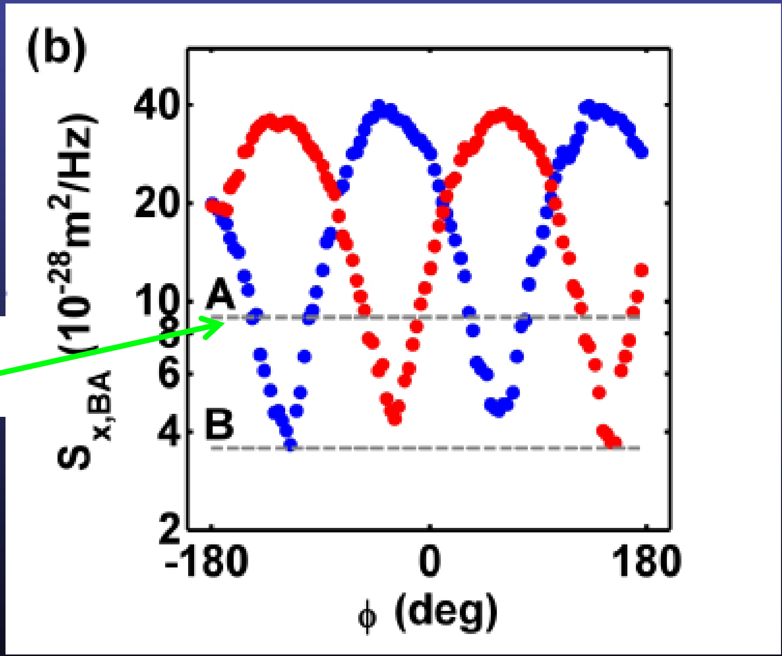


Amplifier back-action noise drives NEMS

Squeeze noise in one quadrature, and amplify noise in the other quadrature (line 'A' is noise level w/o pump applied)

Classical Noise Squeezing

Quadrature noise w/o pump applied



Resonator Amplitude

$$A = \frac{FQ\omega_{NR}}{K_{NR}}\left[\frac{\cos\varphi}{1+Q\delta\omega_{NR}/\omega_{NR}} + j\frac{\sin\varphi}{1-Q\delta\omega_{NR}/\omega_{NR}}\right]$$

conclusions

- Have demonstrated tunable coupling between CPB charge qubit and nanoresonator. Well-described by a simple dispersive model.
- Used the dispersive interaction to perform spectroscopy of CPB and observe LZS interference effects.
- Demonstrated use of CPB for parametric amplification of the mechanics.
- Lots of room for improvement in engineering coupling strength, as well as qubit quality (e.g. moving to higher E_j/E_c, embedding coupled device in SMR). Prospects for using the CPB to manipulate/measure quantum states of the mechanics looks promising.

CPB Quantum Capacitance

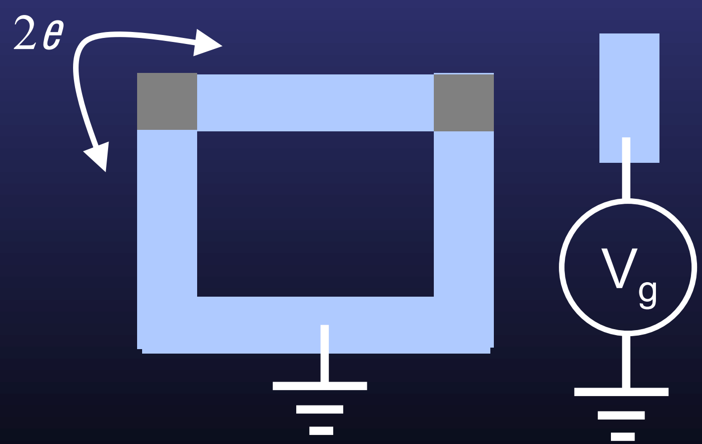
Ground State

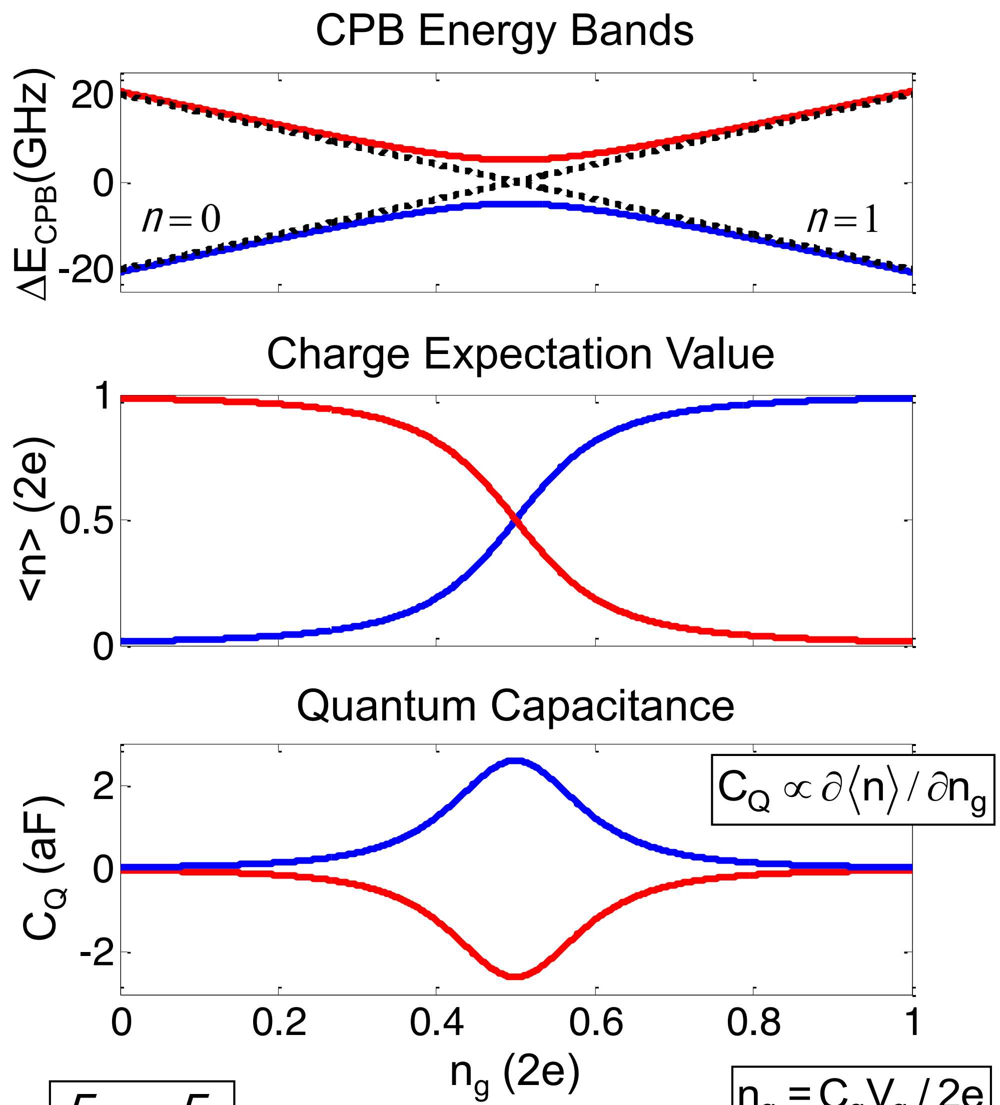
Excited State

Electrostatic Energy alone

CPB Hamiltonian

$$\hat{H}_{CPB} = 2E_C(1-2n_g)\hat{\sigma}_Z - \frac{E_J}{2}\hat{\sigma}_X$$





$E_C = E_J$

$n_g = C_g V_g / 2e$

CPB Quantum Capacitance

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