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Uncertainty of moment tensor determination in case of shallow earthquake: existence condition and description of equivalent double-couples

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We consider the uncertainty of moment tensor determination from surface wave records if the wave length is much larger then the source depth (Bukchin *et al.*, 2010).

Such uncertainty for two special cases of double-couple, namely for pure normal (or reverse) fault and for pure strike-slip, was investigated by Kanamori and Given (1981).

It is well known that in case of shallow earthquake moment tensor cannot be uniquely determined from long period surface waves. Only four out of six elements defining a symmetric moment tensor may be reliably determined by such inversion. We consider the consequences of this fact.

We give an existence condition for double-couples radiating the same long period surface waves as the deviatoric moment tensor (symmetric 3x3 matrix with zero trace) obtained by linear inversion.

We describe the family of such double-couples and show that they may provide better estimates of double-couple mechanisms than the traditional ‘‘best double-couple’’ solution.

We describe a family of shallow double-couples which can be uniquely determined from long period surface waves

We demonstrate the results of application to four large shallow earthquakes with different source signatures.

Definition of the ‘best double-couple’

In many routine determinations of CMT solutions the best double-couple is calculated from the deviatoric moment tensor. The best double-couple has identical eigenvectors to the deviatoric moment tensor, and seismic moment is given by the average of the absolute values of the most positive and most negative eigenvalues of that moment tensor.

The best double-couple may be shown to be the double-couple moment tensor which deviates least from the original deviatoric moment tensor.

Radiation of surface waves by shallow source

We consider surface waves radiated by an instantaneous point source in a horizontally uniform earth. For the spectrum of displacement in surface wave $\mathbf{u}(\mathbf{r},\omega)$ at a point $\mathbf{r}$ we have

$$\mathbf{u}(\mathbf{r},\omega)=\mathbf{q}(\omega)P(\mathbf{M},h,\omega,\varphi)\exp[-i\psi(\mathbf{r},\omega)], \qquad (1)$$

where $\mathbf{q}(\omega)$ is a complex vector depending on earth structure, $\mathbf{M}$ is the moment tensor, h the source depth, φ the azimuth of surface wave radiation, $\psi(\mathbf{r},\omega)$ the propagation phase, and ω the circular frequency. The factor P determines the radiation pattern of the source (azimuth dependence of spectral amplitude) and the initial (source) phase.

For a Love wave this factor is given by

$$\begin{aligned}P(\mathbf{M},h,\omega,\varphi)&=\xi V^{(\tau)}(\omega,h)[0.5(M_{33}-M_{22})\sin 2\varphi+M_{23}\cos 2\varphi]\\&+i\frac{\partial V^{(\tau)}(\omega,h)}{\partial z}(M_{12}\sin\varphi-M_{13}\cos\varphi),\end{aligned} \qquad (2)$$

where $V^{(\tau)}$ (z) is the transverse eigenfunction, ξ is the wave number, i is the imaginary unit, and the coordinate system is defined in the following way: 1 – vertical down, 2 – north, and 3 – east.

For a Rayleigh wave the function P is given by

$$\begin{aligned}P(\mathbf{M},h,\omega,\varphi)&=\frac{\partial V^{(z)}(\omega,h)}{\partial z}M_{11}\\&-0.5\xi V^{(r)}(\omega,h)[M_{22}+M_{33}+(M_{22}-M_{33})\cos 2\varphi+2M_{23}\sin 2\varphi]\\&+i[\xi V^{(z)}(\omega,h)+\frac{\partial V^{(r)}(\omega,h)}{\partial z}](M_{12}\cos\varphi+M_{13}\sin\varphi),\end{aligned} \qquad (3)$$

where $V^{(z)}$ (z) , $V^{(r)}$ (z) are vertical and radial components of the eigenfunction, respectively.

The source rotated around the vertical axis by 180° radiates in the direction with azimuth φ the same surface waves as the original one in the direction with azimuth φ -180°. As can be seen from formulas (2) and (3), the result of this rotation is that the function $P(\mathbf{M},h,\omega,\varphi)$ becomes its complex conjugate.

The coefficients $\dfrac{\partial V^{(\tau)}(\omega,h)}{\partial z}$ for Love waves and $[\xi V^{(z)}(\omega,h)+\dfrac{\partial V^{(r)}(\omega,h)}{\partial z}]$ for Rayleigh waves are proportional to the shear traction acting on the horizontal plane. But such a force is vanishing at the free surface ($h = 0$). As a consequence, if the source depth h is much smaller than the wave length, the moment tensor elements M_{12} and M_{13} almost do not affect the surface wave radiation pattern and the source phase, and they can-not be resolved from the observed spectra. At the same time the imaginary part of $P(\mathbf{M},h,\omega,\varphi)$ is small and the rotation of the source around the vertical axis by 180° doesn't change the radiated surface waves. This property of shallow sources was studied by Henry *et al.* (2002). They explained for the double-couple case the two-fold rotational symmetry of the misfit function around the vertical axis of the moment tensor and demonstrated it for a set of earthquakes.

Note that the elements M_{12} and M_{13} do not affect the surface wave radiation so long as they do not exceed significantly (in absolute value) the other elements of the moment tensor. But if these two elements are dominant, then they do contribute into the surface wave radiation and can be resolved for any nonzero h. This takes place for the double-couple in case one of its nodal planes is subhorizontal (Bukchin, 2006). It is important to note that such a source radiates relatively weak surface waves. So, all moment tensor elements for such a shallow double-couple can be resolved from long period surface waves, provided the magnitude of the event is high enough, and as a result, the surface wave records are characterized by high signal-to-noise ratios.

Existence condition and description of equivalent double-couples

Only four out of six elements defining a symmetric moment tensor may be reliably determined from surface wave records if the wave length is much larger then the source depth. It has in general case nonzero non-double-couple component. It is shown (Bukchin *et al.*, 2010) that given four reliably determined elements is enough to answer the question of existence of pure double-couples radiating similar long period surface waves as original deviatoric moment tensor.

Let elements M_{22}, M_{33} and M_{23} are given. The element M_{11} is defined by zero-trace condition $M_{11} + M_{22} + M_{33} = 0$.

Expressing these moment tensor elements through seismic moment and focal mechanism angles we obtained existence condition for double-couples with given values of these three moment tensor elements, and formulas describing the set of such double-couples.

The existence condition for double-couples with given values of moment tensor elements M_{22}, M_{33} and M_{23} has form of inequality

$$M_{22}M_{33} \le M_{23}{}^2 . \tag{4}$$

If this condition is satisfied, then such double-couples exist and have the same strike angle given by formulas

$$\psi = 0.5(\pm\arccos\frac{A_1}{\sqrt{A_3{}^2 + A_2{}^2}} - \varphi) , \tag{5}$$

where A_1, A_2, A_3 and φ are given by

$$A_1 = M_{22} + M_{33} ,$$

$$A_2 = M_{33} - M_{22} ,$$

$$A_3 = 2M_{23},$$

$$\sin\varphi = \frac{A_3}{\sqrt{{A_3}^2 + {A_2}^2}},$$

$$\cos\varphi = \frac{A_2}{\sqrt{{A_3}^2 + {A_2}^2}}.$$

+ and – in formula (5) corresponds to two nodal planes.

The dip, rake angles and seismic moments for the set of double-couples characterized by similar long period surface wave radiation patterns and source phases are defined by identities

$$\tan\lambda\cos\delta \equiv C_1, \tag{6}$$

$$M_0 \sin\delta\cos\lambda \equiv C_2, \tag{7}$$

where constants C_1 and C_2 are given by formulas

$$\begin{cases} C_1 = -\dfrac{A_1}{A_2\sin 2\psi + A_3\cos 2\psi} \\ C_2 = 0.5(A_2\sin 2\psi + A_3\cos 2\psi) \end{cases}. \tag{8}$$

Adding to triples of seismic moment, dip and rake angle values the value of strike angle we describe the first branch of equivalent double-couples. Substitution of the strike angle value ψ by value ψ+180° gives us the second branch of equivalent double-couples.

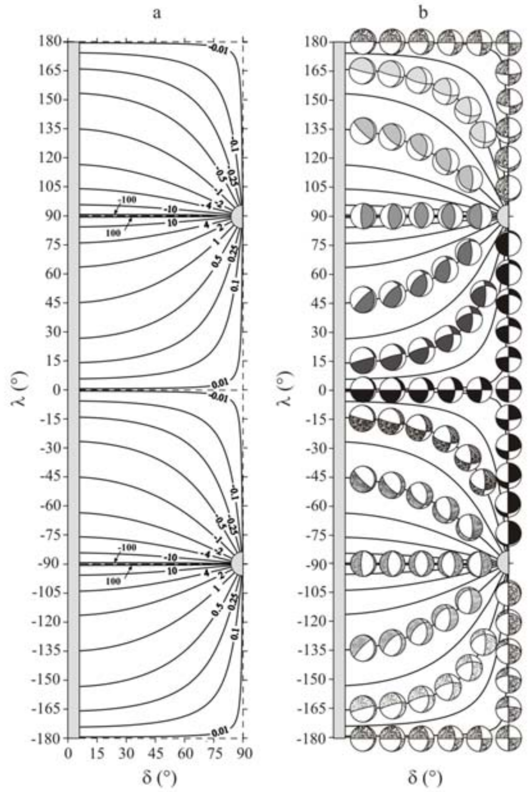


Fig. 1. Examples of families of equivalent double-couples. (a) Contour plot of $f = \tan\lambda\cos\delta$. Contours are marked by the corresponding value of C_1 in (6). Every isoline of this function defines dip and rake angles of a family of equivalent double-couples. (b) The same contour plot with superimposed lower hemispherical projections of focal solutions for some of the isolines. The value of strike angle is fixed equal to 0. Equivalent double-couples are given by focal solutions filled by the same color. Gray strip along the axis $\delta = 0$ and gray sectors centered at points of intersection of the axis $\delta = 90°$ with axes $\lambda = -90°$ and $\lambda = 90°$ show the regions where one of the nodal planes is subhorizontal. Equivalent dip-slips ($\lambda = -90°$ and $\lambda = 90°$) and slips on a vertical fault ($\delta = 90°$) contain the symmetric double-couples rotated around the vertical axis by 180°. All other families of equivalent double-couples must be completed by symmetric solutions with the same corresponding values of dip and rake angles, and with strike angle equal to 180°.

Summing up, if the deviatoric moment tensor **M** in the case of a shallow source is obtained by inversion of long period surface waves, by the CMT method, say, then only the values of four elements M_{11}, M_{22}, M_{33} and M_{23} are reliable. The elements M_{12} and M_{13} are not resolved and incorrect values of these elements can lead to a spurious non-double-couple component even if all other moment tensor elements are correct (Henry *et al.*, 2002).

But it turns out that four reliably determined elements are sufficient to provide the answer to the question of the existence of pure double-couples radiating the same long period surface waves as the original deviator **M** does. If the condition (4) is fulfilled, then using formulas (5-8) one can obtain a complete description of required double-couples. Examples of families of equivalent double-couples are shown in figure 1.

If, on the contrary, the condition (4) does not hold, then there is no double-couple radiating the same long period surface waves as the original deviator **M**. In that case we search for equivalent double-couples with values of the elements M_{22}, M_{33} and M_{23} which deviate the least from the given values. Such double-couples are found by minimizing $\sqrt{d_{22}^2 + d_{33}^2 + d_{23}^2}$, where d_{22}, d_{33} and d_{23} are the differences between the corresponding values of the three elements.

Consider the equality

$$M_{23}^2 = M_{22}M_{33} \tag{9}$$

as the equation of a surface in the 3D Euclidean space M_{22}, M_{33}, M_{23}. This equation describes an elliptic cone. This surface is symmetric with respect to the plane $M_{23} = 0$. The upper part of this surface ($M_{23} \geq 0$) is shown in figure 2. The surface separates the points (M_{22} , M_{33} , M_{23}) corresponding to the moment tensors that satisfy the double-couple existence condition from the points corresponding to the moment tensors that do not satisfy this condition. The existence condition is valid for the exterior of the surface, including the surface itself. To sum up, if for any given values of M_{22} , M_{33} , M_{23} the existence condition (4) is not true, then the double-couples with the least-deviation values of these three elements correspond to a point on the surface under consideration. It is shown (Bukchin *et al.*, 2010) that all such double-couples are pure dip-slips ($\delta = \pm 90°$).

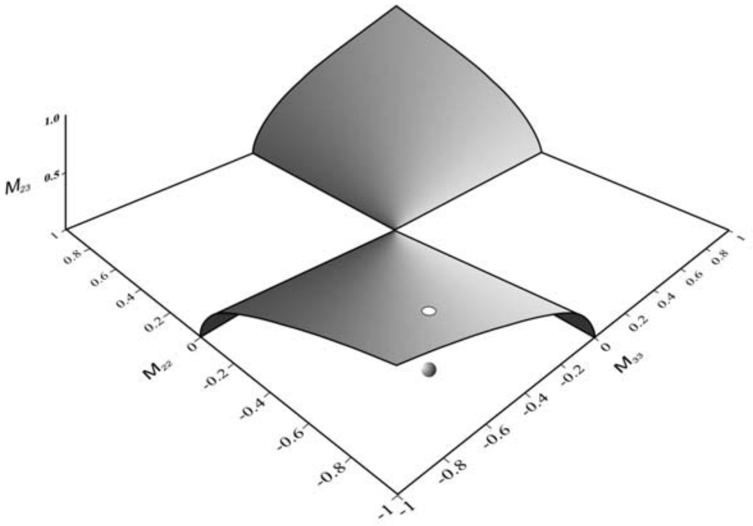


Fig. 2. Upper half of the surface , $M^2_{23} = M_{22} M_{33}$. The small ball specifies the values of moment tensor elements M_{22} , M_{33} , M_{23} not satisfying the existence condition. The small ellipse marks the double-couple with the least deviating values of these elements.

Described double-couples and reference deviatoric moment tensor are radiating similar surface wave fields if the depth of the source is much smaller than the wave length. To control the similarity of radiated wave fields for given values of source depth and period we calculate the normalized misfit between synthetic surface wave spectra calculated for any double-couple with current values of dip and rake angles and for the reference deviatoric moment tensor. This misfit is defined as the ratio of the root mean square misfit to the root mean square spectra calculated for the reference moment tensor.

The spectra of the fundamental Love and Rayleigh modes are calculated for fixed period and source depth for 72 points uniformly located around the source at a fixed distance.

We present the misfit contour plot in plane (δ, λ) in the same way as Henry *et al.* (2000) . The left and the right parts of the picture correspond to two values of strike angle different from each other by 180°. These two parts are rotated by 180° with respect to each other. This allows us to consider these two misfit plots as a single map which is continuous at the line $\delta = 90°$. The continuity follows from the equality

$$\mathbf{m}(\psi, 90°, \lambda) = \mathbf{m}(\psi+180°, 90°, -\lambda,),$$

where $\mathbf{m}(\psi,\delta, \lambda)$ is a double-couple moment tensor with given values of strike angle ψ, dip angle δ and rake angle λ.

Application to four large shallow earthquakes

We consider three large shallow earthquakes studied in detail by (Henry *et al.*, 2002, Henry and Das, 2002, Henry *et al.*, 2000, Robinson *et al.*, 2001). They compared the best fitting double-couple mechanisms obtained from mantle wave inversion with the traditional 'best double-couples' obtained from the best fitting deviatoric moment tensor and with the results of body wave analysis (Henry *et al.*, 2002).

We present here a complete description of double-couples radiating long period surface waves similar to the radiation pattern of the deviatoric moment tensors from the Global CMT catalog, and compare them with the results reported by Henry *et al.* (2002).

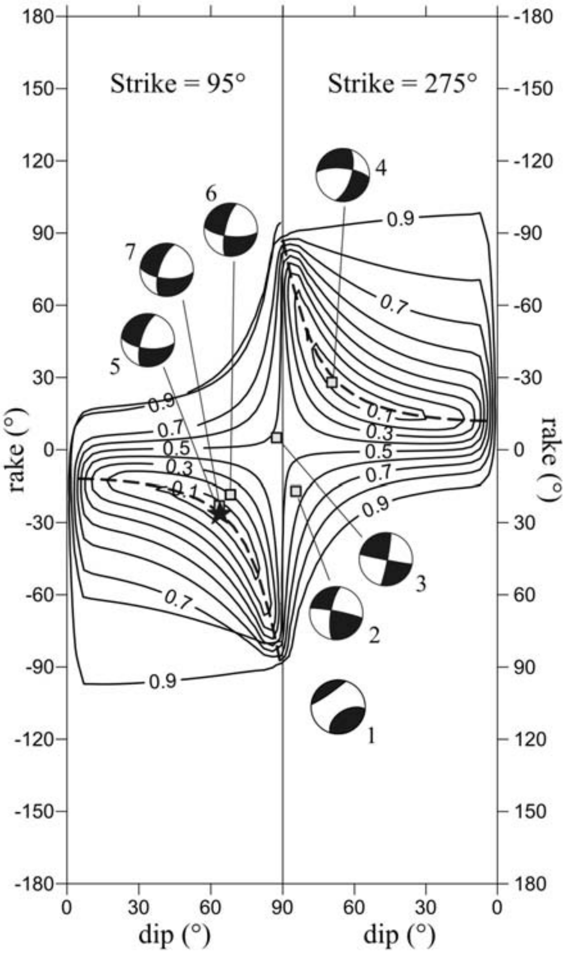
The fourth earthquake is the Hokkaido-Nansei-Oki earthquake of 1993, which we show as an example of the case where existence condition is not true, and we search for equivalent double-couples with the elements M_{22}, M_{33} and M_{23} deviating the least from given values.

We introduce the same notation as in Henry *et al.* (2002). We shall refer to the best fitting deviatoric moment tensor as the optimal deviatoric (ODV) moment tensor. We shall refer to the best fitting double-couple as the optimal pure double-couple (OPDC). And finally we shall refer to the so-called best double-couple which is calculated from the ODV moment tensor in many routine determinations of CMT solutions as the BDC.

The March 25, 1998 Antarctic Plate earthquake

The ODV solution for this $M_w = 8.1$ earthquake from the Global CMT catalog, as well as the ODV solution of Henry *et al.* (2000), both involve large non-double-couple components, characterized by the parameter ε, which is the ratio of minimum (in absolute magnitude) eigenvalue to the maximum (in absolute magnitude) eigenvalue. For these two solutions $\varepsilon = 0.41$ and $\varepsilon = 0.32$, respectively. We use the ODV solution from the Global CMT catalog as the reference deviatoric moment tensor. The values of its elements M_{22}, M_{33} and M_{23} satisfy the equivalent double-couple existence condition (4). Using these values we calculate from (5) the strike angle 95° for one of the two branches of equivalent double-couples. Then we use (6-8) to find the set of pairs of dip and rake angle values and the corresponding seismic moments. Replacing the strike angle value 95° by the value 95° +180° = 275° gives us the second branch of equivalent double-couples.

A contour plot of the misfit function for period 200 s and source depth 10 km is shown in figure 3. The same figure shows two branches of equivalent double-couples and the different moment tensor solutions for this earthquake. It can be seen that the misfit is negligible on long segments of the equivalent double-couple curves. Both optimal pure double-couples obtained by Henry *et al.* (2000) from mantle wave inversion (solutions 3 and 4) fit well the curves of equivalent double-couples, and the values of the strike angle for these solutions practically coincide with the predicted values. On the contrary, both of the BDC (from the Global CMT catalog and obtained by Henry *et al.* (2000)) are far from the lines of minimum misfit and their values of strike angle differ by 6° from the predicted values. The large misfit for the BDC solutions can be explained by the dependence of all the BDC moment tensor elements on the values of ODV elements M_{12} and M_{13} which are not estimated reliably. Large non-double-couple components for both ODV solutions can be explained as well by inaccurate estimates of M_{12} and M_{13}. As a preferable estimate of the double-couple mechanism for this earthquake we suggest the double-couple (marked in figure 3 by the star) from the family of equivalent double-couples nearest to the solution obtained from body wave inversion by Henry *et al.* (2000).



Superimposed moment tensors: 1 - ODV moment tensor from Global CMT catalog; 2 - BDC from Global CMT catalog (strike 281°, dip 84°, rake 17°); 3 - BDC obtained by Henry *et al.* (2000) (strike 101°, dip 88°, rake 5°); 4 - OPDC obtained by Henry *et al.* (2000) (strike 276°, dip 69°, rake -28°); 5 - OPDC obtained by Henry *et al.* (2000) (strike 96°, dip 64°, rake -23°); 6 - double-couple obtained from body wave inversion by Henry *et al.* (2000) (strike 96°, dip 69°, rake -18°); 7 (marked by star) is the double-couple from the family of equivalent double-couples nearest to the solution from body wave inversion (strike 95° , dip 63° , rake -26°).

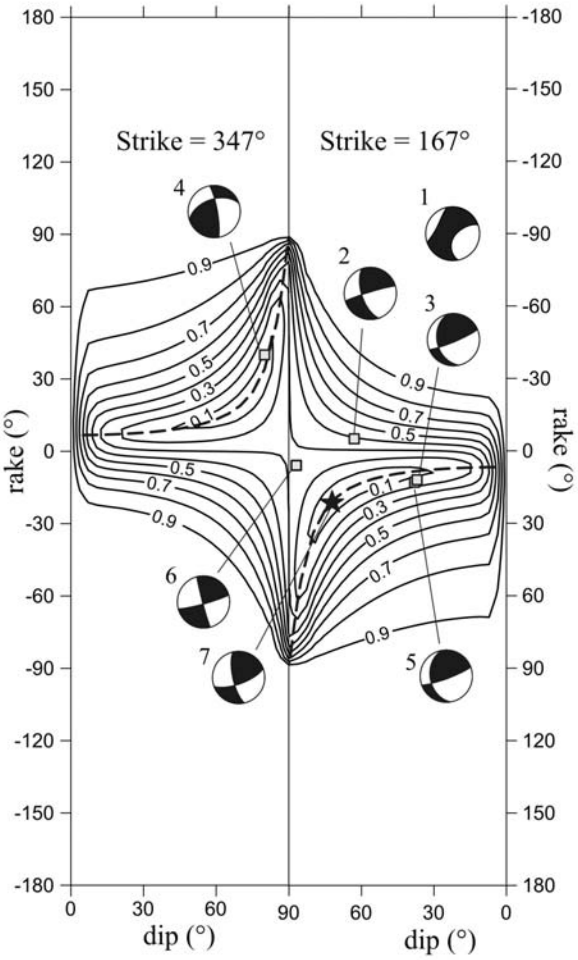
Fig. 3. The 1998 Antarctic earthquake. Misfit function calculated with respect to the radiation of ODV moment tensor from Global CMT catalog for period 200 s and source depth 10 km. Red dashed lines show two branches of equivalent double-couples. Points to the left of the vertical line at the center of the figure have strike 95° and rakes corresponding to the left ordinate. Points to the right of the line have the strike 275° and rakes corresponding to the right ordinate.

The June 18, 2000 Wharton Basin earthquake

The Global CMT ODV solution for this $M_w = 7.8$ earthquake has a large non-double-couple component ($\varepsilon = 0.33$). The non-double-couple component of the ODV moment tensor btained by Henry *et al.* (2002) is about twice as small ($\varepsilon = 0.16$). As in the previous example we used the ODV solution from the Global CMT catalog as the reference deviatoric moment tensor. The values of its elements M_{22}, M_{33} and M_{23} satisfy the equivalent double-couple existence condition. Calculated values of strike angles of two branches of equivalent double-couples are equal to 347° and 167°.

A contour plot of the misfit function for period 200 s and source depth 10 km is shown in the Fig. 4. Two branches of equivalent double-couples and the different moment tensor solutions for this earthquake are shown in the same figure. As for the Antarctic plate event, the misfit is negligible on long segments of the equivalent double-couple curves. Both optimal pure double-couples obtained by Henry *et al.* (2002) from mantle wave inversion (solutions 4 and 5) fit well the curves of equivalent double-couples and the values of the strike angle for these solutions fit well the predicted values.

The BDC obtained by Henry *et al.* (2002) fits well the curve of equivalent double-couples, while the misfit for the BDC from the Global CMT catalog is large. This instability in the BDC determination can be caused by large errors in the estimates of the ODV elements M_{12} and M_{13}. The instability in determination of the non-double-couple component of the ODV solutions and its significant value can also be explained by inaccurate estimates of the elements M_{12} and M_{13}. We show in the same figure the preferable solution (marked by the star) from the family of equivalent double-couples which is nearest to the solution obtained from body wave inversion by Robinson *et al.* (2001).

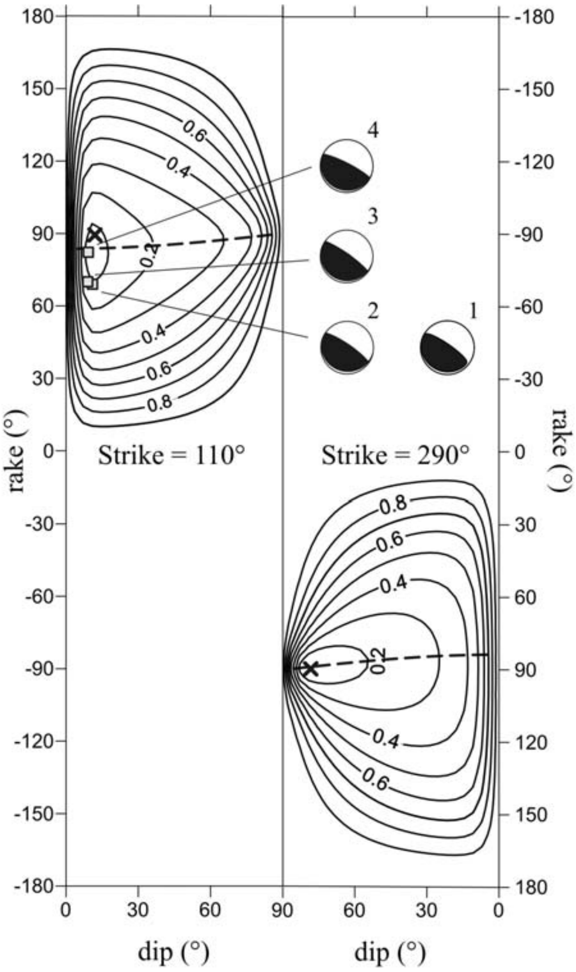


Superimposed moment tensors: 1 - ODV moment tensor from Global CMT catalog; 2 - BDC from Global CMT catalog (strike 161°, dip 63°, rake -5°); 3 - BDC obtained by Henry *et al.* (2000) (strike 167°, dip 37°, rake 12°); 4 - OPDC obtained by Henry *et al.* (2000) (strike 350°, dip 80°, rake 40°); 5 - OPDC obtained by Henry *et al.* (2000) (strike 170°, dip 38°, rake 13°); 6 - double-couple obtained from body wave inversion by Robinson *et al.* (2001) (strike 165°, dip 87°, rake 6°); 7 (marked by star) is the double-couple from the family of equivalent double-couples nearest to the solution from body wave inversion (strike 167°, dip 72°, rake 21°).

Fig. 4. The 2000 Wharton Basin earthquake. Misfit function calculated with respect to the radiation of ODV moment tensor from Global CMT catalog for period 200 s and source depth 10 km. Dashed lines show two branches of equivalent double-couples. Points to the left of the vertical line at the center of the figure have strike 347° and rakes corresponding to the left ordinate. Points to the right of the line have strike 167° and rakes corresponding to the right ordinate.

The February 17, 1996 Biak, Indonesia earthquake

The ODV solution for this $M_w = 8.2$ earthquake from the Global CMT catalog, as well as the ODV solution of Henry *et al.* (2002) have negligible non-double-couple components ($\varepsilon = 0.02$ and $\varepsilon = 0.01$, respectively). We use the ODV solution from the Global CMT catalog as the reference deviatoric moment tensor. The values of its elements M_{22}, M_{33} and M_{23} satisfy the equivalent double-couple existence condition. The calculated values of the strike angles of the two branches of equivalent double-couples are equal to 110° and 290°. A contour plot of the misfit function for period 200 s and source depth 15 km is shown in Fig. 5. The two branches of equivalent double-couples and the four moment tensor solutions for this earthquake are shown in the same figure. It can be seen that in contrast to the first two events considered here, the minimum of the misfit function is not elongated along the curves of the asymptotically equivalent double-couples. In the case of the Biak earthquake it is localized in two relatively small regions. Note that the point of minimum misfit for strike 290° marked by a cross and the point marked by a cross on the left side of the figure correspond to the two nodal planes of the same double-couple (strike 290°, dip 80°, rake 90°; and strike 110°, dip 10°, rake 90°). So, we can consider the double-couples from the left small region only as good fitting solutions. It can be seen that all three double-couple solutions shown in the Fig. 5 differs by a few degrees from each other, and the misfit for all of them is negligible. It means that in spite of a small source depth as compared with the wave length, the solution for the Biak earthquake is practically unique. This uniqueness is due to the fact that the ODV moment tensor for this event is practically a pure double-couple with a subhorizontal nodal plane; under these conditions, as mentioned above, all the moment tensor elements are well resolved.



Superimposed moment tensors: 1 - ODV moment tensor from Global CMT catalog; 2 - BDC from Global CMT catalog (strike 103°, dip 11°, rake 69°); 3 - BDC obtained by Henry *et al.* (2002) (strike 105°, dip 9°, rake 70°); 4 - OPDC obtained by Henry *et al.* (2002) (strike 109°, dip 9°, rake 82°).

Fig. 5. The 1996 Biak, Indonesia earthquake. Misfit function calculated with respect to the radiation of ODV moment tensor from Global CMT catalog for period 200 s and source depth 15 km. Dashed lines show two branches of equivalent double-couples. Points to the left of the vertical line at the center of the figure have strike 110° and rakes corresponding to the left ordinate. Points to the right of the line have strike 290° and rakes corresponding to the right ordinate.

The July 12, 1993 Hokkaido Nansei-Oki, Japan earthquake

The ODV solution for this $M_w = 7.7$ earthquake from the Global CMT catalog has small non-double-couple components ($\varepsilon = 0.06$). We use this solution as the reference deviatoric moment tensor. The values of its elements M_{22}, M_{33} and M_{23} do not satisfy the existence condition (4). We determined a set of double-couples with values of these elements least deviating from the given values. In this case the deviation is less than 5%. The calculated value of the strike angle for these double-couples is equal to 0.07°. As was shown, all of these are pure dip-slips, in this case thrust faults. In the case of pure dip-slips it makes no sense to consider the second branch of symmetric double-couples with a strike angle different by 180°, because all symmetric sources are present in the first branch. A contour plot of the misfit function for period 200 s and source depth 16.5 km is shown in the Fig. 6. The equivalent double-couples with revised values of M_{22}, M_{33} and M_{23} are shown in the same figure by a dashed line and the three moment tensors are superimposed: the ODV solution and the BDC from the Global CMT catalog and the symmetric double-couple rotated around the vertical axis by 180°. As for the Antarctic and Wharton Basin events the misfit is negligible on long segments of the equivalent double-couple line $\lambda = 90°$. Equivalent double-couples form a complete set of thrust-faults with west-plunging nodal planes, as well as with east-plunging nodal planes.

These focal mechanisms fit well the effect observed for this event by Kuge *et al.* (1996). They performed source inversion from long period surface wave analysis. For the inversion of Rayleigh waves, they obtained a thrust-fault mechanism characterized by a shallow dipping west-plunging nodal plane, which is consistent with the P-wave first-motion data and the distribution of aftershocks. For the inversion of both Rayleigh and Love waves, they obtained a thrust-fault mechanism characterized by a shallow east-dipping nodal plane. They explained this effect by spatiotemporal changes in the focal mechanism and the moment release during the rupture process. We suggest that it could be explained by an equivalence of all these solutions and by the uncertainty of the inversion based on long period surface waves.

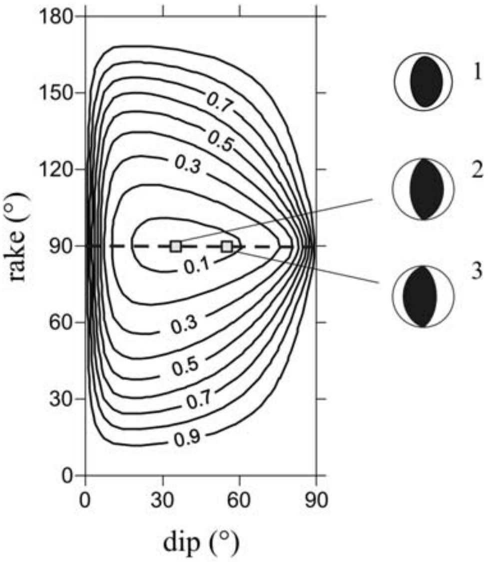


Fig. 6. Hokkaido Nansei-Oki, Japan earthquake. Misfit function calculated with respect to the radiation of ODV moment tensor from Global CMT catalog for period 200 s and source depth 16.5 km. The dashed line shows the family of equivalent double-couples. Superimposed moment tensors: 1 - ODV solution from Global CMT catalog, 2 – BDC from Global CMT catalog (strike 0°, dip 35°, rake 90°); 3 – double-couple symmetric to solution 2 (strike 0°, dip 55°, rake 90°).

Conclusions

(1) Traditional ‘‘best double-couple’’ may provide inadequate estimate of double-couple mechanism.

(2) The existence of equivalent double-couples shows that non-double-couple component of moment tensor may be spurious and can be explained by errors in estimates of elements M_{12} and M_{13}.

(3) Four reliably determined moment tensor elements is enough to answer the question of existence of equivalent double-couples.
(4) If the existence condition is valid a complete set of equivalent double-couples can be described analytically.
(5) If the existence condition is not valid, equivalent double-couples can be determined for values of moment tensor elements M_{22}, M_{33} and M_{23} nearest to given values.
(6) To select the preferable solution from the set of equivalent double-couples, additional data are required. These can be solution obtained from body wave inversion, first motion data or long period P-wave polarities.
(7) If one of nodal planes of shallow pure double-couple is subhorizontal then all elements of moment tensor are well resolved and the solution is unique.

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