



Advanced School on Direct and Inverse Problems of Seismology

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**The study of fluid induced
and triggered seismicity
Theory**

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The study of fluid-induced and triggered seismicity: Theory

ICTP Course 2010

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Overview

- **Lecture A:** Theory - fundamentals, brittle failure, earthquake occurrence
- **Lecture B:** Case studies
 - Long-term, low pressure injection
 - Hydrofracturing & magmatic dikeing
 - Hydrocarbon field depletion
- **Lecture C:** Practical issues & methods



Lecture A; theory

- I. Introduction to the problem
- II. Elastic and brittle deformation
- III. Porous media: fluid flow and pore pressure
- IV. Earthquake trigger and seismicity models
- V. Single fractures: fundamental solutions



Examples of fluid-related seismicity

natural:

- magmatic dike-intrusion (vertical, lateral, horizontal)
- deep degassing through the crust
- water intrusions into glaciers (downward)
- natural hydraulic fracturing (e.g. vein formation)
- rain-induced seismicity

.... initiated by “reservoir” instability or natural inflow

man-made:

- hydraulic fracturing from boreholes
- massiv fluid-injections (waste, CO₂, etc.)
- fluid extraction from hydrocarbon reservoirs
- water reservoir filling (impoundment)

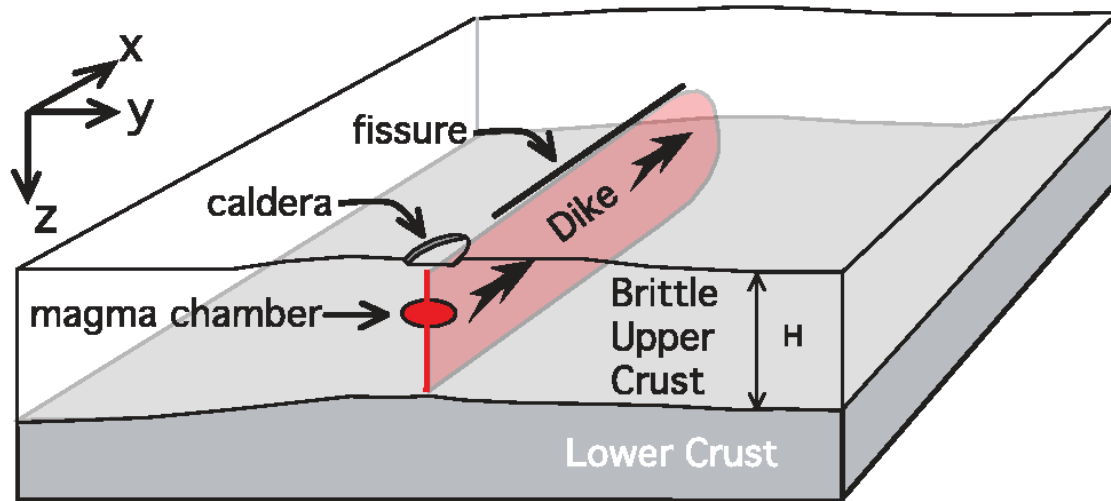
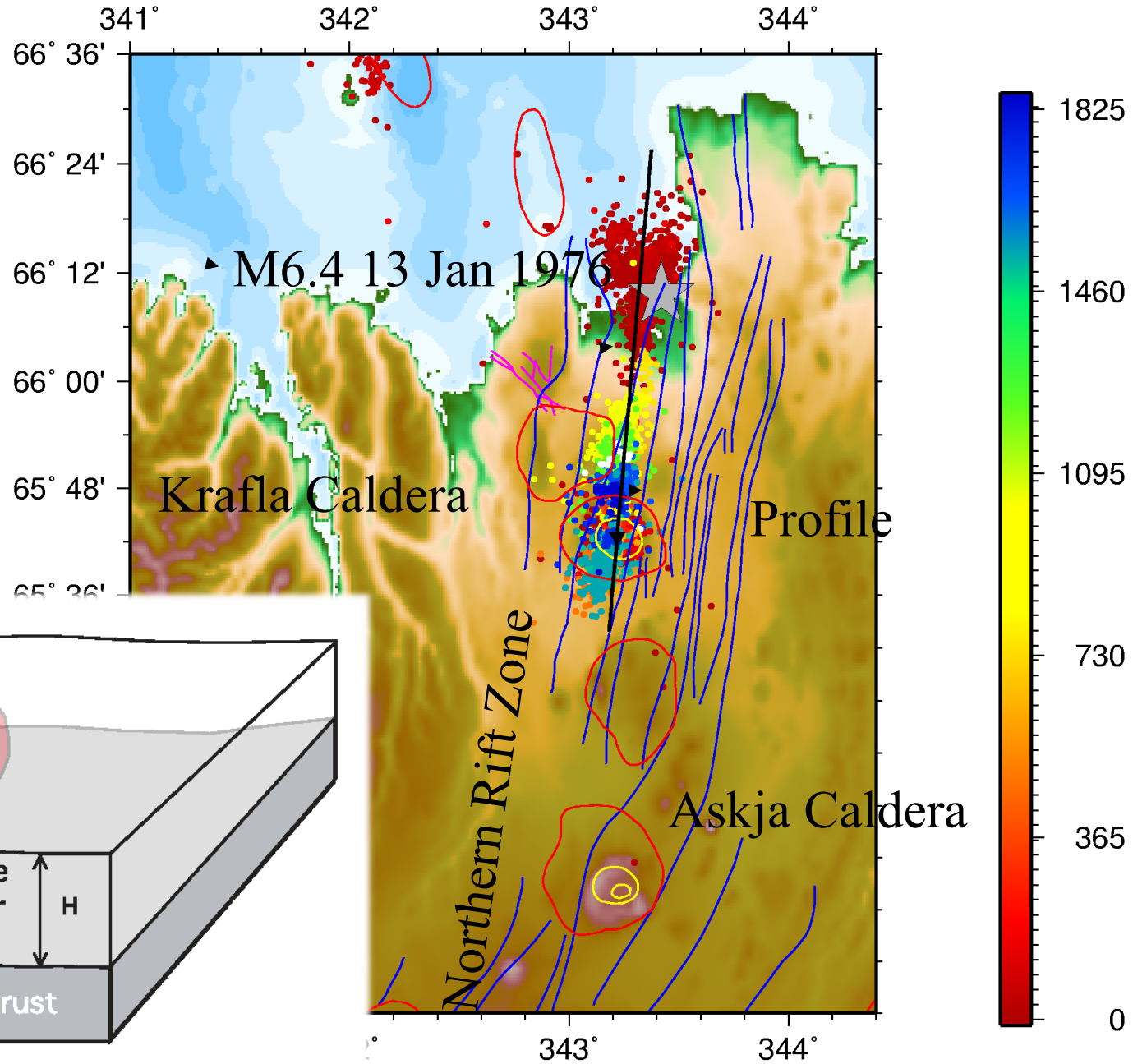
... the fluid flow and loading is induced by humans



Rifting episodes (Krafla 75 – 81): 10–70 km scale

Example of triggered significant earthquake

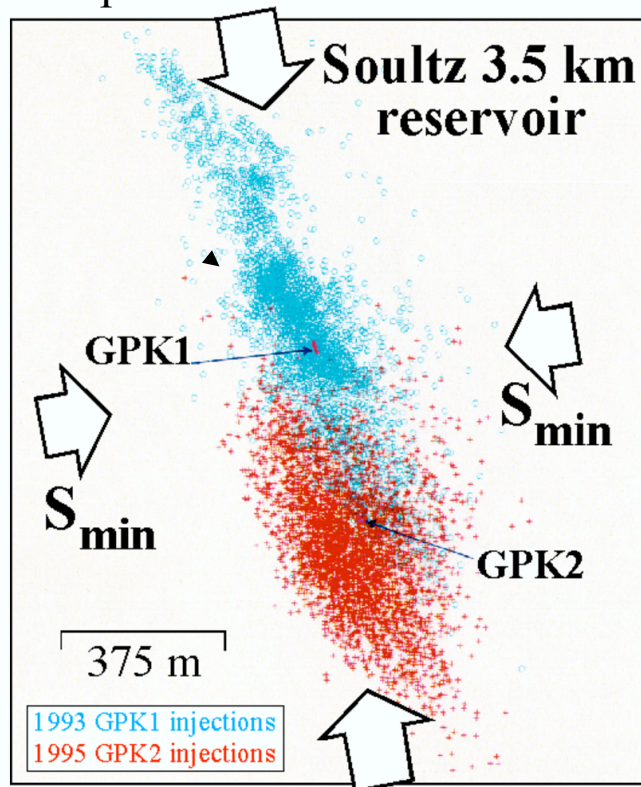
Induced earthquake swarms



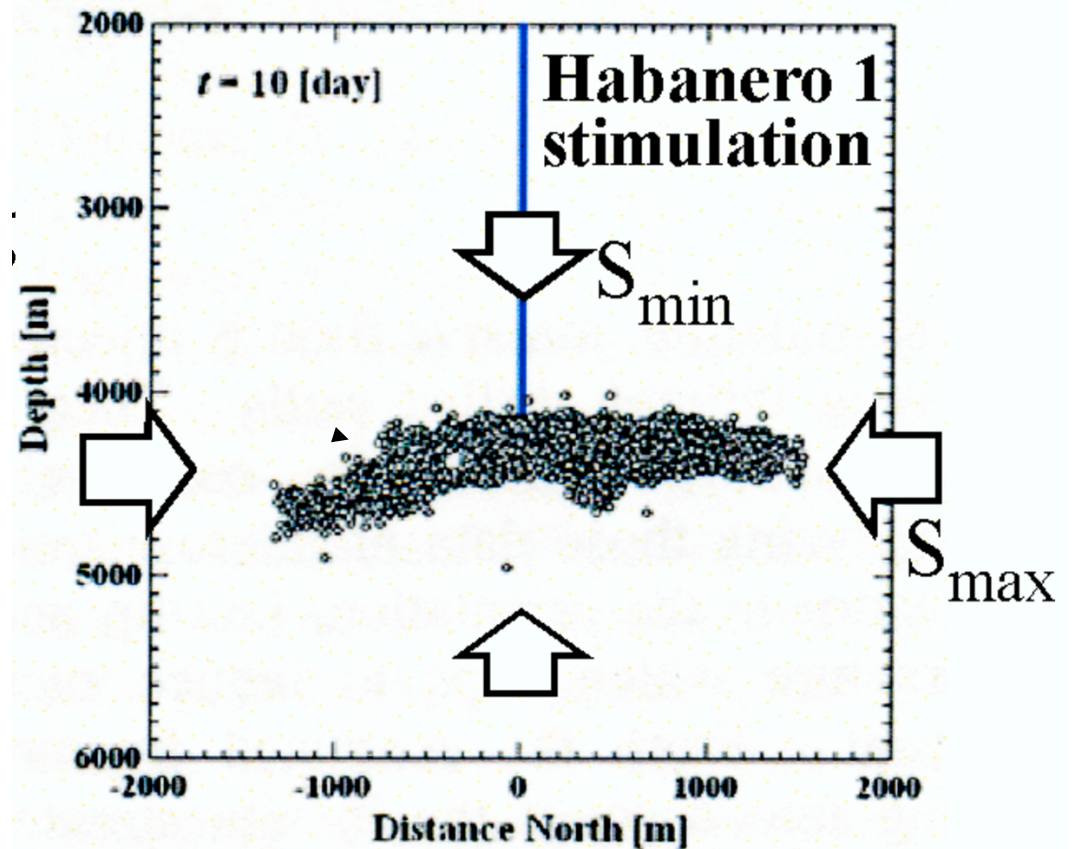
Hydrofracture induced swarms related to Hot Fractured Rock (HFR) experiments

Soultz sous foret (Rhinegraben)

Seismic cloud associated with stress / pressure change

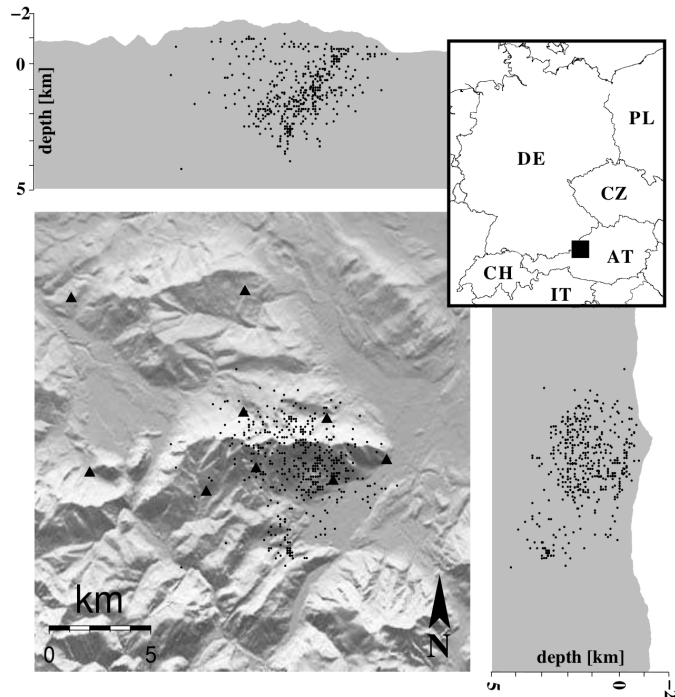


Cooperbasin (Australia)



Ito et al., 2006

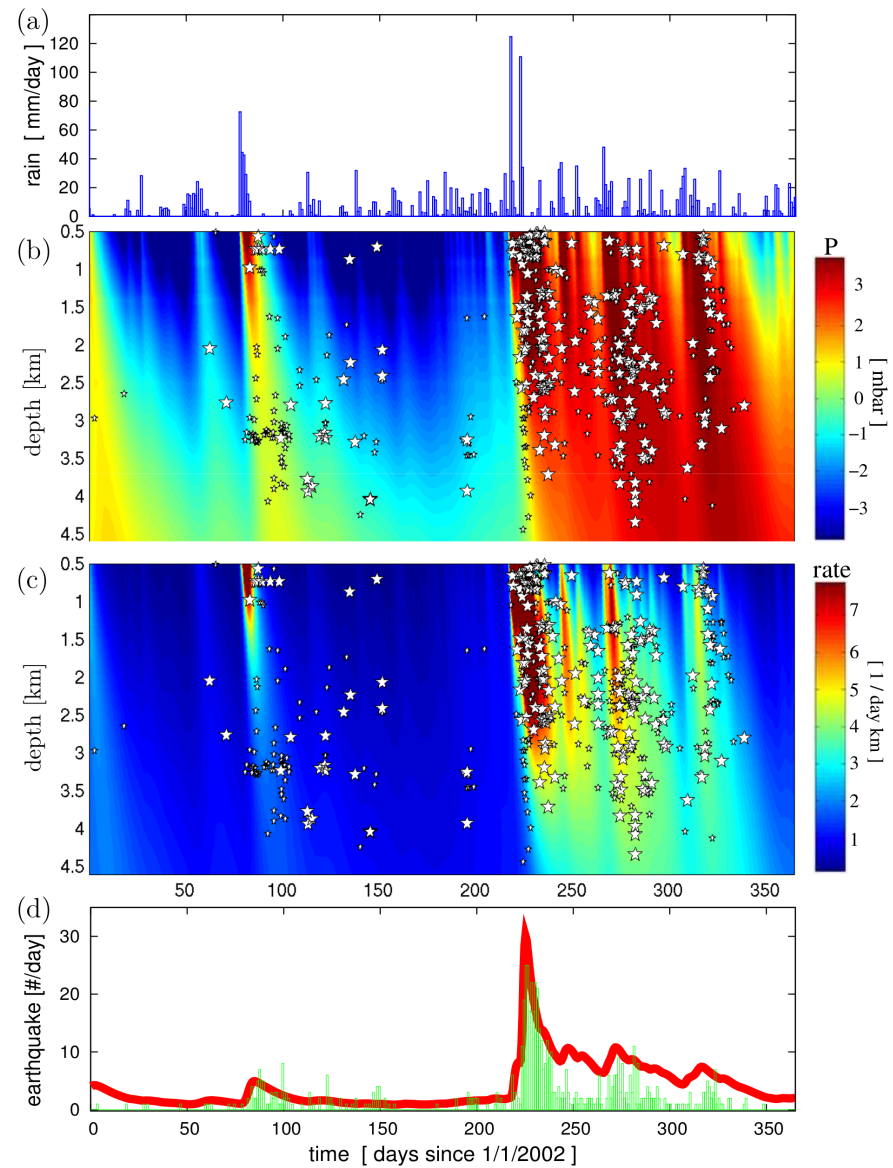
Rainfall-triggered earthquake activity



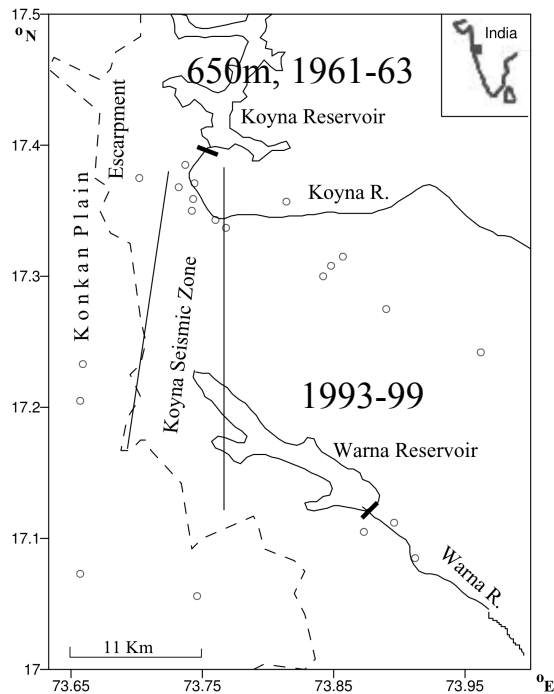
Hochstaufen, S-Germany

1D fluid diffusion model indicates tiny pressure changes of < 0.3 kPa at 4 km depth (diffusivity: $D=3.3 \text{ m}^2/\text{s}$)
 High correlation to seismicity observed !

from Hainzl et al., 2006: GRL 33, L19303, doi: 10.1029/2006GL0276427

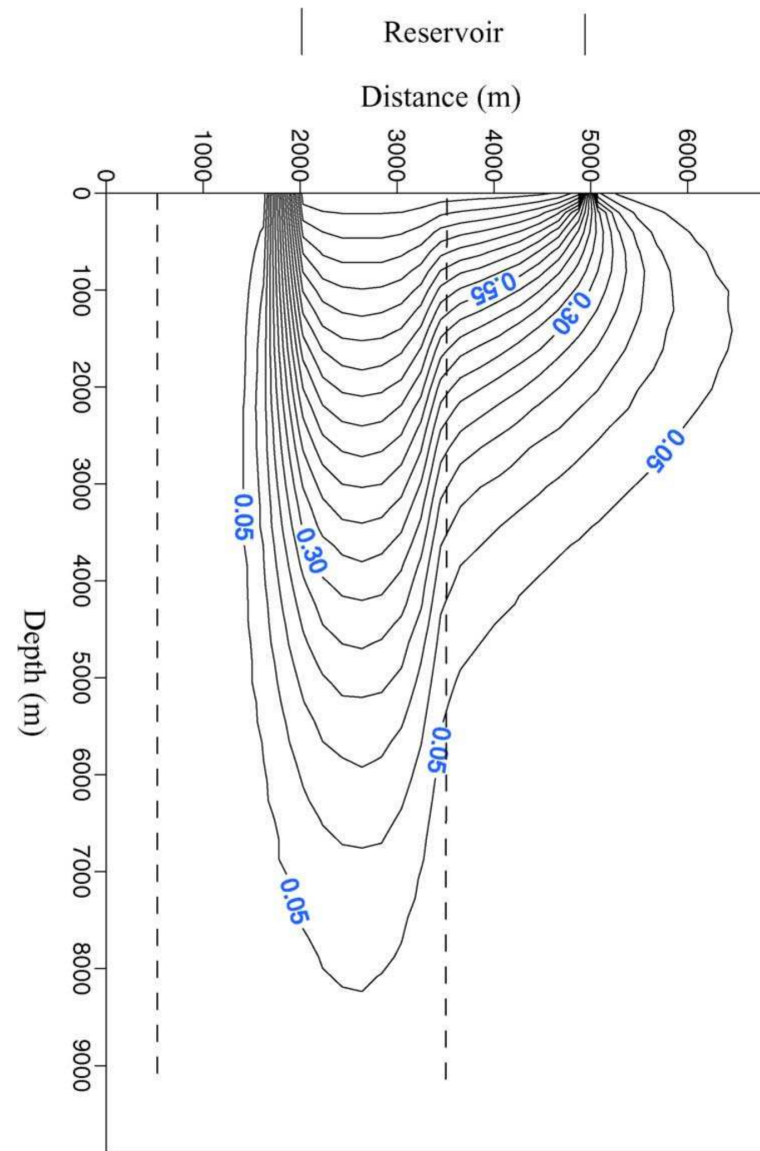


Earthquakes related to reservoir impoundment



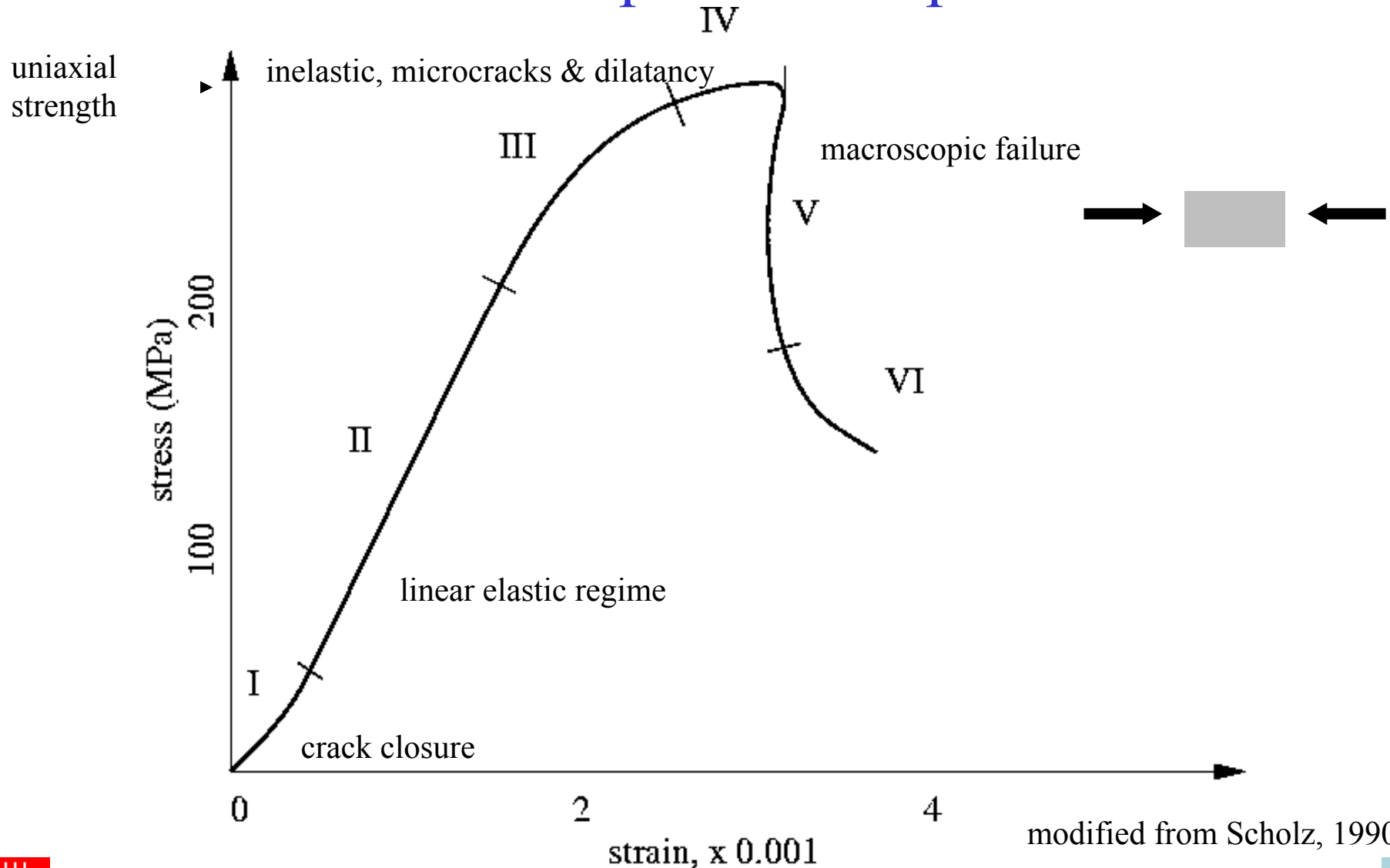
Surface loading and triggered earthquakes in the Koyana-Warna region, W-India

Extended seismic zone with $M < 6.3$ in 1967. 2D fluid diffusion model explains pressure changes of < 200 kPa at 6 km depth (15% of load) ($k_z = 2$ m/day, permeability $E-15$ m²) from Pandey & Chadha, 2003: PEPI 139, 10.1016/j.pepi.2003.08.003



II) Elastic and brittle deformation: a brief review

stress-strain curve uniaxial compression experiment



elastic strain stress relation (sum. convention)

shear modul

Poisson's ratio

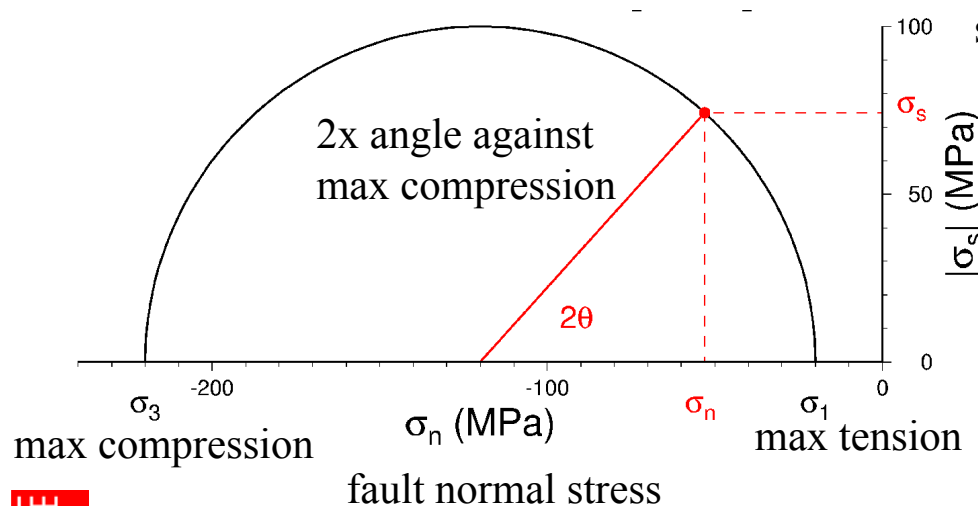
$$2\mathcal{N}\epsilon_{ij} = \frac{-\nu}{1 + \nu}\sigma_{kk}\delta_{ij} + \sigma_{ij}$$

strain tensor

stress tensor (tension >0)

see e.g. textbooks on seismology

Mohr circle representation of normal and shear stress on a fault



$$\sigma_n = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\Theta$$

$$\sigma_s = -\frac{(\sigma_{xx} - \sigma_{yy})}{2} \sin 2\Theta$$

see ICTP lecture of F. Cornet



III) Porous media: some basics

a.

Stress = Force/Area_{Total}
 $S = F/A_T$
(Acting outside an impermeable boundary)

Pore pressure acting in pore space

porosity

$$\Phi = \frac{V_p}{V}$$

b.

Stresses acting on grains

poroelastic strain stress relation

excess pore pressure

Biot's constant

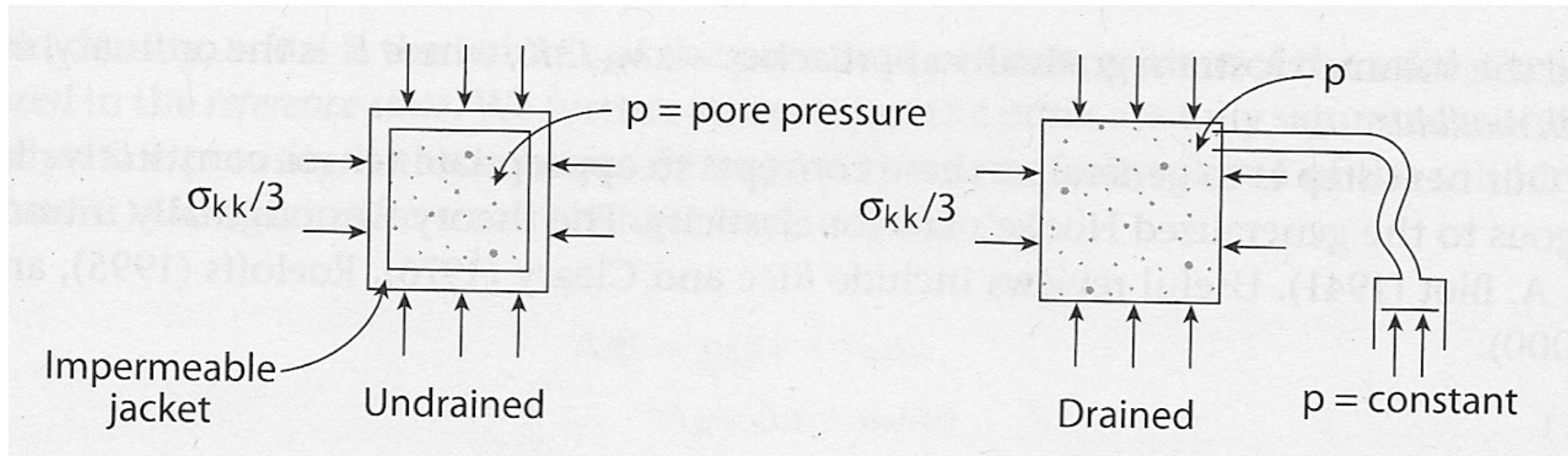
$$2\mathcal{N}\epsilon_{ij} = \underbrace{\sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\delta_{ij}}_{\text{elastic equation}} + \underbrace{\frac{(1-2\nu)}{1+\nu}\alpha\Delta P_f\delta_{ij}}_{\text{additional term}}$$

..... introducing effective stress $\sigma_{ij}^{\text{eff}} = \sigma_{ij} + \alpha\Delta P_f\delta_{ij}$

$$2\mathcal{N}\epsilon_{ij} = \dots = \sigma_{ij}^{\text{eff}} - \frac{\nu}{1+\nu}\sigma_{kk}^{\text{eff}}\delta_{ij}$$



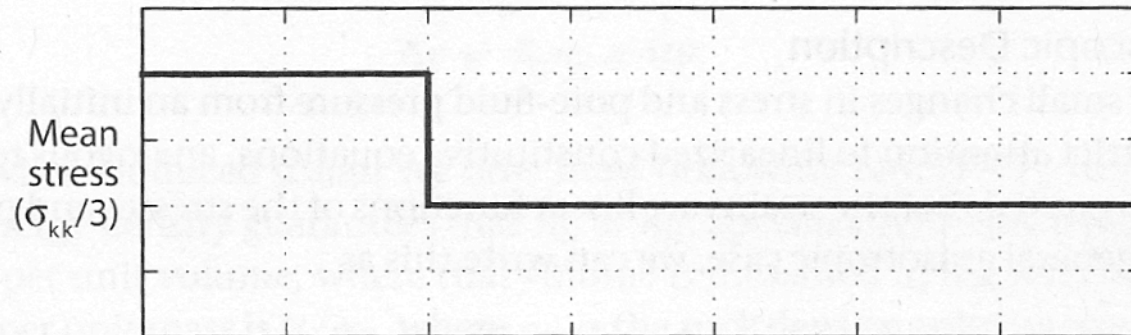
Poroelastic response: drained or undrained?



e.g. short time after loading

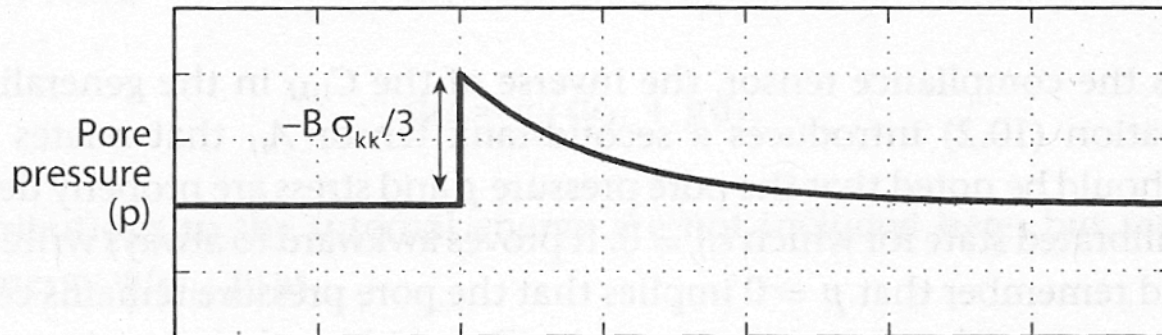
e.g. long time after loading

result of an idealized step-load compression experiment



step-like increase of confining compression

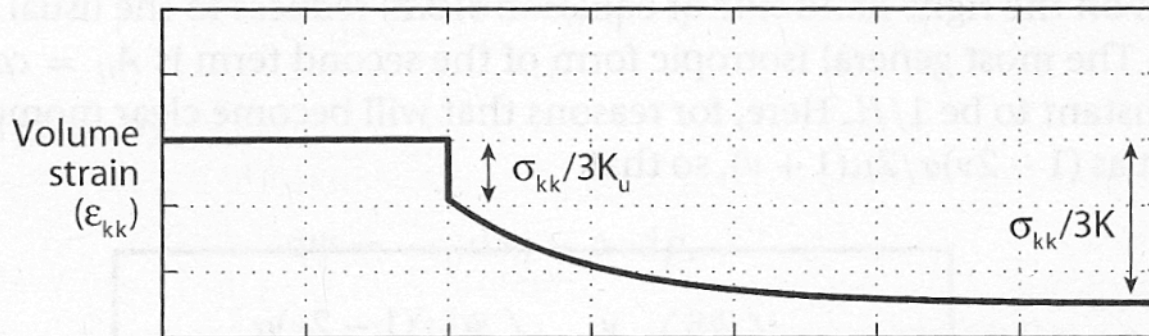
Skempton coefficient



Excess pore pressure increases instantaneously and is zero after long time

undrained bulk modulus

drained bulk modulus



Volume strain is first small due to resistance of pore fluid, but increases with time when fluid flows out

Time

modified from Segall (2010)

poro-elastic modules (among others)

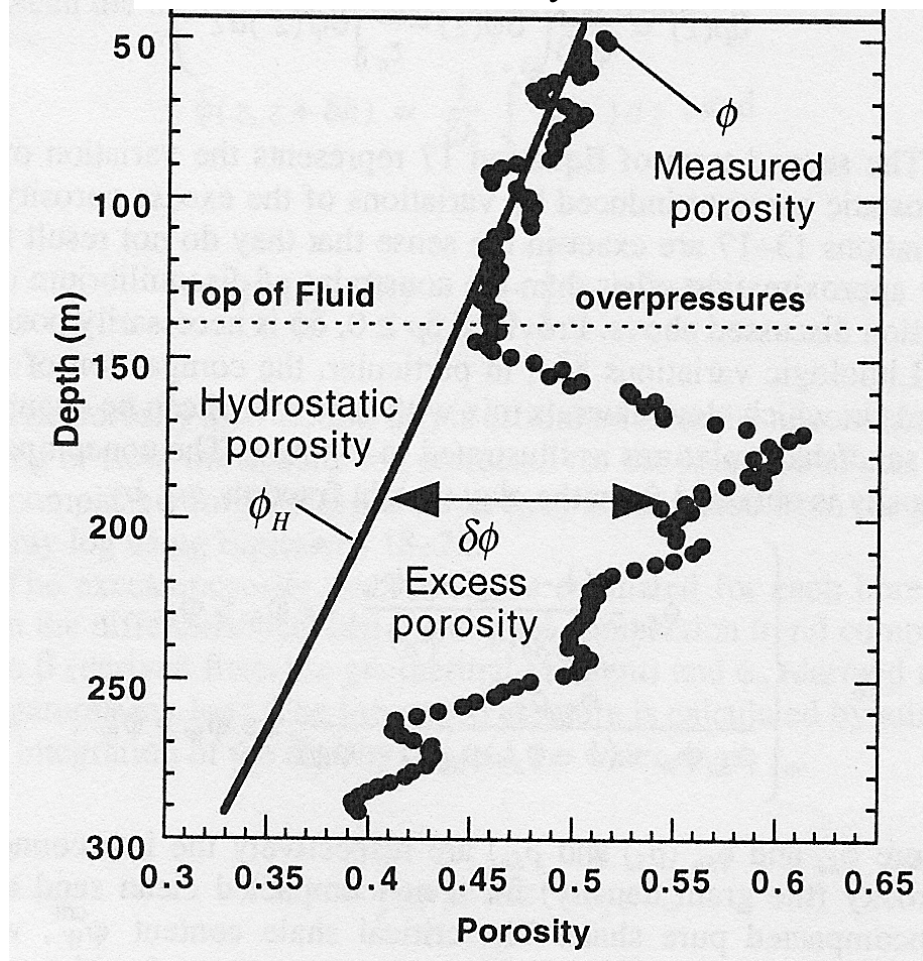
Biot's constant $\alpha = \frac{\Delta V_p}{\Delta V} = \frac{\text{change in pore volume}}{\text{change in total volume}}$ (drained)

Skempton coeff. $B = \frac{dP_p}{d\sigma} = \frac{\text{change in pore pressure}}{\text{change in confining stress}}$ (undrained)
subscript “u”



example: excess porosity and pore pressure in marine sediments at 150 m depth

bulk density $\rho = \frac{\Phi \rho_f V + (1 - \Phi) \rho_m V}{V}$ or $\Phi = \frac{\rho_m - \rho}{\rho_m - \rho_f}$



matrix density

overpressure and excess porosity may be generated by fast sedimentation and impermeable layers (e.g. clay)

pore pressure gradients cause flow

hydraulic diffusivity coupling term

< > < >

1. pore pressure diffusion:
$$\underbrace{kQ}_{\text{permeability}} \nabla^2 (\Delta P_f) = \frac{\partial \Delta P_f}{\partial t} + \alpha Q \frac{\partial \sigma_{kk}}{\partial t}$$

with $Q = \frac{2\mathcal{N}(\nu_u - \nu)}{\alpha^2(1 - 2\nu)(1 - 2\nu_u)}$ and $Q > 0$

2. force equilibrium (static):
$$\frac{\partial \sigma_{ij}}{\partial x_j} + \underbrace{f_i}_{\text{external force}} = 0$$

seepage force (from $f=0$):
$$\alpha \frac{\partial \Delta P_f}{\partial x_i} = \frac{\partial \sigma_{ij}^{\text{eff}}}{\partial x_j}$$



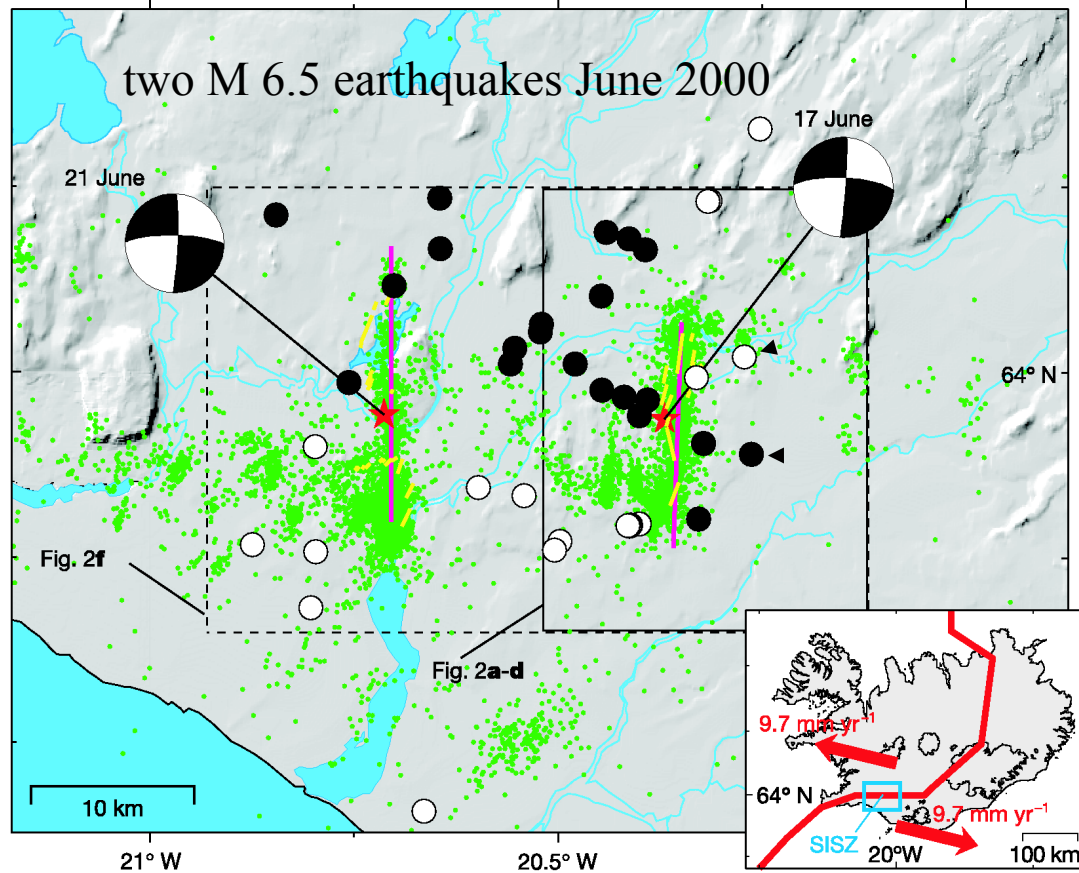
pore pressure gradients cause flow: summary in words

- Pressure diffusion and poroelastic deformation is coupled
- Uncoupled special case is analogue to thermal diffusion problems
- Pore pressure change, matrix deformation and seepage force may affect stability of rocks



Example: post-seismic ground movement related to pore pressure gradients

Jonsson et al. (2003): Nature 424, 10 July



wells with water-level decrease

wells with water-level increase

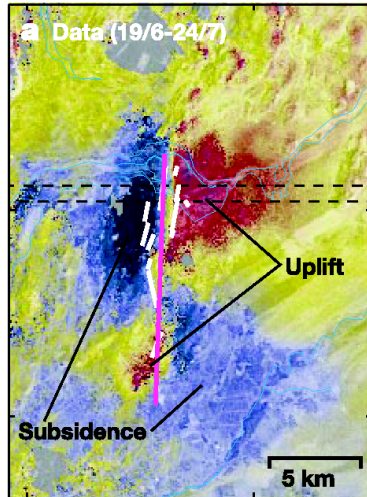
Additional:

InSAR ground displacement

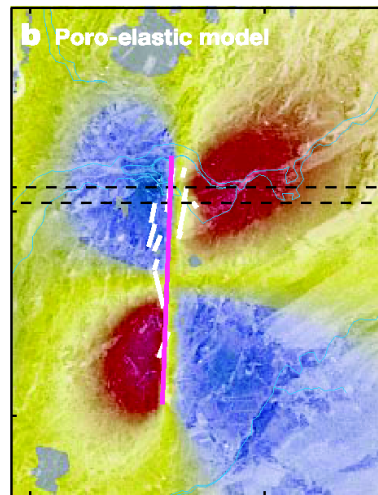
Observed and predicted uplift & water level change

InSAR uplift (postseismic)

pore pressure change and water level in wells

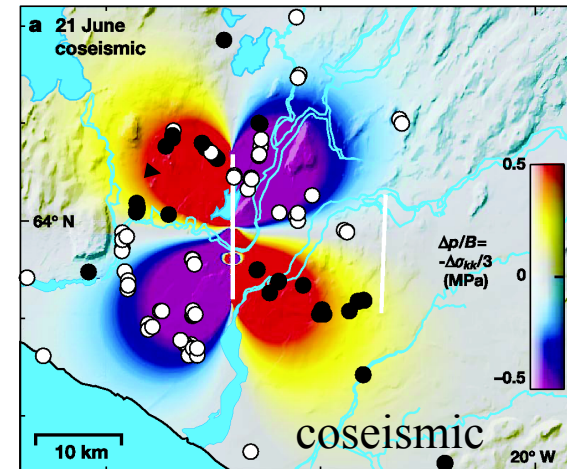


observed

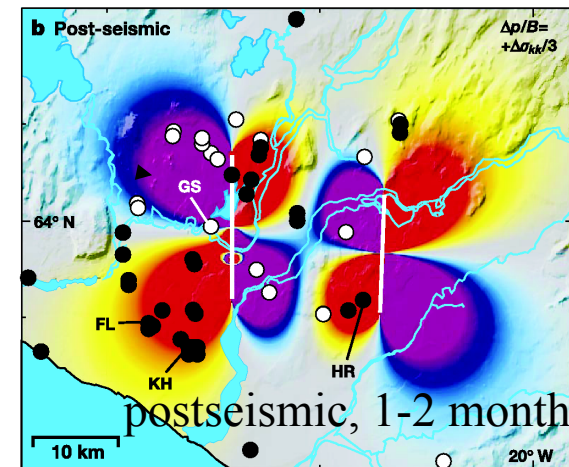


predicted

increase



decrease



Jonsson et al. (2003): Nature 424, 10 July

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Interpretation

- Earthquake related poroelastic effects became evident since deformation and pore pressure was measured simultaneously
- Pore pressure transients and poroelastic effects controlled the postseismic deformation in the first few month
- Pore pressure transients in shallow crust cannot explain long lasting aftershock sequences in Iceland



IV) Earthquake trigger and seismicity models

How are earthquakes triggered ?

The internal friction concept (Coulomb failure)

Rupture is

- driven by applied shear stress
- resisted by cohesive strength and normal stress

$$|\sigma_s| = S_0 - \mu_i (\sigma_n + P_p)$$

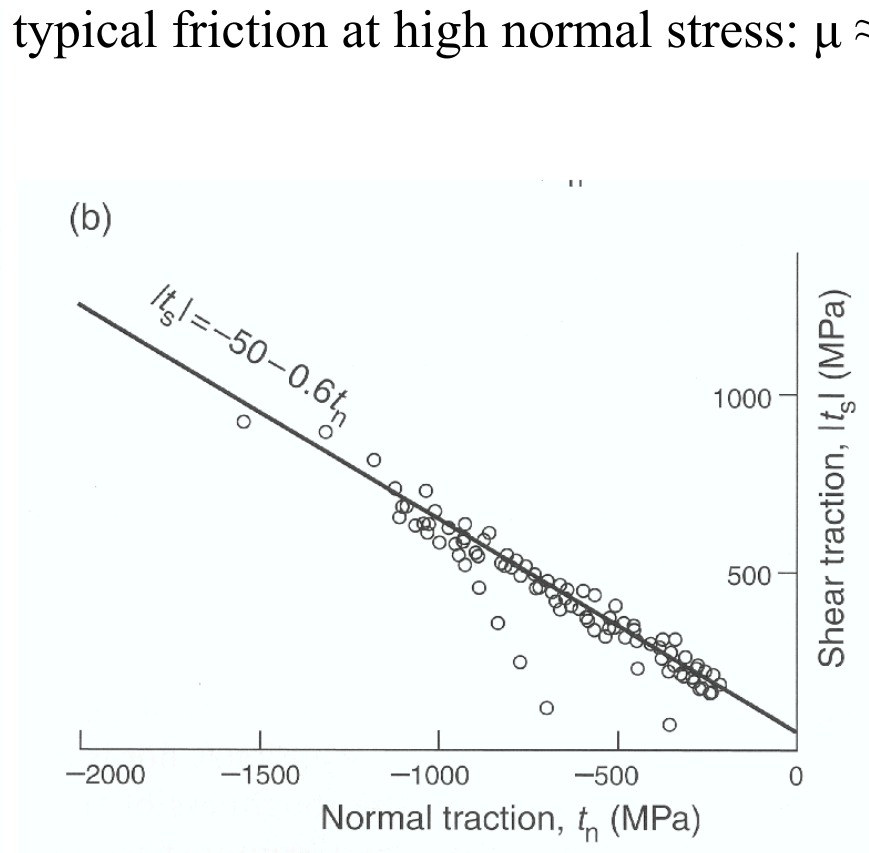
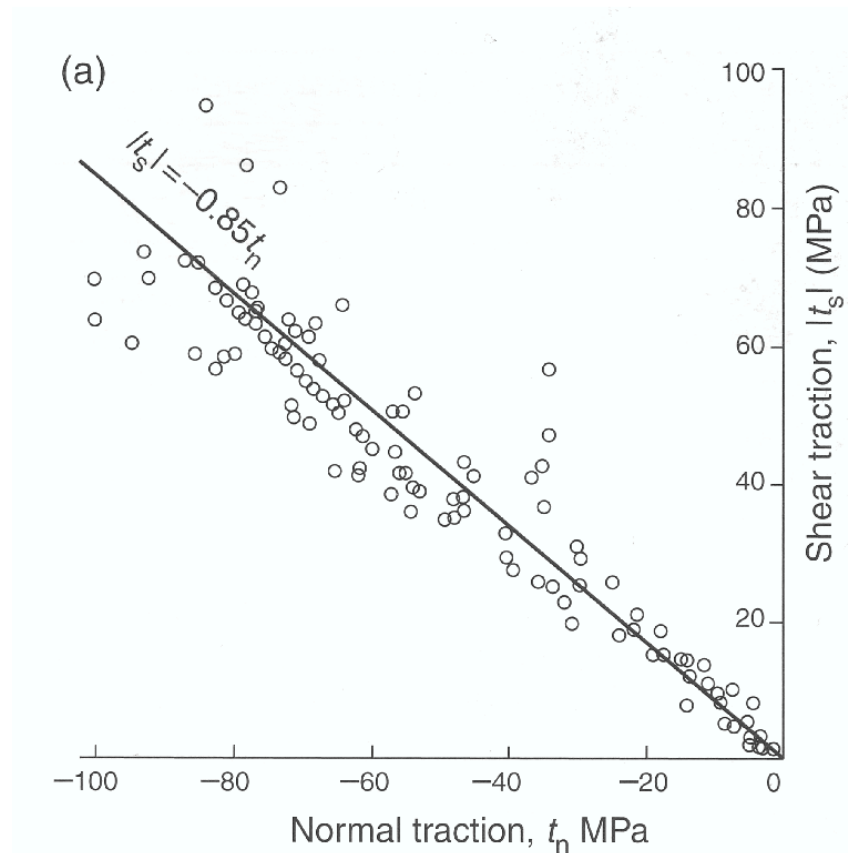
Shear stress inherent cohesion normal stress pore pressure

internal friction



... frictional strength from “sliding experiments”

typical friction at high normal stress: $\mu \approx 0.6$



Coulomb stress and Coulomb failure criteria

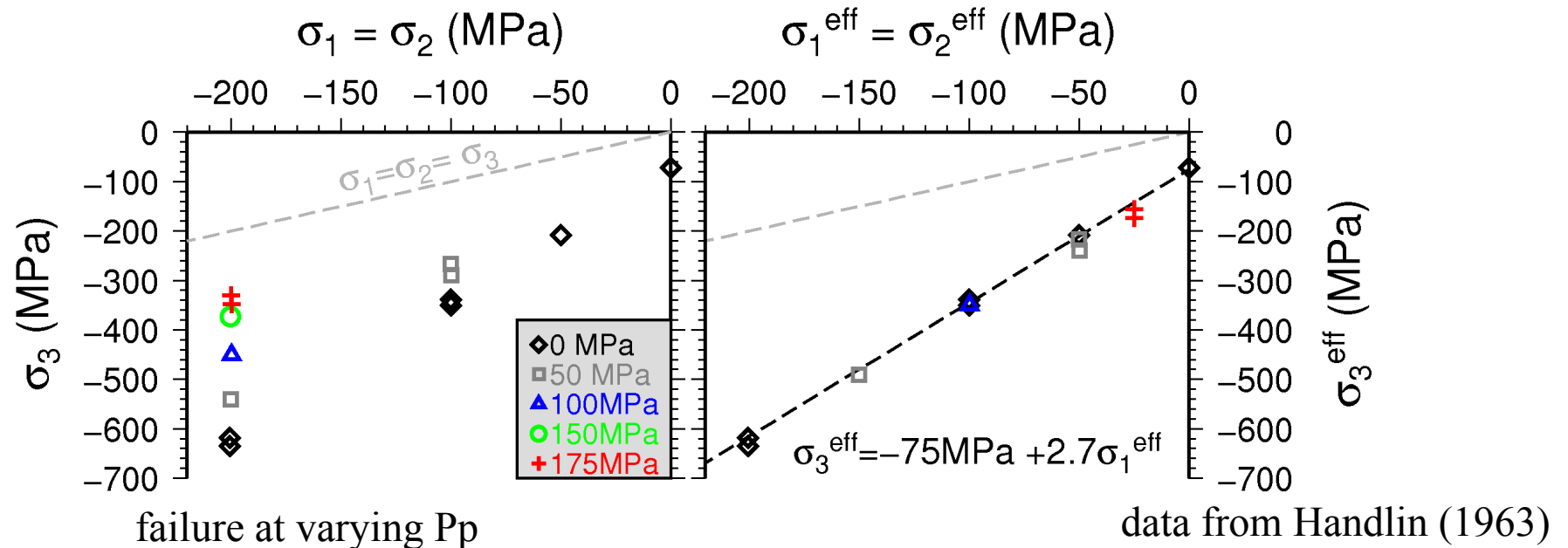
$$\sigma_c = |\sigma_s| + \mu_i (\sigma_n + P_p)$$

Coulomb stress

$$\sigma_c \leq S_0 \quad \text{“stable if Coulomb stress is smaller than } S_0 \text{”}$$

- Shear and normal stress depend on local stress and fault orientation
- P depends on local stress and pore pressure
- The internal friction is rate and state and moisture dependent

Note: Terzaghi's effective stress controls failure



failure point in tri-axial compression depends on confining stress and pore pressure P_p

Terzaghi's effective stress:
$$\sigma_i^{\text{eff}} = \sigma_i + P_p$$

... does not depend on Biots constant α !

Coulomb stress as a function of principal effective stress and fault orientation

$$\begin{aligned}\sigma_c &= \left| + \frac{1}{2}(\sigma_1^{\text{eff}} - \sigma_3^{\text{eff}}) \sin 2\Theta \right| + \mu_i \left(\frac{1}{2}(\sigma_1^{\text{eff}} + \sigma_3^{\text{eff}}) + \frac{1}{2}(\sigma_1^{\text{eff}} - \sigma_3^{\text{eff}}) \cos 2\Theta \right) \\ &= \frac{1}{2}(\sigma_1^{\text{eff}} - \sigma_3^{\text{eff}})(\pm \sin 2\Theta + \mu_i \cos 2\Theta) + \frac{1}{2}(\sigma_1^{\text{eff}} + \sigma_3^{\text{eff}})\mu_i\end{aligned}\quad (3.56)$$

where $\sigma_1^{\text{eff}} < T_u$ and $\sigma_c \leq S_0$.

Plane orientation for max. Coulomb stress

orientation from setting $d\sigma/d\Theta = 0$:

$$\frac{d\sigma_c}{d\Theta} = \frac{1}{2}(\sigma_1^{\text{eff}} - \sigma_3^{\text{eff}})(\pm 2 \cos 2\Theta_c - 2\mu_i \sin 2\Theta_c) = 0$$

or $\tan 2\Theta_c = \frac{\sin 2\Theta_c}{\cos 2\Theta_c} = \frac{1}{\pm \mu_i}$.

Eliminating the critical angle yields the maximal Coulomb stress:

$$\sigma_{cc} = \frac{1}{2}(\sigma_1^{\text{eff}} - \sigma_3^{\text{eff}})\sqrt{1 + \mu_i^2} + \frac{1}{2}(\sigma_1^{\text{eff}} + \sigma_3^{\text{eff}})\mu_i$$

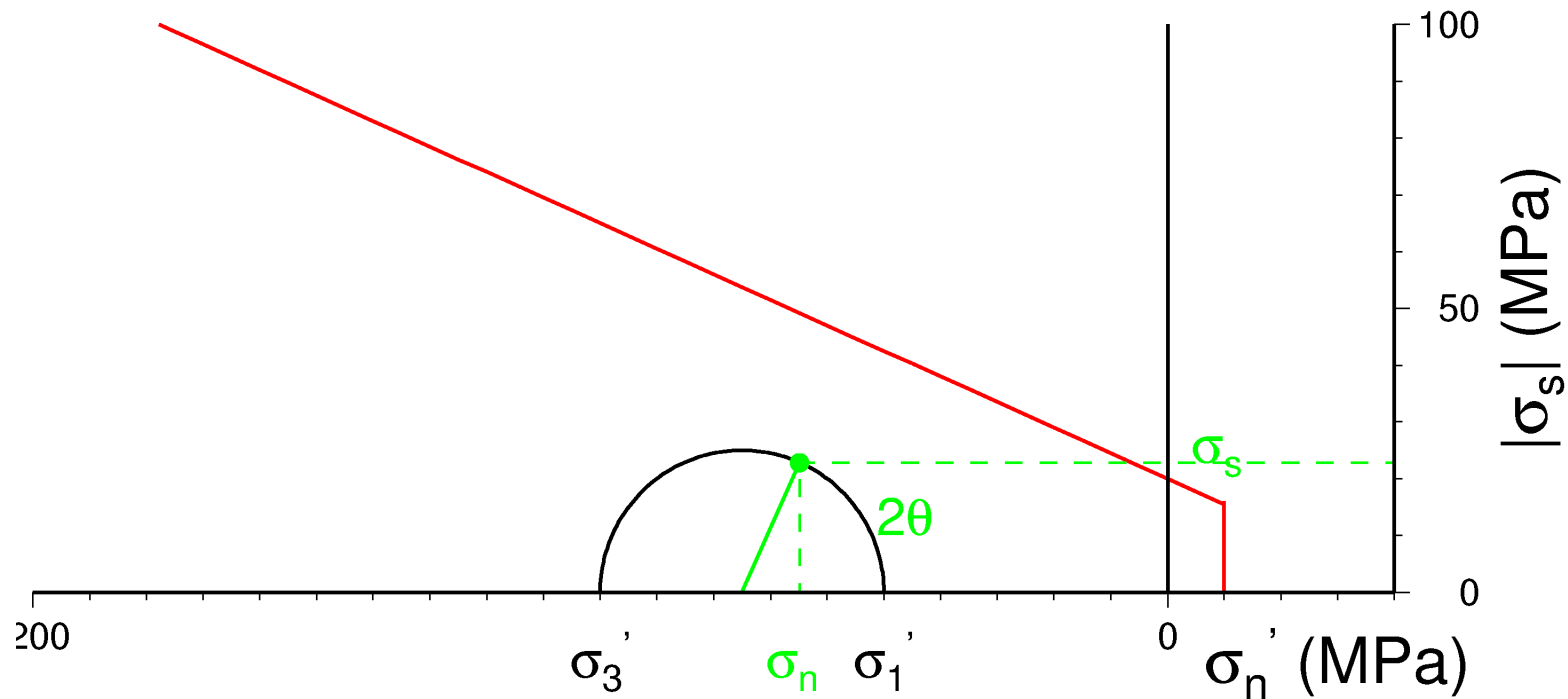
where $\sigma_1^{\text{eff}} < T_u$ and $\sigma_{cc} \leq S_0$

or

$$2\sigma_{cc} = \sigma_1^{\text{eff}} \left[\sqrt{1 + \mu_i^2} + \mu_i \right] + \sigma_3^{\text{eff}} \left[\sqrt{1 + \mu_i^2} - \mu_i \right]$$

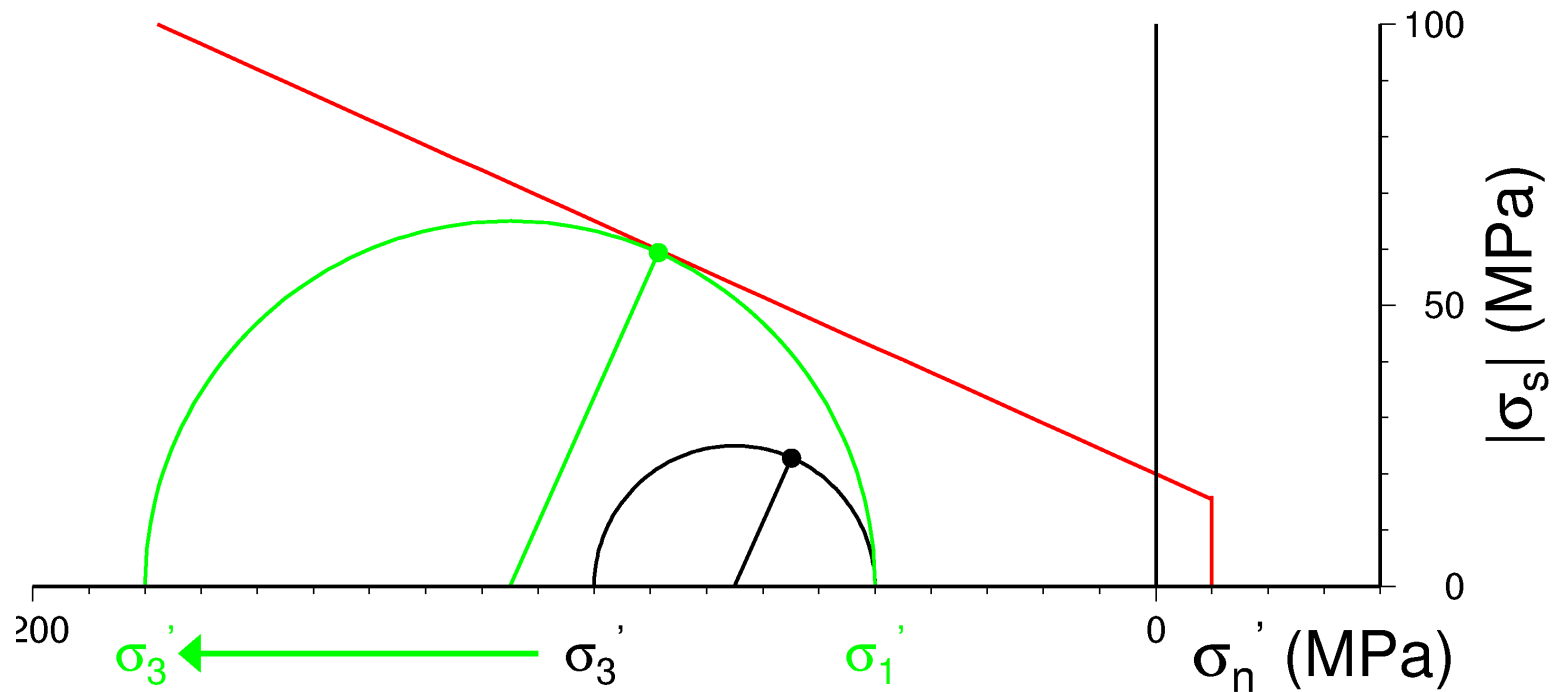
Coulomb criterion and Mohr's circle

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

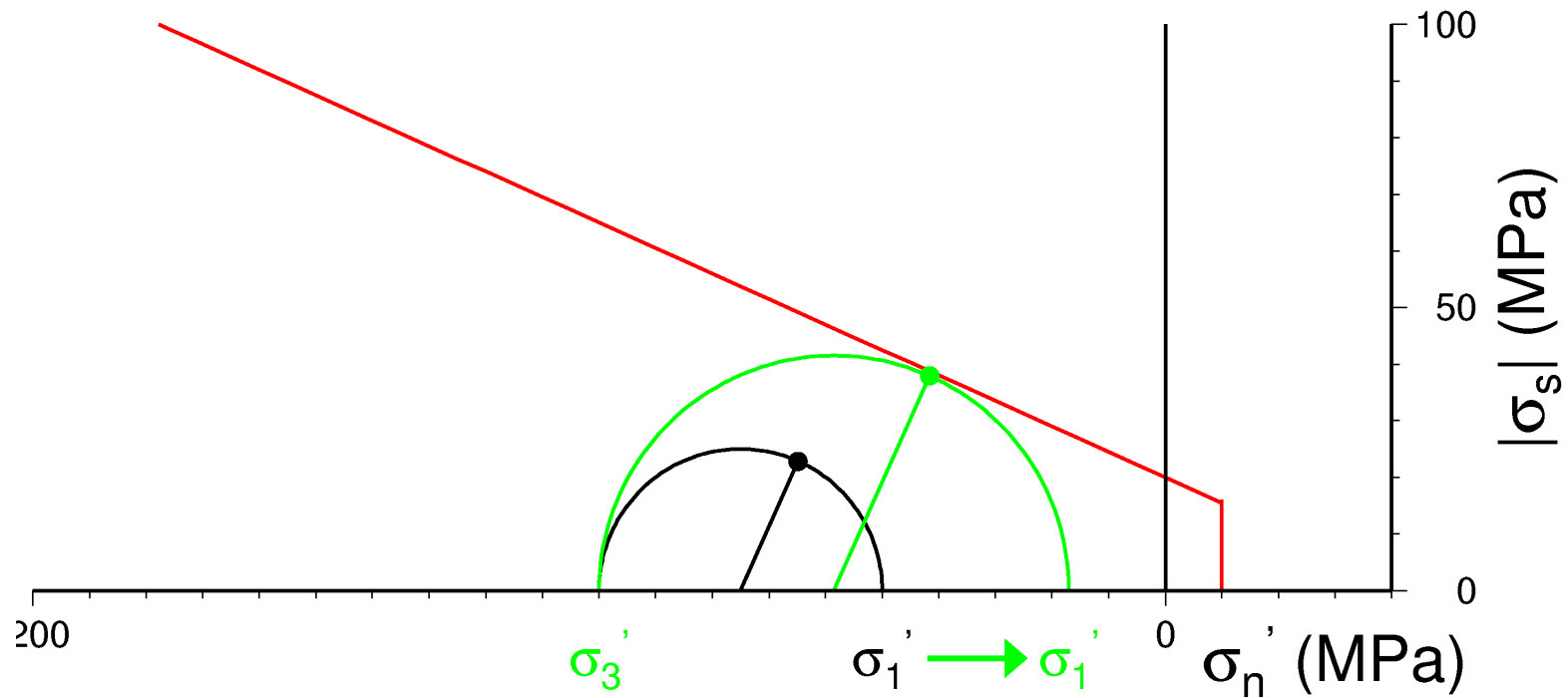


four ways to trigger an earthquake

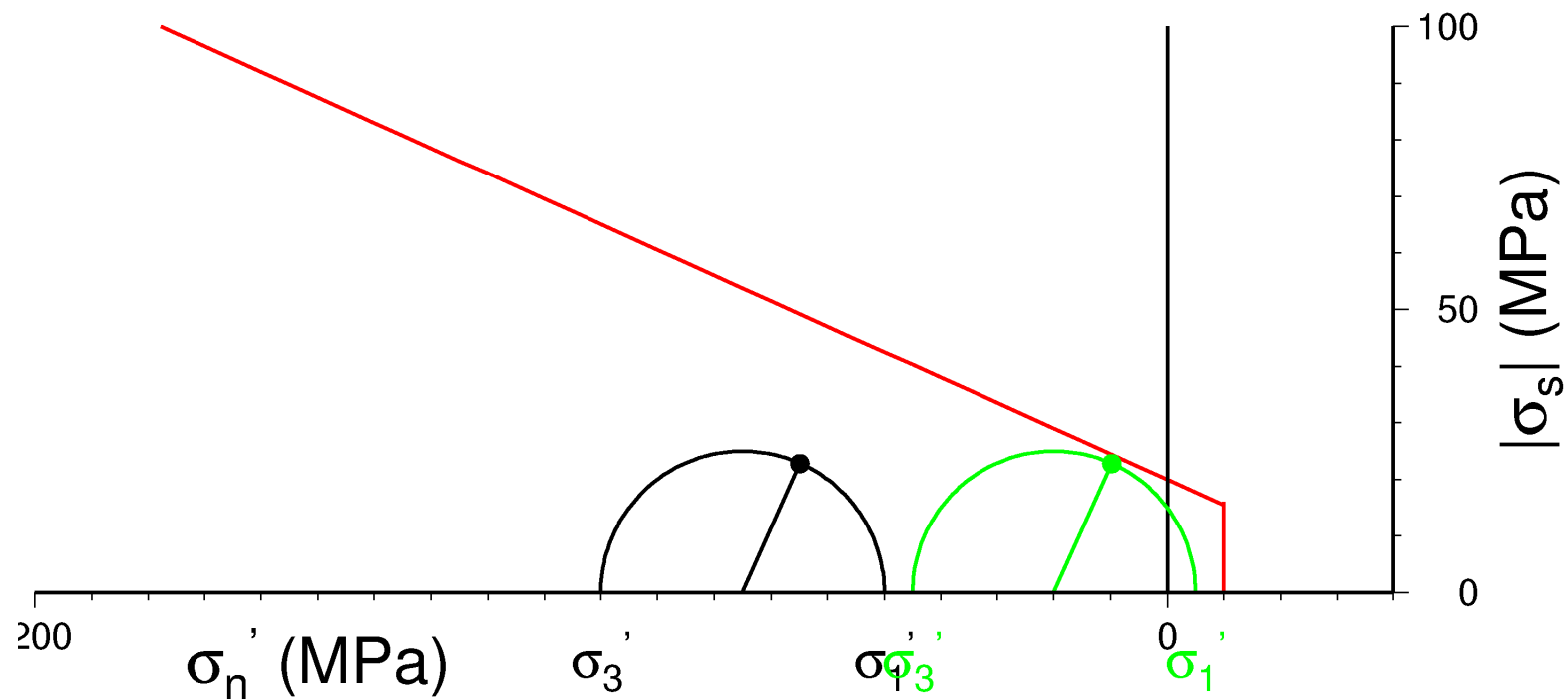
(1) increasing the maximal compressive stress



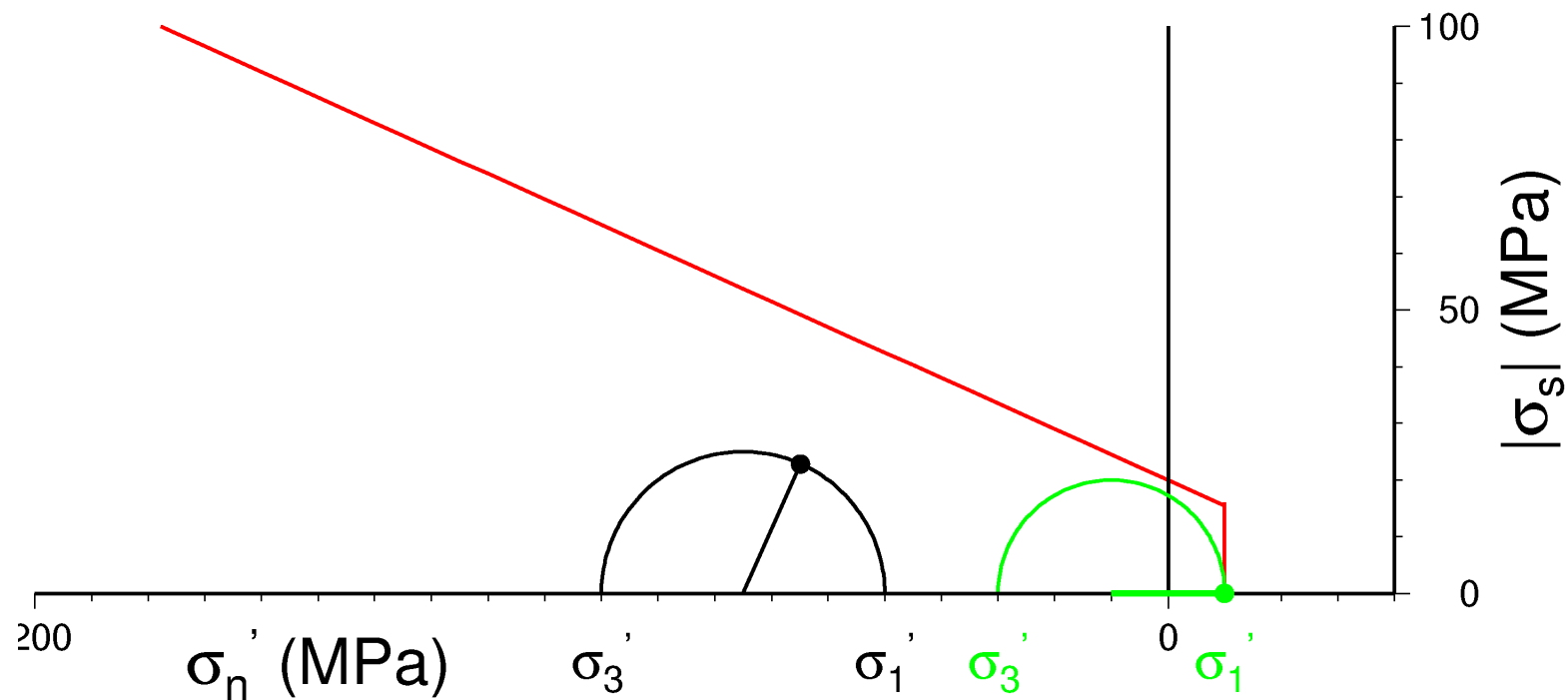
(2) decreasing the least compressive stress



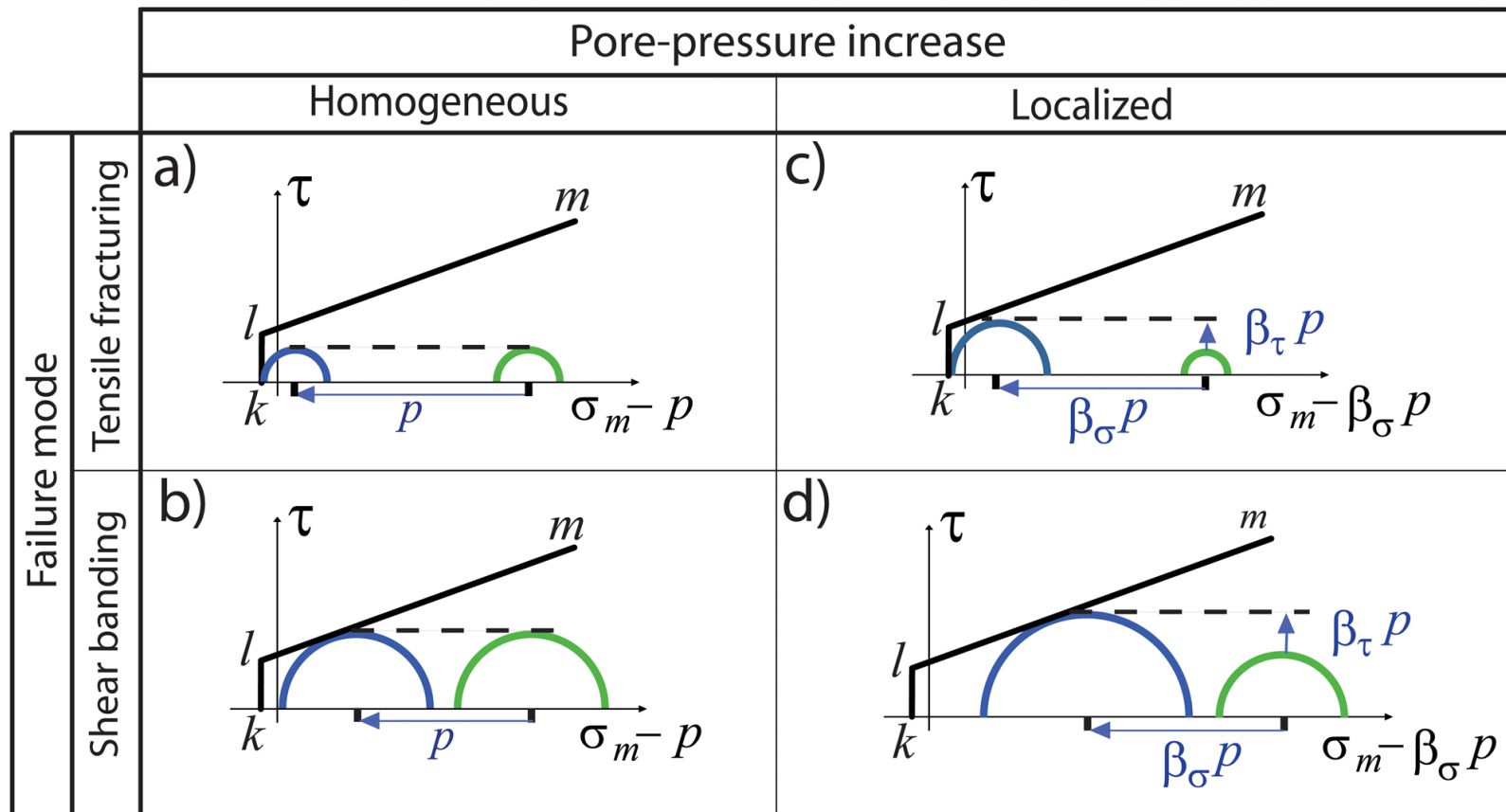
(3) increasing the pore pressure & shear fracturing



(4) increasing the pore pressure & tensional fracturing



Note: localized loading of porous media affects pressure and stress deviator!



from Rozhko (2007) GRL 34, 10.1029/2007GL031696

How large are deviatoric stresses in the crust ?

vertical stress

horizontal stress

$$\sigma_{zz} = -\rho g z \quad \text{and} \quad \sigma_{xx} = -\rho g z + \Delta\sigma_{xx}$$

deviatoric stress

thrust faulting regime:

$$\sigma_{zz} = \sigma_1 = -\rho g z$$

$$\sigma_{xx} = \sigma_3 = -\rho g z + \Delta\sigma_{xx}, \quad \text{with } \Delta\sigma_{xx} < 0.$$

normal faulting:

$\Delta\sigma_{xx} > 0$ and σ_1 and σ_2 are interchanged

resolving for $\Delta\sigma_{xx}$

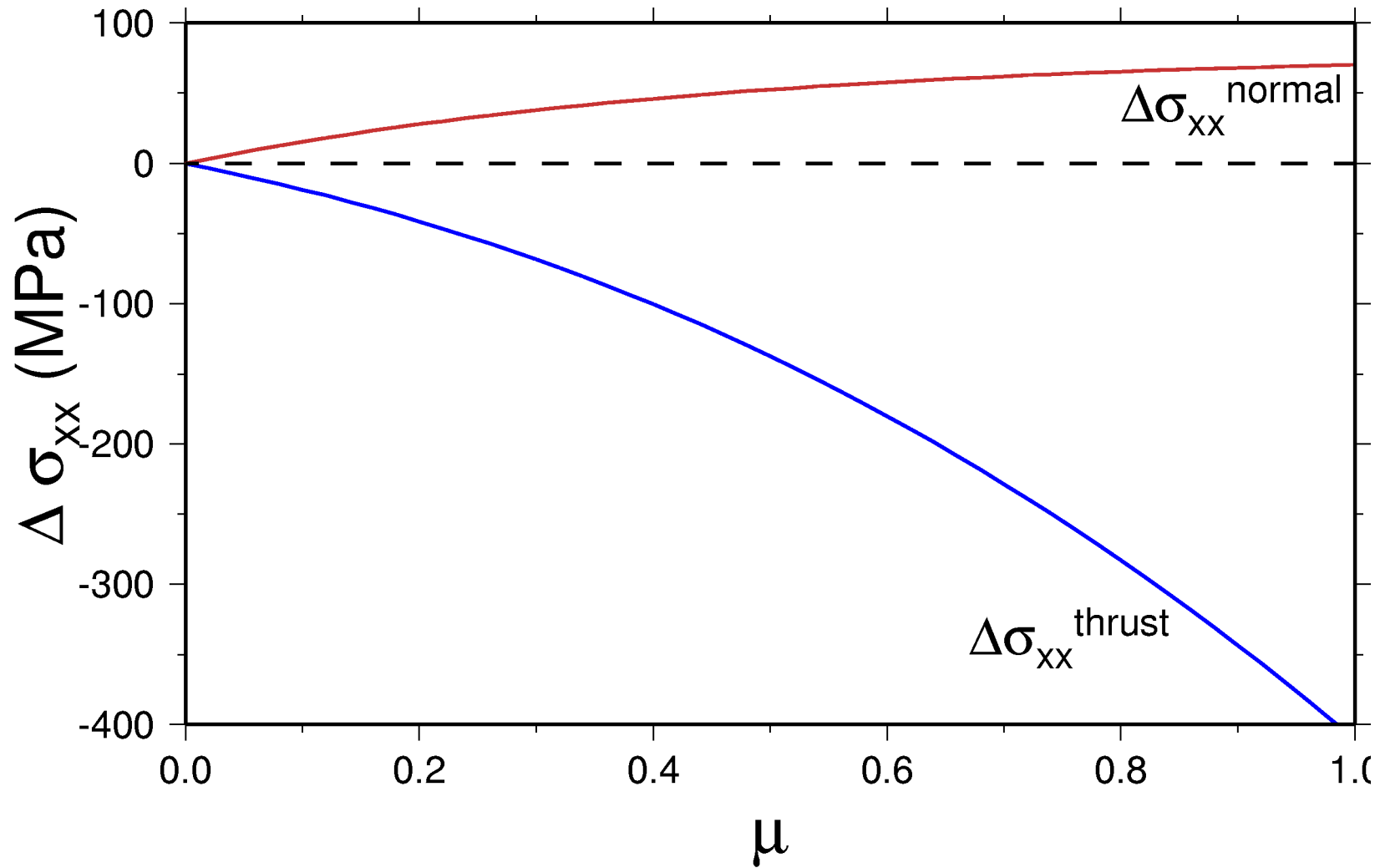
thrust faulting regime:

$$\Delta\sigma_{xx} = +C_u - \left(1 - \left[\sqrt{1 + \mu_i^2} + \mu_i\right]^2\right) (\rho g z - P_p)$$

normal faulting regime:

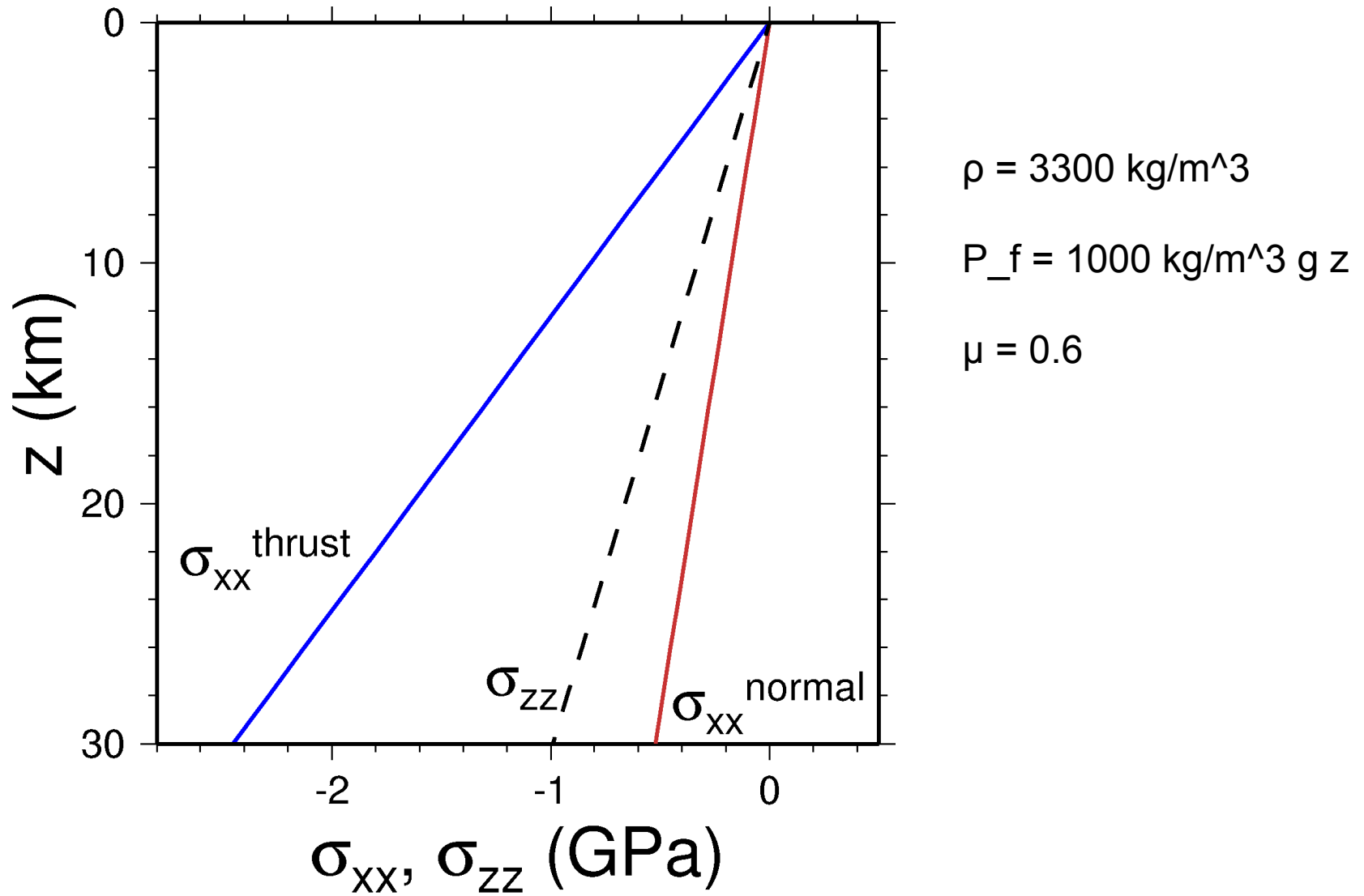
multiply μ_i by -1

friction-dependent deviatoric stress in crust at 5 km depth



assuming $\rho = 2700 \text{ kg/m}^3$, $P_f = 1000 \text{ kg/m}^3$ g z, $\mu = 0.6$

strength of oceanic lithosphere



Note: below ≈ 30 km depth visco-elastic processes relax high deviatoric stresses

Are rocks in a critical stage to failure? Increasing Coulomb stress: how large to triggering rupture?

Several studies indicate that 0.01 MPa (0.1 bar) is sufficient to trigger an earthquake (e.g. Seeber and Armbruster, 2000)

Compare with typical values:

- Average stress drop during earthquake: 1-10 MPa
- Pore pressure reduction in reservoirs up to 10 MPa
- Head pressure for hydrofrac exper. up to 10 MPa
- Head pressure for waste fluid injection up to 10 MPa

Triggered or induced seismicity ?

Triggered:

- the **nucleation** of rupture is controlled by the loading (human-related, natural intrusion, ...)
- the **growth/length** of the rupture is mainly controlled by the pre-existing tectonic stress on the fault

-> the earthquake would have occurred in any case, but now it was slightly earlier

-> the size of the event is not related to the loading-related stress perturbation

Induced:

- both, **nucleation and rupture** are controlled by the induced stress perturbation from the operation/process

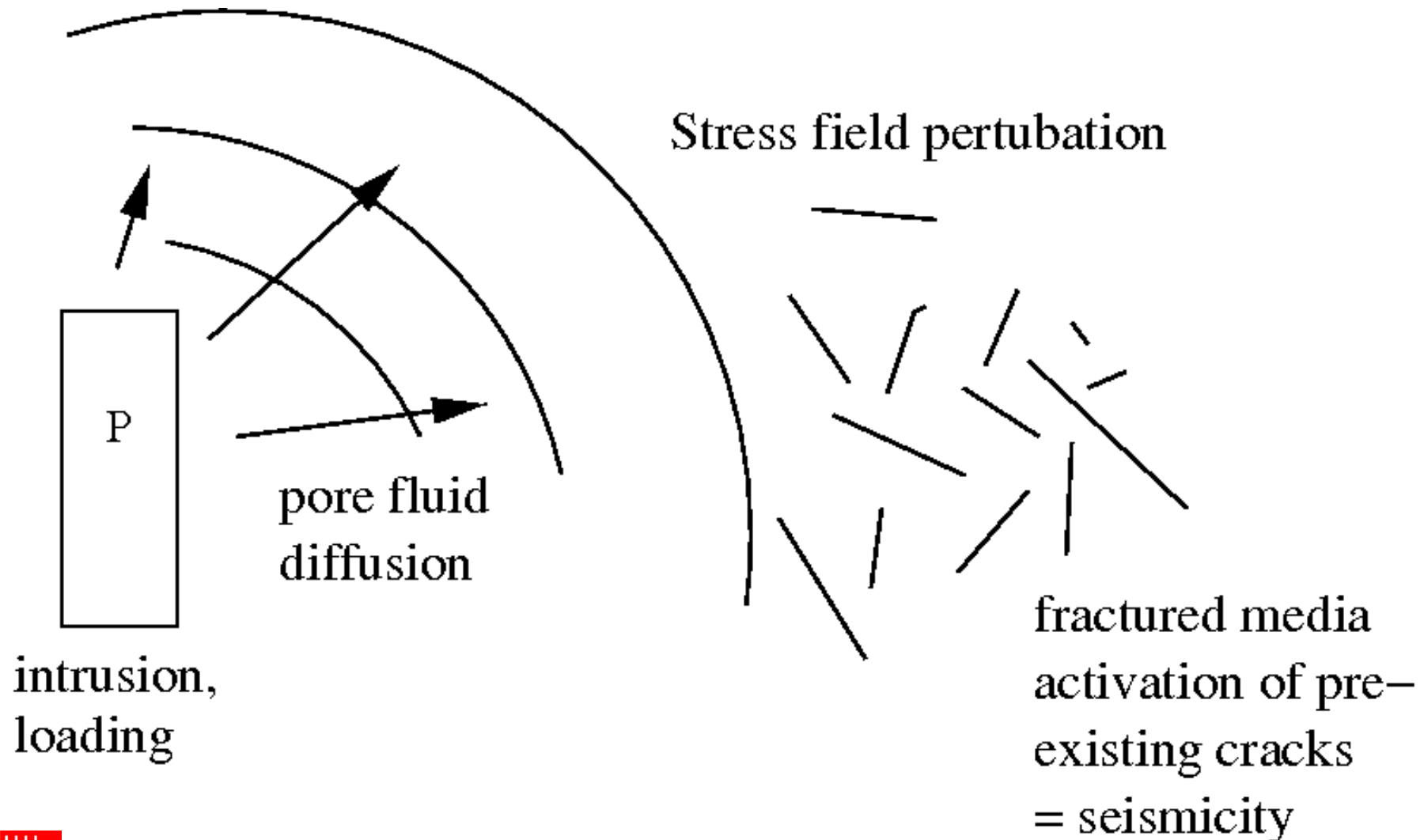


Seismicity models

- Earthquake nucleation from frictional instability (Dietrich model)
- Earthquake nucleation from stress corrosion (not discussed here, see Scholz 1990)



concept of pre-existing cracks



Pre-existing faults with different orientation are everywhere



Faulting in recent volcanic deposits along a road cutting near San Martín

Statistical earthquake relations

(1) magnitude-frequency relation (Gutenberg-Richter)

$$\log N = a_1 - b M_S$$

equivalently moment-frequency relation

$$\log N = \alpha - \frac{b}{1.5} \log(M_0) = \alpha - \beta \log(M_0)$$

----> for tectonic earthquakes b is about 1. For earthquake swarms b is often higher



(2) Omori law for aftershock occurrence

$$n = \frac{C}{(K + t)^P}$$

n : number of aftershocks

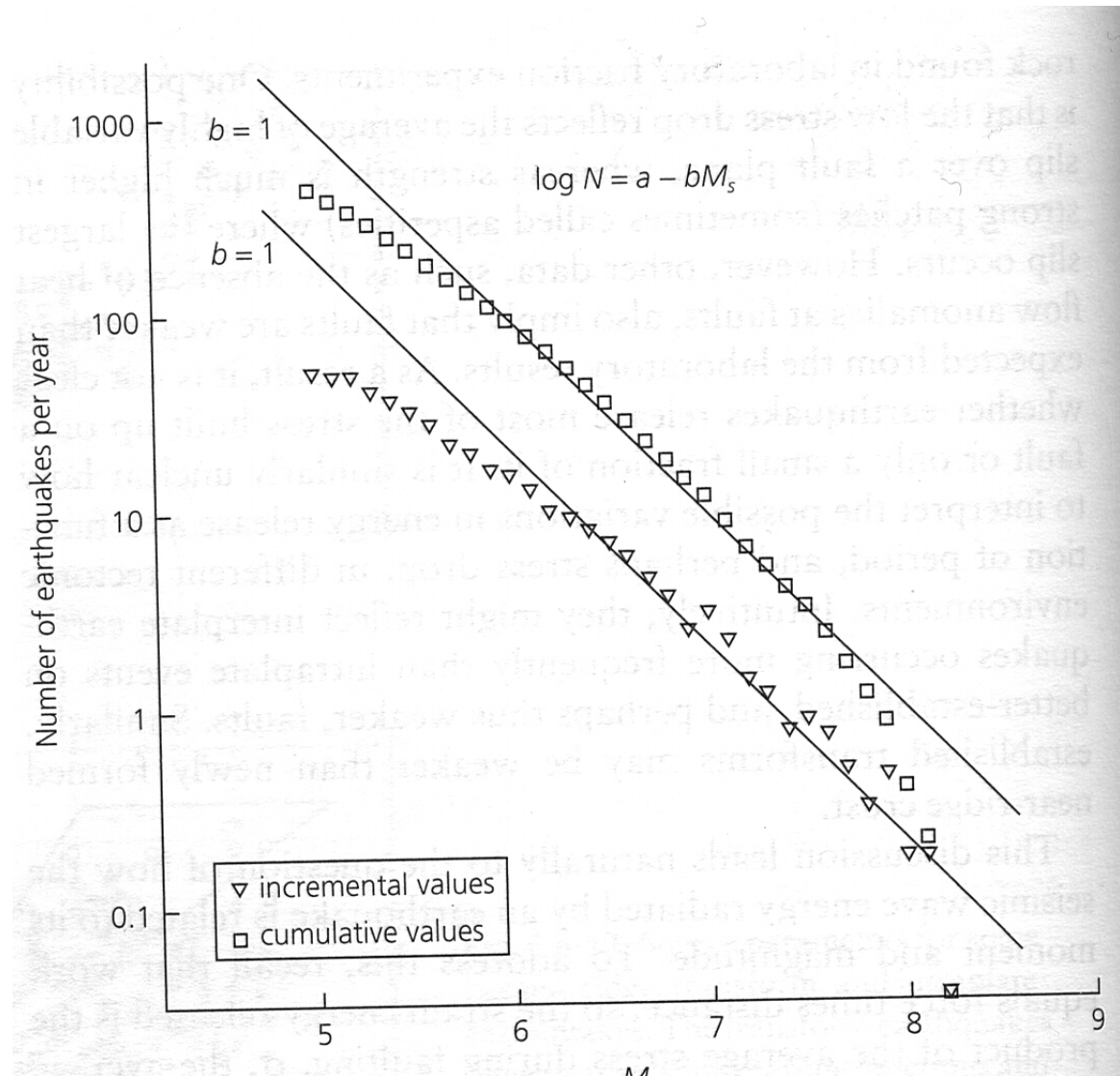
t : time

C, K, p : constants

Three brief examples

- A) Tectonic case
- B) Volcanic earthquake swarm
- C) Cyclic thermal loading of rock

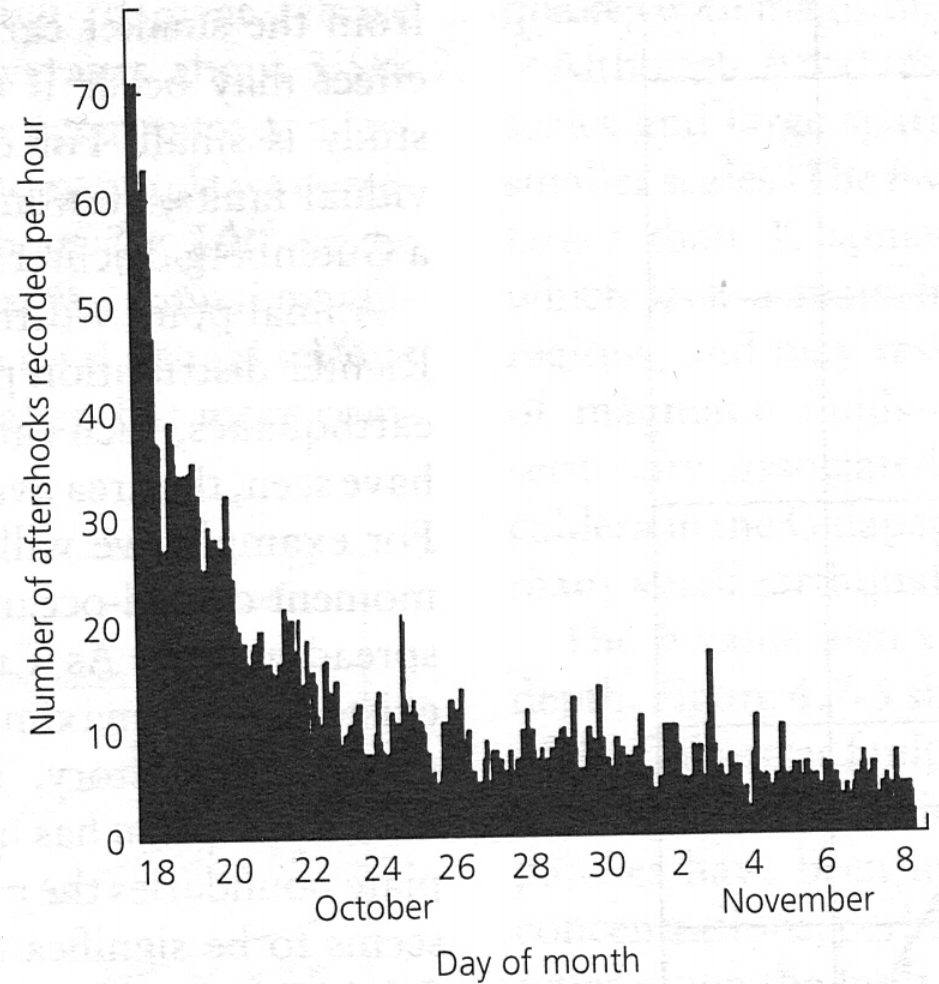
A) Gutenberg Richter (global data)



Loma Prieta, California

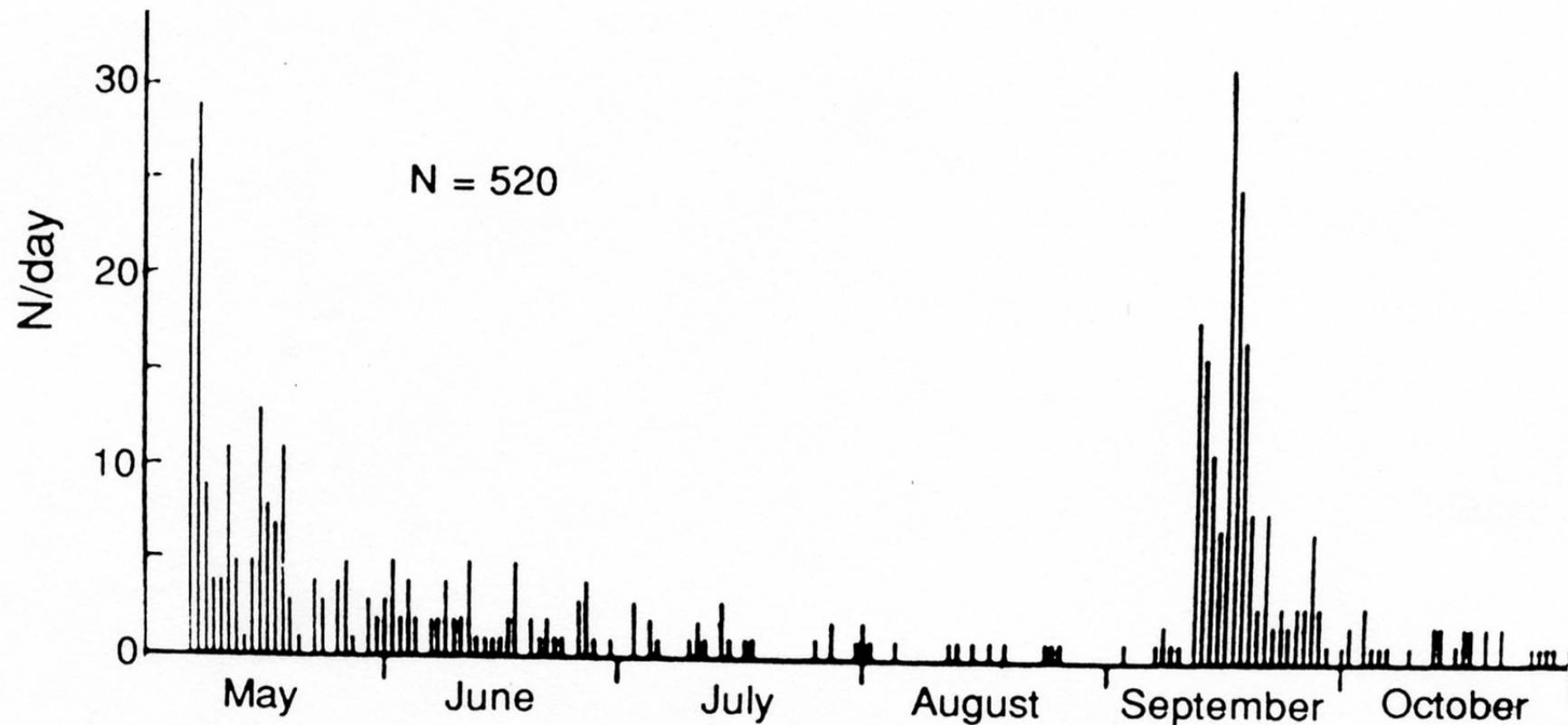
Magnitude	Number	Effect
5	2	Damaging
4	20	Strong
3	65	Perceptible
2	384	Not felt
1	1855	Not felt
<1	2434	Not felt
Total	4760	

4760 aftershocks of the Loma Prieta earthquake had been recorded by noon on November 7, 1989. The diminishing number of aftershocks with time is typical for large California earthquakes.

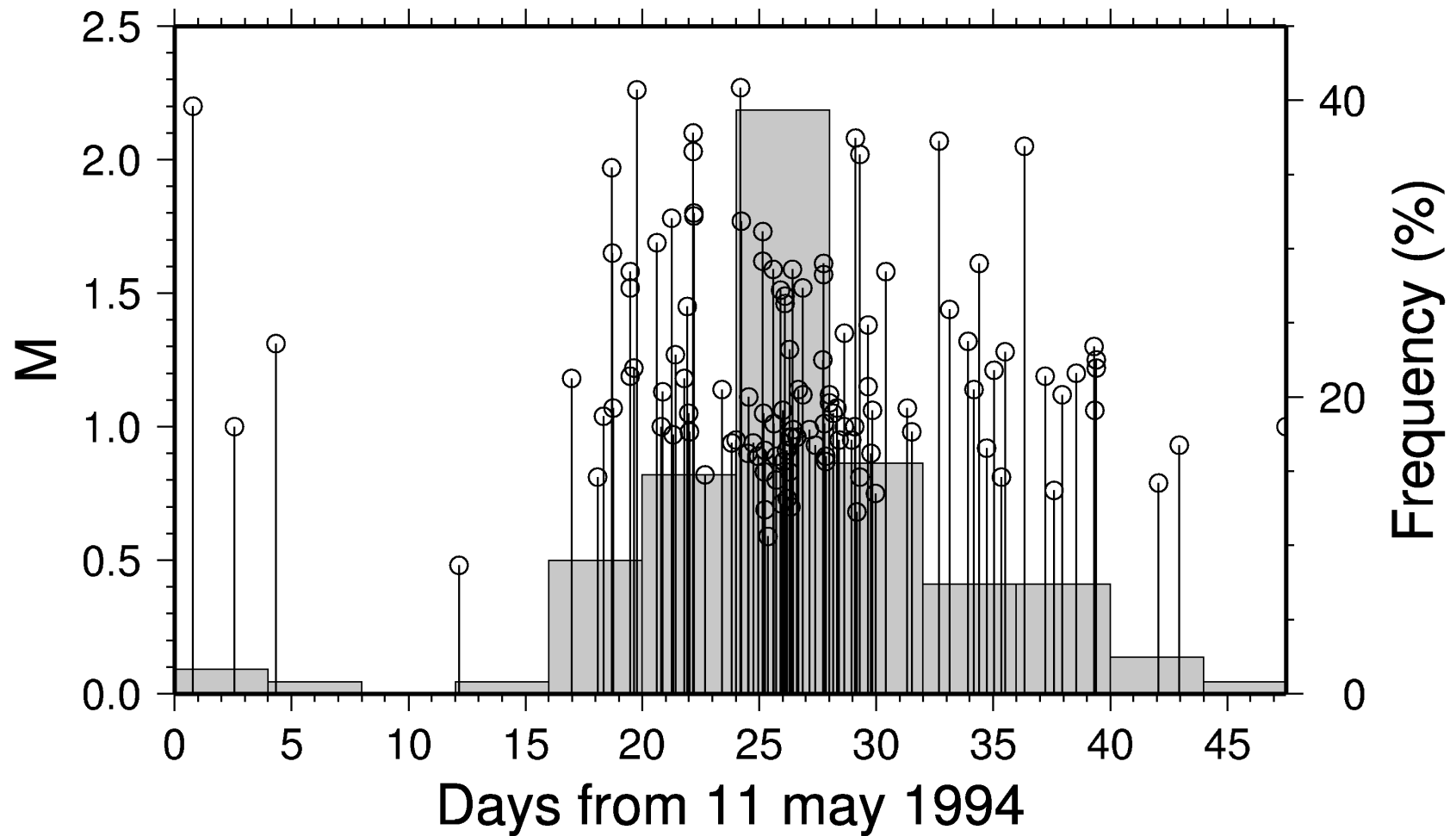


Typical main- & aftershocks: Friuli 1976, Italy

Number of earthquakes per day



B) Typical “volcanic swarm”: Eyjafjallajökull 1994, Iceland

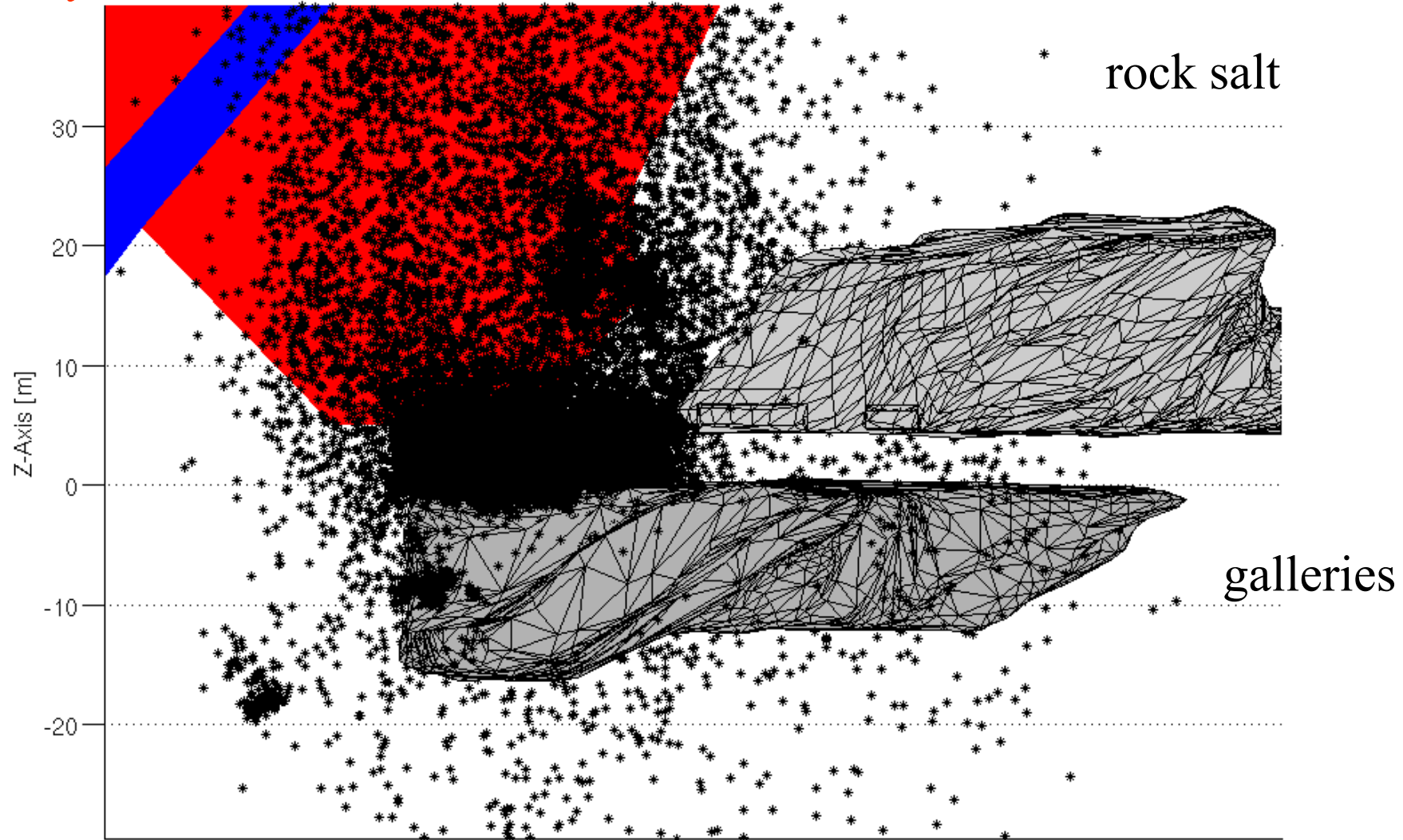


Dahm & Brandsdottir (1997) GJI 130, 183-192

C) Thermal loading of a salt mine

anydrite block

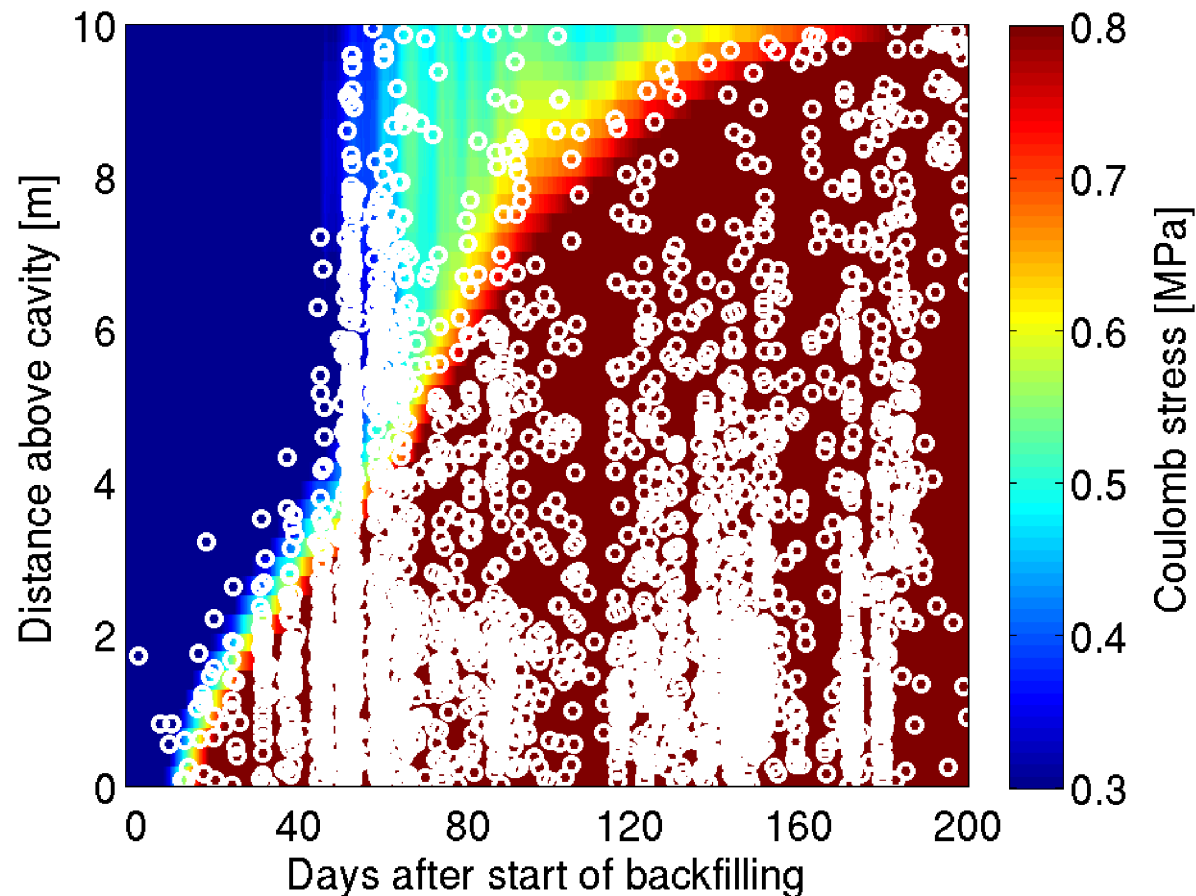
Activity during backfilling days 52 to 63



Köhler, Spies, Dahm (2009) GJI 10.1111/j.1365-246X.2009.04303.x

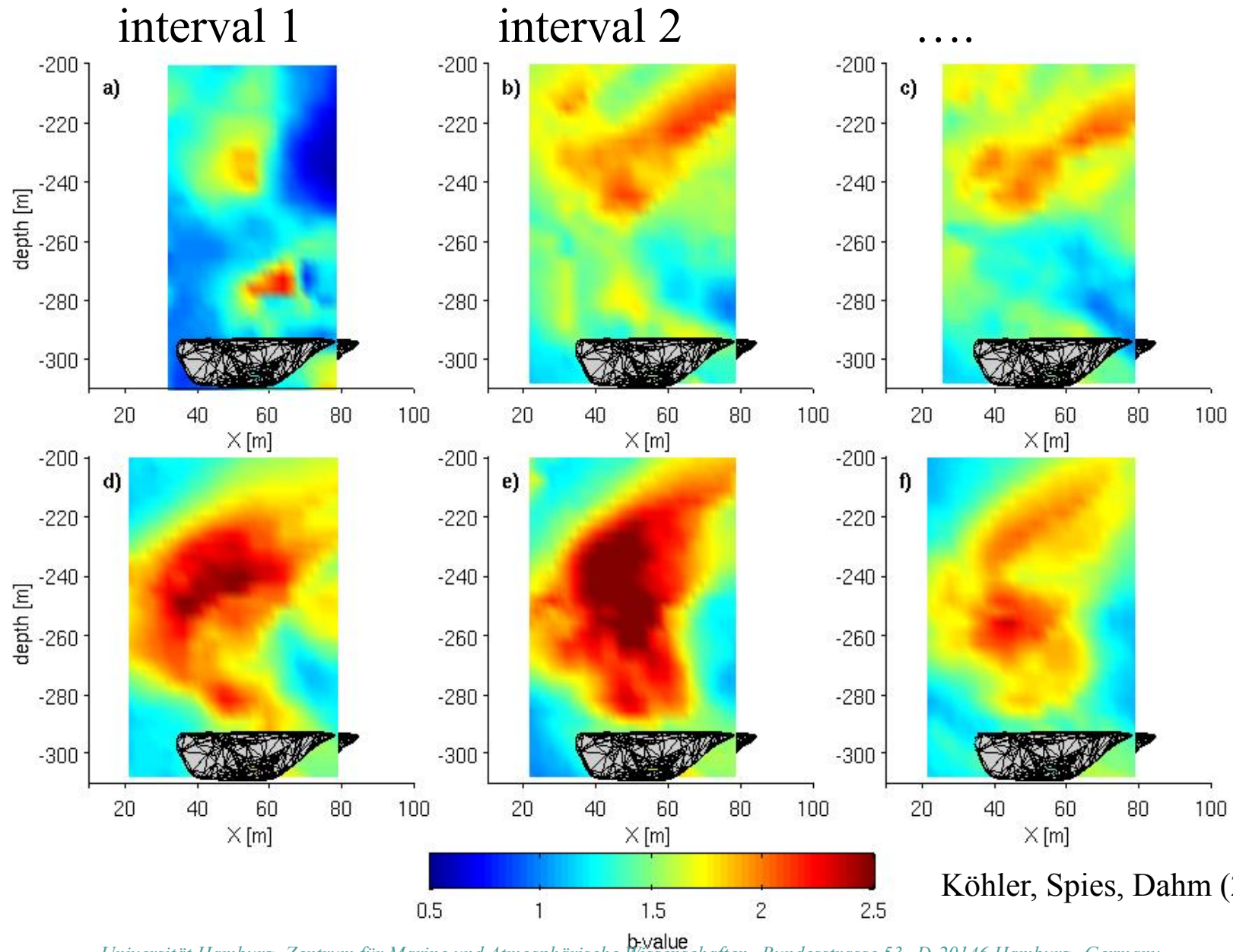
millions of microcracks have been located during 1 year

Event rate & spatial pattern correlates with stress and stress rate



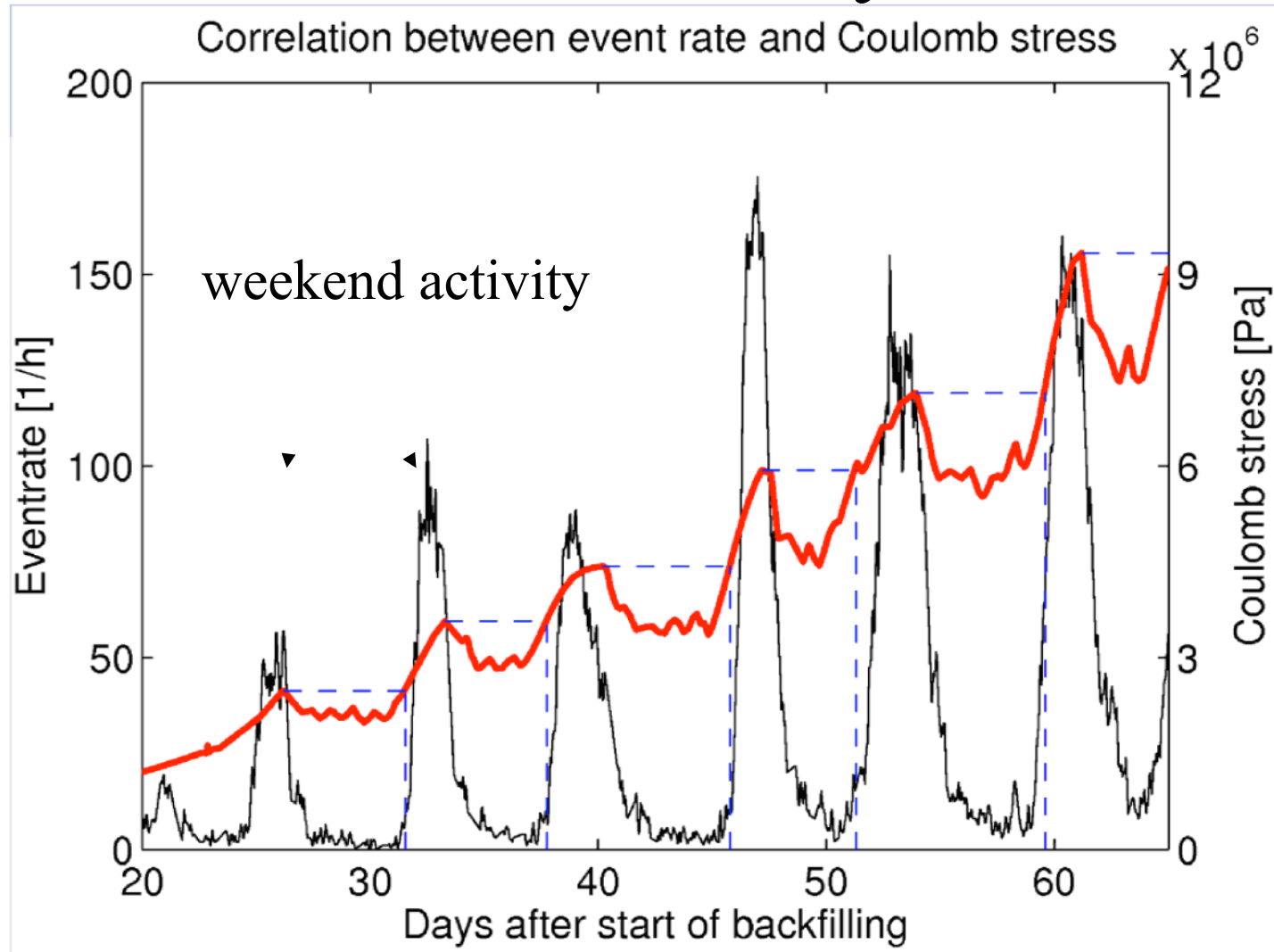
Becker et al., 2010. GJI 10.1111/j.1365-246X.2010.04642.x

extreme high b-values !



Köhler, Spies, Dahm (2009)

Kaiser effect for cyclic loading



D. Becker et al. 2010
60

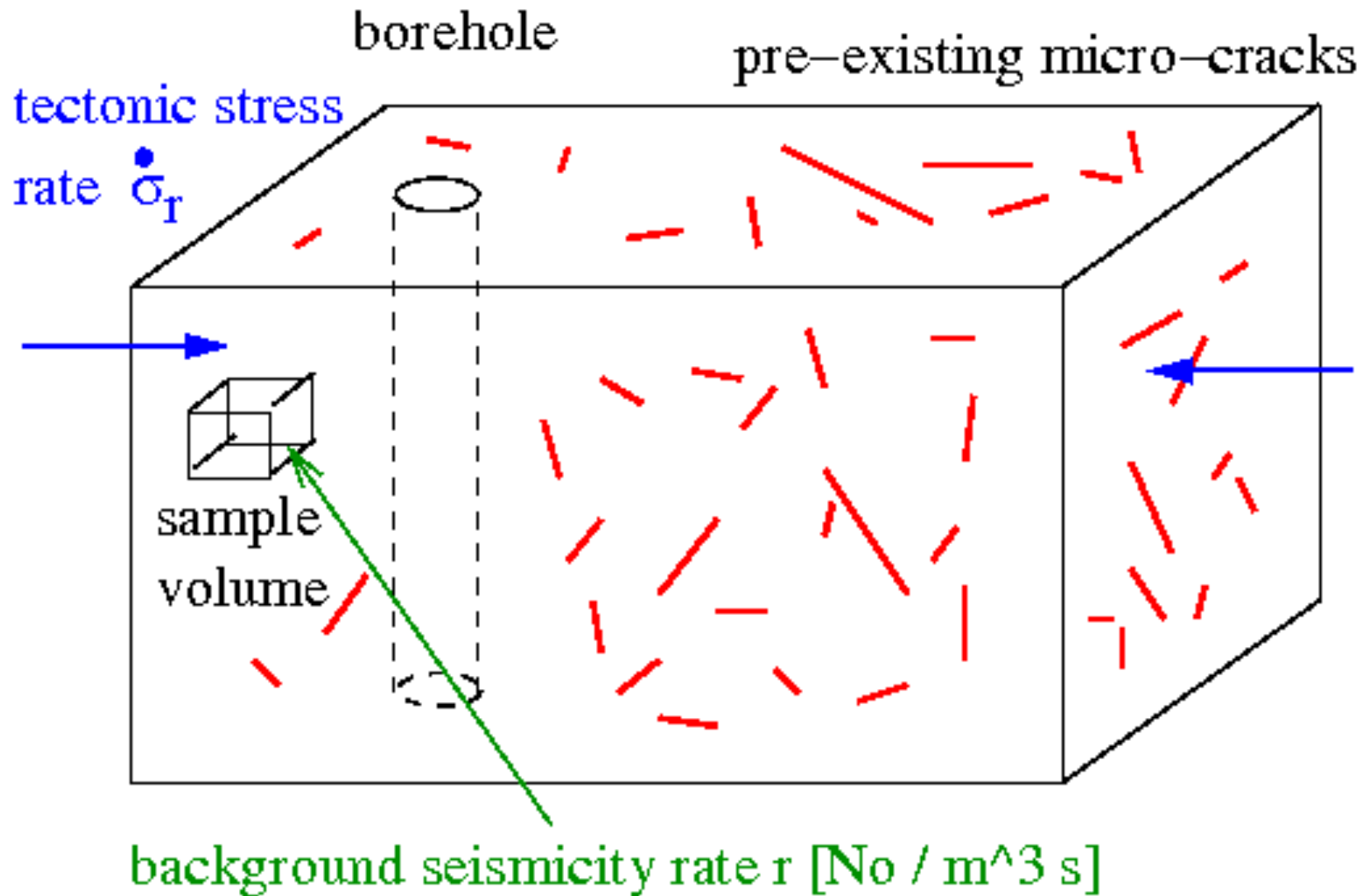
Kaiser effect for cyclic loading

.... events are triggered only after “the stress” exceeds the level of the previous cycle

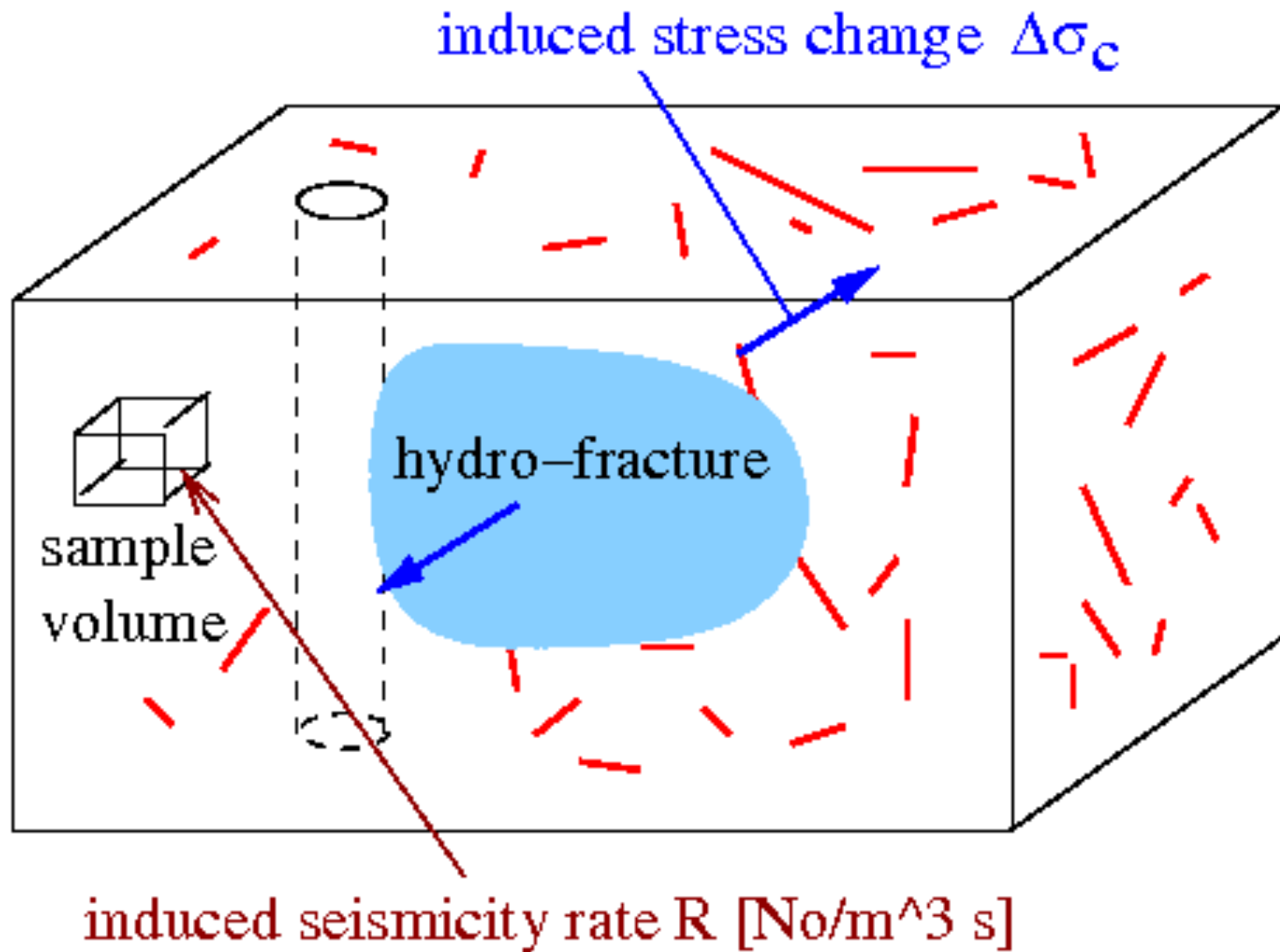


Dietrich (1994) rate and state seismicity model

1) Steady state loading and background seismicity $R=r$



2) transient loading by fracturing: seismicity rate R



Dietrich (1994) model: nucleation assisted by rate-state controlled friction

seismicity rate $R = \frac{r}{\gamma \dot{\sigma}_r}$

background seismicity (points to r)

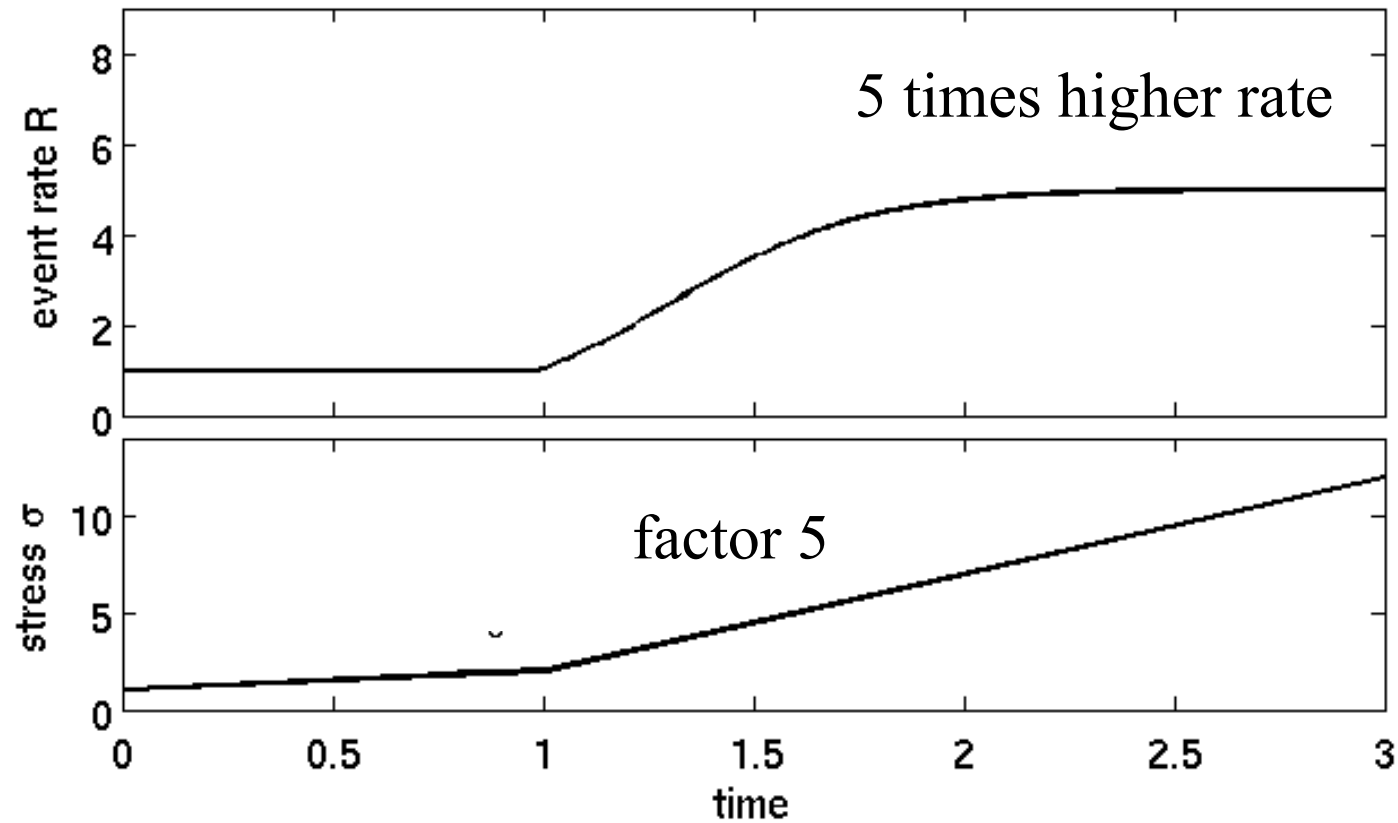
background loading (points to $\dot{\sigma}_r$)

state variable (points to γ)

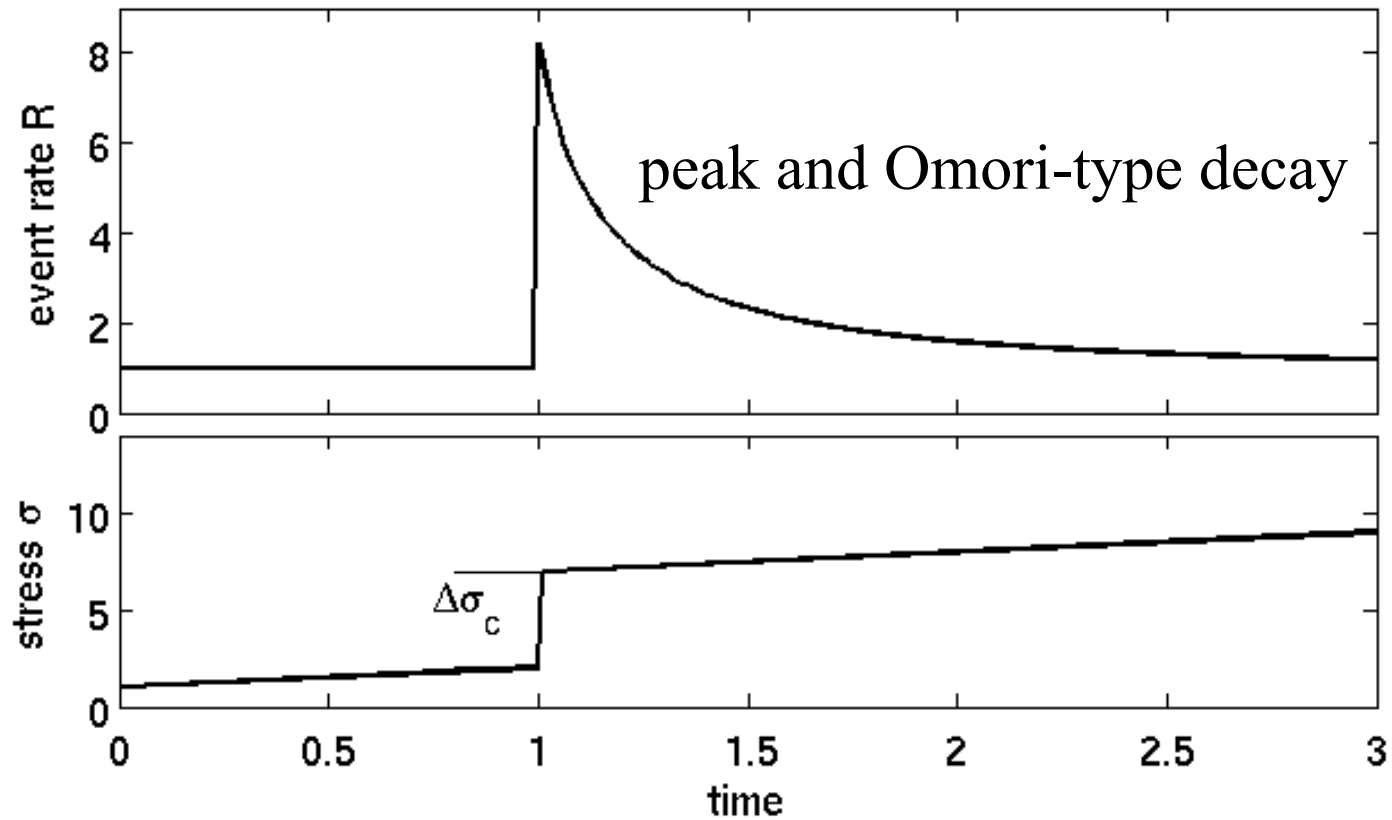
with $d\gamma = \frac{1}{A\sigma} (dt - \gamma d\sigma_c)$

Coulomb stress change !

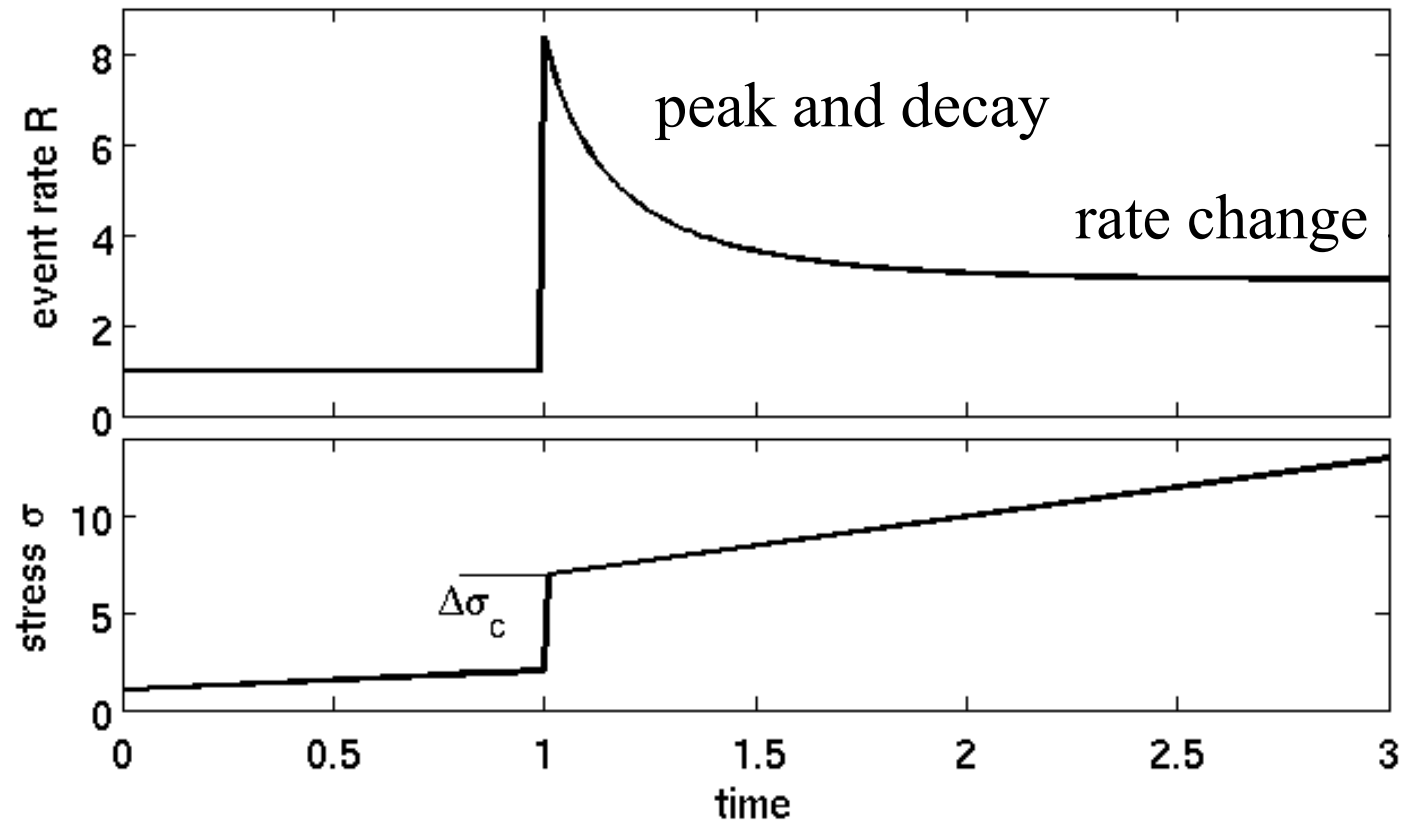
Effect of changing stress rate



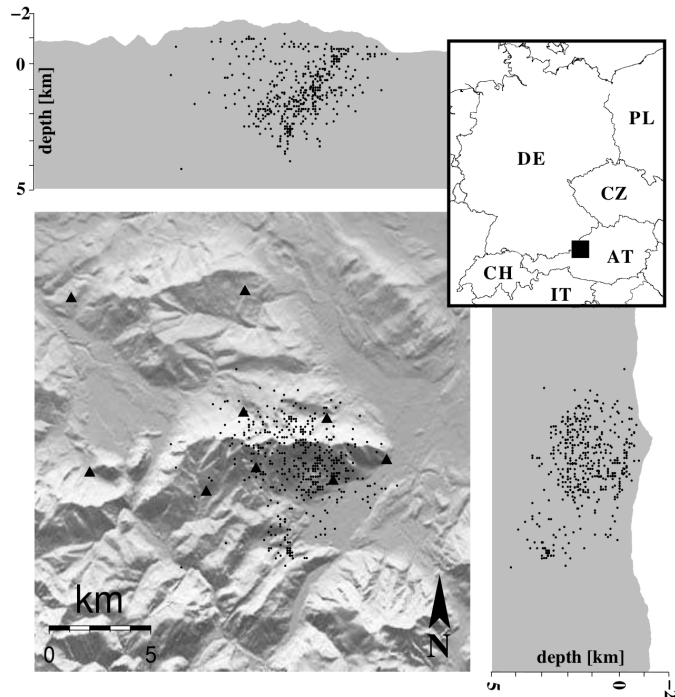
Effect of stress jump



Stress jump and changing stress rate



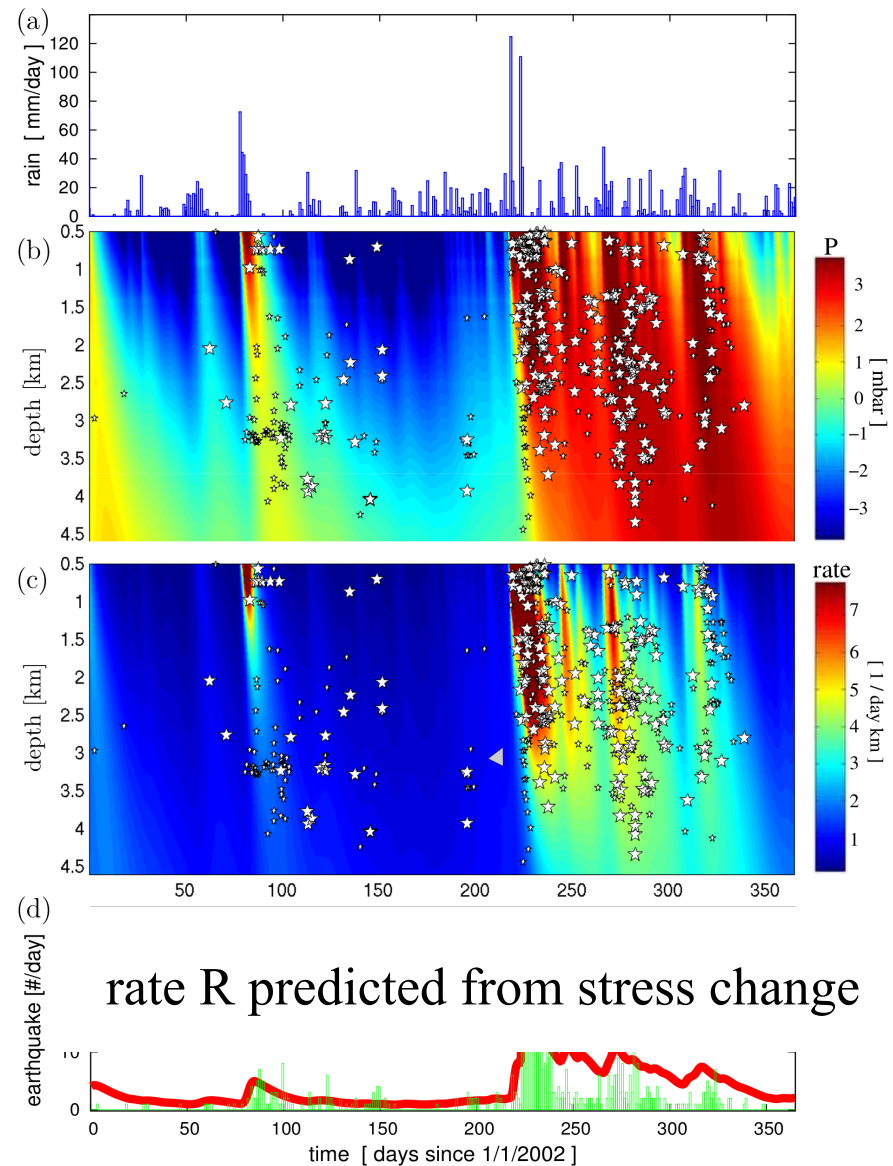
Example: rainfall-triggered earthquake activity



Hochstaufen, S-Germany

1D fluid diffusion model indicates tiny pressure changes of < 0.3 kPa at 4 km depth (diffusivity: $D=3.3 \text{ m}^2/\text{s}$)
 High correlation to seismicity observed !

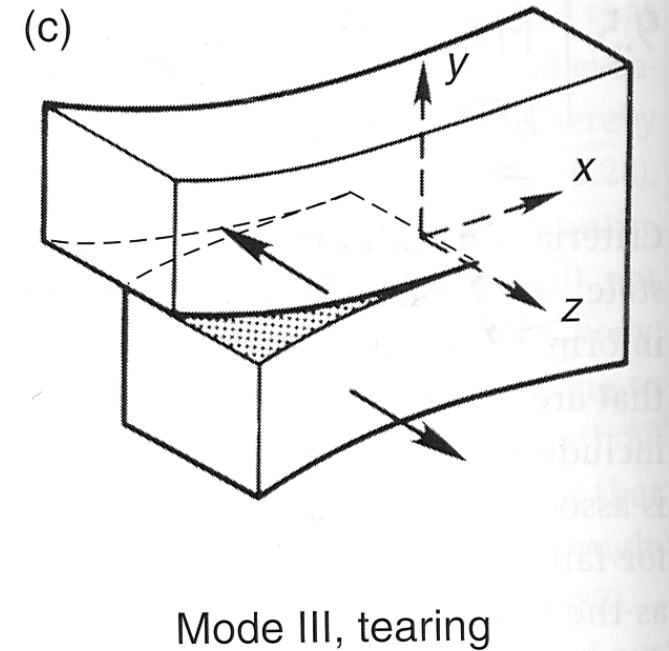
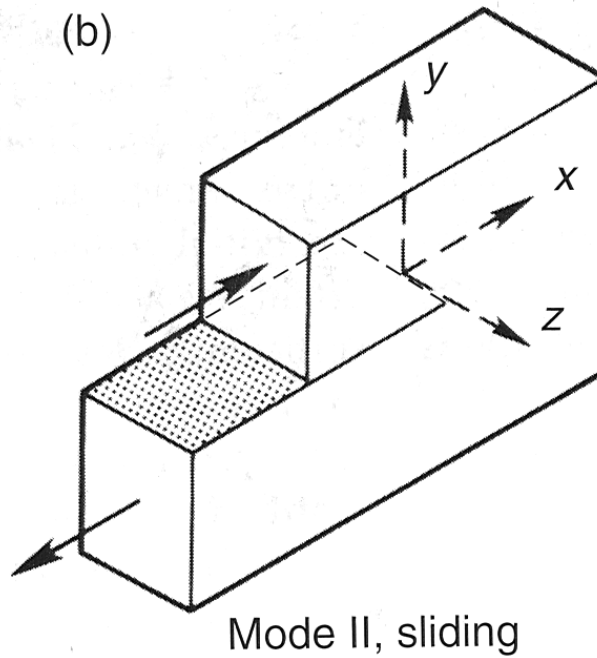
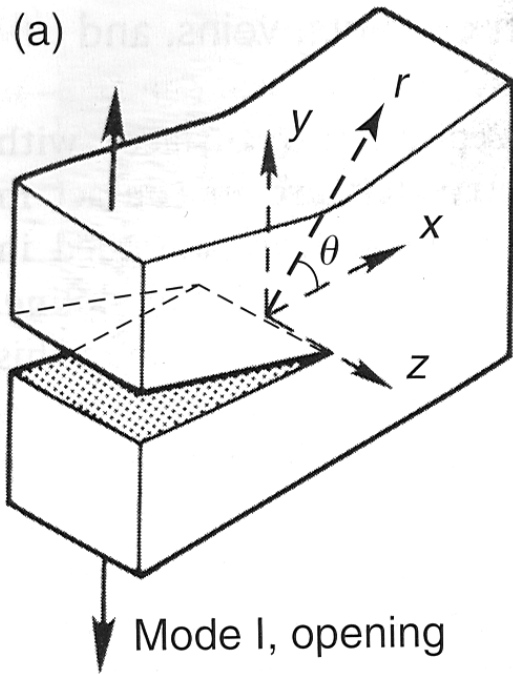
from Hainzl et al., 2006: GRL 33, L19303, doi: 10.1029/2006GL0276427



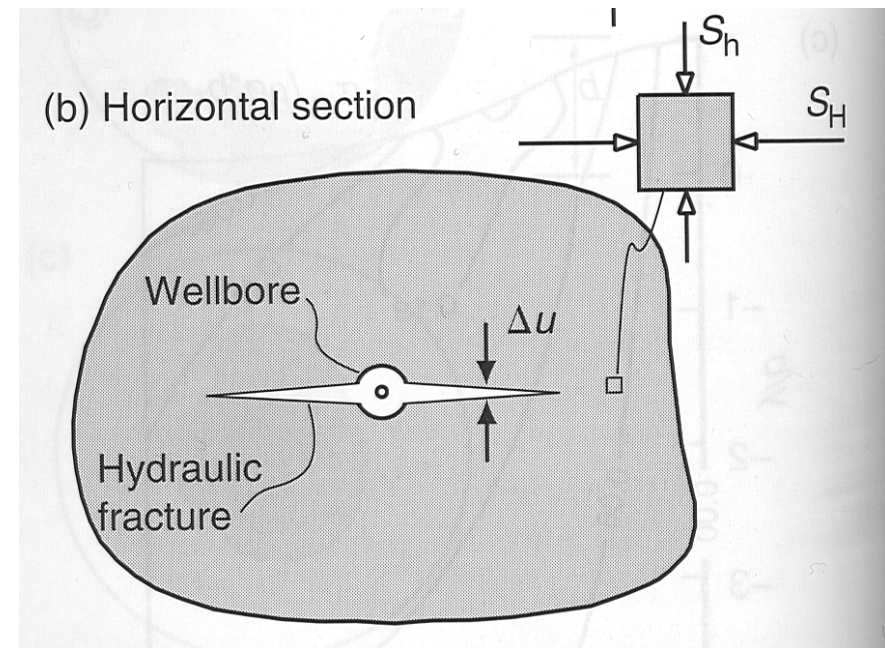
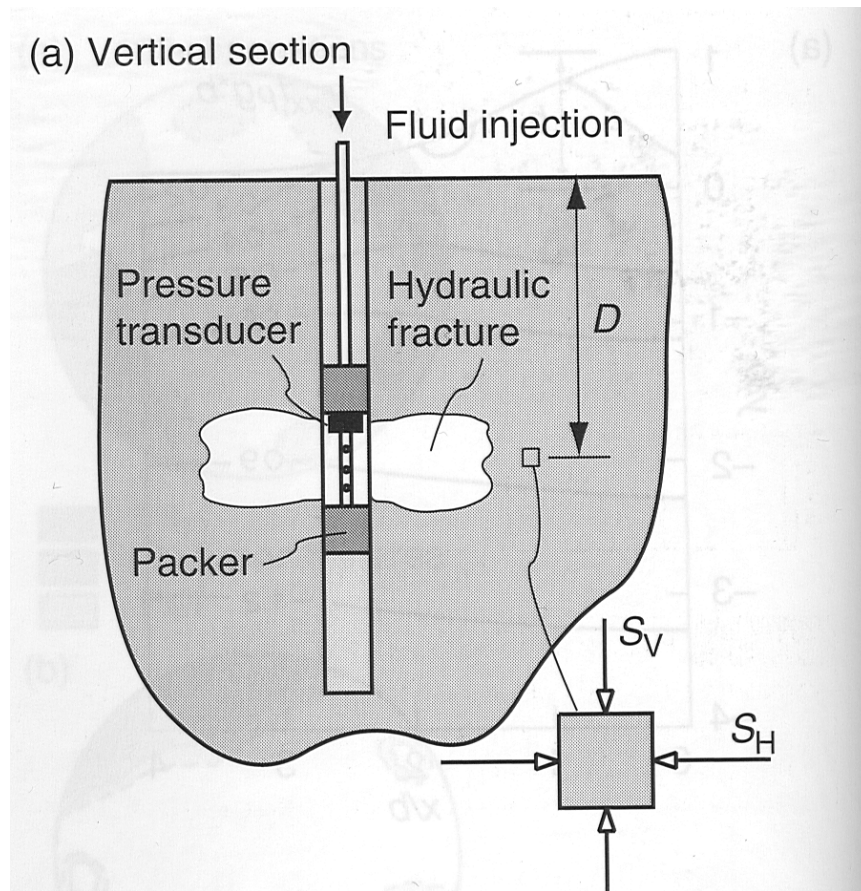
rate R predicted from stress change

V) single fractures: some fundamental solutions

three modes of brittle failure



Hydraulic fracture experiment





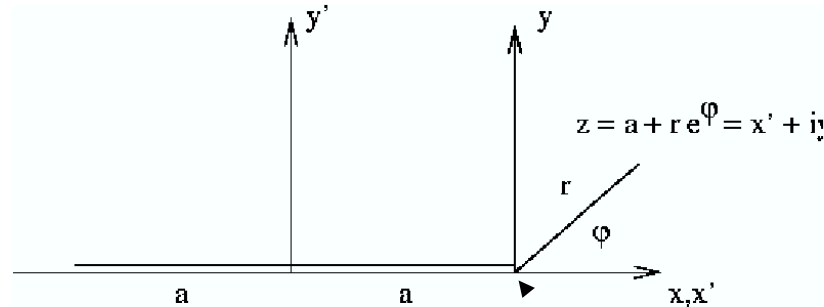
How do intrusions
look like ?

Solidified dike in Iceland

Solidified, eroded dikes are several km high and long and a few tens of cm thick



Stress intensity factor and intrusion tip



crack tip

square root singularity

‘form factor’

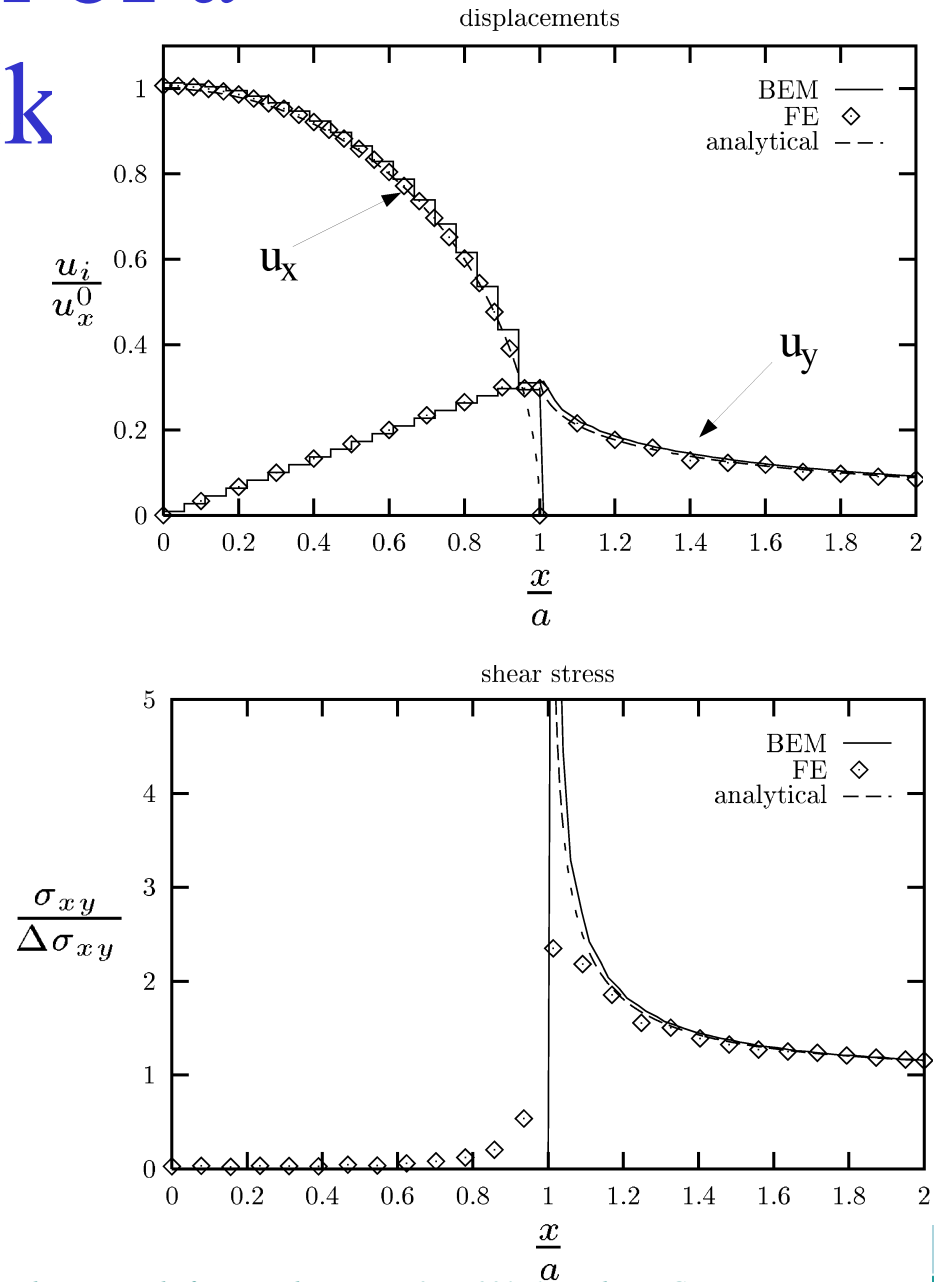
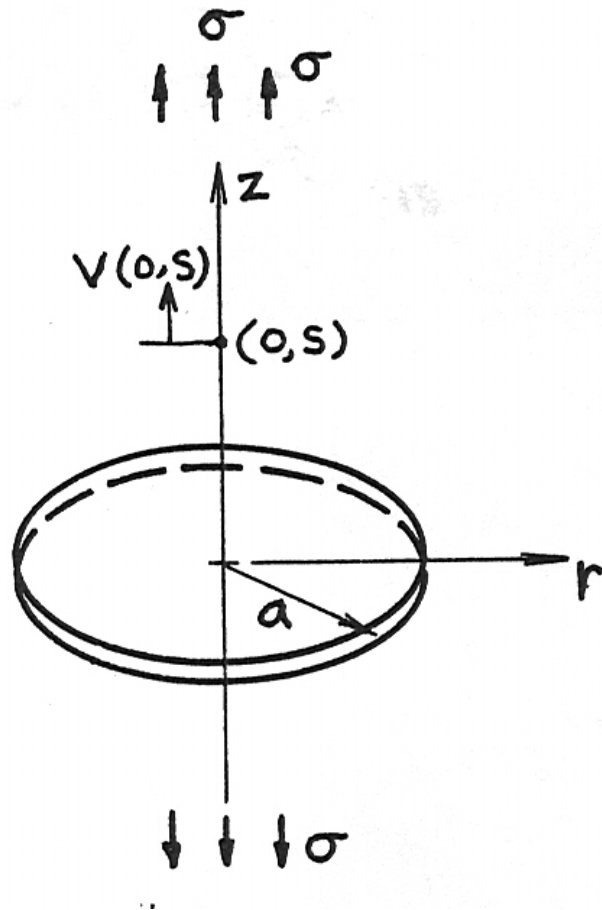
$$\sigma_{ij} = \Delta\sigma^{(m)} \sqrt{\pi a} \frac{1}{\sqrt{2\pi r_1}} f_{ij}^{(m)}(\theta_1)$$

stress intensity loading stress

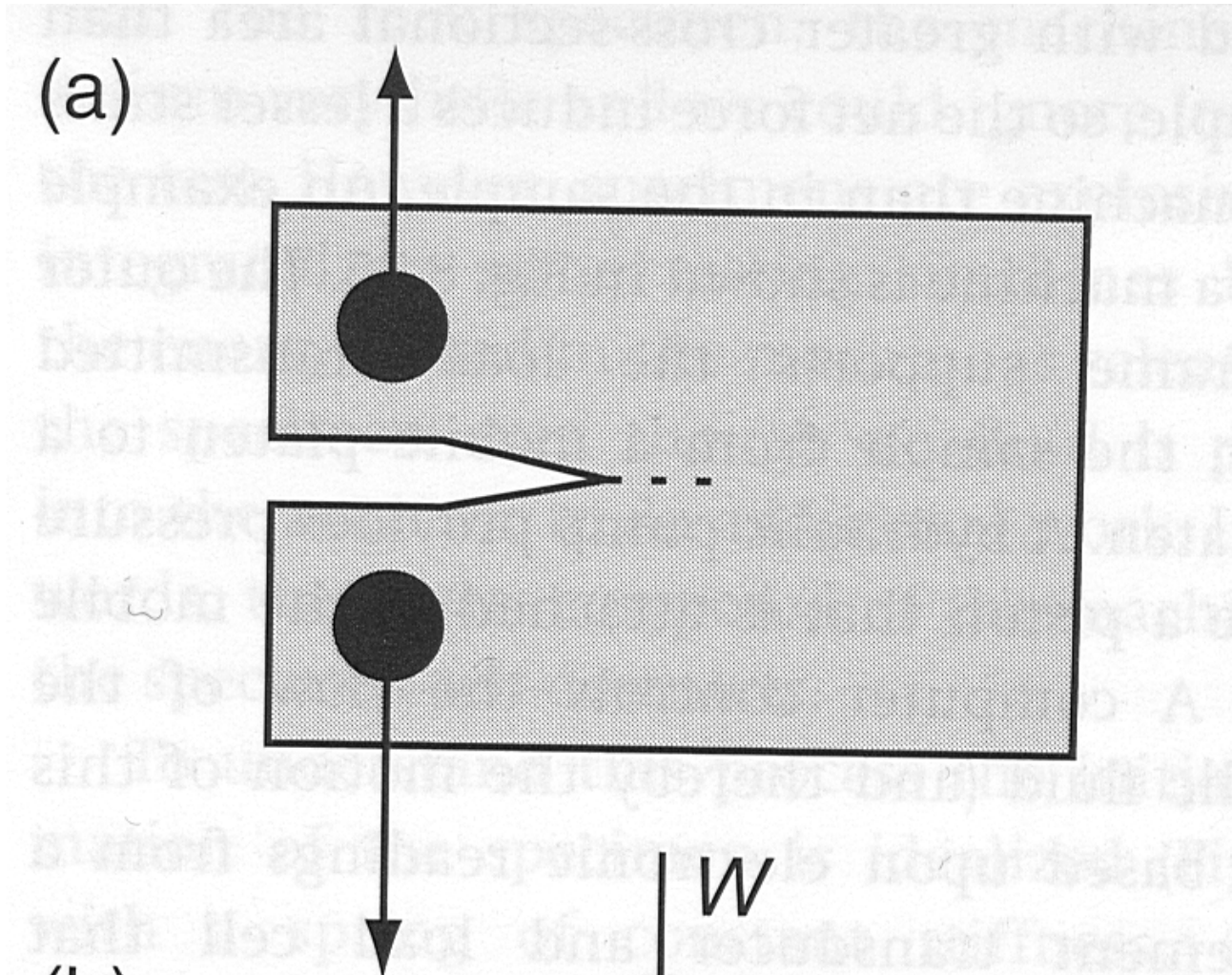
$$K = \Delta\sigma^{(m)} \sqrt{\pi a}$$

crack halflength

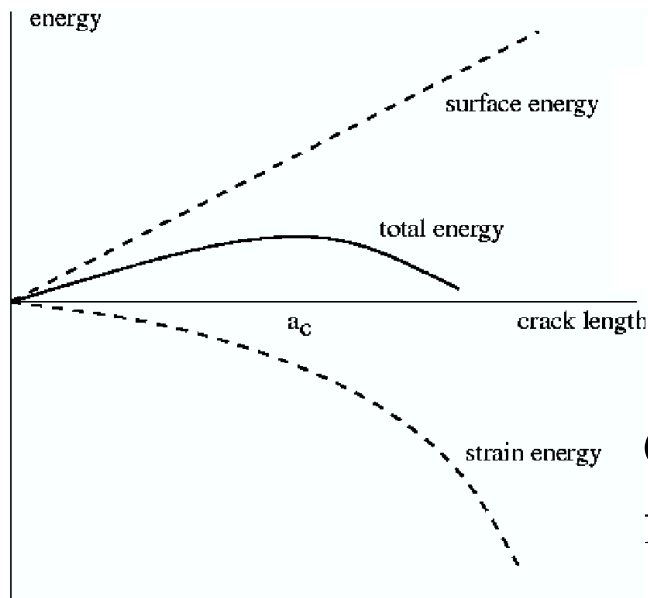
theoretical solution of a penny-shaped crack



Failure criteria for tensile fracture?



Griffith concept of instability



$$\mathcal{G} = \frac{\pi(1-\nu)}{2\mathcal{N}} \Delta\sigma^2 a \geq 2\gamma_0$$

Griffith force

loading stress

Surface energy density

shear modulus

$$K(a) \geq K_c = \left[\frac{4\gamma_0\mathcal{N}}{(1-\nu)} \right]^{1/2}$$

Poisson ratio

stress

fracture

intensity

toughness

Examples for lab-derived fracture toughness

rock type	K_c range ($MPa m^{1/2}$)		
Granite	1.66 - 3.52		
Basalt	0.99 - 3.75		
Quarzite	1.31 - 2.10		
Marble	0.87 - 1.49		
Limestone	0.86 - 1.65		
Sandstone	0.34 - 2.66		
Shale	0.17 - 2.61		

why is Griffith criterion important?

... it is (later) used to understand the growth of hydraulic fractures or magmatic intrusions