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Surface wave inversion for determination of seismic source parameters: theory and application for a study of the main shock and aftershocks of the 2010 Chilean earthquake off-shore Maule

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I. Parameterization of seismic source

The description of seismic source we will consider is based on the formalism developed by Backus and Mulcahy, 1976.

Statement of the problem.
Motion equation

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i \qquad (1.1)$$

Hook's law for isotropic medium

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu\varepsilon_{ij} \qquad (1.2)$$

Initial conditions

$$\dot{\mathbf{u}} \equiv \mathbf{u} \equiv 0, t < 0 \qquad (1.3)$$

Boundary conditions

$$\sigma_{ij} n_j |_{S_0} = 0 \qquad (1.4)$$

Here **u** – displacement vector; σ_{ij} – elements of symmetric 3x3 stress tensor; i,j=1,2,3 and the summation convention for repeated subscripts is used; $\sigma_{ij,j} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}$; ε_{ij} – elements of symmetric 3x3 strain tensor and $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$; ρ - density; f_i – components of external force; n_j – components of the normal to the free surface S_0.

Solution of the problem (1.1)-(1.4) can be given by formula

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega G_{ij}(\mathbf{x},\mathbf{y},t-\tau) f_j(\mathbf{y},\tau) dV_y \qquad (1.5)$$

or

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij}(\mathbf{x},\mathbf{y},t-\tau) \dot{f}_j(\mathbf{y},\tau) dV_y \qquad (1.6)$$

Here G_{ij} is the Green's function,

$$H_{ij}(\mathbf{x},\mathbf{y},t) = \int_0^t G_{ij}(\mathbf{x},\mathbf{y},\tau) d\tau, \qquad (1.7)$$

$\mathbf{x} \in \Omega$ and $0 < t < T$ are the space region and time interval where $\dot{f}$ is not identically zero.

Seismic sources

We will consider internal sources only (earthquakes). In this case any external forces are absent. We must then set $\mathbf{f} \equiv 0$ in equation (1.1), so that the only solution that satisfies the homogeneous initial (1.3) and boundary (1.4) conditions, as well as Hook's law (1.2) will be $\mathbf{u} \equiv 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true.

Following Backus and Mulcahy, 1976, we assume seismic motion to be caused by a departure from ideal elasticity (from Hook's law) within some volume of the medium Ω at some time interval $0 < t < T$.

Let $\mathbf{u}(\mathbf{x},t)$ be the actual displacements, $\boldsymbol{\sigma}(\mathbf{x},t)$ - correspondent stresses, if Hook's law is valid, $\mathbf{s}(\mathbf{x},t)$ - actual stresses.

Let the difference

$$\boldsymbol{\Gamma}(\mathbf{x},t) = \boldsymbol{\sigma}(\mathbf{x},t) - \mathbf{s}(\mathbf{x},t), \qquad (1.8)$$

called the *stress glut tensor* or *moment tensor density*, is not identically zero for $0 < t < T$ and $\mathbf{x} \in \Omega$.
T we define as source duration, and Ω - source region. Within this region and time interval (and only there) the tensor $\dot{\mathbf{\Gamma}}(\mathbf{x}, t)$ is not identically zero as well.

Replacing $\boldsymbol{\sigma}(\mathbf{x},t)$ by $\mathbf{s}(\mathbf{x},t)$ in equation (1.1), using definition (1.8) and the absence of external forces ($\mathbf{f} \equiv 0$) we can rewrite the motion equation (1.1) in form

$$\rho \ddot{u}_i = s_{ij,j}$$

or

$$\rho \ddot{u}_i = \sigma_{ij,j} + g_i \tag{1.9}$$

where

$$g_i = -\Gamma_{ij,j} \quad . \tag{1.10}$$

Equation (1.10) defines the equivalent force $\mathbf{g}$. Using formula (1.6) with f_i replaced by g_i , definition (1.10) and Gauss theorem we have for displacements

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)dV_y , \tag{1.11}$$

where H_{ij} is differentiated with respect to y_k .
If the inelastic motions are concentrated at a surface Σ, then

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Sigma H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)d\Sigma_y . \tag{1.12}$$

Relation of stress glut (moment tensor density) with classic definition of moment tensor M :

$$\mathbf{M} = \int_0^T dt \int_\Omega \dot{\mathbf{\Gamma}}(\mathbf{y},t)dV_y \quad . \tag{1.13}$$

Normalizing moment tensor we define seismic moment M_0 :

$\mathbf{M}=M_0\mathbf{m}$, where tensor $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{m}) = \sum_{i,j=1}^{3} m_{ij}^2 = 2$, $\mathbf{m}^{\mathrm{T}}$ is transposed tensor $\mathbf{m}$.

Stress glut moment for special types of seismic sources

1. Discontinuity of displacement $\Delta\mathbf{u}$ at a surface Σ in isotropic medium (stress is continuous):

$$\Gamma_{ij}(\mathbf{x},t) = \lambda \Delta u_k(\mathbf{x},t) n_k(\mathbf{x}) \delta_{ij} + \mu [n_i(\mathbf{x}) \Delta u_j(\mathbf{x},t) + n_j(\mathbf{x}) \Delta u_i(\mathbf{x},t)]. \tag{1.14}$$

Here $\mathbf{n}(\mathbf{x})$ is the normal to the surface Σ, and seismic disturbances are given by formula (1.12).

2. In the case of tangential (shear) dislocation we have
$\Delta u_k n_k \equiv 0$ and formula (1.14) takes form

$$\Gamma_{ij}(\mathbf{x},t) = \mu [n_i(\mathbf{x}) \Delta u_j(\mathbf{x},t) + n_j(\mathbf{x}) \Delta u_i(\mathbf{x},t)]. \tag{1.15}$$

3. Instant point tangential dislocation occurred in the point $\mathbf{x}=\mathbf{0}$ at time t=0:

$$\dot{\Gamma}_{ij}(\mathbf{x},t) = M_0 m_{ij} \delta(t) \delta(\mathbf{x}), \tag{1.16}$$

where $m_{ij} = n_i a_j + n_j a_i$, $\mathbf{a} = \Delta\mathbf{u}/|\Delta\mathbf{u}|$ and $M_0 = \mu \, |\Delta \mathbf{u}|$.

Phenomena of matrix **m**

Tr**m** = 0. The eigenvalues of matrix **m** are: 1, -1 and 0. The eigenvector correspondent to 1 defines the direction of maximum extension, and the eigenvector correspondent to -1 defines the direction of maximum compression. Such a source is called double couple.

As it follows from formula (1.12) an instant point double couple excites a displacement field of the form

$$u_i(\mathbf{x},t) = M_0 H_{ik,l}(\mathbf{x},\mathbf{0},t) m_{kl} . \tag{1.17}$$

We have for Fourier transforms **H**(**x**,**y**,ω) and **G**(**x**,**y**,ω) from equation (1.7):

$$\mathbf{H}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{\mathrm{i}\omega}\mathbf{G}(\mathbf{x},\mathbf{y},\omega), \tag{1.18}$$

where i is the imaginary unit, and ω is angular frequency.
As result the spectrum of displacements is given by formula

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 m_{kl} G_{ik,l}(\mathbf{x},\mathbf{0},\omega) . \tag{1.19}$$

Relation between the displacement field and stress glut moments

We assume that following product can represent the time derivative of stress glut tensor:

$$\dot{\Gamma}(\mathbf{x},t) = f(\mathbf{x},t)\mathbf{m} , \tag{1.20}$$

where $f(\mathbf{x},t)$ is non-negative function and **m** is a uniform normalized moment tensor.
The moment $f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{q},\tau)$ of spatial degree l and temporal degree n with respect to point **q** and instant of time τ is a tensor of order l and is given by formula

$$f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{q},\tau) = \int_V dV \int_0^{\infty} f(\mathbf{x},t)(x_{k_1} - q_{k_1}) \cdots (x_{k_l} - q_{k_l})(t-\tau)^n \, dt , \tag{1.21}$$

$k_1,\ldots,k_l$=1,2,3.

Replacing $H_{ij}(\mathbf{x},\mathbf{y},t\text{-}\tau)$ in equation (1.11) by its Taylor series in powers of **y** and in powers of τ, we get:

$$u_i(\mathbf{x},t) = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{0},0) \frac{\partial^n}{\partial t^n} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} H_{ij}(\mathbf{x},\mathbf{y},t)\Big|_{\mathbf{y}=\mathbf{0}} . \tag{1.22}$$

Using formulae (1.18) and (1.22) we have following equation for the spectrum of displacements:

$$u_i(\mathbf{x},\omega) = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty} \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{0},0)(\mathrm{i}\,\omega)^{n-1} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} G_{ij}(\mathbf{x},\mathbf{y},\omega)\Big|_{\mathbf{y}=\mathbf{0}} . \tag{1.23}$$

Here we assume that the point **y**=**0** and the instant t=0 belong to the source region and the time of the source activity respectively.

When the spectra of displacements $u_i(\mathbf{x},\omega)$ and Green's function $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ have been low pass filtered, the terms in equation (1.23) start to decrease with l and n increasing at least as rapidly as $(\omega T)^{l+n}$ (T is the source duration, and ωT<1), and one might then restrict to considering finite sums only.

We will take into account in the following sections only the first terms in formula (1.23) for $l + n \le 2$.

II. Source inversion in moment tensor approximation

The first term in (1.23) corresponding to l=0, n=0, describes the spectra of displacements $u_i(\mathbf{x},\omega)$ excited by an instant point source (compare with formula (1.19) taking into account that seismic moment is equal to zero moment of function $f(\mathbf{x},t)$: $M_0 = f^{(0,0)}$). For a source with nonzero size and duration this term approximates $u_i(\mathbf{x},\omega)$ with high accuracy for periods much longer then source duration. Performing the inversion of long period seismic waves we describe the earthquake by an instant point source. As it was mentioned in previous section, an instant point source can be given by moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation $M_0 = \sqrt{\frac{1}{2}\mathrm{tr}(\mathbf{M}^\mathrm{T}\mathbf{M})}$, where $\mathbf{M}^\mathrm{T}$ is transposed moment tensor $\mathbf{M}$, and $\mathrm{tr}(\mathbf{M}^\mathrm{T}\mathbf{M}) = \sum_{i,j=1}^{3} M_{ij}{}^2$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0\mathbf{m}$, where matrix $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^\mathrm{T}\mathbf{m}) = 2$.

We'll consider a double couple instant point source (a pure tangential dislocation) at a depth h. Such a source can be given by 5 parameters: double couple depth, its focal mechanism which is characterizing by three angles: strike, dip and slip or by two orthogonal unit vectors (direction of principal tension **T** and direction of principal compression **P**) and seismic moment M_0. Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimization of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and moment tensor $\mathbf{M}$ can be expressed by formula (1.19) rewritten below in slightly different form:

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\,\omega}[M_{jl}\frac{\partial}{\partial y_l}G_{ij}(\mathbf{x},\mathbf{y},\omega)] \qquad (2.1)$$

i,j = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (2.1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location. We will discuss the inversion of surface wave spectra, so $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ is the spectrum of surface wave Green function. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. We assume that waves propagate from the source to station along great circles. Under these assumptions the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (2.1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods. Correcting the spectra for attenuation we use laterally homogeneous model for quality factor.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known, the representation (2.1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media be given. Using formula (2.1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{x},\omega)$ be any observed value of the spectrum, *i* = 1,…,N; $\varepsilon^{(i)}_{\mathrm{amp}}$ - corresponding residual of $|u^{(i)}(\mathbf{x},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon_{\mathrm{amp}}(h, \mathbf{T}, \mathbf{P}) = \left[\left(\sum_{i=1}^{N} \varepsilon_{\mathrm{amp}}^{(i)\,2} \right) \Big/ \left(\sum_{i=1}^{N} | u^{(i)}(\mathbf{x}, \omega |^2 \right) \right]^{1/2} . \qquad (2.2)$$

The optimal values of the parameters that minimize ε_{amp} we consider as estimates of these parameters. We search them by a systematic exploration of the four-dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε_{amp} on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes. A sketch illustrating the definition of partial residual functions is given in figure 1.

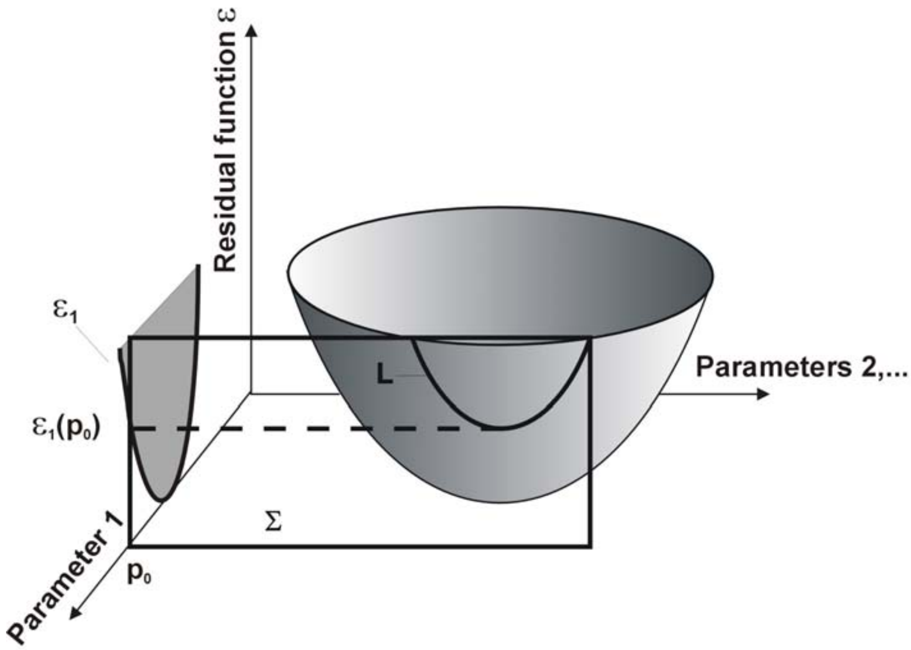


Figure 1. A sketch illustrating the definition of partial residual functions.

Here one of 4 parameters is picked out as ‘parameter 1’, and one of coordinate axis corresponds to this parameter. Another coordinate axis we consider formally as 3-D space of the rest 3 parameters. Plane Σ is orthogonal to the axis ‘parameter 1’ and cross it in a point p_0 . Curve L is the intersection of the plane Σ and the surface of functional ε. As one can see from the figure the point $\varepsilon_1(p_0)$ belong to the boundary of projection of the surface of functional ε, and at the same time it corresponds to a minimal value of the residual ε on the set of all possible values of the other 3 parameters while ‘parameter 1’ is equal to the value p_0. So, as it is accepted in engineering we characterize our surface by its 4 projections on coordinate planes.

It is well known that the focal mechanism cannot be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms radiating the same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of P wave first arrivals.

Joint inversion of surface wave amplitude and phase spectra

Using formula (2.1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual $\varepsilon_{ph}^{(i)}$ for every point of observation, every wave and every frequency ω. We define the normalized phase residual by formula

$$\varepsilon_{ph}(h, \varphi, \mathbf{T}, \mathbf{P}) = \frac{1}{\pi}\left[\left(\sum_{i=1}^{N} \varepsilon_{ph}^{(i)\,2}\right) / \mathrm{N}\right]^{1/2} . \qquad (2.3)$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{ph})(1 - \varepsilon_{amp}) . \qquad (2.4)$$

To characterize the resolution of source characteristics we calculate partial residual functions in the same way as was described above.

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{amp} be the residual of surface wave amplitude spectra, ε_{p} - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{p})(1 - \varepsilon_{amp}) . \qquad (2.5)$$

For this type of inversion we calculate partial residual functions to characterize the resolution of parameters under determination in the same way as it was described for two first types.

Before inversion we apply to observed polarities a smoothing procedure (see Lasserre *et al.*, 2001), which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger then the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of *n* observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if |*m*| is larger then its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial low. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value *m* is a linear function of n_+

such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$M(m) = 2M(n_+) - n = n - n = 0$, $D(m) = 4D(n_+) = n$, and $\sigma(m) = \sqrt{n}$.

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

III. Second moments approximation. Characteristics of source shape and evolution in time.

We present here a technique based on the description of seismic source distribution in space and in time by integral moments (see Bukchin *et al.*, 1994; Bukchin, 1995; Gomez, 1997 a, b). We assume that the time derivative of stress glut tensor $\dot{\mathbf{\Gamma}}$ can be represented in form (1.20). Following Backus and Mulcahy, 1976 we will define the source region by the condition that function $f(\mathbf{x},t)$ is not identically zero and the source duration is the time during which nonelastic motion occurs at various points within the source region, i.e., $f(\mathbf{x},t)$ is different from zero.

Spatial and temporal integral characteristics of the source can be expressed by corresponding moments of the function $f(\mathbf{x},t)$ (Backus, 1977a; Bukchin *et al.*, 1994). These moments can be estimated from the seismic records using the relation between them and the displacements in seismic waves, which we will consider later. In general case stress glut rate moments of spatial degree 2 and higher are not uniquely determined by the displacement field (Pavlov, 1994; Das & Kostrov, 1997). But in the case when equation (1.20) is valid such uniqueness takes place (Backus, 1977b; Bukchin, 1995).

Following equations define the spatio-temporal moments of function $f(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2 with respect to point $\mathbf{q}$ and instant of time τ.

$$f^{(0,0)} = \int_V dV \int_0^\infty f(\mathbf{x},t)dt\,, \quad f_i^{(1,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)dt\,,$$

$$f^{(0,1)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)dt\,, \quad f^{(0,2)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)^2 dt\,,$$

$$f_i^{(1,1)}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(t-\tau)dt\,, \tag{3.1}$$

$$f_{ij}^{(2,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(x_j - q_j)dt$$

Using these moments we will define integral characteristics of the source. Source location is estimated by the spatial centroid $\mathbf{q}_c$ of the field $f(\mathbf{x},t)$ defined as

$$\mathbf{q}_c = \mathbf{f}^{(1,0)}(\mathbf{0}) / M_0\,, \tag{3.2}$$

where $M_0 = f^{(0,0)}$ is the scalar seismic moment.

Similarly, the temporal centroid τ_c is estimated by the formula

$$\tau_c = f^{(0,1)}(0) / M_0\,. \tag{3.3}$$

The source duration is Δt estimated by $2\Delta\tau$, where

$$(\Delta\tau)^2 = f^{(0,2)}(\tau_c) / M_0\,. \tag{3.4}$$

The spatial extent of the source is described by matrix $\mathbf{W}$,

$$\mathbf{W} = \mathbf{f}^{(2,0)}(\mathbf{q}_c) / M_0\,. \tag{3.5}$$

The mean source size in the direction of unit vector $\mathbf{r}$ is estimated by value $2l_r$, defined by formula

$$l_r^2 = \mathbf{r}^{\mathrm{T}}\mathbf{W}\mathbf{r}\,, \tag{3.6}$$

where $\mathbf{r}^{\mathrm{T}}$ is the transposed vector. From (3.5) and (3.6) we can estimate the principal axes of the source. There directions are given by the eigenvectors of the matrix $\mathbf{W}$. The square of the length of the minor semi-axis is equal to the least eigenvalue, and the square of the length of the major semi-axis is equal to the greatest eigenvalue.

In the same way, from the coupled space time moment of order (1,1) the mean velocity $\mathbf{v}$ of the instant spatial centroid (Bukchin, 1989) is estimated as

$$\mathbf{v} = \mathbf{w} / (\Delta\tau)^2\,, \tag{3.7}$$

where $\mathbf{w} = \mathbf{f}^{(1,1)}(\mathbf{q}_c, \tau_c) / M_0$.

The relation between integral estimates and real characteristics of source duration and spatial extent depends on the distribution of moment rate density in time and over the fault. Figure 2 illustrates this relation in the case of Gaussian distributions. In this case 99% confidence duration is 2.5 times larger then the integral estimate, and 99% confidence axis length is 3 times larger then correspondent integral estimate.

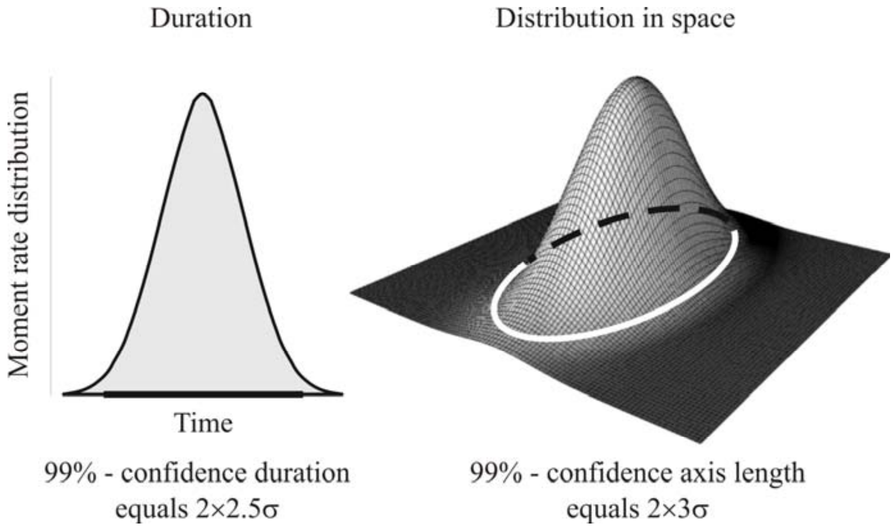


Fig. 2. Relation between integral estimates and real characteristics of source duration and spatial extent.

Now we will consider the low frequency part of the spectra of the i^{th} component of displacements in Love or Rayleigh wave $u_i(\mathbf{x},\omega)$. It is assumed that the frequency ω is small, so that the duration of the source is small in comparison with the period of the wave, and the source size is small as compared with the wavelength. It is assumed that the origin of coordinate system is located in the point of spatial centroid $\mathbf{q}_c$ (i.e. $\mathbf{q}_c = \mathbf{0}$) and that time is measured from the instant of temporal centroid, so that $\tau_c = 0$. With this choice the first degree moments with respect to the spatial origin $\mathbf{x}$=$\mathbf{0}$ and to the temporal origin t=0 are zero, i.e. $\mathbf{f}^{(1,0)}(\mathbf{0}) = \mathbf{0}$ and $f^{(0,1)}(0) = 0$.

Under this assumptions, taking into account in formula (1.23) only the first terms for $l + n \le 2$ we can express the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and the spatio-temporal moments of the function $f(\mathbf{x},t)$ by following formula (Bukchin,1995)

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{1}{2\mathrm{i}\omega} f_{mn}^{(2,0)}(\mathbf{0}) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)$$

$$- f_m^{(1,1)}(\mathbf{0},0) M_{jl} \frac{\partial}{\partial y_m} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{i\omega}{2} f^{(0,2)}(0) M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega), \qquad (3.8)$$

i,j,l,m,n = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (3.8) is the spectrum of Green function for the chosen model of medium and wave type. We assume that the paths from the earthquake source to seismic stations are well approximated by weak laterally inhomogeneous model. Under this assumption, as it was mentioned above, the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (3.8) does not depend on the average phase velocity of the wave, and the errors in source location do not affect the amplitude spectrum.

If all characteristics of the medium, depth of the best point source and seismic moment tensor are known (determined, for example, using the spectral domain of longer periods) the representation (3.8) gives us a system of linear equations for moments of the function $f(\mathbf{x},t)$ of total degree 2. But as we mentioned considering moment tensor approximation the average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining these moments, in spite of non-linear relation between them.

Let us consider a plane source. All moments of the function $f(\mathbf{x},t)$ of total degree 2 can be expressed in this case by formulas (3.2)-(3.7) in terms of 6 parameters: Δt - estimate of source duration, $l_{\max}$ - estimate of maximal mean size of the source, φ_l - estimate of the angle between the direction of maximal size and strike axis, $l_{\min}$ - estimate of minimal mean size of the source, v - estimate of the absolute value of instant centroid mean velocity $\mathbf{v}$ and φ_v - the angle between $\mathbf{v}$ and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain the following constrain for the parameters considered above (Bukchin, 1995):

$$v^2 \Delta t^2 \left(\frac{\cos^2 \varphi}{l_{\max}^2} + \frac{\sin^2 \varphi}{l_{\min}^2} \right) \le 1, \qquad (3.9)$$

where φ is the angle between major axis of the source and direction of $\mathbf{v}$.
Assuming that the source is a plane fault and representation (1.20) is valid let us consider a rough grid in the space of 6 parameters defined above. These parameters have to follow inequality (3.9). Let models of the media be given and the moment tensor be fixed as well as the depth of the best point source. Let the fault plane (one of two nodal planes) be identified. Using formula (3.8) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{r},\omega)$ be any observed value of the spectrum, i = 1,…,N; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{r},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{\max}, l_{\min}, \varphi_l, v, \varphi_v) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2} \right) \Big/ \left(\sum_{i=1}^{N} |u^{(i)}(\mathbf{r},\omega|^2 \right) \right]^{1/2}. \qquad (3.10)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the six dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions in the same way as was described in previous section. We define 6 functions of the residual corresponding to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{\max}}(l_{\max})$, $\varepsilon_{l_{\min}}(l_{\min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters.

IV. Example of application

We illustrate the technique by results of its application for a study of the main shock and largest aftershocks of the 2010 Chilean earthquake off-shore (Satriano, 2010). The study is in progress.

Main shock, 27.02.2010, 06:34:10, M_w = 8.8

At the first step inverting long period (from 250 s to 500 s) records of fundamental Love and Rayleigh modes we obtained parameters characterizing the event in point instant double-couple approximation: seismic moment, focal mechanism, and source depth. The records were processed by the frequency-time and polarization analysis package FTAN (Lander, 1989). We selected 20 Love wave records and 22 Rayleigh wave records of a good quality from IRIS, GEOSCOPE and GEOFON stations. Their azimuthal distribution is given in figure 3.

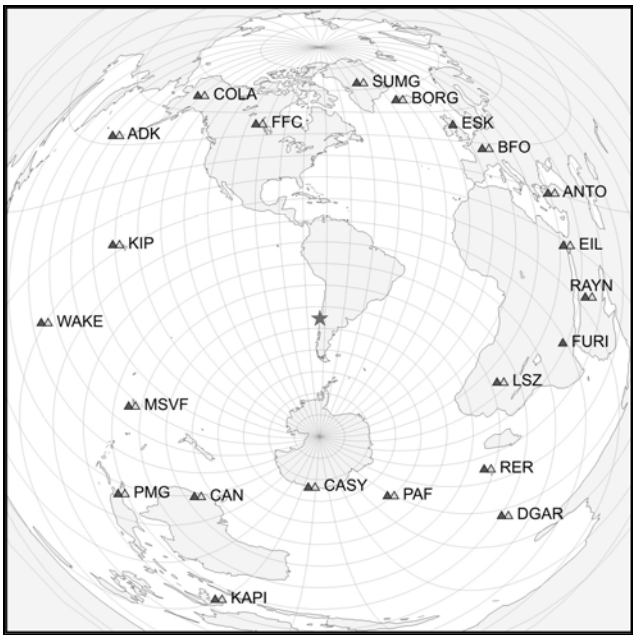


Fig.3. Distribution of stations used for moment tensor inversion. Dark triangles mark fundamental Rayleigh modes, light triangles mark fundamental Love modes. The star marks the epicenter.

To improve the resolution we used polarities of direct P-waves as additional information. In the source region and under the receivers, we used the 3SMAC model (Ricard et al. 1996) for the crust and the PREM model below. We used the quality factor given by the PREM model for attenuation correction. The moment tensor describing the source in instant point source approximation is obtained by joint inversion of surface wave amplitude spectra and first arrival polarities at worldwide stations (Lasserre *et al*. 2001). The solution gives a mechanism described by the following values of strike, dip and slip: 15°, 15°, 105° respectively (see figure 4). The source depth resolution curve is shown in the same figure. The estimate of source depth takes values from 15 to 25 km. The estimated value of seismic moment is equal to $0.18 \cdot 10^{23}$ N·m.

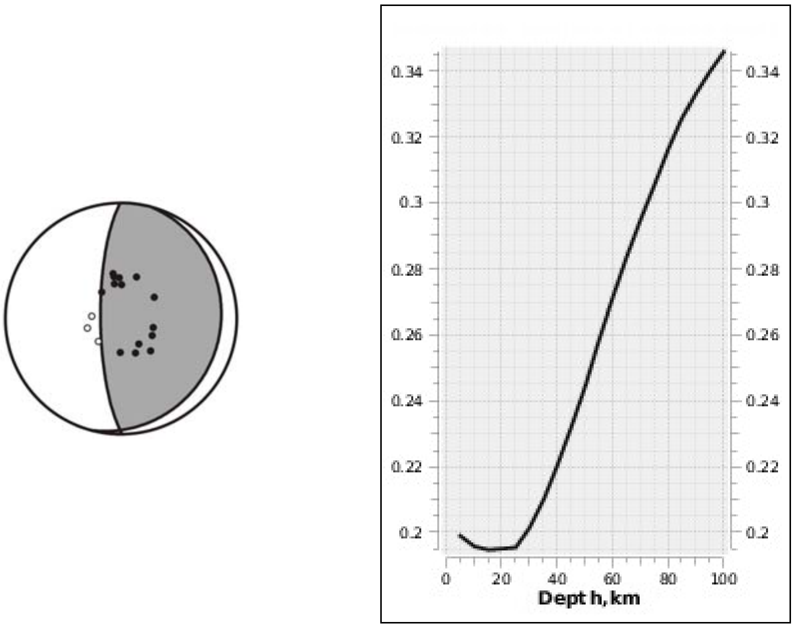


Fig. 4. Double couple solution with polarities of direct P-waves superimposed and source depth resolution curve.

Determining 2-nd moments of moment tensor density we consider the nodal plane dipping to the east as a fault plane. We fixed focal mechanism and seismic moment obtained in instant point source approximation. The source depth is recomputed when determining the source 2nd-order moments. Its final estimate takes values from 15 km to 20 km.

The duration and the geometry of the source is estimated from the amplitude spectra of fundamental Love and Rayleigh modes in the period band from 200 to 300 seconds. We selected 19 Love wave records and 21 Rayleigh wave records of a good quality.

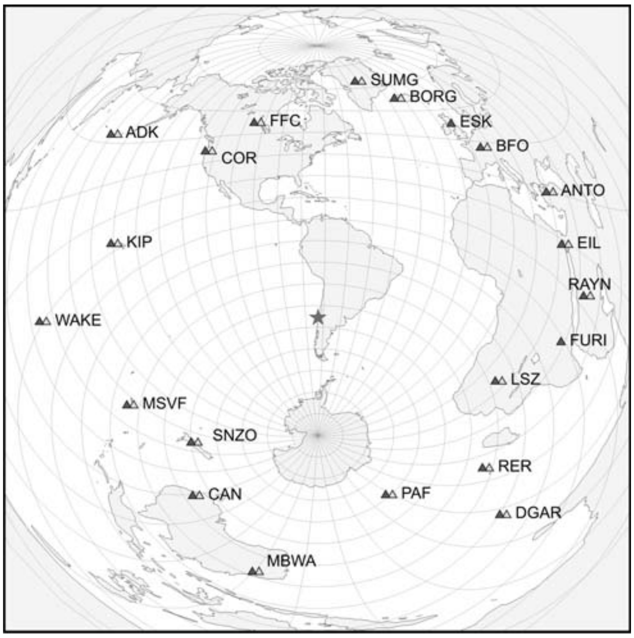


Fig.5. Distribution of stations used for 2nd moments. Dark triangles mark fundamental Rayleigh modes, light triangles mark fundamental Love modes. The star marks the epicenter.

The distribution of stations used for this inversion is shown in figure 5. The residual functions for the integral estimates of the source are given in figure 6.

The inversion yields the integral estimate of duration being about 50 s, a characteristic source length (major axis length) of 160 km. The minor axis length is poorly resolved, lying between 0 and 50 km. The average instant centroid velocity estimate is about 2.4 km/s. The angles giving the major axis and velocity vector orientations are measured clockwise on the footwall starting from the strike axis. They are consistent with each other and correspondent residual functions attain their minimum values at 0°.

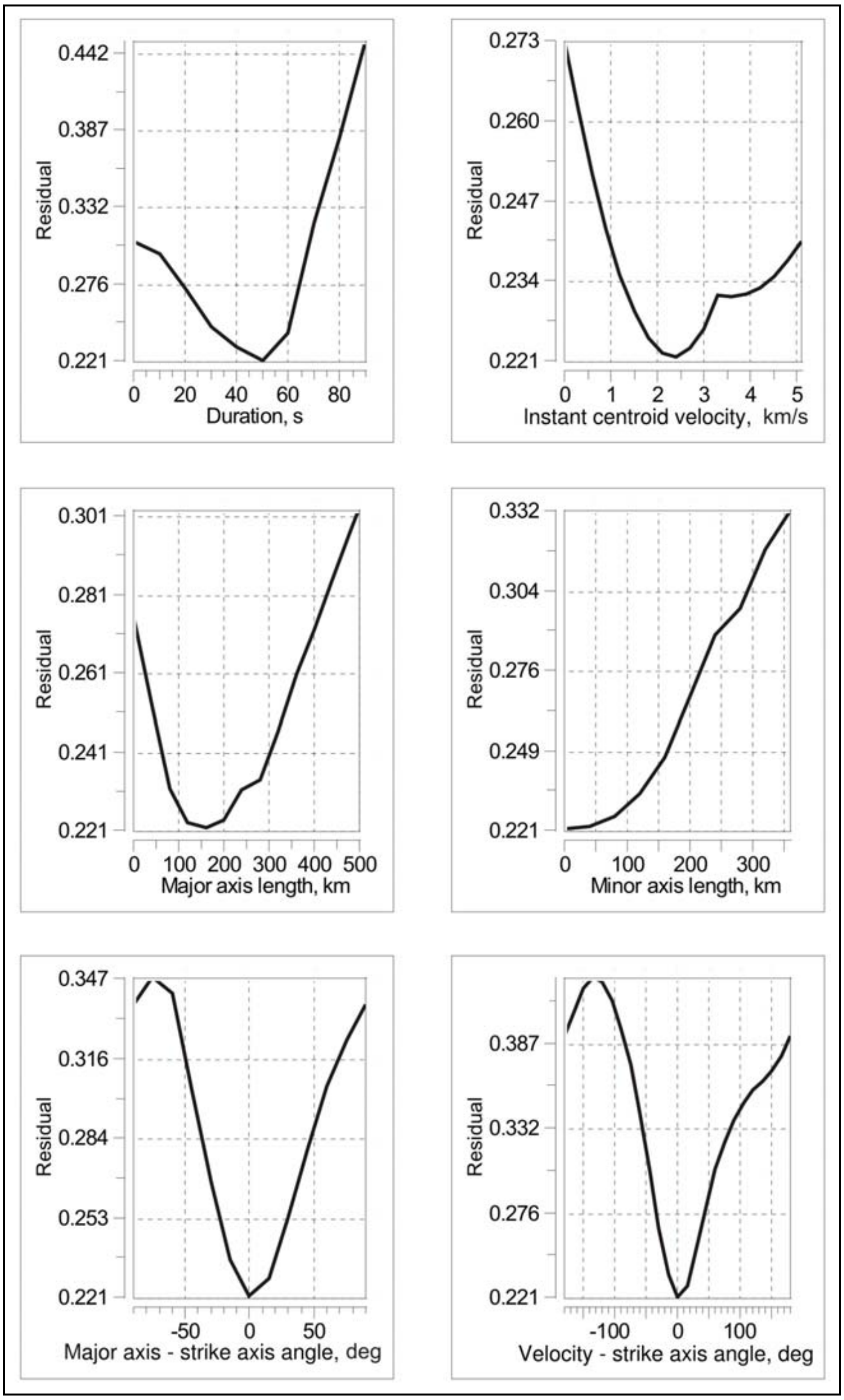


Fig. 6. Residual functions for source integral characteristics.

The propagation of rupture may be characterized by directivity ratio *d* proposed by McGuire (2002). This parameter is defined as the ratio of the average velocity of the instant centroid over the apparent rupture velocity equal to $l_{max}/\Delta t$. For a unilateral rupture where slip nucleates at one end of a rectangular fault and propagates to the other at a uniform rupture velocity with a uniform slip distribution, $d = 1$. For a symmetric bilateral rupture that initiates in the middle and propagates to both ends of a fault at uniform rupture velocity with uniform slip distribution, $d = 0$. Predominantly bilateral ruptures correspond to $0 \le d < 0.5$ while predominantly unilateral ruptures correspond to $0.5 < d \le 1$. We find $d = 0.75$ for our model. This value shows predominantly unilateral (northward) rupture propagation.

The relation between integral estimates and real characteristics of source duration and spatial extent depends on the distribution of moment rate density in time and over the fault. In the case of Gaussian distributions the 99% confidence duration is 2.5 times larger then the integral estimate, and 99% confidence axis length is 3 times larger then correspondent integral estimate. Multiplying the integral estimate of duration by factor 2.5 we get for source process duration the value being equal to 125s. Multiplying the integral estimates for principal axes length by factors 3 we get for major axis size 480 km, and for minor axis less than 150 km.

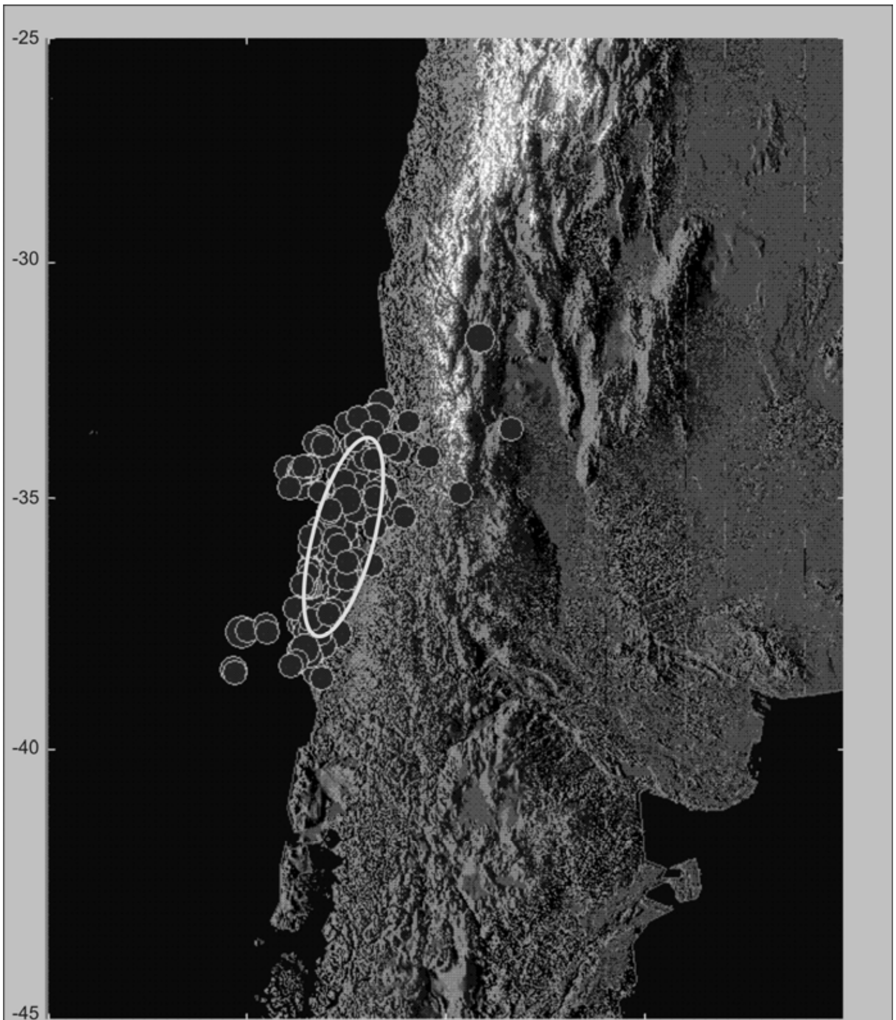


Fig. 7. The first 3 days aftershocks distribution and ellipse describing the characteristic source dimensions.

The comparison of estimates for principal axes lengths with aftershocks distribution is presented in figure 7. We consider aftershocks occurred in the first three days after the earthquake. The axes of ellipse in the figure are equal to estimated principal axes lengths multiplied by 3. As one can see the ellipse delineates the most of the aftershock region.

Largest aftershocks

Seismic waves radiated by most of aftershocks propagated on the background of wave field radiated by the preceding much stronger events. As result the seismic records are difficult to interpret, and for four of studied aftershocks, including the strongest one (M_w=7.5) occurred in 1.5 hours after the main shock, there are no solutions in the Global CMT catalog.

We interpreted these noisy seismograms and isolated surface waves by filtering them in frequency-time domain.

The map of 16 studied aftershocks is shown in figure 8. Results of inversion are given in the table 1.

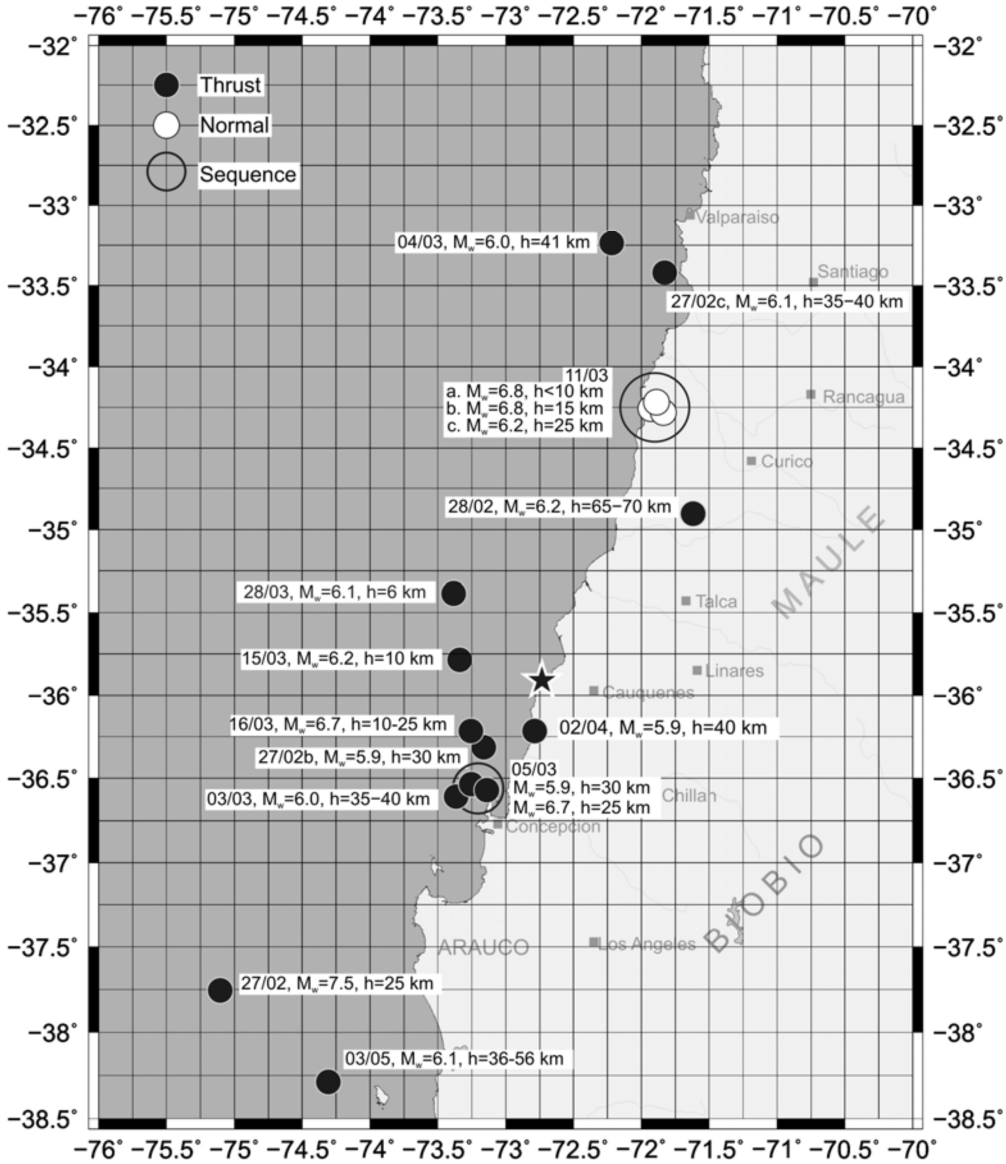


Fig. 8. Map of studied aftershocks. Black circles mark epicenters of thrust faults, white circles mark epicenters of normal faults, the star mark epicenter of the main shock.

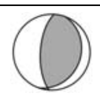
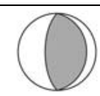
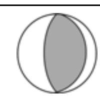
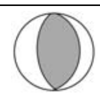
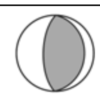
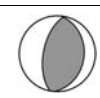
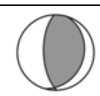
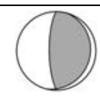
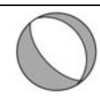
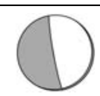
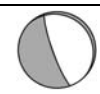
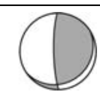
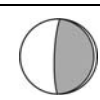
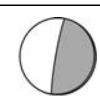
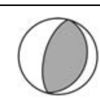
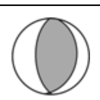
Date	**Time**	**Latitude**	**Longitude**	M_w	**Depth, km**	**Focal mechanism**
27.02.2010	08:01:24	37.754 S	75.104 W	7.5	26	
27.02.2010	17:24:31	36.314 S	73.160 W	5.9	30	
27.02.2010	19:00:07	33.422 S	71.829 W	6.1	35-40	
28.02.2010	11:25:36	34.903 S	71.617 W	6.2	65-70	
03.03.2010	17:44:25	36.610 S	73.360 W	6.0	30	
04.03.2010	01:59:50	33.238 S	72.220 W	6.0	41	
05.03.2010	09:19:38	36.535 S	73.253 W	5.9	30	
05.03.2010	11:47:08	36.570 S	73.139 W	6.7	25	
11.03.2010	14:39:44	34.259 S	71.929 W	6.8	0-10	
11.03.2010	14:55:27	34.282 S	71.837 W	6.8	15	
11.03.2010	15:06:03	34.218 S	71.889 W	6.2	25	
15.03.2010	11:08:29	35.785 S	73.339 W	6.2	10	
16.03.2010	02:21:58	36.217 S	73.257 W	6.7	10-25	
28.03.2010	21:38:28	35.387 S	73.385 W	6.1	6	
02.04.2010	22:58:10	36.216 S	72.788W	5.9	40	
03.05.2010	23:09:38	38.271 S	74.309 W	6.1	36-56	

Table 1. Origins and solutions for largest aftershocks

Strongest aftershock, 27.02.2010, 08:01:24, M_w = 7.5

As an example we consider in more detail the strongest aftershock occurred 1.5 hours after the main shock. The seismograms are very complicated by the presence of waves radiated by the preceding main shock. USGS didn't report any solution for this event. Their estimate for the hypocenter depth is equal to 37.9 km. We filtered the seismograms by the frequency-time and polarization analysis package FTAN to isolate fundamental Love and Rayleigh modes. Example of filtering is given in figure 9.

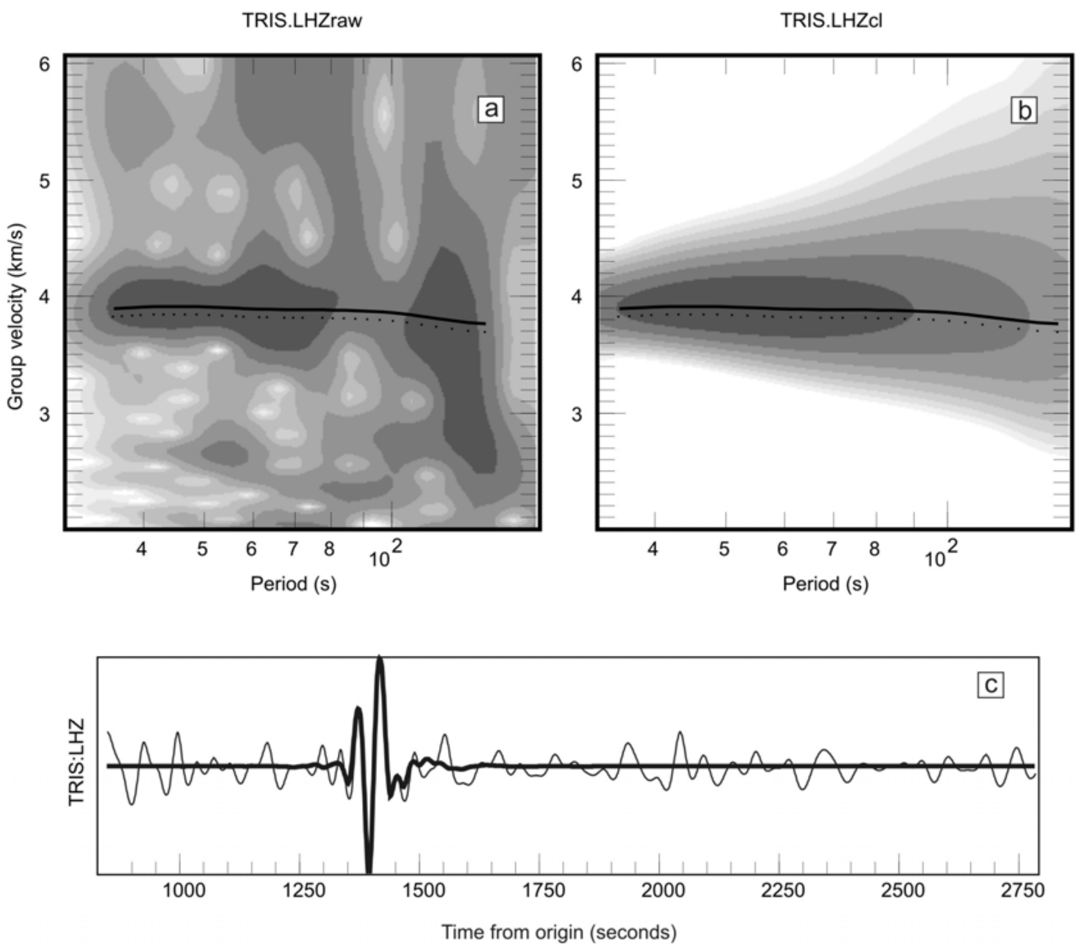


Fig.9. Example of filtering of vertical component of station TRIS. Raw and cleaned FTAN diagrams are shown in figures (a) and (b) respectively. The raw record is shown by thin line in figure (c), the cleaned fundamental Rayleigh wave is shown by thick line.

Inverting isolated long period (from 50 s to 140 s) amplitude spectra of fundamental Love and Rayleigh modes we obtained four equivalent double-couples radiating the same surface wave amplitude spectra. The selection between these four an optimal double-couple could be performed by analysis of polarities of direct P-waves, but these polarities can't be picked up because the P-waves are propagating on the background of seismic waves radiated by the preceding much stronger main shock. We selected for surface wave amplitude spectra inversion 7 Love wave records and 10 Rayleigh wave records of a reasonable quality from IRIS, GEOSCOPE and GEOFON stations. Their azimuthal distribution is given in figure 10. Our solution gives four focal mechanisms presented in figure 11. The left upper double-couple (P1: 20°, 25°, 105°; P2: 184°, 66°, 83°) is very similar to the double-couple obtained for the main shock. Analysis of surface wave phase spectra in period band from 100 s to 140 s confirms the selection of mentioned solution as an optimal. The estimated value of seismic moment is $0.2 \cdot 10^{21}$ Nm. That corresponds to M_w=7.5. Our estimate of best double-couple depth is equal to 30 km and shown in figure 12. Here the depth is measured from the free surface. The water layer in the source epicenter is about 4 km (model 3SMAC).

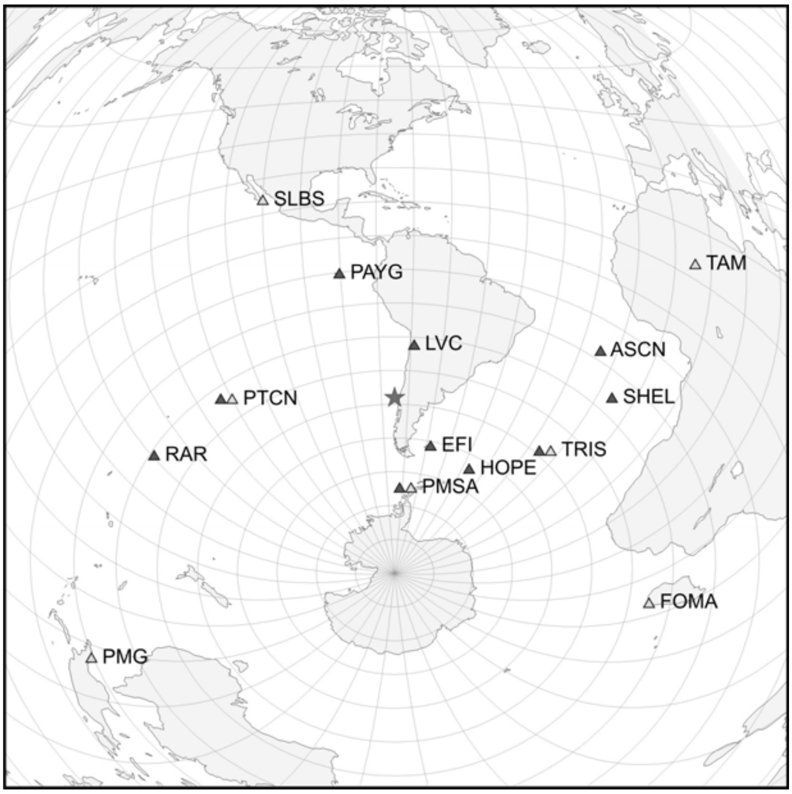


Fig.10. Distribution of stations used for moment tensor inversion. Dark triangles mark fundamental Rayleigh modes, light triangles mark fundamental Love modes. The star marks the epicenter.

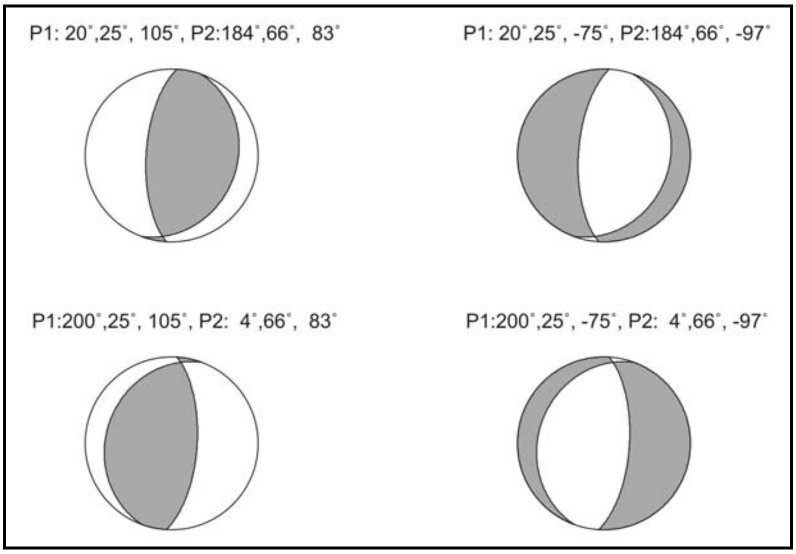


Fig. 11. Four equivalent best double-couples.

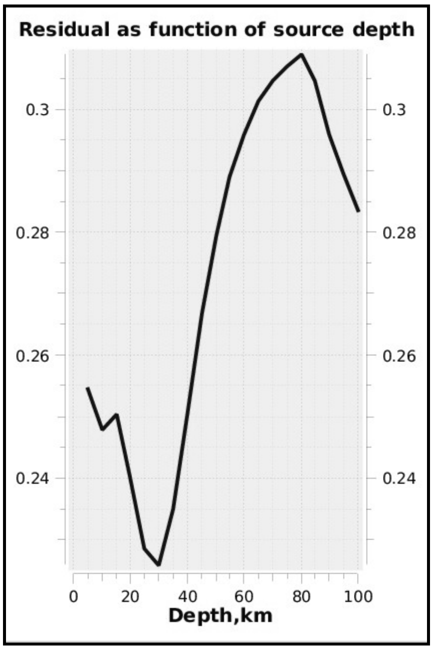


Fig.12. Best double-couple depth residual function.

The study will be continued. The aftershocks will be discussed integrating GPS and seismological information on the co-seismic slip distribution.

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