The Abdus Salam
International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA

2164-8

Workshop on Nano-Opto-Electro-Mechanical Systems Approaching the Quantum Regime

6 - 10 September 2010

Multimode Optomechanics

Florian MARQUARDT
Institute of Theoretical Physics II
University of Erlangen-Nuremberg, Staudtstr.7, 91058
Erlangen
GERMANY

Strada Costiera 11, 34151 Trieste, Italy - Tel.+39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it

Multimode Optomechanics

Florian Marquardt
[just moved from: Ludwig-Maximilians-Universität München]

University of Erlangen-Nuremberg and
Max-Planck Institute for the Physics of Light (Erlangen)

DIP project "Dynamics of Electrons and Collective Modes in Nanostructures"

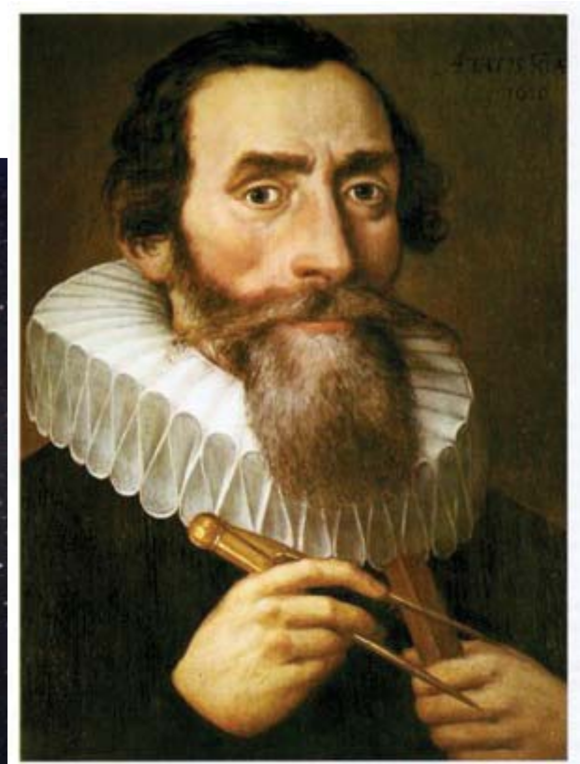
SFB | TR12

SFB 631

nanosystems initiative munich

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

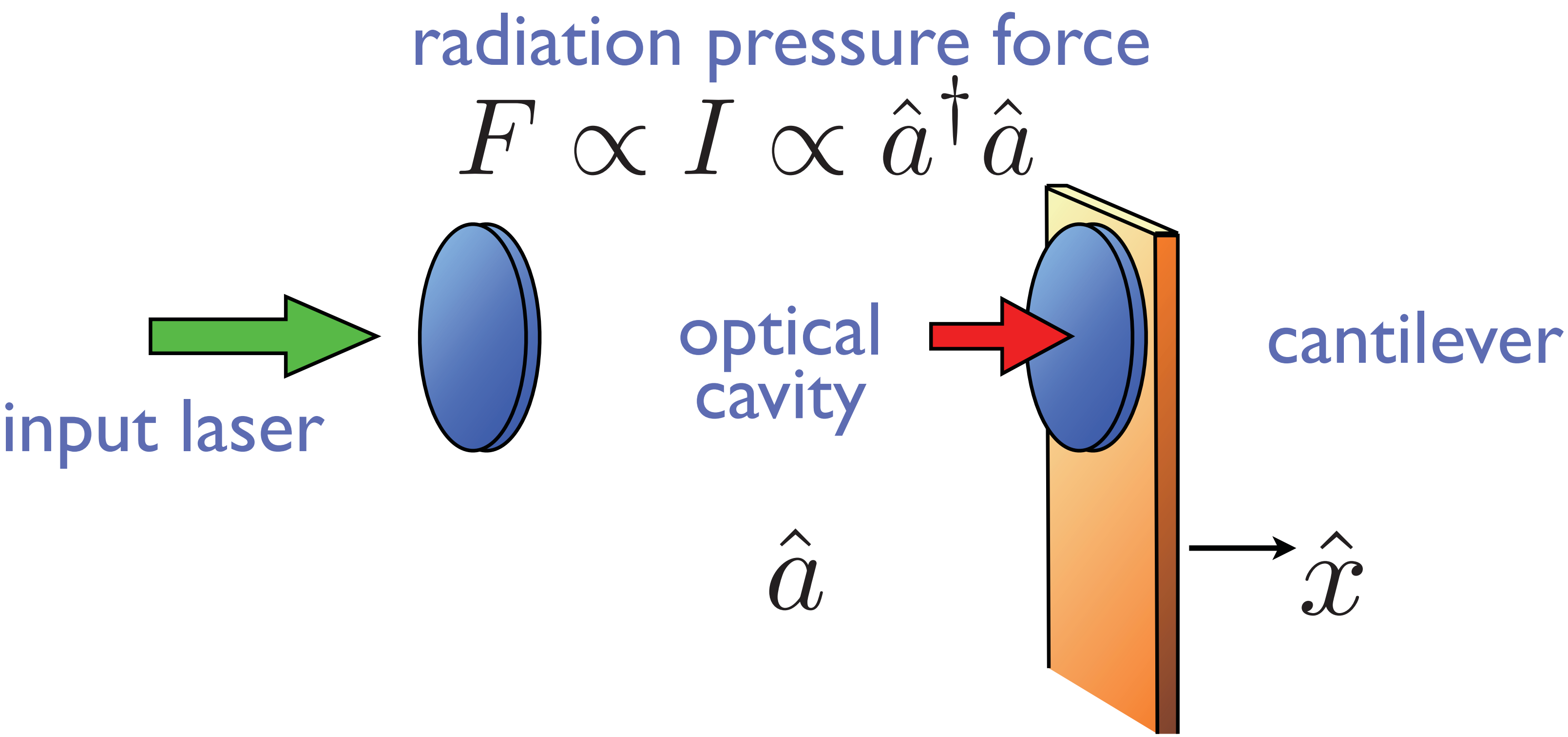
Radiation pressure



Johannes Kepler
De Cometis, 1619

(Comet Hale-Bopp; by Robert Allevo)

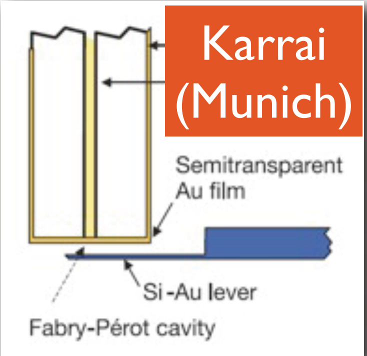
Optomechanical systems

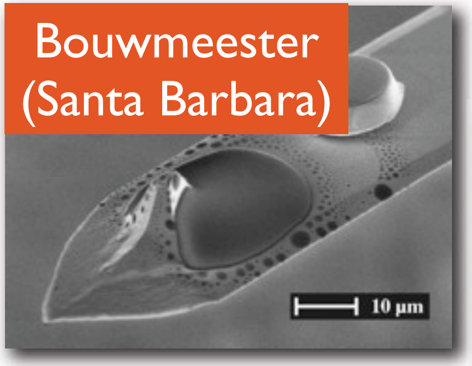


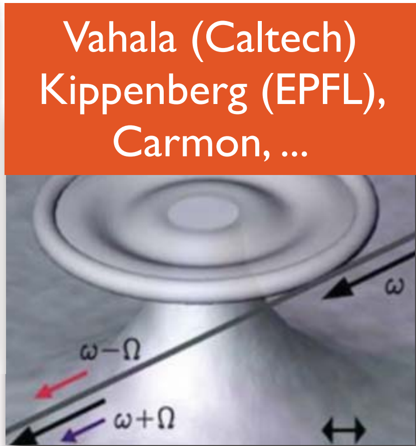
$$\hat{H} = \hbar\omega_R(1 - \hat{x}/L)\hat{a}^\dagger\hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} + \ldots$$

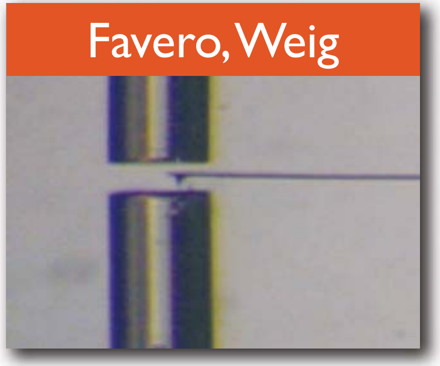
$$\hat{x} = x_{\mathrm{ZPF}}(\hat{b}^\dagger + \hat{b})$$

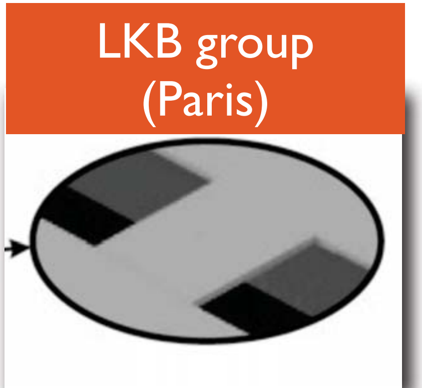
The zoo of optomechanical (and analogous) systems

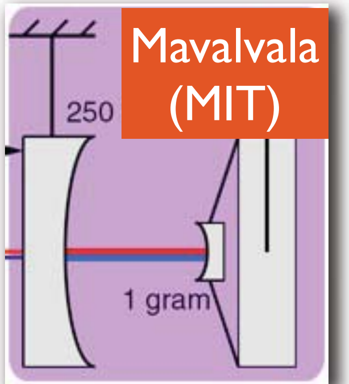


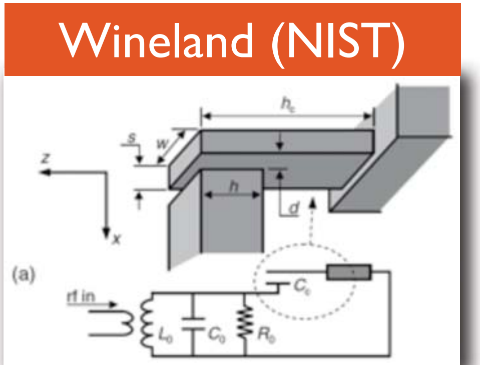


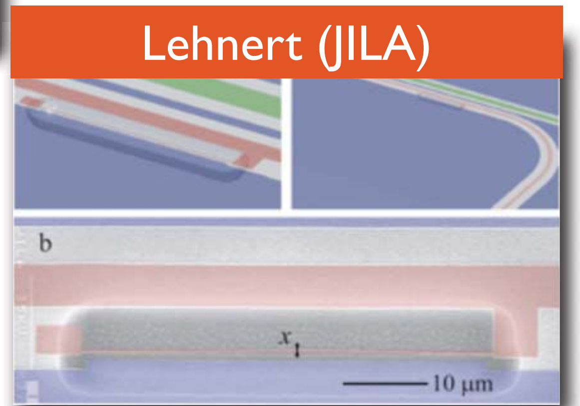












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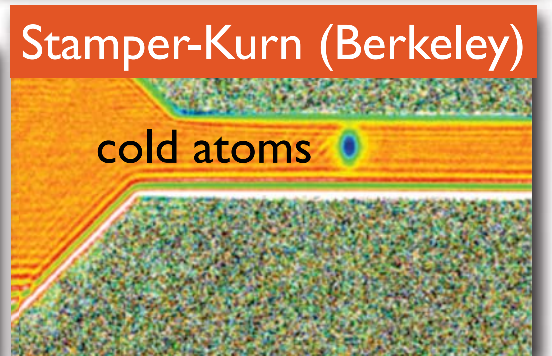
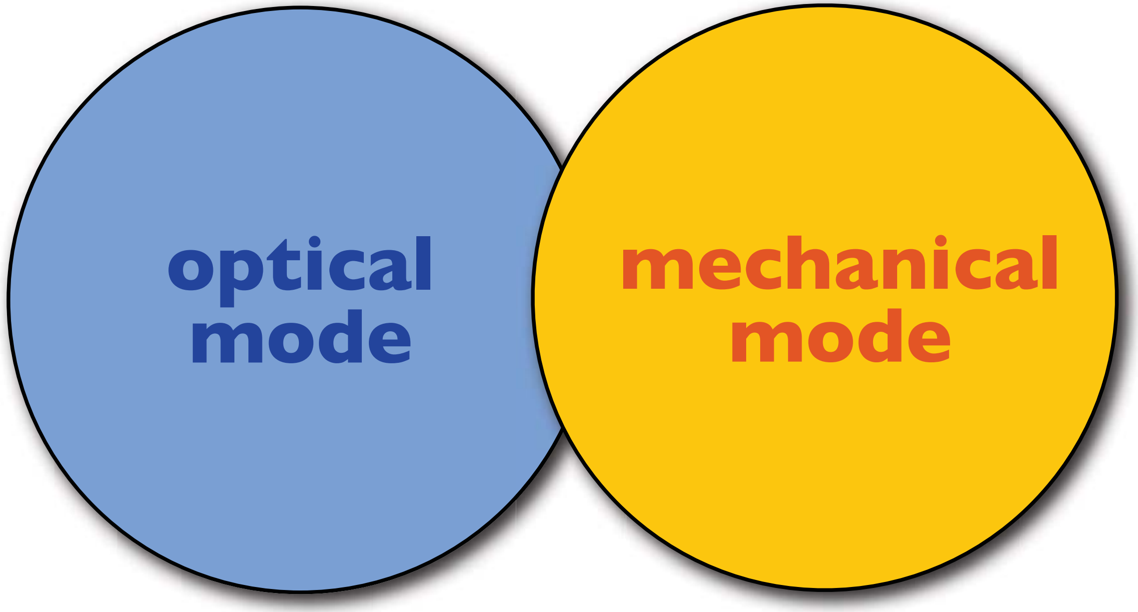


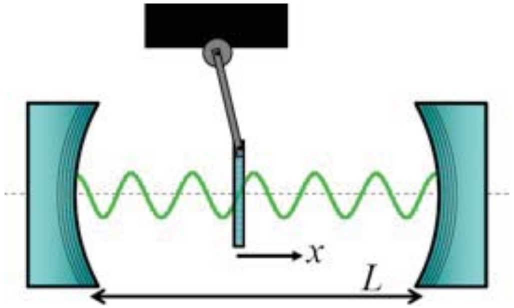
Single-mode optomechanics



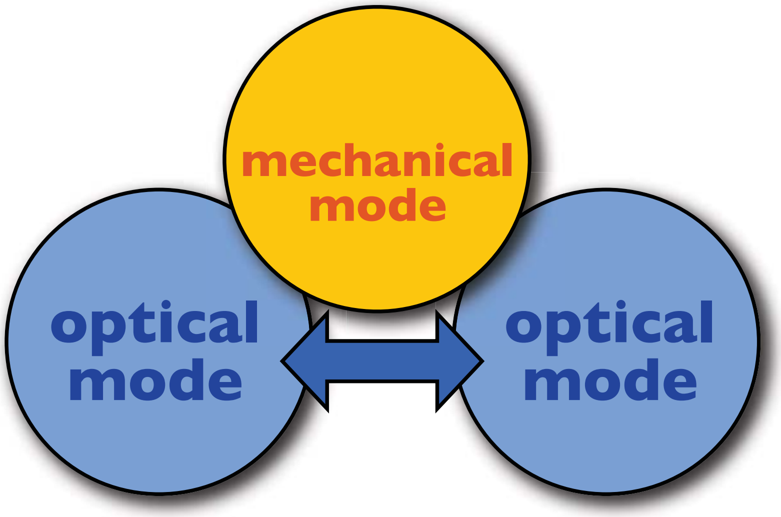
- displacement sensing
- cooling
- strong coupling
- self-oscillations

Multimode optomechanics

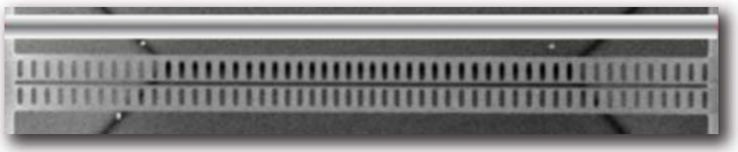
Harris (Yale)



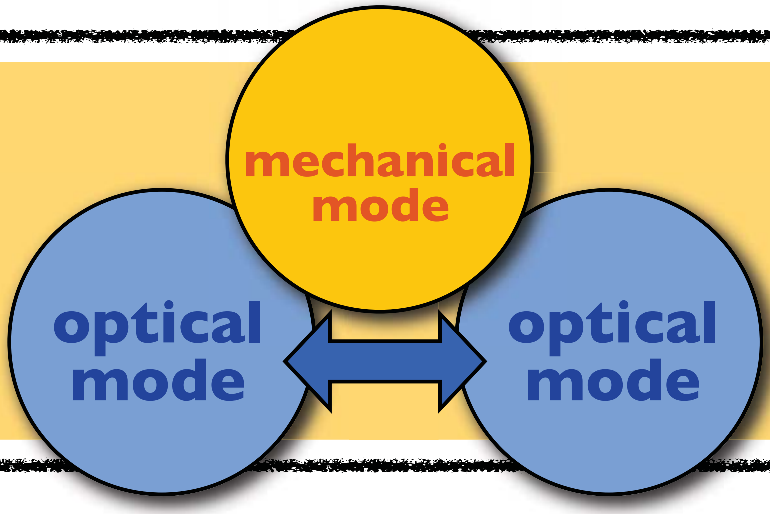
Multimode optomechanics



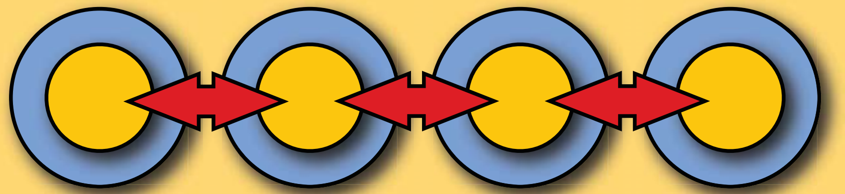
Painter (Caltech)



Multimode optomechanics



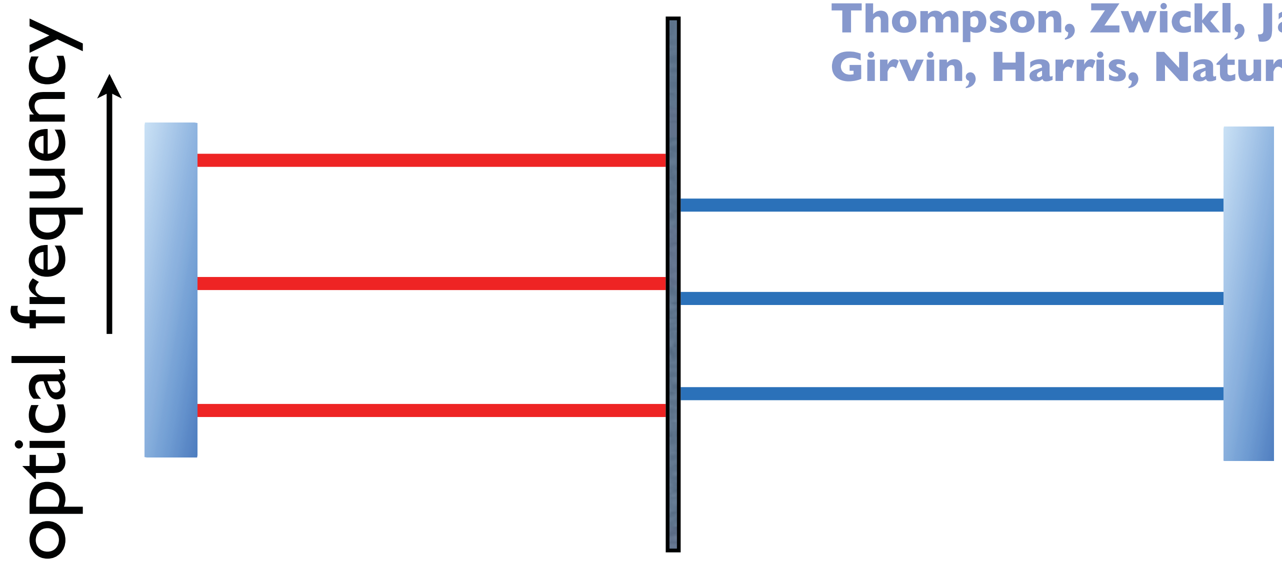
The photon shuttle

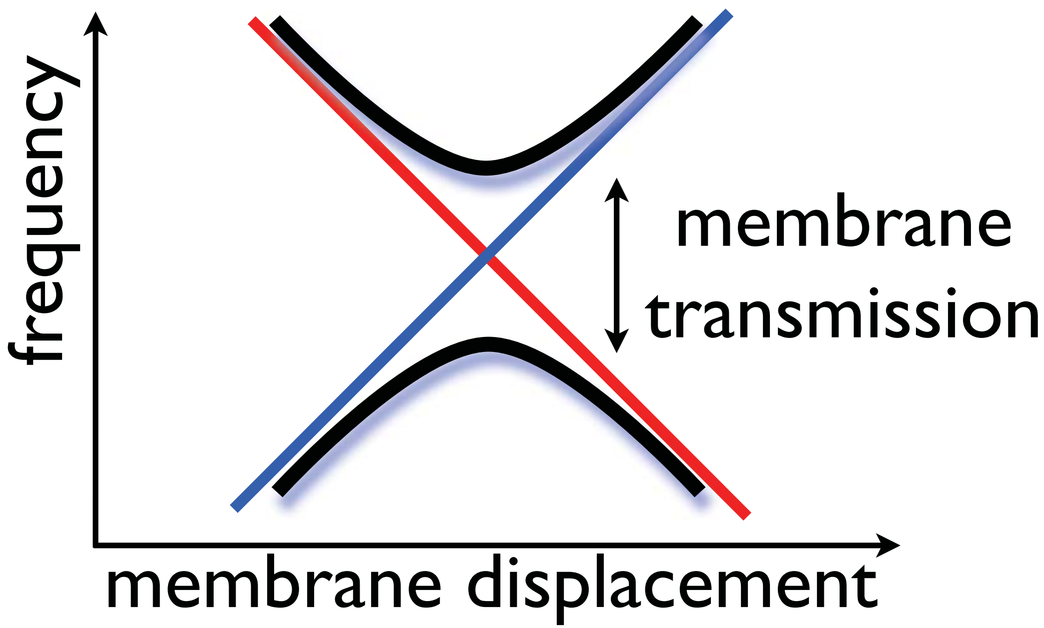


Collective dynamics in optomechanical arrays

“Membrane in the middle” setup

Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)



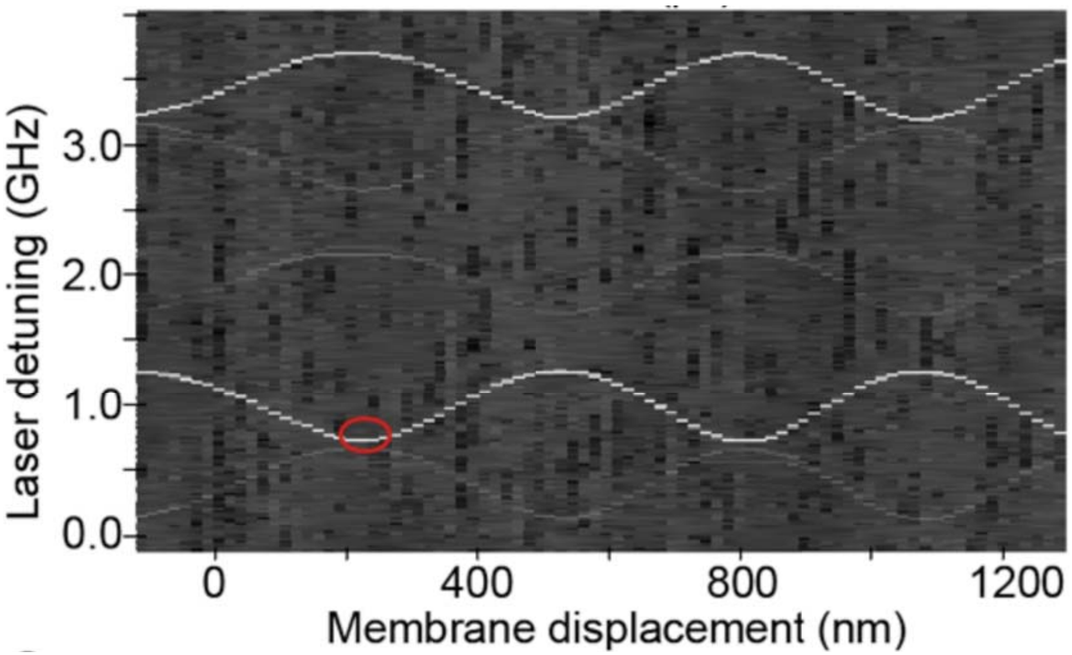


Experiment (Harris group, Yale)

Mechanical frequency:
$\omega_M = 2\pi \cdot 134$ kHz

Mechanical quality factor:
$Q = 10^6 \div 10^7$

Current optical finesse:
>10000

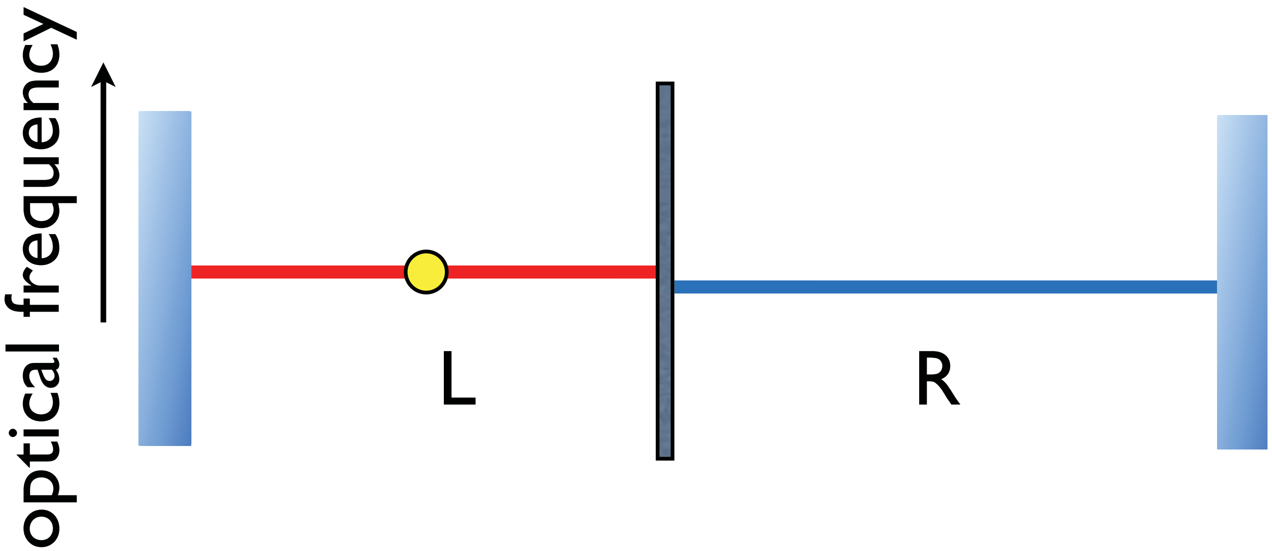


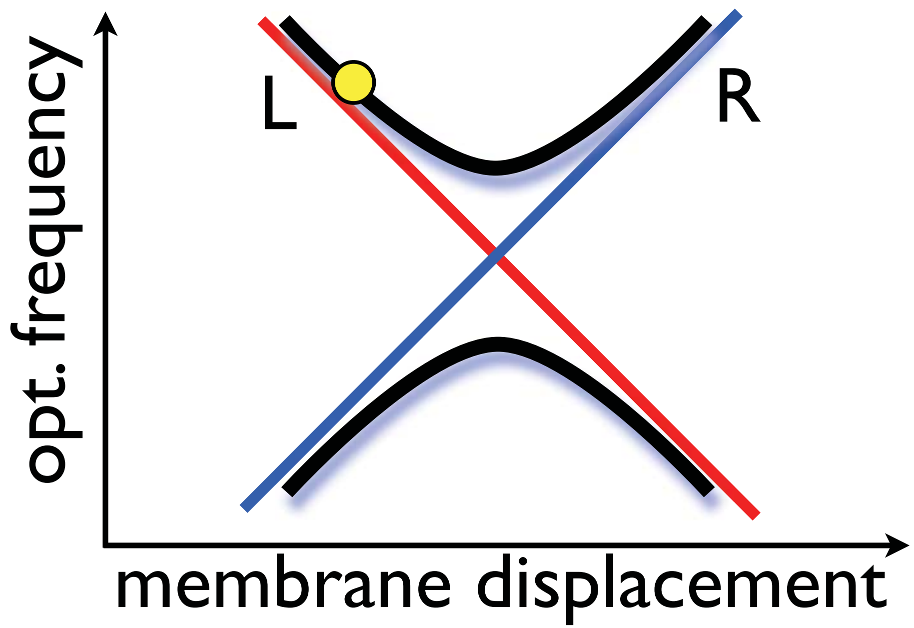
Optomechanical cooling from **300K** to **7mK**

Potential for mechanical quantum jumps / phonon shot noise measurement

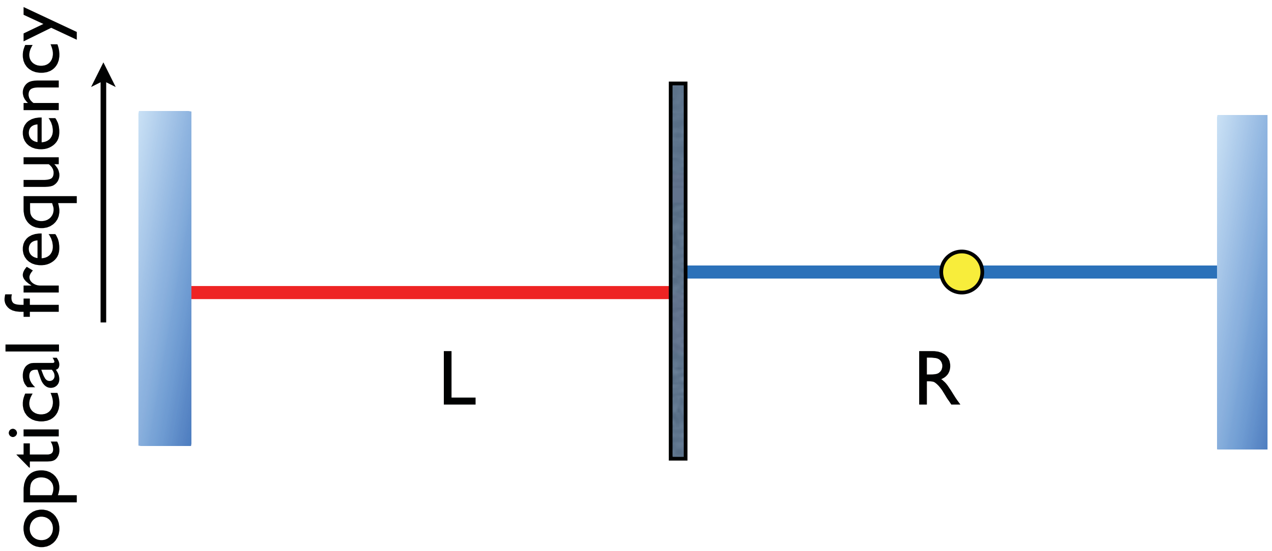
Thompson, Zwickl, Jayich, Marquardt, Girvin, Harris, Nature 72, 452 (2008)

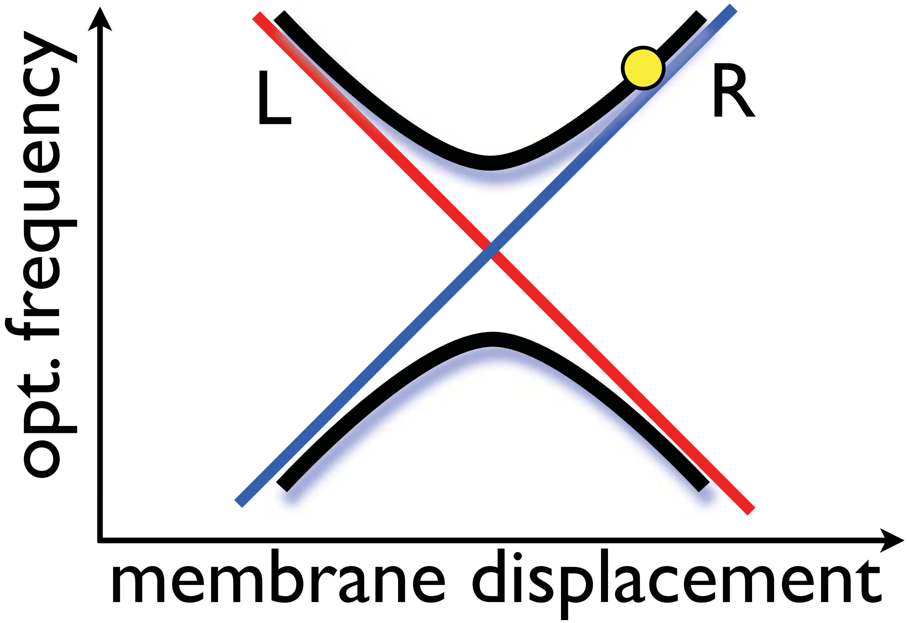
The photon shuttle



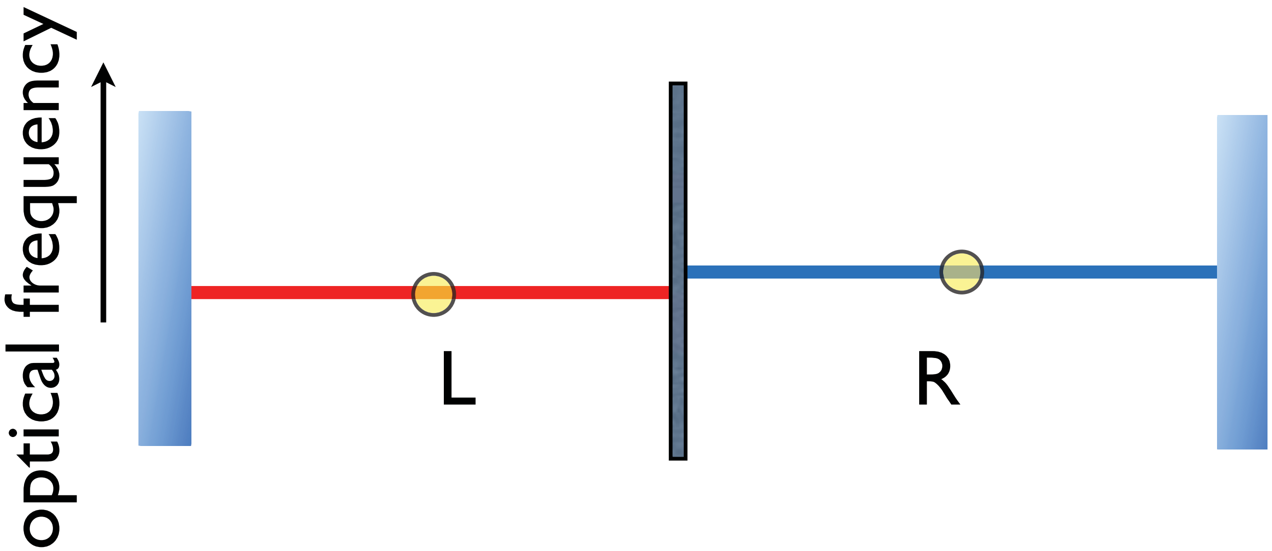


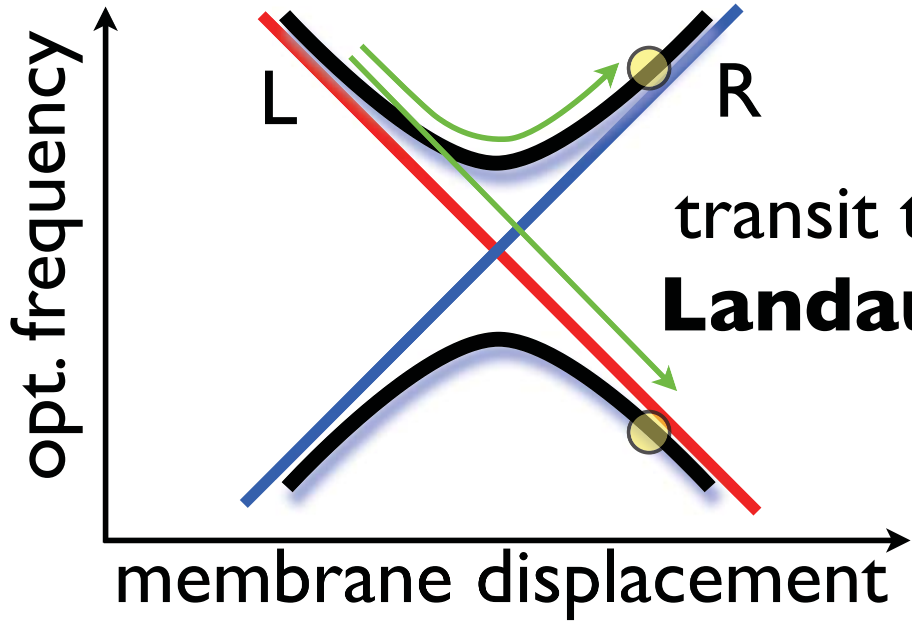
The photon shuttle





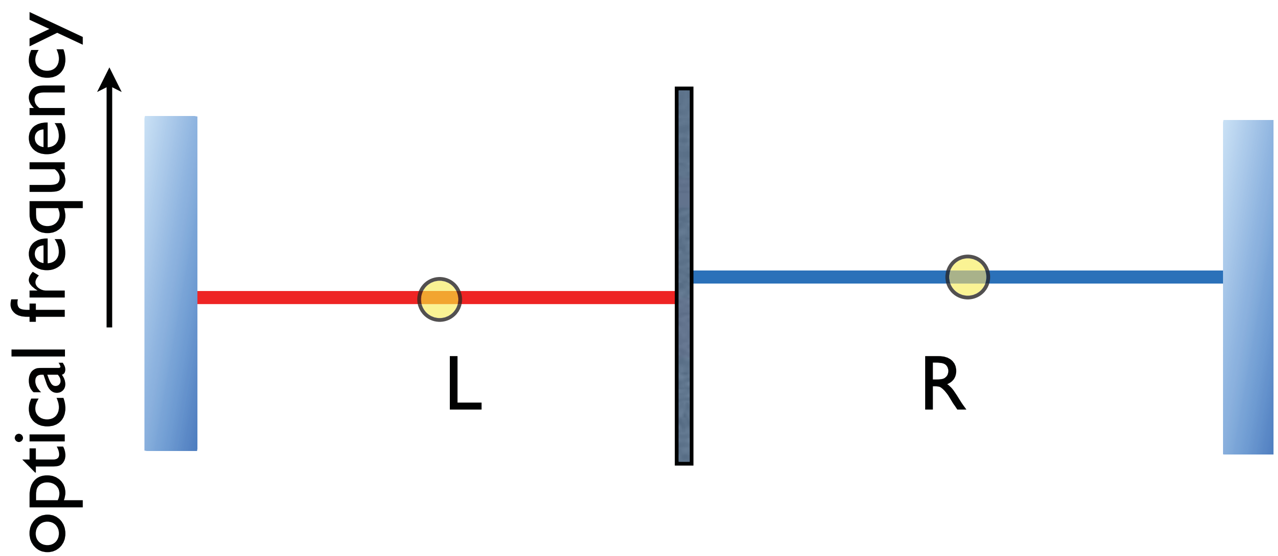
The photon shuttle

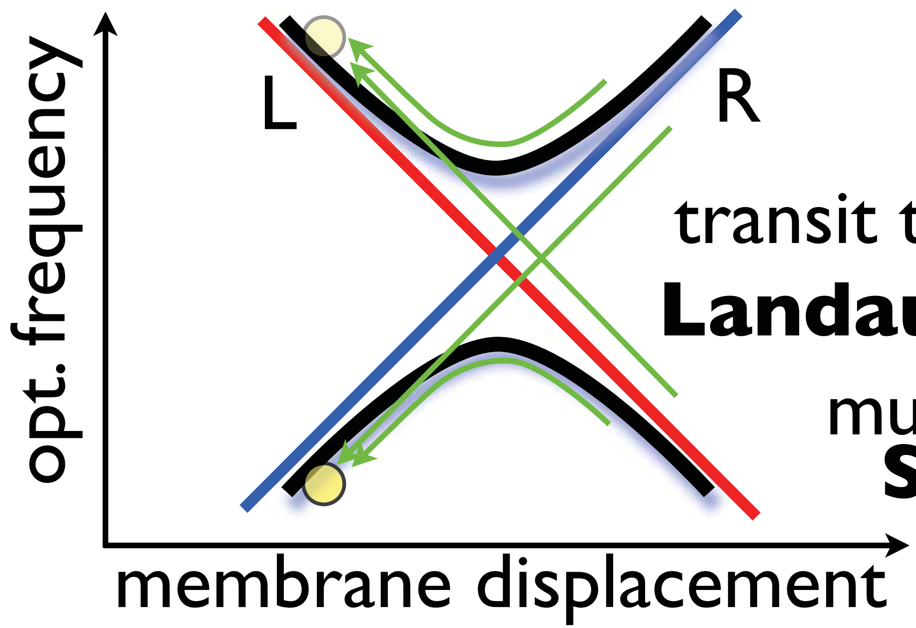




transit time < 1/splitting:
Landau-Zener physics

The photon shuttle

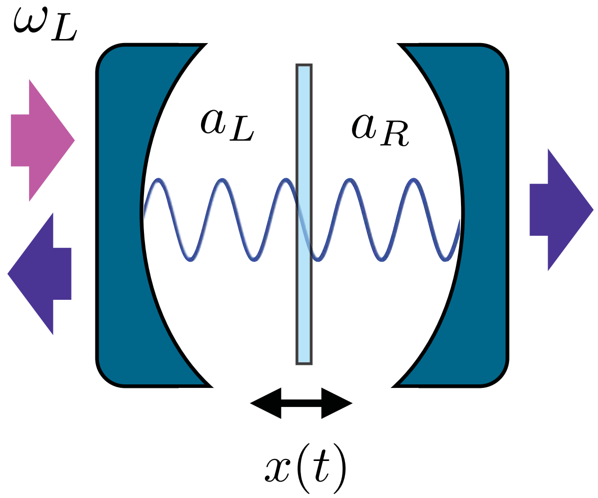


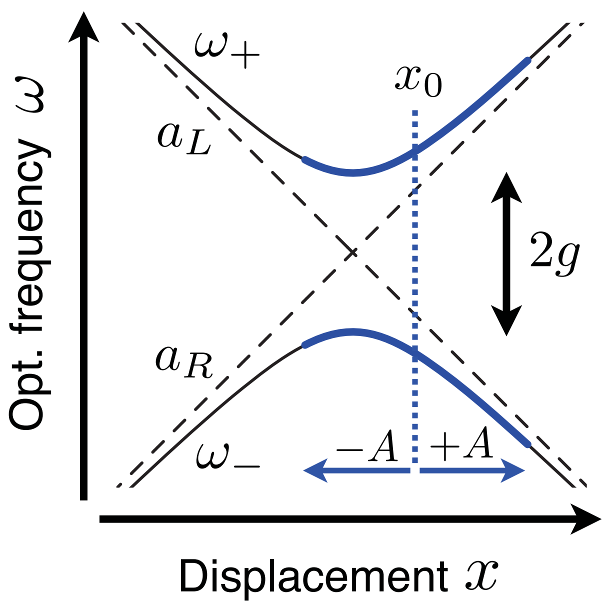


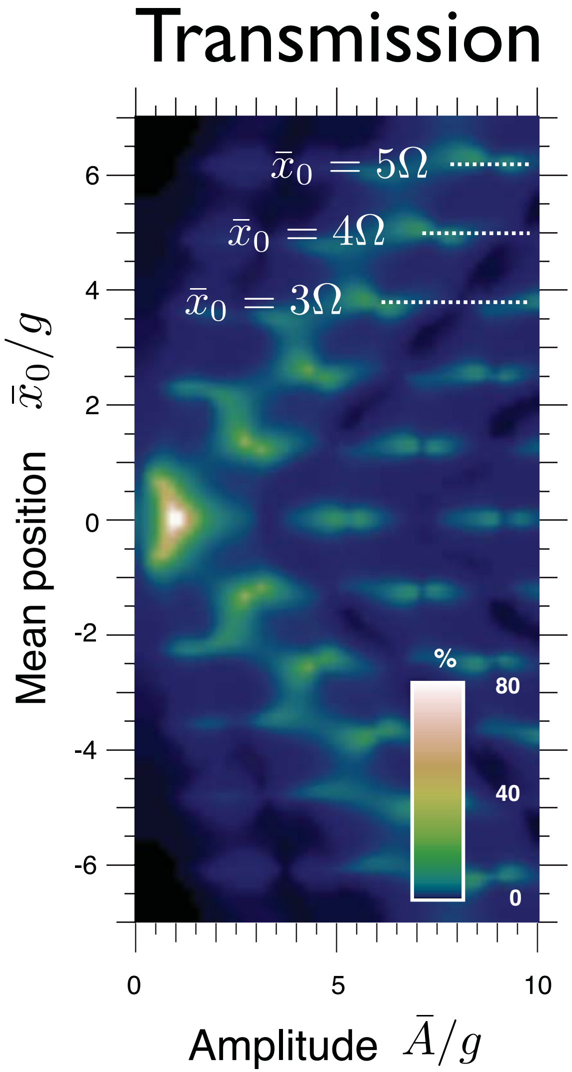
transit time < 1/splitting:
Landau-Zener physics

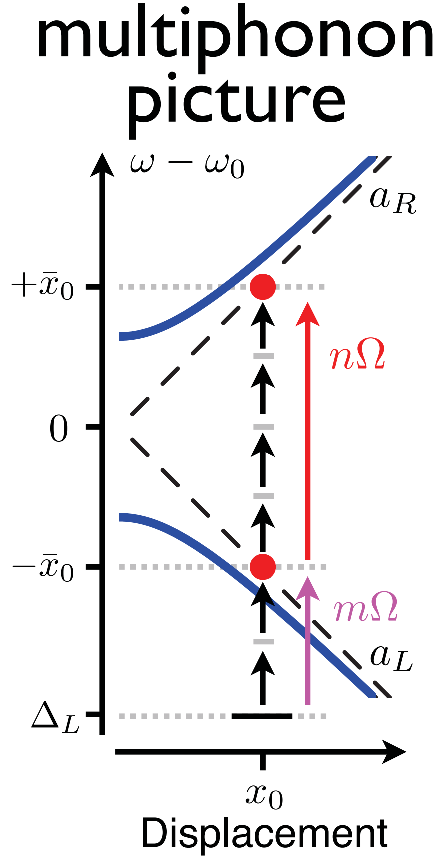
multiple transitions:
Stueckelberg
interference

Landau-Zener-Stueckelberg physics in an optomechanical setup









cf. LZS in driven qubits (W. Oliver / M. LaHaye)

Transmission under drive

Transmission

$$T(t) = \kappa^2 \left| \int_{-\infty}^{t} G(t,t')\, e^{-i\Delta_L t' - (\kappa/2)(t-t')}\, dt' \right|^2$$

Linear system: Transmission for single photon equals that of coherent state

Amplitude for transmission into right cavity mode

$$G(t,t') = \tilde{a}_R(t,t') e^{-i\phi(t')} \qquad \phi(t') = (\bar{A}/\Omega)\sin(\Omega t')$$

Equations of motion in time-dependent rotating frame

$$i\frac{d}{dt}\begin{pmatrix} \tilde{a}_R \\ \tilde{a}_L \end{pmatrix} = \begin{pmatrix} \bar{x}_0 & g e^{+2i\phi(t)} \\ g e^{-2i\phi(t)} & -\bar{x}_0 \end{pmatrix} \begin{pmatrix} \tilde{a}_R \\ \tilde{a}_L \end{pmatrix}$$

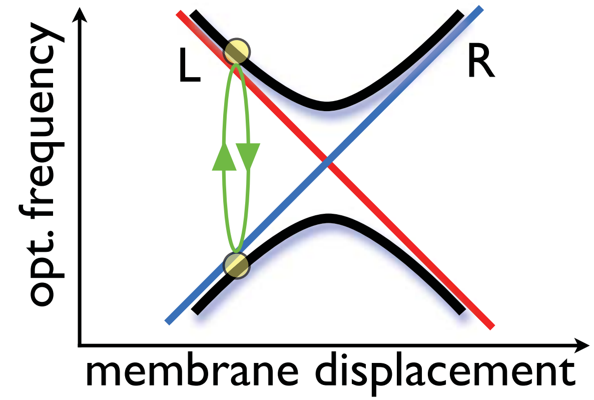
$$\tilde{a}_R(t',t') = 0$$

$$\tilde{a}_L(t',t') = 1$$

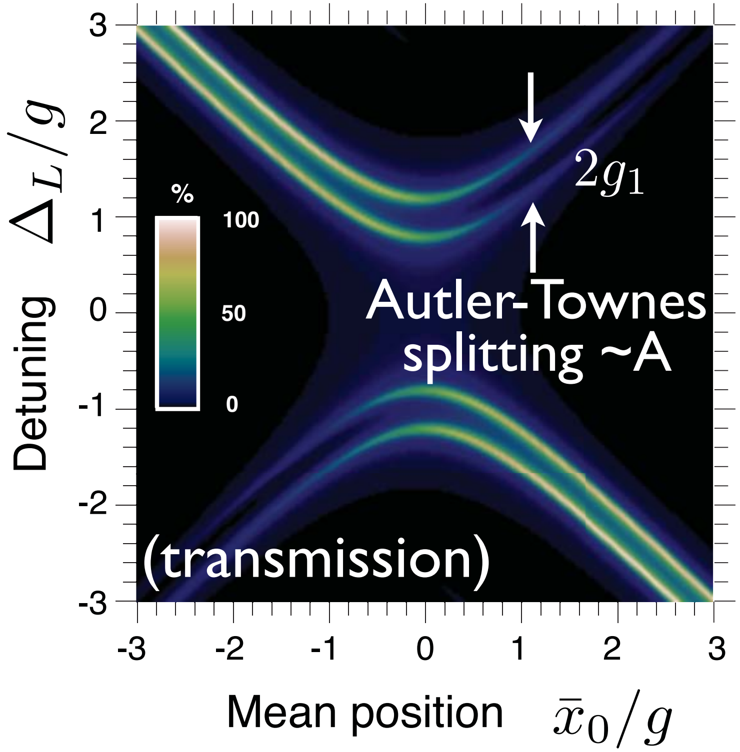
$$g e^{2i\phi(t)} = g \sum_n J_n(2\bar{A}/\Omega) e^{in\Omega t}$$

Characteristic Bessel function dependence on drive amplitude

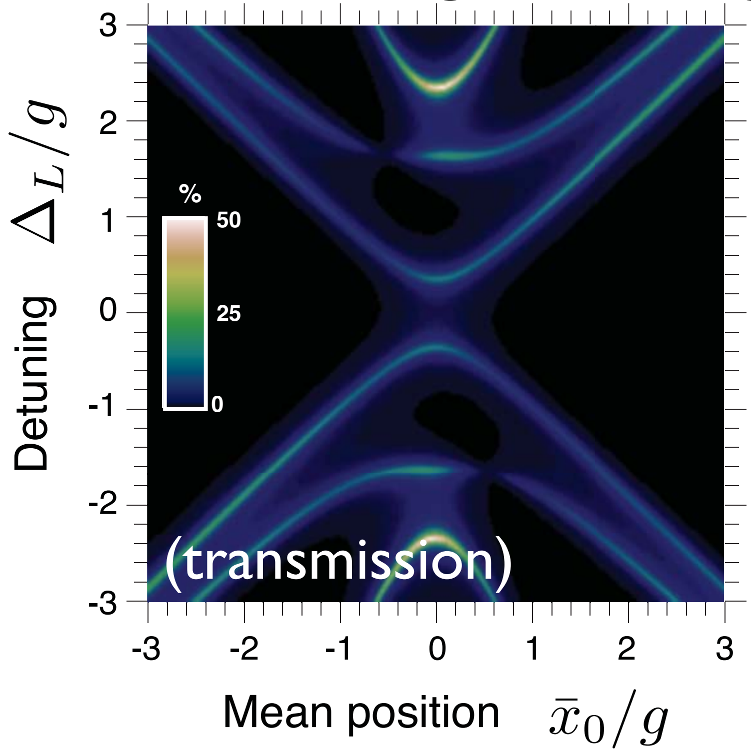
Mechanically driven Rabi oscillations and Autler-Townes splitting



weak driving

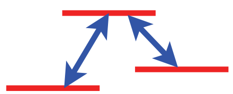


stronger driving



Applications

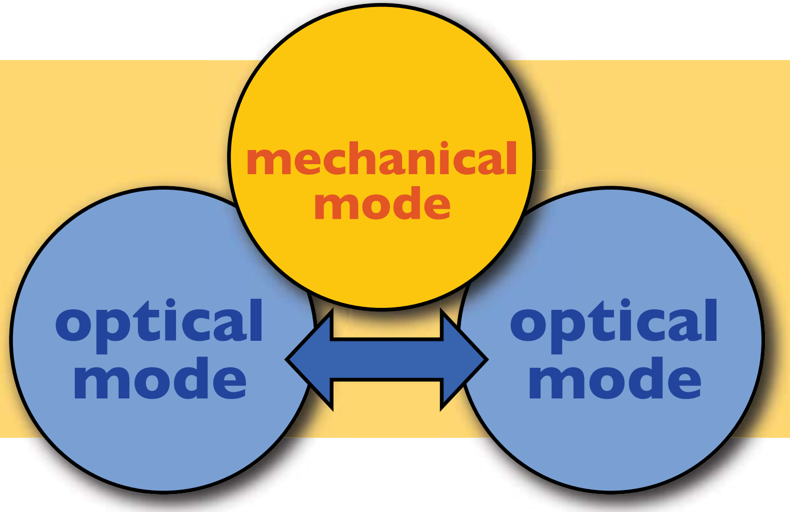
- Optomechanical circuits: transfer photons between modes (coherent, switchable, frequency conversion)
- Implement atomic multi-level physics for optical modes



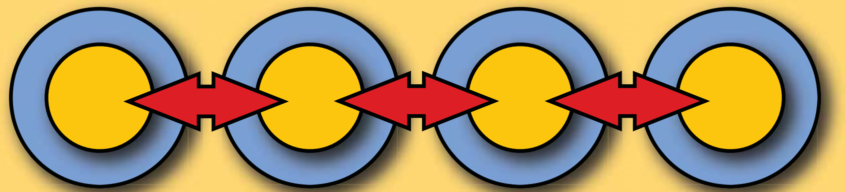
- Self-oscillations: Interference pattern determines stable attractors

cf. poster by Huai-Zhi Wu & G. Heinrich

Multimode optomechanics



The photon shuttle



Collective dynamics in optomechanical arrays

Scaling down

cm

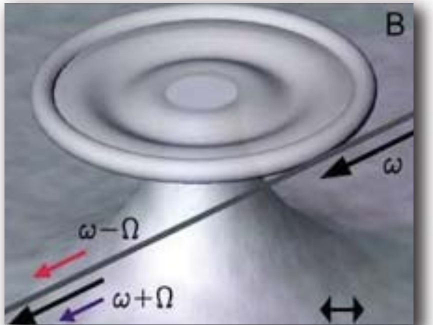
usual optical cavities

10 µm

LKB, Aspelmeyer, Harris, Bouwmeester, ...

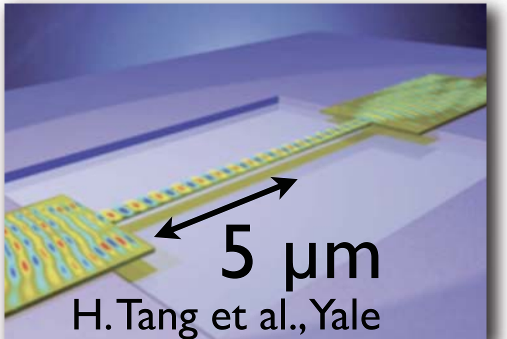
50 µm

microtoroids

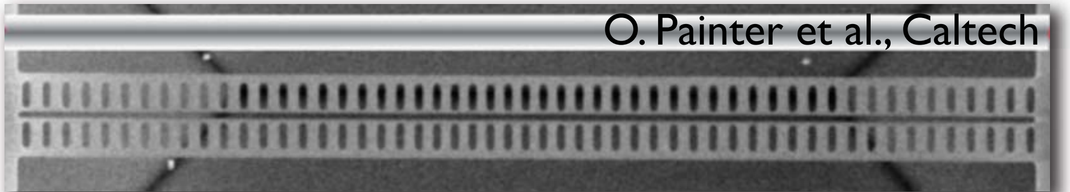


Vahala,
Kippenberg,
Carmon, ...

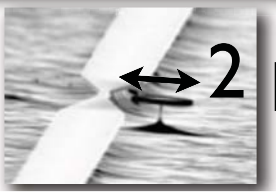
optomechanics in photonic circuits



optomechanical crystals



10 µm



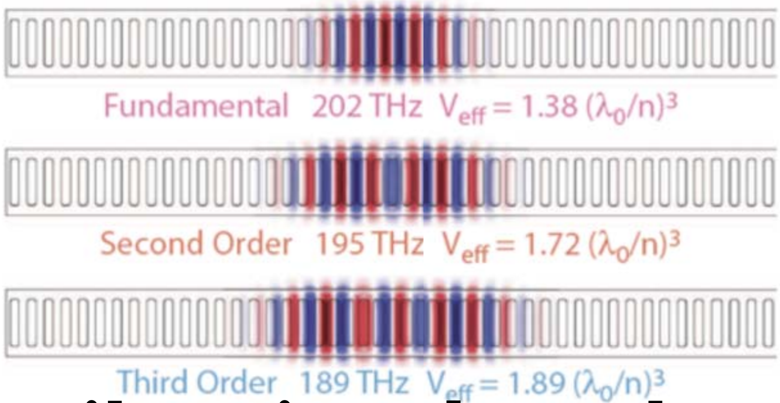
2 µm

Favero, Paris GaAs disks

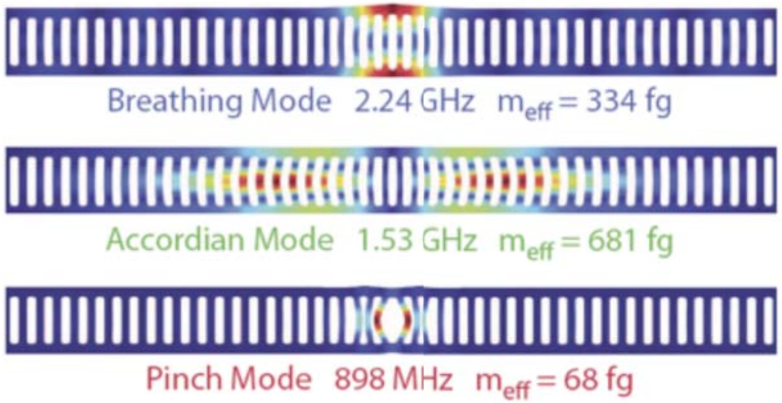
Optomechanical crystals

free-standing photonic crystal structures

optical modes



vibrational modes



advantages:

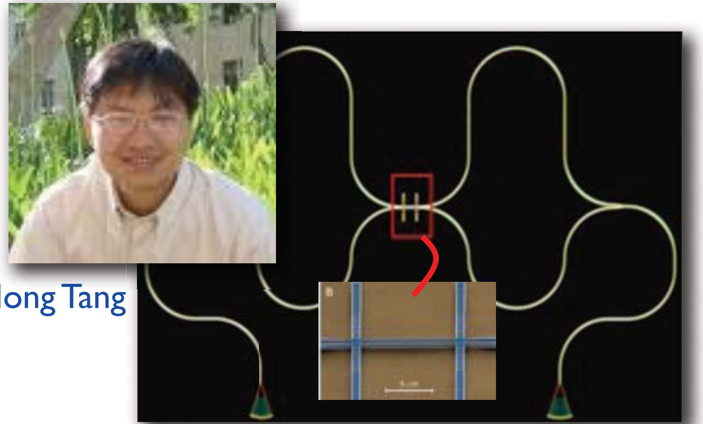
tight vibrational confinement:
high frequencies, small mass
(stronger quantum effects)

tight optical confinement:
large optomechanical coupling
(100 GHz/nm)

integrated on a chip

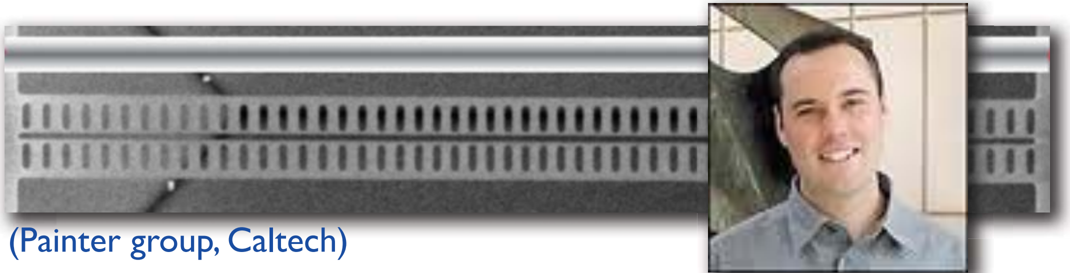
from: M. Eichenfield et al., Optics Express 17, 20078 (2009), Painter group, Caltech

Integrated optomechanical circuits on the chip



Hong Tang

(courtesy Hong Tang, Yale)



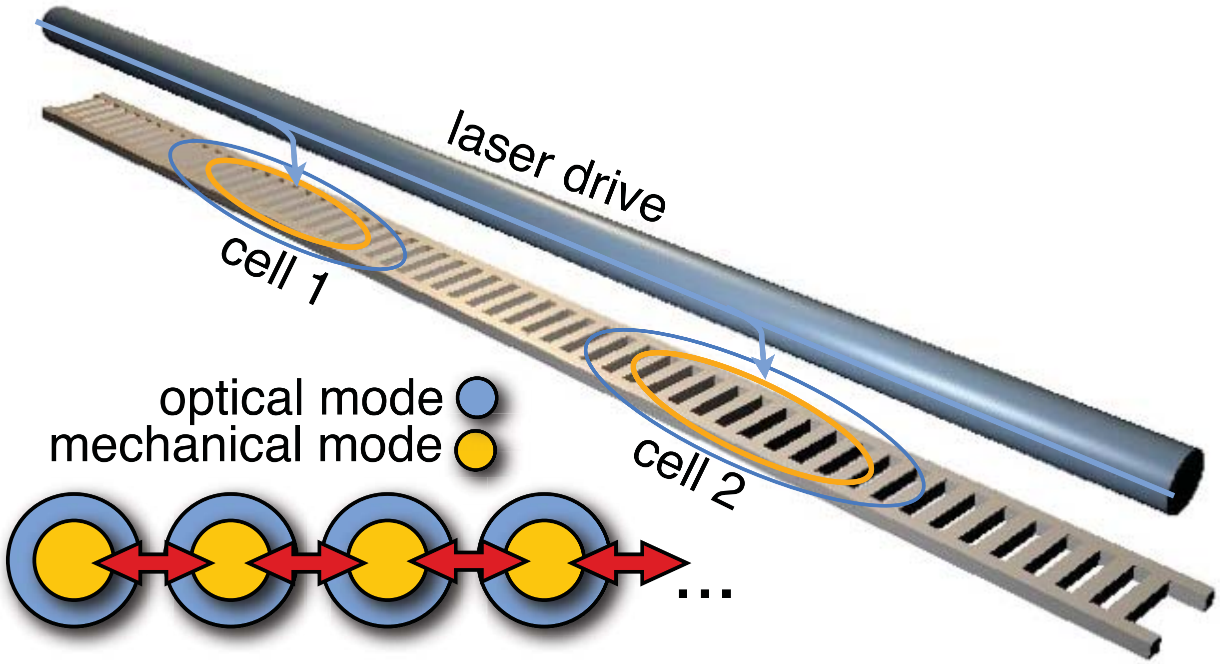
(Painter group, Caltech)

Oskar Painter

Integrated optomechanical circuits:

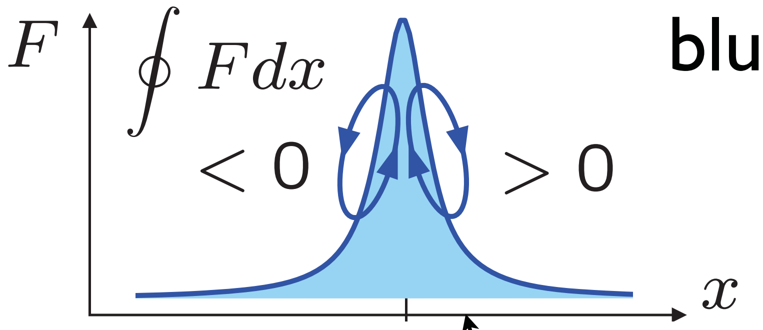
- on-chip manipulation of light (squeezing, nonlinear interactions, ...)
- signal amplification (combined with detection of small masses/forces/displacements)
- general information processing (classical or quantum)

Optomechanical arrays



Collective dynamics:
classical / quantum

Nonlinear dynamics of a single optomechanical cell: Self-induced oscillations



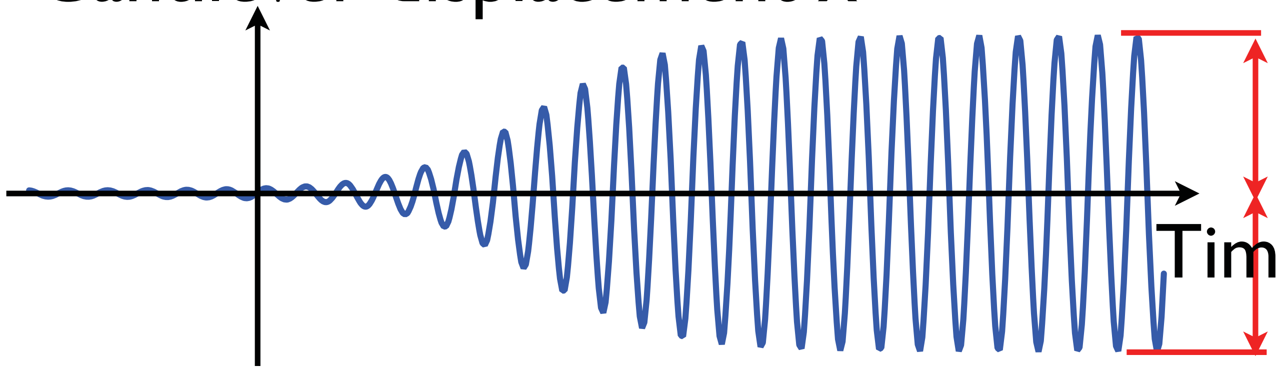
blue-detuned laser:
anti-damping

red-detuned laser:
damping (cooling)

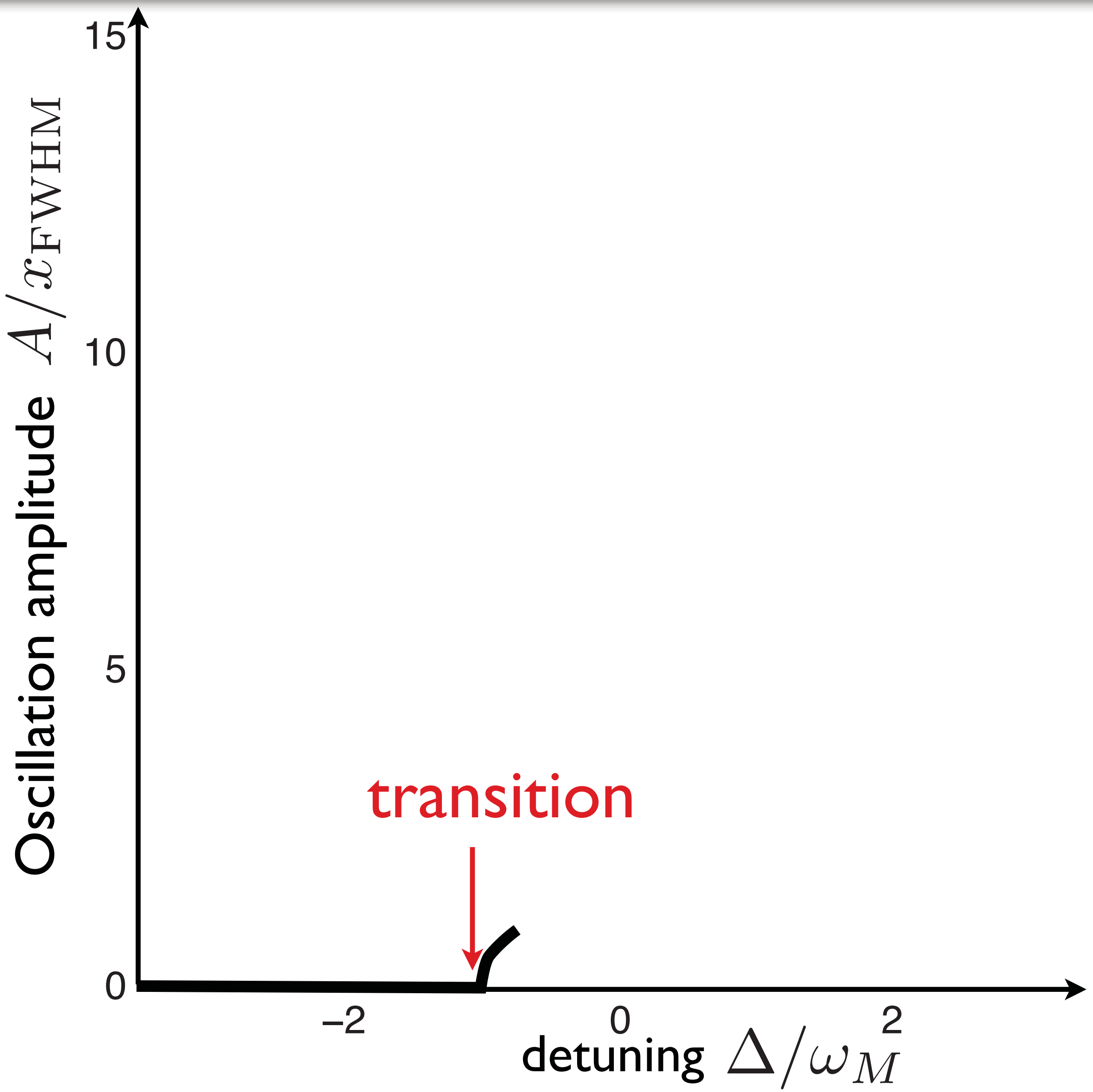
$$\Gamma = \Gamma_M + \Gamma_{\mathrm{opt}} < 0$$

Beyond some laser input power threshold: instability

Cantilever displacement x



Attractor diagram



Höhberger, Karrai, IEEE proceedings 2004

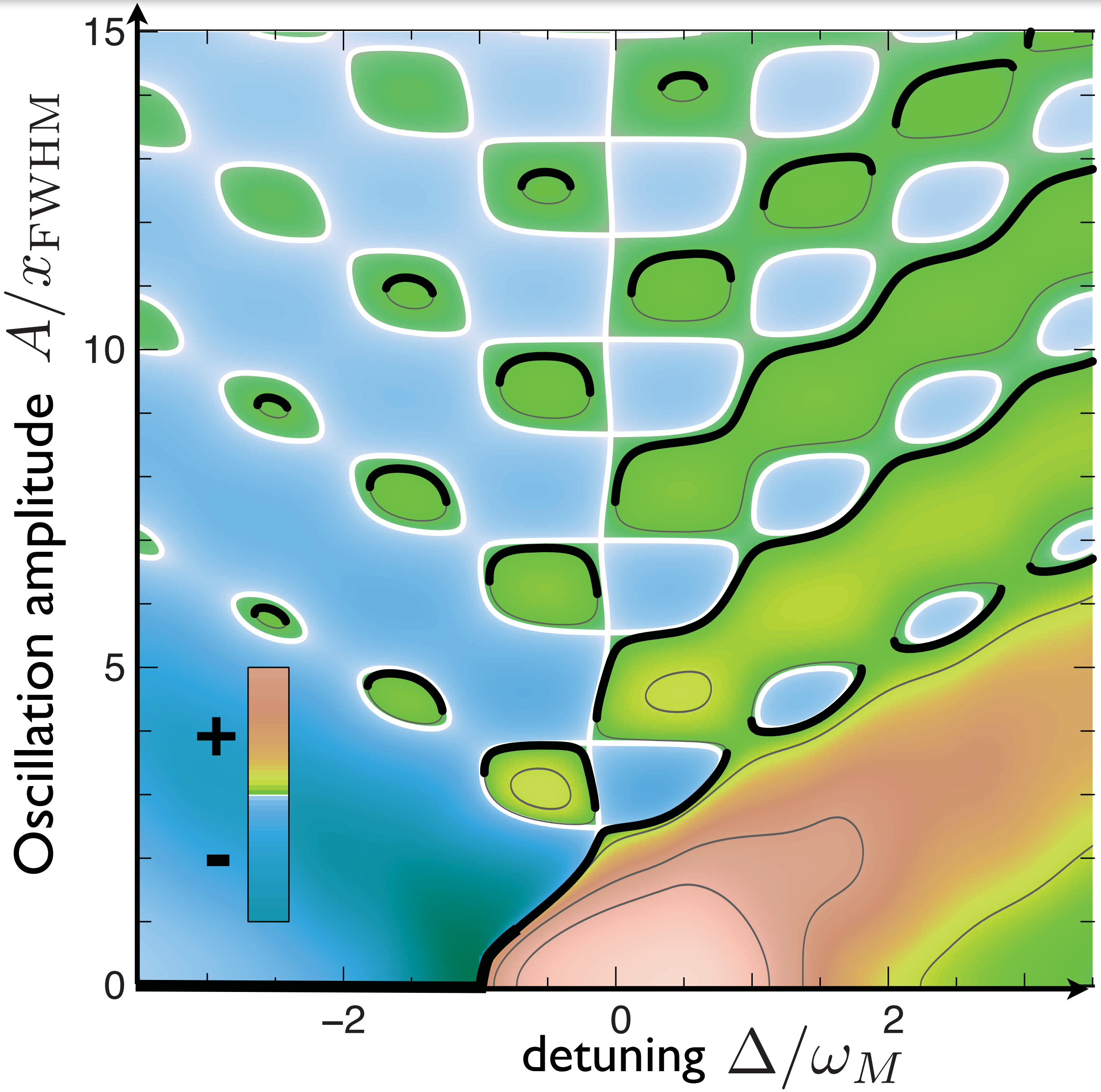
Carmon, Rokhsari, Yang, Kippenberg, Vahala, PRL 2005

FM, Harris, Girvin, PRL 2006

Metzger et al., PRL 2008

related physics: see talks by A. Armour and E. Buks

Attractor diagram



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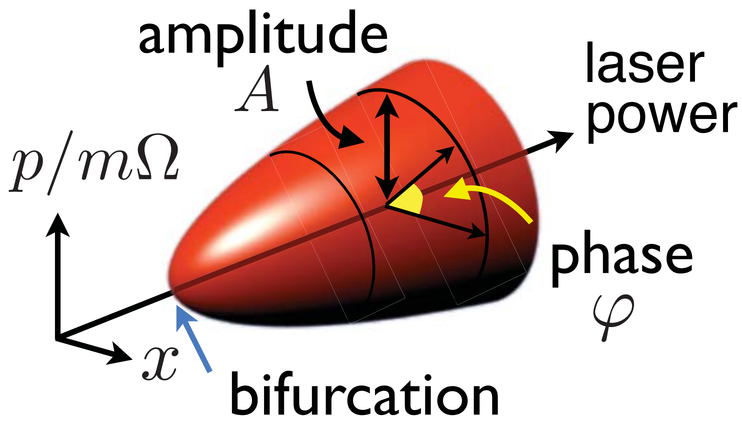
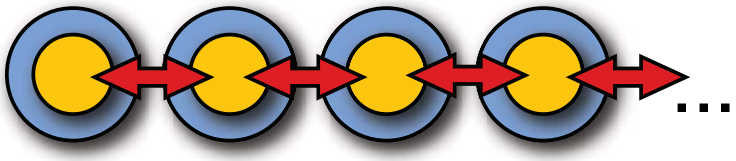
An optomechanical cell as a Hopf oscillator



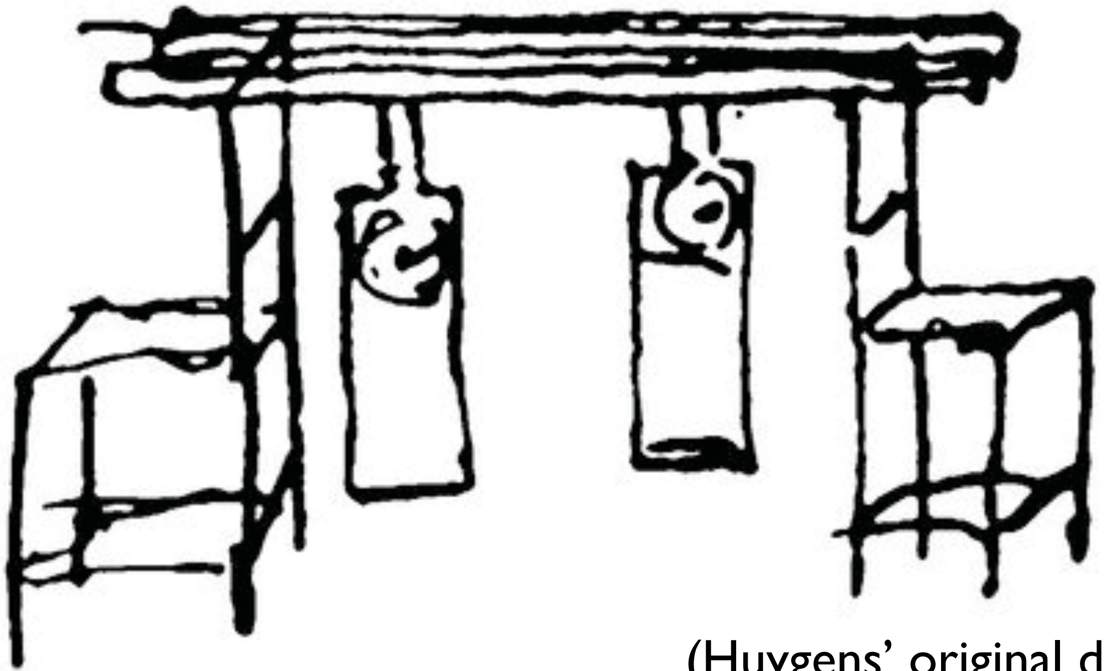
Amplitude fixed, phase undetermined!



Collective dynamics in an array of coupled cells?

Phase-locking: **synchronization**!

Synchronization: Huygens' observation

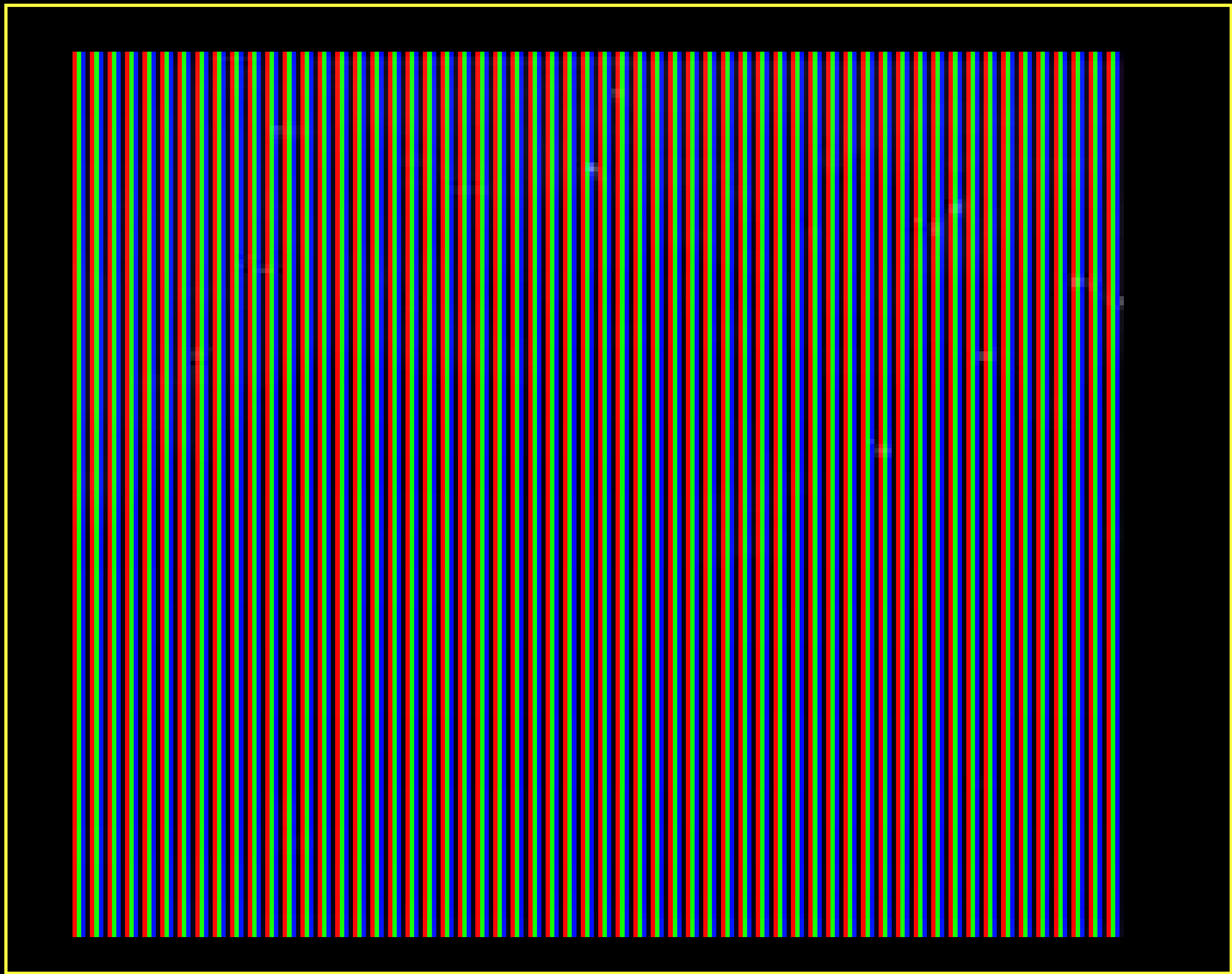


(Huygens' original drawing!)

Coupled pendula synchronize...

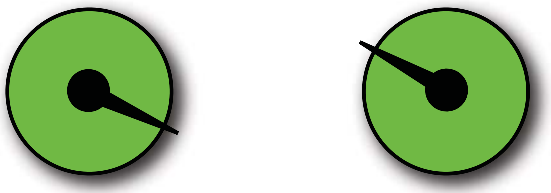
...even though frequencies slightly different
...due to nonlinear effects

Fireflies synchronizing (Source: YouTube)



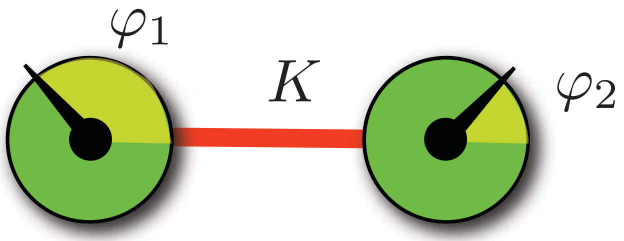
Coupled phase oscillators

Coupled phase oscillators



Coupled phase oscillators

The Kuramoto model



Kuramoto model:

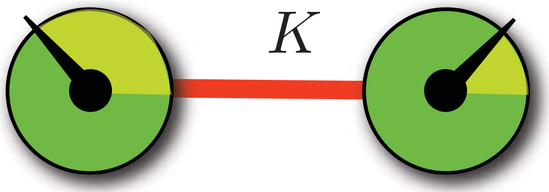
$$\dot{\varphi}_1 = \Omega_1 + K \sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K \sin(\varphi_1 - \varphi_2)$$

- captures essential features
- often found as limiting model

Kuramoto 1975, 1984

Acebron et al. , Rev. Mod. Phys. 77, 137 (2005)

The Kuramoto model



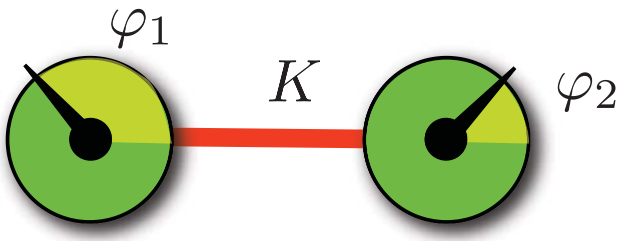
$$\dot{\varphi}_1 = \Omega_1 + K\sin(\varphi_2 - \varphi_1)$$
$$\dot{\varphi}_2 = \Omega_2 + K\sin(\varphi_1 - \varphi_2)$$

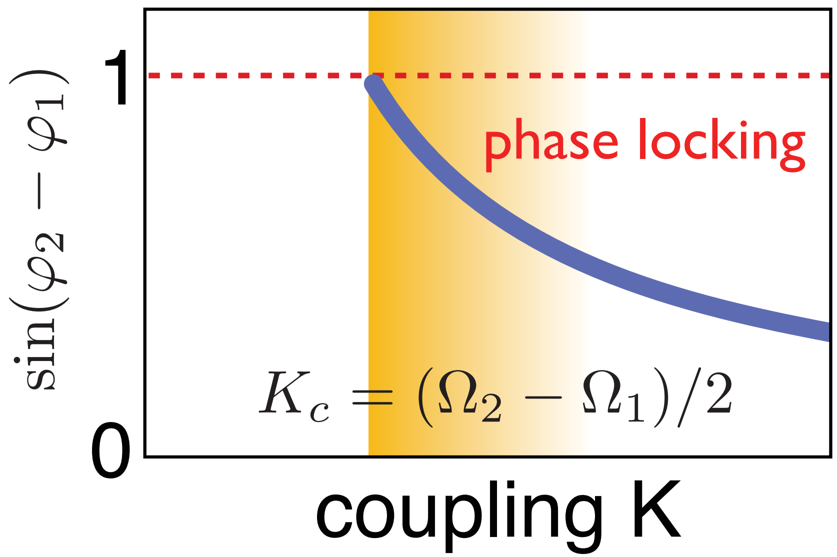
Synchronization:

$$\dot{\varphi}_1 = \dot{\varphi}_2 \quad \Rightarrow \quad \sin(\varphi_2 - \varphi_1) = \frac{\Omega_2 - \Omega_1}{2K}$$

phase lag

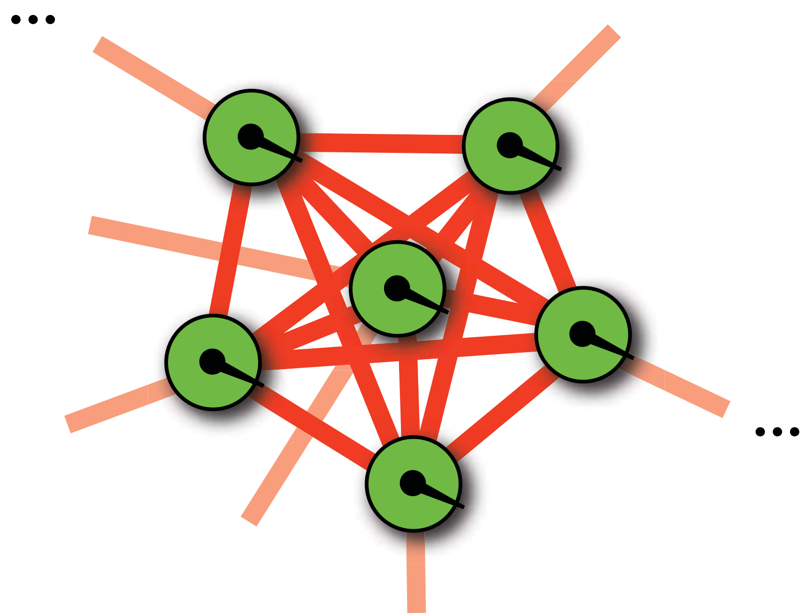
The Kuramoto model

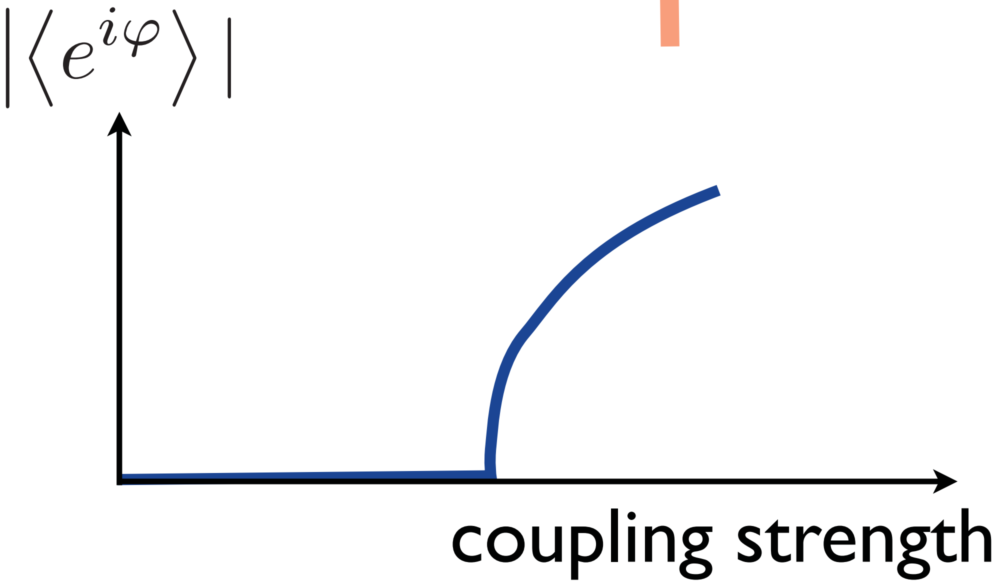




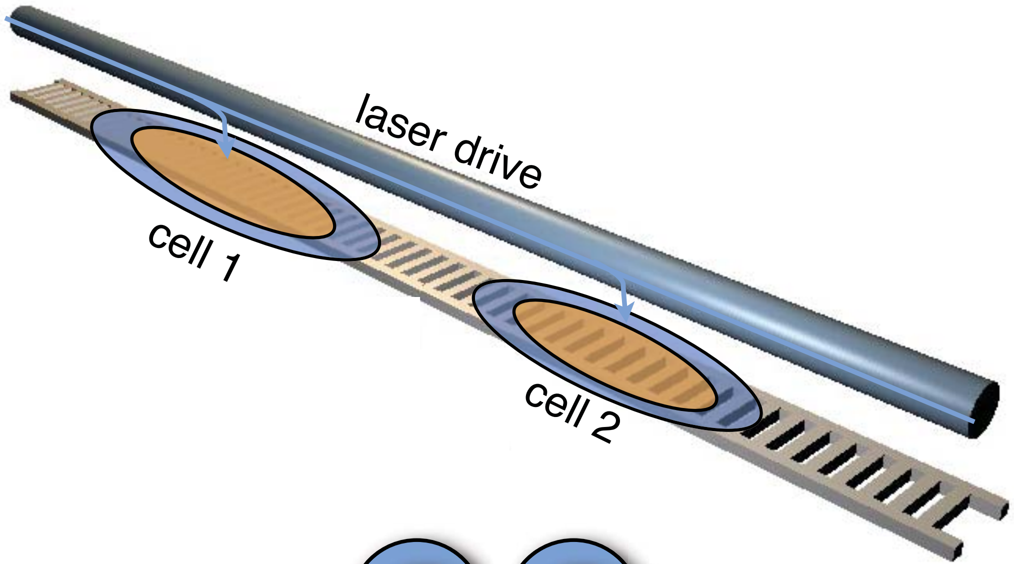
Kuramoto model: Phase-locking transition

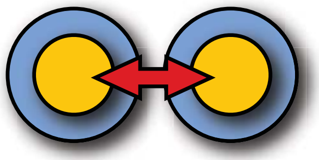
infinite-range coupling
Kuramoto model
displays phase
transition





Phase locking of two optomechanical cells

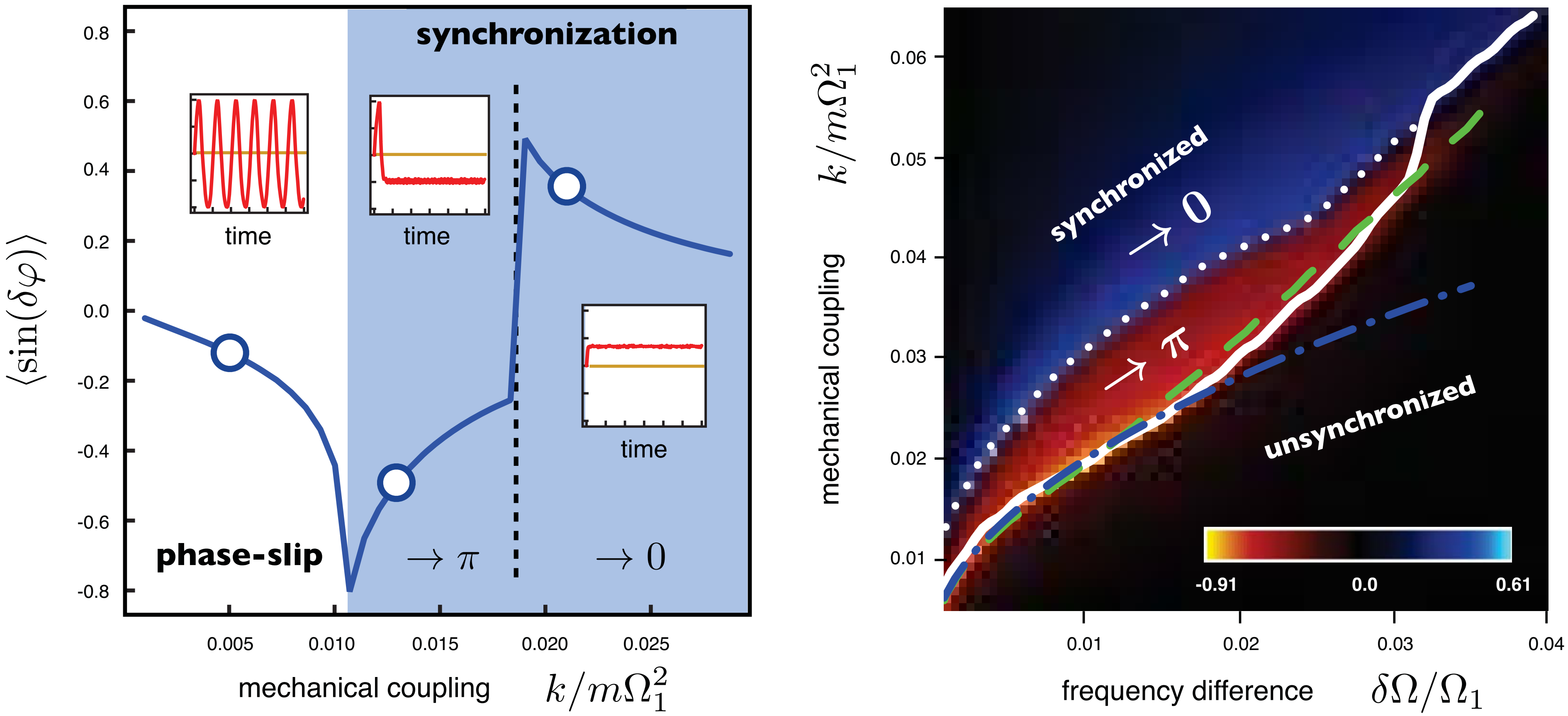




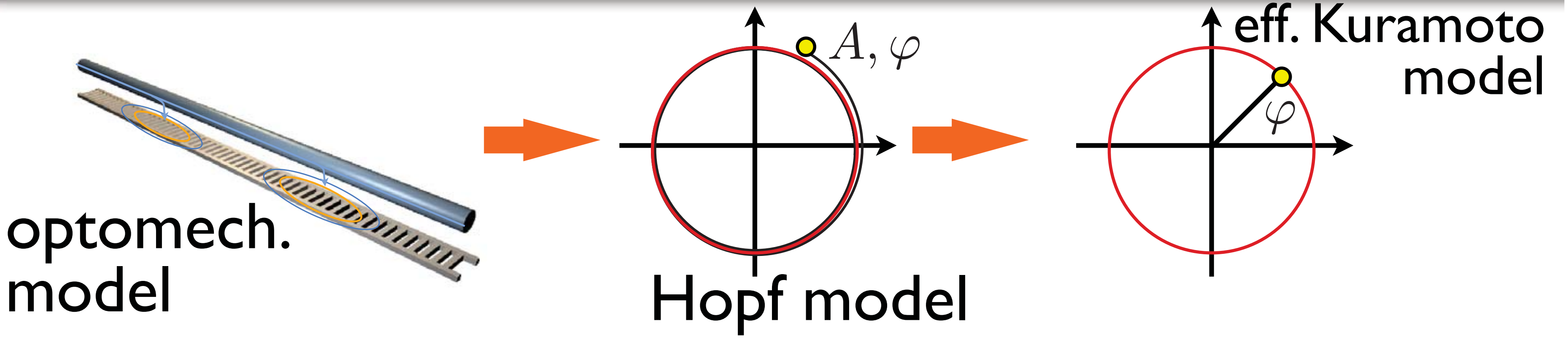
Two optomechanical cells,
fixed laser drive,
increasing mechanical coupling

Phase locking of two optomechanical cells

(direct numerical simulation)

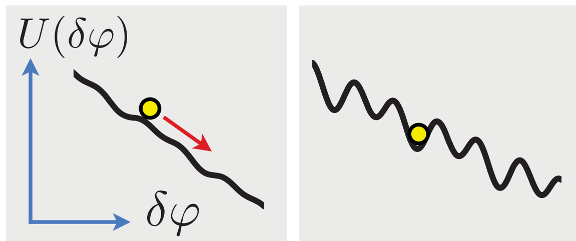


Effective Kuramoto model



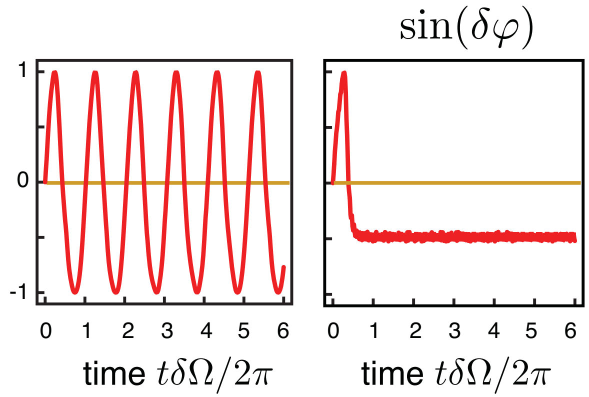
Standard Kuramoto model:

$$\delta\dot{\varphi} = \delta\Omega - 2K\sin(\delta\varphi)$$



Effective Kuramoto model for coupled Hopf oscillators:

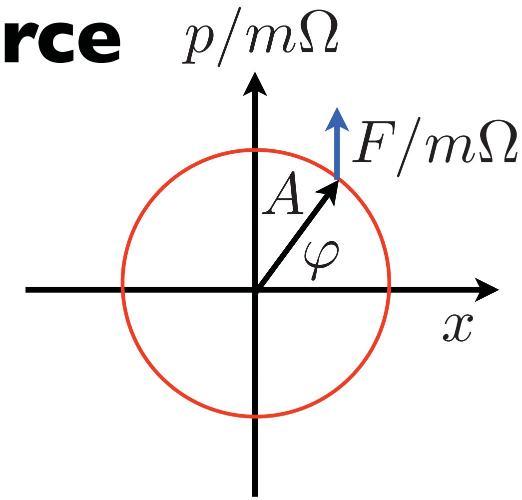
$$\delta\dot{\varphi} = \delta\Omega - 2K_s\sin(2\delta\varphi) - 2K_c\cos(2\delta\varphi)$$



$$K = K(k, \Omega, P_{\text{in}}, \ldots)$$

Hopf oscillator

Hopf oscillator with external force



$$\dot{A} = -\gamma(A - A_0) + \frac{F}{m\Omega}\sin\varphi$$

$$\dot{\varphi} = -\Omega \qquad\qquad + \frac{F}{m\Omega A}\cos\varphi$$

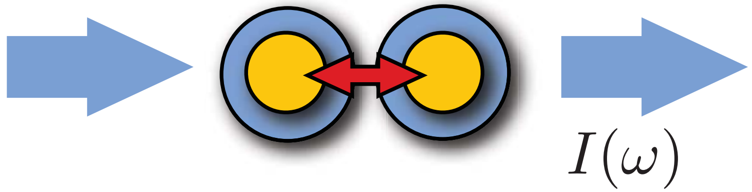
$$\delta A = A - A_0$$

Coupling two such oscillators (1,2) mechanically:

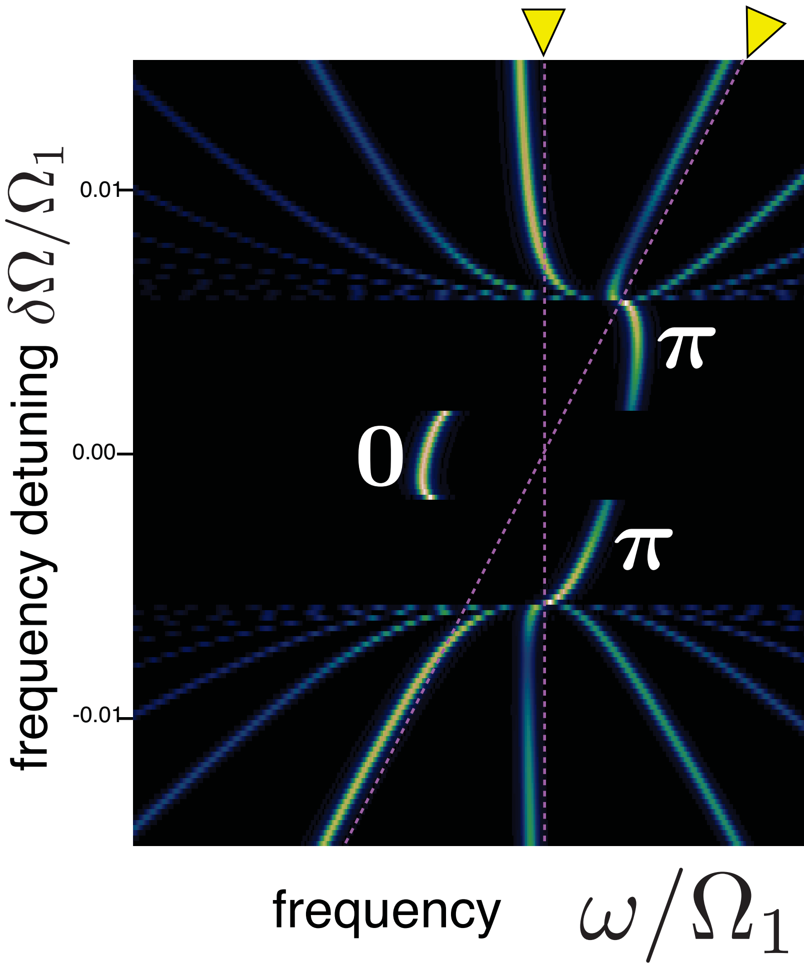
$$F_1 = kx_2 = kA_2\cos\varphi_2$$

Eliminate A-dynamics, time-average phase dynamics

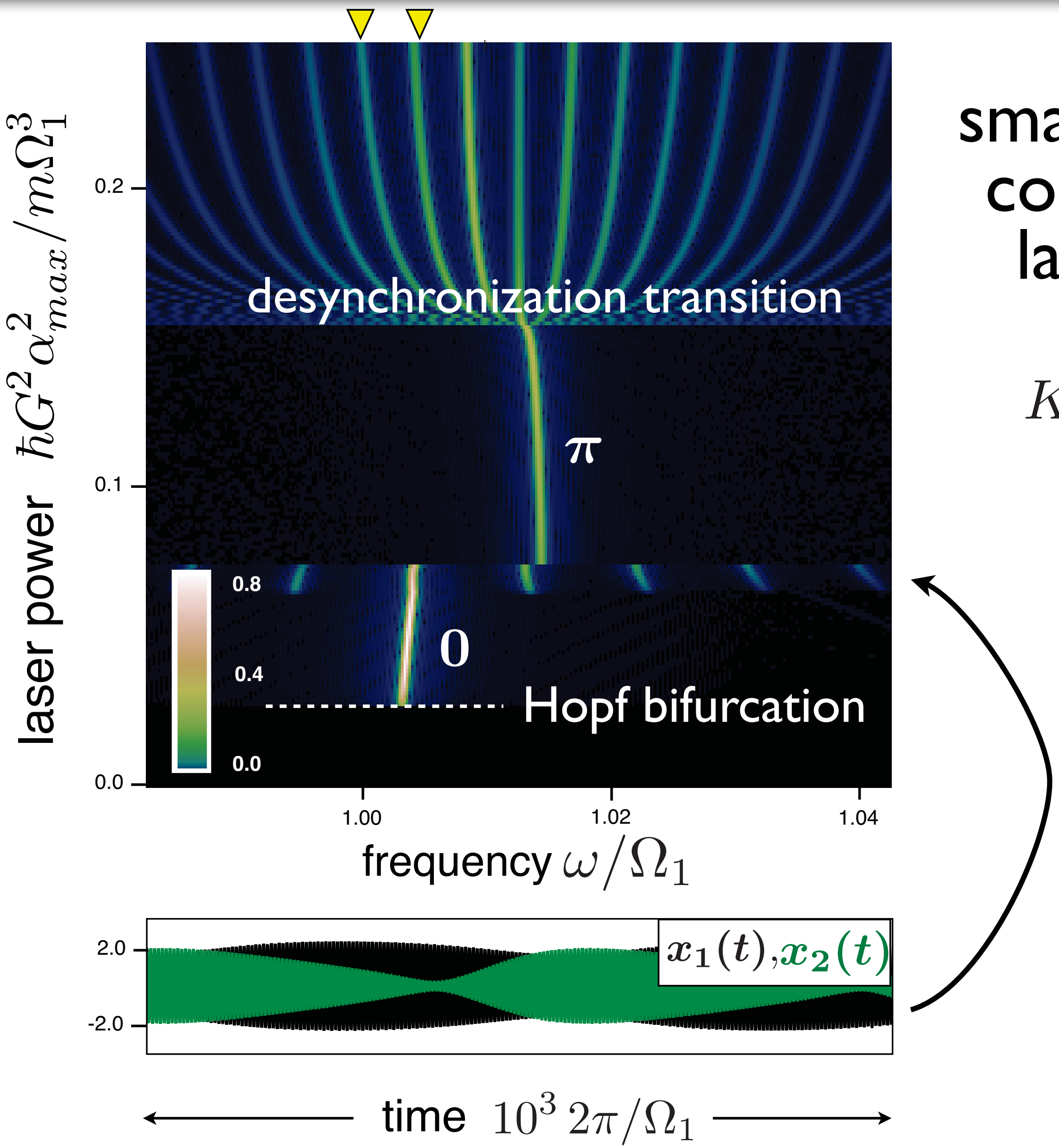
Frequency locking



Mechanical spectrum in light field intensity fluctuations

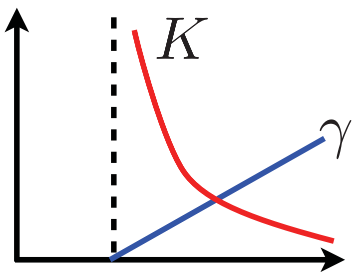


Spectrum as a function of laser drive

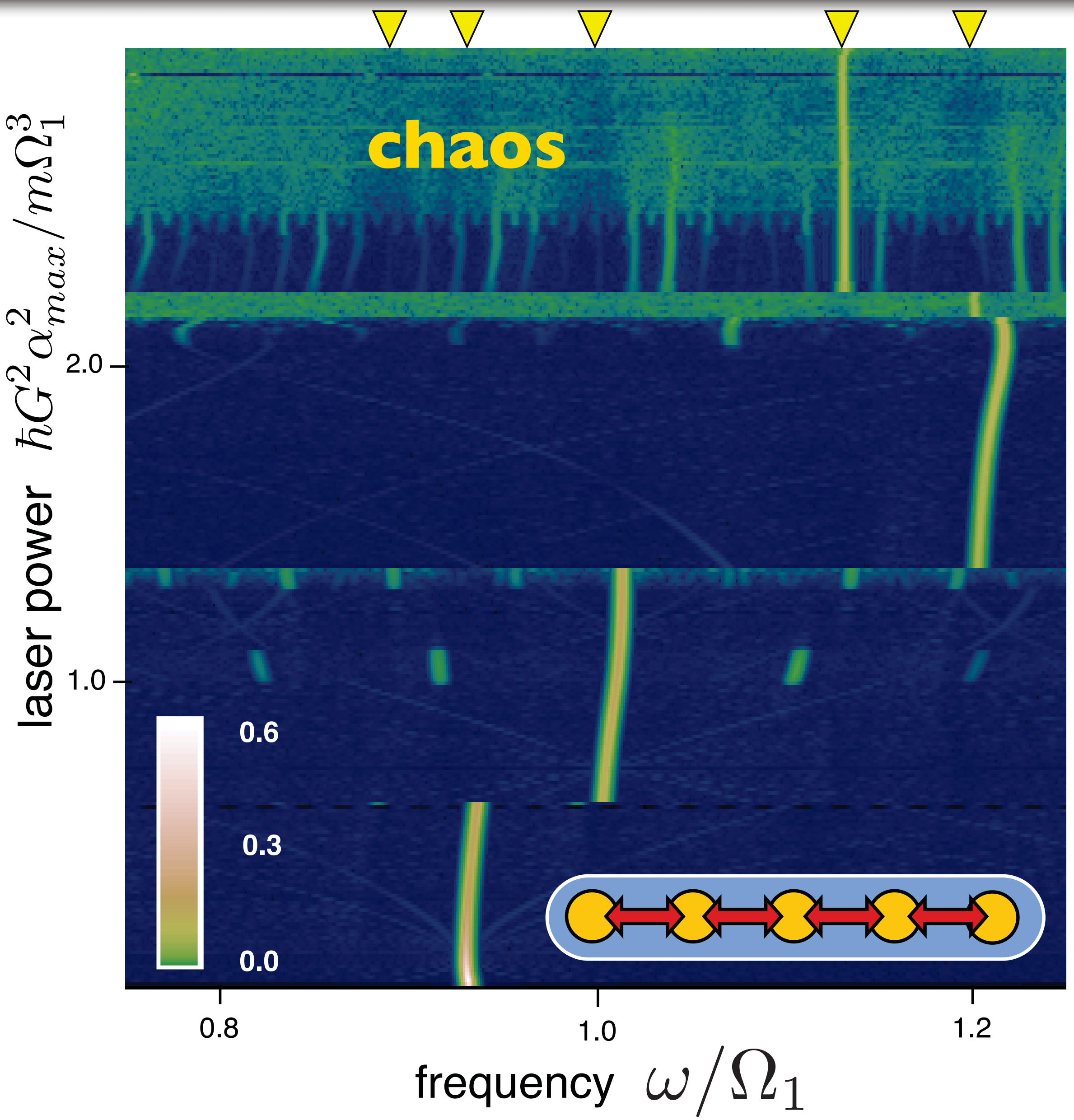


smaller effective coupling K for larger drive!

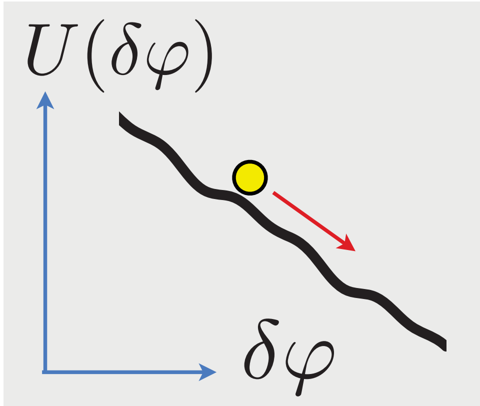
$$K = \frac{k^2}{2m^2\Omega^2\gamma}$$



Array with common optical mode



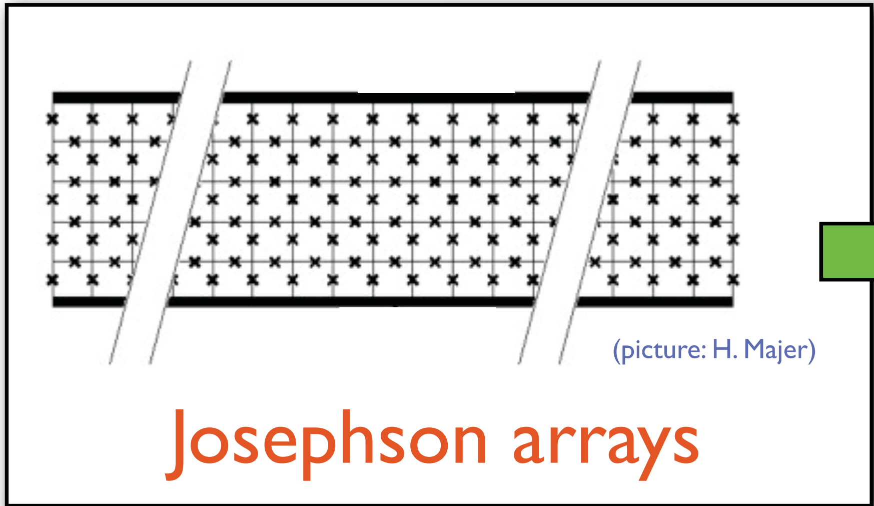
Connection to Josephson physics

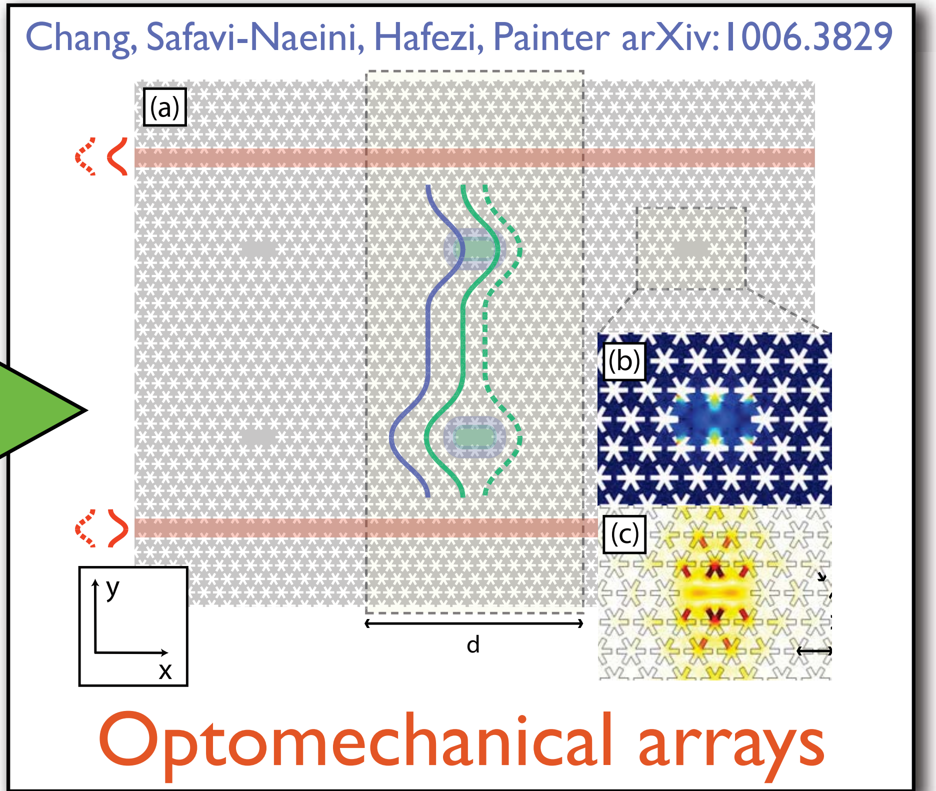


Josephson junction (overdamped)	(Optomechanical) self-oscillations
Superconducting phase	Mechanical osc. phase
Voltage	Relative phase velocity
Bias current	Mech. freq. difference
Supercurrent	Synchronization

e.g.: "(Opto-)mechanical Shapiro steps"

Optomechanical arrays: Outlook

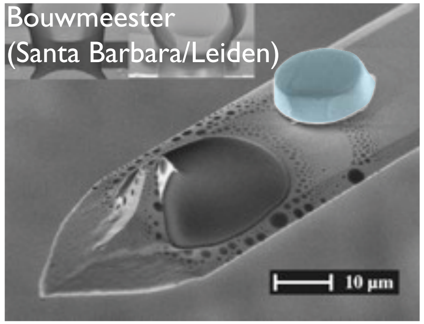




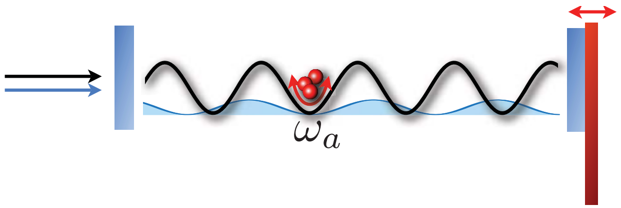
Outlook

- 2D geometries
- Information storage and classical computation
- Dissipative quantum many-body dynamics

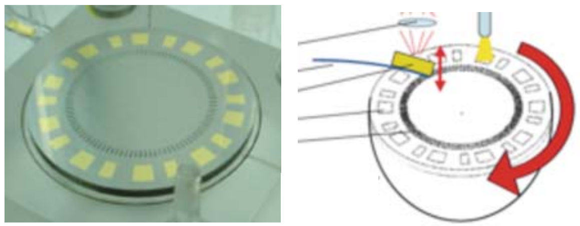
Optomechanics: general outlook



Fundamental tests of quantum mechanics in a new regime: entanglement with 'macroscopic' objects, unconventional decoherence? [e.g.: gravitationally induced?]

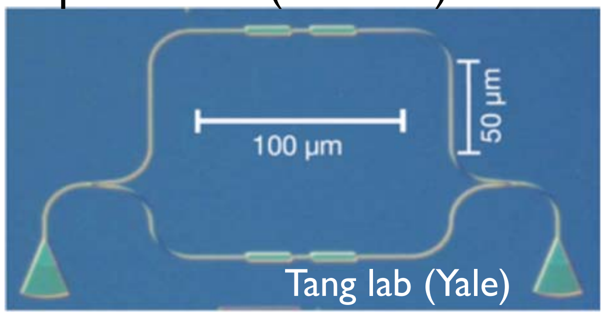


Mechanics as a 'bus' for connecting hybrid components: superconducting qubits, spins, photons, cold atoms, ...



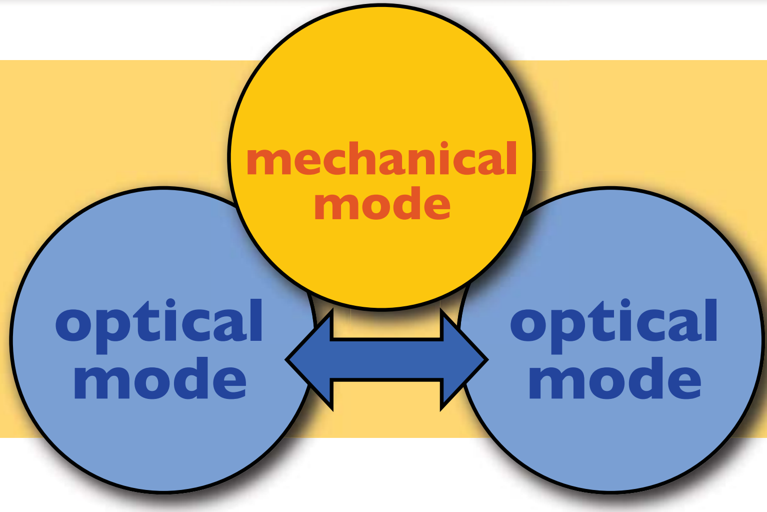
Kapitulnik lab (Stanford)

Precision measurements [e.g. testing deviations from Newtonian gravity due to extra dimensions]

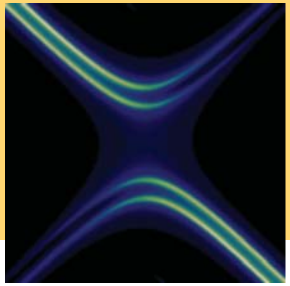


Optomechanical circuits & arrays Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

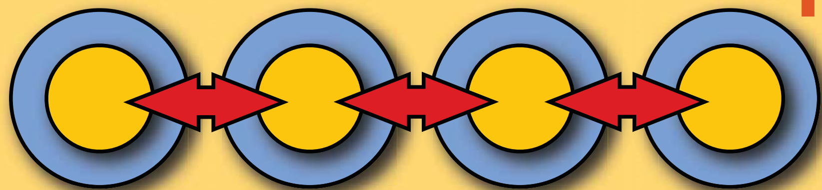
Multimode optomechanics



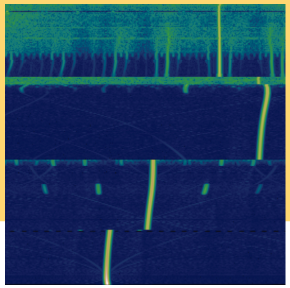
The photon shuttle



G. Heinrich, J. G. E. Harris, F. Marquardt, Phys. Rev. A **81**, 011801(R) (2010)



Collective dynamics in optomechanical arrays



G. Heinrich, M. Ludwig, J. Qian, B. Kubala, F. Marquardt, **arXiv:1007.4819**

Postdoc / PhD positions available

Group: M. Ludwig, G. Heinrich, H.-Z. Wu, J. Qian, B. Kubala, S. Kessler, C. Neuenhahn, O. Viehmann