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Relative location, relative moment tensor inversion

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ICTP Course 2010 Trieste

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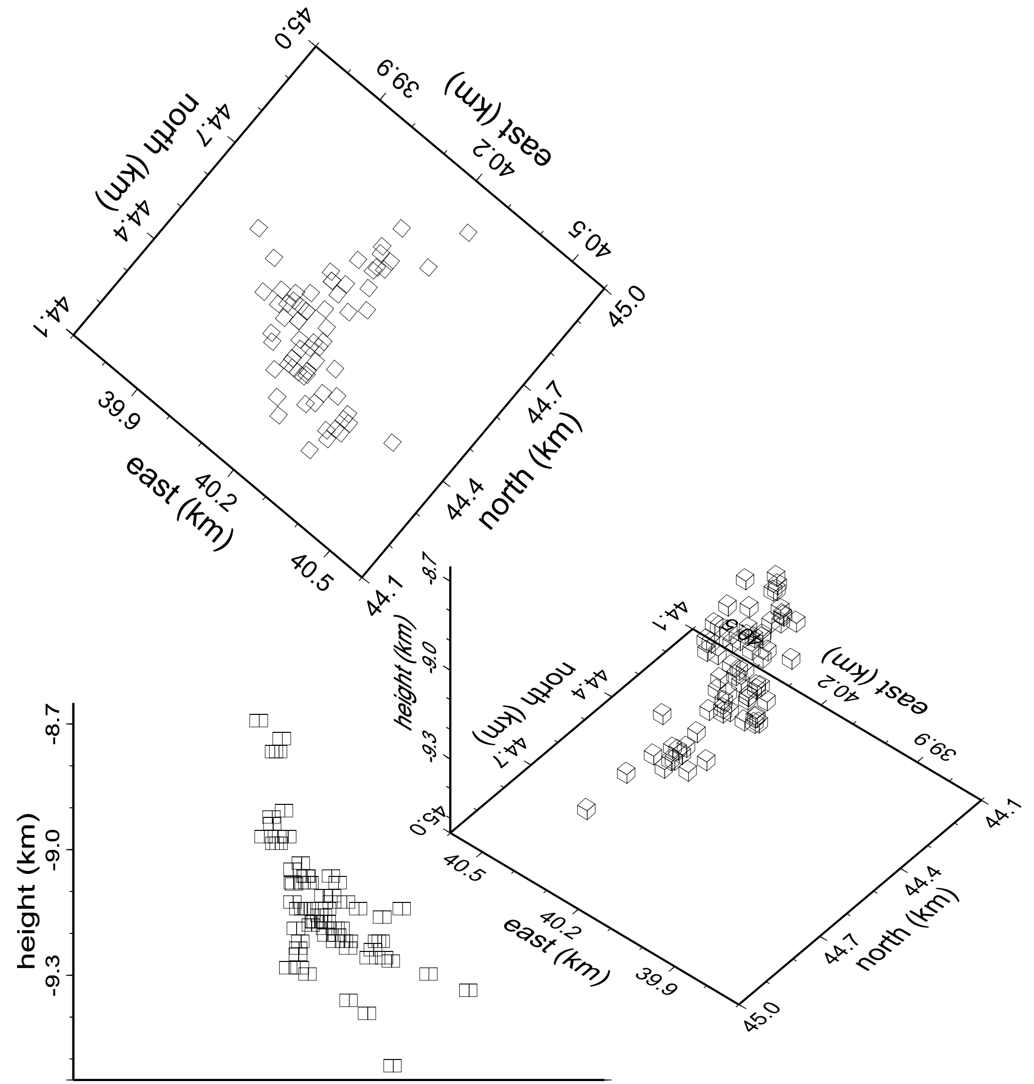
Earthquake swarms & cluster



700 clustered events from the 1997 Bohemian massive earthquake swarm (see Dahm, Sileny and Horalek, 2000).

A. Precise earthquake location

The simultaneous location of clustered earthquakes and multiplets increases the location accuracy and reduces model-dependent bias.

Single source location (Geigers method)

Equation for one observation at $\vec{X} = (X, Y, Z)$:

$$t = t^{\text{obs}} = h + T(\vec{x}, \vec{X}, \text{velocity})$$

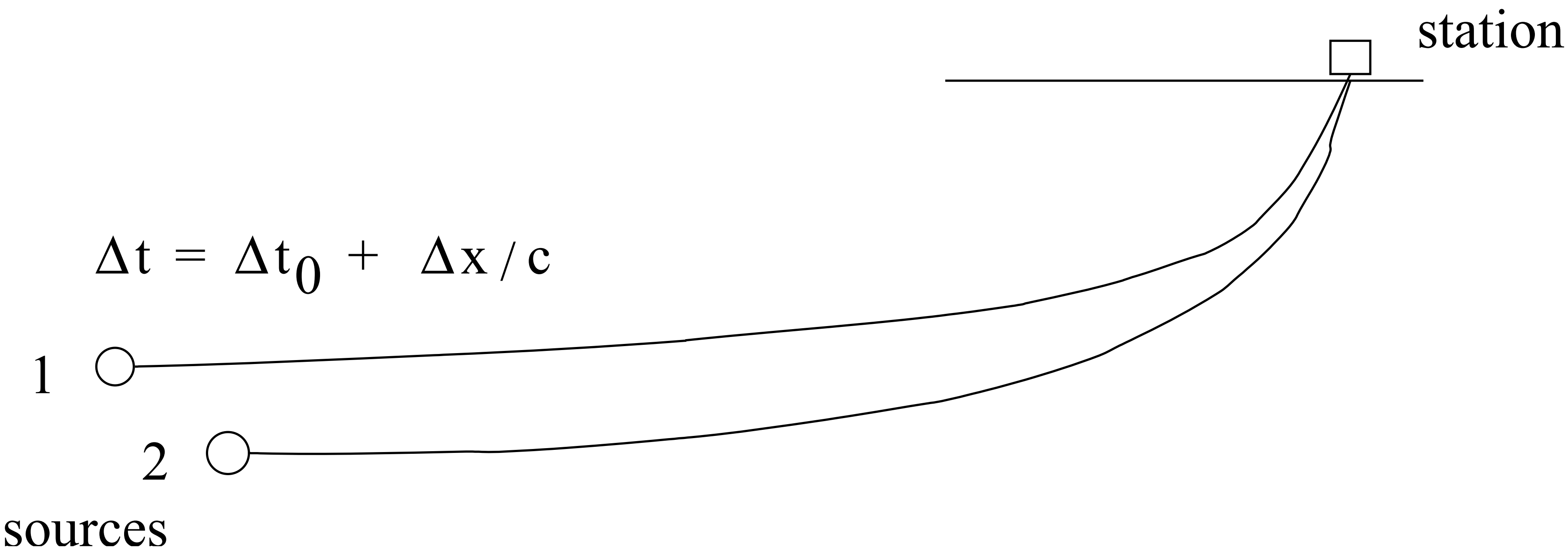
linearizing around starting model $\vec{m0}$ (summ. conv.)

$$t^{\text{obs}} - t^{\text{theo}}(\vec{m0}) \approx \frac{\partial t}{\partial m_k}(m0_k - m_k) = \frac{\partial t}{\partial m_k}\Delta m_k$$

with

$$\begin{aligned} t &: \text{ arrival time} \\ h &: \text{ origin time} \\ T &: \text{ travel time} \\ \vec{x} = (x, y, z) &: \text{ location vector} \\ \vec{m} = (h, X - x, Y - y, Z - z) &: \text{ model vector} \end{aligned}$$

master event location



far-field equation (i=slave event, j=master event):

$$t_i^{\text{obs}} - t_j^{\text{obs}} = (h_i - h_j) + \frac{\vec{n}}{c}(\vec{x}_i - \vec{x}_j)$$

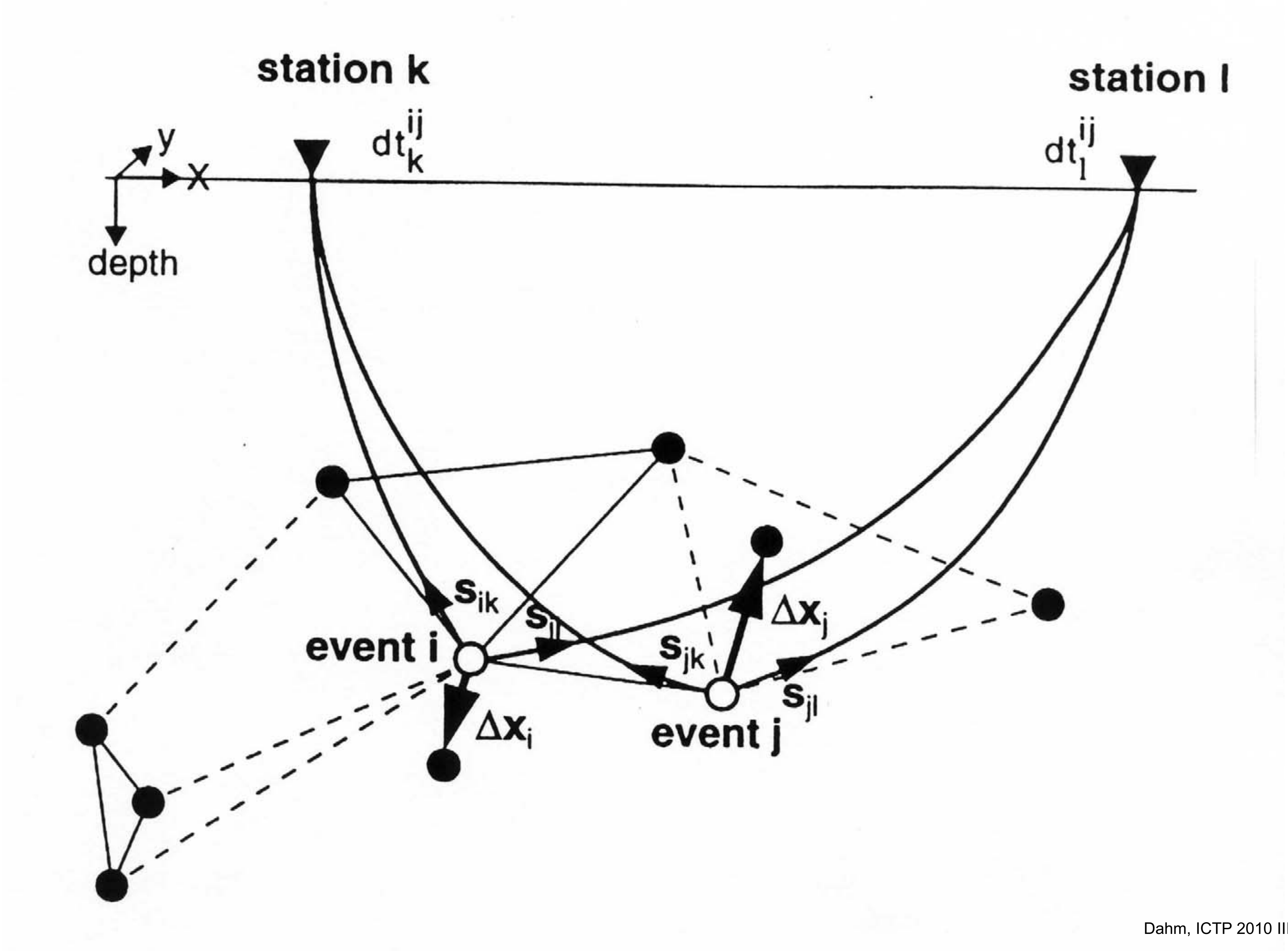
with

$\vec{n}$: unit vector of ray to $\vec{X}$ $\quad$ c : velocity at source

master event location

- relative locations with high accuracy, but absolute location is not improved
- location of master event is not changed
- high precision time-difference measurements when waveforms are similar
- a minor influence of the unmodelled unknown structure far away from the cluster
- systematic time errors at a station have no effect on the results

Double difference method



Double difference method (linearized)

The parallel ray assumption is relaxed. For each observation at $\vec{X}$:

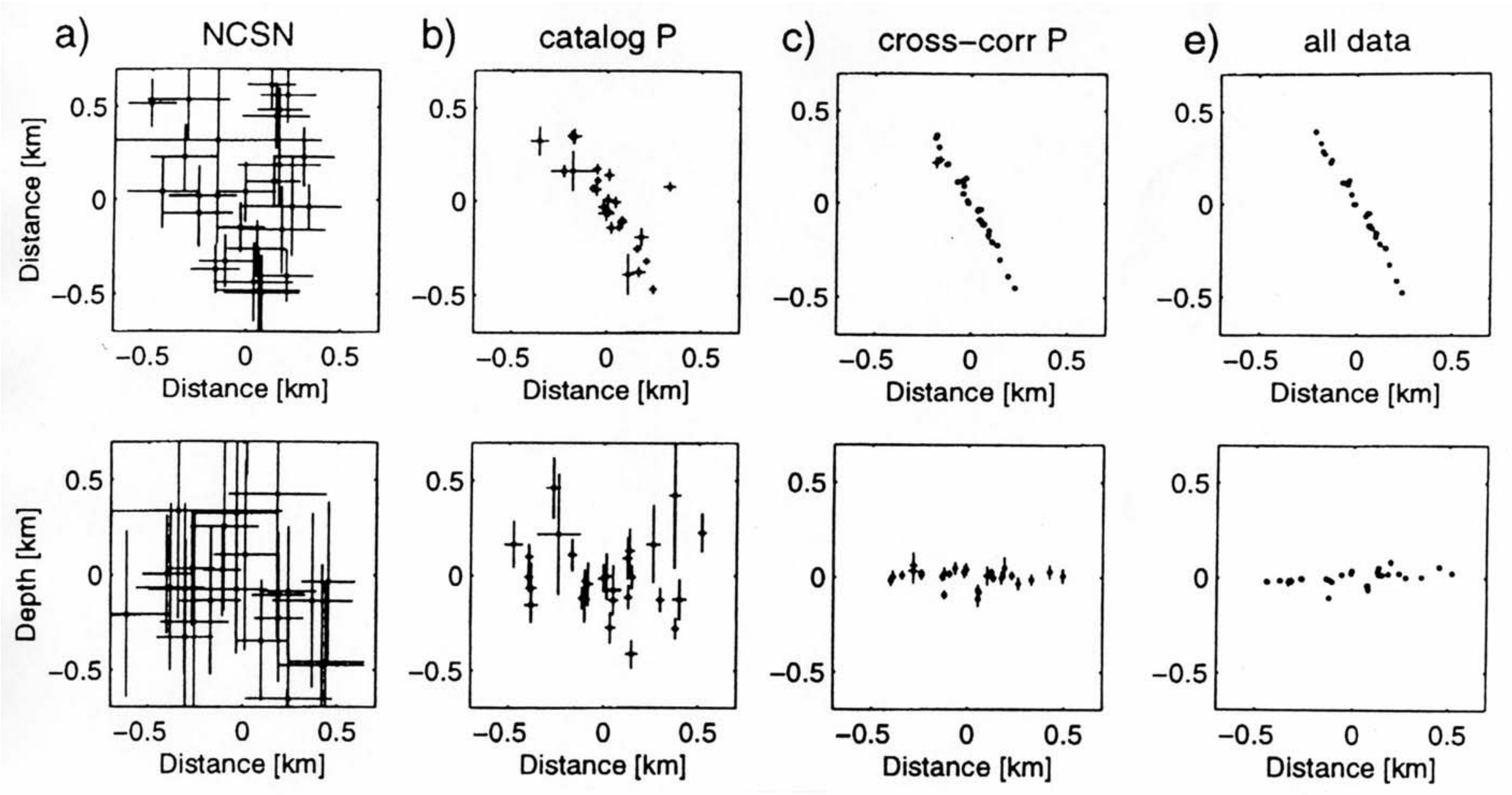
$$(t_{(i)}^{\text{obs}} - t_{(j)}^{\text{obs}}) \quad - \quad (t_{(i)}^{\text{theo}} - t_{(j)}^{\text{theo}}) \quad =$$

$$= \quad \frac{\partial t^{(i)}}{\partial m_k} \Delta m_k^{(i)} - \frac{\partial t^{(j)}}{\partial m_k} \Delta m_k^{(j)}$$

advantage	:	bended rays, mixed input data
disadvantage	:	nonlinear, iterative scheme
		accurate theoretical traveltime differences

(see Waldhauser and Ellsworth, BSSA, 2000)

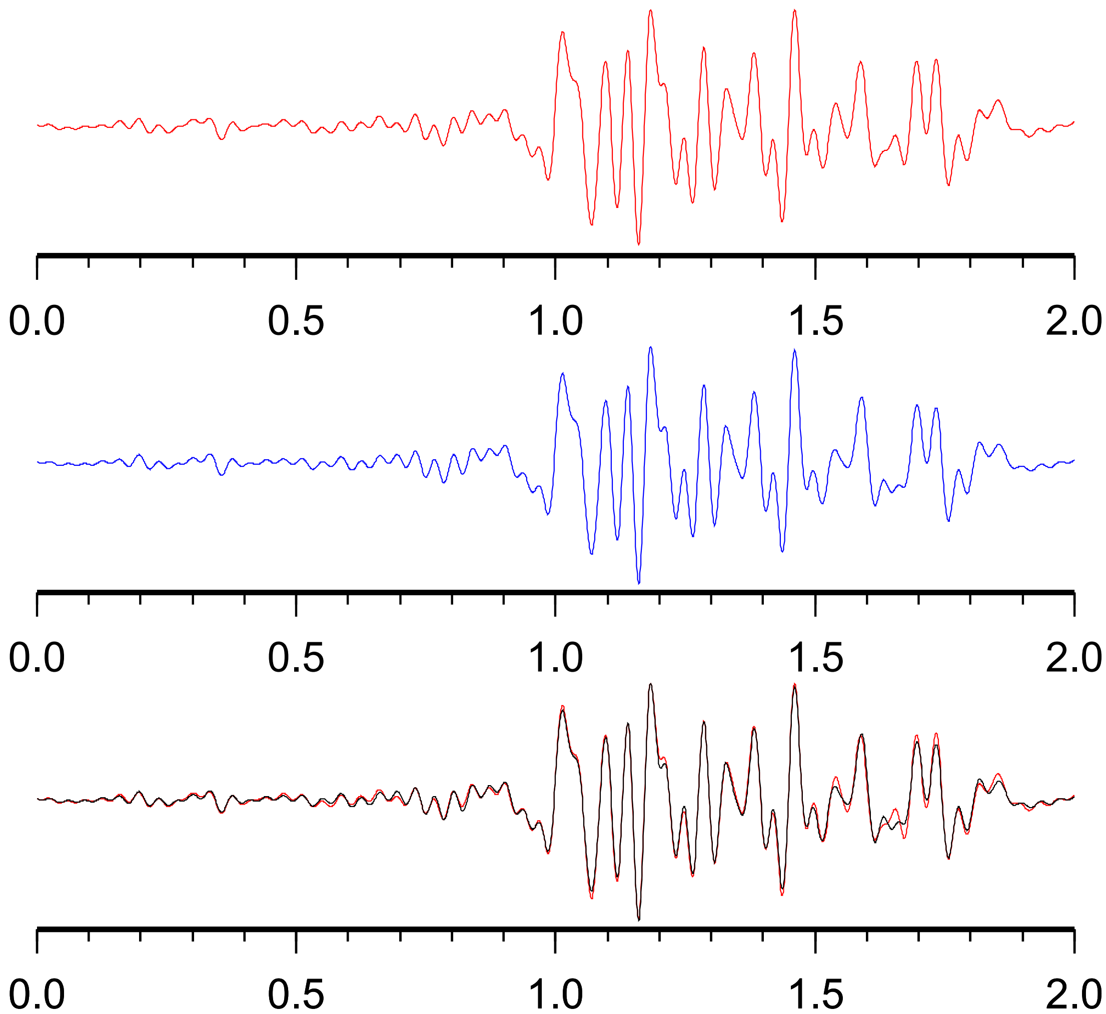
Double difference method



Relocation of 28 events from the Berkeley cluster

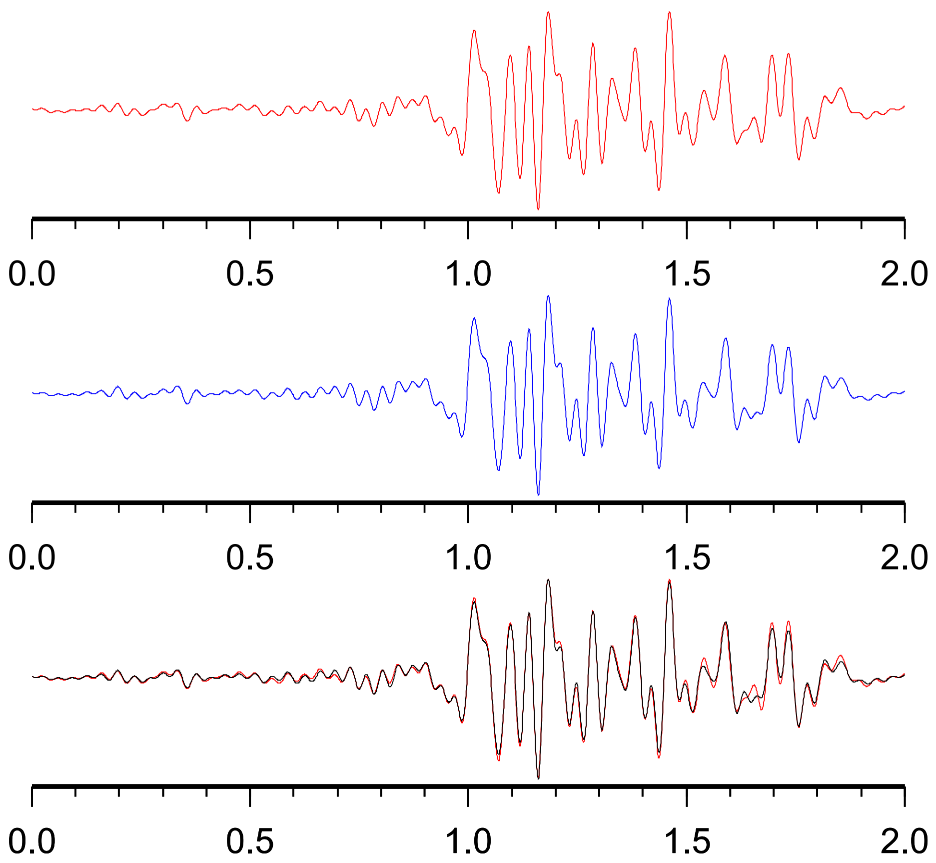
(Waldhauser and Ellsworth, BSSA, 2000)

Application to 1997 Vogtland swarm



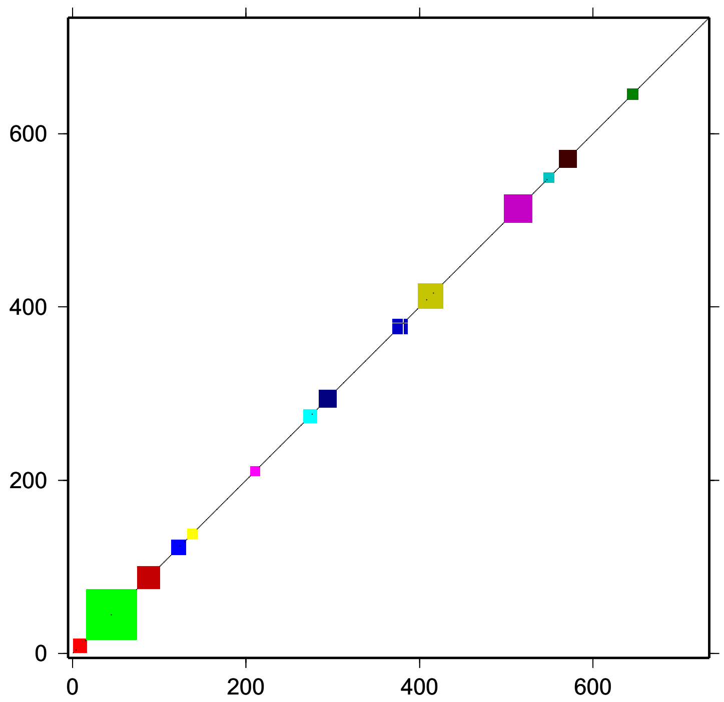
- WEBNET waveform data available (Horálek, pers. comm.)
- Precise estimate of time shift for best coherence
  - Arrival time differences with high accuracy
  - Easy amplitude picking

A. accurate time difference measurment



very accurate arrival time difference measurements when waveforms are similar (correlation approach)

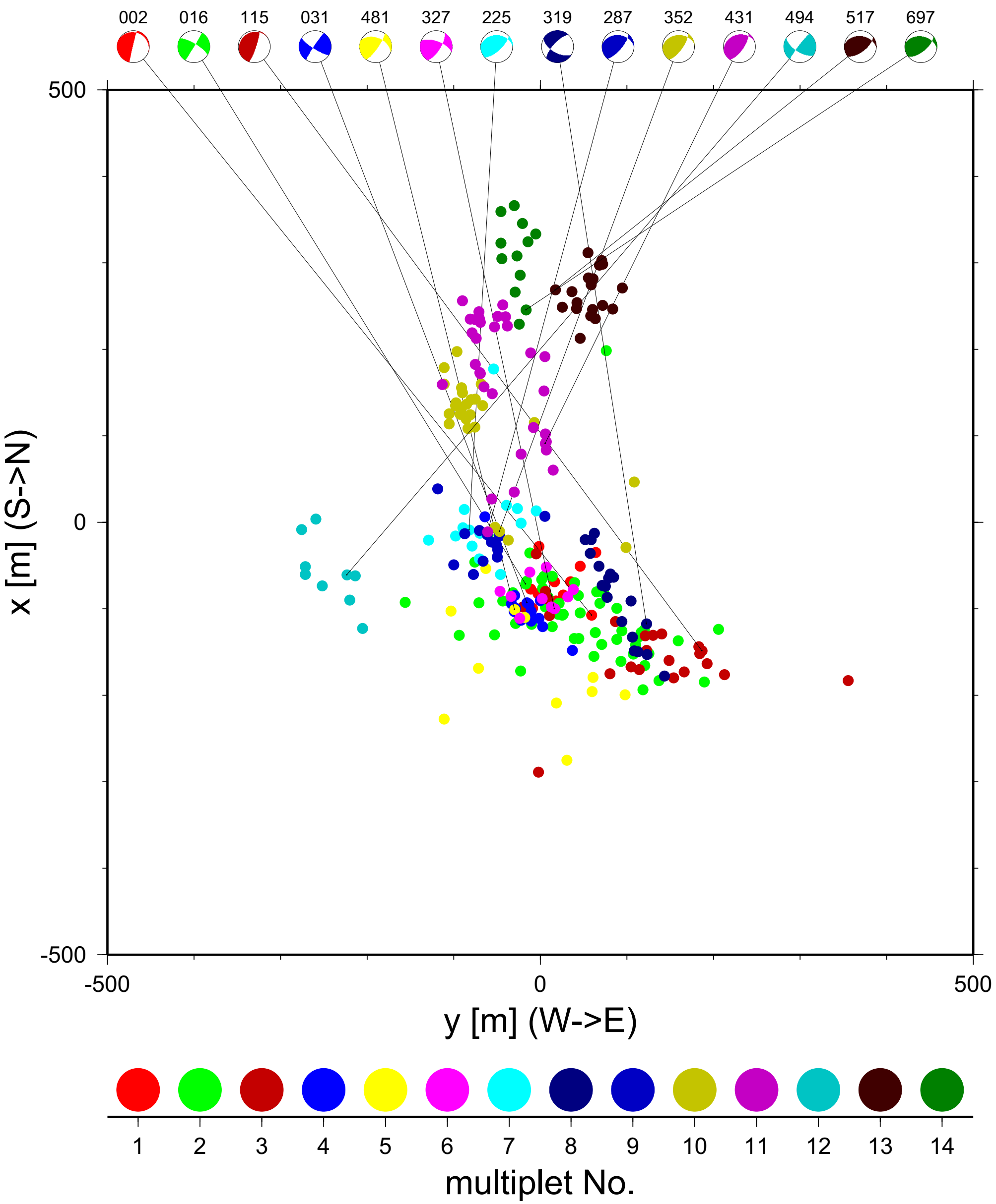
B. multiplet analysis

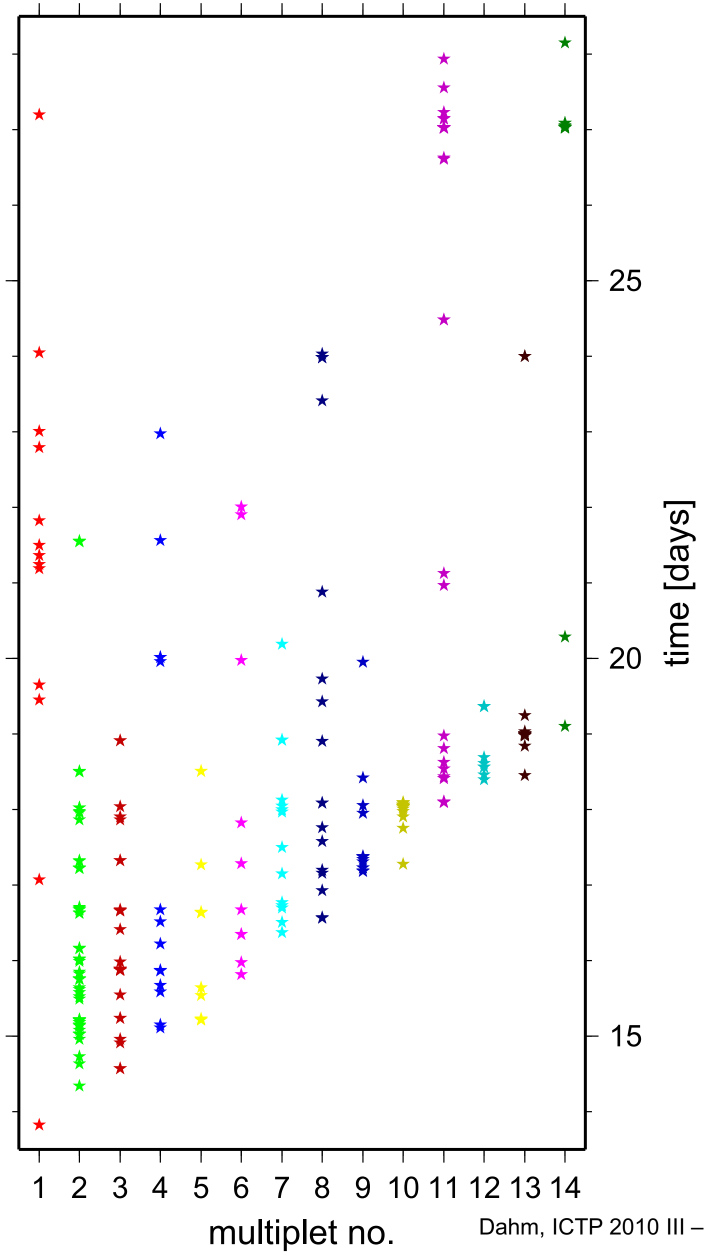


Coherence analysis (Maurer and Deichmann, 1995) with events of the 1997 earthquake swarm beneath Novy Kostel

- 37 % of the events (274) grouped in 14 multiplets
- relative time differences extracted
- correlation coefficients calculated

C. high precision relocation

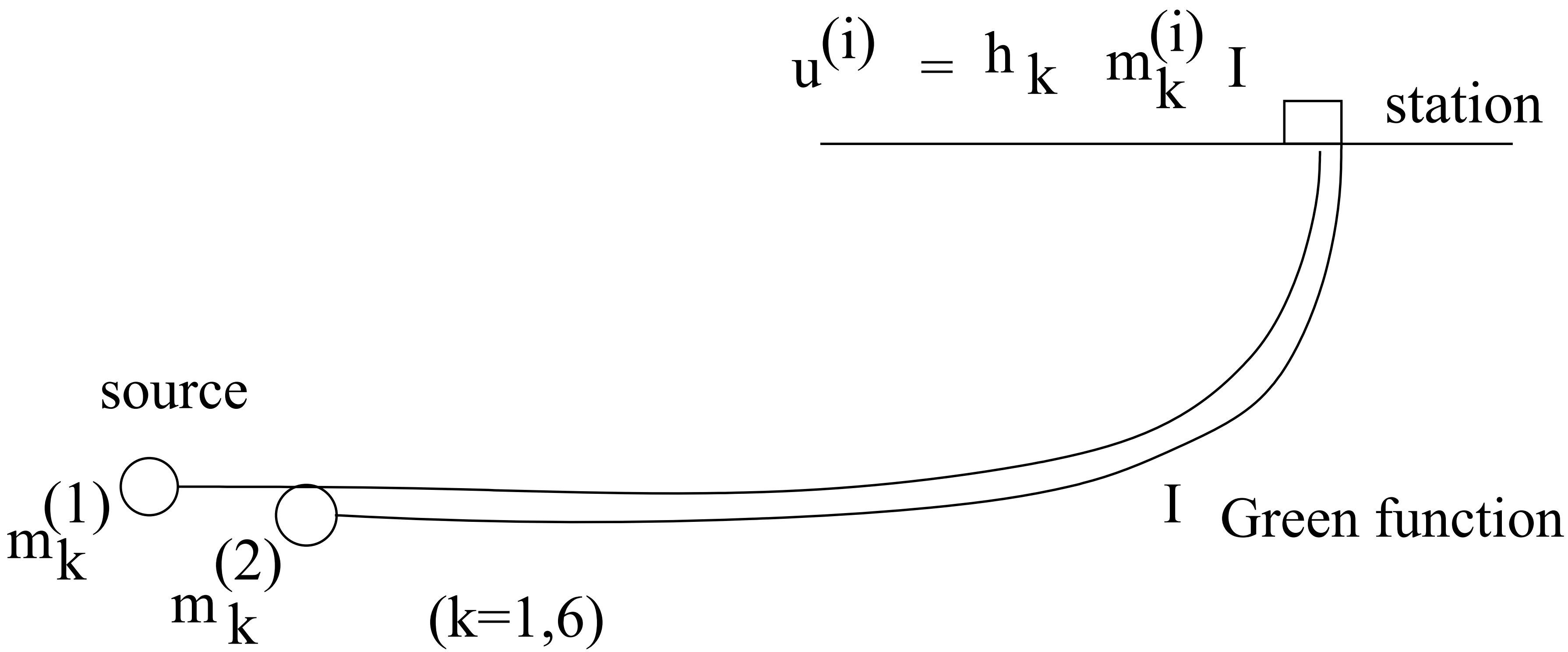




D. Precise source mechanisms

For clustered earthquakes and for multiplets a relative amplitude inversion (relative moment tensor) is possible and allows to analyse weak events. Having accurate locations and source mechanism is useful for the identification of micro-faults and stress inversion

relative moment tensor inverson



Body waves amplitudes depend on a single scalar Green function, that can be eliminated when one master event mechanism is known

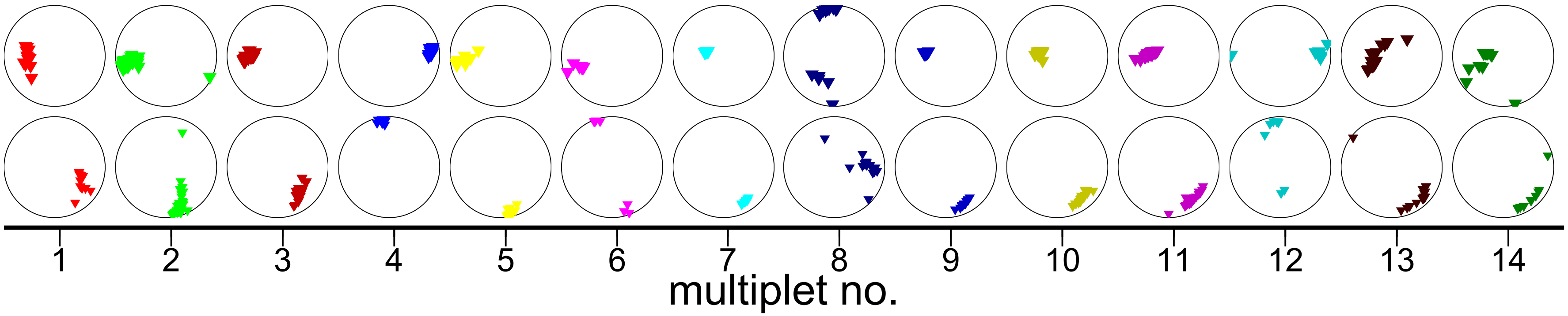
relative moment tensor inversion

- moment tensors relative to the tensor of a master event, but absolute tensors (moments) are not improved
- moment tensor of the master event is not improved
- relative amplitudes are measured with high precision when waveforms are similar
- unmodelled unknown structure far away from the cluster has a minor influence on the results
- systematic station effects have no impact on the results

assumptions

- the mechanism of the reference event is well known a priori
- events are narrow clustered and seismograms are lowpass filtered (temporal and spatial point source, all events occur within approx. one wavelength from the master event)
- used frequency range below the corner frequency of the largest studied event of the cluster
- isolated, non-interfering body-waves are used

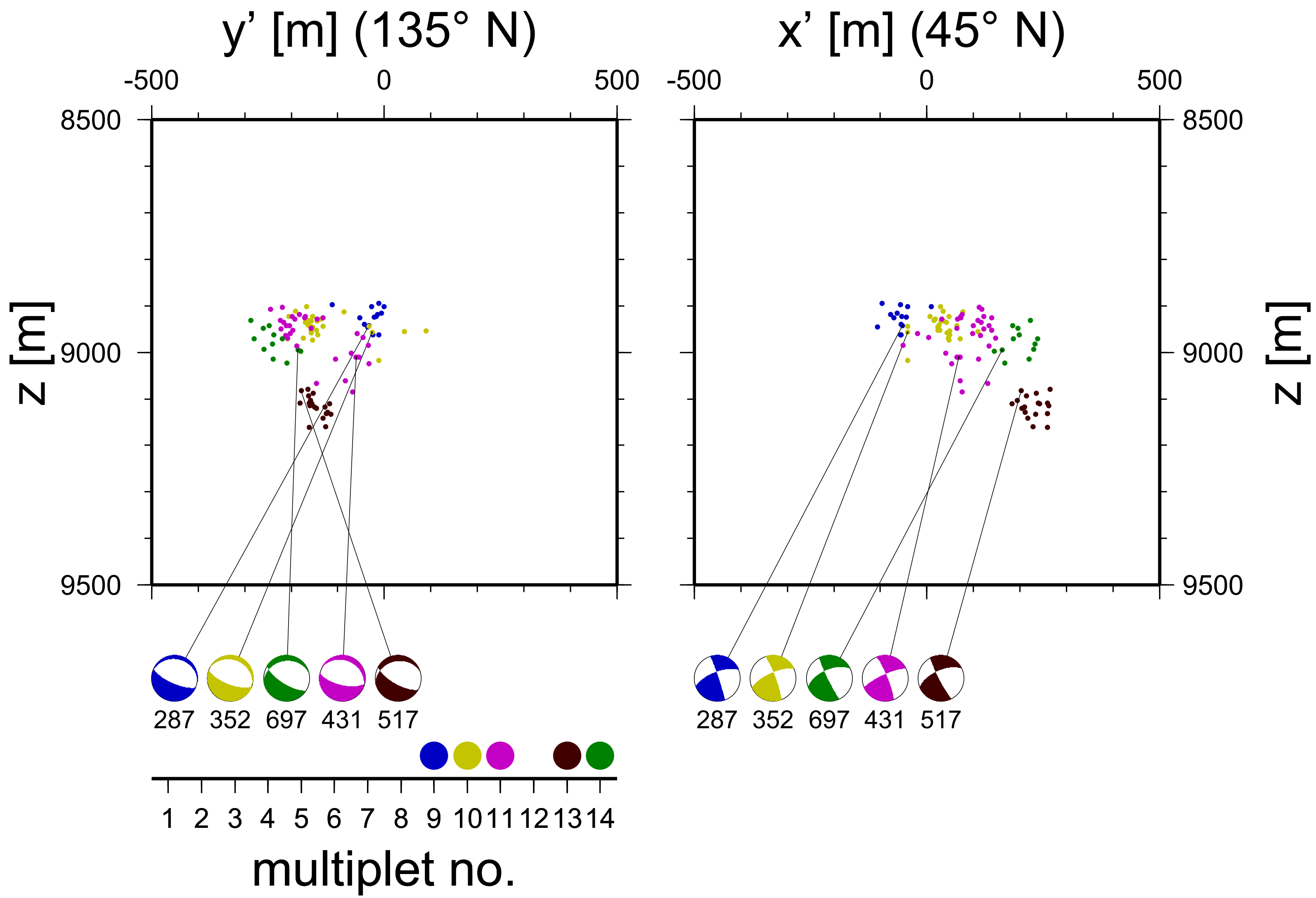
Estimation of Moment Tensors



Relative moment tensor inversion (Dahm, 1996)

- automated picking of P- and S-phase amplitudes
- 522 moment tensors inverted
- Classification with multiplets possible

Plane with en echelon faults



body-wave approach

for each ray (P, SV, SH): $\tilde{u} \;=\; h_k m_k \; I \quad ,$

with

$$
\begin{aligned}
h_1^P &= -\cos(2\varphi) sin^2\alpha\,, & h_1^{SV} &= -0.5\cos(2\varphi) sin2\alpha\,,\\
h_2^P &= \sin(2\varphi) sin^2\alpha\,, & h_2^{SV} &= 0.5\sin(2\varphi) sin2\alpha\,,\\
h_3^P &= \cos\varphi sin2\alpha\,, & h_3^{SV} &= \cos\varphi cos2\alpha\,,\\
h_4^P &= \sin\varphi sin2\alpha\,, & h_4^{SV} &= \sin\varphi cos2\alpha\,,\\
h_5^P &= 2 - 3sin^2\alpha\,, & h_5^{SV} &= -1.5sin2\alpha\,,\\
h_6^P &= 1\,, & h_6^{SV} &= 0\,,
\end{aligned}
$$

and similar for SH-waves. (e.g. Aki & Richards, 1982)

used convention

take-off angle: α

azimuth angle: φ

moment tensor:

$$
\begin{aligned}
m_1 &= \frac{1}{2}(M_{22} - M_{11}) \\
m_2 &= M_{12} \\
m_3 &= M_{13} \\
m_4 &= M_{23} \\
m_5 &= \frac{1}{3}(0.5(M_{22} - M_{11}) - M_{33}) \\
m_6 &= \frac{1}{3}(M_{11} + M_{22} + M_{33})
\end{aligned}
$$

what is different?

- the "Green function I" is **a scalar** for each body-wave mode

$\Longrightarrow$ it can be eliminated when using two earthquakes from the same source region

$$\tilde{u}^{(1)} = h_k^{(1)}\, m_k^{(1)}\, I$$

$$I = \frac{\tilde{u}^{(2)}}{h_k^{(2)}\, \mathbf{m_k^{(2)}}}$$

leading to

$$\tilde{u}^{(2)}\, h_l^{(1)}\, m_l^{(1)} = \tilde{u}^{(1)}\, h_k^{(2)}\, m_k^{(2)}$$

$$[\text{data}] = [\text{vector}\,]^T\, [\text{unknown model}]$$

additional polarity constraints

$$\tilde{u}^{(2)}\, h_l^{(1)}\, m_l^{(1)} = \tilde{u}^{(1)}\, h_k^{(2)}\, m_k^{(2)}$$

$$0 \leq \frac{\tilde{u}^{(2)}}{|\tilde{u}^{(2)}|}\, h_k^{(2)}\, m_k^{(2)}$$

$$[\text{data}] = [\text{vector}\,]^T\, [\text{unknown model}\,]$$

$$0 \leq \text{polarity} \cdot [\,\text{geometry}\,]^T\, [\text{unknown model}\,]$$

relative method without master

From

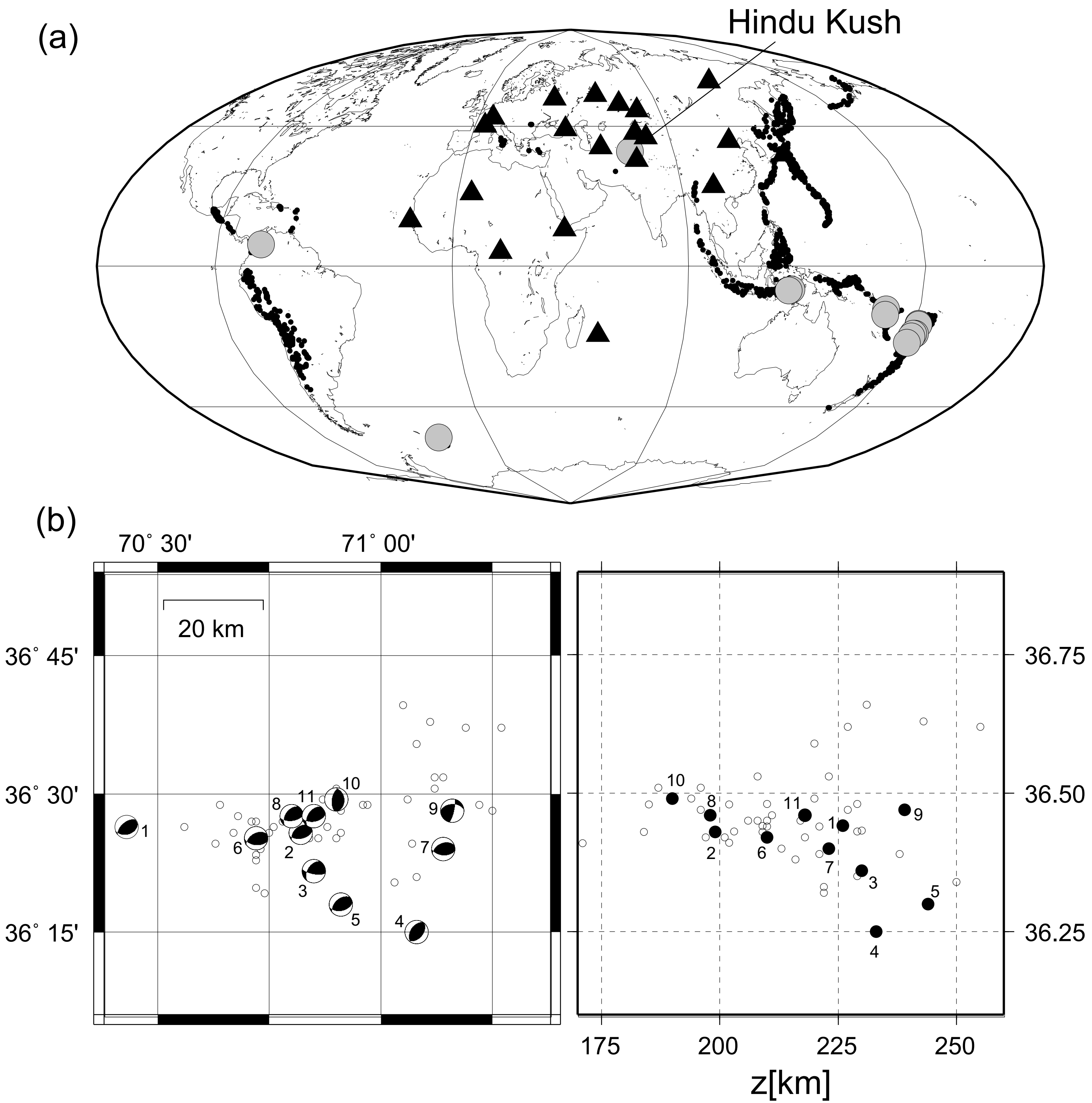
$$\tilde{u}^{(2)}\, h_l^{(1)}\, m_l^{(1)} \quad = \quad \tilde{u}^{(1)}\, h_k^{(2)}\, m_k^{(2)}$$

we formally write

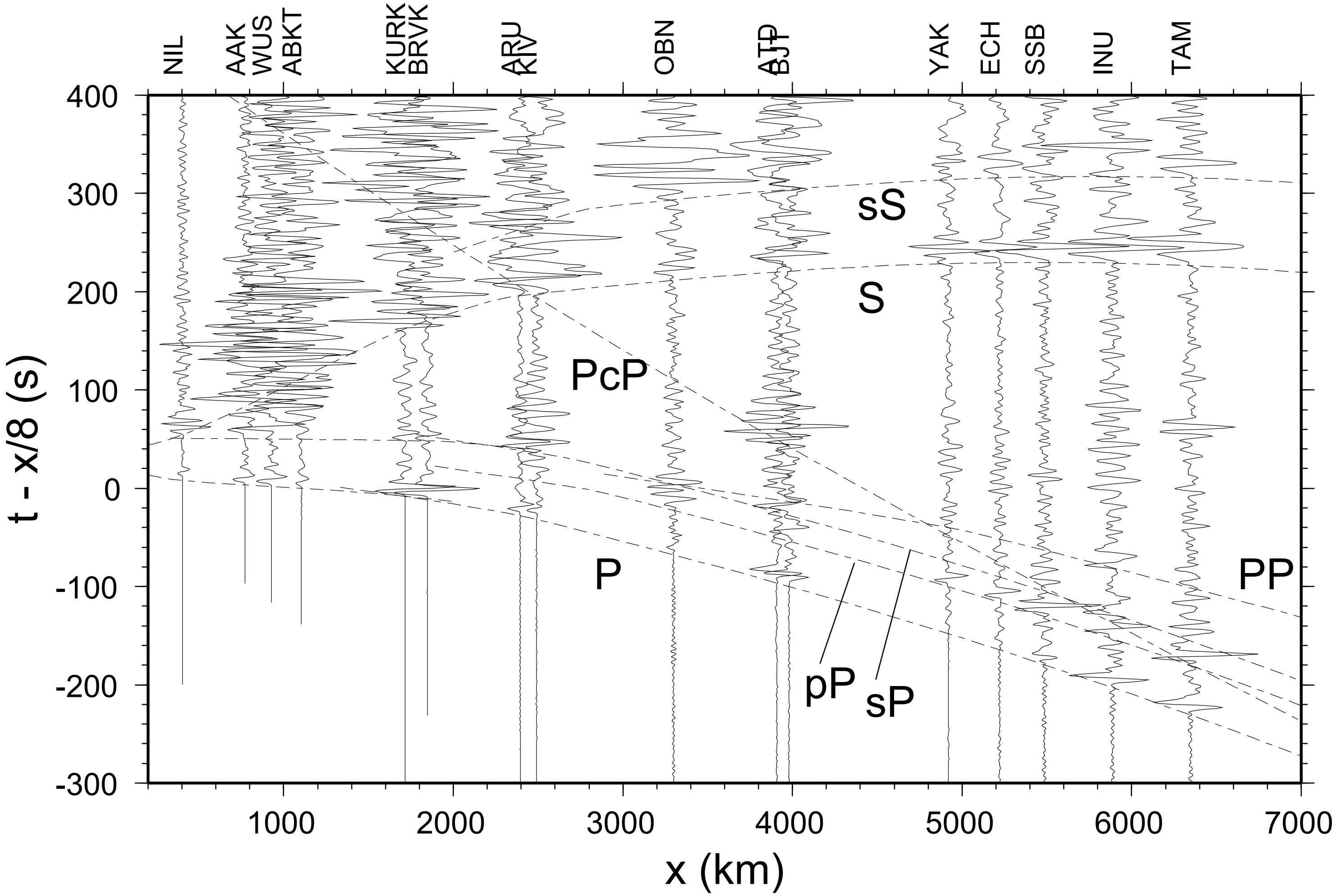
$$0 \quad = \quad -\tilde{u}^{(2)}\, h_l^{(1)}\, m_l^{(1)} \; + \; \tilde{u}^{(1)}\, h_k^{(2)}\, m_k^{(2)}$$

$$\begin{bmatrix} 0 \\ \text{const} \end{bmatrix} = \begin{bmatrix} -\tilde{u}^{(2)} h_1^{(1)} & \ldots & -\tilde{u}^{(2)} h_6^{(2)} & \tilde{u}^{(1)} h_1^{(2)} & \ldots & \tilde{u}^{(1)} h_6^{(2)} \\ 1 & \ldots & 1 & 1 & \ldots & 1 \end{bmatrix} \begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ \vdots \\ m_6^{(1)} \\ m_1^{(2)} \\ m_2^{(2)} \\ \vdots \\ m_6^{(2)} \end{bmatrix}$$

Hindu Kush deep cluster



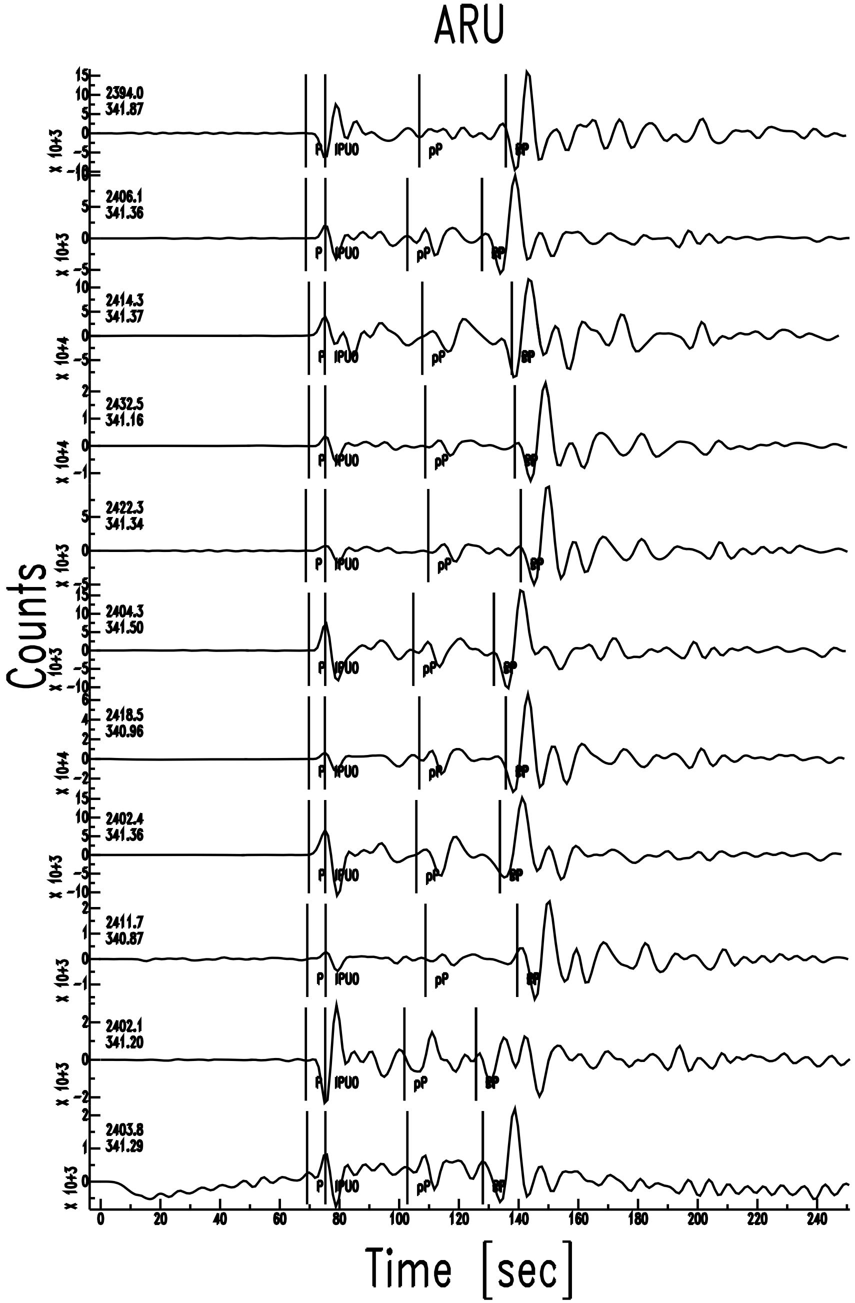
record section for event No 1

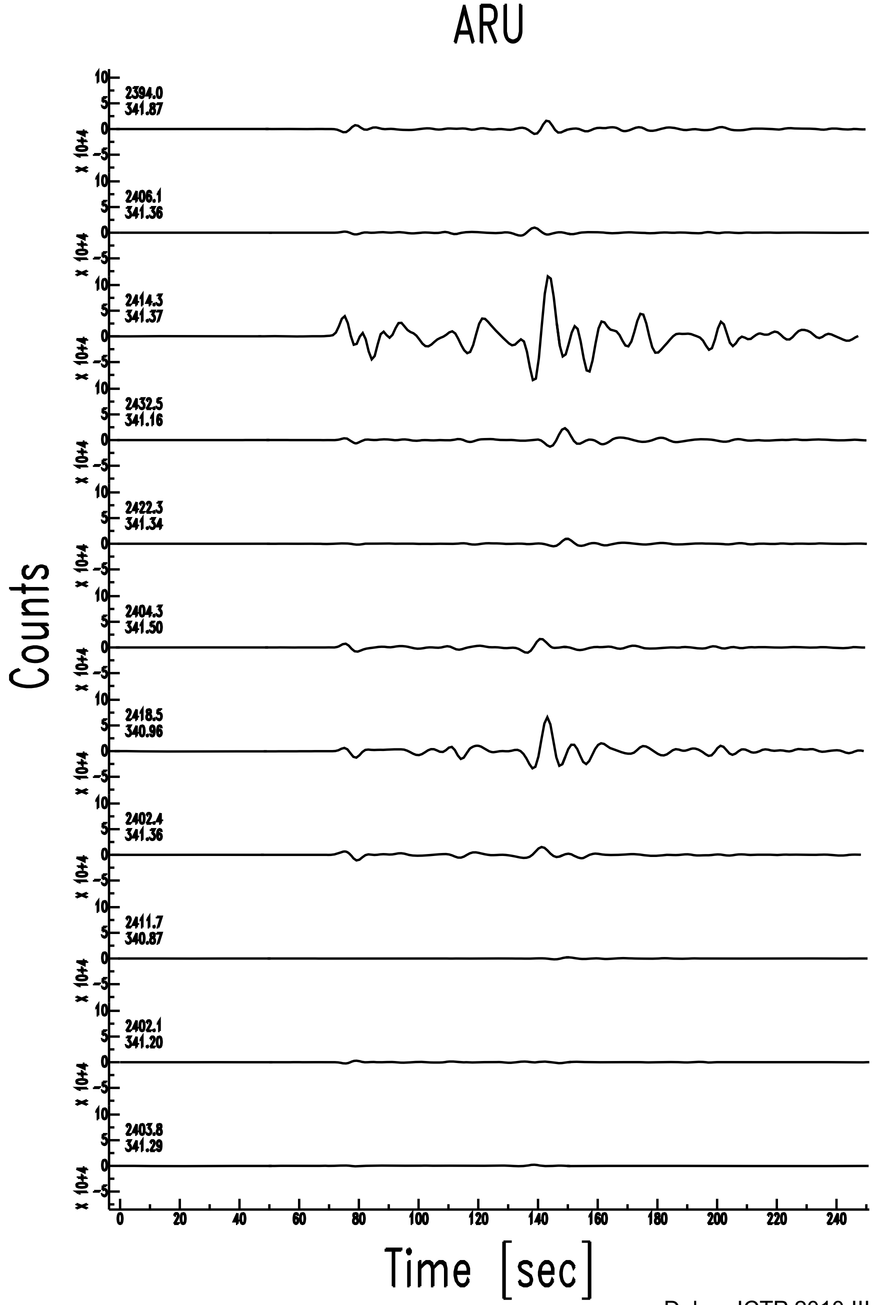


data processing

- 11 events with max. distance of 35 km.
- band-pass filter between 0.05 Hz and 0.1 Hz; wavelength between $\approx 80\,km$ (P) and $\approx 46\,km$ (S)
- P-, S-, pP-, sP-, PP-phases selected where no interference
- peak amplitudes measured

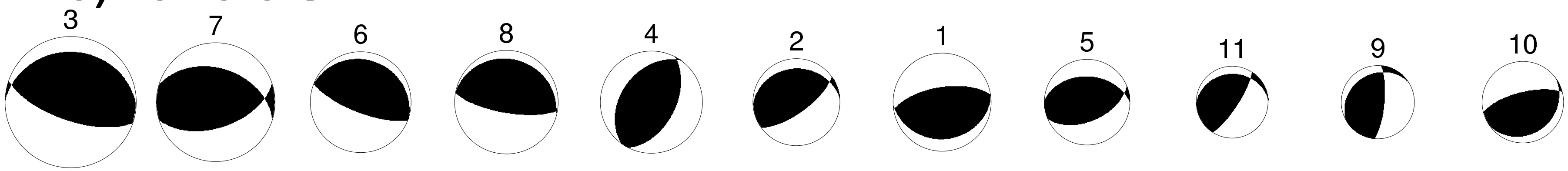
measuring amplitudes



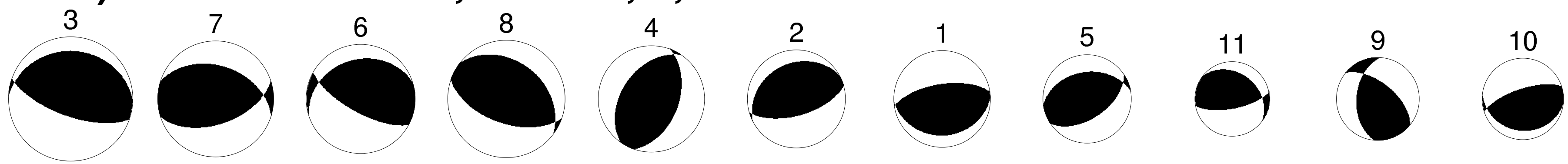


inversion results

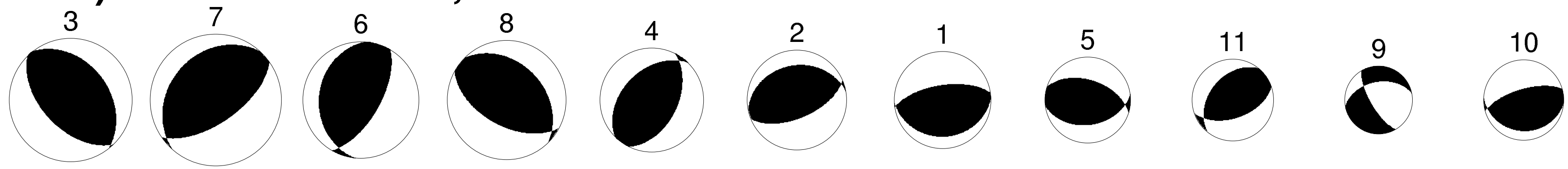
a) Harvard CMT



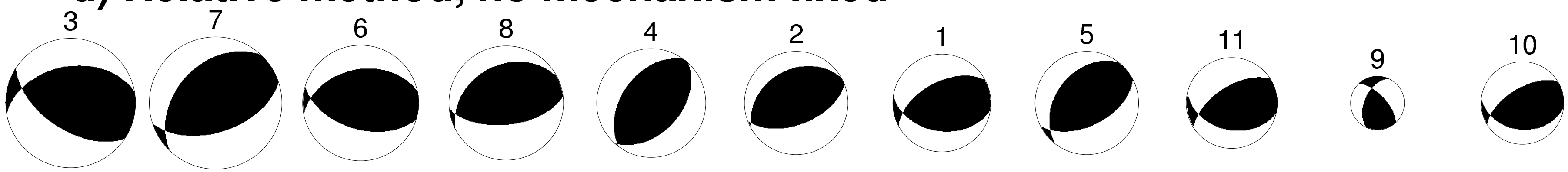
b) Relative method, event 1, 3, and 7 fixed



c) Relative method, event 1 fixed



d) Relative method, no mechanism fixed



programs and directories

directory *example*

1. *relref.exe*: relative inversion with reference mechanism
2. *relrpos.exe*: relative inversion without reference mechanism
3. *syndat4relef.f*: generate synthetic input to test geometry
4. *pltbeachball.cmd*: plot solutions in comparison to Harvad CMT

relref.f

Input: *relref.inp*

Output:

1. *relref.out*: full listing of result
2. *result.par*: short version of result for plotting
3. *relray.pola*: polarities used
4. ...

description of input file

```
HK, 11ev, 28.07.99, Pol
  iev ibit inorm idyn iclust imat idev ipol
   11    56    +1     0     0     0     6     0
                  fact(i),i=1,iev (16f5.1)
  1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1.0   1

   .00E+00      .0000




        wt     cmp     azi     toff / u(i,j), i=1,iev
 1.00     1     23.062   102.630            AAK   P       0.000      0.000
0 -7.952200000000e+03    25.644    98.997         0.0000 1.000e+00 -1
0 -1.928310000000e+05    23.072    99.817         0.0000 1.000e+00 -1
1  0.000000000000e+00    21.440   100.23 4        0.0000 1.000e+00  0
0 -7.277300000000e+04    23.992   106.039         0.0000 1.000e+00 -1

    ......                .....                  .....

    Reference Mechanisms
   1 0     -0.76       0.07       0.00      -0.63      0.12       0.76     0.4(
```

parameter description (formatted input!)

line 1	:	text, not used
line 3	:	*iev:* No of events
	:	*ibit:* No of extracted phases per event (max)
	:	*inorm:* 1 defines *fact(i)=1*
	:	*idyn:* 0 defines weighting with **wt(j)** of each phase
	:	*iclust:* 0 azimuth (φ) and take-off angle (α) defined only by event 1
	:	*iclust:* 1 take-off and azi as given in phase reading line
	:	*idummy* not used
	:	*idev* 5 deviatoric moment tensor constraint
	:	*ipol* 1 if polarity constraints should be used
line 5	:	*(16f5.1) (fact(i),i=1,iev)* to weight individual events (not recommended)
line 13 ff	:	event block-wise reading of phase amplitudes
last line(s)	:	definining moment tensor of reference event(s)

```
HK, 11ev, 28.07.99, Pol
  iev ibit inorm idyn iclust imat idev ipol
   11   56   +1     0     0     0    6     0
                     fact(i),i=1,iev (16f5.1)
  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1
```

ibit blocks of phase amplitudes

head line	:	*wt:* weight,
	:	*cmp:* compenent (1=ver,2=rad,3=trans),
	:	*azi:* mean azimuth,
	:	*toff:* mean take-off,
	:	*cstat:* station name,
	:	*cph:* phase name, must begin with either *P* (P-wave) or *S* (S-wave)
block of size *iev*	:	*isk(i,j):* phase reading may be scipped (except polarity constraint)
	:	*u(i,j):* measured amplitude
	:	*azi(i,j):* azimuth of ray
	:	*toff(i,j):* take-off angle of ray
	:	*rm:* dummy value, not used
	:	*tdiff:* relative arrival time difference (not used here)
	:	*f1:* geometrical spreading correction factor (if *iclust*=2)

```
       wt     cmp     azi     toff / u(i,j), i=1,iev
 1.00      1     23.062    102.630            AAK  P       0.000      0.000
0 -7.952200000000e+03     25.644     98.997       0.0000 1.000e+00 -1
0 -1.928310000000e+05     23.072     99.817       0.0000 1.000e+00 -1
1  0.000000000000e+00     21.440    100.23 4      0.0000 1.000e+00  0
0 -7.277300000000e+04     23.992    106.039       0.0000 1.000e+00 -1
```

reference event(s) moment tensors

add reference event lines as needed, each containing

kref	:	No of reference event as given in the data-block
is	:	(0) give moment tensor as $M_{11}, M_{12}, M_{22}, M_{13}, M_{23}, M_{33}$
xmref(1-6)	:	six moment tensor components as defined by *is*
rmref	:	scalar moment of reference event

```
 Reference Mechanisms
1 0    -0.76      0.07      0.00     -0.63      0.12      0.76    0.4(
```

practical

1. reproduce results using relref.exe
2. introduce deviatoric source constraint
3. introduce polarity constraint
4. use only event 1 as reference event (relref.exe)
5. change moment tensor of reference event

```
1. edit input file (relref.inp) and make changes

2. run relref.exe

3. view output files, e.g. relref.out

4. plot results in comparison to CMT solutions
(e.g. pltbeachball.cmd)
```